Algorithms:
Greedy Method

An Activity-Selection Problem

Greedy Algorithms: Principles

- A *greedy algorithm* always makes the choice that looks best at the moment.
- A greedy algorithm works in phases.

  At each phase:
  - You take the best you can get right now, without regard for future consequences.
  - You hope that by choosing a local optimum at each step, you will end up at a global optimum.
  - For some problems, it works.

An Activity Selection Problem

- Input: A set of activities S = $\{a_1, \ldots, a_n\}$
  - Each activity a_i has a start time s_i and a finish time f_i, where $0 \leq s_i < f_i < \infty$
  - If selected, activity a_i takes place during the half-open time interval $[s_i, f_i)$

- Two activities are compatible if and only if their intervals do not overlap

- Output: a maximum-size subset of mutually compatible activities

The Activity Selection Problem

- Here are a set of start and finish times

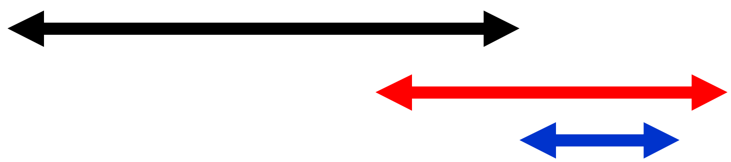
i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	8	9	10	11	12	13	14

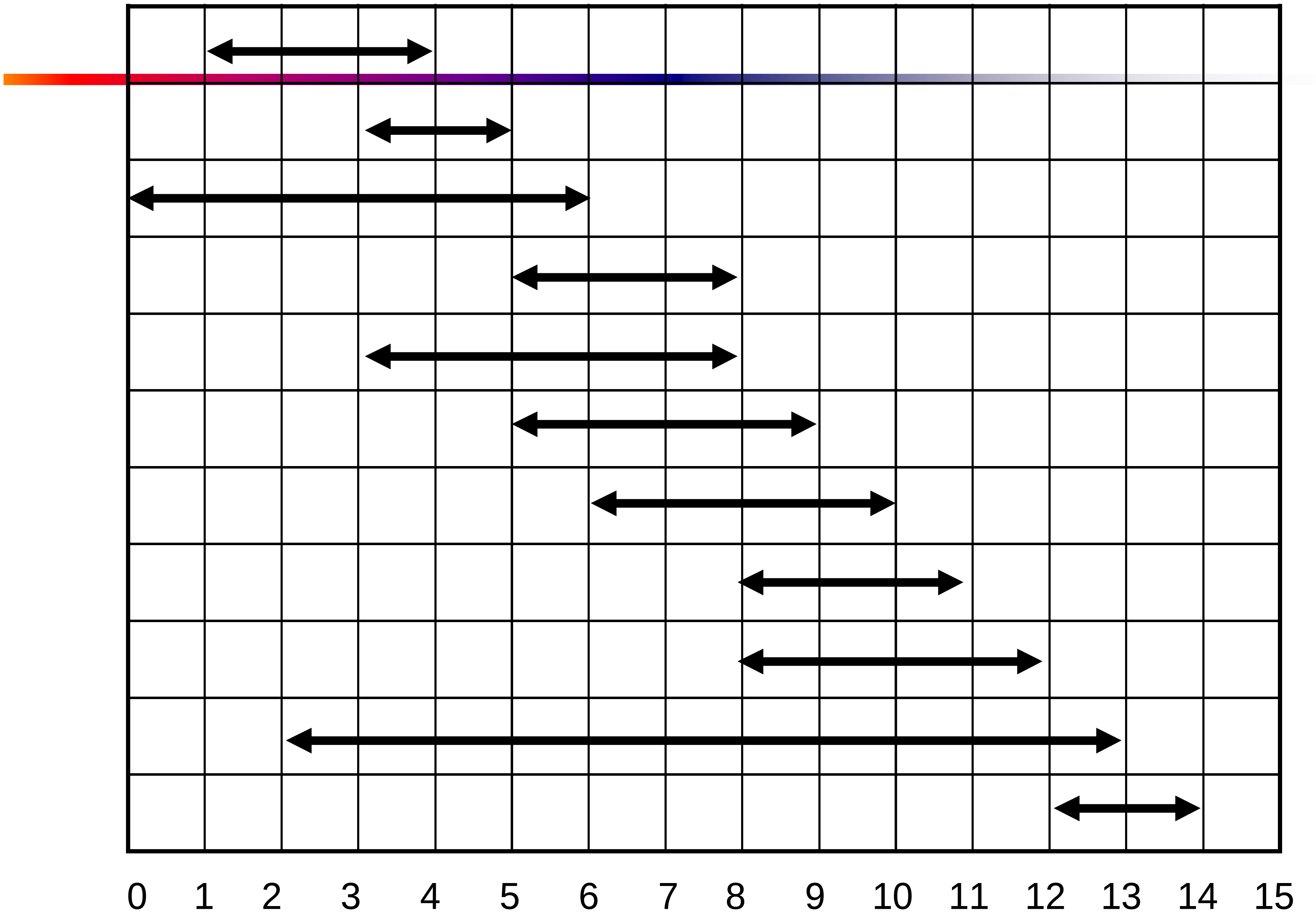
- What is the maximum number of activities that can be completed?
  - $\{a_3, a_9, a_{11}\}$ can be completed
  - But so can $\{a_1, a_4, a_8, a_{11}\}$ which is a larger set
  - But it is not unique, consider $\{a_2, a_4, a_9, a_{11}\}$

