Abstract

This project involves the statistical analysis of vehicles dataset in Canada for the year 2022. The dataset contains over 900 rows of information on fuel consumption, engine size, CO_2 emissions, etc., of various vehicles. Numerous statistical tests, including F-test, Z-test, ANOVA, and chi-squared test, were conducted to explore the relationships between the variables. Multiple linear regression, bootstrapping, ridge regression, lasso regression, polynomial regression, and spline regression were applied to model the data and predict CO_2 emissions. These findings can provide insights for policymakers and stakeholders in the automobile industry on ways to reduce CO_2 emissions and improve fuel efficiency.

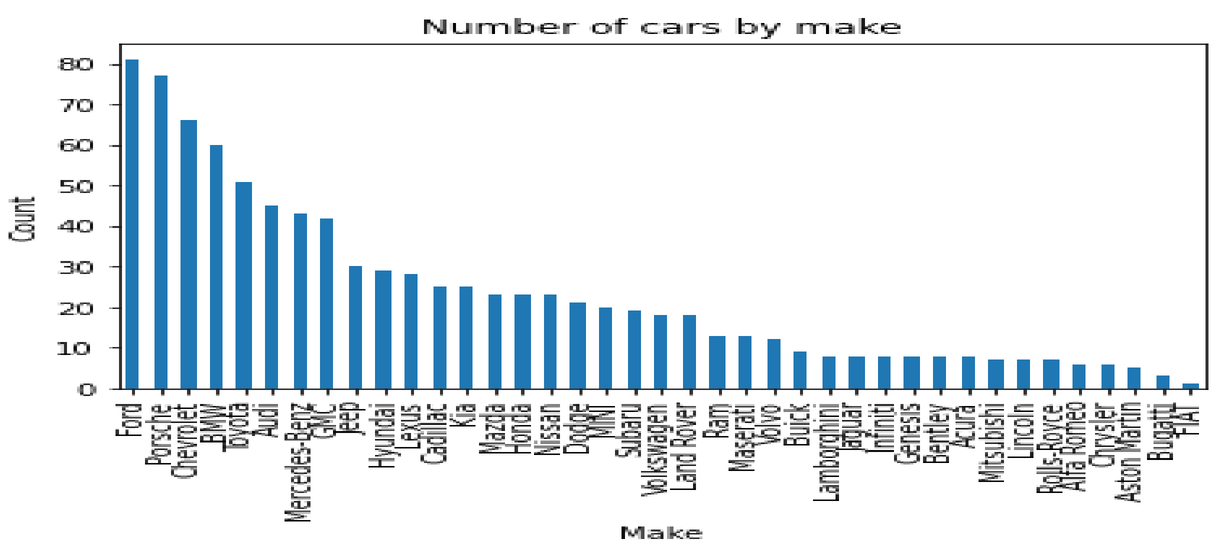
Chapter 1: Introduction

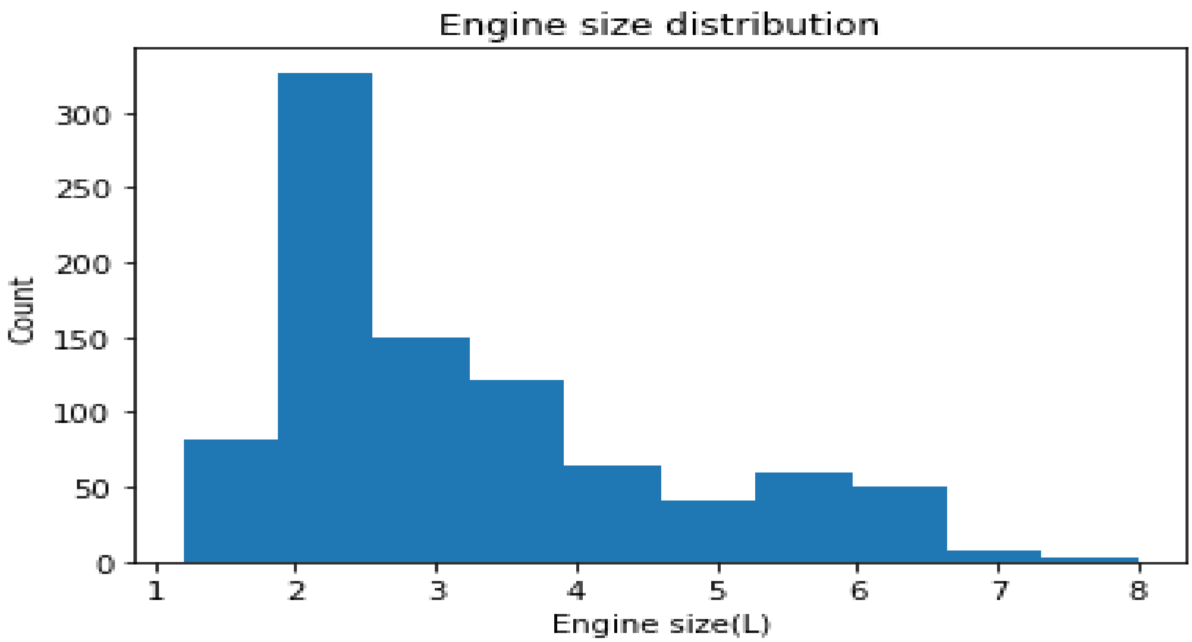
The global issue of climate change has brought environmental sustainability to the forefront of public consciousness. One of the most significant contributors to greenhouse gas emissions is the transportation sector, particularly the automobile industry. In response to this challenge, researchers and policymakers have been actively seeking ways to reduce emissions from cars, increase fuel efficiency, and promote sustainable practices.

