

BUS 310 – Lesson 5 & 6 Notes

Hypothesis Testing

1. What is Hypothesis Testing?

Hypothesis testing is a statistical procedure that uses sample data to evaluate a claim about a population parameter. The goal is to determine whether there is enough evidence to support a new claim against the current belief.

2. Types of Hypotheses

Null Hypothesis (H_0)

- Represents the **status quo** or existing belief
- Always stated about a **population parameter**, not a sample statistic
- Always contains $=, \leq, \text{ or } \geq$
- May or may not be rejected

Examples:

- $H_0: \mu = 30$
- $H_0: \pi \leq 0.5$

Alternative Hypothesis (H_1 or H_a)

- Represents the claim the researcher wants to prove
- Challenges the status quo
- **Never contains $=, \leq, \text{ or } \geq$**
- May or may not be proven

Examples:

- $H_1: \mu \neq 30$
- $H_1: \pi > 0.5$

3. One-Sample Hypothesis Tests

There are **three types** of one-sample tests:

1. Right-tailed (Upper-tailed) Test

$H_0: \mu \leq \mu_0$ vs $H_1: \mu > \mu_0$

2. Left-tailed (Lower-tailed) Test

$H_0: \mu \geq \mu_0$ vs $H_1: \mu < \mu_0$

3. Two-tailed Test

$H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$

4. Level of Significance (α)

- α is the **maximum probability of making a Type I error**
- Common values: **0.10, 0.05, 0.01**
- Chosen **before** conducting the test

5. Errors in Hypothesis Testing

Type I Error

- Rejecting a **true** null hypothesis
- Probability = α

Type II Error

- Failing to reject a **false** null hypothesis
- Probability = β

6. Test Statistic Formulas

Population Mean (σ known) — Z Test

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Population Mean (σ unknown) — t Test

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}, \quad df = n - 1$$

Population Proportion

$$Z = \frac{p - \pi}{\sqrt{\frac{\pi(1 - \pi)}{n}}}$$

7. P-value

The p-value is the probability of obtaining a test statistic at least as extreme as the one observed, assuming the null hypothesis is true. It measures the strength of evidence against the null hypothesis.

8. Critical Value & Rejection Region

- The **critical value** defines the cutoff between rejection and non-rejection
- Determined by:
 - α
 - Type of test (left, right, two-tailed)
 - Distribution used (Z or t)

8. Decision Rules

Critical Value Approach

- Reject H_0 if test statistic falls in rejection region

p-Value Approach

- Reject H_0 if:

$$p\text{-value} \leq \alpha$$

- Fail to reject H_0 if:

$$p\text{-value} > \alpha$$

9. Final Decision & Conclusion

- **Reject H₀:**
“There is enough evidence to support the alternative hypothesis.”
- **Fail to reject H₀:**
“There is not enough evidence to support the alternative hypothesis.”

10. General Hypothesis Testing Procedure

1. State H₀ and H₁
2. Choose significance level α
3. Select test statistic (Z or t)
4. Determine critical value or p-value
5. Make a decision
6. State conclusion in words