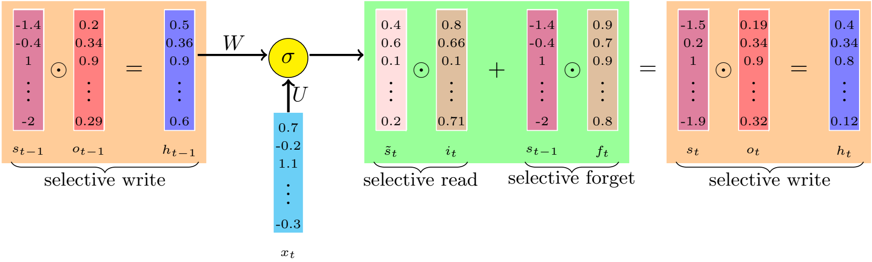
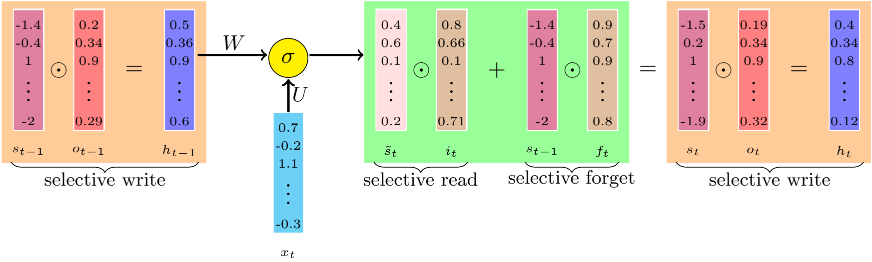
Module 15.3: How LSTMs avoid the problem of vanishing gradients



Intuition

- During forward propagation the gates control the flow of information



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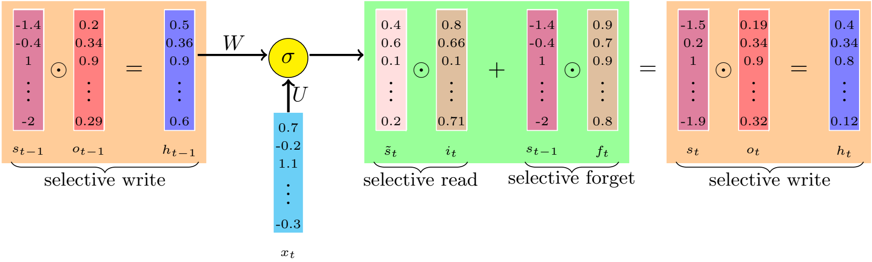
- During forward propagation the gates control the flow of information
- They prevent any irrelevant information from being written to the state



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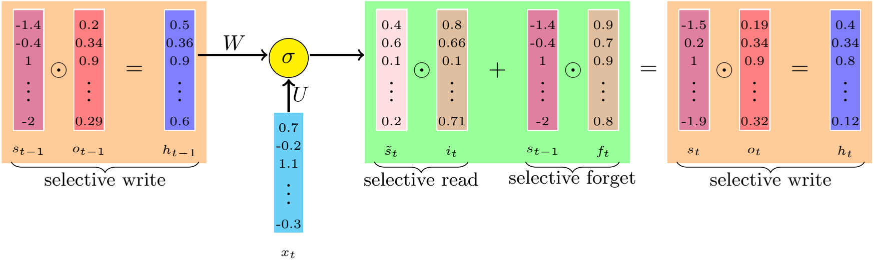
- Similarly during backward propagation they control the flow of gradients



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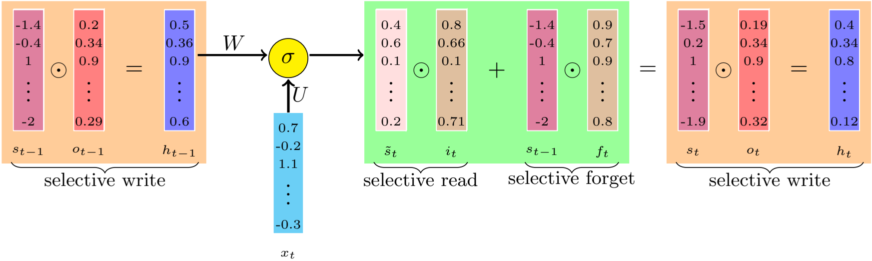
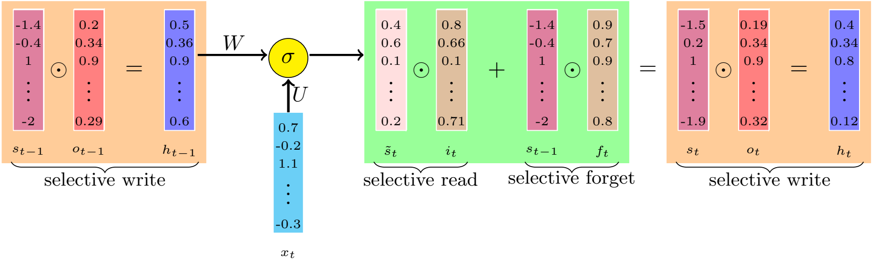
- Similarly during backward propagation they control the flow of gradients
- It is easy to see that during backward pass the gradients will get multiplied by the gate



- If the state at time $t-1$ did not contribute much to the state at time t (i.e., if $\|f_t\| \to 0$ and $\|o_{t-1}\| \to 0$) then during backpropagation the gradients flowing into s_{t-1} will vanish



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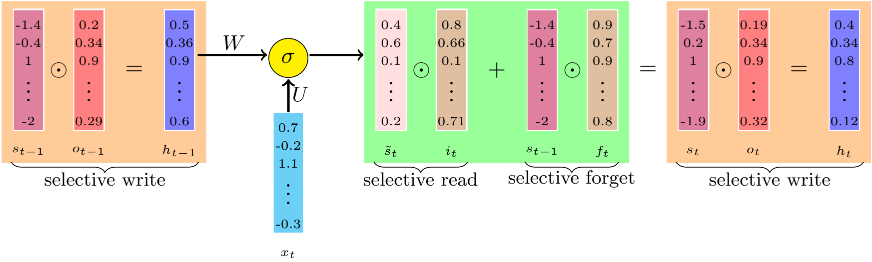
- But this kind of a vanishing gradient is fine (since s_{t-1} did not contribute to s_t we don't want to hold it responsible for the crimes of s_t)



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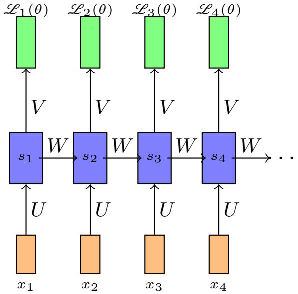
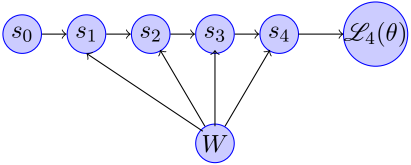
- But this kind of a vanishing gradient is fine (since s_{t-1} did not contribute to s_t we don't want to hold it responsible for the crimes of s_t)
- The key difference from vanilla RNNs is that the flow of information and gradients is controlled by the gates which ensure that the gradients vanish only when they should (i.e., when s_{t-1} didn't contribute much to s_t)

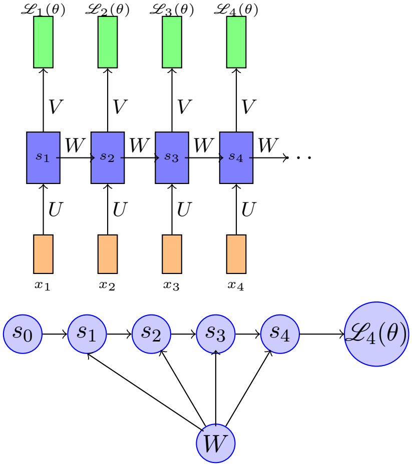
We will now see an illustrative proof of how the gates control the flow of gradients