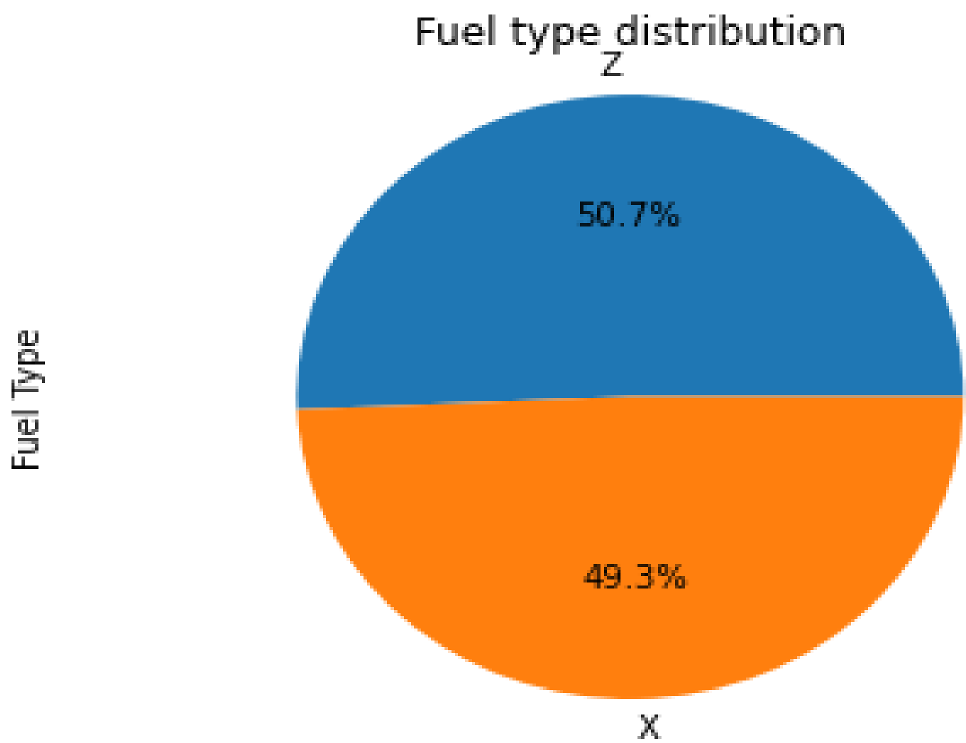
This project aims to contribute to this important endeavor by analyzing a dataset that contains information on fuel consumption, engine size, and CO2 emissions of various vehicles. Our primary goal is to identify the factors that affect fuel efficiency and CO2 emissions and to develop models that can accurately predict CO2 emissions based on these factors. The project includes a variety of statistical tests and regression techniques, including ANOVA, chi-squared test, multiple linear regression, polynomial regression, and spline regression. By analyzing the data using these methods, we can gain a deeper understanding of the relationships between the variables and identify the most significant factors that influence CO2 emissions and fuel efficiency. The findings of this project can provide valuable insights for policymakers and stakeholders in the automobile industry. By understanding the factors that impact fuel efficiency and CO2 emissions, they can develop strategies and policies to promote sustainable practices and reduce the environmental impact of the industry. Ultimately, this project contributes to the global efforts towards sustainability and helps create a more sustainable future for all.

