BUS 310 – Lessons 7 & 8 Notes

Two-Sample Hypothesis Tests

1. Purpose of Two-Sample Tests

Two-sample hypothesis tests are used to **compare two population parameters**, most commonly **two population means**, to determine whether there is a statistically significant difference between them.

2. Types of Two-Sample Hypothesis Tests

Based on Direction of Test

- **Lower-tailed test**
  Tests whether population parameter (1) is **less than** population parameter (2)
- **Upper-tailed test**
  Tests whether population parameter (1) is **greater than** population parameter (2)
- **Two-tailed test**
  Tests whether population parameter (1) is **different from** population parameter (2)

3. Independent vs Paired Samples

Independent Samples

- Two samples are **unrelated**
- Sample sizes may be equal or different
  $n1 = n2$ or $n1 \neq n2$
- Example:
  - Tourists from Japan vs Tourists from the UK
  - NYSE stocks vs NASDAQ stocks

Paired (Related) Samples

- Observations are **matched or repeated**
- Sample sizes are the same
  $n1 = n2 = n$
- Examples:
  - Same students rated **before and after** lectures
  - Product ratings **before and after** a commercial

4. Two-Sample Tests for Means

Case 1: Equal Variances (Pooled t-Test)

Assumptions

1. Both populations are normally distributed
   (or total sample size n1+n2 ≥ 30)
2. Population variances are equal

Hypotheses

H0: μ1−μ2 = 0

H1: μ1−μ2 ≠ 0

Test Statistic

$$t = \frac{\overline{X_1} - \overline{X_2}}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where pooled variance is:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Degrees of Freedom

$$df = n_1 + n_2 - 2$$

Case 2: Unequal Variances (Separate Variance t-Test)

- Used when:
  - Population variances are **not equal**
  - Sample sizes are small and unequal

Test Statistic

$$t = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

- Degrees of freedom are **adjusted** (Satterthwaite approximation)

5. Paired Sample t-Test

When to Use

- Same individuals measured **twice**
- Focus is on **mean of differences**

Define Differences

D = After−Before

Hypotheses

H0: $\mu D \leq 0$

H1: $\mu D > 0$

Test Statistic

$$t = \frac{\bar{D}}{s_D/\sqrt{n}}$$

Degrees of Freedom

df = n−1

6. Decision Rules

Critical Value Approach

- Reject H0 if test statistic falls in rejection region

p-Value Approach

- Reject H0 if:

$$\text{p-value} \leq \alpha$$

7. Interpreting Excel Output (Key Rules)

- If **test statistic ≥ 0**:
  - Upper-tailed test → p-value = one-tailed p-value
  - Lower-tailed test → p-value = 1− one-tailed p-value
- If **test statistic < 0**:
  - Lower-tailed test → p-value = one-tailed p-value
  - Upper-tailed test → p-value = 1− one-tailed p-value

8. Summary: Types of Tests

1. One-Sample Mean Test
   - Z-test (σ known)
   - t-test (σ unknown)
2. One-Sample Proportion Test
   - Z-test
3. Two-Sample Mean Tests
   - Pooled t-test (equal variances)
   - Separate variance t-test (unequal variances)
4. Paired Two-Sample t-Test