- Recall that RNNs had this multiplicative term which caused the gradients to vanish

$$\frac{\partial \mathscr{L}_t(\theta)}{\partial W} = \frac{\partial \mathscr{L}_t(\theta)}{\partial s_t} \sum_{k=1}^{t} \color{red}{\prod_{j=k}^{t-1} \frac{\partial s_{j+1}}{\partial s_j}} \color{black}{\frac{\partial^+ s_k}{\partial W}}$$



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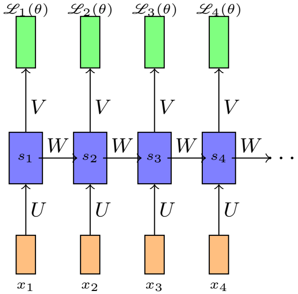
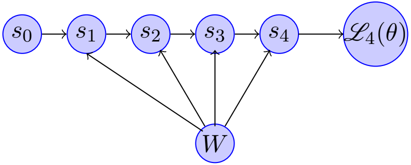
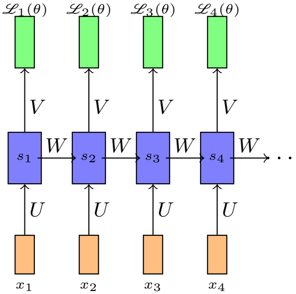
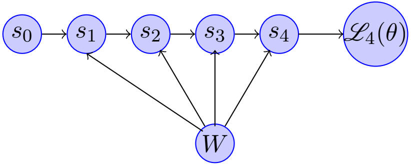
- In particular, if the loss at $\mathscr{L}_4(\theta)$ was high because W was not good enough to compute s_1 correctly then this information will not be propagated back to W as the gradient $\frac{\partial \mathscr{L}_t(\theta)}{\partial W}$ along this long path will vanish



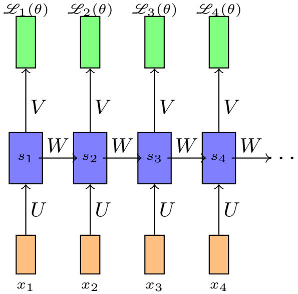


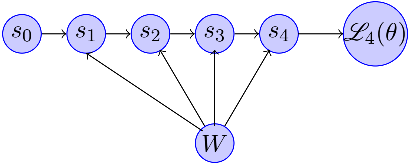
- In general, the gradient of $\mathscr{L}_t(\theta)$ w.r.t. θ_i vanishes when the gradients flowing through **each and every path** from $L_t(\theta)$ to θ_i vanish.





- In general, the gradient of $\mathscr{L}_t(\theta)$ w.r.t. θ_i vanishes when the gradients flowing through **each and every path** from $L_t(\theta)$ to θ_i vanish.
- On the other hand, the gradient of $\mathscr{L}_t(\theta)$ w.r.t. θ_i explodes when the gradient flowing through **at least one path** explodes.





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- We will first argue that in the case of LSTMs there exists at least one path through which the gradients can flow effectively (and hence no vanishing gradients)

- We will start with the dependency graph involving different variables in LSTMs

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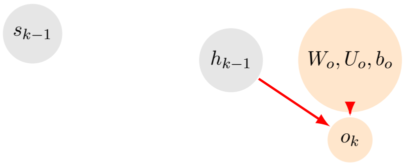

- We will start with the dependency graph involving different variables in LSTMs
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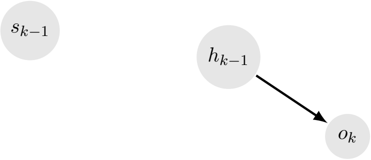
$$o_k = \sigma(W_o h_{k-1} + U_o x_k + b_o)$$



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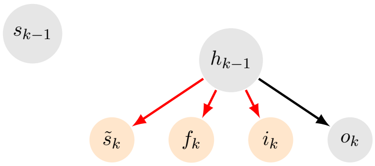

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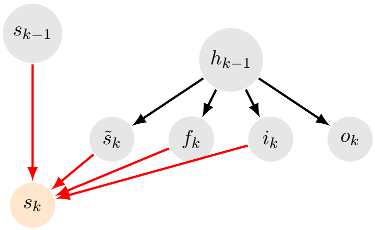
$$i_k = \sigma(W_i h_{k-1} + U_i x_k + b_i)$$
$$f_k = \sigma(W_f h_{k-1} + U_f x_k + b_f)$$
$$\tilde{s_k} = \sigma(W h_{k-1} + U x_k + b)$$



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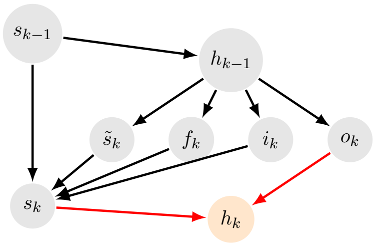
$$s_k = f_k \odot s_{k-1} + i_k \odot \tilde{s_k}$$



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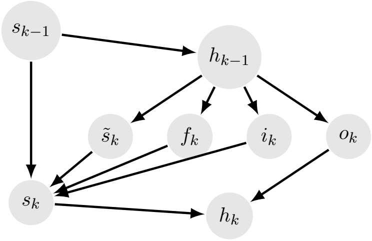
$$s_k = f_k \odot s_{k-1} + i_k \odot \tilde{s_k}$$
$$h_k = o_k \odot \sigma(s_k)$$



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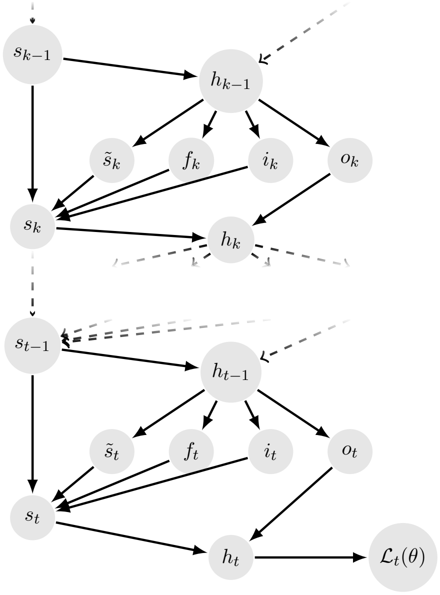

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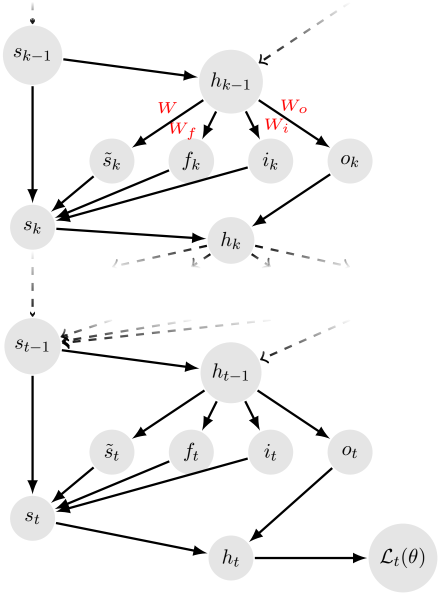

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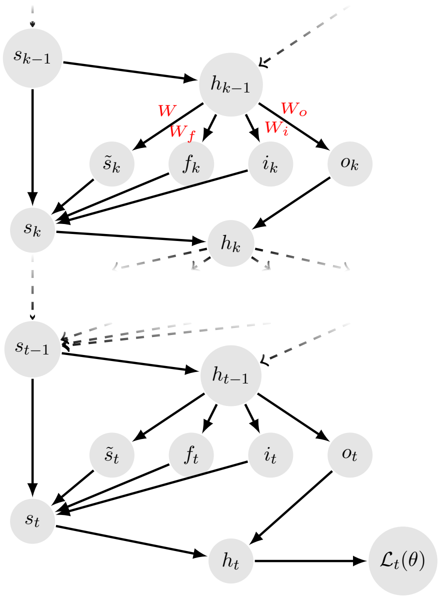
- For simplicity and ease of illustration, instead of considering the parameters (W, W_o, W_i, W_f, U, U_o, U_i, U_f) as separate nodes in the graph we will just put them on the appropriate edges. (We show only a few parameters and not all)



