

Let's initialise the values as,

$$\begin{aligned} w_{01} &= -2 & w_{02} &= -1 & w_{11} &= 1 & w_{12} &= 2 & w_{21} &= -3 & w_{22} &= 3 \\ w_{03} &= -1 & w_{23} &= 2 & w_{03} &= 4 & x_0 &= 1 & x_1 &= 2 & x_2 &= 2 & h_0 &= 1 \\ & & & & & & & & & & & & & \alpha = 0.1 \end{aligned}$$

To find, $\hat{y} = w_{03} h_0 + w_{13} h_1 + w_{23} h_2$ ——— ①

$$Z_1 = w_{01} x_0 + w_{11} x_1 + w_{21} x_2$$
 ——— ②

$$Z_2 = w_{02} x_0 + w_{12} x_1 + w_{22} x_2$$
 ——— ③

From ② finding Z_1 ,

$$Z_1 = -2(1) + 1(2) + -3(2) = -6$$
 ——— ④

$$h_1 = \text{Sigmoid}(Z_1) = 0.00247$$

From ③ finding Z_2 ,

$$Z_2 = -1(1) + 2(2) + 3(2) = 9$$
 ——— ⑤

$$h_2 = \text{Sigmoid}(Z_2) = 0.99987$$

From ①, Finding \hat{y} ,

$$\hat{y} = 4(1) + (-3)(0.00247) + 2(0.99987)$$

$$\hat{y} = 5.9923$$

$$L = \frac{1}{2} (\hat{y} - y)^2 = \frac{1}{2} (5.9923 - 2)^2 = \frac{1}{2} (15.9387) = 7.969$$

Finding $\frac{\partial L}{\partial \hat{y}} = \frac{\partial}{\partial \hat{y}} \left(\frac{1}{2} (\hat{y} - y)^2 \right) = \hat{y} - y = (5.9923 - 2) = 3.9923$

$$\frac{\partial L}{\partial w_{03}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{03}}$$

For $\frac{\partial \hat{y}}{\partial w_{03}}$, From ① $\hat{y} = w_{03} h_0 + w_{13} h_1 + w_{23} h_2$ ——— ⑥

$$\frac{\partial \hat{y}}{\partial w_{03}} = h_0$$

Similarly

$$\frac{\partial \hat{y}}{\partial w_{13}} = h_1$$

$$\frac{\partial \hat{y}}{\partial w_{23}} = h_2$$

Finding $\frac{\partial L}{\partial w_{03}}$, $\frac{\partial L}{\partial w_{13}}$, $\frac{\partial L}{\partial w_{23}}$

$$\frac{\partial L}{\partial w_{03}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{03}} = (3.9923)(1) = 3.9923.$$

From (9),

$$\frac{\partial L}{\partial w_{13}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{13}} = (3.9923)(0.00247) = 0.00984$$

From (9),

$$\frac{\partial L}{\partial w_{23}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{23}} = (3.9923)(0.99987) = 3.9918$$

Updated Weights,

$$\begin{aligned} w_{03} &= w_{03} - \alpha \frac{\partial L}{\partial w_{03}} \\ &= 4 - (0.1)(3.9923) \\ &= 3.6077 \end{aligned}$$

$$\begin{aligned} w_{13} &= w_{13} - \alpha \frac{\partial L}{\partial w_{13}} \\ &= -1 - (0.1)(0.0098) \\ &= -1.00098 \end{aligned}$$

$$\begin{aligned} w_{23} &= w_{23} - \alpha \frac{\partial L}{\partial w_{23}} \\ &= 2 - (0.1)(3.9918) \\ &= 2 - 0.39918 \\ &= 1.6008 \end{aligned}$$

$$\frac{\partial L}{\partial w_{01}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h_1} = (3.992)(-1) = -3.992.$$

$$\frac{\partial \text{sigmoid}}{\partial z_1} = 0.2499$$

$$\begin{aligned} \frac{\partial L}{\partial z_1} &= \frac{\partial L}{\partial h_1} \cdot \frac{\partial \text{sig}}{\partial z_1} = (-3.992)(0.2499) \\ &= -0.998 \end{aligned}$$