Interval Representation

- Here are a set of start and finish times

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	8	9	10	11	12	13	14





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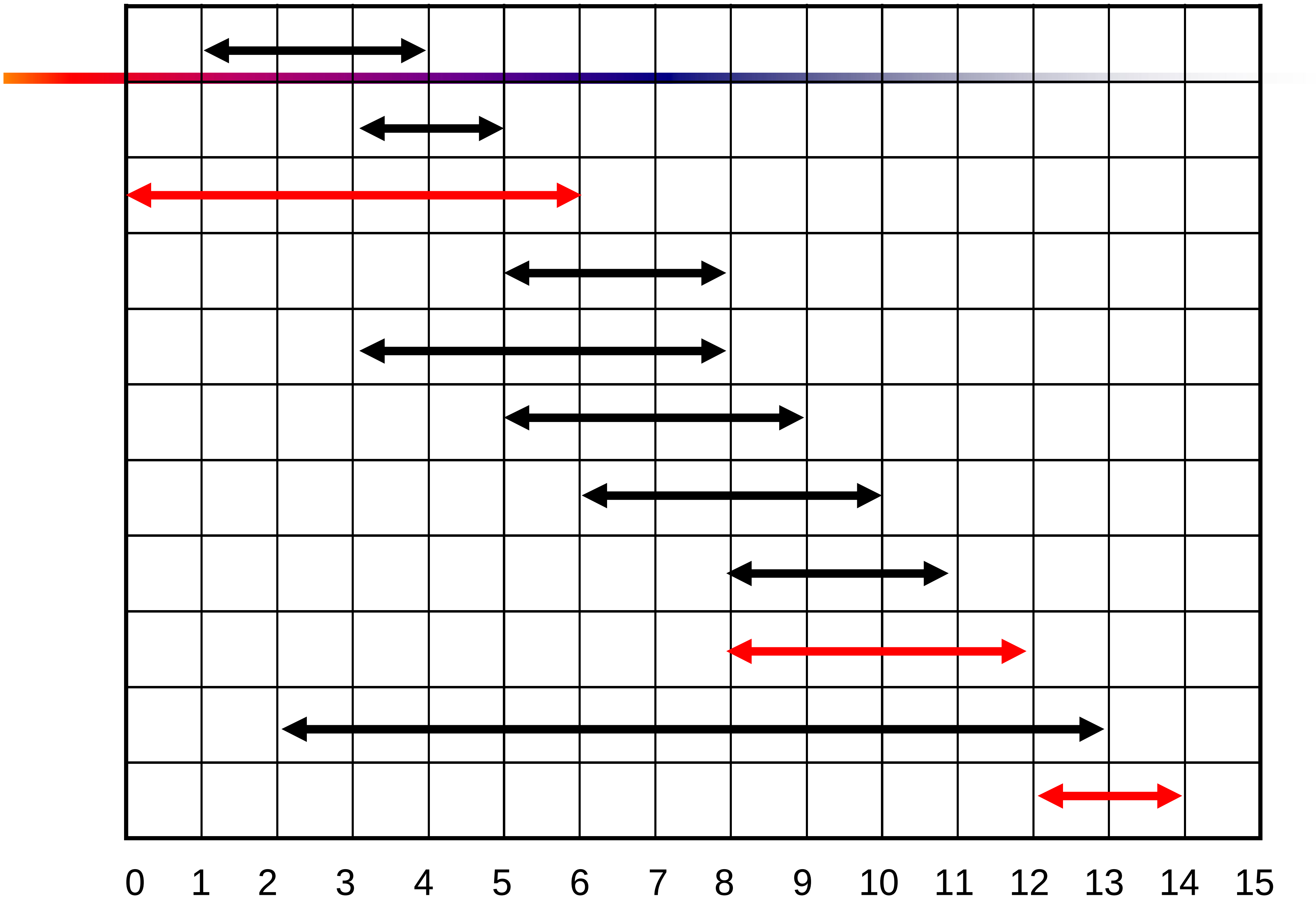