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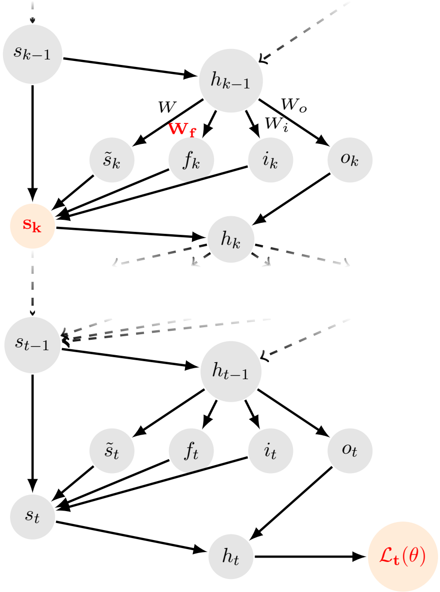
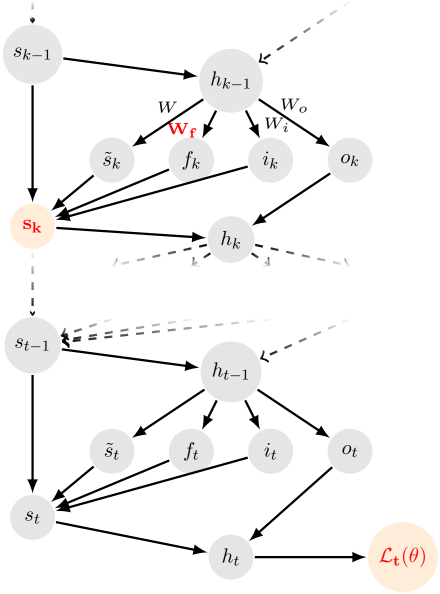
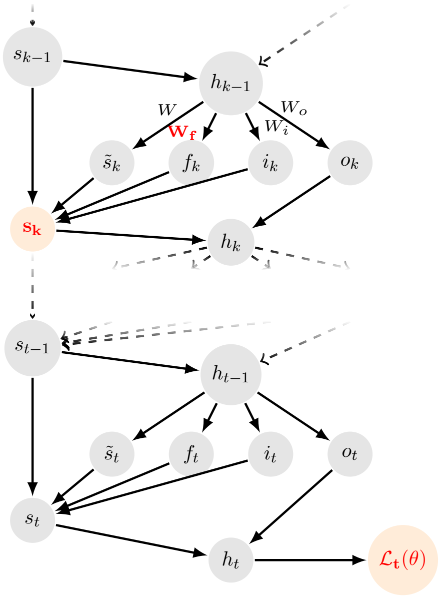
- We are now interested in knowing if the gradient from $\mathscr{L}_t(\theta)$ flows back to an arbitrary timestep k



- For example, we are interested in knowing if the gradient flows to W_f through s_k



- For example, we are interested in knowing if the gradient flows to W_f through s_k
- In other words, if $\mathscr{L}_t(\theta)$ was high because W_f failed to compute an appropriate value for s_k then this information should flow back to W_f through the gradients



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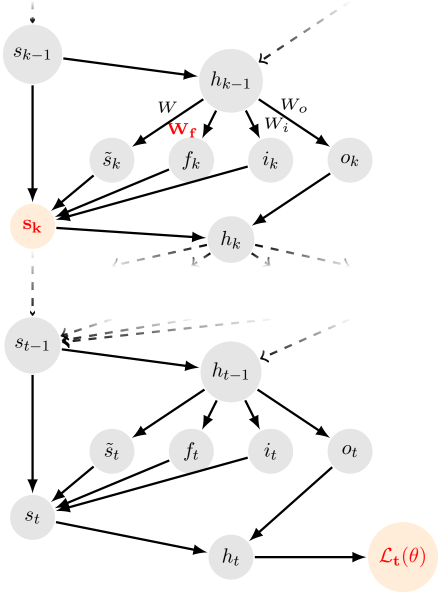
- We can ask a similar question about the other parameters (for example, W_i, W_o, W, etc.)



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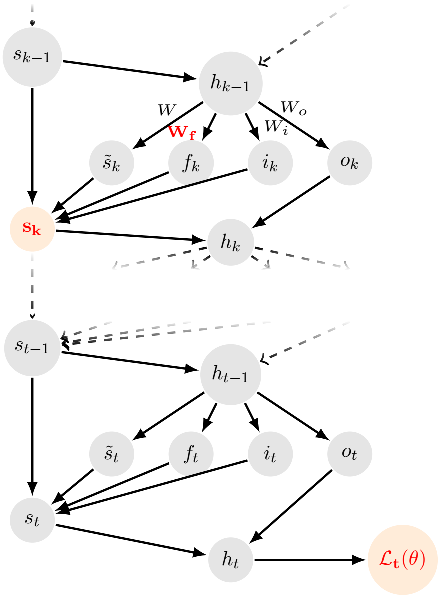
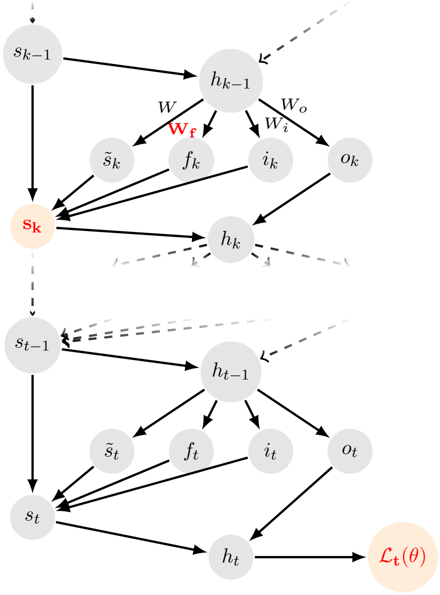
- How does LSTM ensure that this gradient does not vanish even at arbitrary time steps? Let us see



- It is sufficient to show that $\frac{\partial \mathcal{L}_t(\theta)}{\partial s_k}$ does not vanish (because if this does not vanish we can reach W_f through s_k)



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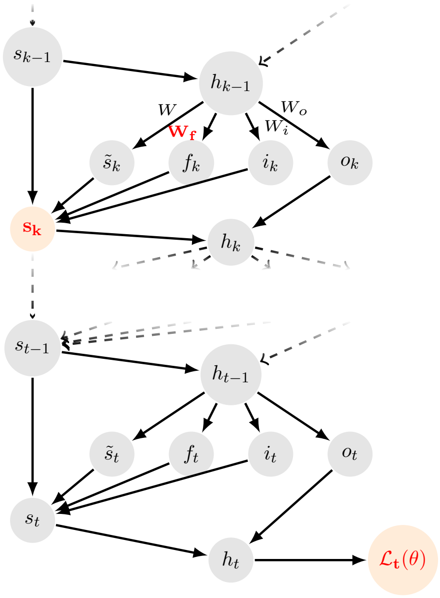
- First, we observe that there are multiple paths from $\mathscr{L}_t(\theta)$ to s_k (you just need to reverse the direction of the arrows for backpropagation)



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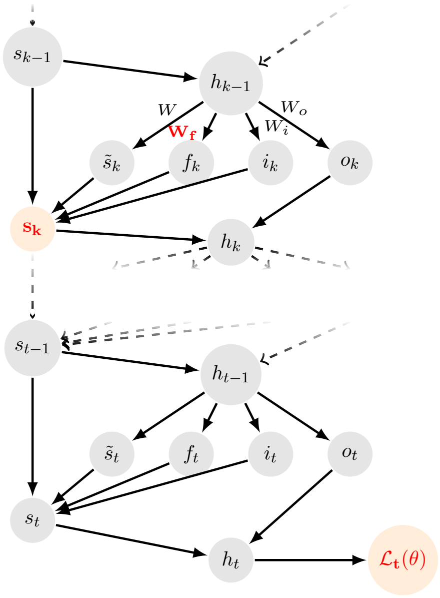
- For example, there is one path through s_{k+1}, another through h_k