$$\frac{\partial L}{\partial w_0} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_0}$$

$$= (-0.998) \frac{\partial}{\partial w_0} (w_0 x_0 + w_{11} x_1 + w_{12} x_2)$$

$$\frac{\partial L}{\partial w_0} = (-0.998) (x_0) = (-0.998) (1) = -0.998$$

$$\text{Updated } w_{01} = w_{01} - \alpha \left(\frac{\partial L}{\partial w_{01}} \right)$$

$$w_{01} = -2 - 0.1(-0.998)$$

$$= -2 + 0.099 = -1.901$$

$$w_{01} = -1.901$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{11}}$$

$$= (-0.998) \frac{\partial}{\partial w_{11}} (w_{01} x_0 + w_{11} x_1 + w_{12} x_2)$$

$$= (-0.998) (x_1) = -0.998 \times 2 = -1.996$$

$$\text{Updated } w_{11} = w_{11} - \alpha \left(\frac{\partial L}{\partial w_{11}} \right)$$

$$w_{11} = 1 - 0.1(-1.996)$$

$$w_{11} = 1.1996$$

To find Updated w_{02}, w_{12}, w_{22}

$$\frac{\partial \text{sigmoid}}{\partial z_2} = 0.00012$$

$$\frac{\partial L}{\partial z_2} = \frac{\partial L}{\partial h_2} \cdot \frac{\partial g}{\partial z_2}$$

$$\frac{\partial L}{\partial h_2} = \frac{\partial L}{\partial g} \cdot \frac{\partial g}{\partial h_2} = (3.992)(2)$$

$$= 7.984$$

$$\frac{\partial L}{\partial z_2} = (7.984)(0.00012) = 0.00095$$

$$\frac{\partial L}{\partial w_{02}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{02}}$$

$$= (0.00095) \frac{\partial}{\partial w_{02}} (w_{02} x_0 + w_{12} x_1 + w_{22} x_2)$$

$$= 0.00095 \times (x_0) = 0.00095$$

Updated w_{02} .

$$\begin{aligned}w_{02} &= w_{02} - 2 \frac{\partial L}{\partial w_{02}} \\&= 1 - 0.1 (0.00095) \\&= 1.000095\end{aligned}$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{12}}$$

$$\frac{\partial z_2}{\partial w_{12}} = (0.00095) (2) = 0.0019$$

$$\frac{\partial L}{\partial w_{12}} = (7.984) (0.0019) = 0.0151$$

Updated w_{12} .

$$\begin{aligned}w_{12} &= w_{12} - 2 \frac{\partial L}{\partial w_{12}} \\&= 2 - 0.1 (0.0151) \\w_{12} &= 1.99989 \approx 2\end{aligned}$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{22}}$$

$$\frac{\partial L}{\partial w_{22}} = (0.00095) (2) = 0.0019$$

Updated w_{22} .

$$\begin{aligned}w_{22} &= w_{22} - 2 \frac{\partial L}{\partial w_{22}} \\&= 3 - 2 (0.0019) \\&\approx 3\end{aligned}$$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{21}}$$

$$= (-0.998) (2)$$

$$\frac{\partial L}{\partial w_{21}} = -1.996$$

Updated w_{21} ,

$$w_{21} = w_{21} - \alpha \frac{\partial L}{\partial w_{21}}$$
$$= -3 - 0.1 (-1.996)$$

$$w_{21} = -2.8004$$

Iteration 2,

$$\hat{y} = w_{03} h_0 + w_{13} h_1 + w_{23} h_2$$

$$= 3.6$$

$$z_1 = w_{01} x_0 + w_{11} x_1 + w_{21} x_2$$

$$= (-1.901)(1) + (1.1996)(1) + (-2.8)(1)$$

$$z_1 = -3.5$$

$$h_1 = \text{Sigmoid}(z_1) = 0.0292$$

$$z_2 = w_{02} x_0 + w_{12} x_1 + w_{22} x_2$$

$$= (-1)(1) + 2(1) + 3(1)$$

$$= 4$$

$$h_2 = \text{Sigmoid}(z_2) = 0.9820138$$

$$L = \frac{1}{2} (\hat{y} - y)^2 = \frac{1}{2} ($$

$$\hat{y} = (3.6) + (-1.001)(0.0292) + (1.601)(0.982)$$

$$\hat{y} = 5.142$$

$$L = \frac{1}{2} (\hat{y} - y)^2 = \frac{1}{2} (5.142 - 1)^2 = \frac{1}{2} (4.142)^2 = 8.58$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{\hat{y} - y}{\hat{y}} = \frac{5.142 - 1}{5.142} = 0.785$$

