BUS 310 – Lesson 3 Notes

Confidence Interval Estimation (Population Proportion)

1. Confidence Interval: Concept

A confidence interval is an interval of values, calculated from sample data, that is likely to contain the true value of a population parameter.
It is constructed around a **point estimate** and includes a **margin of error** that reflects sampling uncertainty.

2. Point Estimate vs Interval Estimate

- A **point estimate** is a single value used to estimate a population parameter (e.g., sample proportion p).
- A **confidence interval** provides a **range of plausible values** for the parameter and gives more information about variability than a point estimate alone.

3. General Confidence Interval Formula

All confidence intervals follow the same general structure:

$$\text{Point Estimate} \pm (\text{Critical Value})(\text{Standard Error})$$

- **Point Estimate**: Sample statistic estimating the population parameter
- **Critical Value**: Based on the confidence level and sampling distribution
- **Standard Error**: Measures variability of the point estimate

The **margin of error** equals:

$$(\text{Critical Value}) \times (\text{Standard Error})$$

4. Confidence Level

The **confidence level**, denoted by $1-\alpha$, represents how confident we are that the interval contains the true population parameter.

Common confidence levels:

- 90%
- 95%
- 99%

Interpretation:
If many samples are taken and intervals constructed in the same way, **$1-\alpha$% of those intervals will contain the true parameter**.

5. Effect of Confidence Level on Interval Width

- Higher confidence level → **wider interval**
- Lower confidence level → **narrower interval**

There is a **trade-off**:

- Higher confidence gives less precision (wider intervals)
- Lower confidence gives more precision (narrower intervals)

Increasing **sample size** can reduce interval width without lowering confidence.

6. Confidence Interval for a Population Proportion

To estimate a population proportion p, we use the sample proportion p.

Formula

$$\mathrm{p} \pm Z_{\alpha/2}\sqrt{\frac{\mathrm{p}(1-\mathrm{p})}{n}}$$

Where:

- p = sample proportion
- $Z\alpha/2$ = critical value from the standard normal distribution
- n= sample size

7. Conditions for Using the Proportion Interval

The normal approximation is valid when:

- $np>5$
- $n(1-p)>5$

These conditions ensure the sampling distribution of p is approximately normal.

8. Interpreting a Confidence Interval for a Proportion

A 95% confidence interval for a population proportion means:

We are 95% confident that the true population proportion lies between the lower and upper bounds of the interval.

This does not mean there is a 95% probability that the specific interval contains the parameter; the confidence refers to the long-run performance of the method.

9. Using Confidence Intervals for Decision Making

Confidence intervals can be used to:

- Predict election outcomes
- Assess whether a population proportion exceeds or falls below a benchmark (e.g., 50%)

If the entire interval lies above or below a threshold, a clear conclusion can be made. If the interval includes the threshold, conclusions are uncertain.