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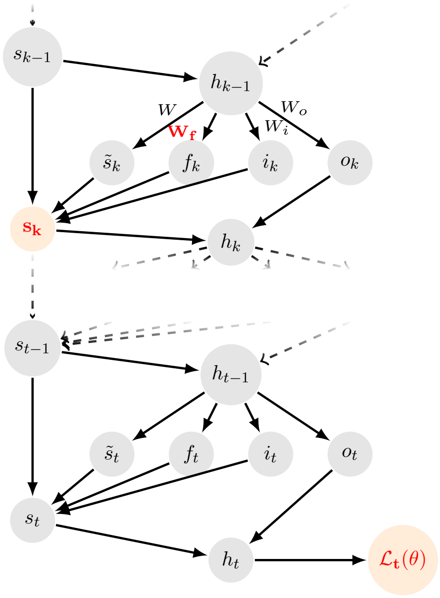
- Further, there are multiple paths to reach to h_k itself (as should be obvious from the number of outgoing arrows from h_k)



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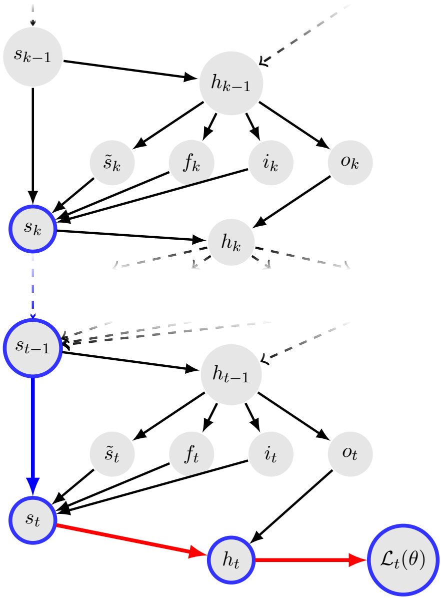
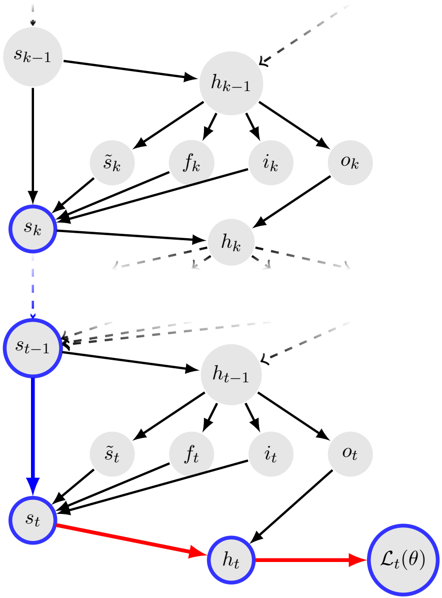
- First, we observe that there are multiple paths from $\mathscr{L}_t(\theta)$ to s_k (you just need to reverse the direction of the arrows for backpropagation)
- For example, there is one path through s_{k+1}, another through h_k
- Further, there are multiple paths to reach to h_k itself (as should be obvious from the number of outgoing arrows from h_k)
- So at this point just convince yourself that there are many paths from $\mathscr{L}_t(\theta)$ to s_k



- Consider one such path (highlighted) which will contribute to the gradient



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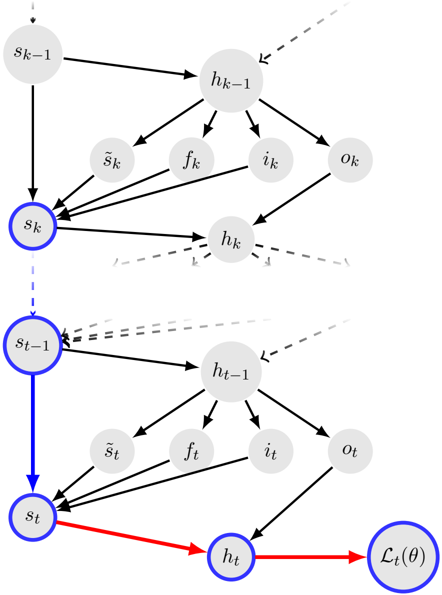
- Let us denote the gradient along this path as t_0



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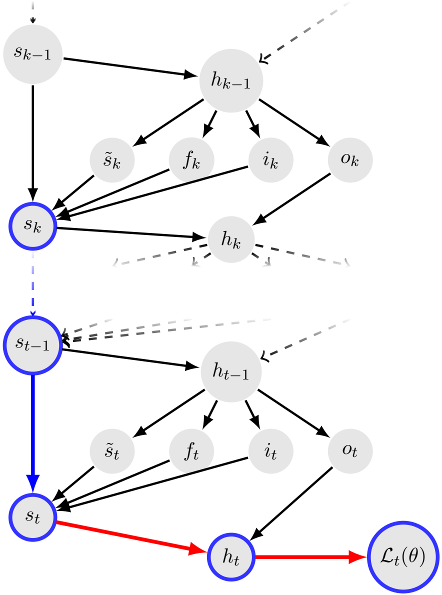
$$t_0 = \frac{\partial \mathscr{L}_t(\theta)}{\partial h_t} \frac{\partial h_t}{\partial s_t} \frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k}$$



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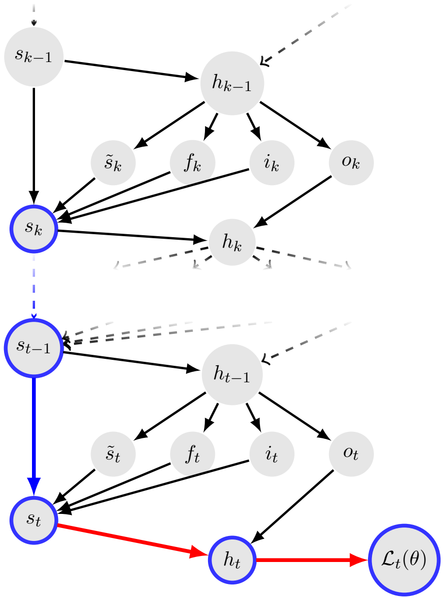
- The first term $\frac{\partial \mathscr{L}_t(\theta)}{\partial h_t}$ is fine and it doesn't vanish (h_t is directly connected to $\mathscr{L}_t(\theta)$ and there are no intermediate nodes which can cause the gradient to vanish)



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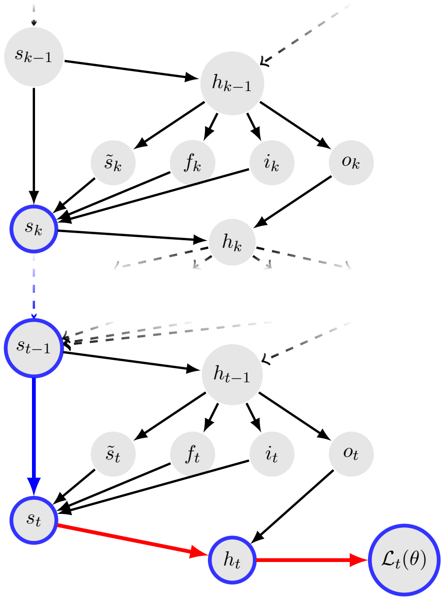
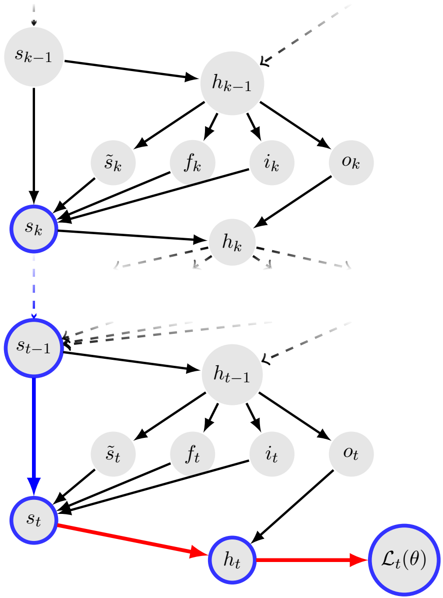
- The first term $\frac{\partial \mathscr{L}_t(\theta)}{\partial h_t}$ is fine and it doesn't vanish (h_t is directly connected to $\mathscr{L}_t(\theta)$ and there are no intermediate nodes which can cause the gradient to vanish)
- We will now look at the other terms $\frac{\partial h_t}{\partial s_t}$ $\frac{\partial s_t}{\partial s_{t-1}}$ $(\forall t)$



