



Carnegie Mellon University
Tepper School of Business

46-886: Machine Learning Fundamentals

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Recommender Systems: Model-Based Collaborative Filtering – Part 2

Much of this slide deck is derived/borrowed from course material
I've co-taught at MIT



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Jose's predicted rating of movie Q

User	Archetype								
	1	2	3	4	5	6	7	8	9
John
Isabel									
Charles									
Paul									
Rahul
Jose	.2	.3	.1	.9	-.3	-.4	.8	-.8	.7
Caroline
Stephanie									
...

Archetype	Movie																										
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	...
1																...	3
2																...	2
3																...	4
4																...	5
5																...	3
6																...	2
7																...	1
8																...	4
9	3

$$R_{\text{Jose}, Q} = .2 \times 3 + .3 \times 2 + .1 \times 4 + .9 \times 5 - .3 \times 3 - .4 \times 2 + .8 \times 1 - .8 \times 4 + .7 \times 3 = 4.1$$

Jose's observed rating of movie Q

User	Movie																											
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	...	
John			5						1				3			4												...
Isabel														4				4						1		2		...
Charles			3				2				2				5													...
Paul					1				1										4		5							...
Rahul												2													4			...
Jose		5				3											2			2							1	...
Caroline								4							1								4					...
Stephanie	3				2					2	2							3							5			...
...

Residual for model's predicted value for Jose of movie Q is

$$OBSR_{Jose,Q} - 4.1 = 2 - 4.1 = -2.1$$

Least Squares Fitting

- Question: Which objects do we need to determine?
 - $S_{a,m}$: supposed rating of movie m by archetype a
 - $w_{u,a}$: weight of archetype a in the linear combination of user u
- Let us compute these values to minimize the sum of squared residuals, namely the difference between observed and predicted user-movie ratings for all of the user-movie pairs in the observed ratings

$$\text{minimize} \sum_{\substack{\text{all observed} \\ \text{user-movie} \\ \text{pairs } (u, m)}} (\text{OBSR}_{u,m} - (w_{u,1}S_{1,m} + w_{u,2}S_{2,m} + \cdots + w_{u,9}S_{9,m}))^2$$

Summary of Model-Based CF

1. Observed user-movie ratings

User	Movie																											
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	...	
John				5					1				3		4												...	
Isabel													4				4						1		2		...	
Charles			3				2			2					5												...	
Paul					1	<div>OBSR_{u,m}</div>														4		5						...
Rahul												2												4		...		
Jose		5				3										2			2							1	...	
Caroline							4								1							4				...		
Stephanie	3				2					2	2							3						5		...		
...	

2. Archetype-movie ratings

Archetype	Movie									
	Hillbilly Elegy	The Dig	Girls	...	The Midnight Sky	I Care a Lot	Rebecca	Moxie	The King	...
1	3	5	2	5	5	3	3	4	1	...
2	5	4	5	2	3	1	5	4	5	...
...
...
9	2	5	3	5	4	5	1	4	3	...

3. User-archetype weights

User	Archetype								
	1	2	3	4	5	6	7	8	9
John	.7	.12
Isabel									-.3
Charles									1.1
Paul
Rahul
Jose	.2	.3	.1	.9	-.3	-.4	.8	-.8	.7
Caroline
Stephanie
...

4. Least Squares Fitting

minimize

$$\sum_{\text{all observed user-movie pairs } (u,m)} (\text{OBSR}_{u,m} - (w_{u,1}S_{1,m} + w_{u,2}S_{2,m} + \cdots + w_{u,9}S_{9,m}))^2$$

Model-Based Collaborative Filtering

- This modeling approach is an example of “model-based collaborative filtering”
 - collaborative: leverages other users’ ratings to predict unobserved ones
 - filtering: of all the choices, filter the ones that’s best for the individual
- We specify the number of archetypes k , and the model determines the archetypes and the user-archetype weights
 - This approach shares similarities with clustering in the sense that it produces customer archetypes that can be easy to “interpret”
- Active area of research and development