From (7)

$$\frac{\partial L}{\partial w_{03}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{03}} = (0.785) (h_0) = 0.785 \times 1 = 0.785$$

$$\frac{\partial L}{\partial w_{13}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{13}} = (0.785) (h_1) = 0.785 \times 0.0292 = 0.0229$$

$$\frac{\partial L}{\partial w_{23}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{23}} = (0.785) (h_2) = 0.785 \times 0.982 = 0.771$$

Updated w_{03}, w_{13}, w_{23} :

$$w_{03} = w_{03} - \alpha \frac{\partial L}{\partial w_{03}}$$

$$w_{03} = 3.6 - 0.1(4.142) = 3.185$$

$$w_{13} = w_{13} - \alpha \frac{\partial L}{\partial w_{13}}$$

$$= -1 - 0.1(1.21)$$

$$= -1.121$$

$$w_{23} = w_{23} - \alpha \frac{\partial L}{\partial w_{23}}$$

$$= 1.601 - 0.1(4.07)$$

$$= 1.195$$

$$\frac{\partial L}{\partial h_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h_1}$$

$$= 4.142 \times (-1)$$

$$\frac{\partial L}{\partial h_1} = -4.142$$

$$\frac{\partial \hat{y}}{\partial h_1} = w_{13}$$

$$\frac{\partial \text{sigmoid}}{\partial z_1} = \text{Sigmoid}(-3.5) = 0.0293$$

$$\frac{\partial L}{\partial z_1} = \frac{\partial L}{\partial h_1} \cdot \frac{\partial \text{sig}}{\partial z_1} = (-4.142)(0.0293)$$

$$= -0.1213$$

$$\frac{\partial L}{\partial w_{01}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{01}}$$

$$= (-0.1213)(1) = -0.1213$$

Updated w_{01} ,

$$w_{01} = w_{01} - \alpha \frac{\partial L}{\partial w_{01}}$$

$$= -1.901 - 0.1(-0.1213)$$

$$w_{01} = -1.897$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{11}}$$

$$= (-0.1213)(1) = -0.1213$$

Updated

$$w_{11} = w_{11} - \alpha \frac{\partial L}{\partial w_{11}}$$

$$= 1.1996 - 0.1(-0.1213) = 1.3209$$

$$\frac{\partial L}{\partial z_1} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{21}}$$

$$= (-0.1213) \cdot 1 = -0.1213$$

Updated w_{21} .

$$w_{21} = w_{21} - \alpha \frac{\partial L}{\partial z_1}$$

$$= -2.8 - 0.1(-0.1213)$$

$$= -2.787$$

$$\frac{\partial L}{\partial z_2} = \frac{\partial L}{\partial h_2} \cdot \frac{\partial s_1}{\partial z_2}$$

$$\frac{\partial L}{\partial h_2} = \frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial h_2}$$

$$= 4.42 \times (1.601)$$

$$= 6.631$$

$$\frac{\partial L}{\partial z_2} = 6.631 \times 0.98201$$

$$= 6.512$$

$$\frac{\partial L}{\partial w_{02}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{02}}$$

$$= 6.512 (1) = 6.512$$

$$\text{Updated } w_{02} = w_{02} - \alpha \frac{\partial L}{\partial w_{02}} = -1 - 0.1(6.512) = -1.6512$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_2}{\partial w_{12}}$$

$$= (6.512) (1) = 6.512$$

$$\text{Updated } w_{12} = w_{12} - \alpha \frac{\partial L}{\partial w_{12}} = 2 - 0.1(6.512) = 1.3488$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{22}}$$

$$= 6.512 \times 1 = 6.512$$

$$\text{Updated } w_{22} = w_{22} - \alpha \frac{\partial L}{\partial w_{22}} = 3 - 0.1(6.512) = 2.3488$$

