Problem 3

Design HPF using bilinear transformation

Given:

Pass band attenuation $\alpha P - 3dB$

Stop band attenuation $\alpha s = 10dB$

Pass band freq^n $\omega p = 2\pi \times 1000 = 2000\pi$ rad/sec

Stop band freq^n $\omega s = 2\pi \times 350 = 700\pi$ rad/sec

$$T = \frac{1}{f} = \frac{1}{5000} = 2\times10^{-4} \text{ sec}$$

Prewarping freq^n

$$\Omega P = \frac{2}{T} \tan \frac{\omega T}{2} = \frac{2}{2\times10^{-4}} \tan \frac{(2000\pi \times 2\times10^{-4})}{2}$$

$$= 10^4 \tan (0.2\pi) = 7265 \text{ rad/sec}$$

$$\Omega s = \frac{2}{T} \tan \frac{\omega_s t}{2} = \frac{2}{2\times10^{-4}} \tan \frac{(700\pi \times 2 \times 10^{-4})}{2}$$

$$= 10^4 \tan (0.07\pi) = 2235 \text{ rad/sec}$$

The order of filter -

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha s}-1}{10^{0.1\alpha p}-1}}}{\log \frac{\Omega s}{\Omega P}}$$

$$= \frac{\log \sqrt{10^{0.1\times10}-1}}{\log \frac{7265}{2235}}$$

$$= \frac{\log(3)}{\log(3.25)} = \frac{0.4771}{0.5118} = 0.932$$

1st order butter worth filter for $\Omega c - 1$ rad/sec is $H(s) = \frac{1}{s+1}$

High pass filter for $\Omega c = \Omega p = 7265$ rad/sec can be obtained by

$$s \rightarrow \frac{\Omega c}{s}$$

$$s \rightarrow \frac{7265}{s}$$

Transfer the function of high pass filter

$$H(s) = \frac{1}{s+1} \Big|_{s = \frac{7265}{s}}$$

$$\boxed{H(s) = \frac{s}{s+7265}}$$

Using bilinear transformation

$$H(z) = H(s) \Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265} \Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

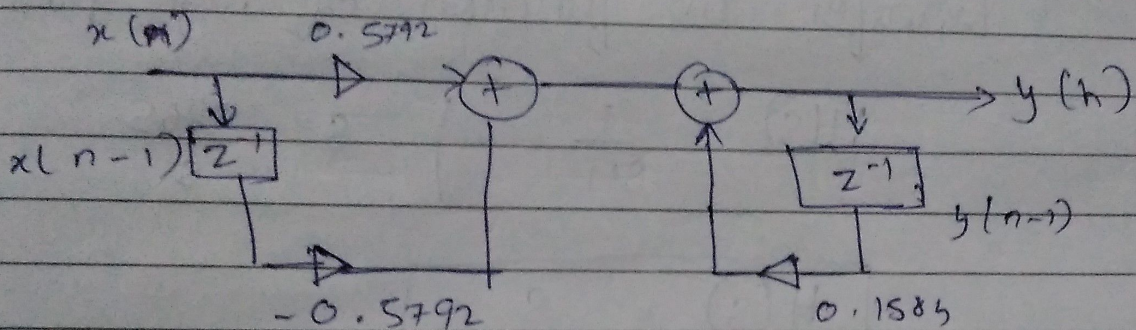
$$= \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$H(z) = \frac{0.5792\,(1-z^{-1})}{1 - 0.1584\,z^{-1}}$$

$$H(z) = \frac{0.5792\,(1-z^{-1})}{1-0.1584z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{0.5792 - 0.5792\,z^{-1}}{1-0.1584z^{-1}}$$



$$y(n) = 0.1584\,y(n-1) - 0.5792\,x(n-1) + 0.5792\,x(n)$$