Information Difussion, Interdependence and Stock Market Comovement

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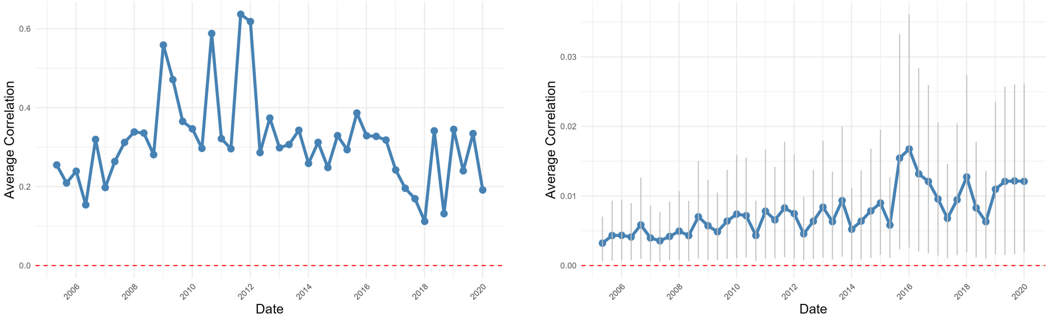
Motivation

- Numerous patterns of comovement have been observed in asset returns.
- Origins of comov. is relevant: asset pricing, propagation of shocks, risk management, etc.
- Some explanations and channels have been studied in the literature.
  - Some theoretical: Calvo and Mendoza [2000], Veldkamp [2006], Gorton and Ordoñez [2014]
  - Some empirical: Barberis et al. [2005], Bekaert et al. [2014], Israelsen [2016]
- Diffusion of information, agents' learning process' and beliefs are key in determining prices.
  - learning frictions/biases (e.g. costs to learn, limited attention capaity) can generate correlated beliefs across stocks

This paper

Can excess comovement in stock returns be driven by firm-to-firm ***informational*** linkages?

Motivation



(a) Avg. Comovement in Returns

(b) Avg. Comovement in Idios. Returns

Idiosyncratic stock return comovement is positive. Theoretically, it should be 0.

The informational channel

- Investors' form beliefs and trade with the info they receive [Giglio et al., 2021]
  - $\rightarrow$ their learning process (formation of beliefs) will determine stock returns.
- Due to biases in the agents' learning process, agents' beliefs are correlated - based on Bernales et al. [2024]
  - two sets of info: common info and idiosyncratic. info.
  - agents' focus more on new info related to common info $\rightarrow$ corr. errors in agents' beliefs.

$$\text{new info.} \rightarrow \underbrace{\text{learning process}}_{\text{biased to. common info}} \rightarrow \underbrace{\text{correlated errors in agents' beliefs}}_{\text{informational linkages}} \rightarrow \text{comov. stock returns}$$

This paper

- Novel measure for informational linkages between stocks based on analysts forecast errors
  - estimate empirically its relevance in explaining idios. stock returns
- The informational channel explains idios. returns $\rightarrow$ interdependence across stocks
  - with diff. effects in periods of higher uncertainty.
  - category-learning amplyfing the effect (S&P, industry, geog. proximity, common analysts).
- We estimate and quantify the propagation of simulated and real world (idios.) shocks.
  - how do shocks over idios. returns of one firm spillover ***other*** firms?
  - how different is the impact of a highly connected stock v/s a less connected one?
  - how well can our informational linkages capture the propagation of unexpected climate events?

Related Literature

- Beliefs and Trading Behavior:
  - Coibion and Gorodnichenko [2015], Coibion et al. [2018, 2019], Giglio et al. [2021]
- Stock Market Comovement & Mechanisms
  Lee et al. [1991], Pindyck and Rotemberg [1993], Froot and Dabora [1999], Barberis and Shleifer [2003], Barberis et al. [2005], Veldkamp [2006], Greenwood [2007], Antón and Polk [2014], Muslu et al. [2014], Hameed et al. [2015], Israelsen [2016], Kumar et al. [2016]
- Information Production/Acquisiton, Herding & contagion:
  - Scharfstein and Stein [1990], Froot et al. [1992], Kaminsky and Schmukler [1999], Calvo and Mendoza [2000], De Gregorio and Valdés [2001], Forbes and Rigobon [2001], Basu [2002], Diether et al. [2002], Gorton and Ordoñez [2014], Debarsy et al. [2018], Chousakos et al. [2023], Bernales et al. [2024], Cole et al. [Forthcoming]