Iteration 3,

$$Z_1 = w_{01}x_0 + w_{11}x_1 + w_{21}x_2$$

$$= 4.2808 - 1.8978(1) + 1.321(-1) + 2.187(-2)$$

$$Z_1 = 4.2808 - 1.035$$

$$Z_2 = w_{02} + w_{12}x_1 + w_{22}x_2$$

$$= -4.244 - 1.6512 + 1.3488(2) + 2.3488(-2)$$

$$Z_2 = -9.0464$$

$$h_1 = \text{sigmoid}(Z_1) = 0.9863$$

$$h_2 = \text{sig}(Z_2) = 0.00011$$

$$\hat{y} = 3.185(1) + 0.7398(-1.121) + 0.00011(1.195)$$

$$\hat{y} = 2.358$$

$$L = \frac{1}{2}(\hat{y} - y)^2 = \frac{1}{2}(2.358 - 10)^2 = \frac{1}{2}(7.642)^2 = 29.09$$

$$\frac{\partial L}{\partial w_{03}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{03}} = (-7.642)(1) = -7.642$$

$$\frac{\partial L}{\partial w_{13}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{13}} = (-7.642)(-1.121) = 8.566$$

$$\frac{\partial L}{\partial w_{23}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{23}} = (-7.642)(0.00011) = -0.000840$$

Updated w_{03}, w_{13}, w_{23} :

$$w_{03} = w_{03} - \eta \frac{\partial L}{\partial w_{03}} = 3.185 - 0.1(-7.642) = 3.185 + 0.7642 = 3.949$$

$$w_{13} = w_{13} - \eta \frac{\partial L}{\partial w_{13}} = -1.121 - 0.1(8.566) = -1.121 - 0.8566 = -1.9776$$

$$w_{23} = w_{23} - \eta \frac{\partial L}{\partial w_{23}} = 1.195 - 0.1(-0.000840) = 1.195 + 0.000840 = 1.19584$$

$$\frac{\partial L}{\partial h_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h_1}$$

$$\frac{\partial \hat{y}}{\partial h_1} = w_{13}$$

$$= (-7.642)(-1.121) = 8.566$$

$$\frac{\partial \text{sigmoid}}{\partial Z_1} = 0.1934$$

$$\frac{\partial L}{\partial Z_1} = 0.1934 \times 8.566 = 1.66$$

$$\frac{\partial L}{\partial w_{01}} = \frac{\partial L}{\partial Z_1} \cdot \frac{\partial Z_1}{\partial w_{01}} = 1.66 \times 1 = 1.66$$

Updated w_{01} :

$$w_{01} = w_{01} - \eta \frac{\partial L}{\partial w_{01}} = -1.897 - 0.1(1.66) = -1.897 - 0.166 = -2.063$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{11}} = 1.269 \times (-2) = -2.538$$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{21}} = 1.269 \times (-2) = -2.538$$

$$w_{11} = 1.321 - 0.1(-2.538) = 1.5748$$

$$w_{21} = 2.3488 - 0.1(-2.538) = 2.6026$$

$$\frac{\partial \text{sigmoid}}{\partial z_2} = \frac{\partial}{\partial z_2} (\text{sigmoid}(z_2 = -0.0064)) = 0.00012$$

$$\frac{\partial \hat{y}}{\partial h_2} = w_{23}$$

$$\frac{\partial L}{\partial h_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h_2}$$

$$= (-7.642)(1.195) = -9.132$$

$$\frac{\partial L}{\partial z_2} = (-9.132 \times 0.00012) = -0.0011$$

$$\frac{\partial L}{\partial w_{02}} = \frac{\partial L}{\partial z_2} \times \frac{\partial z_2}{\partial w_{02}} = -0.0011(1) = -0.0011$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_2} \times \frac{\partial z_2}{\partial w_{12}} = -0.0011(-2) = 0.0022$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_2} \times \frac{\partial z_2}{\partial w_{22}} = -0.0011(-2) = 0.0022$$

