Physics Problem Set 3

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1 Answers

1. Done reading.
2. Done reading.
3. We know that $V = Q/C$, and n this case when the switch is thrown to the left,

$$V_1 = 16.0V \tag{1}$$

$$C_1 = 4.00\mu F$$

$$Q_1 = V_1 \times C_1 = 64.0\mu C$$

When the switch is thrown to the right, this charge Q_1 is redistributed among the capacitors. We know the following about this system:

$$V_{23} = V_1 \tag{2}$$

$$Q_1 = Q_{1new} + Q_{23} \tag{3}$$

We then find the combined capacitance of C_2 and C_3:

$$C_{2+3} = (\frac{1}{C_2} + \frac{1}{C_3})^{-1} = 2.0\mu F$$

From (2), we can now find the ratio of the new charge distribution:

$$\frac{Q_{1new}}{C_1} = \frac{Q_{23}}{C_{2+3}}$$

$$\therefore Q_{1new} = 2 \times Q_{23}$$

From this, we can now find that

(a) The final charge on capacitor 1:

$$Q_{1new} = \frac{2}{3} \times Q_1$$

$$Q_{1new} = 42.6\mu C$$

(b) Final charge on capacitor 2:

$$Q_2 = \frac{1}{3} \times Q_1$$

$$Q_2 = 21.3\mu C$$

(c) Since capacitor 2 and 3 are in parallel, we know that the magnitude of the charge across both of them must be the same. Hence:

$$Q_3 = 21.3\mu C$$

4. (a) The easiest way to build the resistance network when you know the desired value is to start with a decade-counter setup, whereby you have multiple units of resistance of values 1Ω, 0.1Ω and 0.01Ω. We can achieve this by having 1, 10 and 100 resistances of value 1Ω in parallel, respectively. Once this is done, we can simply combine them in the number we need such that the total resistance equals $1.41k\Omega$. However, in this case, it would require a large number of resistors. We could simply use a resistance ladder in order to make this work, and an extension of this ladder, as we will see in (b), can solve for the square root of 2 to infinity. We have the following network:

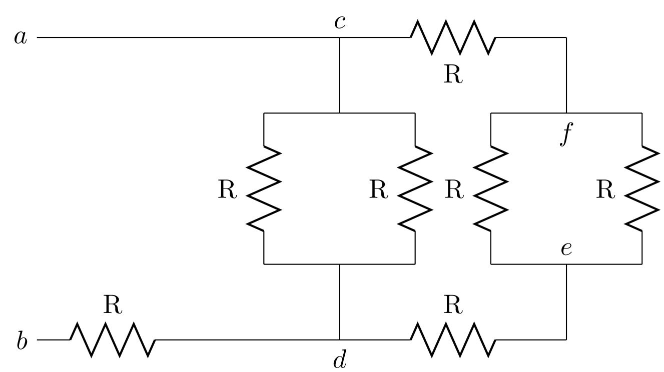


Figure 1: Basic Unit

When $R = 1k\Omega$,

$$R_{fe} = \frac{1}{R} = 500\Omega$$

$$R_{cd} = \frac{2.5k\Omega \times 0.5k\Omega}{2.5k\Omega + 0.5k\Omega} = 0.41\overline{6}k\Omega$$

$$R_{ab} = 1k\Omega + R_{cd} \approx 1.41k\Omega$$

(b) We can simply extend this ladder to find the resistance of $\sqrt{2}$ to any precision we require, as follows:

