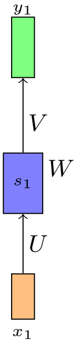
CS7015 (Deep Learning) : Lecture 15

Long Short Term Memory Cells (LSTMs), Gated Recurrent Units (GRUs)

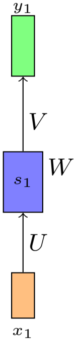
Mitesh M. Khapra

Department of Computer Science and Engineering
Indian Institute of Technology Madras

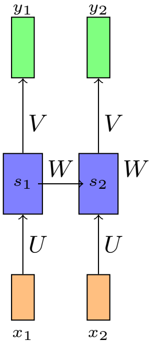
Module 15.1: Selective Read, Selective Write, Selective Forget - The Whiteboard Analogy



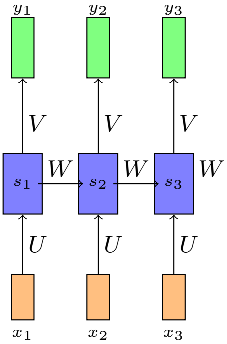
- The state (s_i) of an RNN records information from all previous time steps



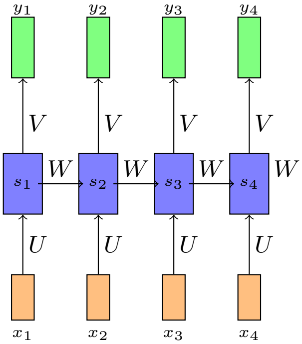
- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input



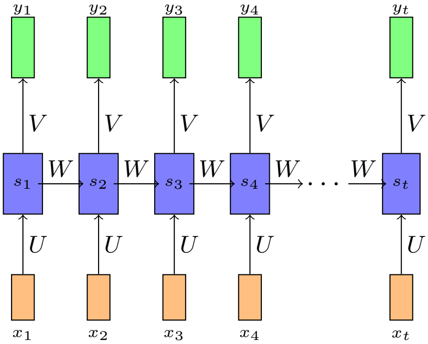
- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input



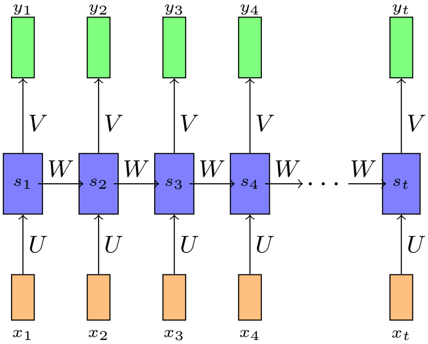
- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input



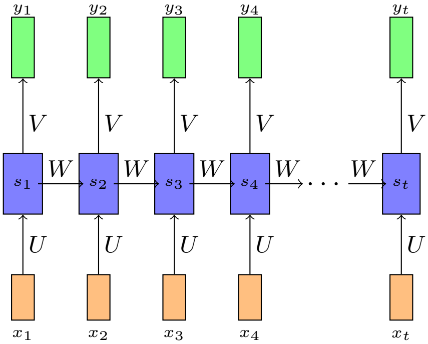
- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input



- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input

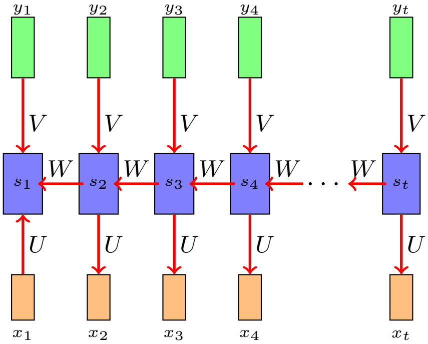


- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input
- One could imagine that after t steps the information stored at time step $t-k$ (for some $k < t$) gets completely morphed

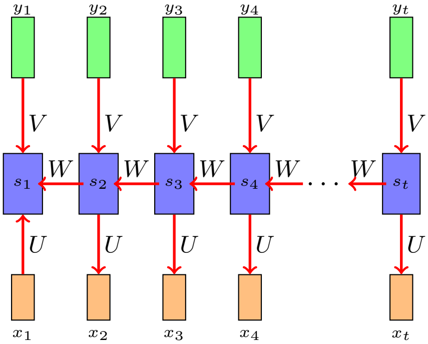


- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input
- One could imagine that after t steps the information stored at time step $t-k$ (for some $k < t$) gets completely morphed