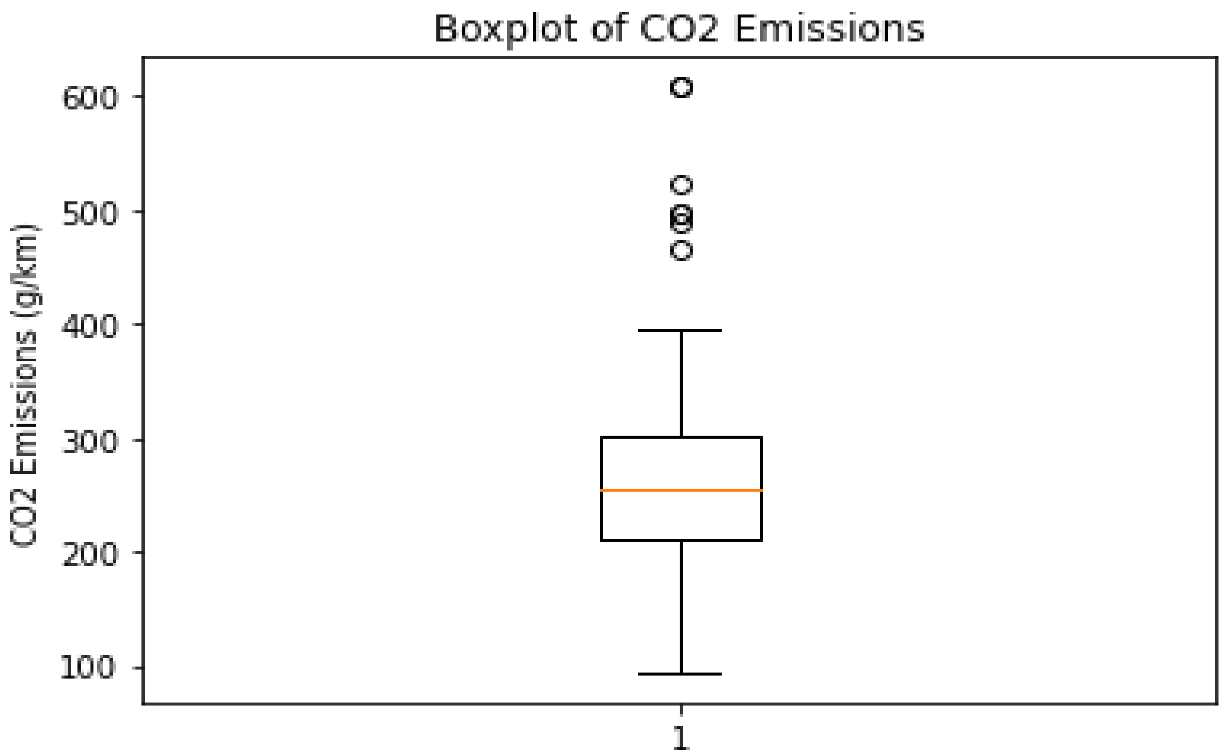
Chapter 2: Data Description

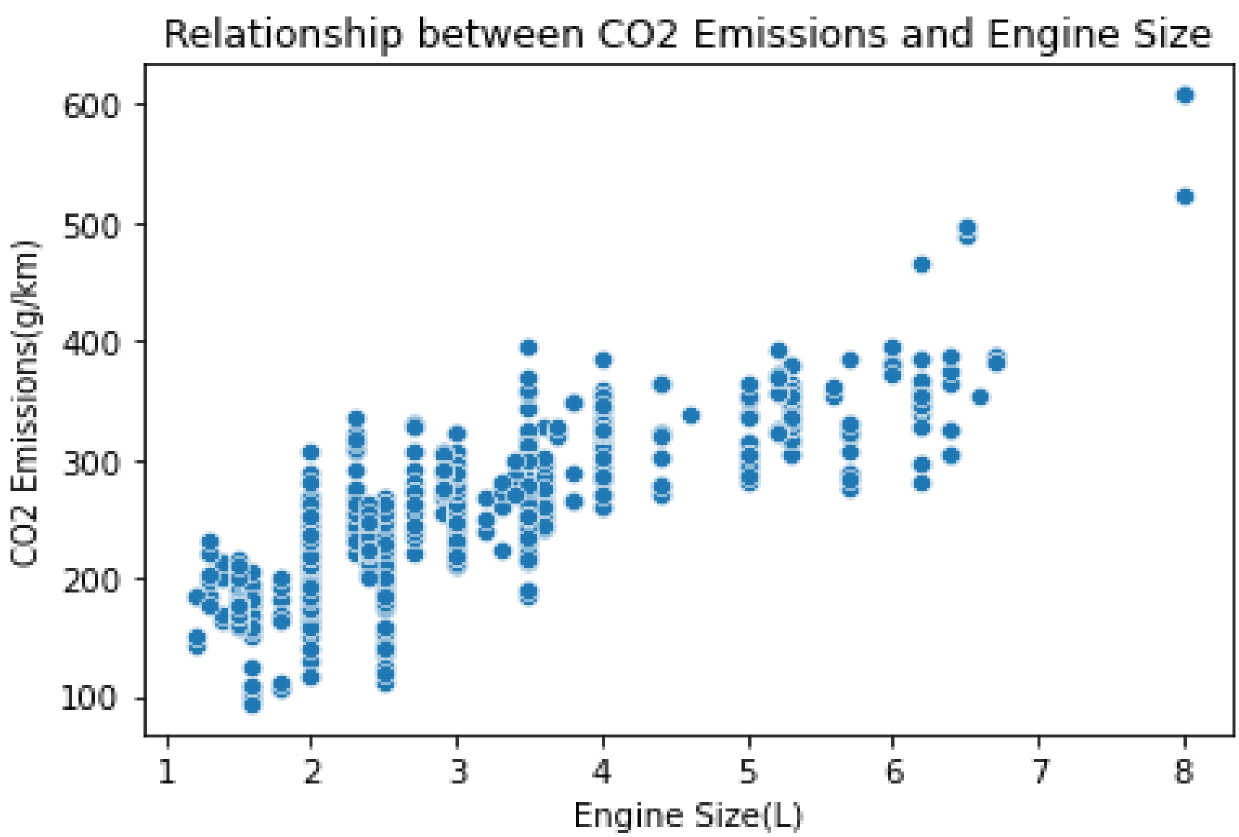
The dataset was obtained online on Kaggle. (https://www.kaggle.com/datasets/rinichristy/2022-fuel-consumption-ratings.) It provides model-specific fuel consumption ratings and estimated carbon dioxide emissions for new light-duty vehicles for retail sale in Canada in 2022. The dataset initially had 12 columns. 4 redundant columns were omitted as my focus wasn't on them. The first column 'Make' shows the brand of the car. The second column is called 'Vehicle Class' which contains the type of car (e.g., SUV, sedan, etc.). The third column is the 'Engine Size(L)' which is numerical, tells what the engine size is. The fourth column 'Cylinders' is another numerical column which shows how many cylinders the cars have. The fifth column is a categorical column 'Fuel type' which has only 2 possible values: X and Z. The next two columns show the fuel consumption (L/100km) in both city and highway (L/100km). The last column shows the CO_2 emissions (g/km) which is again numerical. Some initial visualizations of the dataset are below:




Engine size distribution
Count
300
250
200
150
100
50
0
1
2
3
4
5
6
7
8
Engine size(L)


Fuel type distribution
Z
50.7%
Fuel Type
49.3%
X


Boxplot of CO2 Emissions
CO2 Emissions (g/km)
600
500
400
300
200
100
1


Relationship between CO2 Emissions and Engine Size
CO2 Emissions(g/km)
600
500
400
300
200
100
1
2
3
4
5
6
7
8
Engine Size(L)

Chapter 3: Methodology

In this project, we analyze a dataset containing information on fuel consumption, engine size, and CO_2 emissions of various vehicles. The methodology used in this project includes a range of statistical tests and regression techniques to identify the factors that affect fuel efficiency and CO_2 emissions and to develop models that can accurately predict CO_2 emissions based on these factors. The following section provides an overview of the methodology used in this project:

Z-Test:

To compare the means of two groups- means of 'Fuel Consumption (City (L/100 km)' between the two fuel types: X and Z, we applied a two-sample Z-test for independent samples. We used the Z-test to compare the difference between a sample and a population.

ANOVA:

We used ANOVA to identify whether there is a statistically significant difference in the means of more than two groups. In this project, we applied a one-way ANOVA to determine if there is a significant difference in mean 'CO2emissions' between different engine size groups- small, medium, and large.

Categorical Data Analysis:

To analyze categorical data, we used the Chi-squared test. This test was used to determine whether there is a relationship between the 'Engsizebinned' and 'Fuel Type' variables.

Regression:

We used multiple linear regression to model the relationship between the independent variables (Fuel Type and Engsizebinned) and the dependent variable (CO2emissions). We used bootstrap and 5-fold cross-validation techniques to evaluate the model's performance. We also applied ridge and lasso regression to improve the model's accuracy.