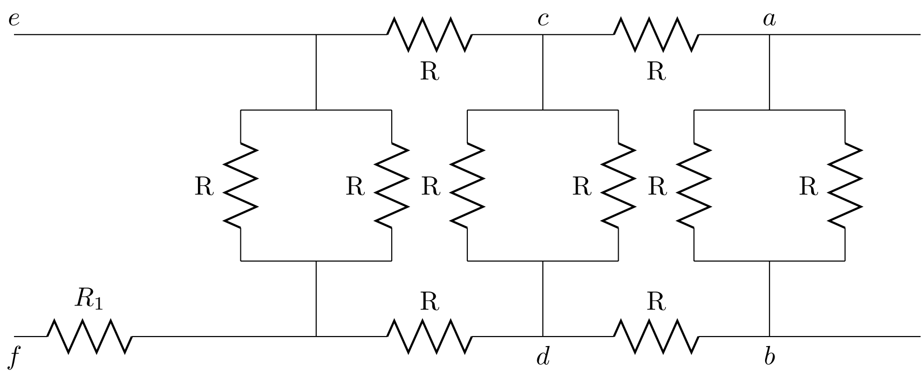


Figure 2: Basic Unit

If we repeate the segment of the circuit between $a - b$, we can adjust the precision to which the $\sqrt{2}$ can be found. Let's prove this: Assuming this ladder to extend to infinity, we can then make this reasonable assumption -

$$R_{cd} = R_{ef} \tag{4}$$

Forgetting the initial resistor R_1, we get the following:

$$R_{ef} = 2 + 0.5||R_{cd}$$

From (4),

$$R_{ef} = 2 + 0.5||R_{ef}$$

Simplifying, we get

$$R_{ef}^2 - 2R_{ef} - 1 = 0$$

Solving,

$$R_{ef} = \sqrt{2} - 1$$

Adding R_1, we get

$$R_{total} = \sqrt{2}$$

5. (a) If we consider effective voltage and capacitance as across R_3:

$$R_{eff} = (R_1 + R_2)||R_3$$

$$R_{eff} = 7.5k\Omega$$

(b)

$$C_{eff} = 5.0\mu F||10.0\mu F$$

$$C_{eff} = \frac{10.0 \times 5.0}{10.0 + 5.0} = 3.\overline{3}\mu F$$

(c) From the RC time constant we know that it takes approximately $0.7RC$ to charge the capacitor to half capacity. If we ignore the current from the charging of one capacitor as affecting the charge time of the other, we find the following:

$$\tau = R_1 \times C_2 = R_2 \times C_1 = 5.0 \times 10^3 \Omega \times 10.0 \times 10^{-6} F$$

$$\tau = 0.05s$$

$$0.7\tau = 0.035s$$

Upon simulating the circuit, we find that this holds true:

Figure 3: Circuit Simulation

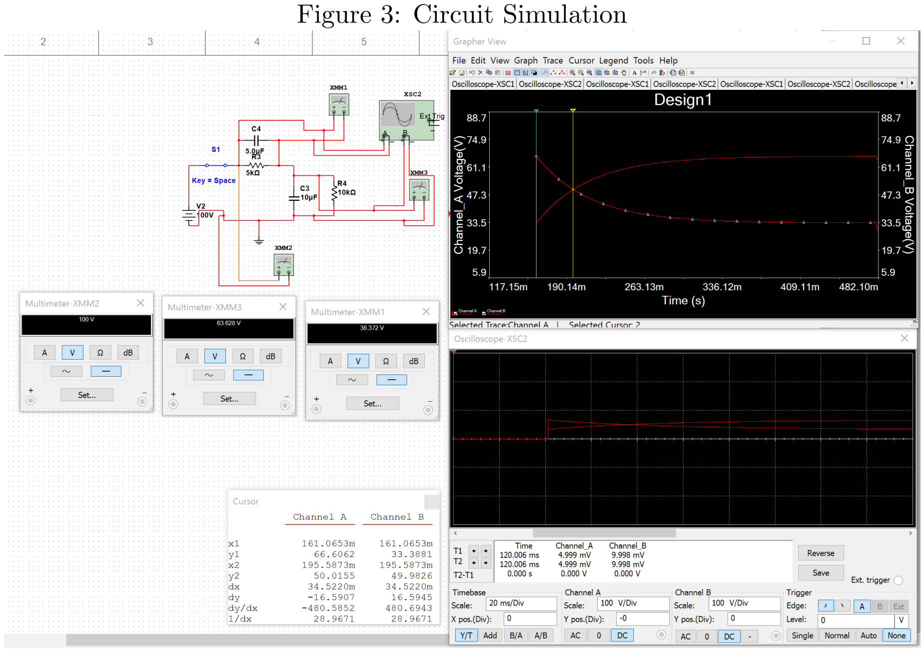


Figure 4: The Circuit

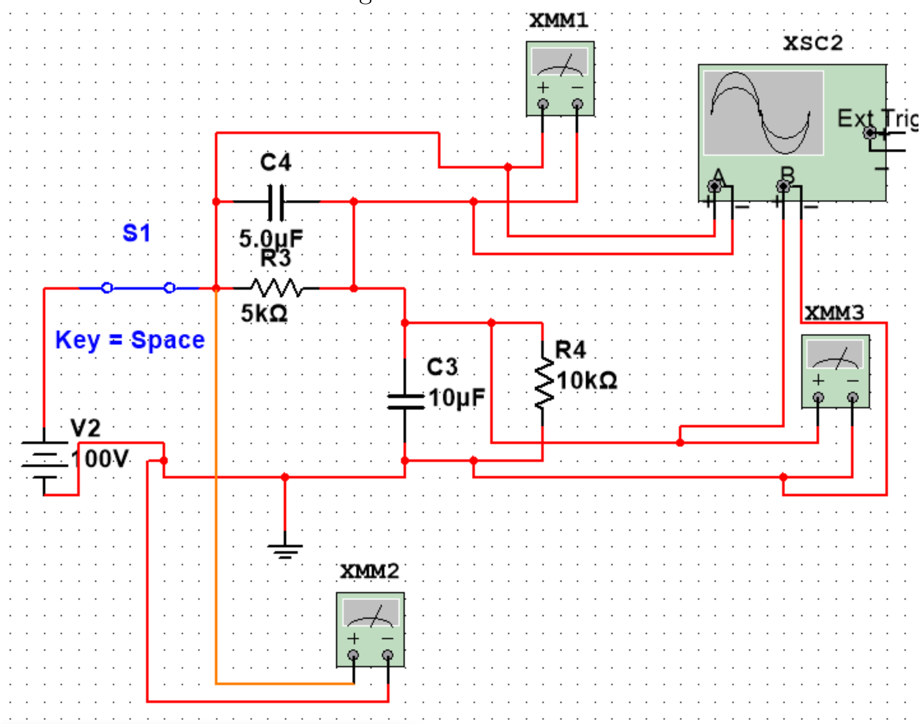


Figure 5: Oscilloscope Reading

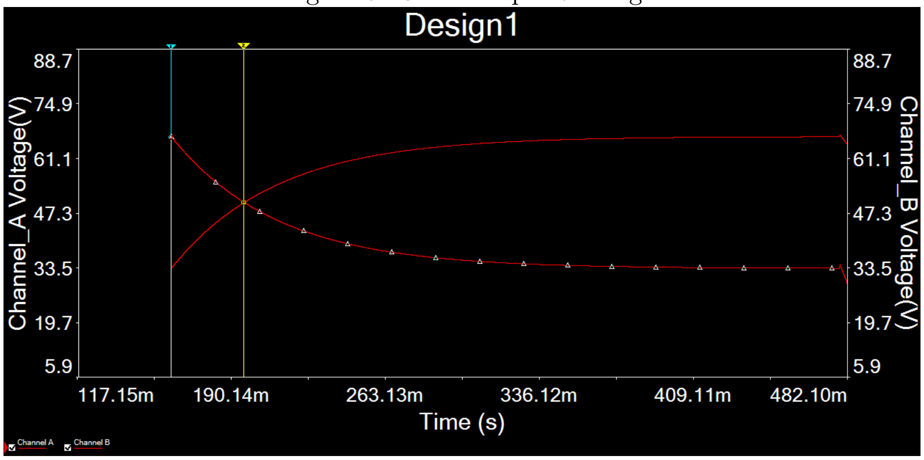


Figure 6: Final Readings

Cursor

	Channel A	Channel B
x1	161.0653m	161.0653m
y1	66.6062	33.3881
x2	195.5873m	195.5873m
y2	50.0155	49.9826
dx	34.5220m	34.5220m
dy	-16.5907	16.5945
dy/dx	-480.5852	480.6943
1/dx	28.9671	28.9671