Updated weights:

$$w_{02} = w_{02} - \alpha \frac{\partial L}{\partial w_{02}} = -1.6512 - 0.1(-0.0011) = -1.6501$$

$$w_{12} = w_{12} - \alpha \frac{\partial L}{\partial w_{12}} = 1.3488 - 0.1(0.0022) = 1.3486$$

$$w_{22} = w_{22} - \alpha \frac{\partial L}{\partial w_{22}} = 2.3488 - 0.1(0.0022) = 2.3486$$

Iteration 4,

$$z_1 = w_{01}x_0 + w_{11}x_1 + w_{21}x_2$$

$$= (-2.029)1 + (1.575)3 + (2.6026)(-3)$$

$$z_1 = -2.029 + 4.725 - 7.8138 = -5.1178$$

$$z_2 = w_{02}x_0 + w_{12}x_1 + w_{22}x_2$$

$$z_2 = (-1.6507)1 + (1.35)3 - 3(2.35) = -4.6507$$

$$h_1 \xrightarrow{\text{Sig}(z_1)} 0.006$$

$$h_2 \xrightarrow{\text{Sig}(z_2)} 0.0095$$

$$\hat{y} = h_0 \cdot w_{03} + h_1 \cdot w_{13} + h_2 \cdot w_{23}$$

$$= 1(3.947) + 0.006(-0.56) + 0.0095(1.195)$$

$$= 3.955$$

$$L = \frac{1}{2} (\hat{y} - y)^2 = \frac{1}{2} (3.955 - 17)^2 = 85.1$$

$$\frac{\partial L}{\partial \hat{y}} = -13.05$$

$$\frac{\partial L}{\partial w_{03}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{03}} = (-13.05)1 = -13.05 \quad \frac{\partial \hat{y}}{\partial w_{03}} = h_0$$

$$\frac{\partial L}{\partial w_{13}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{13}} = (-13.05)(0.006) = -0.0783$$

$$\frac{\partial L}{\partial w_{23}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{23}} = (-13.05)(0.0095) = -0.124$$

Updated w_{03}, w_{13}, w_{23}

$$w_{03} = w_{03} - \alpha \frac{\partial L}{\partial w_{03}} = 3.947 - 0.1(-13.05) = 5.252$$

$$w_{13} = w_{13} - \alpha \frac{\partial L}{\partial w_{13}} = -0.558 - 0.1(-0.0783) = -0.565$$

$$w_{23} = w_{23} - \alpha \frac{\partial L}{\partial w_{23}} = 1.195 - 0.1(-0.124) = 1.207$$

$$\frac{\partial L}{\partial h_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h_1} = (-13.05)w_{13} = 7.282$$

$$\frac{\partial \hat{y}}{\partial h_1} = w_{13}$$

$$\delta \text{ Sigmoid} = 0.00592$$

$$\frac{\partial L}{\partial z_1} = \frac{\partial L}{\partial h_1} \cdot \frac{\partial \text{Sigmoid}}{\partial z_1} = 7.282 \times 0.00592 = 0.0431$$

$$\frac{\partial L}{\partial w_{01}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{01}} = 0.0431(1) = 0.0431$$

Updated w_{01} ,

$$w_{01} = w_{01} - \alpha \frac{\partial L}{\partial w_{01}} = -2.029 - 0.1(0.0431) = -2.1559$$

$$\frac{\partial L}{\partial w_{01}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{01}} = (0.0431) \cdot 1 = 0.0431$$

$$\text{Updated } w_{01}, \quad w_{01} = w_{01} - \alpha \frac{\partial L}{\partial w_{01}} = 0. - 2.029 - 0.1(0.0431) \Rightarrow -2.033$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{11}} = (0.0431) \cdot 3 = 0.13$$

$$\text{Updated } w_{11}, \quad w_{11} = w_{11} - \alpha \frac{\partial L}{\partial w_{11}} \Rightarrow 1.575 - 0.1(0.13) = 1.562$$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_1} \cdot \frac{\partial z_1}{\partial w_{21}} = (0.0431) \cdot (-3) = -0.13$$

