Robotics: Planning and Navigation - Motion planning for UAV

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I. Rotary wing UAV

Figure: Multi-rotor VTOL

II. Fixed wing UAV

Figure: Fixed wing UAV

Key elements in Unmanned operation

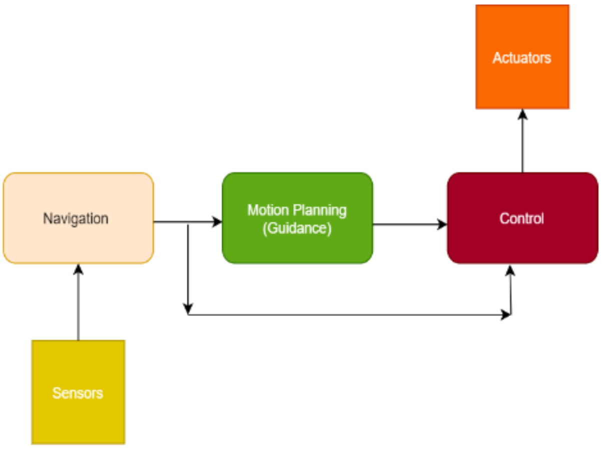


Figure: Navigation, Guidance and Control

Inertial Frame

Position (x, y, z), Velocity (v_x, v_y, v_z), Acceleration (a_x, a_y, a_z).

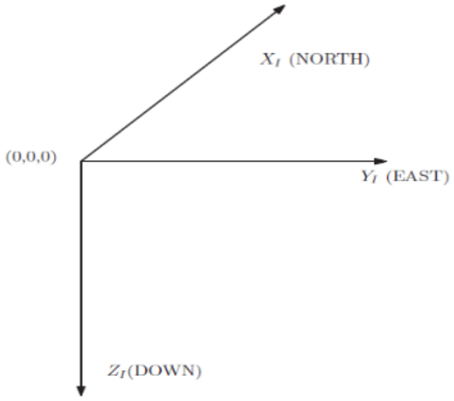


Fig. 1. Diagram showing inertial $X_I Y_I Z_I$ co-ordinate sytem

Velocity Control

Motion in 2D plane $(X_I - Y_I)$.
Altitude $(h = -z)$ is constant, i.e. $\frac{dh}{dt} = \dot{h} = 0$.
Current position $(x,\ y)$ and the desired position is $(x_d,\ y_d)$.

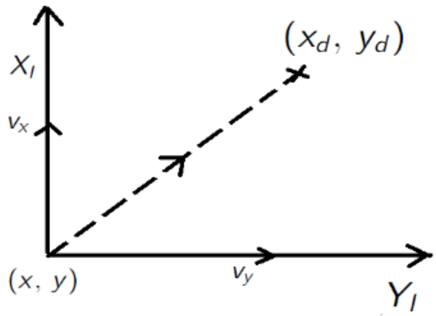
$$\frac{dx}{dt} = \dot{x} = v_x \tag{1}$$

$$\frac{dy}{dt} = \dot{y} = v_y \tag{2}$$

Input is $(v_x,\ v_y)$ and output is $(x,\ y)$.
NB: The desired $(v_x,\ v_y)$ is tracked by inner loop control system.

Velocity Control - Reaching a point

$$v_x = k(x_d - x) \tag{3}$$
$$v_y = k(y_d - y) \tag{4}$$



Acceleration Control

$$\frac{dx}{dt} = \dot{x} = v_x \tag{5}$$
$$\frac{d^2x}{dt^2} = \ddot{x} = a_x \tag{6}$$

$$\frac{dy}{dt} = \dot{y} = v_y \tag{7}$$
$$\frac{d^2y}{dt^2} = \ddot{y} = a_y \tag{8}$$

Reaching a point (x_d, y_d).

$$a_x = k_1(x_d - x) - k_2 v_x, \; a_y = k_1(y_d - y) - k_2 v_y \tag{9}$$

Acceleration Control

Trajectory $(x(t), y(t))$

$$\ddot{x} + k_2\dot{x} + k_1 x = k_1 x_d, \; \ddot{y} + k_2\dot{y} + k_1 y = k_1 y_d \tag{10}$$

At steady-state, $\ddot{x} = \dot{x} = 0 \implies x = x_d$ and $\ddot{y} = \dot{y} = 0 \implies y = y_d$.