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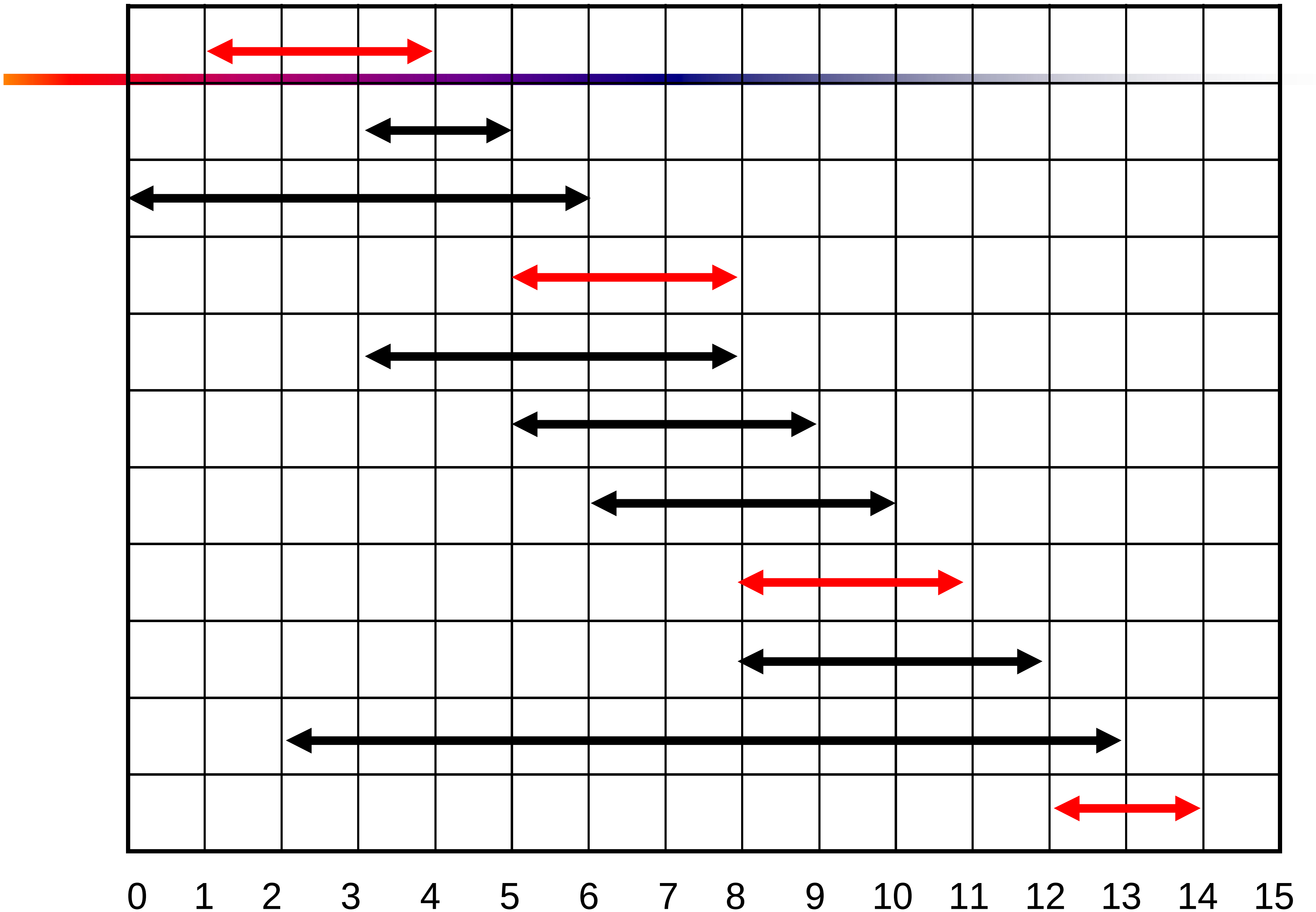
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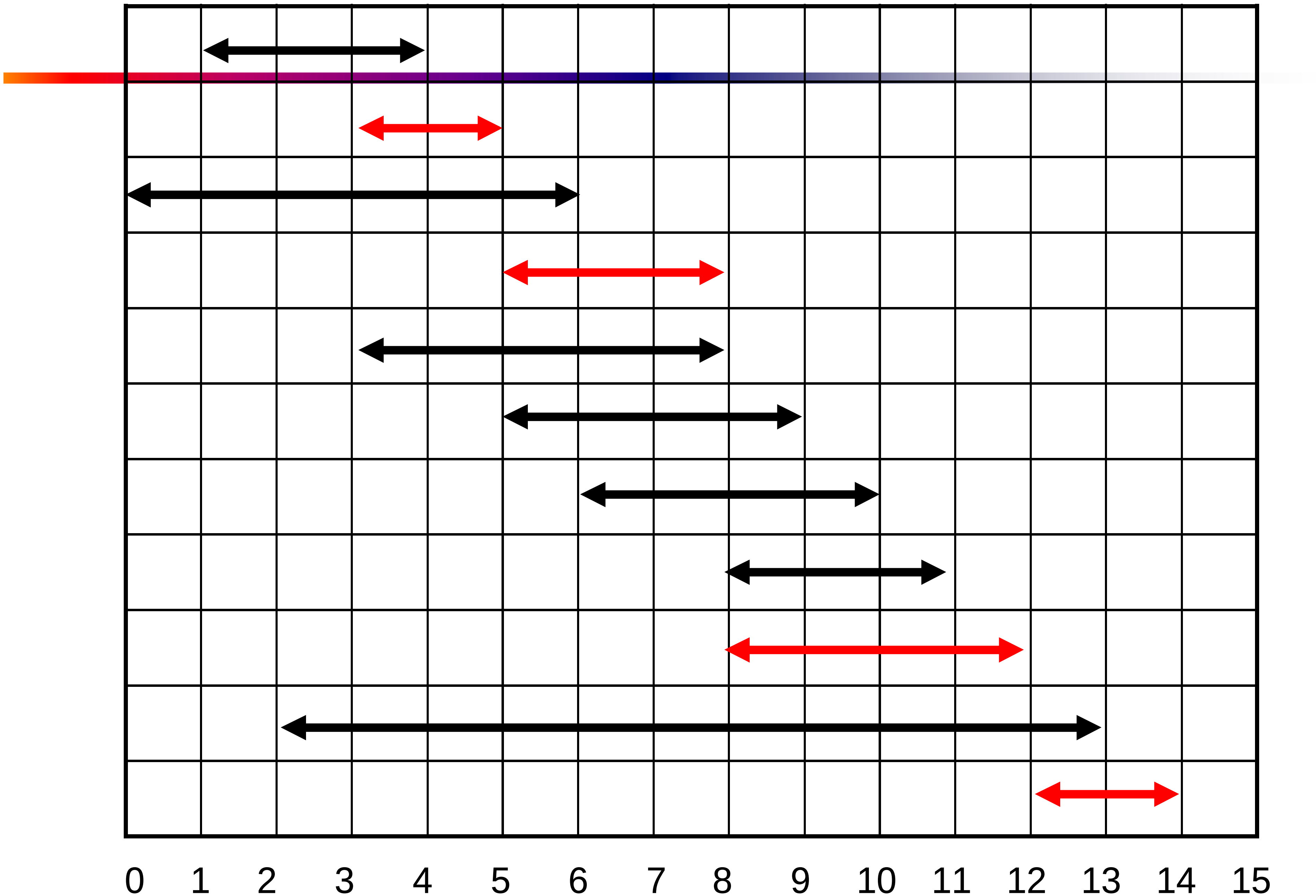