Data

- Combine information from security prices, financial ratios and analyst forecasts. Data from 2000 until 2019.
  - CRSP: Data on securities daily returns.
  - Fama-French Factors: We are interested in explaining the idiosyncratic component of returns.
  - Compustat: Financial ratios as control variables.
  - I/B/E/S: Analysts forecast on Earnings per Share (EPS).
- After some filters, this gives us a total of 771 US firms', at a monthly freq.

Summary Statistics

Methodology

The informational linkage - Metric

- We analyze if forecast errors (proxy for beliefs) are correlated across stocks. On a monthly basis, we calculate the EPS one year ahead forecast error of analysts for firm i as follows:

$$NMRSE_{i,t}^{EPS} = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\left(\frac{\hat{EPS}_{k,i,t+1} - EPS_{i,t+1}}{EPS_{i,t+1}}\right)^2} \tag{1}$$

- Where k is the analyst index, $\hat{EPS}_{k,i,t+1}$ is the forecast from analyst k for firm i at period $t+1$, $EPS_{i,t+1}$ is the realized value. $NMRSE$ denotes the Normalized Mean Root Squared Error.
- Based on the $NMRSE_{i,t}^{EPS}$ by firm, we extract common factors and keep the residual.
- Finally, we **estimate the correlations of residualized beliefs between securities** and keep those relevant connections.
  - This gives origin to an $N \times N$ matrix of informational linkages.

Common Factors | Relevant Connections

Empirical implementation - Spatial Autoregression

- Spatial Two-Stage Least Squares (S2SLS) methodology.
- N firms, T periods. r_t is a vector of idios. returns at $t \in T$. The model to be estimated is:

$$\mathrm{r}_t = \rho \boldsymbol{W}^{I}_{t-1}\mathrm{r}_t + \boldsymbol{X}_{t-1}\beta_i + \mu + \gamma_t + \mathrm{v}_t \tag{2}$$

- W^{I}_{t-1} is our $N \times N$ matrix of informational linkages, $\boldsymbol{X}_{t-1}$ contains financial ratios and lagged return and risk as controls, $\boldsymbol{\mu}, \gamma_t$ are firm and time fixed effects, and v_t is an error term.
- ρ ($|\rho|$<1) is the avg. intensity with which info. linkages explain the variation on idios. asset returns: intensity of interdependence

Idiosyncratic Returns | S2SLS Details | Why Spatial Econometrics?

Results - Informational Channel Relevance

Results - Informational Channel Relevance

Table 1:

	Dep. Var.: Monthly Idios. Return		
	(i) Random Controls	(ii) Financial Ratios	(iii) Full Controls
W	0.73	0.72	**0.74**
	29.32	23.69	30.29
Observations	138,780	138,780	138,780
Controls	Random	Ratios	Full
Stock-Month FE	Yes	Yes	Yes

Notes: The Dep. Var. is the residualized monthly return of stocks after accounting Fama-French 6-Factor model. The first column includes 5 randomly normally distributed controls. The second includes as controls: Market-Book, Cash-Assets, Ebitda-Assets, Debt-Assets and Log(Sales), all lagged. The third column includes the previously financial ratios controls plus lagged Return and Risk with firm-specific coefficients. T-stats are below each coefficient.

Interpretation

- Table 1 shows that there is interdependence between stock returns.
  - robust to including different sets of controls
- Coefficient do not have a direct interpretation, more on the next section.
  - interpretation: avg. intensity of interdependence (due to info. links)
- Is this evidence of stock market comovement as an aggregate?
  - implicitly, yes. the avg. intensity of info. links is way above 0

Do learning biases change in periods with higher uncertainty?

Does the learning process change in periods of higher uncertainty?

