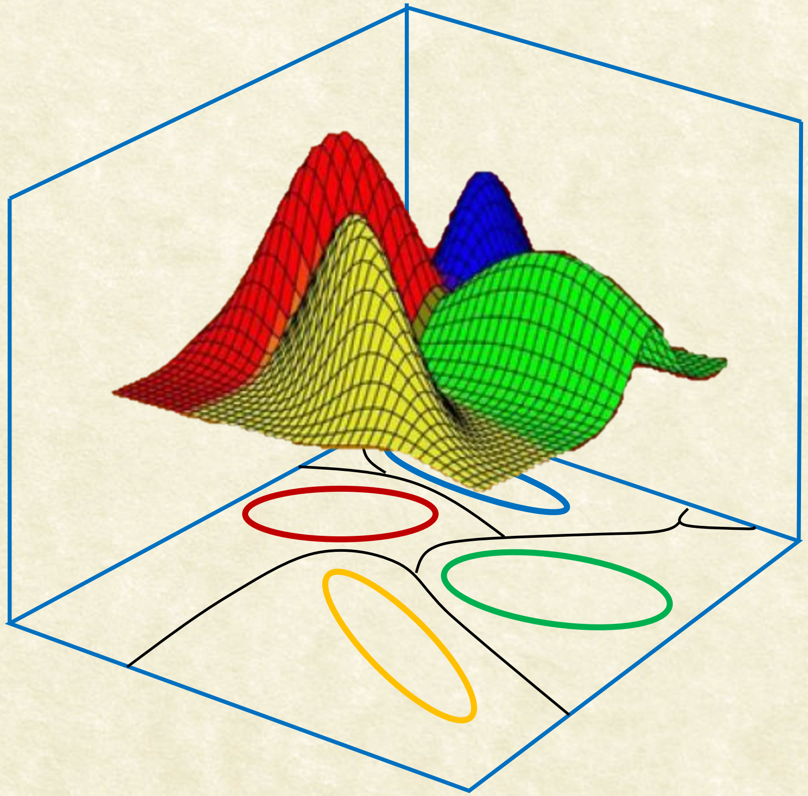
CS7.404: Digital Image Processing

Monsoon 2023: Linear Filters



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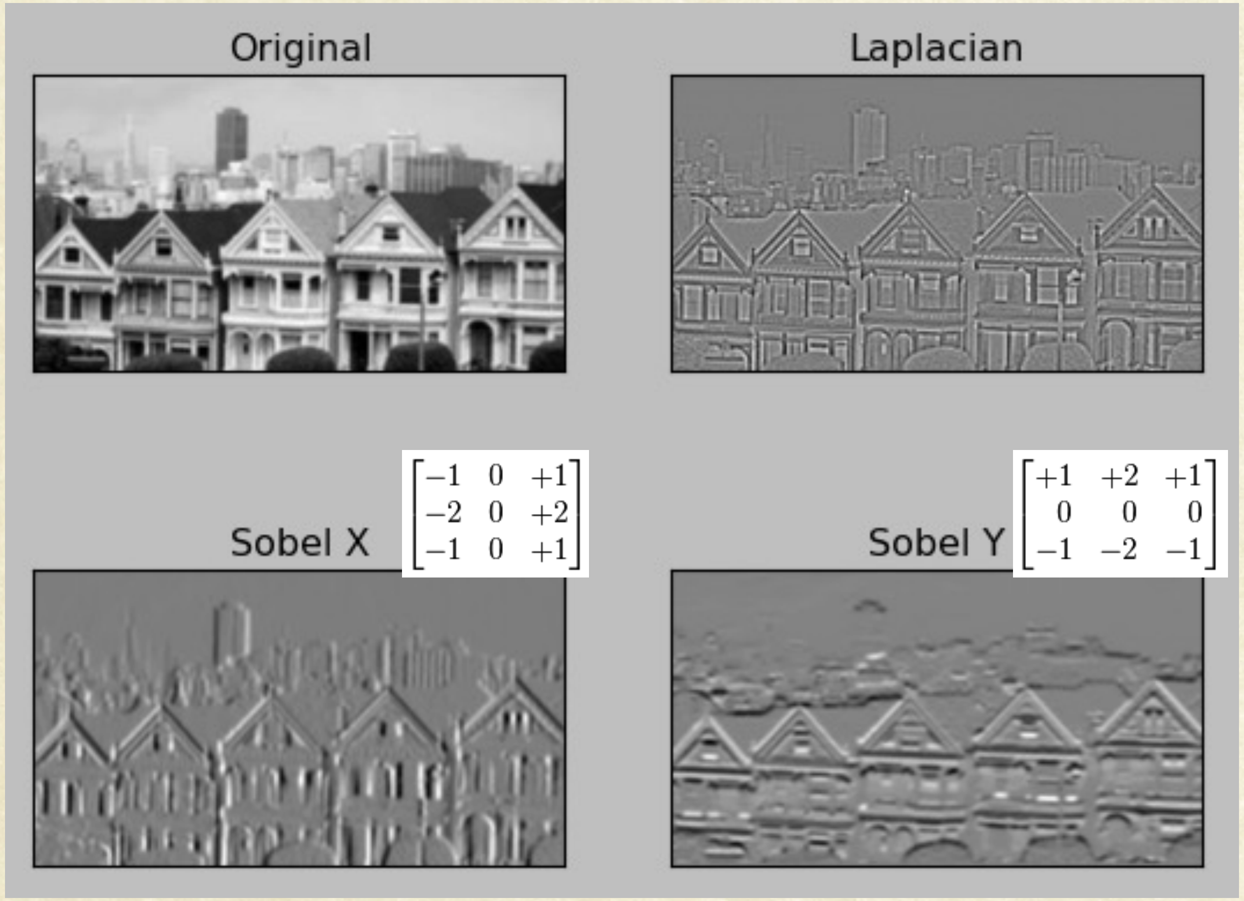
Recap: 2-D Laplacian Filter

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

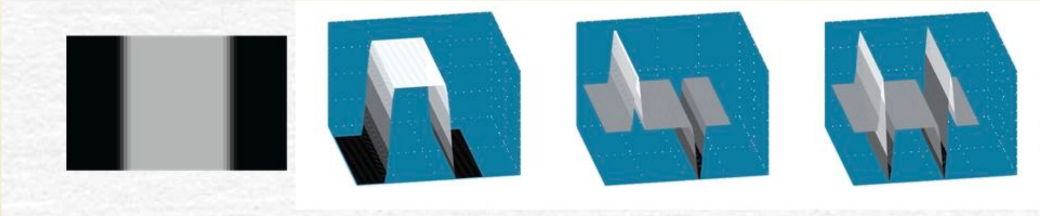
0	1	0
1	-4	1
0	1	0

Edge Masks: Sobel, Laplacian

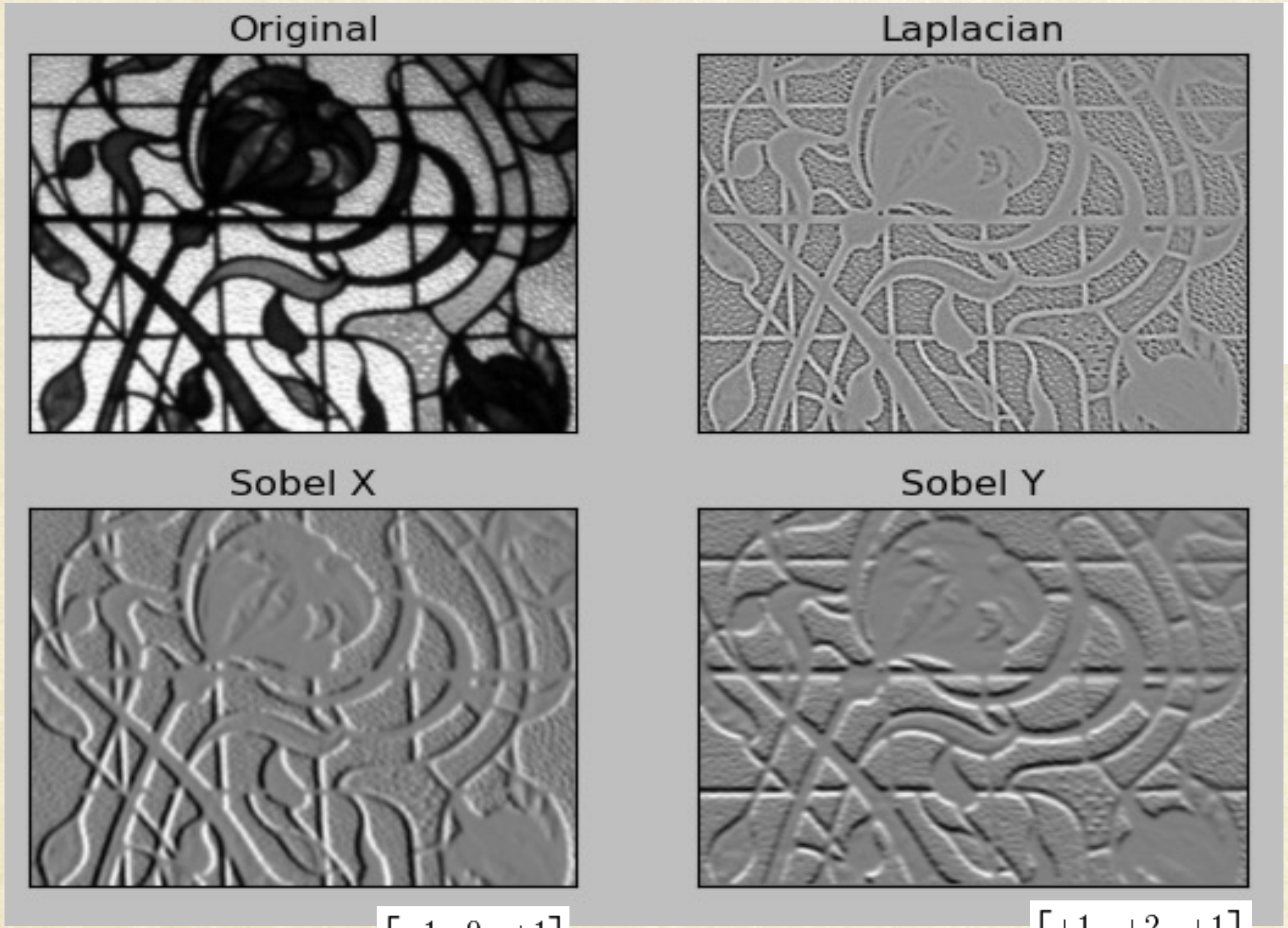


0	−1	0
−1	4	−1
0	−1	0

Note: Coefficients sum to 0



Edge Masks: Sobel, Laplacian



0	−1	0
−1	4	−1
0	−1	0

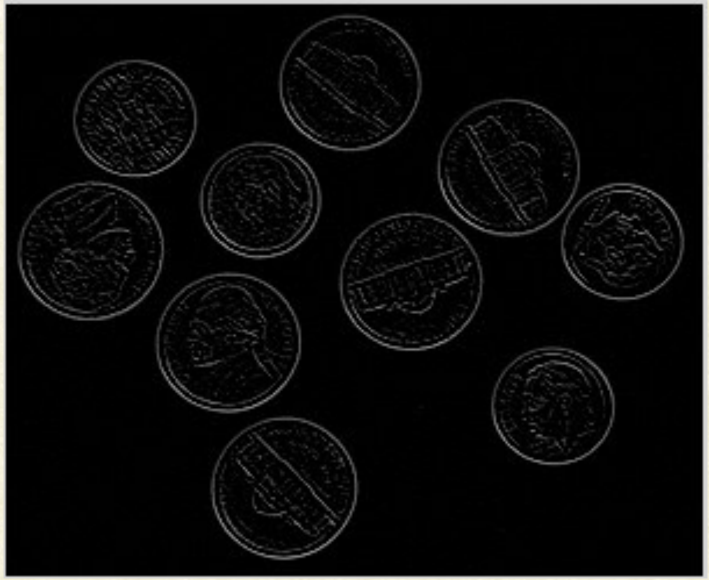
$$\begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

