Problem 3

Design HPF using bilinear transformation

Given:

Pass band attenuation $\alpha_P = 3dB$

Stop band attenuation $\alpha_S = 10dB$

Pass band freq^n $\omega_p = 2\pi \times 1000 = 2000\pi$ rad/sec

Stop band freq^n $\omega_s = 2\pi \times 350 = 700\pi$ rad/sec

$$T = \frac{1}{f} = \frac{1}{5000} = 2\times10^{-4} \text{ sec}$$

Prewarping freq^n

$$\Omega_P = \frac{2}{T}\tan\frac{\omega T}{2} = \frac{2}{2\times10^{-4}}\tan\left(\frac{2000\pi \times 2\times10^{-4}}{2}\right)$$

$$= 10^4 \tan(0.2\pi) = 7265 \text{ rad/sec}$$

$$\Omega_S = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2\times10^{-4}}\tan\left(\frac{700\pi \times 2 \times 10^{-4}}{2}\right)$$

$$= 10^4 \tan(0.07\pi) = 2235 \text{ rad/sec}$$

The order of filter -

$$N \geq \frac{\log\sqrt{\frac{10^{0.1\alpha_S}-1}{10^{0.1\alpha_P}-1}}}{\log\frac{\Omega_S}{\Omega_P}}$$

$$= \frac{\log\sqrt{\frac{10^{0.1\times10}-1}{1}}}{\log\frac{7265}{2235}}$$

$$= \frac{\log(3)}{\log(3.25)} = \frac{0.4771}{0.5118} = 0.932$$

1st order butter worth filter for $\Omega c = 1$ rad/sec is $H(s) = \frac{1}{s+1}$

High pass filter for $\Omega c = \Omega p = 7265$ rad/sec can be obtained by

$$s \rightarrow \frac{\Omega c}{s}$$

$$s \rightarrow \frac{7265}{s}$$

Transfer the function of high pass filter

$$H(s) = \frac{1}{s+1} \Big| \; s = \frac{7265}{s}$$

$$\boxed{H(s) = \frac{s}{s+7265}}$$

Using bilinear transformation

$$H(z) = H(s) \Big| \; s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$= \frac{s}{s+7265} \Big| \; s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

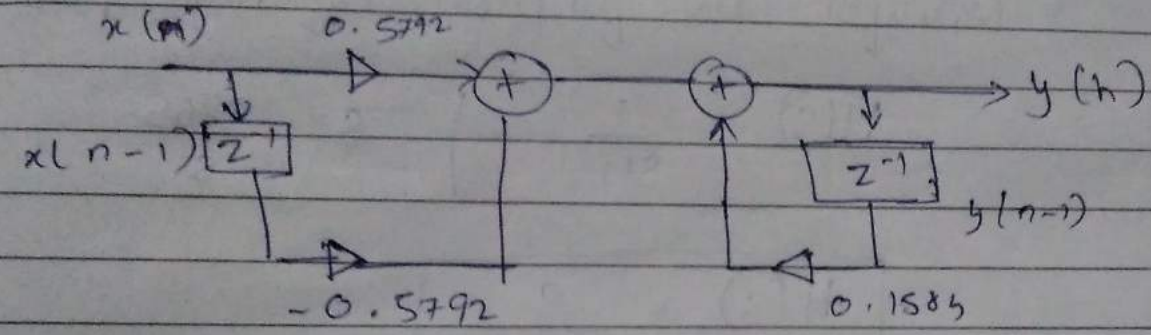
$$= \frac{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$H(z) = \frac{0.5792\,(1-z^{-1})}{1-0.15842\,z^{-1}}$$

$$H(z) = \frac{0.5792\ (1 - z^{-1})}{1 - 0.1584 z^{-1}}$$

$$H(z) = \frac{y(z)}{x(z)}$$

$$= \frac{0.5792 - 0.5792 \cdot z^{-1}}{1 - 0.1584 z^{-1}}$$



$$y(n) = 0.1584\, y(n-1) - 0.5792\, x(n-1) + 0.5792\, x(n)$$