- Information production [Gorton and Ordoñez, 2014]:
  - Producing information is costly, not optimal to produce at every period.
  - When high negative shocks arise, there are more incentives to processing information correctly.
- Herding and contagion theories [Calvo and Mendoza, 2000]:
  - Investors follow the market instead of making their own assessments of fundamentals, market portfolios embodied relevant information.
  - The cost of acquiring information can be higher than the potential gains over 'market' portfolios.
  - Investors have less incentive to gather good news in a bull market, incentives to verify good news increasing in a bear market.
  - Hence, good rumors generate more herding behavior and thus possibly over-reaction in good times.

Most empirical studies focus on crisis periods to study these theories. We include 'normal' periods in order to see differences it from high uncertainty periods.

Results - Crisis Behavior

Table 2: Informational Linkages and Uncertainty Periods

	Dep. Var.: Monthly Idios. Return							
	Baseline	GFC	VIXQ75	VIXQ90	Unc. PolQ75	Unc. PolQ90	S&P Ret.Q25	S&P Ret.Q10
W	0.74	0.72	0.71	0.73	0.69	0.72	0.73	0.75
	30.29	27.67	26.43	27.7	25.16	27.00	27.06	28.07
W*Crisis		-0.24						
		-3.08						
W*VIX			-0.15	-0.35				
			-2.9	-4.99				
W*Unc Policy					-0.16	-0.49		
					-3.38	-8.09		
W*S&P Ret							-0.2	-0.56
							-4.31	-9.29
Observations	138780	138780	138780	138780	138780	138780	138780	138780
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Stock-Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The Dep. Var. is the residualized monthly return of stocks after accounting Fama-French 6-Factor model. Each estimation controls for Market-Book, Cash-Assets, Debt-Assets, Ebitda-Assets, Log(Sales), Return and Risk (all lagged) with firm-specific coefficients, besides stock and monthly FE. Each column includes an interaction for crises periods. The second column, Global Financial Crisis (GFC) considers 2007m7-2009m3. VIXQ75 considers periods where the VIX index distribution was in its upper 25%, same for VIXQ90, but with the upper 10%. Unc. Pol. uses the Uncertainty Policy Index. S&P Ret uses periods where the monthly return of the S&P index was in its lower 25% and 10% values. T-stats are below each coefficient.

Category Learning

- Investors might think in terms of categories. If two companies are in the same group, investments might flow in similar ways to both companies.
- The informational channel has different effects in securities that share groups, such as:
  - Companies in S&P 500 [Barberis et al., 2005, Boyer, 2011]
  - Companies in the same industry[1] [Ozdagli and Weber, 2023]
  - Companies Geog. Connected[2] [Froot and Dabora, 1999]
  - Companies with common analyst coverage[3] [Israelsen, 2016, Hameed et al., 2015, Muslu et al., 2014]
  - We show that, even after controlling for each of these groups, the informational channel is still relevant.

[1] At the 2-digit NAICS level.

[2] Two measures: (i) Headquarters (HQ) at a 100km radius; (ii) HQ in the same state.

[3] Securities with at least one common analyst covering it in at least 2 out of the 5 years in which W is constructed.

Results - Category Learning

Table 3: Informational Linkages and Category Learning

	Dep. Var.: Monthly Idios. Return					
Groups	Baseline	S&P	Industry	Geog.	Geog.	Common Analy.
W	0.74	0.48	0.35	0.64	0.60	0.63
	30.29	18.27	13.4	25.86	24.54	23.41
W*S&P		0.76				
		13.40				
W*Industry			0.85			
			16.14			
W*GeogThres				9.12		
				3.70		
W*GeogState					12.89	
					4.67	
$W^{ComAnalys}$						0.75
						28.07
$W*W^{ComAnalys}$						6.11
						1.05
Observations	138,780	138,780	138,780	138,780	138,780	138,780
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Stock-Month FE	Yes	Yes	Yes	Yes	Yes	Yes
log-Lik						

Notes: The Dep. Var. is the residualized monthly return of stocks after accounting Fama-French 6-Factor model. Each estimation controls for Market-Book, Cash-Assets, Debt-Assets, Ebitda-Assets, Log(Sales), Return and Risk (all lagged) with firm-specific coefficients, besides stock and monthly FE. Each column includes an interaction for pair of firms that belong to a group. The second column interacts the W matrix keeping only pair of firms that have ever been part of the S&P 500 index. The third column keeps connections between pairs of firms that are in the same industry. The fourth column keeps connections between stoctks hat are at most 100km. distanced (between their headquarters). The fifth column keeps connections that are in the same state. The sixth column builds a matrix of common analyst coverage (see the Appendix) and then interacts our base W with the common analyst W. T-stats are below each coefficient.