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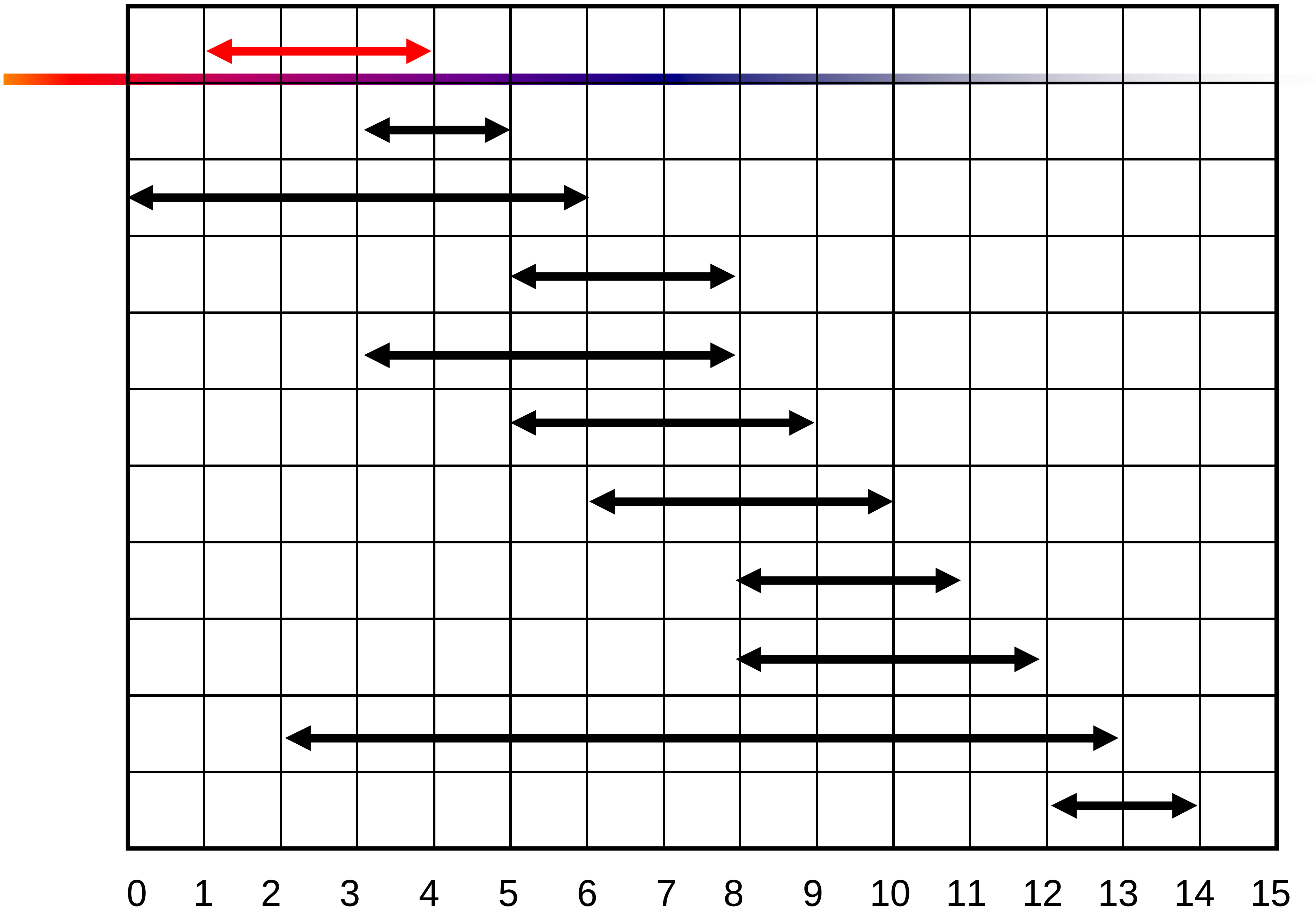
Early Finish Greedy

