

# Introduction to Language Modeling

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Topic 6

- Why probability to explain theory of language?
- Language Modeling
- Basic idea
- Examples
- N-gram Language Modeling

# Language and cognition as probabilistic phenomena

- Human cognition is probabilistic and that language must therefore be probabilistic too since it is an integral part of cognition.
- We live in a world filled with uncertainty and incomplete information.
- We make decisions by processing these information
- These cognitive processes are best formalized as probabilistic processes
- Processing the words, forming an idea of the overall meaning of the sentence, and weighing it in making a decision is no different in principle
- Meaning of the word is defined by the circumstances of its use → uncertainty
- So Probability plays an important role in tackle the questions of meaning

- A language model is a probability distribution over strings on an alphabet

# Language Modeling

- A language model is a probability distribution over strings on an alphabet
- Language Models are the development of probabilistic models that are able to predict the next word in the sequence given the words that precede it.

- A language model is a probability distribution over strings on an alphabet
- Language Models are the development of probabilistic models that are able to predict the next word in the sequence given the words that precede it.
- Language models explain various linguistics phenomena using probability
- It is a root problem for a large range of natural language processing tasks
  - Machine Translation
  - Spell checking
  - Next word prediction
  - Text generation

**The cat**

## Example: Word Prediction





# Example: Word Prediction



## Example: Word Prediction



## Example: Word Prediction



# Language Model- Other Applications

- LM can assign probability to the sentence (how probable the sequences are?)

$$P(\textit{The cat sat on the mat}) = \\ P(\textit{The}).P(\textit{cat}|\textit{the}).P(\textit{sat}|\textit{the cat}).P(\textit{on}|\textit{The cat sat})...$$

- This will give higher probability to common/syntactically/semantically correct usages

Large wind tonight  $\rightarrow 0.31$

High wind tonight  $\rightarrow 0.97$

- Spelling mistake or correct word?

$P(I \text{ was late by fifteeen minuets}) > P(I \text{ was late by fiteen minutes})$

- Speech Recognition

$P(I \text{ saw a van}) \gg P(\text{eyes awe of an})$

- Machine Translation: which usage is more plausible in target language

$P(\text{high wind}) > P(\text{large wind})$

$P(\text{He immediately moved to hospital}) >$

$P(\text{He quickly moved to hospital})$

- Spelling mistake or correct word?

$P(I \text{ was late by } \textit{fifteen minuets}) > P(I \text{ was late by } \textit{fiteen minutes})$

- Speech Recognition

$P(I \text{ saw a } \textit{van}) \gg P(\textit{eyes awe of an})$

- Machine Translation: which usage is more plausible in target language

$P(\textit{high wind}) > P(\textit{large wind})$

$P(\textit{He immediately moved to hospital}) >$

$P(\textit{He quickly moved to hospital})$

# Language Model-Other Applications

- Context sensitive spelling correction
- Natural Language generation
- Next word prediction
- Auto completion (In mail, search engines)
  - Please, call .....
  - Can you .... ..

# Language Model

- Goal: Compute the probability of a sentence or sequence of words  
 $P(W) = P(w_1, w_2, \dots, w_n)$
- Related Task:  
 $P(w_{n+1} | w_1, \dots, w_n)$
- Language model compute either of the above



- Probability of a sentence is the joint probability of the words

$$P(W) = P(w_1, w_2, \dots, w_n)$$

- This rely on conditional probability

$$P(A|B) = \frac{P(A,B)}{P(B)}$$

$$P(A, B) = P(B)P(A|B)$$

- This can be expanded

$$P(A, B, C, D) = P(A)P(B|A)P(C|A, B)P(D|A, B, C)$$

# Counting Probabilities

- $P(\text{the cat sat on the mat}) = P(\text{the})P(\text{cat}|\text{the})P(\text{sat}|\text{the cat})\dots P(\text{mat}|\text{the cat sat on the})$
- How do you compute this probability?  
$$P(\text{sat}|\text{the cat}) = \frac{\text{Count}(\text{the,cat,sat})}{\text{Count}(\text{the,cat})}$$
- We may never see enough data to estimate the long sequences

# Markov Assumption and Language Model

- In the previous examples, the next word is represented by a conditional probability of previous word

$$P(W_{n+1}|W_1...W_n) = \frac{P(W_1...W_{n+1})}{P(W_1...W_n)}$$

- Exponential number of sequences, counting is difficult
- The question here is how much history is required?
  - It depends only a limited history (3 or 4 words)

- **Markov Assumption**

$$P(W_{n+1}|W_1...W_n) = P(W_{n+1}|W_n)$$

- Generally, we can say, it depends only on previous  $k$  words (k-order Markov model,  $k < n$ )

$$P(W_{n+1}|W_1...W_n) = P(W_{n+1}|W_{n-k}...W_n)$$

# Markov Assumption and n-gram model

Looks only at n words at a time.