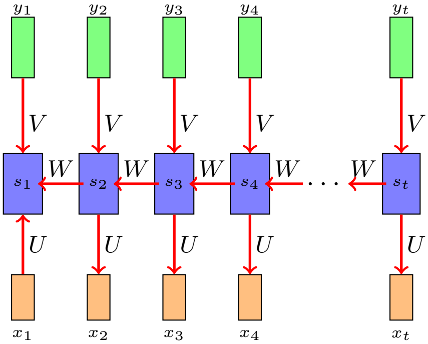
  so much that it would be impossible to extract the original information stored at time step $t-k$



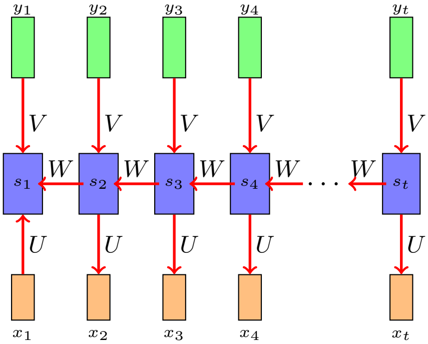
- A similar problem occurs when the information flows backwards (backpropagation)



- A similar problem occurs when the information flows backwards (backpropagation)
- It is very hard to assign the responsibility of the error caused at time step t to the events that occurred at time step $t-k$



- A similar problem occurs when the information flows backwards (backpropagation)
- It is very hard to assign the responsibility of the error caused at time step t to the events that occurred at time step $t-k$
- This responsibility is of course in the form of gradients and we studied the problem in backward flow of gradients



- A similar problem occurs when the information flows backwards (backpropagation)
- It is very hard to assign the responsibility of the error caused at time step t to the events that occurred at time step $t-k$
- This responsibility is of course in the form of gradients and we studied the problem in backward flow of gradients
- We saw a formal argument for this while discussing vanishing gradients

- Let us see an analogy for this

- Let us see an analogy for this
- We can think of the state as a fixed size memory

- Let us see an analogy for this
- We can think of the state as a fixed size memory
- Compare this to a fixed size white board that you use to record information

- Let us see an analogy for this
- We can think of the state as a fixed size memory
- Compare this to a fixed size white board that you use to record information
- At each time step (periodic intervals) we keep writing something to the board

- Let us see an analogy for this
- We can think of the state as a fixed size memory
- Compare this to a fixed size white board that you use to record information
- At each time step (periodic intervals) we keep writing something to the board
- Effectively at each time step we morph the information recorded till that time point

- Let us see an analogy for this
- We can think of the state as a fixed size memory
- Compare this to a fixed size white board that you use to record information
- At each time step (periodic intervals) we keep writing something to the board
- Effectively at each time step we morph the information recorded till that time point
- After many timesteps it would be impossible to see how the information at time step $t-k$ contributed to the state at timestep t

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard
- We follow the following strategy at each time step

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard
- We follow the following strategy at each time step
- Selectively write on the board

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard
- We follow the following strategy at each time step
- Selectively write on the board
- Selectively read the already written content

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard
- We follow the following strategy at each time step
- Selectively write on the board
- Selectively read the already written content
- Selectively forget (erase) some content

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard
- We follow the following strategy at each time step
- Selectively write on the board
- Selectively read the already written content
- Selectively forget (erase) some content
- Let us look at each of these in detail

Selective write

- There may be many steps in the derivation but we may just skip a few

Selective write

- There may be many steps in the derivation but we may just skip a few
- In other words we select what to **write**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

Selective write

- There may be many steps in the derivation but we may just skip a few
- In other words we select what to **write**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$

Selective write

- There may be many steps in the derivation but we may just skip a few
- In other words we select what to **write**

$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$

$$bd = 33$$

Selective write

- There may be many steps in the derivation but we may just skip a few
- In other words we select what to **write**

$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$

Selective read

- While writing one step we typically read some of the previous steps we have already written and then decide what to write next

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$

$$bd = 33$$

Selective read

- While writing one step we typically read some of the previous steps we have already written and then decide what to write next
- For example at Step 3, information from Step 2 is important

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$

$$bd = 33$$

Selective read

- While writing one step we typically read some of the previous steps we have already written and then decide what to write next
- For example at Step 3, information from Step 2 is important
- In other words we select what to **read**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$
$$bd + a = 34$$

Selective read

- While writing one step we typically read some of the previous steps we have already written and then decide what to write next
- For example at Step 3, information from Step 2 is important
- In other words we select what to **read**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$
$$bd + a = 34$$