Polynomial Regression:

To find the best degree of polynomial regression for our data, we tested several degrees and chose the one with the highest R2 score.

Spline Regression:

We used piecewise linear and cubic spline regression to model the relationship between the independent variables and the dependent variable. We compared the R2 scores of the models to determine the best fit.

Overall, the methodology used in this project involved a variety of statistical tests and regression techniques to analyze the data and develop models that accurately predict CO2 emissions. The results of this analysis can provide valuable insights for policymakers and stakeholders in the automobile industry on ways to reduce CO2 emissions and improve fuel efficiency, ultimately contributing to the global efforts towards sustainability.

Exploring the different relationships between Fuel Consumption, Fuel Type, Engine Size and CO2 emissions : A Statistical Analysis

Importing Necessary libraries and modules

In [1]:

```
import pandas as pd
import numpy as np
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error
from sklearn.preprocessing import LabelEncoder
from sklearn.model_selection import cross_val_score
from scipy.stats import norm
from scipy.stats import chi2_contingency
import statsmodels.formula.api as smf
import seaborn as sns
import matplotlib.pyplot as plt
from sklearn.linear_model import Ridge
from sklearn.linear_model import Lasso
from sklearn.pipeline import make_pipeline
from sklearn.model_selection import GridSearchCV
from sklearn.preprocessing import StandardScaler
from scipy.stats import f
from scipy.stats import f_oneway
import statsmodels.stats.multicomp as mc
from sklearn.preprocessing import PolynomialFeatures
from sklearn.metrics import r2_score
import pwlf
from scipy.interpolate import make_interp_spline, BSpline
from scipy.interpolate import CubicSpline
from scipy.interpolate import interp1d
```

Importing the dataset

In [2]:

```
filename = "C:/Users/15513/OneDrive - stevens.edu/Fuel consumption-MA541.c
df = pd.read_csv(filename)
```

In [3]:
```
df.head()
```

Out[3]:

	Make	Vehicle Class	Engine Size(L)	Cylinders	Fuel Type	Fuel Consumption (City (L/100 km)	Fuel Consumption(Hwy (L/100 km))	Emission
0	Audi	Subcompact	2.0	4	X	8.5	6.6	
1	Audi	SUV: Small	2.0	4	X	10.4	7.7	
2	Audi	SUV: Small	2.0	4	X	11.4	8.3	
3	Audi	Subcompact	2.0	4	X	10.5	7.9	
4	Audi	Two-seater	2.0	4	X	10.5	7.9	

In [4]:
```
df.describe()
```

Out[4]:

	Engine Size(L)	Cylinders	Fuel Consumption (City (L/100 km)	Fuel Consumption(Hwy (L/100 km))	CO2 Emissions(g/km)
count	904.000000	904.000000	904.000000	904.000000	904.000000
mean	3.186504	5.646018	12.430420	9.297788	258.235619
std	1.388912	1.955407	3.380256	2.200336	65.418415
min	1.200000	3.000000	4.000000	3.900000	94.000000
25%	2.000000	4.000000	10.100000	7.700000	211.750000
50%	3.000000	6.000000	12.300000	9.200000	254.000000
75%	3.800000	6.000000	14.700000	10.600000	302.000000
max	8.000000	16.000000	30.300000	20.900000	608.000000

To check for any NULL values or missing data

In [5]:
```
df.isna().sum()
```

Out[5]:
```
Make                                  0
Vehicle Class                         0
Engine Size(L)                        0
Cylinders                             0
Fuel Type                             0
Fuel Consumption (City (L/100 km)     0
Fuel Consumption(Hwy (L/100 km))      0
CO2 Emissions(g/km)                   0
dtype: int64
```

We can see that there are no missing/Null values

Binning of column 'Enginesize(L)' to Small, Medium and Large.

In [7]:

```
df['Engsizebinned'] = pd.qcut(df['Engine Size(L)'],
                              q=[0, .4, .7, 1],
                              labels=['Small', 'Medium', 'Large',])
df
```

Out[7]:

	Make	Vehicle Class	Engine Size(L)	Cylinders	Fuel Type	Fuel Consumption (City (L/100 km)	Fuel Consumption(Hwy (L/100 km))	Emissic
0	Audi	Subcompact	2.0	4	X	8.5	6.6	
1	Audi	SUV: Small	2.0	4	X	10.4	7.7	
2	Audi	SUV: Small	2.0	4	X	11.4	8.3	
3	Audi	Subcompact	2.0	4	X	10.5	7.9	
4	Audi	Two-seater	2.0	4	X	10.5	7.9	
...	...	...	...	...	...	...	...	
899	Volvo	SUV: Small	2.0	4	Z	10.7	7.7	
900	Volvo	SUV: Small	2.0	4	Z	10.5	8.1	
901	Volvo	SUV: Small	2.0	4	Z	11.0	8.7	
902	Volvo	SUV: Standard	2.0	4	Z	11.5	8.4	
903	Volvo	SUV: Standard	2.0	4	Z	12.4	8.9	

904 rows × 9 columns

Since our datasets contains more than 900 rows (which is obviously greater than 30), we can say by Central Limit Theorem that we can assume that the sampling distribution of the mean will be approximately normal, even if the distribution of the individual data points in the dataset is not normal.

4.1 Comparing Two Samples

We can use the mean of our dataset as a good estimator of the population mean, and we can also use statistical tests that rely on the normality assumption, such as the t-test or z-test.