Velocity tracking through acceleration control

Desired velocity (v_{xd}, v_{yd})

$$\frac{d^2x}{dt^2} = \ddot{x} = \frac{dv_x}{dt} = a_x \tag{11}$$

$$a_x = k_3(v_{xd} - v_x) \tag{12}$$

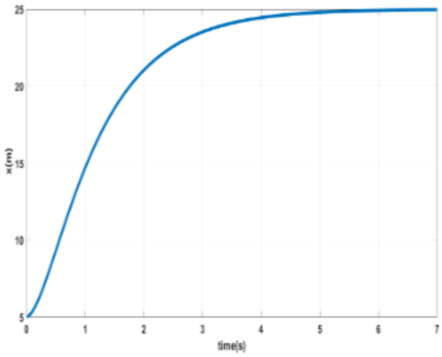
$$\frac{dv_x}{dt} = k_3(v_{xd} - v_x) \tag{13}$$

$$\frac{dv_x}{dt} + k_3 v_x = k_3 v_{xd} \tag{14}$$

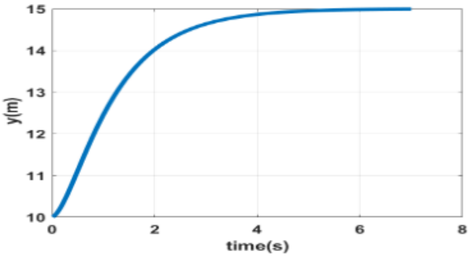
At steady-state $dv_x/dt = 0 \implies v_x = v_{xd}$

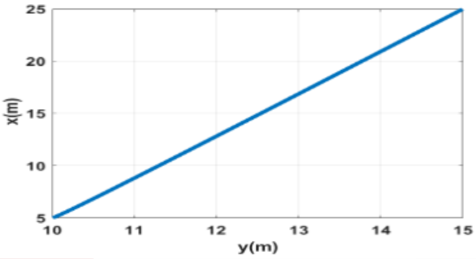
Example

$(x, y) = (5, 10)$, $(v_x, v_y) = (1, 0.5)$,
$(x_d, y_d) = (25, 15)$, $k_1 = 3$, $k_2 = 4$.



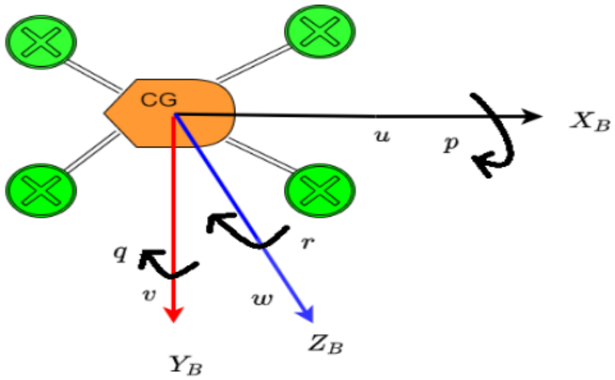
Example





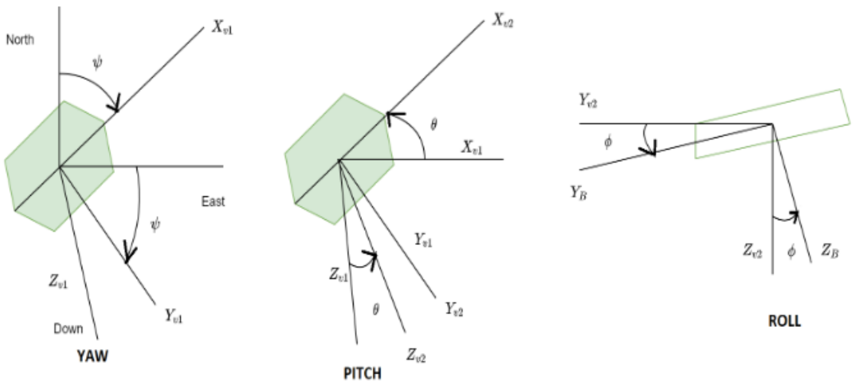
Variables- Body Frame

- Velocity: $V_B = [u,\ v,\ w]^T$
- Acceleration: $\dot{u} = \frac{du}{dt},\ \dot{v} = \frac{dv}{dt},\ \dot{w} = \frac{dw}{dt}$
- Angular velocity: $\Omega = [p,\ q,\ r]^T$



Euler angles - $\phi,\ \theta,\ \psi$

First rotation (i to v_1)- $\dot{\phi} = 0,\ \dot{\theta} = 0,\ \dot{\psi} \neq 0$
Second rotation (v_1 to v_2) - $\dot{\phi} = 0,\ \dot{\theta} \neq 0,\ \dot{\psi} = 0$
Third rotation (v_2 to B) - $\dot{\phi} \neq 0,\ \dot{\theta} = 0,\ \dot{\psi} = 0$