- Let us first look at $\frac{\partial h_t}{\partial s_t}$



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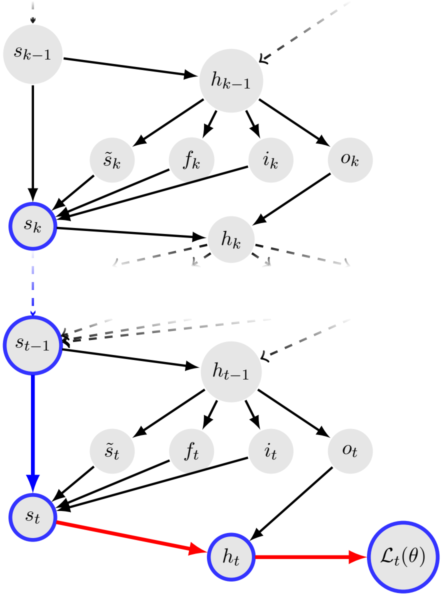
- Recall that

$$h_t = o_t \odot \sigma(s_t)$$



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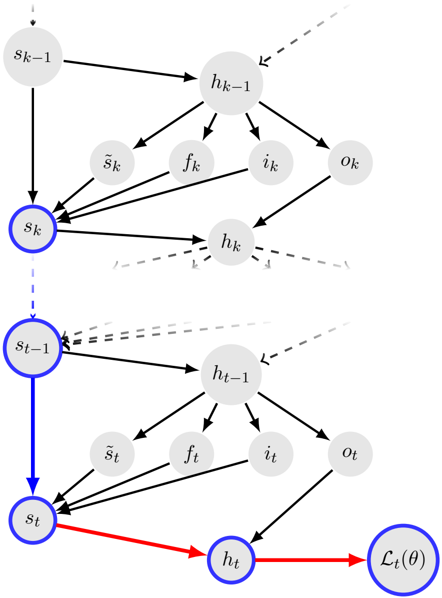
- Note that h_{ti} only depends on o_{ti} and s_{ti} and not on any other elements of o_t and s_t



- Let us first look at $\frac{\partial h_t}{\partial s_t}$
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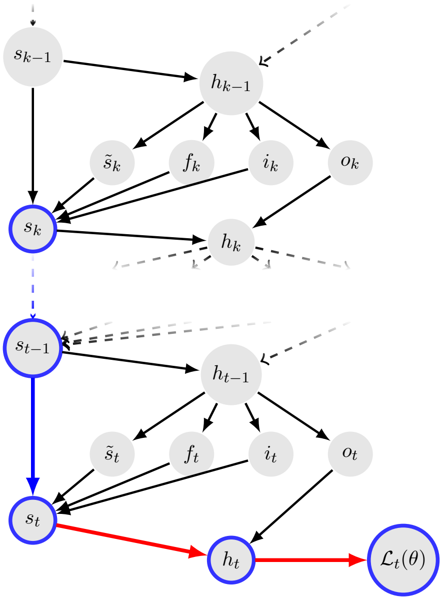
- $\frac{\partial h_t}{\partial s_t}$ will thus be a square diagonal matrix $\in \mathbb{R}^{d \times d}$ whose diagonal will be $o_t \odot \sigma'(s_t) \in \mathbb{R}^d$ (see slide 35 of Lecture 14)



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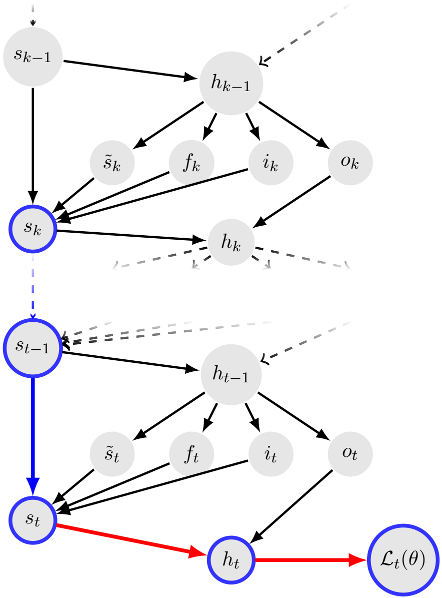
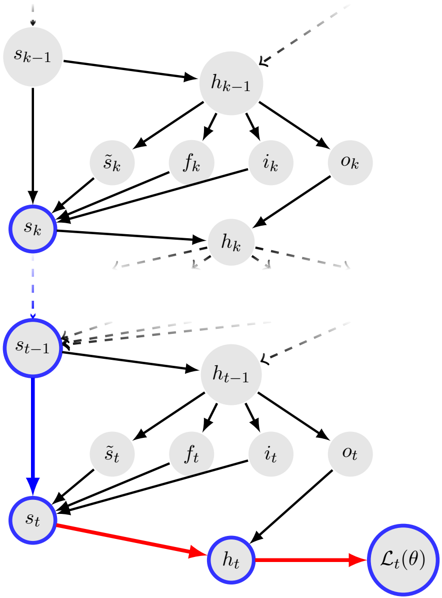
- We will represent this diagonal matrix by $\mathcal{D}(o_t \odot \sigma'(s_t))$



- Now let us consider $\frac{\partial s_t}{\partial s_{t-1}}$



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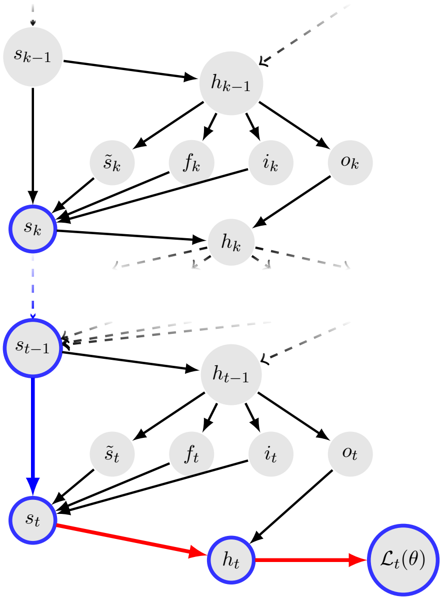

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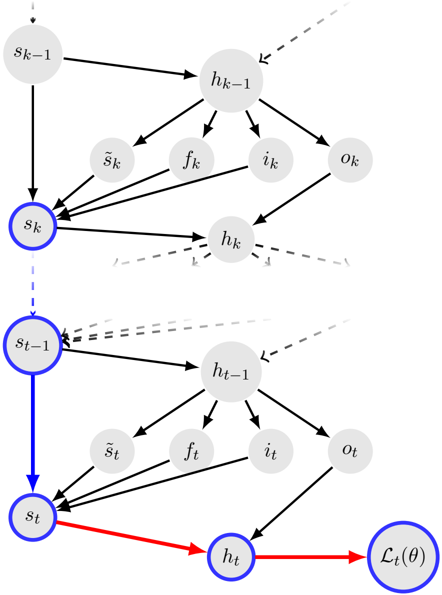