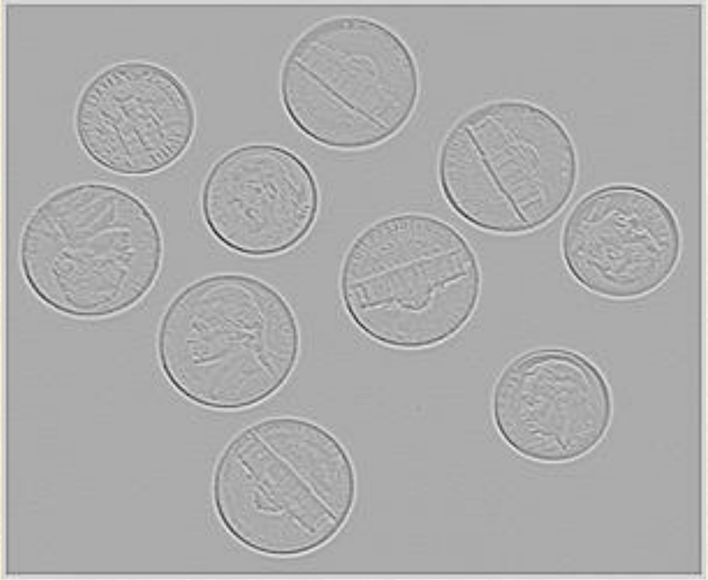
$$\begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Image Sharpening

$I(u,v)$

$\nabla^2 I(u,v)$





$\nabla^2 I(u,v) + 128$

(For visualization)

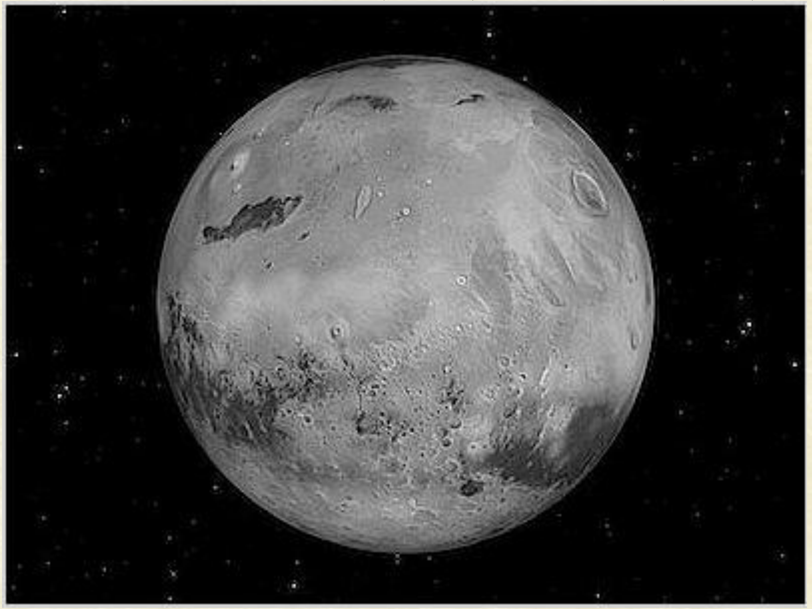
$I'(u,v) = I(u,v) + \nabla^2 I(u,v)$

Sharpening (Unsharp Masking)

$I(u,v)$

$\nabla \ I(u,v)$

$I'(u,v) = I(u,v) + \nabla^2 I(u,v)$



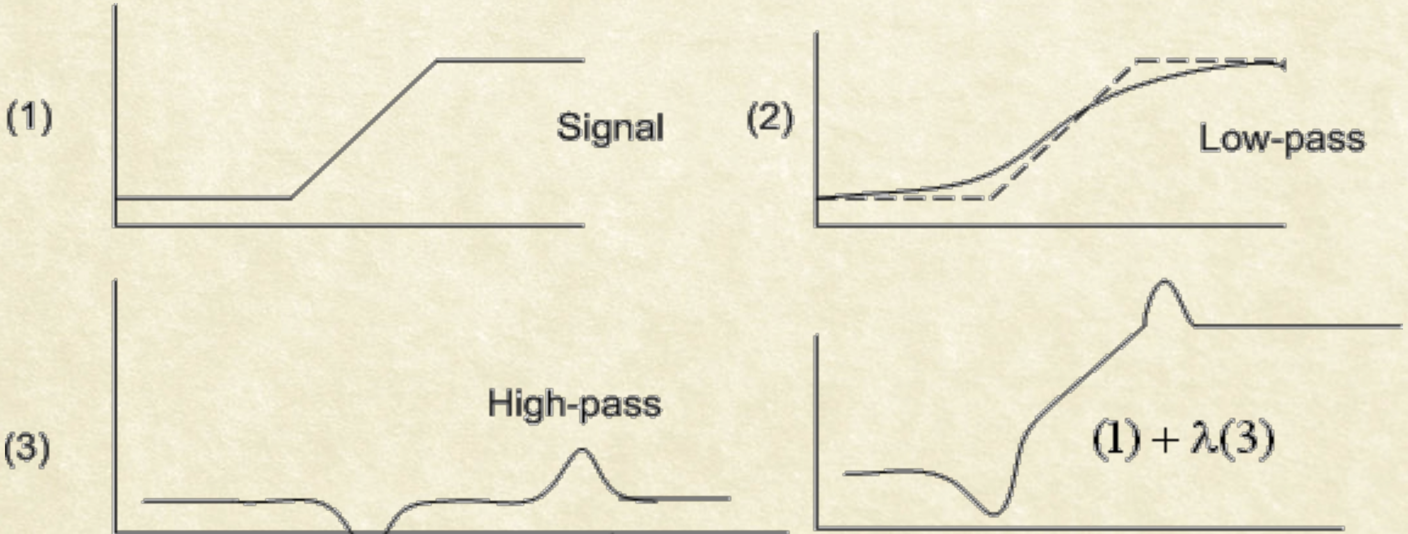
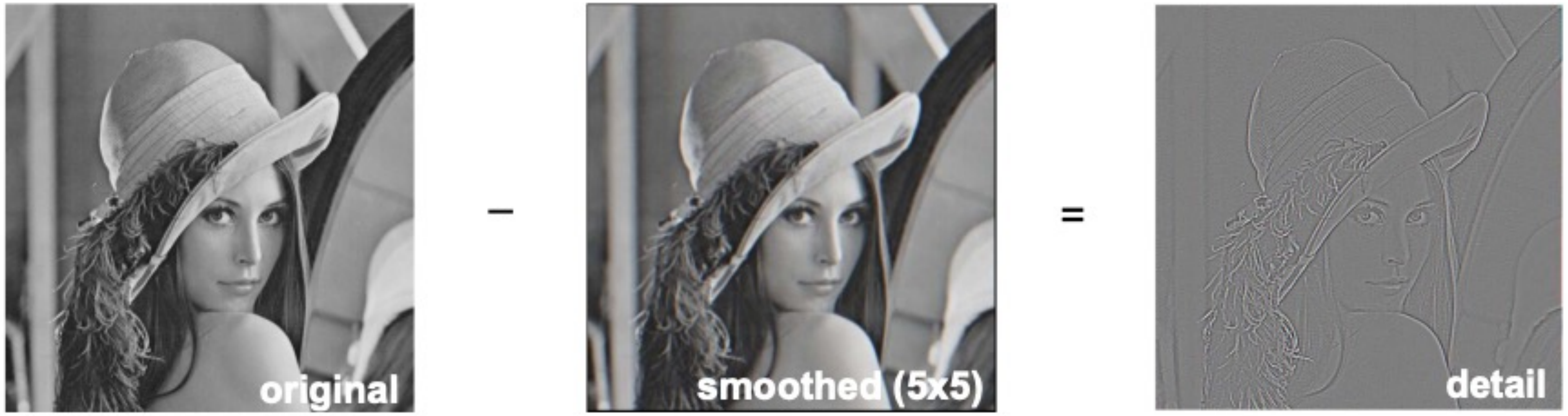


Image Courtesy:NASA

Highboost Filtering

- What does blurring take away?



- Let's add it back:

Unsharp Masking vs Highboost Filtering

USM vs. HBF as Spatial Filters

A=1

w = 9A-1

-1	-1	-1
-1	w	-1
-1	-1	-1

A=2

w = 17

-1	-1	-1
-1	17	-1
-1	-1	-1

- If **A=1**, we get unsharp masking. $I'(u,v) = I(u,v) + \nabla^2 I(u,v)$
- If **A>1**, original image is added back to detail image (HBF).

Corner Cases: Padding

```
M = 3
For each valid location [x,y] in S
        a ← Average of intensities in a M x M neighborhood centered on [x,y]
        D[x,y] = round(a)
```

120	190	140	150	200
17	21	30	8	27
89	123	150	73	56
10	178	140	150	18
190	14	76	69	87

x

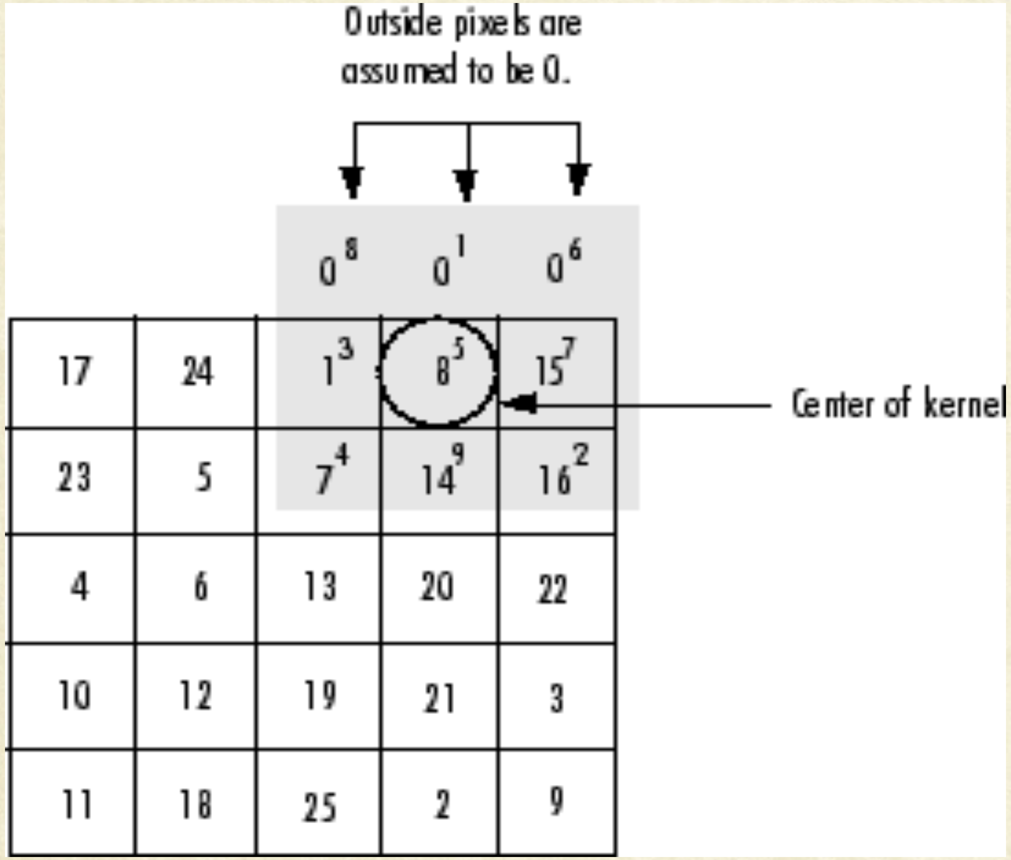
1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