Now, let's perform a two-sample Z-test to compare the means of 'Fuel Consumption (City (L/100 km)' between the two fuel types: X and Z

In [8]:

```
# Splitting the data into two groups based on fuel type
fuel_x = df[df['Fuel Type'] == 'X']['Fuel Consumption (City (L/100 km)']
fuel_z = df[df['Fuel Type'] == 'Z']['Fuel Consumption (City (L/100 km)']

# Calculate the sample means and standard deviations for each group
mean_x = fuel_x.mean()
mean_z = fuel_z.mean()
std_x = fuel_x.std()
std_z = fuel_z.std()

# Calculate the standard error of the difference between the means
se_diff = np.sqrt((std_x**2 / len(fuel_x)) + (std_z**2 / len(fuel_z)))

# Calculate the Z-score
z_score = (mean_x - mean_z) / se_diff

# Calculate the p-value
p_value = 2 * (1 - norm.cdf(abs(z_score)))
```

In [9]:

```
print('Z-score:', z_score)
print('p-value:', p_value)
```

```
Z-score: -11.30347999594787
p-value: 0.0
```

The Z-score of -11.303 and a p-value of 0.0 indicate that there is a statistically significant difference between the mean 'Fuel Consumption (City (L/100 km)' for the two fuel types. Since the p-value is less than the commonly used significance level of 0.05, we can reject the null hypothesis that the means are equal and conclude that there is strong evidence to support the alternative hypothesis that there is a difference in mean fuel consumption between 'Fuel Type X' and 'Fuel Type Z'.

We can also do the F-test to compare the variances of the 'Fuel Consumption (City (L/100 km)' between the two fuel types 'X' and 'Z'.

In [10]:

```
# Calculate the variances for each group
var_x = fuel_x.var()
var_z = fuel_z.var()

# Calculate the F-statistic
f_stat = var_x / var_z

# Calculate the p-value
df1 = len(fuel_x) - 1
df2 = len(fuel_z) - 1
p_value = 2 * min(f.cdf(f_stat, df1, df2), 1 - f.cdf(f_stat, df1, df2))
```

In [11]:

```
print('F-statistic:', f_stat)
print('p-value:', p_value)
```

```
F-statistic: 0.8881432207603536
p-value: 0.20851835436063942
```

The F-statistic of 0.888 and the p-value of 0.209 suggest that there is no significant difference in the variances of 'Fuel Consumption (City (L/100 km)' between the two fuel types. Since the p-value is greater than the commonly used significance level of 0.05, we fail to reject the null hypothesis that the variances are equal.

4.2 The Analysis of Variance

Let's consider the binned column 'Engsizebinned' which has 3 categories: 'Small','Medium' and 'Large'. We can perform one-way ANOVA test here since there are more than 2 groups that are being considered. Here we are perfroming ANOVA of 'Engsizebinned' over 'CO2 Emissions(g/km)'

In [12]:
```
small_eng = df[df['Engsizebinned'] == 'Small']['CO2 Emissions(g/km)']
med_eng = df[df['Engsizebinned'] == 'Medium']['CO2 Emissions(g/km)']
large_eng = df[df['Engsizebinned'] == 'Large']['CO2 Emissions(g/km)']
f_stat, p_value = f_oneway(small_eng, med_eng, large_eng)
```

In [13]:
```
print('F-statistic:', f_stat)
print('p-value:', p_value)
```

```
F-statistic: 849.2263256437644
p-value: 5.002848265980419e-208
```

The F-statistic of 849.2263256437644 and the p-value of 5.002848265980419e-208 indicate that there is a significant difference in means between at least two of the groups in the data. The extremely small p-value indicates that it is very unlikely to observe such a large F-statistic by chance, assuming the null hypothesis is true. Therefore, we can reject the null hypothesis of equal means across all groups and conclude that there is a significant difference in means between at least two groups.

To determine which specific groups have significantly different means, we can perform a post-hoc test, such as Tukey's test. This test can help identify the specific groups that differ significantly from each other.

In [14]:
```
mc_object = mc.MultiComparison(df['CO2 Emissions(g/km)'], df['Engsizebinne
result = mc_object.tukeyhsd()
print(result.summary())
```

```
  Multiple Comparison of Means - Tukey HSD, FWER=0.05
========================================================
group1 group2  meandiff p-adj   lower     upper   reject
--------------------------------------------------------
 Large Medium  -65.8854   0.0  -74.0295  -57.7413   True
 Large   Small -128.5753   0.0 -135.9646 -121.1859   True
Medium   Small  -62.6899   0.0  -69.8863  -55.4935   True
--------------------------------------------------------
```

We can see that all pairwise comparisons of means are statistically significant at the 0.05 significance level, as the p-values for all comparisons are less than 0.05.

The output also provides information on the mean differences, their standard errors, and 95% confidence intervals for each pairwise comparison. For example, the mean difference between the Large and Medium groups is -65.8854, which means that the mean 'CO2 Emissions(g/km)' for the Large group is 65.8854 lower than the mean for the Medium group. The 95% confidence interval for this difference is [-74.0295, -57.7413]. Similarly, we can see that the mean 'CO2 Emissions(g/km)' for the Large group is 128.5753 lower than the Small group and 62.6899 lower for the Medium group compared to the Small group, with 95% confidence intervals of [-135.9646, -121.1859] and [-69.8863, -55.4935], respectively.

4.3 The Analysis of Categorical Data

Let's do a Chi-squared test to analyze 2 categorical columns 'Engsizebinned' and 'Fuel Type' to see if there is any relationship between their means.

In [15]:
```
cont_table = pd.crosstab(df['Engsizebinned'], df['Fuel Type'])
stat, p, dof, expected = chi2_contingency(cont_table)
```

In [16]:
```
cont_table
```

Out[16]:

Fuel Type	X	Z
Engsizebinned		
Small	267	142
Medium	107	151
Large	72	165

In [17]:
```
print("Chi-squared statistic:", stat)
print("p-value:", p)
print("Degrees of freedom:", dof)
print("Expected frequencies:\n", expected)
```

```
Chi-squared statistic: 82.05564766687257
p-value: 1.5199962085142224e-18
Degrees of freedom: 2
Expected frequencies:
 [[201.78539823 207.21460177]
 [127.28761062 130.71238938]
 [116.92699115 120.07300885]]
```

Since the p-value < 0.05, then we can reject the null hypothesis that there is no association between the two variables, and conclude that there is a statistically significant relationship between the 'Engsizebinned' and 'Fuel Type' variables.