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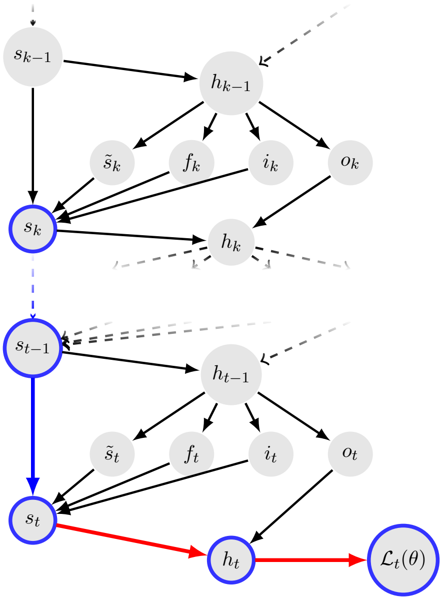

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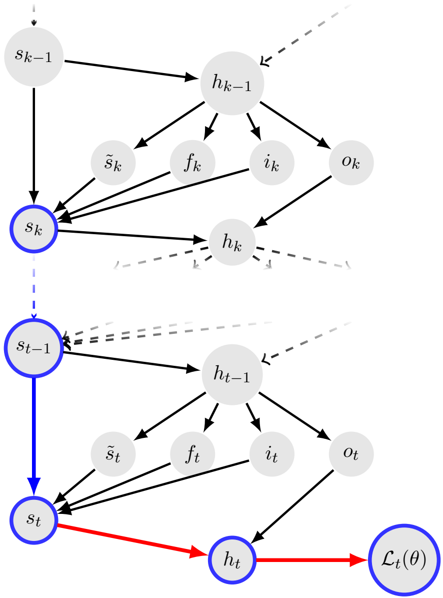

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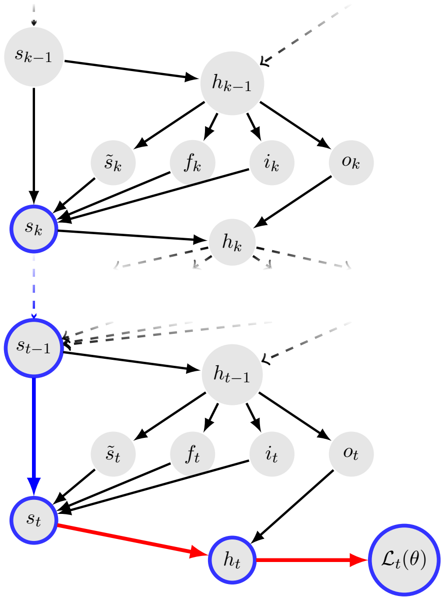
- And the gradient from the explicit term (treating $\tilde{s}_t$ as a constant) is given by $\mathcal{D}(f_t)$



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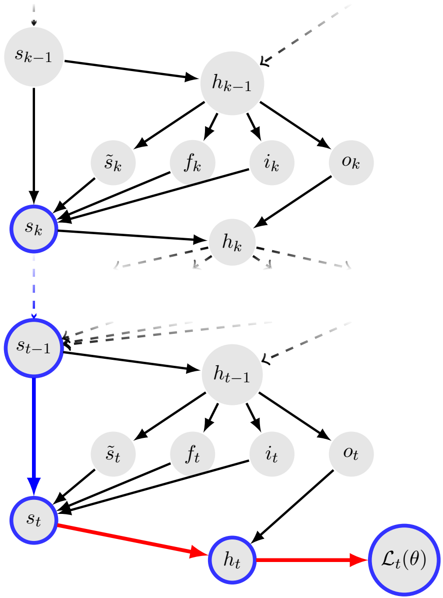

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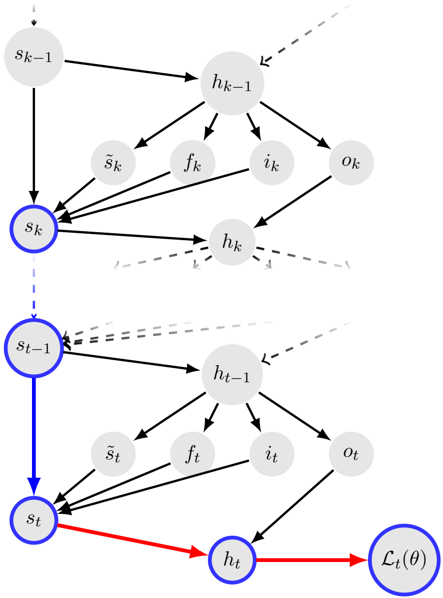

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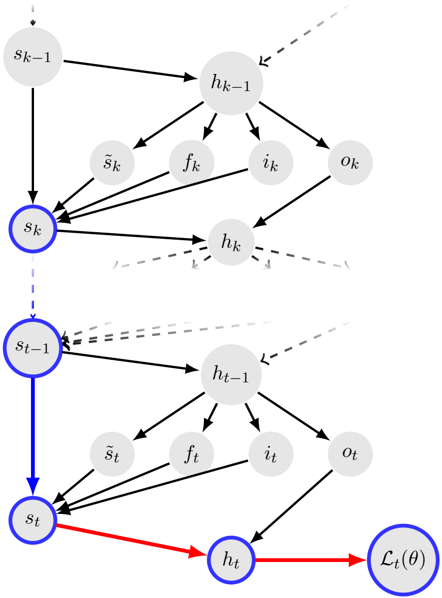

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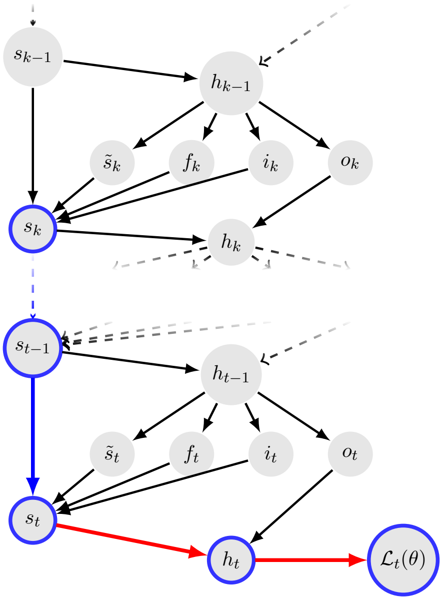

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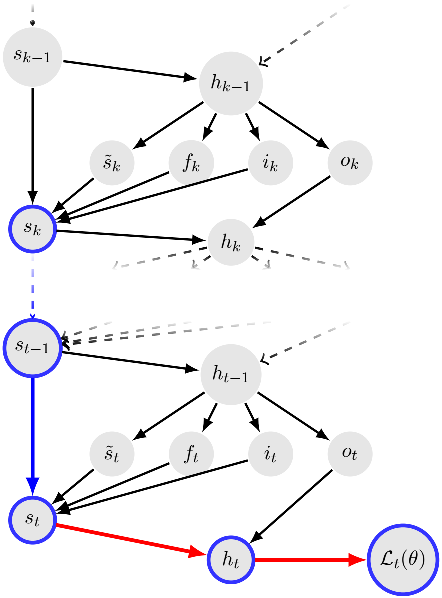

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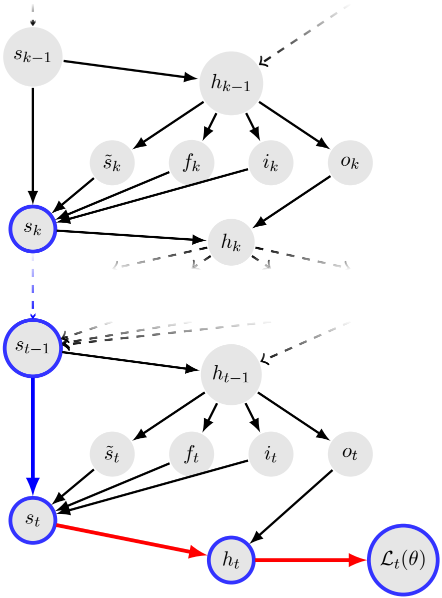
- The red terms don't vanish and the blue terms contain a multiplication of the forget gates
- The forget gates thus regulate the gradient flow depending on the explicit contribution of a state (s_t) to the next state s_{t+1}