=

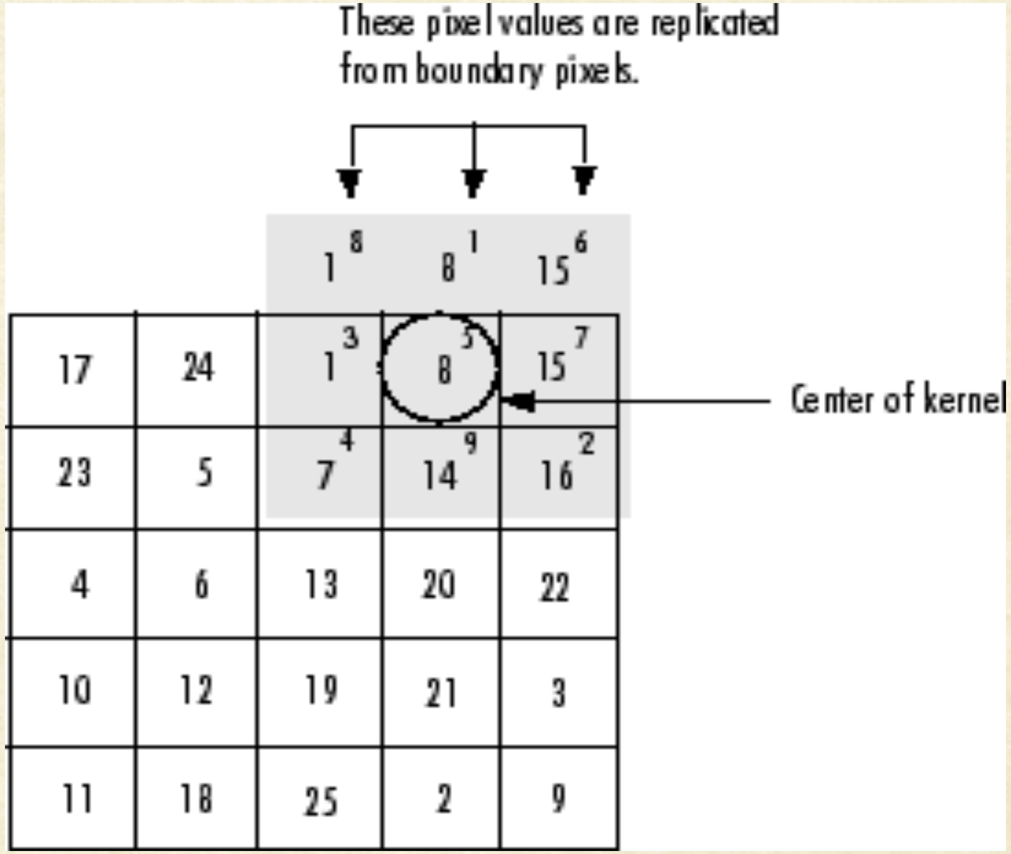
	98			

http://199.116.233.101/index.php?title=File:Animated-kernels.gif

Image Padding



Zero/Constant

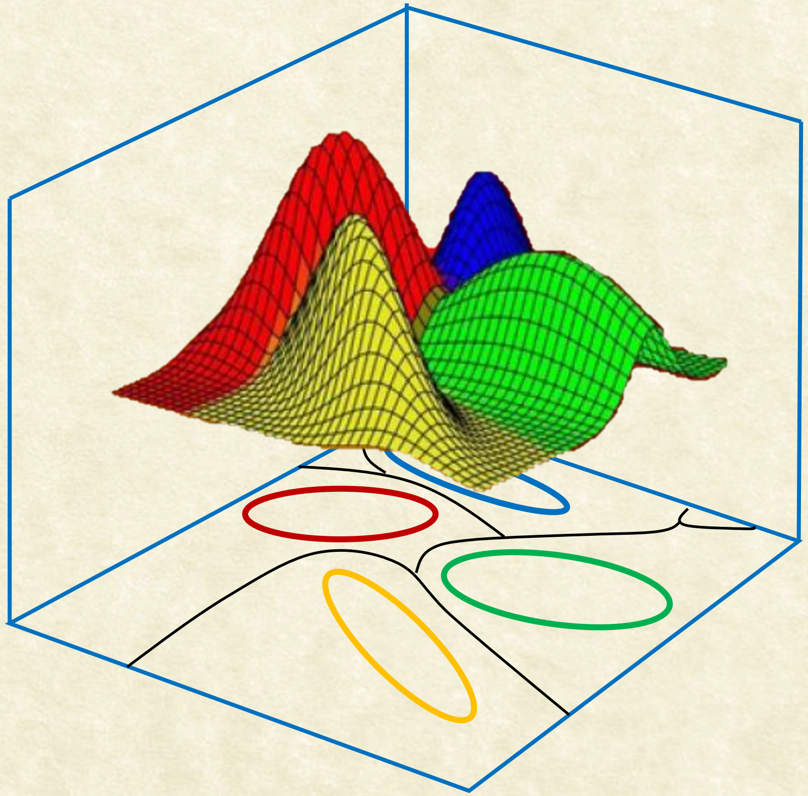


Replicate/Reflect

Questions?

CS7.404: Digital Image Processing

Monsoon 2023: Linear Shift Invariant Systems



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Linear Systems

$$g(x) = T_H(f(x))$$

$$f_1(x) \longrightarrow \boxed{T_H(\,)} \longrightarrow g_1(x) \qquad f_2(x) \longrightarrow \boxed{T_H(\,)} \longrightarrow g_2(x)$$

$$\alpha f_1(x) + \beta f_2(x) \longrightarrow \boxed{T_H(\,)} \longrightarrow \alpha g_1(x) + \beta g_2(x)$$

Shift Invariant Systems

$$f(x) \longrightarrow \boxed{T_H(\;)} \longrightarrow g(x)$$

$$f(x+\delta) \longrightarrow \boxed{T_H(\;)} \longrightarrow g(x+\delta)$$

Linear Shift Invariant Systems

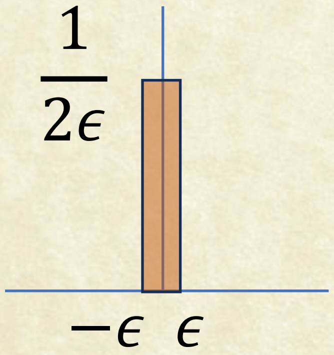
- Systems that are both Linear and Shift Invariant
- All convolution operations are Linear Shift Invariant
- All Linear Shift Invariant Systems can be expressed as Convolutions
- Characterized by Impulse Response or PSF
- Additional Properties
    - Separability

Impulse Response in Practice

$$f(x) \longrightarrow \boxed{T_H(\,)} \longrightarrow g(x)$$

$$f(x+\delta) \longrightarrow \boxed{T_H(\,)} \longrightarrow g(x+\delta)$$

Impulse Response in Practice

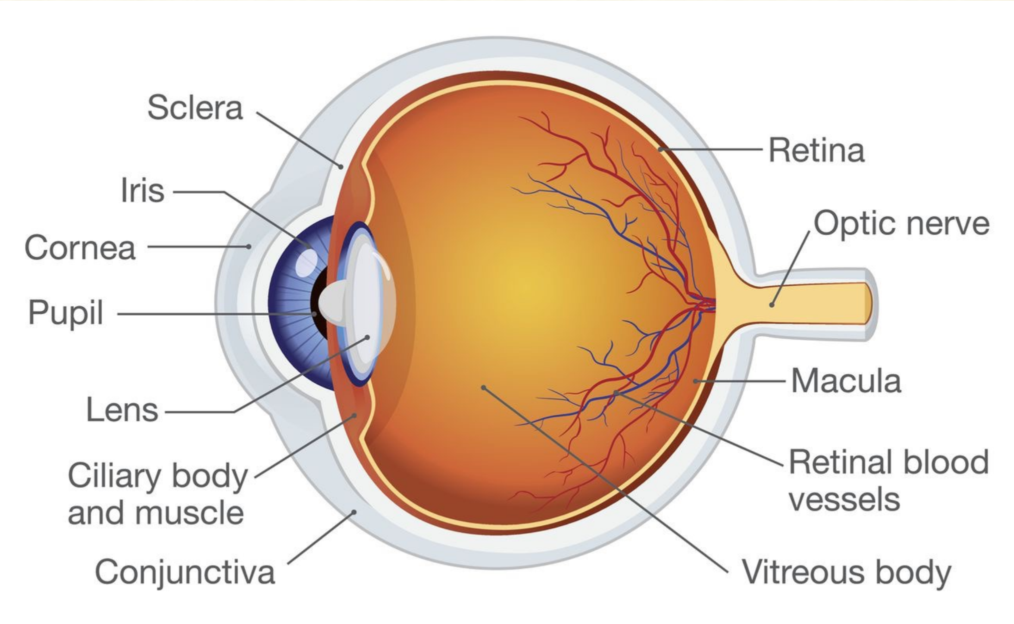


$$I(x) \longrightarrow \boxed{T_H(\,)} \longrightarrow H(x)$$

: IR or PSF

Impulse Response of Human Eye

- Input: Point Light Source ?
  - A bright (distant) star
- System: Cornea, Eye Lens, Vitreous, Retina
- Output: PSF



Homework

1. Show that Gaussian Convolutions are both LSI and Separable

2. Show that Impulse Response can fully characterize a LSIS

1. Gaussian convolutions are a common type of filtering operation used in image processing and signal analysis. To show that Gaussian convolutions are both Linear Shift-Invariant (LSI) and separable, we can break down the properties step by step:

1. Linear Shift-Invariance (LSI):
To demonstrate LSI, we need to show that the Gaussian convolution operation satisfies the principles of linearity and shift-invariance.

- **Linearity:** The Gaussian convolution operation is linear because the response to a sum of inputs is the sum of responses to each individual input. Mathematically, if \(f(x)\) and \(g(x)\) are two functions and \(a\) and \(b\) are constants, then the convolution of \(a \cdot f(x) + b \cdot g(x)\) with a Gaussian kernel is the same as \(a\) times the convolution of \(f(x)\) and the Gaussian kernel, plus \(b\) times the convolution of \(g(x)\) and the Gaussian kernel.

- **Shift-Invariance:** The Gaussian convolution operation is shift-invariant because applying a Gaussian kernel to a shifted version of the input signal results in the same output as applying the Gaussian kernel to the original signal and then shifting the output. In other words, the response to a shifted input is the same as shifting the response to the original input.

2. Separability:
Gaussian convolutions are also separable, which means that a two-dimensional Gaussian convolution can be decomposed into two consecutive one-dimensional convolutions along the two axes. This greatly reduces the computational complexity of the operation.

For example, a 2D Gaussian kernel \(G(x, y)\) can be separated into two 1D Gaussian kernels: \(G(x) \cdot G(y)\). This allows you to first convolve the input with \(G(x)\) along one axis and then convolve the result with \(G(y)\) along the other axis. This property is derived from the separability of the Gaussian function itself.

In conclusion, Gaussian convolutions are both Linear Shift-Invariant (LSI) and separable, making them powerful tools for filtering and processing signals and images efficiently while maintaining important mathematical properties.

2. In a Linear Shift-Invariant (LSI) system, the impulse response indeed fully characterizes the system's behavior. Here's how:

Impulse Response:
The impulse response of an LSI system is the system's output when an impulse function (also known as a Dirac delta function) is input to the system. Mathematically, if (h(t)) represents the impulse response of the system, then the output (y(t)) of the system when an input (x(t)) is applied is given by the convolution of the input with the impulse response:

[y(t) = x(t) * h(t)]

Characterization of LSI System:
The key property of an LSI system is that it preserves linearity and shift-invariance. Let's see how the impulse response characterizes these properties:

1.Linearity: The linearity of the system means that if (x_1(t)) produces (y_1(t)) and (x_2(t)) produces (y_2(t)), then (a \cdot x_1(t) + b \cdot x_2(t)) will produce (a \cdot y_1(t) + b \cdot y_2(t)), where (a) and (b) are constants. When an impulse response (h(t)) characterizes the system, the output to (x(t)) is (y(t) = x(t) * h(t)). The linearity property ensures that (a \cdot y_1(t) + b \cdot y_2(t)) can also be expressed as a convolution:
[a \cdot y_1(t) + b \cdot y_2(t) = a \cdot (x_1(t) * h(t)) + b \cdot (x_2(t) * h(t))]
Since convolution is a linear operation, the above expression becomes:
[= x_1(t) * (a \cdot h(t)) + x_2(t) * (b \cdot h(t)) = x(t) * (a \cdot h(t) + b \cdot h(t))]
Which simplifies to:
[= x(t) * (a + b) \cdot h(t)]
This demonstrates the preservation of linearity.

2. Shift-Invariance: The shift-invariance property states that if (x(t)) produces (y(t)), then (x(t - \tau)) will produce (y(t - \tau)), where (\tau) is a time shift. The convolution with the impulse response (h(t)) ensures this property as well, since a time shift of the input (x(t)) also results in a time shift of the output (y(t)) due to the convolution operation.