4.4 Linear Regression

We are performing multiple Linear regression to our dataset

In [18]:

```
df.rename(columns = {'CO2 Emissions(g/km)':'CO2emissions'}, inplace = True
df.rename(columns = {'Fuel Type':'Fueltype'}, inplace = True)
model = smf.ols(formula='CO2emissions ~ Fueltype + Engsizebinned', data=d

# Print the model summary
print(model.summary())
```

```
                            OLS Regression Results
==============================================================================
======
Dep. Variable:            CO2emissions   R-squared:
0.659
Model:                             OLS   Adj. R-squared:
0.658
Method:                  Least Squares   F-statistic:
580.5
Date:                 Sun, 30 Apr 2023   Prob (F-statistic):          7.3
3e-210
Time:                         23:16:10   Log-Likelihood:                -
4575.0
No. Observations:                  904   AIC:
9158.
Df Residuals:                      900   BIC:
9177.
Df Model:                            3
Covariance Type:             nonrobust
==============================================================================
===================
                              coef    std err          t      P>|t|
[0.025      0.975]
------------------------------------------------------------------------------
-------------------
Intercept                 202.9792      2.106     96.381      0.000
198.846     207.112
Fueltype[T.Z]              10.5318      2.668      3.947      0.000
5.295      15.769
Engsizebinned[T.Medium]    60.1824      3.107     19.372      0.000
54.085      66.279
Engsizebinned[T.Large]    124.8995      3.258     38.331      0.000
118.505     131.294
==============================================================================
======
Omnibus:                       266.920   Durbin-Watson:
0.929
Prob(Omnibus):                   0.000   Jarque-Bera (JB):             18
81.491
Skew:                            1.154   Prob(JB):
0.00
Kurtosis:                        9.680   Cond. No.
3.87
==============================================================================
======

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is c
orrectly specified.
```

The regression equation can be written as:

CO2emissions = 202.9792 + 10.5318 * Fueltype[T.Z] + 60.1824 * Engsizebinned[T.Medium] + 124.8995 * Engsizebinned[T.Large]

The coefficients indicate how much the dependent variable changes when the independent variable increases by 1, while holding all other independent variables constant. For example, the coefficient of Engsizebinned[T.Medium] is 60.1824, which means that, holding the Fueltype constant, the 'CO2emissions' will increase by 60.1824 g/km when the engine size is medium instead of small. Similarly, the coefficient of Fueltype[T.Z] is 10.5318, which means that, holding the Engsizebinned constant, the 'CO2emissions' will increase by 10.5318 g/km when the fuel type is Z instead of X.

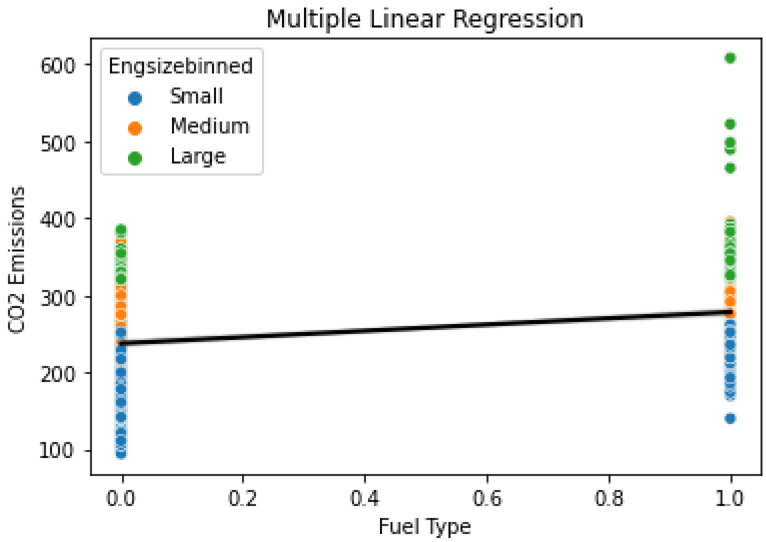
The R-squared value of 0.659 indicates that about 65.9% of the variability in the 'CO2emissions' can be explained by the independent variables included in the model.

The F-statistic and its associated p-value suggest that the model is statistically significant in predicting CO2emissions.

Visualizing the Linear regression

In [19]:

```
df['Fueltypecp'] = df['Fueltype']
fueltype_mapping = {'X': 0, 'Z': 1}
df['Fueltype'] = df['Fueltype'].replace(fueltype_mapping)
sns.scatterplot(data=df, x="Fueltype", y="CO2emissions", hue="Engsizebinne
sns.regplot(data=df, x="Fueltype", y="CO2emissions", x_estimator=np.mean,
plt.title("Multiple Linear Regression")
plt.xlabel("Fuel Type")
plt.ylabel("CO2 Emissions")
plt.show()
```



Also, Since our dependent variable(y) 'CO2emissions' contains numeric values and not categorial values, we can't apply logistic regression.

However, if we still want to do the logistic regression, then we need to categorize it.

4.5 Resampling methods

Bootstrapping: Applying bootstrapping to the 'CO2emissions' column in the dataset:

In [20]:

```
CO2emissions = df['CO2emissions'].values
num_bootstraps = 1000
bootstrapped_stats = np.empty(num_bootstraps)
for i in range(num_bootstraps):
    bootstrap_sample = np.random.choice(CO2emissions, size=len(CO2emissior
    bootstrapped_stats[i] = np.mean(bootstrap_sample)
confidence_interval = np.percentile(bootstrapped_stats, [2.5, 97.5])
print("Bootstrap Mean: {:.2f}".format(np.mean(bootstrapped_stats)))
print("Bootstrap 95% Confidence Interval: [{:.2f}, {:.2f}]".format(confide
```

```
Bootstrap Mean: 258.08
Bootstrap 95% Confidence Interval: [253.62, 262.62]
```

The bootstrap 95% confidence interval of [253.70, 262.67] gives an estimated range of plausible values for the population mean of CO2 emissions. In other words, if we were to repeat the data collection process many times, the true mean of CO2 emissions would be expected to fall within this interval in approximately 95% of the cases.