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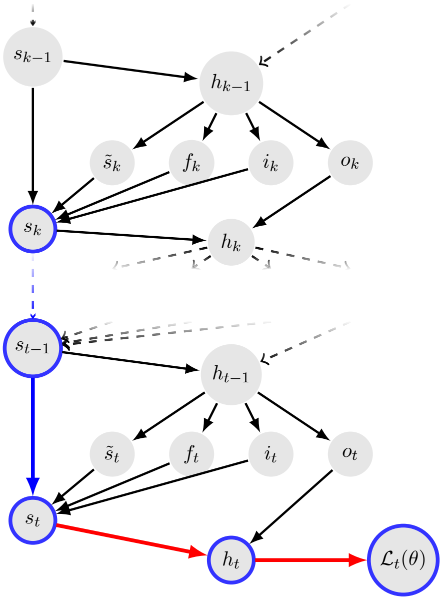

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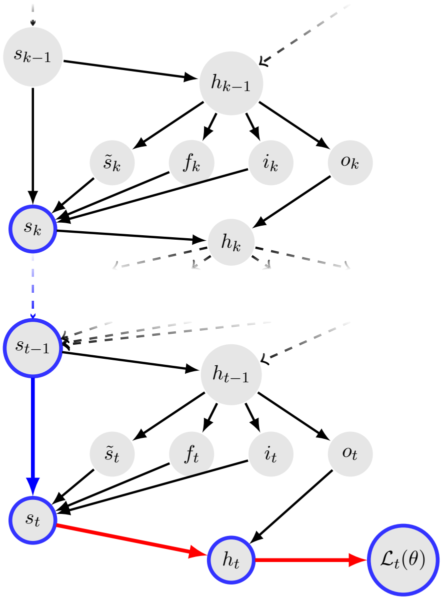

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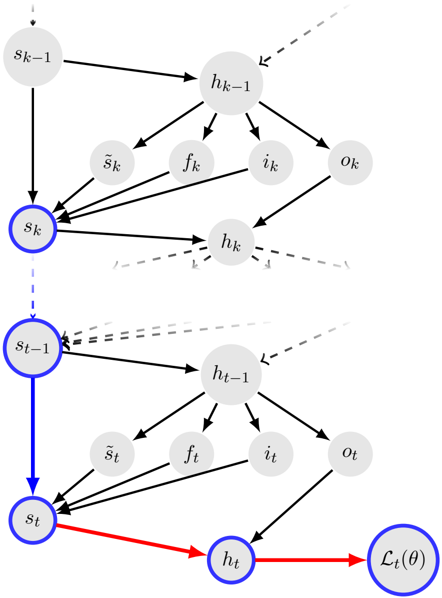

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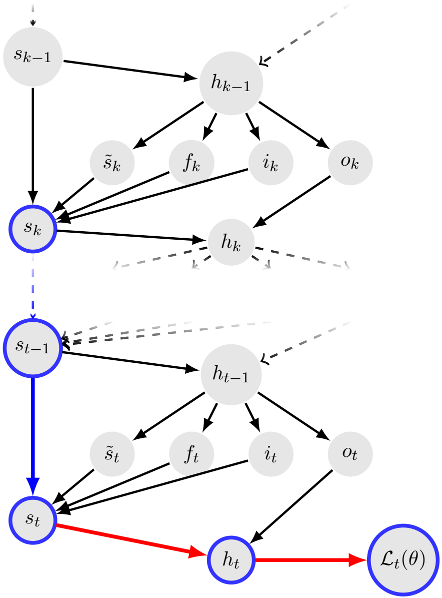

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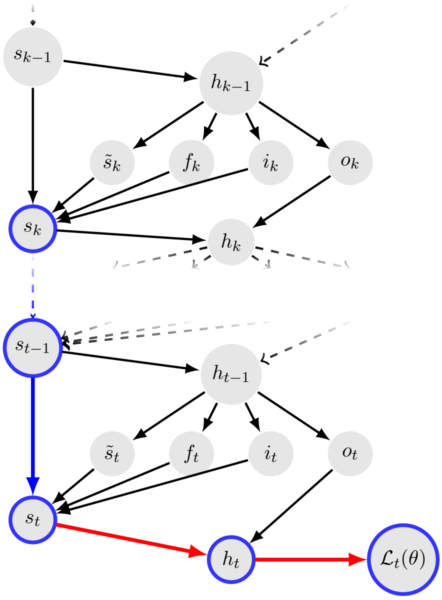
- And as argued as long as the gradient flows back to W_f through one of the paths (t_0) through s_k we are fine !



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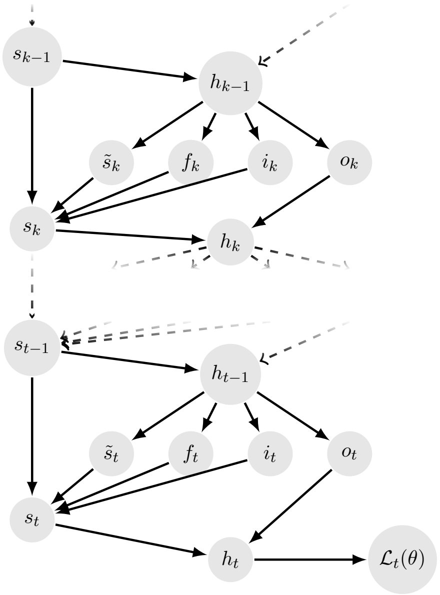
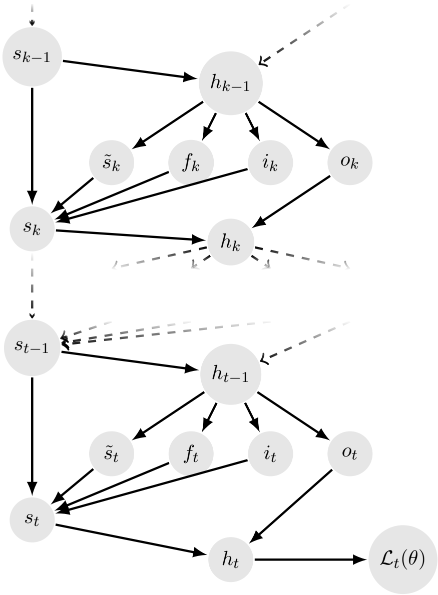
- Of course the gradient flows back only when required as regulated by f_i's (but let me just say it one last time that *this is fair*)



- Now we will see why LSTMs do not solve the problem of exploding gradients



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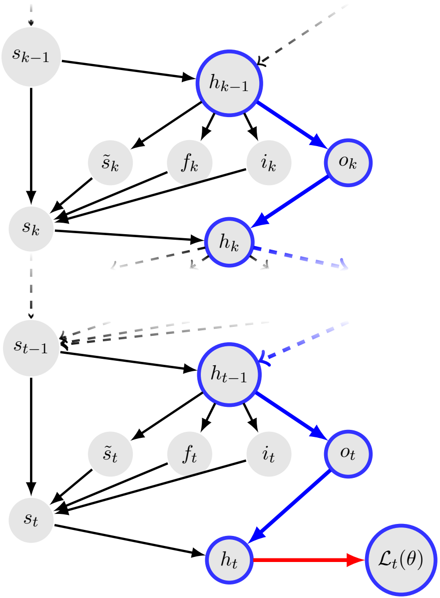

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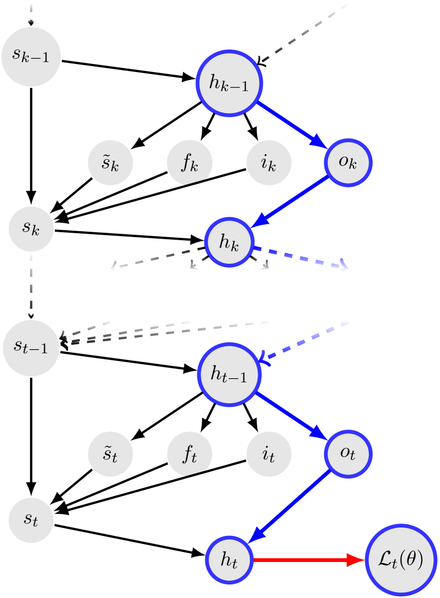