Applications - Quantifying the Propagation of Shocks

Indirect Impacts of Shocks

- Our baseline model allows us to identify the direct and indirect impacts of a shock over one stock. The model can be written as[4]:

$$\mathrm{r}_t = (I_N - \rho \mathrm{W}^l_{t-1})^{-1}(\mathrm{X}_{t-1}\beta_i + \boldsymbol{\mu} + \gamma_t + \mathrm{v}_t) \tag{3}$$

- Then

$$\frac{\partial \mathrm{r}_t}{\partial \mathrm{v}_t} = (I_N - \rho \mathrm{W}^l_{t-1})^{-1}\mathrm{v}_t \tag{4}$$

- Note that the resulting derivative will give us an $N \times N$ matrix. The diagonal of the matrix has the direct effects of a shock over each firm. The non-diagonal elements will include the indirect effects of the shock.

[4] If $|\rho| < 1$, $(I_N - \rho \mathrm{W}^l_{t-1}\mathrm{r}_t)^{-1}$ can be written as: $(I_N + \rho W^l_{t-1} + \rho^2 {W^l_{t-1}}^2 + \ldots)$. The first element includes the direct effect, the second one the impact over direct "neighbours", the third one the impact over second-order "neighbours", and so on.

Indirect Impact of Idios. Shocks - Simulation

- We simulate a 1 sd shock over idiosyncratic returns of one firm. How does it spillover other firms?

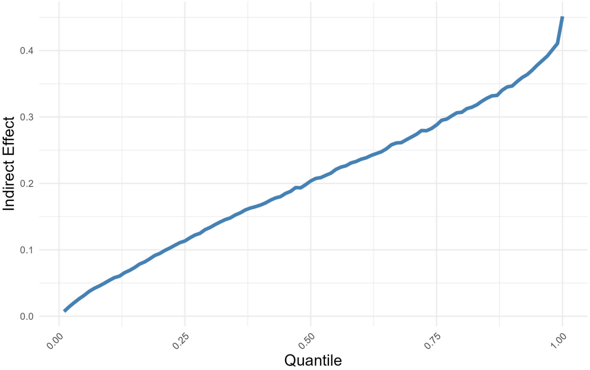
Table 4: Indirect Impact of a 1 sd shock

	Avg. Indirect Impact of a Shock to:			
	(1)	(2)	(3)	(4)
	All Firms	More Connected	Least Connected	First Order
Indirect Impact/Mean(Idios. Ret.)	14.40%	32.65%	0.73%	28.54%

- A 1 sd shock on idios. ret. of one stock, increases idios. ret. of *other* stocks by 14.40%, relative to the mean idios. return.
  - more than twice (32.65%) for the most connected stocks, almost nothing (0.73%) for least connected
  - analyzing the impact only over connected stocks captures higher spillover effects

Indirect Impacts of Shocks

Figure 1: Indirect Impacts of a 1sd shock (7.37%) by Quantiles of Centrality



Indirect Impact of Idios. Shocks - Real World

- How does our network capture indirect effects of real world (exog.) shocks?
- Hurricane Florence, Sep. 2018: Huge ($9M USD uninsured damages) and localized (NC, SC, VA).
  - Stock market's response: affected stocks (in NC, SC, VA) and unaffected firms (other states).
  - Spillovers over unaffected stocks

Table 5: Hurricane Florence Indirect Effects

	(1) Affected	(2) 1st Order (W^I)	(3) Not Affected	(4) All
Avg. Return	-1.55%	-0.59%	-0.26%	-0.34%
Avg. Return/Affected	100 %	37.97%	17.13%	22.00%

Notes: The first row calculates the average monthly return in September 2018 for different set of stocks. (1) does it for stocks with HQ in SC, NC and VA. (2) does it for stocks informationally linked to stocks in (1), but not present in SC, NC and VA. (3) does it for all stocks with HQ not in SC, NC, or VA. (4) does it for all stocks. The second row divides the first row results over the average monthly return in Sept. 2018 for stocks affected (-1.5%).

