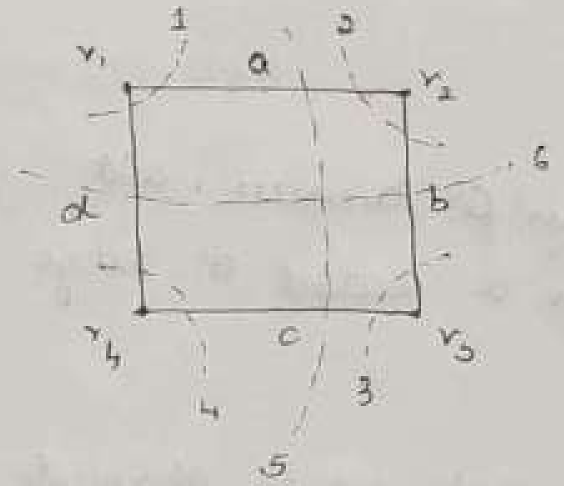
Cutset matrix - A cutset matrix of a graph G denoted by $C(G)$ is defined as $C = [C_{ij}] = \begin{cases} 1 & \text{if } j\text{th edge is included in } i\text{th cutset} \\ 0 & \text{otherwise} \end{cases}$

Ex -



$$C = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

Note - $B \cdot C^T = O = C^T \cdot B \pmod 2$ when columns of B and C are in same order of edges.

Example :

Circuit (1): $v_1 e_1 v_4 e_2 v_3 e_3 v_2 e_4 v_1$

Circuit (2): $v_4 e_5 v_5 e_6 v_3 e_2 v_4$

Circuit (3): $v_1 e_1 v_4 e_5 v_5 e_6 v_3 e_3 v_2 e_4 v_1$

$$B = \begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 & 1 & 1 \end{array}$$

Cut-sets

1 $\{e_1, e_4\}$ $\{e_2, e_4, e_6\}$

2 $\{e_3, e_4\}$ $\{e_1, e_2, e_6\}$

3 $\{e_2, e_3, e_4\}$ $\{e_2, e_3, e_6\}$

4 $\{e_1, e_2, e_5\}$ $\{e_1, e_3\}$

3 $\{e_5, e_6\}$

1 $\{e_2, e_4, e_6\}$

$$C = \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$B \cdot C^T = 0 \pmod 2$

Note - There are many dependent rows in C because ring sum of any 2 cutsets is again a cutset or edge disjoint union of cut-sets.

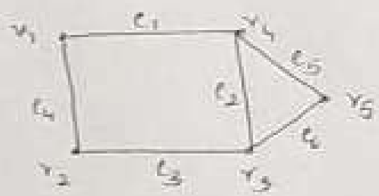
So, if we identify the no. of independent rows through the defn of cut-set or fundamental cut-set, we can determine rank (C).

Similarly, there are many dependent rows in a circuit matrix B, by determining the fundamental circuits we can get all the circuits of a graph

Matrices A_f, B_f and C_f :

Reduced incidence matrix A_f - WKT 1 row of an incidence matrix $A(G)_{n \times e}$ is dependent on others i.e., $A(G)$ contains $(n-1)$ linearly independent rows so information will not be lost if we delete any one row (so rank $(A(G))$ is $(n-1)$) The matrix obtained from incidence matrix by deleting any row is known as reduced incidence matrix, A_f and the ~~row~~ vertex corresponding to the deleted row is known as reference vertex. Rank of A_f is also $(n-1)$.

Fundamental circuit matrix B_f -

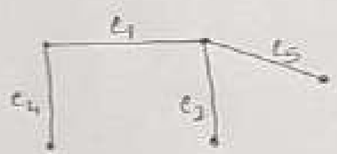


C_1 - $v_1\ e_1\ v_4\ e_2\ v_3\ e_3\ v_2\ e_4\ v_1$

fundamental circuit on adding e_3

C_2 - $v_4\ e_5\ v_5\ e_6\ v_3\ e_2\ v_4$

fundamental circuit on adding e_6



C_3 - $v_1\ e_1\ v_4\ e_5\ v_5\ e_6\ v_3\ e_3\ v_2\ e_4\ v_1$

Ringsum of (1) & (2)

Spanning tree

$$B = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Rank (B) = 2 = Rank (B_f) = nullity (no. of chords)

$$B_f = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$