(d) When discharging, we need to consider R_3 as well:

$$\tau_{C2} = (R_1 + R_3)||R_2 \times C_2 = 6.66k\Omega \times 10.0\mu F$$

$$\tau_{C2} = 0.06s$$

$$0.7\tau_{C2} = 0.04s$$

$$\tau_{C1} = (R_2 + R_3)||R_1 \times C_2 = 6.66k\Omega \times 10.0\mu F$$

$$\tau_{C1} = 0.02s$$

$$0.7\tau_{C2} = 0.015s$$

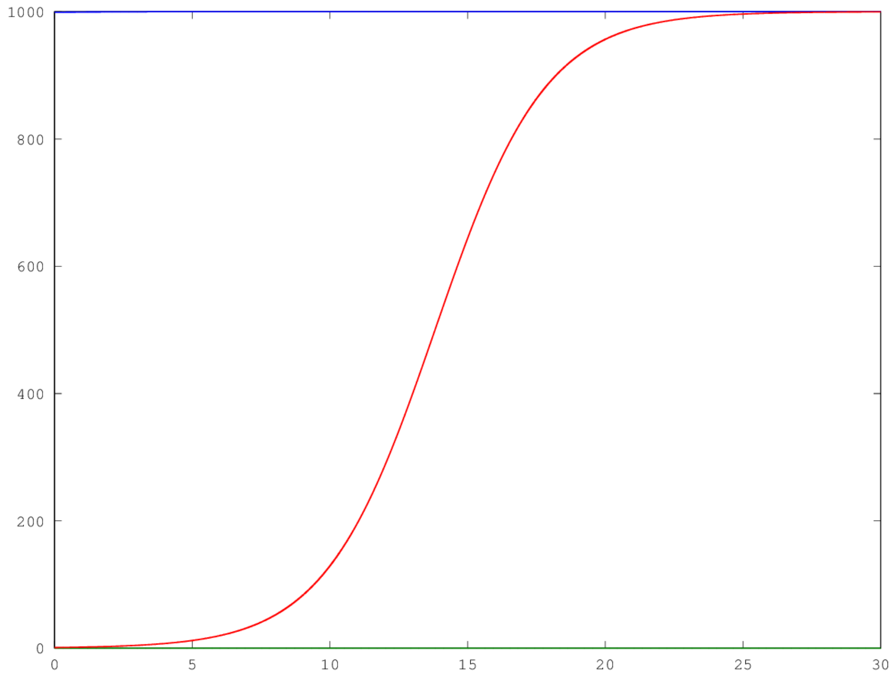
Likewise, simulation holds up the results.