$$\text{Updated } w_{21}, \quad w_{21} = w_{21} - \alpha \frac{\partial L}{\partial w_{21}} \Rightarrow 2.6267 + 0.1(0.13) \Rightarrow 2.6156$$

$$\frac{\partial \text{Sigmoid}}{\partial z_2} = 0.00937$$

$$\frac{\partial L}{\partial w_{23}} = w_{23}$$

$$\frac{\partial L}{\partial z_2} = \frac{\partial L}{\partial h_1} \times \frac{\partial \text{Sigmoid}}{\partial z_2} = 1.195(0.00937) = 0.01195$$

$$\frac{\partial L}{\partial w_{02}} = x_0$$

$$\frac{\partial L}{\partial w_{02}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{02}} = 0.01195(1) = 0.01195$$

$$\frac{\partial L}{\partial w_{12}} = x_1$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{12}} = (0.01195) \cdot 3 = 0.03585$$

$$\frac{\partial L}{\partial w_{22}} = x_2$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{12}} = (0.01195) \cdot 3 = -0.03585$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{22}} = (0.01195) \cdot 3 = -0.03585$$

$$\text{Updated } w_{02}, w_{12}, w_{22}$$

$$w_{02} = w_{02} - \alpha \frac{\partial L}{\partial w_{02}} = -1.6507 - 0.1(0.01195) = -1.652$$

$$w_{12} = w_{12} - \alpha \frac{\partial L}{\partial w_{12}} = 1.3486 - 0.1(0.03585) = 1.345$$

$$w_{22} = w_{22} - \alpha \frac{\partial L}{\partial w_{22}} = 2.3486 - 0.1(-0.03585) = 2.384$$

Iteration 5,

$$z_1 = w_{01}x_0 + w_{11}x_1 + w_{21}x_2$$

$$= (-2.033)1 + (0.562)5 + (2.6156)4 = 16.24$$

$$z_2 = w_{02}x_0 + w_{12}x_1 + w_{22}x_2$$

$$= (-1.852)1 + (1.345)5 + 2.384(4) = 14.61$$

$$h_1 = \text{sigmoid}(z_1) = 0.999$$

$$h_2 = \text{sigmoid}(z_2) = 0.999$$

$$\hat{y} = w_{03}h_0 + w_{13}h_1 + w_{23}h_2 = 5.252 + (-0.565) + 1.207 = 5.89$$

$$L = \frac{1}{2} (\hat{y} - y)^2 = 86$$

$$\frac{\partial L}{\partial \hat{y}} = -13.11$$

$$\frac{\partial L}{\partial w_{03}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{03}} = 13.11(1) = -13.11$$

$$\frac{\partial L}{\partial w_{13}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{13}} = 13.11(1) = -13.11$$

$$\frac{\partial L}{\partial w_{23}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{23}} = 13.11(1) = -13.11$$

Updated w_{03}, w_{13}, w_{23} .

$$w_{03} = w_{03} - 2 \frac{\partial L}{\partial w_{03}} = 5.252 - 0.1(-13.11) = 18.955$$

$$w_{13} = w_{13} - 2 \frac{\partial L}{\partial w_{13}} = -0.565 - 0.1(-13.11) = -12.535$$

$$w_{23} = w_{23} - 2 \frac{\partial L}{\partial w_{23}} = 1.207 - 0.1(-13.11) = 14.307$$

$$\frac{\partial L}{\partial w_{01}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{01}} = 13.11(-0.565) = -7.41 \quad \frac{\partial \hat{y}}{\partial w_{01}} = w_{13}$$

$$\frac{\partial \text{sigmoid}}{\partial z_1} = 0.0001$$

$$\frac{\partial L}{\partial w_{01}} = 0.0001$$

$$\frac{\partial L}{\partial w_{11}} = 0.0001$$

$$\frac{\partial L}{\partial w_{21}} = 0.0001$$

Updated w_{01}, w_{11}, w_{21} .

$$w_{01} = w_{01} - 2 \frac{\partial L}{\partial w_{01}} = -2.033$$

$$w_{11} = w_{11} - 2 \frac{\partial L}{\partial w_{11}} = 1.562$$

$$w_{21} = w_{21} - 2 \frac{\partial L}{\partial w_{21}} = 5.252$$

Similarly, we observe very negligible/non-significant amount for w_{02}, w_{12}, w_{22} .
 so, update the table with same numbers.