- Select the activity with the earliest finish
- Eliminate the activities that could not be scheduled
- Repeat!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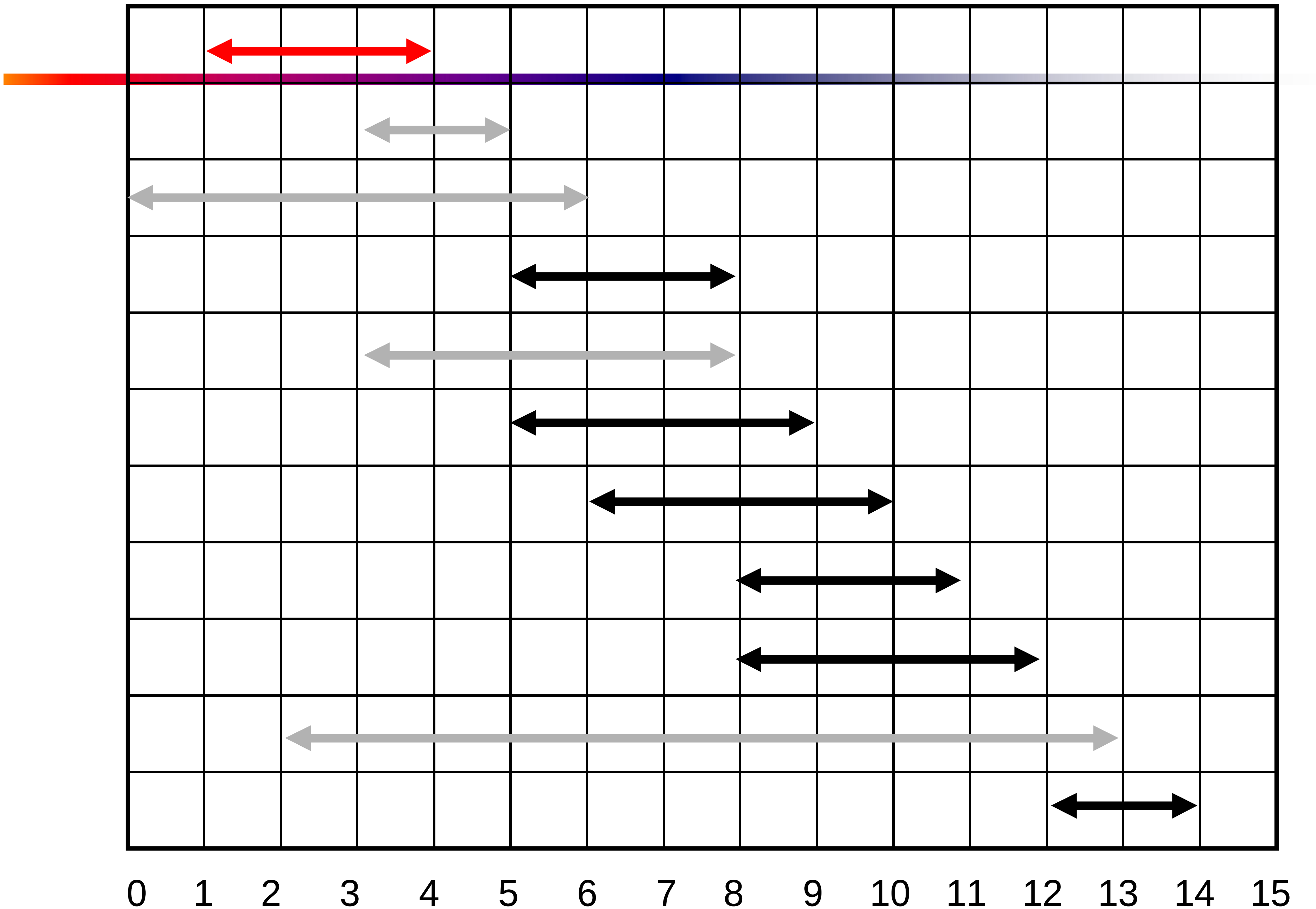
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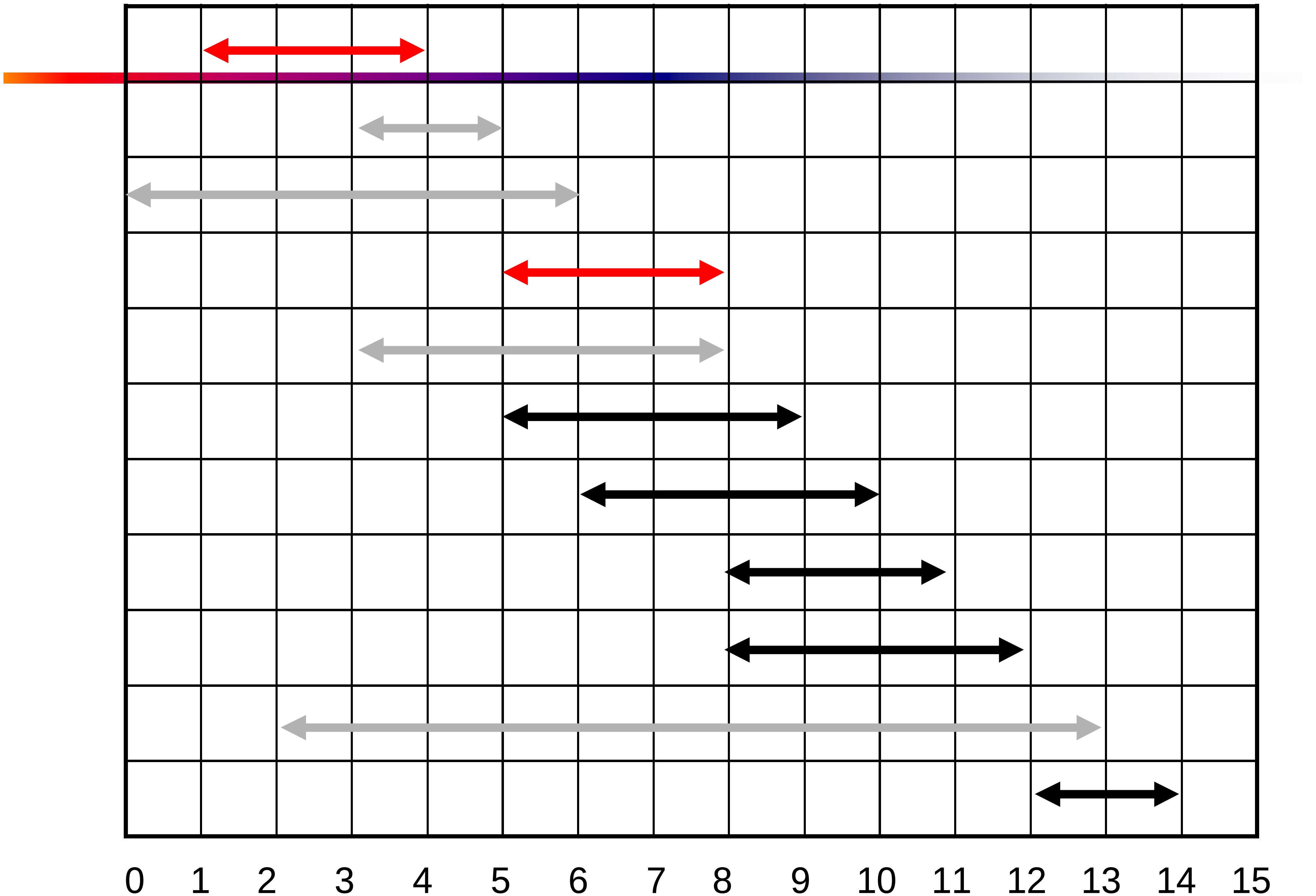