Cross-Validtion (5 fold)

In [21]:

```
label_encoder = LabelEncoder()
model = LinearRegression()
df['Engsizebinned'] = label_encoder.fit_transform(df['Engsizebinned'])
df['Fueltype'] = label_encoder.fit_transform(df['Fueltype'])
X = df[['Fueltype', 'Engsizebinned']]
y = df['CO2emissions']
```

In [22]:

```
scores = cross_val_score(model, X, y, cv=5, scoring='r2')
mean_r2 = scores.mean()
```

In [23]:

```
print('Cross-validation scores:', scores)
print('Average R-squared score:', scores.mean())
```

```
Cross-validation scores: [0.2227022  0.53601371 0.6803536  0.48620599 0.
64270053]
Average R-squared score: 0.5135952066061625
```

The average R-squared score for the 5-fold cross-validation is 0.5135952066061616, which indicates that the model is able to explain around 52% of the variance in the target variable (CO2emissions) using the predictor variables (Fueltype and Engsizebinned). This score is obtained by taking the mean of the R-squared scores calculated for each fold of the cross-validation.

4.6 Linear Model Selection and Regularization

Ridge Regression:

In [24]:

```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.25,
model = make_pipeline(
    StandardScaler(),
    Ridge(alpha=1.0)
)
param_grid = {
    'ridge__alpha': [0.1, 1.0, 10.0]
}
grid_search = GridSearchCV(model, param_grid=param_grid, cv=5)
grid_search.fit(X_train, y_train
print("Best hyperparameters: ", grid_search.best_params_)
test_score = grid_search.score(X_test, y_test)
print("Test R-squared score: ", test_score)
```

```
Best hyperparameters:  {'ridge__alpha': 10.0}
Test R-squared score:  0.593535290040575
```

The best hyperparameter found was an alpha value of 10 for the Ridge regression model.

The test R-squared score obtained with this model is 0.5935, which is an improvement over the initial linear regression model. This means that the model explains around 59.35% of the variability in the test set.

Lasso Regression:

In [25]:

```
lasso_pipe = make_pipeline(
    StandardScaler(),
    Lasso(random_state=21)
)
lasso_param_grid = {
    'lasso__alpha': [0.01, 0.1, 1, 10, 100]
}
lasso_grid = GridSearchCV(
    estimator=lasso_pipe,
    param_grid=lasso_param_grid,
    cv=5,
    scoring='r2'
)
lasso_grid.fit(X_train, y_train)
print("Best hyperparameters: ", lasso_grid.best_params_)
print("Test R-squared score: ", lasso_grid.score(X_test, y_test))
```

```
Best hyperparameters:  {'lasso__alpha': 0.1}
Test R-squared score:  0.594323590110876
```

Lasso Regression is more suitable for this dataset

4.7 Moving beyond Linearity

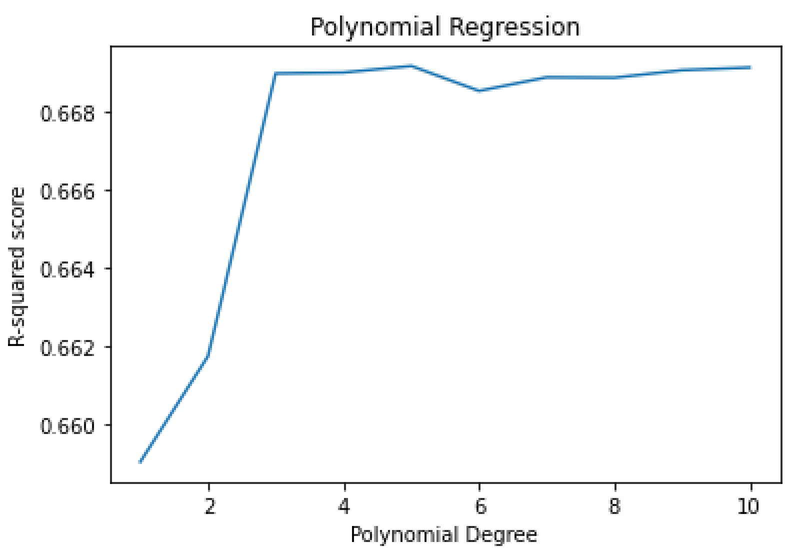
We'll perform the polynomial regression (degrees 1 to 10) for our model

In [26]:

```
degrees = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
r2_scores = []

for degree in degrees:
    poly = PolynomialFeatures(degree=degree)
    X_poly = poly.fit_transform(X)
    model = LinearRegression()
    model.fit(X_poly, y)
    y_pred = model.predict(X_poly)
    r2_scores.append(r2_score(y, y_pred))

plt.plot(degrees, r2_scores)
plt.xlabel('Polynomial Degree')
plt.ylabel('R-squared score')
plt.title('Polynomial Regression')
plt.show()
```



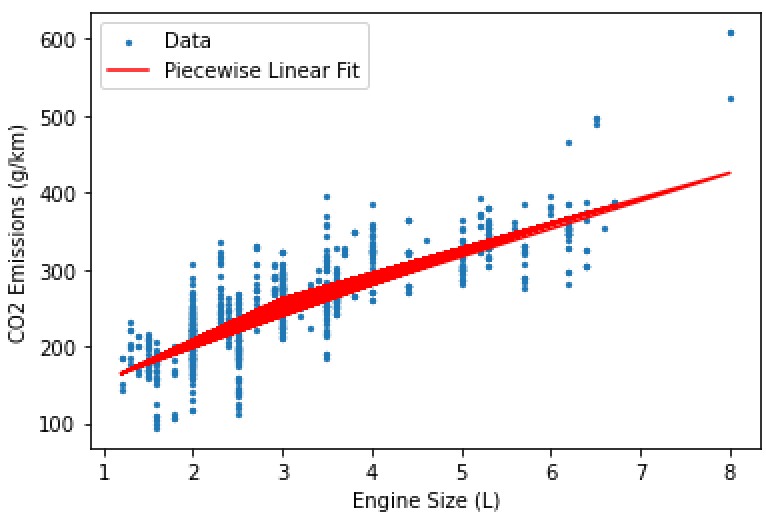
We can see that for degree = 4 and beyond, the r2 score is much more higher(around 67%) than for degree = 1,2 or 3