In conclusion, the impulse response fully characterizes an LSI system's behavior by encapsulating both its linearity and shift-invariance properties. This makes the impulse response a fundamental tool for analyzing and understanding the behavior of LSI systems.

Questions?

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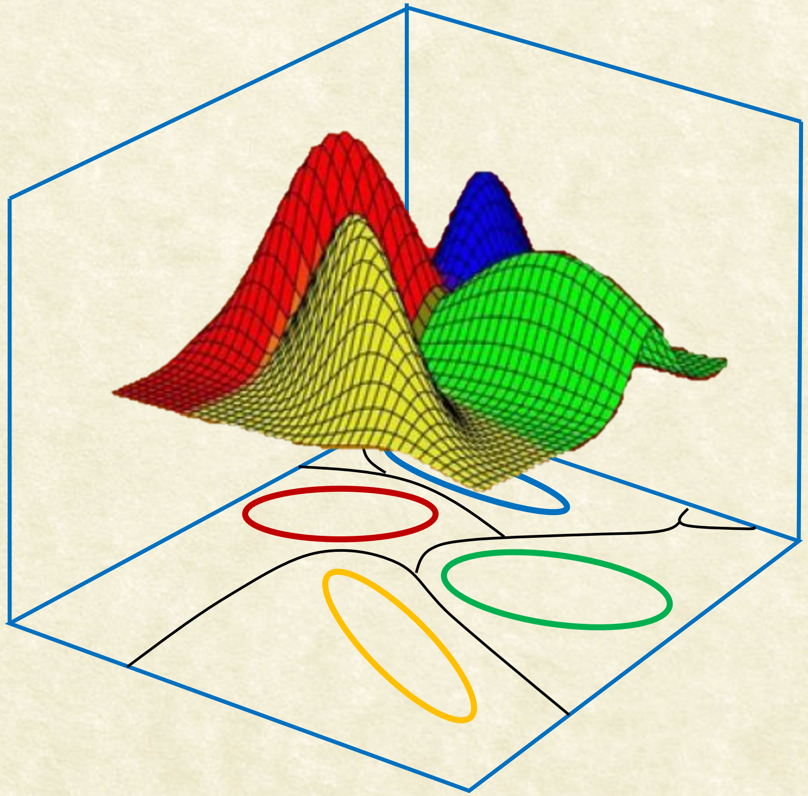
Monsoon 2023: Non-Linear Filters



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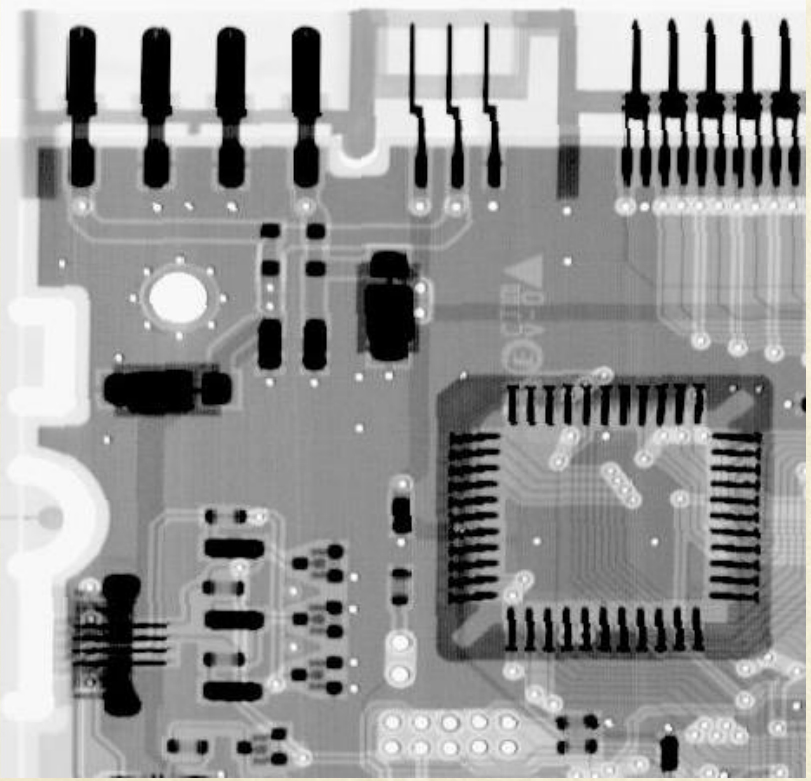
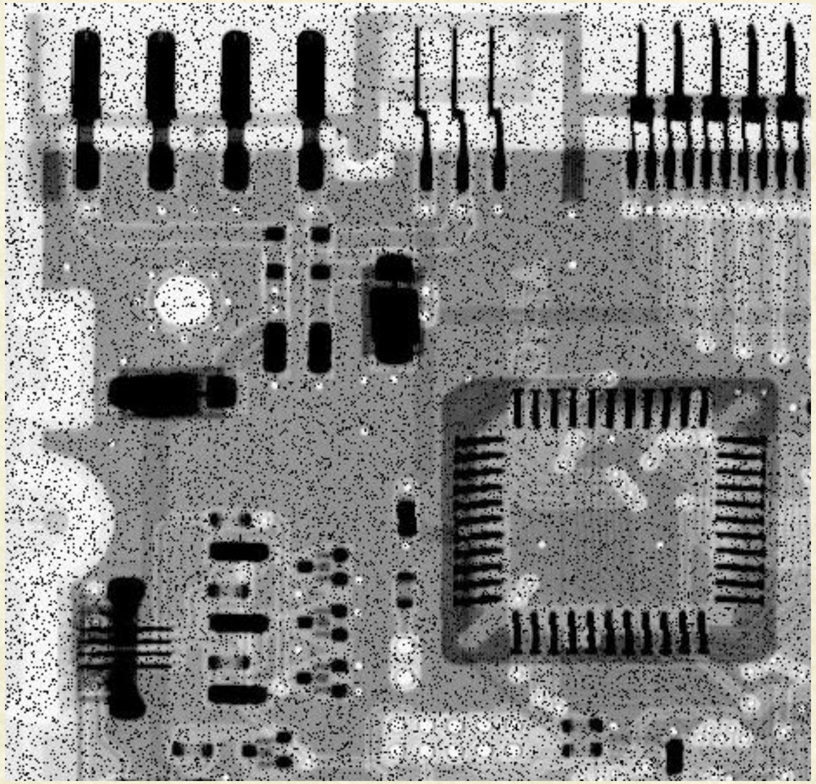
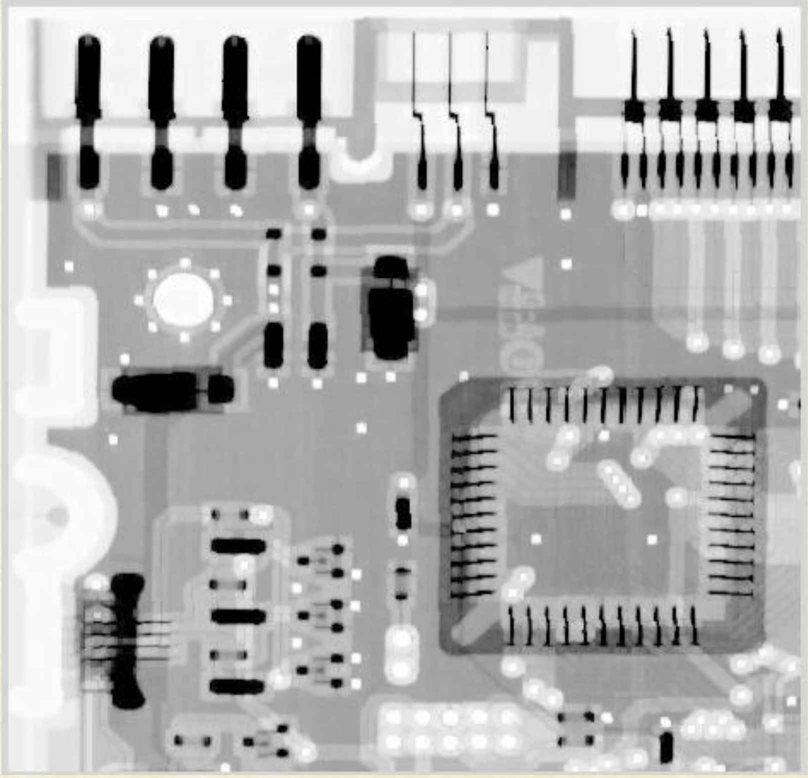
Non-linear Spatial Filters (max)