Iteration 6,

$$z_1 = w_{01}x_0 + w_{11}x_1 + w_{21}x_2$$

$$= (-2.033)(1) + (-1.562)(-4) + (2.615)(2) = 9.45$$

$$z_2 = w_{02}x_0 + w_{12}x_1 + w_{22}x_2$$

$$= (-1.652)(1) + (1.345)(-4) + (2.384)(2) = -2.264$$

$$h_1 = \text{Sigmoid}(z_1) = 0.999$$

$$h_2 = \text{Sigmoid}(z_2) = 0.094$$

$$\hat{y} = w_{03}h_0 + w_{13}h_1 + w_{23}h_2$$

$$= 3.945(1) + (-1.88)(0.99) + (-0.103)(0.094)$$

$$= 2.055$$

$$L = \frac{1}{2} (2.055 - 14)^2$$

$$\hat{y} = w_{03}h_0 + w_{13}h_1 + w_{23}h_2$$

$$\hat{y} = 18.355(1) + (-12.535)(0.99) + 14.307 = 20.127$$

$$L = \frac{1}{2} (20.127 - 14)^2 = 39.54$$

$$\frac{\partial L}{\partial \hat{y}} = 6.127$$

$$\frac{\partial L}{\partial w_{13}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{13}} = 6.127(1) = 6.127$$

$$\frac{\partial L}{\partial w_{03}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{03}} = (6.127)(1) = 6.127$$

$$\frac{\partial L}{\partial w_{23}} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_{23}} = (6.127)(0.094) = 0.575$$

Updated w_{03}, w_{13}, w_{23} :

$$w_{03} = w_{03} - \alpha \frac{\partial L}{\partial w_{03}} = 18.355 - (0.1)(6.127) = 17.742$$

$$= 18.355 - (0.1)(6.127) = -13.15$$

$$w_{13} = w_{13} - \alpha \frac{\partial L}{\partial w_{13}} = -12.535 - (0.1)(6.127) = -13.15$$

$$w_{23} = w_{23} - \alpha \frac{\partial L}{\partial w_{23}} = 14.307 - (0.1)(0.575) = 14.25$$

$$\frac{\partial \text{Sigmoid}}{\partial z_1} = 0.0001$$

Since the change is so less, the w_{01} , w_{11} , w_{21} values, change.

$$w_{01} = -2.033$$

$$w_{11} = -1.562$$

$$w_{21} = 2.615$$

$$\frac{\partial \text{Sigmoid}}{\partial z_2} = -0.0853$$

$$\frac{\partial L}{\partial z_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \text{Sigmoid}}{\partial z_2}$$

$$\frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial z_2} \cdot \frac{\partial \text{Sigmoid}}{\partial z_2}$$

$$\frac{\partial L}{\partial z_2} = 6.127 \times 14.307 \cdot 0.0853 = 7.475$$

$$\frac{\partial \hat{y}}{\partial z_2} =$$

$$\frac{\partial L}{\partial w_{01}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{01}} = 7.475(1) = 7.475$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{12}} = 7.475(-4) = -29.900$$