- $0^{th}$  order:  $P(W_{n+1}|W_1...W_n) = P(W_{n+1})$  (independent of previous words, unigram model)
- $1^{st}$  order:  $P(W_{n+1}|W_1...W_n) = P(W_{n+1}|W_n)$  (bigram model)  
 $P(cat|the), P(sat|cat) ..$
- $2^{nd}$  order:  $P(W_{n+1}|W_1...W_n) = P(W_{n+1}|W_n, W_{n-1})$  (trigram model)

# Simple N-Gram Model

- Goal is to estimate the probability of a word given history,  $P(w|h)$
- $P(\text{apple}|\text{the boy ate})$
- $P(\text{apple}|\text{the boy ate}) = \frac{\text{Count}(\text{the boy ate apple})}{\text{Count}(\text{the boy ate})}$
- Beware!— language is creative and new sentences are added every time
- It is hard to estimate sufficient statistics even from a large corpus (like web)

# Some Notations

- Word sequence  $w_1, \dots w_n$  is represented as  $w_1^n$
- Joint probability  $P(X = w_1, Y = w_2, \dots)$  is represented as  $P(w_1, w_2, \dots w_n)$

# Chain Rule

- $P(X_1, \dots, X_n) = P(X_1)P(X_2|X_1)P(X_3|X_1, X_2) \dots P(X_n|X_1, \dots, X_{n-1})$
- $P(X_1, \dots, X_n) = \prod_{i=1}^n P(X_i|X_1^{i-1})$
- $P(w_1^n) = \prod_{i=1}^n P(w_i|w_1^{i-1})$
- The joint probability of a sequence and computing the conditional probability of a word given previous words
- We don't know any way to compute the exact probability of a word given a long sequence of preceding words
- Approximate: The intuition of the N-gram model is that instead of computing the probability of a word given its entire history, we will approximate the history by just the last few words.

- Bigram model approximates the probability of a word given all the previous words
- $P(w_n | w_1^{n-1}) = P(w_n | w_{n-1})$
- $P(\text{apple} | \text{the boy ate}) = P(\text{apple} | \text{ate})$
- Markov Assumption
- We can generalize bigram to **N-gram** model



# How to estimate N-gram probability

- Maximum Likelihood Estimate (MLE)
- For example to compute a particular bigram probability of a word  $y$  given a previous word  $x$ , we'll compute the count of the bigram  $C(xy)$  and normalize by the sum of all the bigrams that share the same first word  $x$
- $$P(w_n|w_{n-1}) = \frac{\text{Count}(w_{n-1}, w_n)}{\sum_w (\text{Count}(w_{n-1}, w))}$$
- Since the sum of all bigram counts that start with a given word  $w_{n1}$  must be equal to the unigram count for that word  $w_{n1}$
- $$P(w_n|w_{n-1}) = \frac{\text{Count}(w_{n-1}, w_n)}{\text{Count}(w_{n-1})}$$

# Example

<s> I am Sam </s>

<s> Sam I am </s>

<s> I do not like green eggs and ham </s>

Here are the calculations for some of the bigram probabilities from this corpus

$$P(I | <s>) = \frac{2}{3} = .67 \quad P(\text{Sam} | <s>) = \frac{1}{3} = .33 \quad P(\text{am} | I) = \frac{2}{3} = .67$$

$$P(</s> | \text{Sam}) = \frac{1}{2} = 0.5 \quad P(\text{Sam} | \text{am}) = \frac{1}{2} = .5 \quad P(\text{do} | I) = \frac{1}{3} = .33$$

- For N-gram model:  $P(w_n | w_1^{n-1}) = \frac{\text{count}(w_{n-N+1}^{n-1} w_n)}{\text{Count}(w_{n-N+1}^{n-1})}$
- This ratio is called relative frequency

# Example

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

**Figure 4.1** Bigram counts for eight of the words (out of  $V = 1446$ ) in the Berkeley Restaurant Project corpus of 9332 sentences.