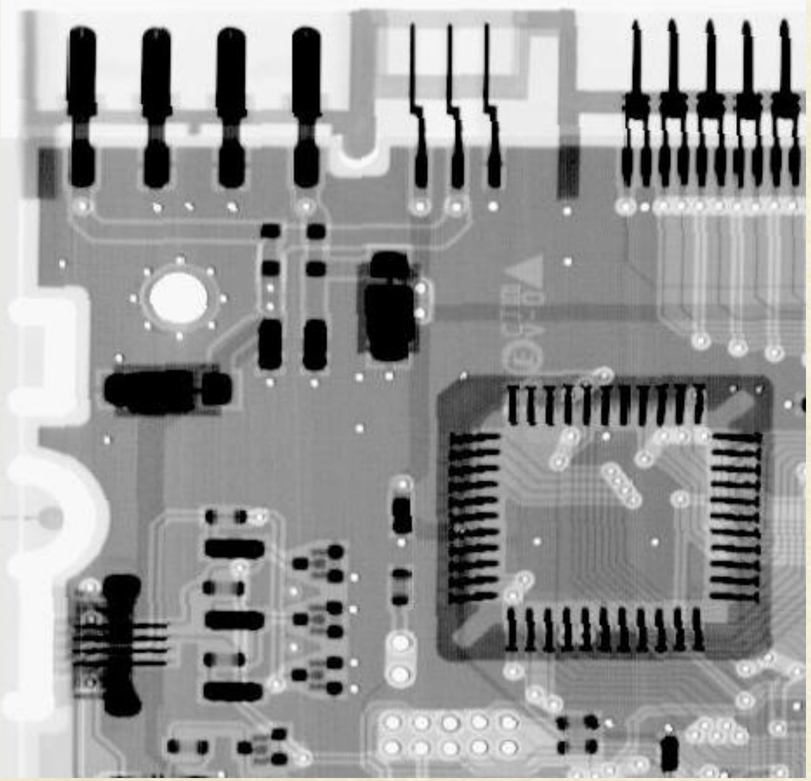
Pepper noise



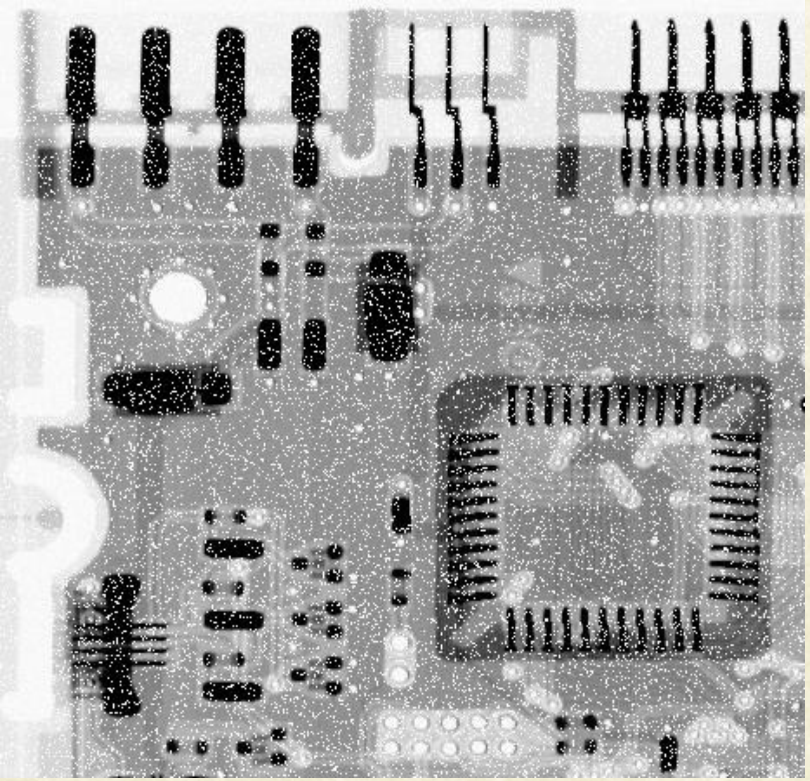
After applying max filter



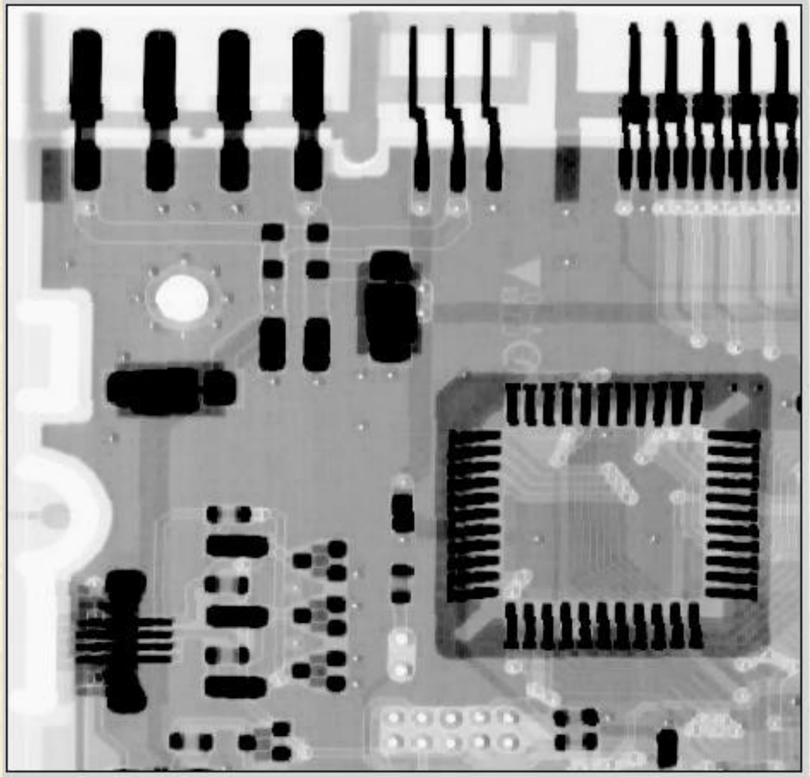
Non-linear Spatial Filters (min)



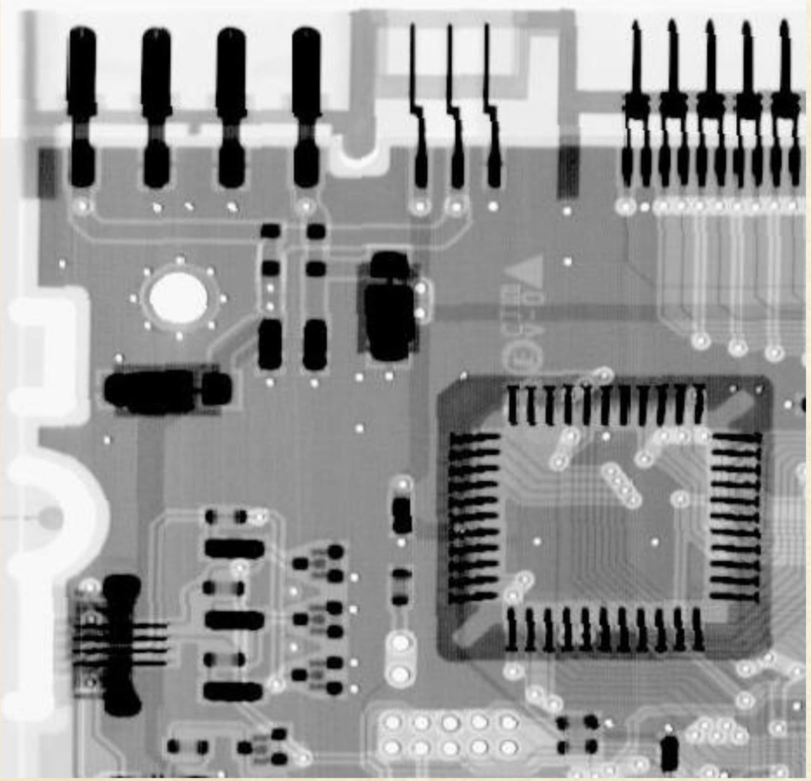
Salt noise



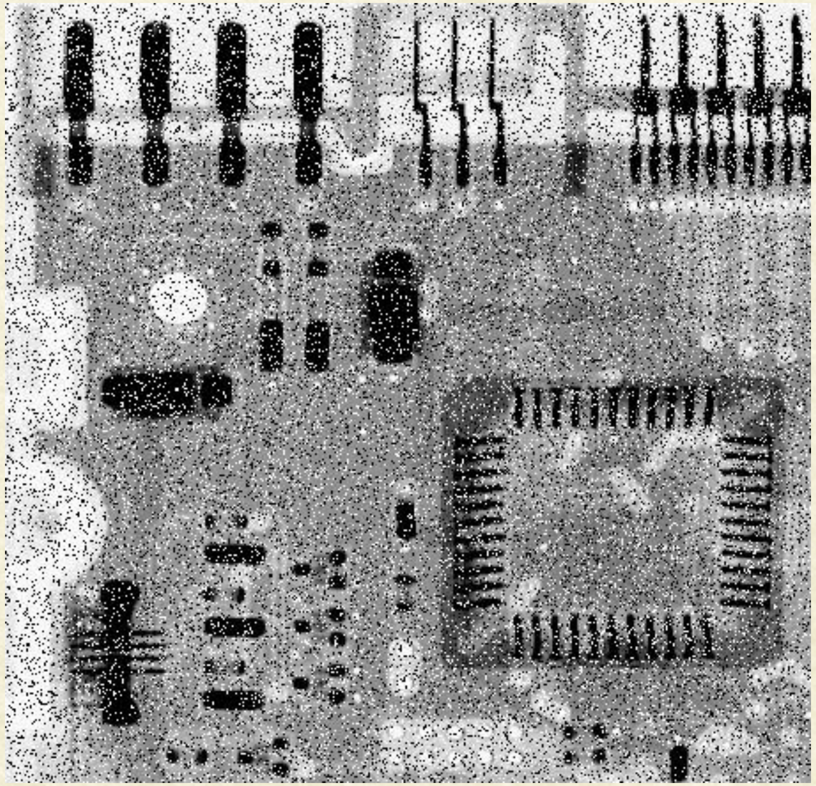
After applying min filter



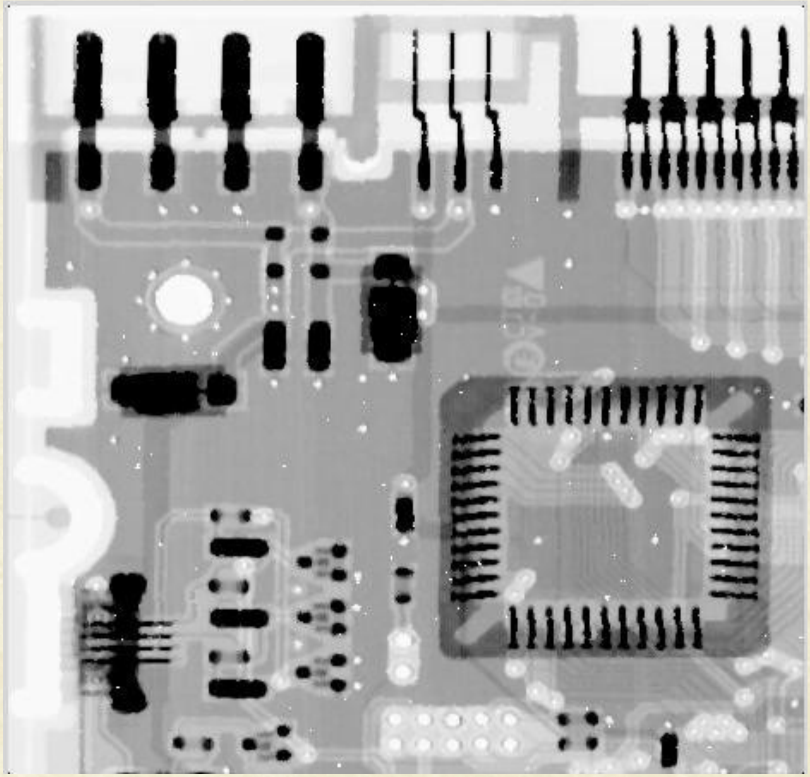
Non-linear Spatial Filters (median)