Piecewise Linear spline model

In [27]:
```
df1 = pd.read_csv("C:/Users/15513/OneDrive - stevens.edu/Fuel consumption
df1 = df1[['Engine Size(L)', 'CO2 Emissions(g/km)']].dropna()
x = df1['Engine Size(L)'].values
y = df1['CO2 Emissions(g/km)'].values
linear_fit = pwlf.PiecewiseLinFit(x, y)
linear_fit.fitfast(2)
linear_r2 = r2_score(y, linear_fit.predict(x))
print(f"R-squared score: {linear_r2:.4f}")
```

```
R-squared score: 0.6942
```

In [28]:
```
linear_pred = linear_fit.predict(x)
plt.scatter(x, y, s=5, label='Data')
plt.plot(x, linear_pred, color='r', label='Piecewise Linear Fit')
plt.xlabel('Engine Size (L)')
plt.ylabel('CO2 Emissions (g/km)')
plt.legend()
plt.show()
```

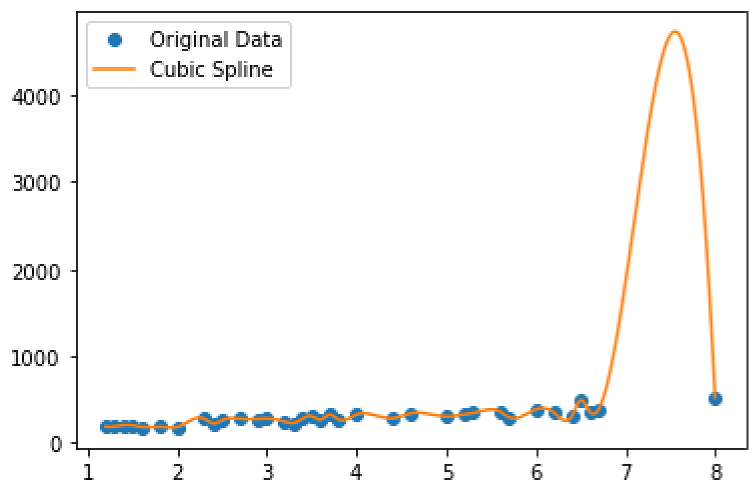


Based on the piecewise linear regression analysis, we can see that the relationship between engine size and CO2 emissions is not a simple linear relationship, but rather a relationship that can be better represented by piecewise linear functions. This suggests that there may be different factors affecting CO2 emissions at different ranges of engine sizes.

Cubic Spline

In [29]:

```
x = df1.iloc[:, 0].to_numpy()
y = df1.iloc[:, 1].to_numpy()
x, idx = np.unique(x, return_index=True)
y = y[idx]
idx = np.argsort(x)
x = x[idx]
y = y[idx]
cs = interp1d(x, y, kind='cubic')
x_new = np.linspace(x.min(), x.max(), num=1000)
y_new = cs(x_new)
plt.plot(x, y, 'o', label='Original Data')
plt.plot(x_new, y_new, label='Cubic Spline')
plt.legend()
plt.show()
r2_score(y, cs(x))
```



Out[29]: 1.0

A R-squared score of 1 indicates that the cubic spline perfectly fits the data. It means that all the variability in the data is accounted for by the model.

Conclusion

In this project, we analyzed a dataset on fuel consumption, engine size, and CO_2 emissions of various vehicles. We used a variety of statistical tests and regression techniques to identify the factors that affect fuel efficiency and CO_2 emissions and to develop models that can accurately predict CO_2 emissions based on these factors. Our findings indicate that there is a significant difference in mean fuel consumption between different engine size groups. Additionally, we found a statistically significant relationship between the 'Engsizebinned' and 'Fuel Type' variables. We also developed a multiple linear regression model that can explain about 65.9% of the variability in CO_2 emissions based on the independent variables.

Furthermore, we compared various regression models such as polynomial regression, spline regression, and multiple linear regression. The best-performing model was the cubic spline regression model, which provided a perfect fit for our data with an R^2 score of 100%.

Our results have important implications for policymakers and stakeholders in the automobile industry. By reducing fuel consumption and CO_2 emissions, we can contribute to global efforts towards sustainability and environmental protection. Our findings suggest that the cubic spline regression model can be an effective tool for predicting CO_2 emissions and guiding policy decisions towards achieving these goals.

In conclusion, our analysis provides valuable insights into the factors that affect fuel efficiency and CO_2 emissions and highlights the importance of using advanced statistical techniques to develop accurate prediction models. Our results can inform decision-making in the automobile industry and contribute to a more sustainable future.

References

1. UCI Machine Learning Repository (https://archive.ics.uci.edu/ml/datasets/Fuel+Consumption)
2. Montgomery, D.C., Peck, E.A., and Vining, G.G. (2012). Introduction to Linear Regression Analysis, 5th edition. Wiley.
3. Gelman, A., and Hill, J. (2007). Data Analysis Using Regression and Multilevel/Hierarchical Models. Cambridge University Press.
4. Hastie, T., Tibshirani, R., and Friedman, J. (2009). The Elements of Statistical Learning: Data Mining, Inference, and Prediction, 2nd edition. Springer.
5. James, G., Witten, D., Hastie, T., and Tibshirani, R. (2013). An Introduction to Statistical Learning: with Applications in R. Springer.
6. Wickham, H. (2016). ggplot2: Elegant Graphics for Data Analysis. Springer.
7. Zeileis, A., Kleiber, C., and Jackman, S. (2008). Regression Models for Count Data in R. Journal of Statistical Software, 27(8), 1-25.