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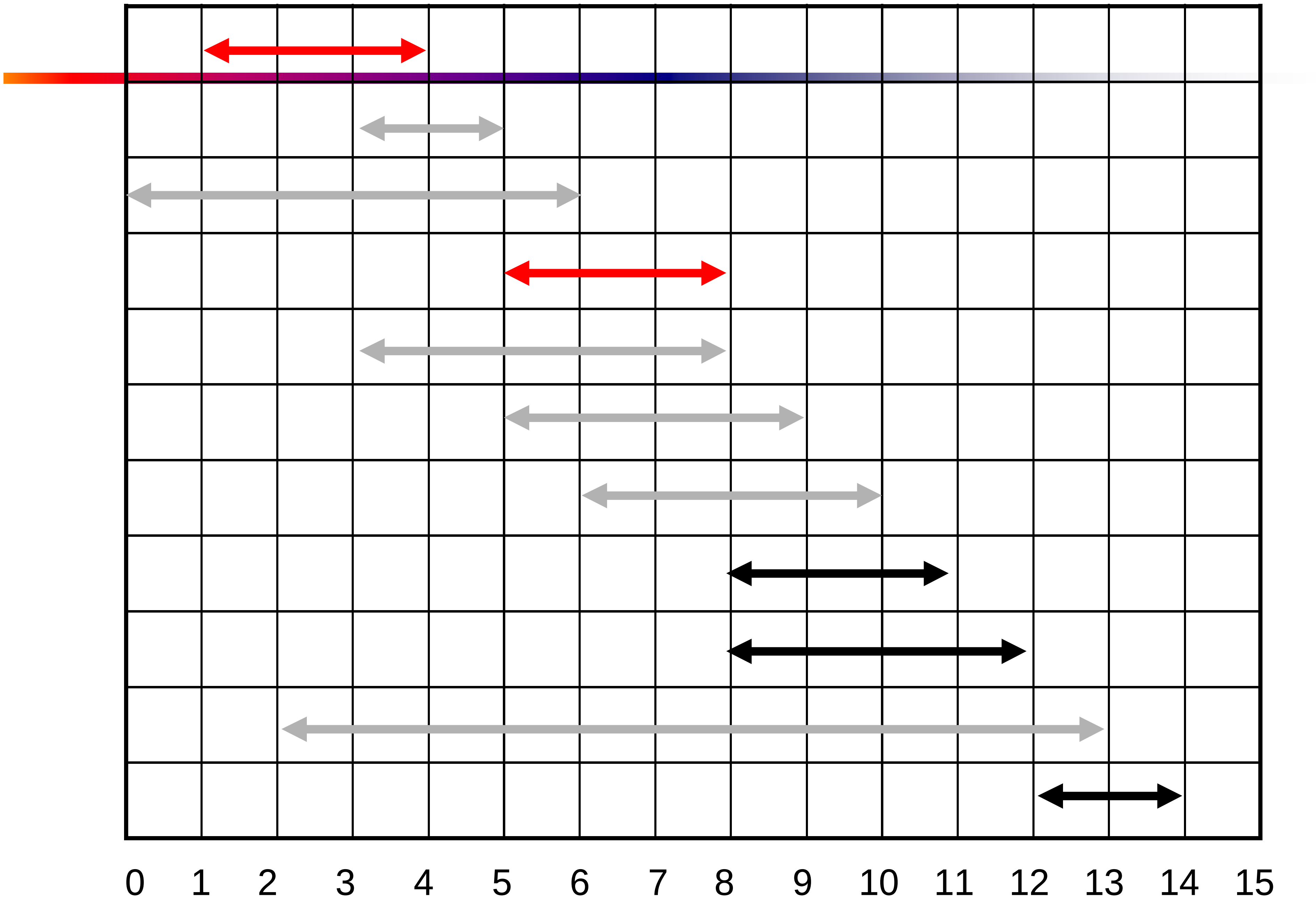
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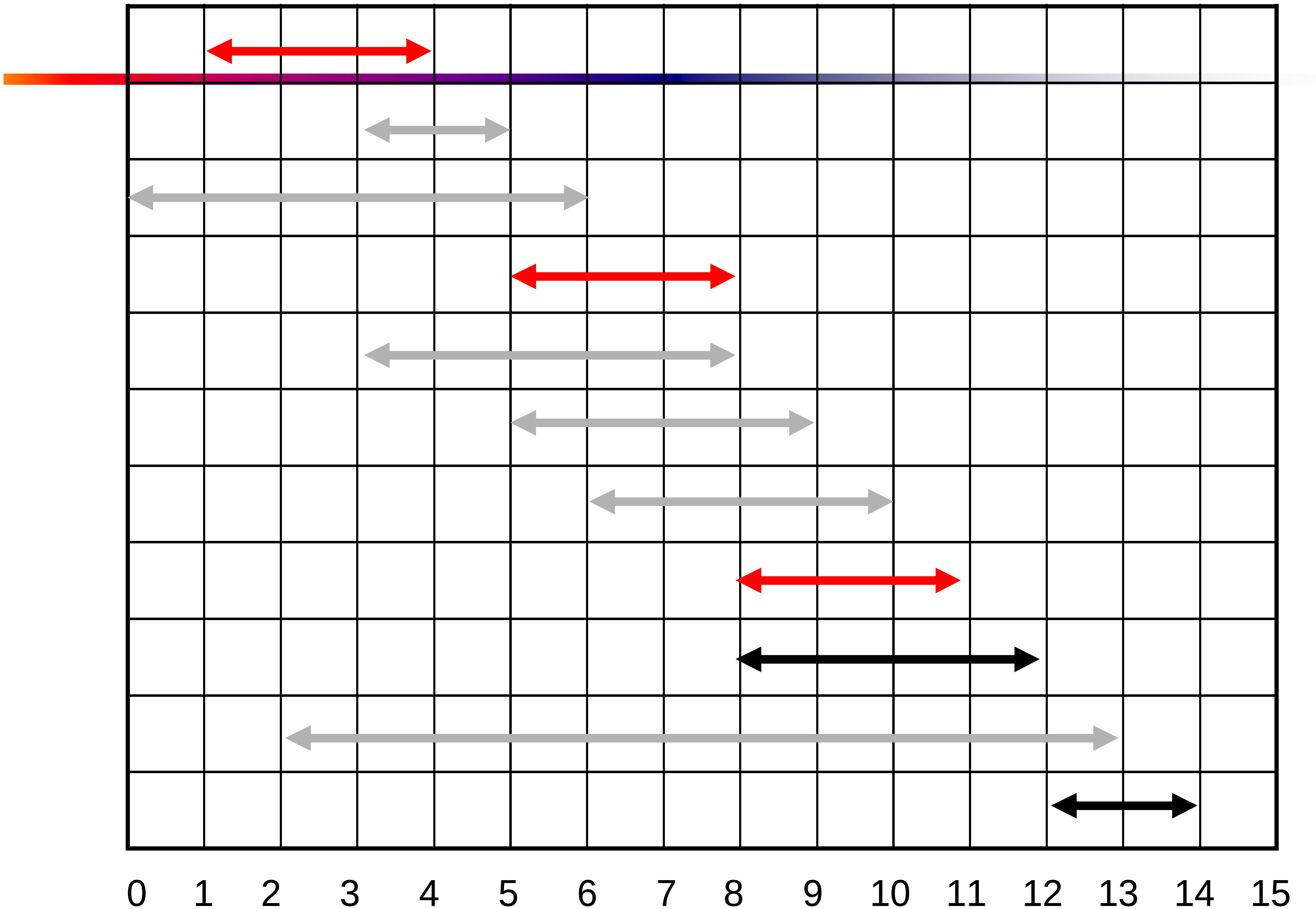