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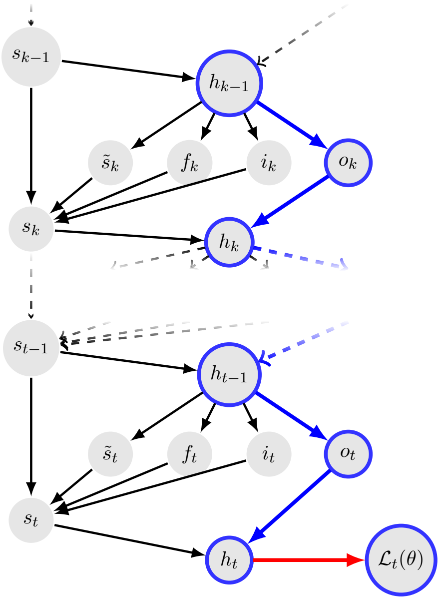

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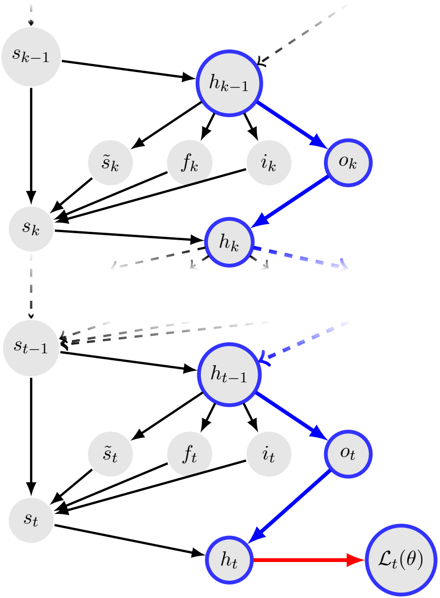

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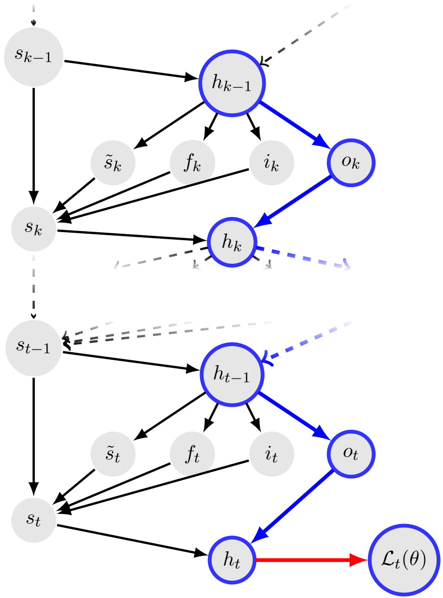

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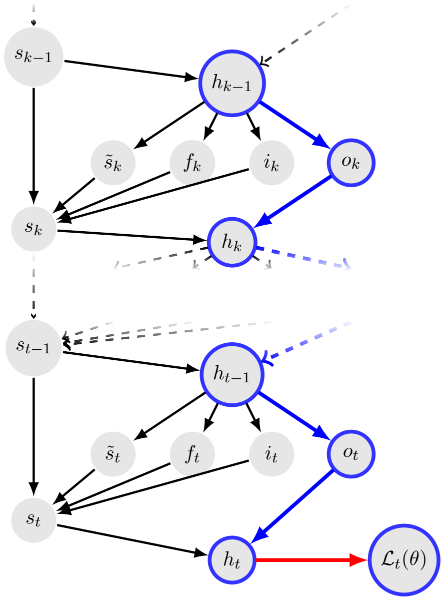
- Depending on the norm of matrix W_o, the gradient $\frac{\partial \mathscr{L}_t(\theta)}{\partial h_{k-1}}$ may explode



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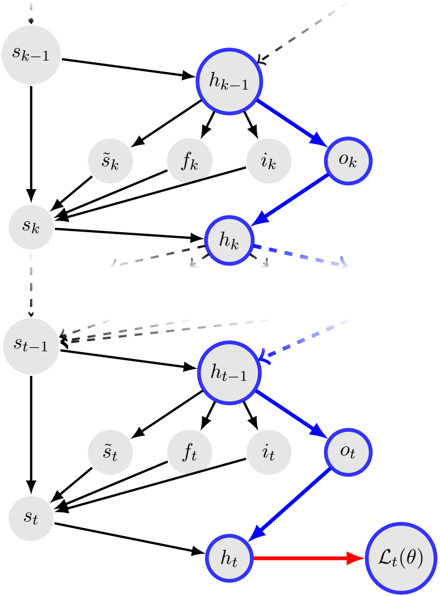
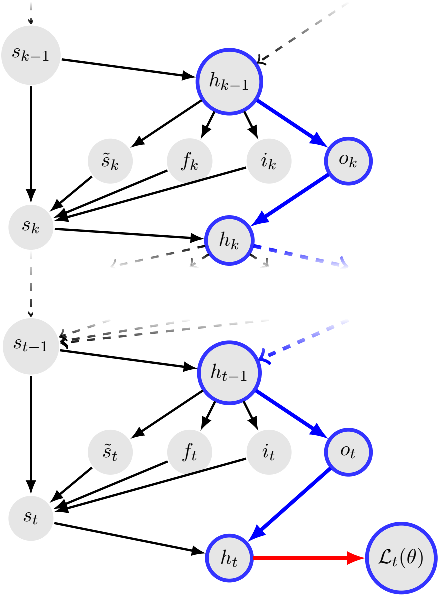
- Depending on the norm of matrix W_o, the gradient $\frac{\partial \mathscr{L}_t(\theta)}{\partial h_{k-1}}$ may explode
- Similarly, W_i, W_f and W can also cause the gradients to explode



- So how do we deal with the problem of exploding gradients ?

*Pascanu, Razvan, Tomas Mikolov, and Yoshua Bengio. "On the difficulty of training recurrent neural networks." ICML(3)28(2013):1310-1318



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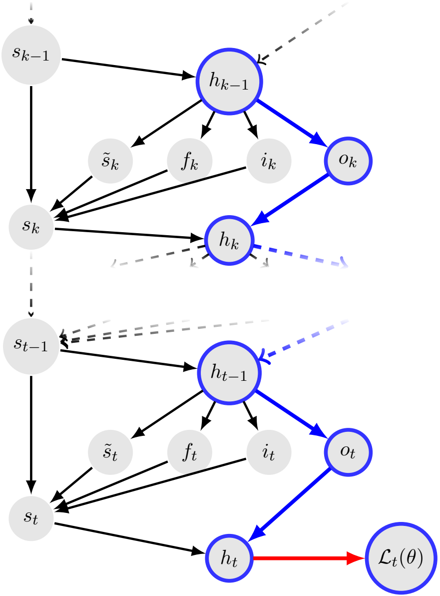

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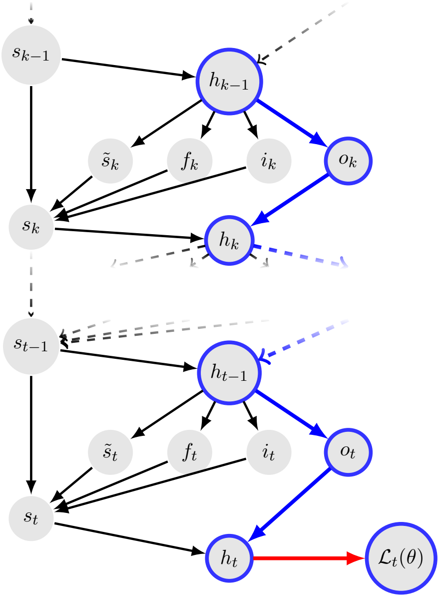

- So how do we deal with the problem of exploding gradients ?
- One popular trick is to use gradient clipping
- While backpropagating if the norm of the gradient exceeds a certain value, it is scaled to keep its norm within an acceptable threshold*
- Essentially we retain the direction of the gradient but scale down the norm

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