Fig. 4.2 shows the bigram probabilities after normalization (dividing each row by the following unigram counts):

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

# Example

$$P(i | <s>) = 0.25$$

$$P(\text{food} | \text{english}) = 0.5$$

$$P(\text{english} | \text{want}) = 0.0011$$

$$P(</s> | \text{food}) = 0.68$$

# Example

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

**Figure 4.2** Bigram probabilities for eight words in the Berkeley Restaurant Project corpus of 9332 sentences.

$$\begin{aligned}P(<s> \text{ i want english food } </s>) \\&= P(i|<s>)P(\text{want}|i)P(\text{english}|\text{want}) \\&\quad P(\text{food}|\text{english})P(</s>|\text{food}) \\&= .25 \times .33 \times .0011 \times 0.5 \times 0.68 \\&= .000031\end{aligned}$$

## So far

- Language Modeling is assigning probabilities to sequence of words/characters
- Markov assumption and N-gram models
- Counting probability
- Example

## Next

- Evaluation - Perplexity
- Smoothing

# Training and Test corpus

- The probabilities of the N-gram model come from the corpus it trained on
- Training on some data and testing on some other data (mutually exclusive sets)
- Divide the dataset into **Training corpus and test corpus**
- Its important not to let the test sentences into the training set

# Training and Test corpus

- The probabilities of the N-gram model come from the corpus it trained on
- Training on some data and testing on some other data (mutually exclusive sets)
- Divide the dataset into Training corpus and test corpus
- **Its important not to let the test sentences into the training set**
- How to split input corpus: 80:20, 70:30,...



# Issues in N-gram modeling

- The probabilities of the N-gram model come from the corpus it trained on
- It may not TRUE for another testing corpus
- Sensitivity to the value of N
- Create a general purpose corpus by collecting different genres

# Treating unknown words

- Test set may contain unknown words (Out of Vocabulary words-OOV)
- Choose a Vocabulary which is fixed
- Convert OOV to  $\langle UNK \rangle$  unknown category
- Estimate the probability of  $\langle UNNK \rangle$

# Evaluating the model

- **Extrinsic** (in vivo) vs intrinsic evaluation
- If know the expected model in prior, we can us extrinsic measures (eg: accuracy)
- For example: We can apply a model on some application (say text classification and predict accuracy)
- **Intrinsic** measures evaluate without any external applications
- Perplexity

# Perplexity

- **Perplexity** is the most common intrinsic evaluation metric for N-gram language models.
- Given two probabilistic models, **the better model is the one that has a tighter fit to the test data**, or predicts the details of the test data better.
- The perplexity (sometimes called PP for short) of a language model on a test set is a function of the probability that the language model assigns to that test set.
- For sequence of  $W = w_1, \dots w_N$
- $PP(W) = P(w_1, \dots w_N)^{-1} = \sqrt[N]{\frac{1}{P(w_1, \dots w_N)}}$

# Perplexity

- For sequence of  $W = w_1, \dots w_N$
- $PP(W) = P(w_1, \dots w_N)^{-1} = \sqrt[N]{\frac{1}{P(w_1, \dots w_N)}}$
- $PP(W) = \sqrt[N]{\prod_{i=1}^n \frac{1}{P(w_i | w_1, \dots w_{i-1})}}$
- $PP(W) = \sqrt[N]{\prod_{i=1}^n \frac{1}{P(w_i | w_{i-1})}}$  For bigram model

# Perplexity



- The best model will be the one that assigns the highest probability to the test set.
- Intuitively, if a model assigns a high probability to the test set, it means that it is not surprised to see it (its not perplexed by it)
- Low perplexity indicates it has a good understanding of how the language works.
- $PP(W) = \sqrt[n]{\prod_{i=1}^n \frac{1}{P(w_i|w_1,..w_{i-1})}}$  (we are taking inverse of probability)
- A lower perplexity indicates a better model

- Perplexity also indicate average branching factor of a language model
- Branching factor is the number of possible next words that can follow a word
- perplexity is high means: *“when trying to guess the next word, our model is as confused as if it had to pick between many different words”*

# Next: Smoothing

- The estimating probabilities from sparse data
- Zero count affect entire MLE computation
- Addressing poor estimation: smoothing
- Different Smoothing techniques
  - Laplace smooting
  - Good-Turing discounting
- Interpolation