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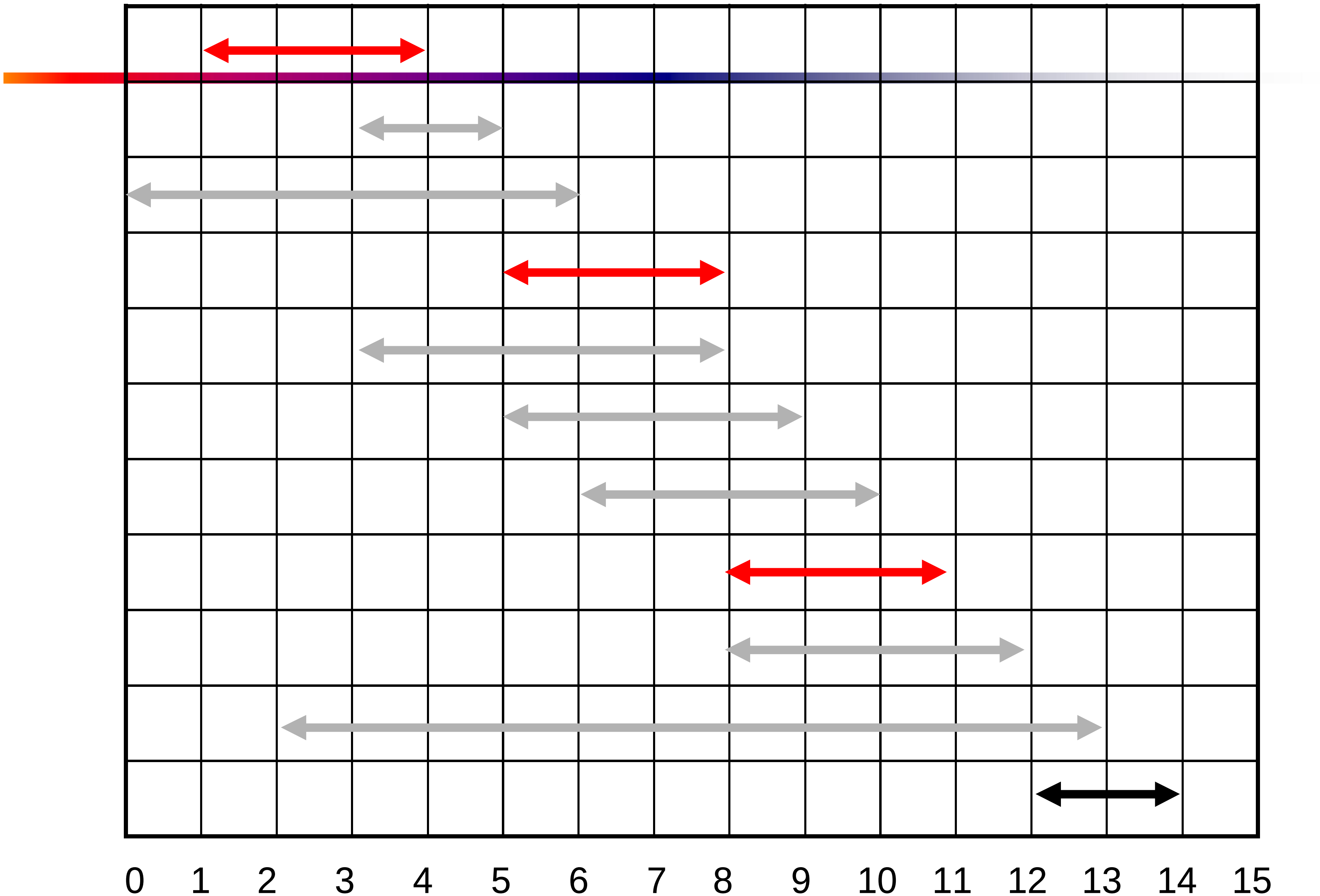
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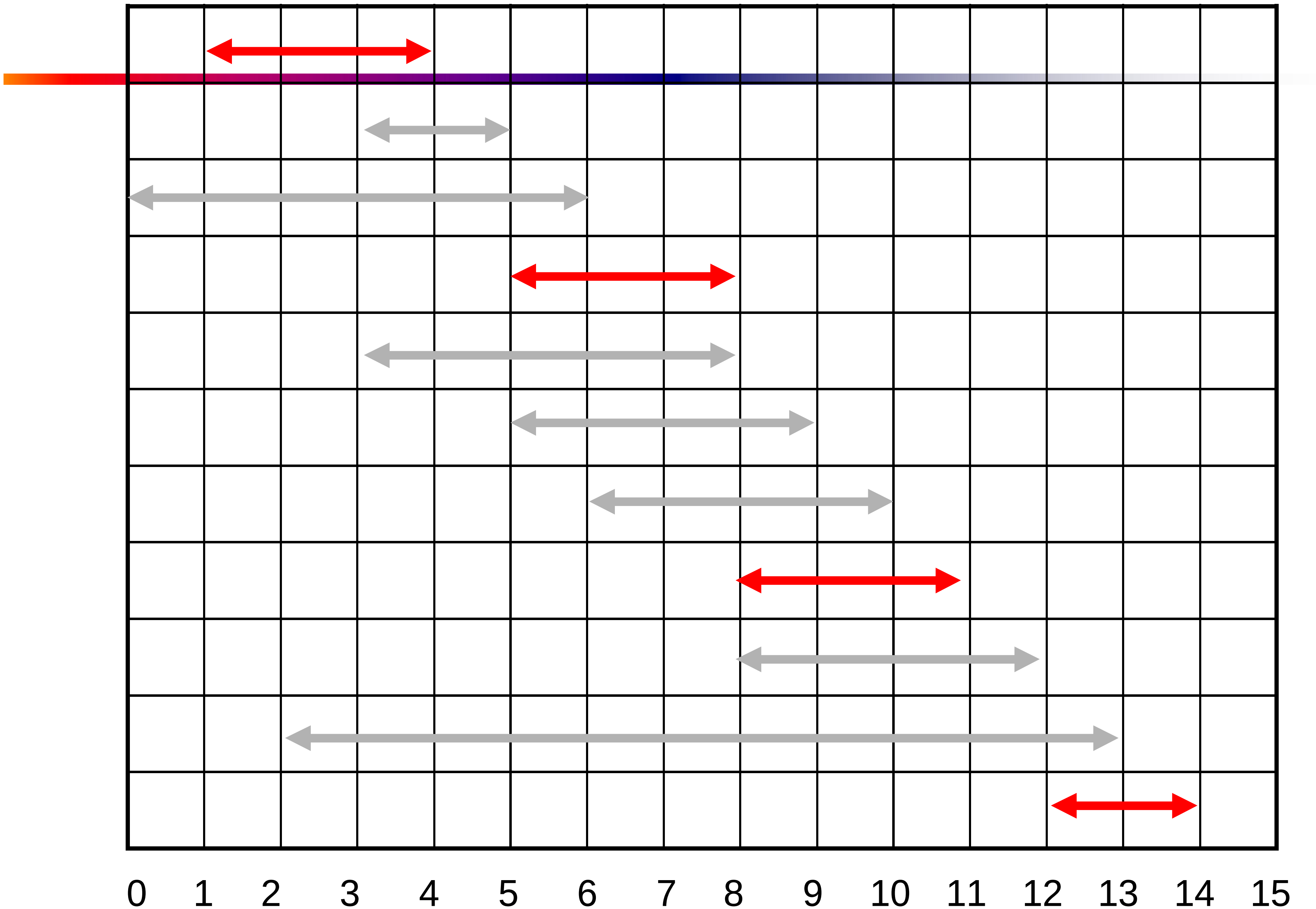