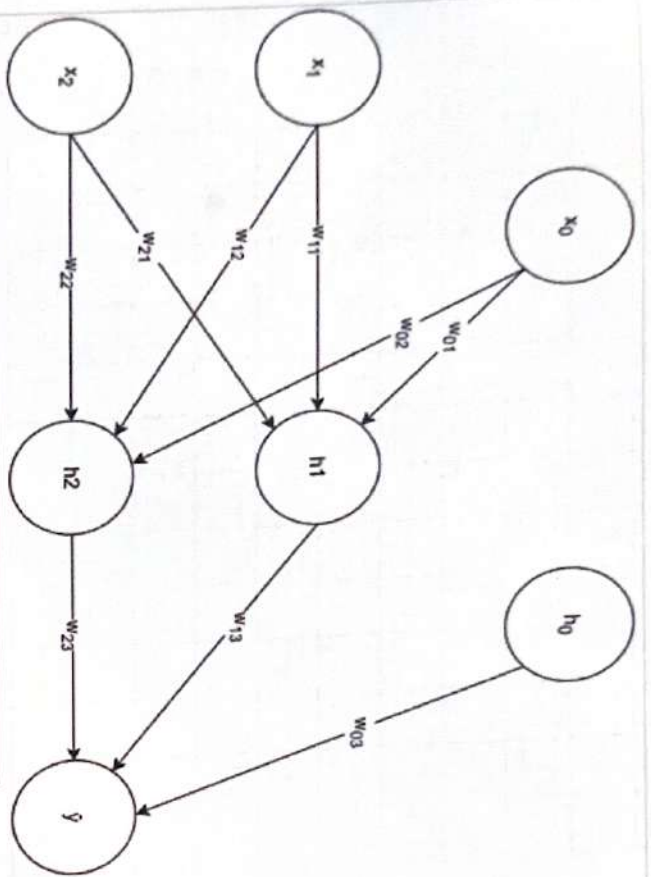
$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_2} \cdot \frac{\partial z_2}{\partial w_{22}} = 7.475(2) = 14.950$$

Updated w_{02} , w_{12} , w_{22} .

$$w_{02} = w_{02} - \eta \frac{\partial L}{\partial w_{02}} = -1.652 - 0.1(7.475) = -2.403$$

$$w_{12} = w_{12} - \eta \frac{\partial L}{\partial w_{12}} = 1.345 - 0.1(-29.900) = 4.335$$

$$w_{22} = w_{22} - \eta \frac{\partial L}{\partial w_{22}} = 2.384 - 0.1(14.95) = 0.889$$



$\partial L / \partial w_{01}$	$\partial L / \partial w_{02}$	$\partial L / \partial w_{03}$	$\partial L / \partial w_{11}$	$\partial L / \partial w_{12}$	$\partial L / \partial w_{13}$	$\partial L / \partial w_{21}$	$\partial L / \partial w_{22}$	$\partial L / \partial w_{23}$	$\partial L / \partial w_{01}$	$\partial L / \partial w_{02}$	$\partial L / \partial w_{03}$	$\partial L / \partial w_{11}$	$\partial L / \partial w_{12}$	$\partial L / \partial w_{13}$	$\partial L / \partial w_{21}$	$\partial L / \partial w_{22}$	$\partial L / \partial w_{23}$
-0.998	0.0005	-1.996	0.0019	-1.996	0.0019	3.9923	0.0028	3.9918	-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007
-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007	-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007
-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007	-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007
-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007	-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007
-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007	-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007
-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007	-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007
-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007	-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007
-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007	-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007
-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007	-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007
-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007	-0.1213	6.512	-0.1213	6.512	-0.1213	6.512	6.1162	1.21	4.007

x_0	w_{01}	w_{02}	x_1	w_{11}	w_{12}	x_2	w_{21}	w_{22}	h_0	w_{03}	h_1	w_{13}	h_2	w_{23}	y	\hat{y}	L
1	-2	-1	2	1	2	2	-3	3	1	4	0.00147	-1	0.9998	2	25.9923	2.969	
1	-1.901	-1	1.1996	2	1	-2.8	3	1	3.6	0.0029	-1.001	0.982	1.601	15.142	8.58		
1	-1.897	-1.6512	-2.1321	1.3488	-2	-2.391	2.348	1	3.185	0.1863	-1.121	1.04	1.195	10.2358	29.02		
1	-2.014	-1.654	3.1525	1.3486	-2	-2.602	2.348	1	5.949	0.006	-0.558	0.0095	1.195	17.3955	-8.5		
1	-2.033	-1.652	5.1562	1.345	4	-2.615	2.384	1	5.252	0.999	-0.555	0.999	1.207	19.589	8.6		
1	-2.033	-2.0662	-4.1562	2.3495	2	-2.615	2.384	1	18.355	0.999	-12.535	0.094	14.349	14.20129	8.754		
1	-2.033	-2.0603	-1.562	4.1335	-2	-6.15	0.889	1	1.7942	-13.15			14.25				