Read: Chapter 4, SLP, Jurafsky



# Language Modeling

Smoothing: Add-one  
(Laplace) smoothing

# The intuition of smoothing (from Dan Klein)

When we have sparse statistics:

$P(w \mid \text{denied the})$

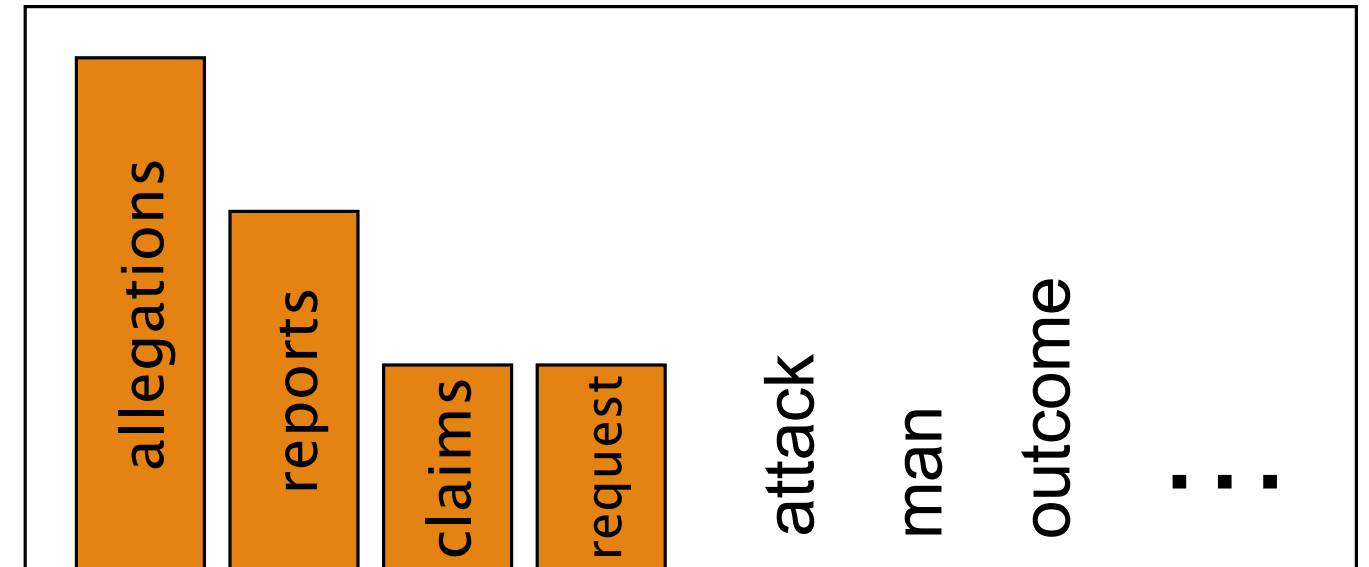
3 allegations

2 reports

1 claims

1 request

7 total



Steal probability mass to generalize better

$P(w \mid \text{denied the})$

2.5 allegations

1.5 reports

0.5 claims

0.5 request

2 other

7 total



# Add-one estimation

Also called Laplace smoothing

Pretend we saw each word one more time than we did

Just add one to all the counts!

MLE estimate:

$$P_{MLE}(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

Add-1 estimate:

$$P_{Add-1}(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

# Maximum Likelihood Estimates

The maximum likelihood estimate

- of some parameter of a model  $M$  from a training set  $T$
- maximizes the likelihood of the training set  $T$  given the model  $M$

Suppose the word “bagel” occurs 400 times in a corpus of a million words

What is the probability that a random word from some other text will be “bagel”?

MLE estimate is  $400/1,000,000 = .0004$

This may be a bad estimate for some other corpus

- But it is the **estimate** that makes it **most likely** that “bagel” will occur 400 times in a million word corpus.

# Berkeley Restaurant Corpus: Laplace smoothed bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

# Laplace-smoothed bigrams

$$P^*(w_n|w_{n-1}) = \frac{C(w_{n-1}w_n) + 1}{C(w_{n-1}) + V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

# Reconstituted counts

$$c^*(w_{n-1}w_n) = \frac{[C(w_{n-1}w_n) + 1] \times C(w_{n-1})}{C(w_{n-1}) + V}$$

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

# Compare with raw bigram counts

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
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chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16



# Add-1 estimation is a blunt instrument

So add-1 isn't used for N-grams:

- We'll see better methods

But add-1 is used to smooth other NLP models

- For text classification
- In domains where the number of zeros isn't so huge.