- Shock propagated with more intensity over linked stocks (according to W^I)

Conclusion

- We show empirically that stock return comovement might arise due to analysts' learning biases, shedding lights on a long-standing puzzle in finance.
  - relevance: understand how assets are priced, the propagation of shocks and how efficient are markets
- Category-thinking is observed and amplifies the effect of informational linkages.
- Learning biases change during periods of uncertainty, there's more at stake.
- Simulated and real world applications to study the propagation of shocks across stocks.
- Future research:
  - how different would this be on an emerging country? how do these learning biases emerge between different stock markets (countries)?
  - policy: what could a planner do to diminish these market frictions?
  - theory: how could we model these interdependence patterns?

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Appendix

Summary Statistics

Table 6: Summary Statistics

	Min.	p25	p50	p75	Max.	Mean	St.Dev.	Obs.
Idios. Return	-18.53%	-4.24%	-0.22%	3.83%	20.46%	-0.04%	8.76%	136,980
Log(Sales)	11.28	13.72	14.81	15.93	18.37	15	1.7	136,807
Cash-Assets	0.00	0.02	0.07	0.18	1.02	0.25	2.2	136721
Ebitda-Assets	-0.41	0.27	0.72	1.74	149.36	23	332	136,704
Debt-Assets	0.12	0.42	0.58	0.74	0.93	0.57	0.22	136,819
Market-Book	0.62	1.36	2.16	3.52	12.99	3.6	15	136,712
Analys. EPS Forecast Errors	0.02	0.07	0.16	0.39	6.18	0.52	1.1	118,954

Notes: Idios. Returns is the residualized monthly returns of stocks after applying a Fama-French 6 Factor Model to excess returns (return-risk free rate). Analys. EPS Forecasts Errors are estimated following Eq. 1. Every variable is winsorized at the 2% of each tail to estimation being driven by outliers.

Back

The informational linkage - Considering Common Factors

- Linkages observed in the previous step might be showing common factors. We follow Bailey, Holly & Pesaran (2016) two-stage methodology to correct for cross-sectional dependence by de-factoring the forecast error.
- First step: Test Cross-Sectional Dependence (CSD). If the degree of CSD is high (over 0.5), defactor until this measure is weakly CSD (below 0.5).

$$MRSE_{i,t}^{EPS} = \alpha_i + \lambda_i \mathrm{f}_t + \varepsilon_{i,t}^{DFact} \tag{5}$$

- Where f_t are unobserved common factors and λ_i are loadings. Our baseline model uses 1 common factor (where de-factored observations are weakly CSD). But we have tested with variations of this number.
- The de-factored observations, $\varepsilon_{i,t}^{DFact}$, are kept to construct our informational linkage matrix.

Back

The informational linkage - Keeping relevant connections

- We define $\hat{\epsilon}_{i,j}$ as the sample estimate of the de-factored pairwise correlation of the EPS forecast error between any two firms i and j over $t \in \{1, 2, \ldots, T\}$
- In order to keep relevant connections, we follow Bailey, Holly and Pesaran (2016) methodology and apply a multiple testing procedure:

$$w^{I}_{i,j} = I\left(\hat{\epsilon}_{i,j} > T^{-1/2} c_p(n_{ij})\right) \tag{6}$$

- where $c_p(n_{ij}) = \Phi^{-1}\left(1 - \frac{p}{2f(n_{ij})}\right)$, p is the size of the test, which is set to $p = 0.01$, $\Phi^{-1}(.)$ is the inverse cumulative standard normal distribution and $f(n_{ij})$ is a function linearly increasing in n_{ij}, the number of firm pairs, that is $n_{ij} = N \times (N - 1)$.
- Intuition: An element w^{I}_{ij} of the matrix of informational linkages W^{I} is non-zero when the informational link between countries i and j is sufficiently strong, that is, when it is statistically different from zero.
- We allow for different degrees of intensity of the link based on $\hat{\epsilon}_{i,j}$, multiplying w^{I}_{ij} by $\hat{\epsilon}_{i,j}$ and row-normalizing.