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Assuming activities are sorted by finish time

```
GREEDY-ACTIVITY-SELECTOR(s, f)
n ← length[s]
A ← {a_1}
i ← 1
for m ← 2 to n
    do if s_m ≥ f_i
        then A ← A ∪ {a_m}
             i ← m
return A
```

Why it is Greedy?

- Greedy in the sense that it leaves as much opportunity as possible for the remaining activities to be scheduled
- The greedy choice is the one that maximizes the amount of unscheduled time remaining

- We will show that this algorithm uses the following properties
  - The algorithm satisfies the greedy-choice property
  - The problem has the optimal substructure property

Elements of Greedy Strategy

- A greedy algorithm makes a sequence of choices, each of the choices that seems best at the moment is chosen
  - NOT always produce an optimal solution
- Two ingredients that are exhibited by most problems that lend themselves to a greedy strategy
  - Greedy-choice property
  - Optimal substructure

Greedy-Choice Property

- A globally optimal solution can be arrived at by making a locally optimal (greedy) choice
  - Make whatever choice seems best at the moment and then solve the sub-problem arising after the choice is made
  - The choice made by a greedy algorithm may depend on choices so far, but it cannot depend on any future choices or on the solutions to sub-problems
- Of course, we must prove that a greedy choice at each step yields a globally optimal solution

Optimal Substructures

- A problem exhibits optimal substructure if an optimal solution to the problem contains within it optimal solutions to sub-problems
  - If an optimal solution A to S begins with activity 1, then $A' = A - \{1\}$ is optimal to $S'=\{i \in S: s_i \geq f_1\}$

Greedy-Choice Property

- Show there is an optimal solution that begins with a greedy choice (with activity 1, which as the earliest finish time)
- Suppose $A \subseteq S$ in an optimal solution
  - Order the activities in A by finish time. The first activity in A is k
    - If k = 1, the schedule A begins with a greedy choice
    - If $k \neq 1$, show that there is an optimal solution B to S that begins with the greedy choice, activity 1
  - Let $B = A - \{k\} \cup \{1\}$
    - $f_1 \leq f_k \rightarrow$ activities in B are disjoint (compatible)
    - B has the same number of activities as A
    - Thus, B is optimal

Optimal Substructure Property

- Once the greedy choice of activity 1 is made, the problem reduces to finding an optimal solution for the activity-selection problem over those activities in S that are compatible with activity 1
  - Optimal Substructure
  - If A is optimal to S, then $A' = A - \{1\}$ is optimal to $S' = \{i \in S: s_i \geq f_1\}$
  - Why?
    - If we could find a solution B' to S' with more activities than A', adding activity 1 to B' would yield a solution B to S with more activities than A ➔ contradicting the optimality of A
- After each greedy choice is made, we are left with an optimization problem of the same form as the original problem
  - By induction on the number of choices made, making the greedy choice at every step produces an optimal solution