BUS 310 – Lesson 4 Notes

Sample Size Questions: Finding n

1. Purpose of Sample Size Determination

In statistics, we often want to estimate a population parameter with a desired level of accuracy. The goal of sample size determination is to find the minimum number of observations (**n**) needed to achieve a specified **margin of error** at a given **confidence level**.

2. Sampling Error (Margin of Error)

- Sampling error, denoted by **e**, represents the amount of imprecision in an estimate.
- It is the value added to and subtracted from the point estimate to form a confidence interval.
- Smaller margins of error require **larger sample sizes**.
- Sample size depends on:
    - Confidence level $(1-\alpha)$
    - Margin of error e
    - Population variability (σ or π)

3. Determining Sample Size for the Mean (σ Known)

Margin of Error Formula

$$\boldsymbol{e} = \boldsymbol{Z}_{\alpha/2} \cdot \frac{\sigma}{\sqrt{\boldsymbol{n}}}$$

Where:

- $Z_{\alpha/2}$= critical value from the standard normal distribution
- σ = population standard deviation
- n = sample size

Solving for Sample Size

- Rearranging the formula to solve for n:

$$n = \frac{Z_{\alpha/2}^2 \sigma^2}{e^2}$$

Information Needed

To determine sample size for the mean, you must know:

- Desired confidence level (1−α)
- Acceptable margin of error e
- Population standard deviation σ

Important Rule

- **Always round up** the calculated sample size.

Example (Mean)

If:

- σ=45
- e=5
- Confidence level $= 90\%$ $\left(Z_{\alpha/2} = Z_{0.05} = 1.645\right)$

$$n = \frac{Z_{\alpha/2}^2 \sigma^2}{e^2}$$

$$n = \frac{(1.645)^2 (45)^2}{5^2}$$

$$n = \frac{(2.705)(2025)}{25}$$

$$n = 220$$

If σ Is Unknown

- Use an estimate of σ that is at least as large as the true σ.
- Alternatively, use a **pilot sample** and estimate σ using the sample standard deviation s.

4. Determining Sample Size for a Proportion

Margin of Error Formula

$$\boldsymbol{e = Z_{\alpha/2} \cdot \sqrt{\frac{\pi(1-\pi)}{n}}}$$

Where:

- π = true population proportion

Solving for Sample Size

$$n = \frac{Z_{\alpha/2}^2\, \boldsymbol{\pi}(1-\boldsymbol{\pi})}{e^2}$$

Information Needed

To determine sample size for a proportion, you must know:

- Desired confidence level ($1-\alpha$)
- Acceptable margin of error e
- Population proportion π

If π is unknown:

- Estimate it using a pilot sample, or
- Use $\pi=0.5$ (most conservative, yields largest sample size)

Example (Proportion)

If:

- $\text{Confidence level} = 95\% \quad \left(Z_{\alpha/2} = Z_{0.025} = 1.96\right)$
- e=0.07
- p=0.15

$$n = \frac{Z_{\alpha/2}^2\, p(1-p)}{e^2}$$
$$n = \frac{(1.96)^2(0.15)(1-0.15)}{(0.07)^2}$$
$$n = \frac{(3.8416)(0.1275)}{0.0049}$$
$$n\ =\ 100$$

5. Key Takeaways

- Sample size depends on **confidence level**, **margin of error**, and **population variability**.
- Larger confidence levels or smaller margins of error require larger samples.
- The logic of confidence intervals provides the basis for determining sample size.
- Always **round sample size up**.

6. Handy Formulas to Remember

Sample Size for Mean

$$\boldsymbol{n = \frac{Z_{\alpha/2}^2 \sigma^2}{e^2}}$$

Sample Size for Proportion

$$n = \frac{Z_{\alpha/2}^2\, \pi(1-\pi)}{e^2}$$