Back

Keeping idiosyncratic returns

- Stock returns might comove due to common factors. We want to explain the idiosyncratic component of stock return comovement. To keep the idios. component, we:
  - Compute the excess return of each asset by subtracting the effective return by a risk-free instrument at a daily basis.
  - Keep idiosyncratic returns by de-factoring the assets excess returns with typical common factors used in finance. We use CAPM's & Fama and French's factor models to de-factor asset returns. These idiosyncratic excess returns are denoted by r_t
  - Aggregate daily observations at a monthly frequency: $\mathrm{r}_t^{Monthly} = \prod_{t \in T}(1 + \mathrm{r}_t^{Daily}) - 1$. Where T is the number of days in each month.
- Theoretically, idios. returns should not comove. We show that they **do** comove and this can be explained by Infor. Linkages.

Back

Spatial Two-Stage Least Squares (S2SLS) Details

We follow and adapt Kelejian and Prucha (1998)'s S2SLS procedure. We consider the following instruments:

$$\mathrm{H}_{\mathbf{n}} = \left\{ \left(\mathrm{W}_t' \otimes \mathrm{I}_{\mathbf{T}} \right) \mathrm{X}_{i,t-\mathbf{1}} \right\}_{t \in \mathbf{T}}$$

which allows the estimation of the following parameters

$$\widehat{\theta} = \begin{bmatrix} \widehat{\rho} \\ \widehat{\beta} \end{bmatrix} = \left[\mathrm{Z}'\mathrm{QH}_{\mathbf{n}} \left(\mathrm{H}_{\mathbf{n}}'\mathrm{QH}_{\mathbf{n}} \right)^{-\mathbf{1}} \mathrm{H}_{\mathbf{n}}'\mathrm{QZ} \right]^{-\mathbf{1}} \mathrm{Z}'\mathrm{QH}_{\mathbf{n}} \left(\mathrm{H}_{\mathbf{n}}'\mathrm{QH}_{\mathbf{n}} \right)^{-\mathbf{1}} \mathrm{H}_{\mathbf{n}}'\mathrm{Qy},$$

where Z denotes the matrix of all LHS variables (including the spatial lags of the dependent variables) and Q is a matrix that sweeps all fixed effects and the intercept, that is,

$$\mathrm{Q} = \left(\mathrm{I}_{\mathbf{N}} - \mathrm{N}^{-\mathbf{1}} \mathrm{J}_{\mathbf{N}} \right) \otimes \left(\mathrm{I}_{\mathbf{T}} - \mathrm{T}^{-\mathbf{1}} \mathrm{J}_{\mathbf{T}} \right),$$

with J a square matrix of ones. The standard errors can be obtained as

$$\mathrm{se}(\widehat{\theta}) = \widehat{\sigma}^{\mathbf{2}} \left[\mathrm{Z}'\mathrm{QH}_{\mathbf{n}} \left(\mathrm{H}_{\mathbf{n}}'\mathrm{QH}_{\mathbf{n}} \right)^{-\mathbf{1}} \mathrm{H}_{\mathbf{n}}'\mathrm{QZ} \right]^{-\mathbf{1}}$$

with

$$\hat{\sigma}^{\mathbf{2}} = \frac{\hat{\mathrm{v}}'\mathrm{Q}\hat{\mathrm{v}}}{(N-1)(T-1) - (k+T)}$$

Back

Empirical Implementation - Why Spatial Econometric Models?

- We are working with a high dimensional (sparse) matrix of firm-to-firm connections (771×771).
- Spatial econometric models enable us to analyze the spillover mechanisms from multiple sources of transmission of shocks across firms in a single model.
- Past literature has effectively implemented spatial econometric models without necessarily using geographical connections.
- An alternative methodology would be Global VAR (GVAR) models.
  - Main difference with spatial econometric models is that the latter focuses on potentially sparse levels of connections, which is important in our context.
  - Also, GVARs are typically used in lower dimensions, it estimates unit-specific coefficients.
- Following Elhorst et al. [2021], Spatial Models are particularly useful in our context, since our treatment ($W\mathrm{r}_t$) is endogenous, we include control variables and we have $N > T$.

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