Euler angles and p,q,r

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} \dot{\phi} \\ 0 \\ 0 \end{pmatrix} + R(\phi)_{v_2}^{b} \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix} + R(\phi)_{v_2}^{b} R(\theta)_{v_1}^{v_2} \begin{pmatrix} 0 \\ 0 \\ \dot{\psi} \end{pmatrix} \quad (15)$$

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 & 0 & -sin(\theta) \\ 0 & cos(\phi) & sin(\phi)cos(\theta) \\ 0 & -sin(\phi) & cos(\phi)cos(\theta) \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} \quad (16)$$

Euler angles and p,q,r

Remark: Rotating about only one of the body axis will directly relate $\dot{\phi} = p\,(q = r = 0)$; $\dot{\theta} = q\,(\phi = 0, p = r = 0)$ and $\dot{\psi} = r\,(\phi = \theta = 0,\ p = q = 0)$.

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & sin(\phi)tan(\theta) & cos(\phi)tan(\theta) \\ 0 & cos(\phi) & -sin(\phi) \\ 0 & sin(\phi)sec(\theta) & cos(\phi)sec(\theta) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \end{pmatrix} \quad (17)$$

why this is important?
Ans. We cannot measure ϕ, θ, ψ directly using a sensor. But we can always measure p, q, r using 3-axis gyroscope mounted on the UAV and integrate the above to get ϕ, θ, ψ.

Connecting inertial and body axis velocities

How do we use the knowledge of ϕ, θ, ψ?

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = R(\phi)^{b}_{v_2} R(\theta)^{v_2}_{v_1} R(\psi)^{v_1}_{i} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \quad (18)$$

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = [R(\phi)^{b}_{v_2} R(\theta)^{v_2}_{v_1} R(\psi)^{v_1}_{i}]^{-1} \begin{pmatrix} u \\ v \\ w \end{pmatrix} \quad (19)$$

How do we get u, v, w ?

Equation of Coriolis

$$V_B = [u,\ v,\ w]^T,\ \Omega = [p,\ q,\ r]^T,\ F = [F_{XB},\ F_{YB},\ F_{ZB}]^T.$$

- Equation of Coriolis: Rate of change of a vector (that is defined in a rotating frame)in inertial frame $=$ Rate of change of the vector in the rotating frame $+$ change due to relative angular velocity between the inertial frame and the rotating frame.

$$(\frac{dV_B}{dt})_I = (\frac{dV_B}{dt})_B + \Omega \times V_B = \frac{F}{m} \quad (20)$$

Translational Dynamics

$V_B = [u,\ v,\ w]^T,\ \Omega = [p,\ q,\ r]^T,\ F = [F_{XB},\ F_{YB},\ F_{ZB}]^T.$

$$(\frac{dV_B}{dt})_I = (\frac{dV_B}{dt})_B + \Omega \times V_B = \frac{F}{m} \qquad (21)$$

$$\dot{u} = rv - qw + \frac{F_{XB}}{m} \qquad (22)$$

$$\dot{v} = pw - ru + \frac{F_{YB}}{m} \qquad (23)$$

$$\dot{w} = qu - pv + \frac{F_{ZB}}{m} \qquad (24)$$

Time-Scale Separation

Basic Idea: Faster variable changes and goes to steady value when compared to the slower variable.

- p,q,r
- $\phi,\ \theta,\ \psi$
- $\dot{u}$, $\dot{v}$ and $\dot{w}$ (a_x, a_y, a_z)
- u, v, w and (v_x, v_y, v_z)
- x,y,z

Simplification

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = R(\phi, \theta, \psi) \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{25}$$

$$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} = R(\phi, \theta, \psi) \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} + \dot{R}(\phi, \theta, \psi) \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{26}$$

$\dot{R}(\phi, \theta, \psi) \approx 0$

Implementation of acceleration command

Compute the desired angles θ_d, ϕ_d and the thrust T_d for a given ψ using the below equations. Alternatively, we can also compute desired θ_d, ψ_d, T_d for a given ϕ (or) compute desired ϕ_d, ψ_d, T_d for a given θ.

$$a_x = (-cos\phi sin\theta cos\psi - sin\phi sin\psi)\frac{T}{m} \tag{27}$$

$$a_y = (-cos\phi sin\theta sin\psi - sin\phi cos\psi)\frac{T}{m} \tag{28}$$

$$a_z = g - (cos\phi cos\theta)\frac{T}{m} \tag{29}$$

NB: The angles θ, ϕ, ψ and the thrust T can be controlled precisely.

THANK YOU