Salt & Pepper noise



After applying median filter

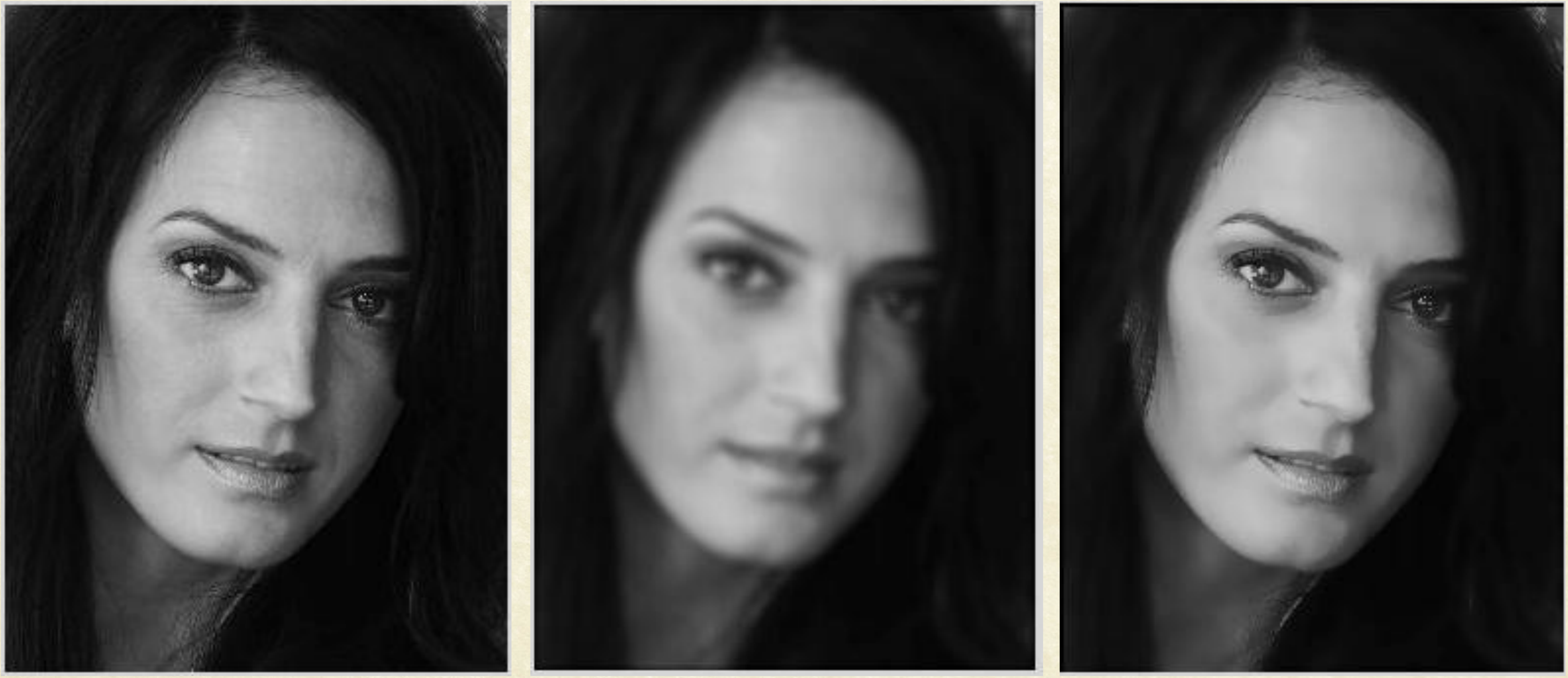


max, min, median → also known as rank / order statistic filters

Other Spatial Filters

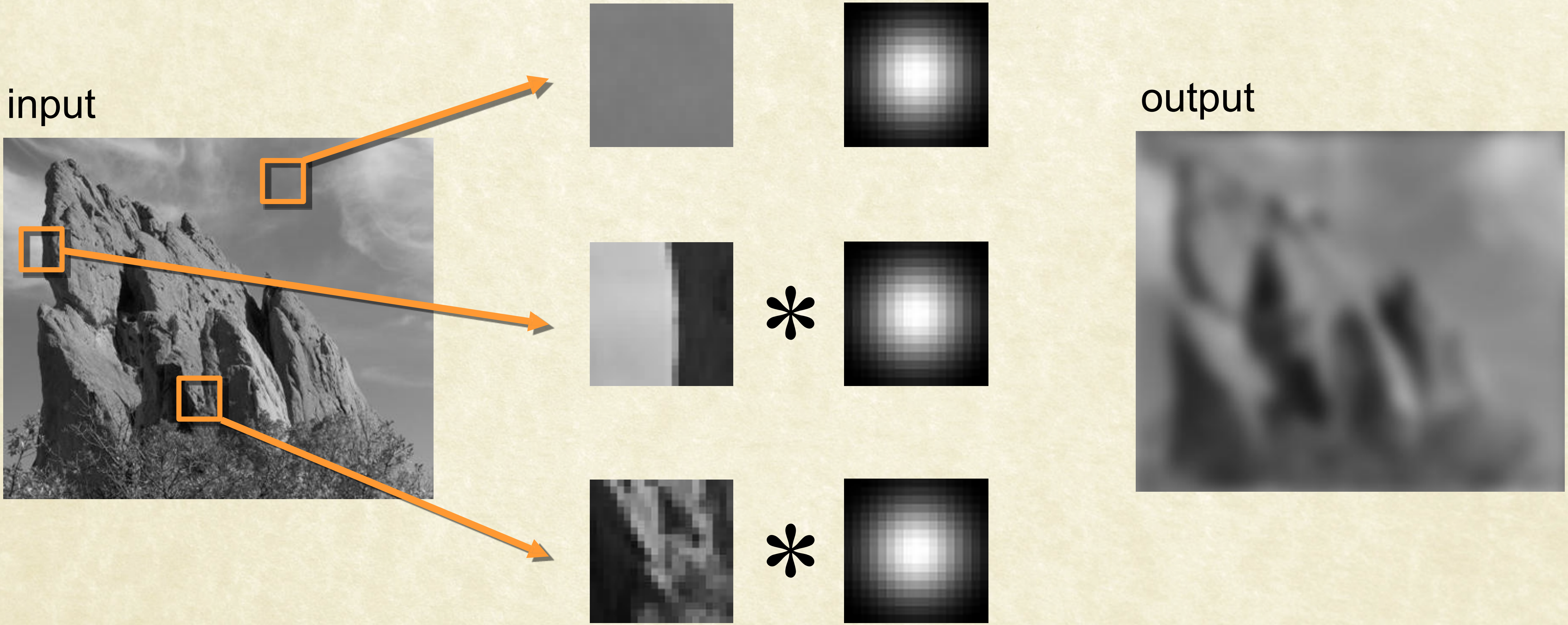
- Geometric mean
- Harmonic mean
- Contra harmonic mean
- Mid Point filter
- Alpha trimmed mean filter
- …..

Bilateral Filtering (Edge preserving smoothing)



Original image taken from cs.cityu.edu.hk

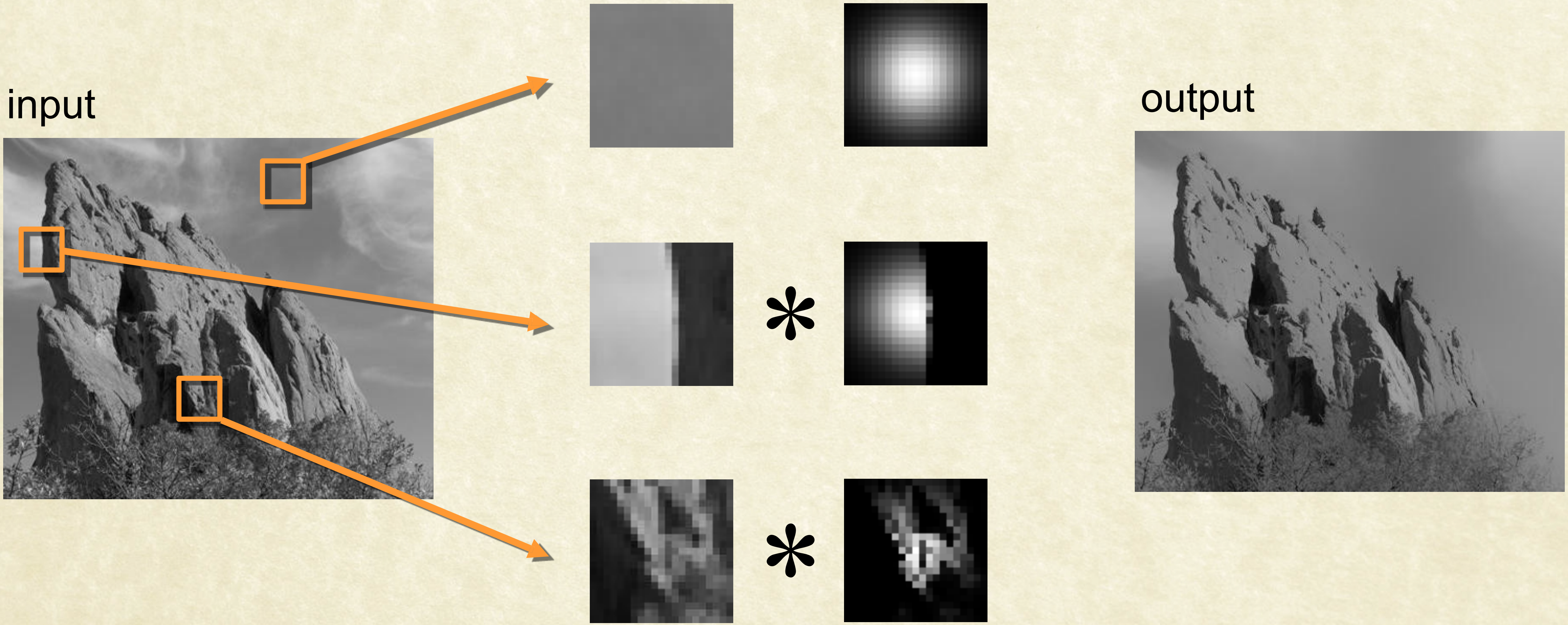
*Usual Gaussian Filtering



Same Gaussian kernel everywhere.

Source: Sylvain Paris

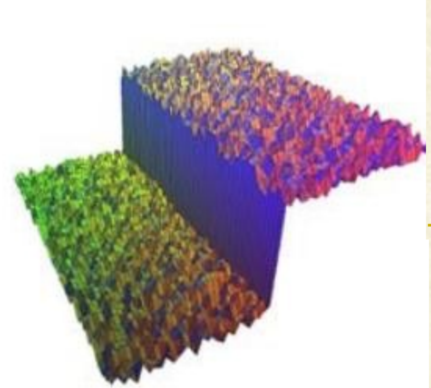
Bilateral Filtering =



The kernel shape depends on the image content.

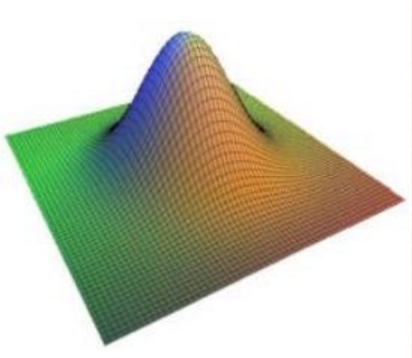
Source: Sylvain Paris

Bilateral Filter



$$g(i,j) = \frac{\sum_{k,l} f(k,l)w(i,j,k,l)}{\sum_{k,l} w(i,j,k,l)}.$$

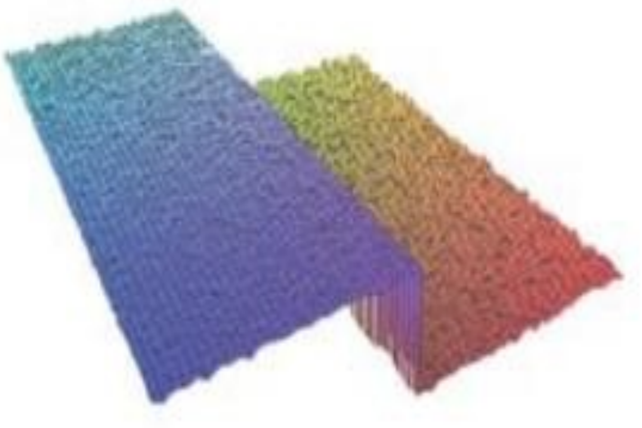
The weighting coefficient $w(i,j,k,l)$ depends on the product of a *domain kernel*

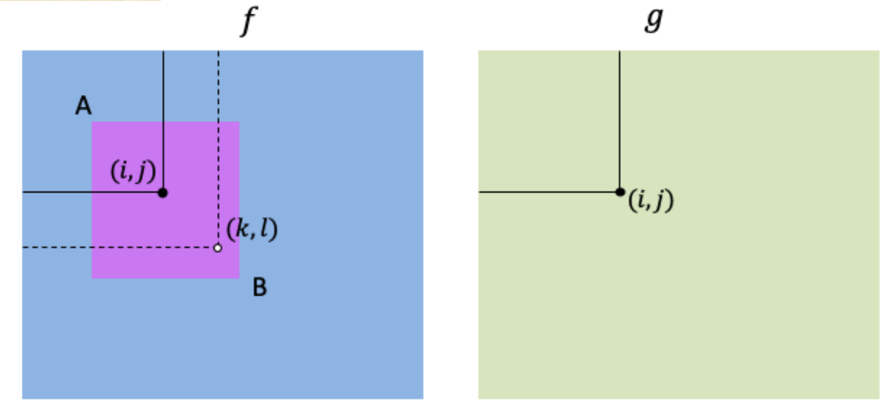


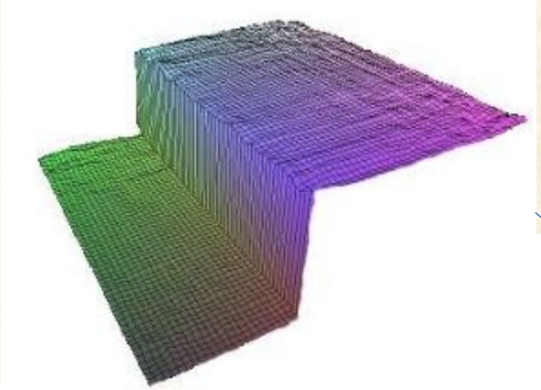
$$d(i,j,k,l) = \exp\left(-\frac{(i-k)^2 + (j-l)^2}{2\sigma_d^2}\right),$$

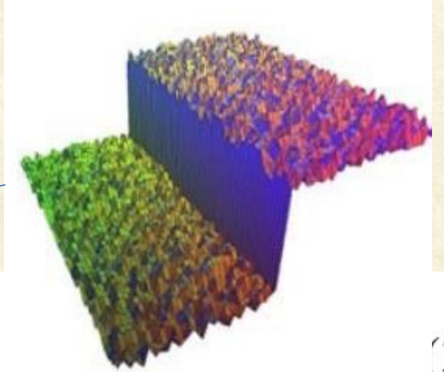
and a data-dependent *range kernel* (Figure 3.19d),

$$r(i,j,k,l) = \exp\left(-\frac{\|f(i,j) - f(k,l)\|^2}{2\sigma_r^2}\right).$$

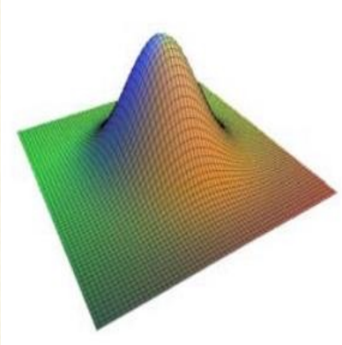








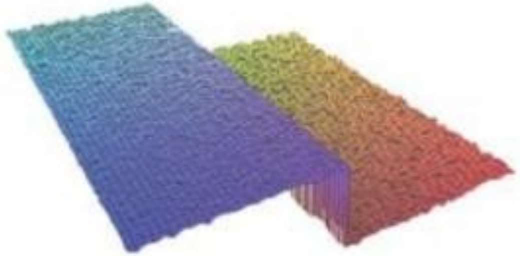
$$g(i,j) = \frac{\sum_{k,l} f(k,l) w(i,j,k,l)}{\sum_{k,l} w(i,j,k,l)}. \tag{3.34}$$



