BUS 310 – Lesson 9 Notes

One-Factor Analysis of Variance (ANOVA)

1. What is One-Factor ANOVA?

One-Factor ANOVA is a hypothesis-testing procedure used to compare the mean responses of **three or more populations** (groups) using a single categorical factor.

Its goal is to determine whether **at least one population mean differs** from the others.

2. When Do We Use ANOVA Instead of t-Tests?

- When comparing **3 or more means**
- Avoids multiple t-tests, which increase **Type I error**
- Uses the **F-test** to compare variability **between groups** to variability **within groups**

3. ANOVA Assumptions

1. Populations are **normally distributed**
2. Population variances are **equal**
3. Samples are **independent**
4. The F-test is **robust** to moderate departures from normality
   (especially when sample sizes are equal)

4. Hypotheses in One-Way ANOVA

Null Hypothesis

$H0: \mu1 = \mu2 = \cdots = \mu c$

(All population means are equal)

Alternative Hypothesis

H1: At least two population means differ

This does NOT mean all means are different.

5. Partition of Total Variation

ANOVA divides total variability into:

SST = SSB + SSW

- **SST** – Total Sum of Squares
- **SSB** – Sum of Squares Between Groups
- **SSW** – Sum of Squares Within Groups (Error)

6. Mean Squares

$$\text{MSB} = \frac{\text{SSB}}{df_b}$$

$$\text{MSW} = \frac{\text{SSW}}{df_w}$$

Degrees of freedom:

$$df_b = c - 1$$
$$df_w = n - c$$

where

- c = number of groups
- n = total sample size

7. F Test Statistic

$$\boldsymbol{F = \frac{\text{MSB}}{\text{MSW}}}$$

- Large F ⇒ more variation between groups than within groups
- Indicates evidence against H0

8. Decision Rules

Critical-Value Approach

- Reject H0 if:

  $F > F_{critical}$

p-Value Approach

- Reject H0 if:

  $p\text{-value} < \alpha$

9. Interpretation of Rejection

If H0 is rejected:

There is sufficient evidence that **at least one population mean differs** from the others.

ANOVA **does not identify which means differ** further tests are required.

10. Example (Machines – From Slides)

- $\alpha = 0.05$
- $c = 3$
- $df1 = 2, df2 = 12$
- $F = 25.6$
- $p < \alpha$

Decision: Reject H0
Conclusion: At least one machine has a different mean filling time.

11. What ANOVA Does NOT Tell Us

- It does **not** say which groups differ
- It does **not** measure size of difference
- Requires **post-hoc tests** for pairwise comparisons