BUS 310 – Lesson 2 Notes

Confidence Interval Estimation

1. Introduction to Confidence Intervals

In statistics, a **confidence interval** is used to estimate an unknown population parameter by providing a **range of plausible values** rather than a single number. This interval is constructed using sample data and reflects the uncertainty inherent in sampling.

A confidence interval is centered around a **point estimate** and extends outward by a **margin of error**.

2. Point Estimates vs. Interval Estimates

Point Estimate

- A **single numerical value** used to estimate a population parameter.
- Examples:
    - Sample mean ($\bar{x}$)
    - Sample proportion (p)

Interval Estimate

- A **range of values** that is likely to contain the true population parameter.
- Provides more information than a point estimate because it shows **variability and uncertainty**.

3. General Form of a Confidence Interval

All confidence intervals follow this general structure:

Point Estimate ± (Critical Value × Standard Error)

Where:

- **Point Estimate**: Sample statistic estimating the population parameter
- **Critical Value**: Value from a probability distribution based on the chosen confidence level

- **Standard Error**: Measures variability of the point estimate

The quantity *(Critical Value × Standard Error)* is called the **margin of error**.

4. Confidence Level

The **confidence level** represents how confident we are that the interval contains the true population parameter.

- Common confidence levels: **90%, 95%, 99%**
- Denoted as $\mathbf{1 - \alpha}$
- Example interpretation:
    - A 95% confidence level means that **95% of similarly constructed intervals would contain the true parameter**

5. Confidence Interval for the Population Mean (σ Known)

When the **population standard deviation (σ) is known**, the confidence interval for the population mean (μ) is:

$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Where:

- $\bar{x}$ = sample mean
- σ = population standard deviation
- n = sample size
- $Z_{\alpha/2}$ = critical value from the standard normal distribution

6. Interpreting a Confidence Interval

A confidence interval does **not** state that the probability the parameter lies in the interval is 95%. Instead, it means:

If we repeatedly take samples and construct confidence intervals in this way, **95% of those intervals will contain the true population mean**.

7. Confidence Interval for the Population Mean (σ Unknown)

When the population standard deviation **σ is unknown**, it is replaced by the **sample standard deviation (S)**.

This introduces additional uncertainty, so we use the **t-distribution** instead of the normal distribution.

The confidence interval becomes:

$$\bar{X} \pm t_{\alpha/2,n-1} \frac{s}{\sqrt{n}}$$

Where:

- $t_{\alpha/2,n-1}$ = critical value from the t-distribution
- Degrees of freedom (df) = n − 1

8. The t-Distribution

- The **t-distribution** is a family of distributions similar to the standard normal distribution
- It depends on **degrees of freedom**
- As sample size increases, the t-distribution **approaches the standard normal distribution**

9. Confidence Interval for a Population Proportion

For estimating a population proportion (π), the confidence interval is:

$$p \pm \left(Z_{\alpha/2}\right)\sqrt{\frac{p(1-p)}{n}}$$

Where:

- p = sample proportion
- n = sample size

10. Properties of Confidence Intervals

- Higher confidence level → **wider interval**
- Lower confidence level → **narrower interval**
- Increasing sample size → **narrower interval**
- There is a trade-off between **confidence** and **precision**

11. Key Takeaways

- Confidence intervals provide more information than point estimates
- Choice of distribution depends on whether σ is known
- Larger samples improve precision
- Interpretation of confidence intervals is critical for exams