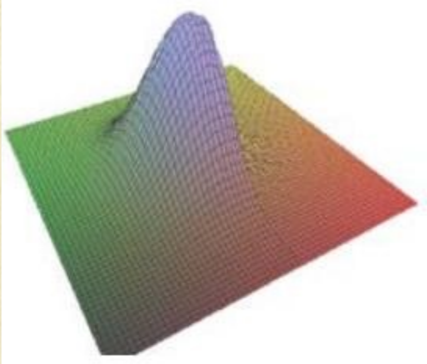
The weighting coefficient $w(i,j,k,l)$ depends on the product of a *domain kernel* (Figure 3.19c),

$$d(i,j,k,l) = \exp\left(-\frac{(i-k)^2 + (j-l)^2}{2\sigma_d^2}\right), \tag{3.35}$$

and a data-dependent *range kernel* (Figure 3.19d),



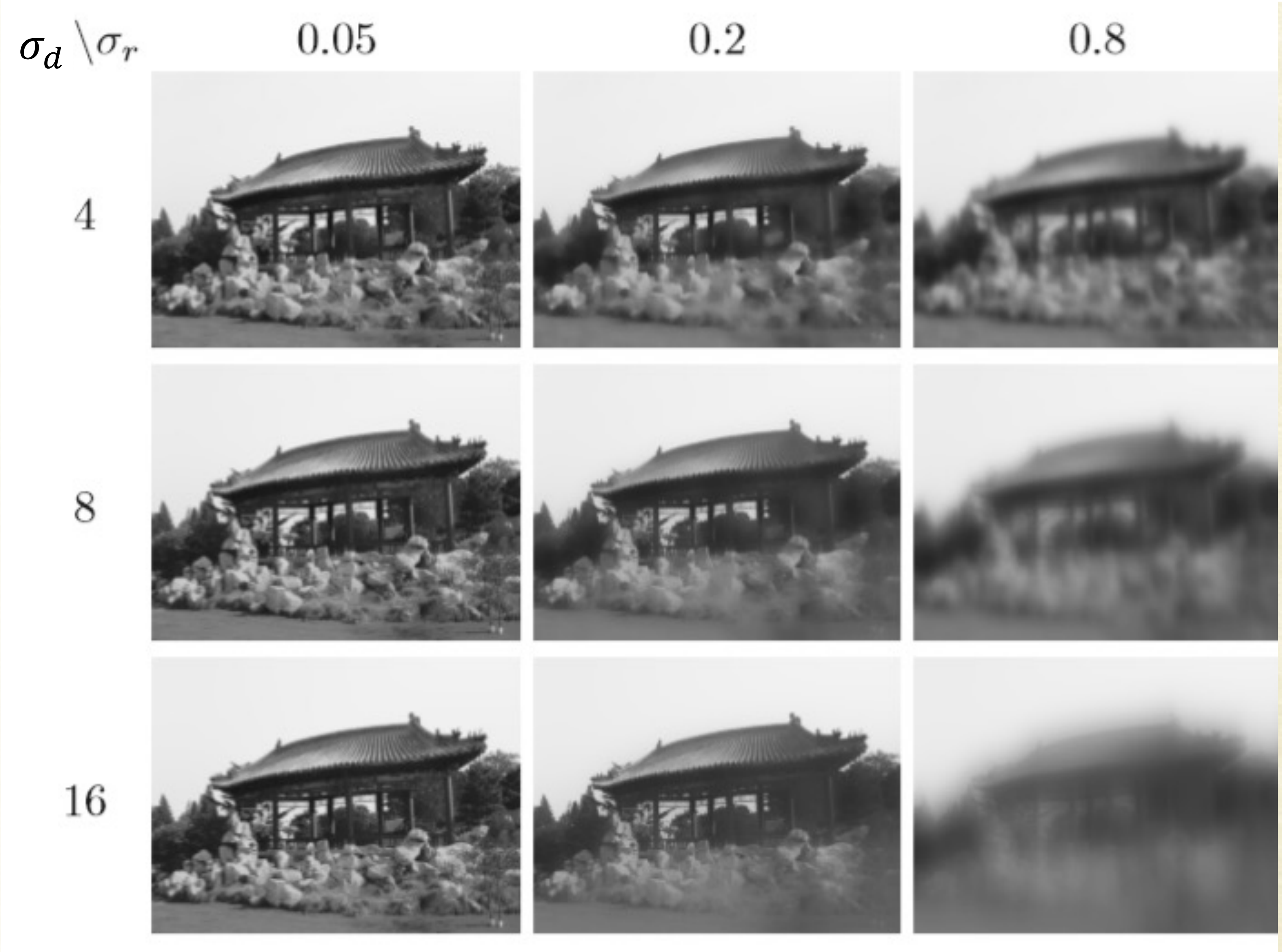
$$r(i,j,k,l) = \exp\left(-\frac{\|f(i,j) - f(k,l)\|^2}{2\sigma_r^2}\right).$$



When multiplied together, these yield the data-dependent *bilateral weight function*

$$w(i,j,k,l) = \exp\left(-\frac{(i-k)^2 + (j-l)^2}{2\sigma_d^2} - \frac{\|f(i,j) - f(k,l)\|^2}{2\sigma_r^2}\right). \tag{3.37}$$

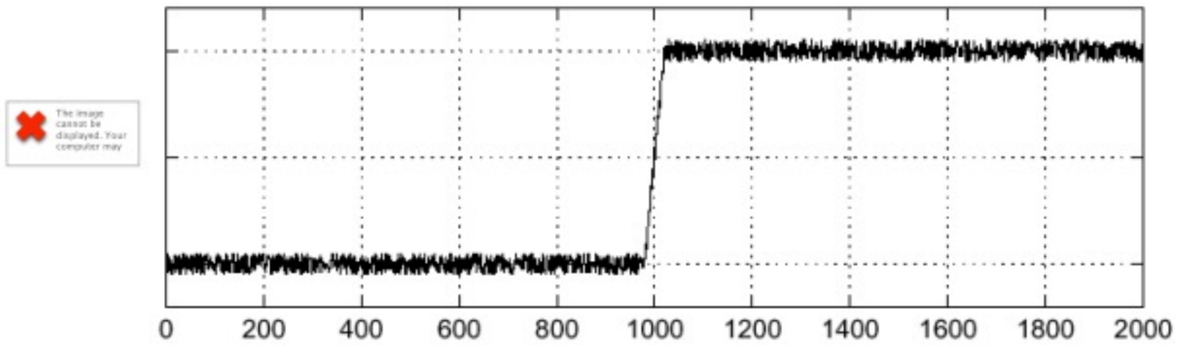
$$w(i,j,k,l) = \exp\left(-\frac{(i-k)^2 + (j-l)^2}{2\sigma_d^2} - \frac{\|f(i,j) - f(k,l)\|^2}{2\sigma_r^2}\right)$$

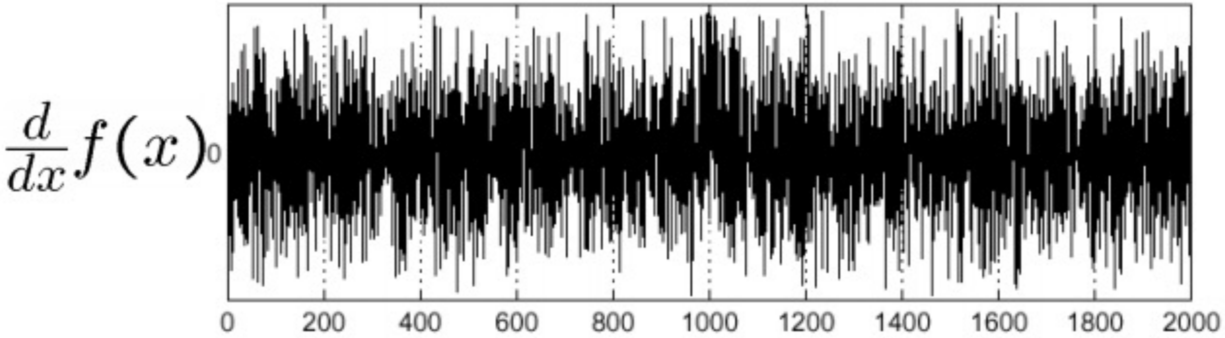


Edge Detection under Noise

Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal

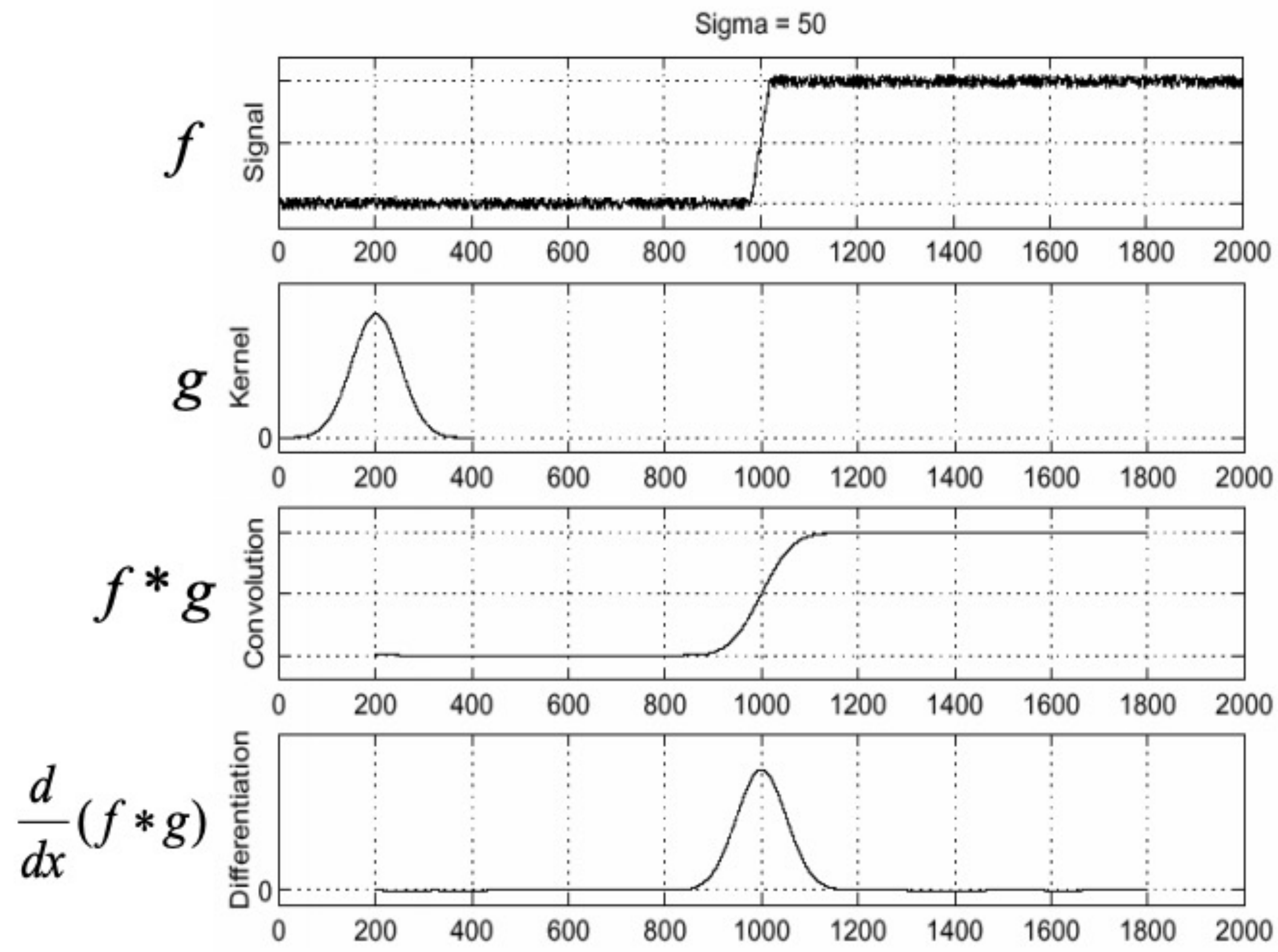




Where is the edge?

