A Brief Overview of Cross-Entropy Method

Initially, cross-entropy method was developed for estimation of rare events probability [1]. However, it was found that it is also appropriate for solving optimization problems.

Algorithm 1 defines a variant of cross-entropy method for optimization.

Algorithm 1 Cross-entropy method for optimization

Input: X – set of elements, $f : X \to \mathbb{R}$ – target function, $u(\cdot, w)$ – probabilistic distribution over X parametrized by vector w.

Output: $\hat{w}$ – approximate solution to the problem $\max_w \mathbb{E}_{x \sim u(\cdot, w)} f(x)$.

Hyperparameters: $w^{(0)}$ – initial value of w; N – number of iterations, n – number of vectors to draw at each iteration, σ – standard deviation for vectors generation, m – number of trials for each vector; ρ – fraction of best vectors to use for update; α – smoothing coefficient of updates.

for all $i \in \{1, \ldots, N\}$ **do**
  for all $j \in \{1, \ldots, n\}$ **do**
    draw $w^{(i,j)} \sim \mathcal{N}(\cdot | w^{(i-1)}, \sigma)$
    $r_j \leftarrow \sum_{k=1}^{m} f(x_k)$ where $x_k \sim u(\cdot, w^{(i,j)})$
  end for
  $r_{\text{threshold}} \leftarrow [\rho n]$-th highest value of $\{r_j : j \in \{1, \ldots, n\}\}$
  $J \leftarrow \{j : r_j \geq r_{\text{threshold}}\}$
  $w^{(i)} \leftarrow \alpha w^{(i-1)} + (1 - \alpha)(\sum_{j \in J} w^{(i,j)})/[\rho n]$
end for
$\hat{w} \leftarrow w^{(N)}$

Sometimes, hyperparameter m is omitted and intermediate results are not aggregated over multiple trials. In case of $u(\cdot, w)$ that acts like a deterministic function of w, m is redundant but, in general case, terminal result can be improved by setting $m > 1$.

More detailed discussion of cross-entropy method can be found in [2].

References

[1] Reuven Y. Rubinstein. Optimization of Computer Simulation Models with Rare Events. *European Journal of Operational Research*, 99(1):89–112, 1997.

[2] P. T. de Boer, D. P. Kroese, S. Mannor, and R. Y. Rubinstein. A Tutorial on the Cross-Entropy Method. *Annals of Operations Research*, 134(1):19–67, 2005.