6. Let us consider the equation:

$$\dot{N} = rN(1 - \frac{N}{K})$$

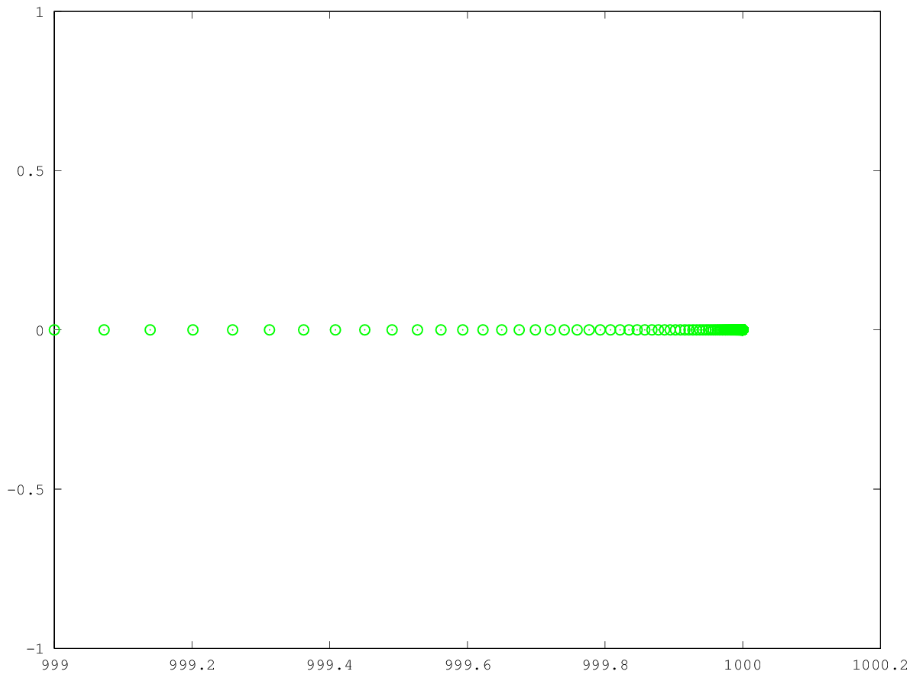
Very quickly, we can simulate this first-order ODE for a few starting values:

Figure 7: Graph of N against T, K = 1000



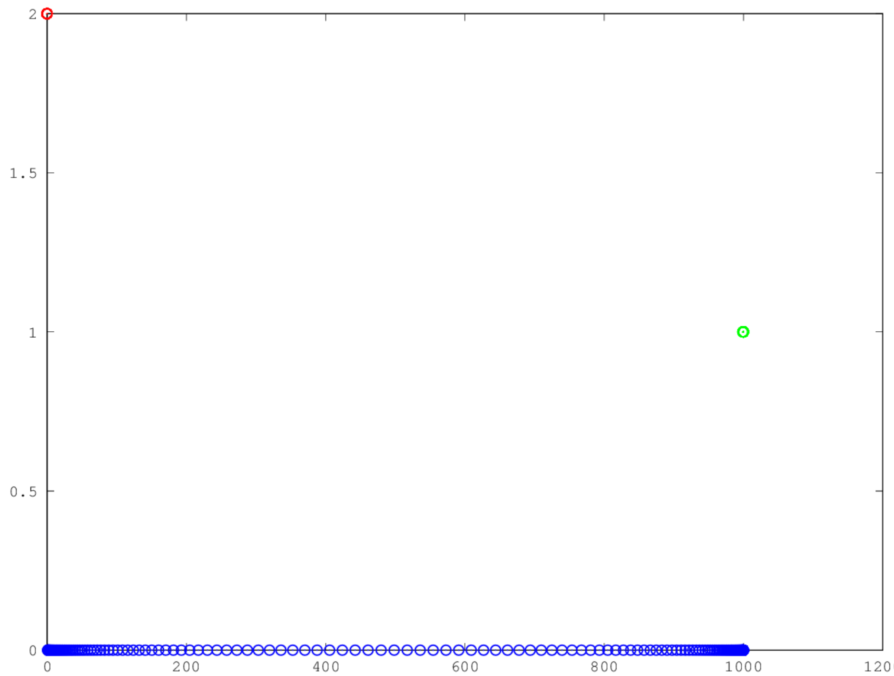
The simulation holds up the conclusions found in Strogatz. N=K is a stable fixed point, where disturbances in either direction will precipitate a return back to it. N=0 is the secondary fixed point, but it is unstable. A negative start value will upset the system whereby it will keep increasing with no fixed point, however when simulating a real populating system N¡0 is impossible. Next we look at the flow on a line plot:

Figure 8: Flow on a Line, $x_0 = 1$, K=1000



Combining the different start values, we see the same result:

Figure 9: Flow on a Line, $x_0 = 0, 1, 1000$, K=1000



From the analysis we can see how a stable fixed point can be added by the addition of the $(1 - \frac{N}{K})$ term. However, it must be noted that even though N=0 is an unstable fixed point in the simulation of the system, it is not so when a real population is considered. A population of zero is has no chance of being altered, if the system is closed.