Source: S. Seitz

Solution: Smooth First



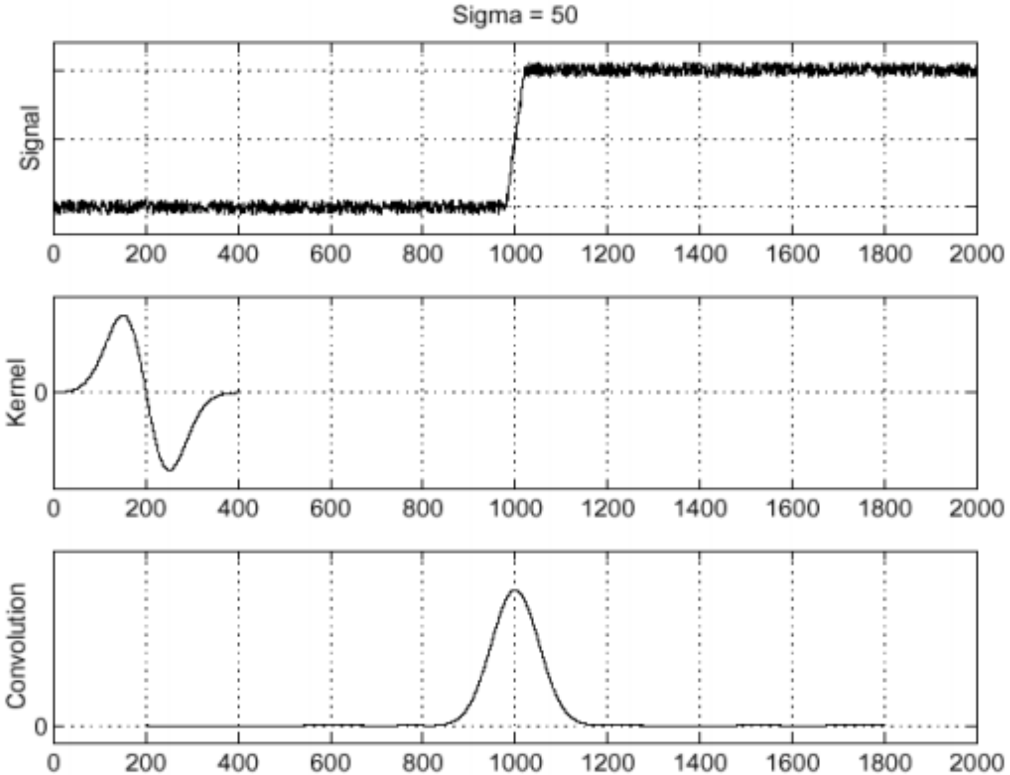
Derivative Theorem of Convolution

- Differentiation is convolution, and convolution is associative: $\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$
- This saves us one operation:

f

$\frac{d}{dx} g$

$f * \frac{d}{dx} g$



Source: S. Seitz

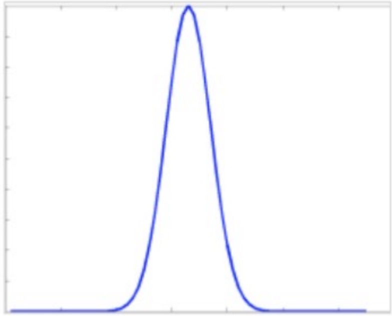
Other Important Filters

- Laplacian of Gaussian (LoG)

Robert Collins
CSE486

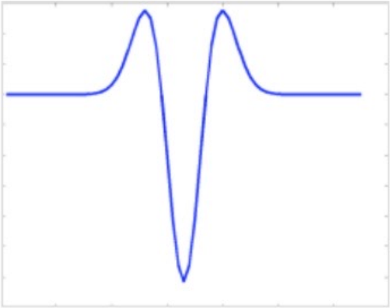
1D Gaussian and Derivatives

$$g(x) = e^{-\frac{x^2}{2\sigma^2}}$$



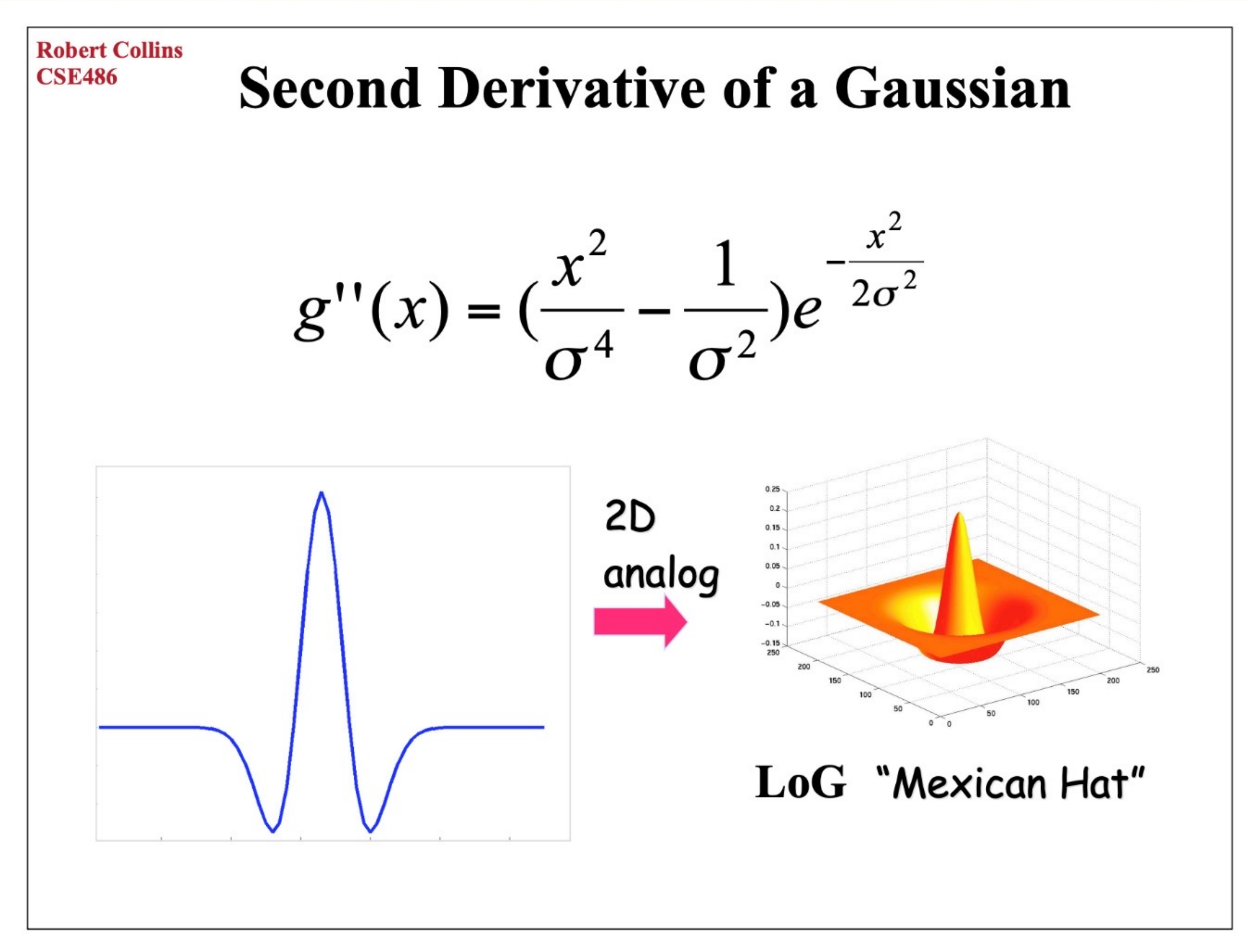
$$g'(x) = -\frac{1}{2\sigma^2} 2xe^{-\frac{x^2}{2\sigma^2}} = -\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$$

$$g''(x) = (\frac{x^2}{\sigma^4} - \frac{1}{\sigma^2}) e^{-\frac{x^2}{2\sigma^2}}$$



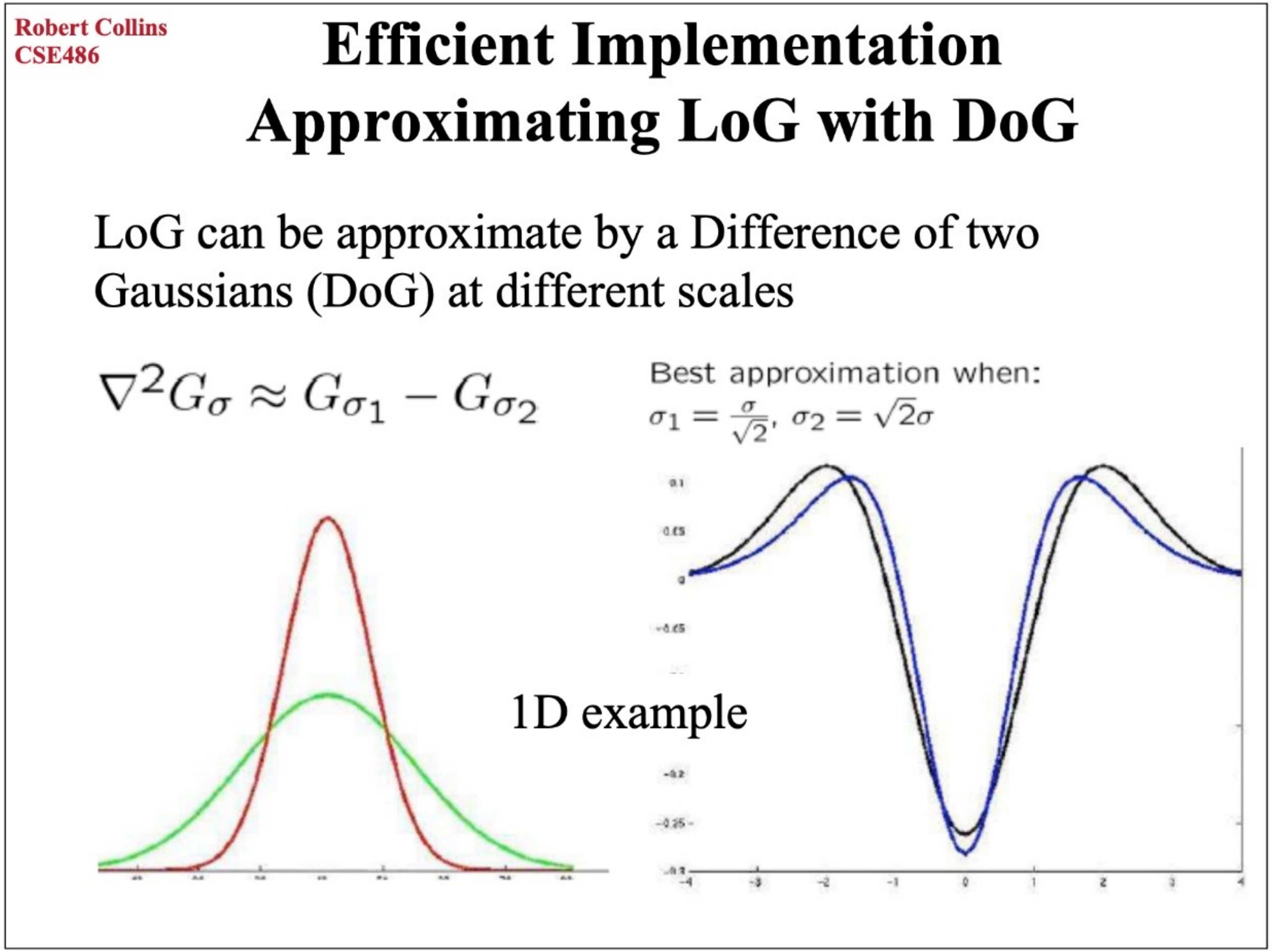
Other Important Filters

- Laplacian of Gaussian (in 2D)



Other Important Filters

- Laplacian of Gaussian (in 2D)
- Difference of Gaussian (DoG)



Questions?