# Language Modeling

Smoothing: Add-one  
(Laplace) smoothing

# Language Modeling

Interpolation, Backoff, and  
Web-Scale LMs

# Backoff and Interpolation

Sometimes it helps to use **less** context

- Condition on less context for contexts you haven't learned much about

## **Backoff:**

- use trigram if you have good evidence,
- otherwise bigram, otherwise unigram

## **Interpolation:**

- mix unigram, bigram, trigram

Interpolation works better

# Linear Interpolation

## Simple interpolation

$$\begin{aligned}\hat{P}(w_n|w_{n-2}w_{n-1}) = & \lambda_1 P(w_n|w_{n-2}w_{n-1}) \\ & + \lambda_2 P(w_n|w_{n-1}) \\ & + \lambda_3 P(w_n)\end{aligned} \qquad \sum_i \lambda_i = 1$$

Lambdas conditional on context:

$$\begin{aligned}\hat{P}(w_n|w_{n-2}w_{n-1}) = & \lambda_1(w_{n-2}^{n-1}) P(w_n|w_{n-2}w_{n-1}) \\ & + \lambda_2(w_{n-2}^{n-1}) P(w_n|w_{n-1}) \\ & + \lambda_3(w_{n-2}^{n-1}) P(w_n)\end{aligned}$$

# How to set the lambdas?

Use a **held-out** corpus

Training Data

Held-Out  
Data

Test  
Data

Choose  $\lambda$ s to maximize the probability of held-out data:

- Fix the N-gram probabilities (on the training data)
- Then search for  $\lambda$ s that give largest probability to held-out set:

$$\log P(w_1 \dots w_n \mid M(\lambda_1 \dots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \dots \lambda_k)}(w_i \mid w_{i-1})$$

# Unknown words: Open versus closed vocabulary tasks

If we know all the words in advanced

- Vocabulary  $V$  is fixed
- Closed vocabulary task

Often we don't know this

- **Out Of Vocabulary** = OOV words
- Open vocabulary task

Instead: create an unknown word token <UNK>

- Training of <UNK> probabilities
  - Create a fixed lexicon  $L$  of size  $V$
  - At text normalization phase, any training word not in  $L$  changed to <UNK>
  - Now we train its probabilities like a normal word
- At decoding time
  - If text input: Use UNK probabilities for any word not in training

# Huge web-scale n-grams

How to deal with, e.g., Google N-gram corpus

## Pruning

- Only store N-grams with count > threshold.
  - Remove singletons of higher-order n-grams
- Entropy-based pruning

## Efficiency

- Efficient data structures like tries
- Bloom filters: approximate language models
- Store words as indexes, not strings
  - Use Huffman coding to fit large numbers of words into two bytes
- Quantize probabilities (4-8 bits instead of 8-byte float)



# Smoothing for Web-scale N-grams

“Stupid backoff” (Brants *et al.* 2007)

No discounting, just use relative frequencies

$$S(w_i | w_{i-k+1}^{i-1}) = \begin{cases} \frac{\text{count}(w_{i-k+1}^i)}{\text{count}(w_{i-k+1}^{i-1})} & \text{if } \text{count}(w_{i-k+1}^i) > 0 \\ 0.4S(w_i | w_{i-k+2}^{i-1}) & \text{otherwise} \end{cases}$$

$$S(w_i) = \frac{\text{count}(w_i)}{N}$$

# N-gram Smoothing Summary

Add-1 smoothing:

- OK for text categorization, not for language modeling

The most commonly used method:

- Extended Interpolated Kneser-Ney

For very large N-grams like the Web:

- Stupid backoff

# Advanced Language Modeling

Discriminative models:

- choose n-gram weights to improve a task, not to fit the training set

Parsing-based models

Caching Models

- Recently used words are more likely to appear

$$P_{CACHE}(w | history) = \lambda P(w_i | w_{i-2} w_{i-1}) + (1 - \lambda) \frac{c(w \in history)}{|history|}$$

- These turned out to perform very poorly for speech recognition (why?)

# Language Modeling

Interpolation, Backoff, and  
Web-Scale LMs