Selective forget

- Once the board is full, we need to delete some obsolete information

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$
$$bd + a = 34$$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$
$$bd + a = 34$$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information
- In other words we select what to **forget**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$

$$bd + a = 34$$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information
- In other words we select what to **forget**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$ac(bd + a) = 170$$
$$bd + a = 34$$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information
- In other words we select what to **forget**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$ac(bd + a) = 170$$
$$ad = 11$$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information
- In other words we select what to **forget**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ad + ac(bd + a) = 181$$
$$ac(bd + a) = 170$$
$$ad = 11$$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information
- In other words we select what to **forget**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ad + ac(bd + a) = 181$$
$$ac(bd + a) = 170$$
$$ad = 11$$

- There are various other scenarios where we can motivate the need for selective write, read and forget

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$\begin{aligned} ad + ac(bd + a) &= 181 \\ ac(bd + a) &= 170 \\ ad &= 11 \end{aligned}$$

- There are various other scenarios where we can motivate the need for selective write, read and forget
- For example, you could think of our brain as something which can store only a finite number of facts

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ad + ac(bd + a) = 181$$
$$ac(bd + a) = 170$$
$$ad = 11$$

- There are various other scenarios where we can motivate the need for selective write, read and forget
- For example, you could think of our brain as something which can store only a finite number of facts
- At different time steps we selectively read, write and forget some of these facts

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$\begin{aligned} ad + ac(bd + a) &= 181 \\ ac(bd + a) &= 170 \\ ad &= 11 \end{aligned}$$

- There are various other scenarios where we can motivate the need for selective write, read and forget
- For example, you could think of our brain as something which can store only a finite number of facts
- At different time steps we selectively read, write and forget some of these facts
- Since the RNN also has a finite state size, we need to figure out a way to allow it to selectively read, write and forget

Module 15.2: Long Short Term Memory(LSTM) and Gated Recurrent Units(GRUs)

Questions

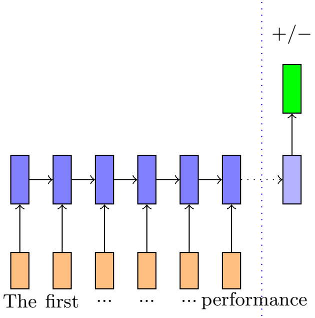
- Can we give a concrete example where RNNs also need to selectively read, write and forget ?

Questions

- Can we give a concrete example where RNNs also need to selectively read, write and forget ?
- How do we convert this intuition into mathematical equations ?

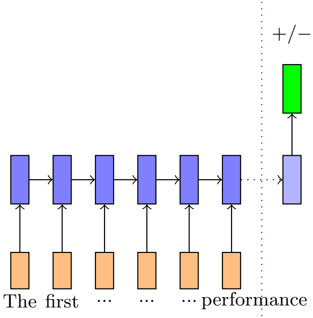
Questions

- Can we give a concrete example where RNNs also need to selectively read, write and forget ?
- How do we convert this intuition into mathematical equations ? We will see this over the next few slides



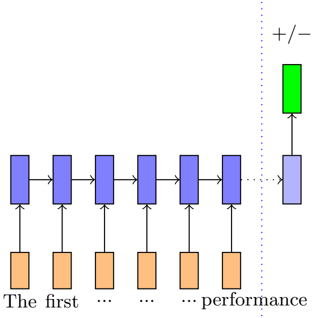
Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review



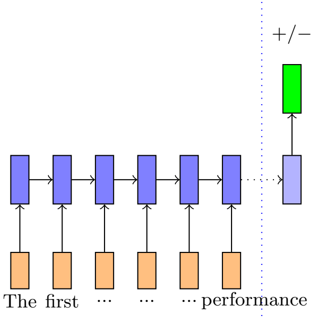
Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state



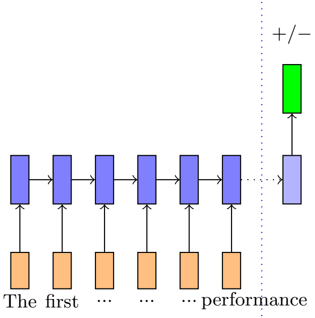
Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state
- By the time we reach the end of the document the information obtained from the first few words is completely lost



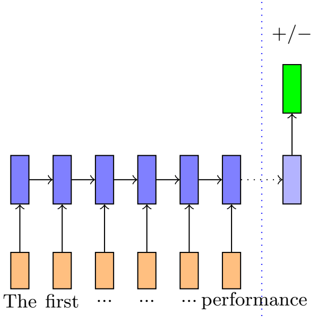
Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state
- By the time we reach the end of the document the information obtained from the first few words is completely lost
- Ideally we want to



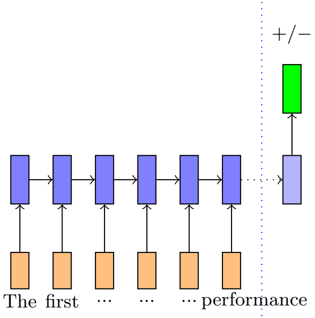
Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state
- By the time we reach the end of the document the information obtained from the first few words is completely lost
- Ideally we want to
  - **forget** the information added by stop words (a, the, etc.)



Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state
- By the time we reach the end of the document the information obtained from the first few words is completely lost
- Ideally we want to
  - **forget** the information added by stop words (a, the, etc.)
  - **selectively read** the information added by previous sentiment bearing words (awesome, amazing, etc.)

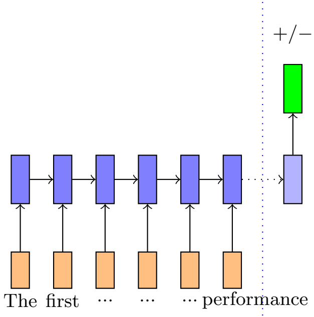


Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state
- By the time we reach the end of the document the information obtained from the first few words is completely lost
- Ideally we want to
  - **forget** the information added by stop words (a, the, etc.)
  - **selectively read** the information added by previous sentiment bearing words (awesome, amazing, etc.)
  - **selectively write** new information from the current word to the state

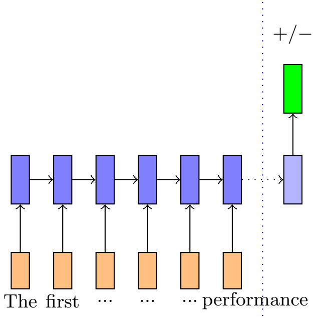
Questions

- Can we give a concrete example where RNNs also need to selectively read, write and forget ?
- How do we convert this intuition into mathematical equations ?



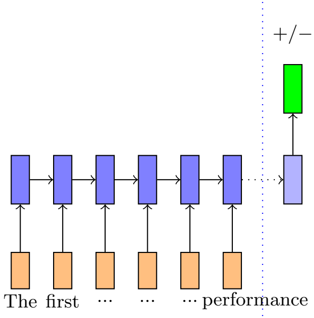
Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Recall that the blue colored vector (s_t) is called the state of the RNN



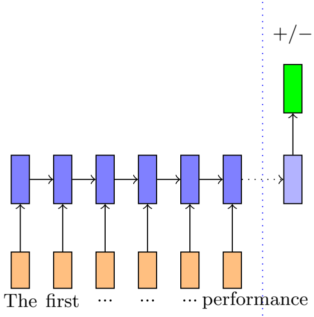
Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Recall that the blue colored vector (s_t) is called the state of the RNN
- It has a finite size ($s_t \in \mathbb{R}^n$) and is used to store all the information upto timestep t



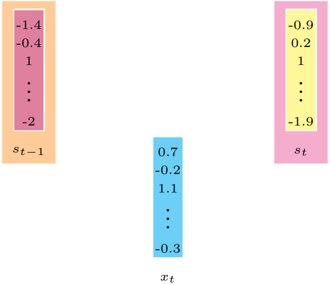
Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Recall that the blue colored vector (s_t) is called the state of the RNN
- It has a finite size ($s_t \in \mathbb{R}^n$) and is used to store all the information upto timestep t
- This state is analogous to the whiteboard and sooner or later it will get overloaded and the information from the initial states will get morphed beyond recognition

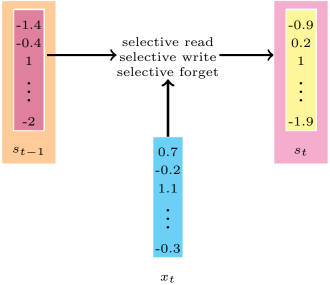


Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

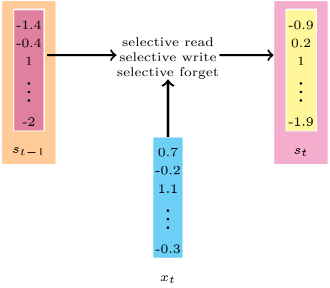
- Recall that the blue colored vector (s_t) is called the state of the RNN
- It has a finite size ($s_t \in \mathbb{R}^n$) and is used to store all the information upto timestep t
- This state is analogous to the whiteboard and sooner or later it will get overloaded and the information from the initial states will get morphed beyond recognition
- **Wishlist:** selective write, selective read and selective forget to ensure that this finite sized state vector is used effectively



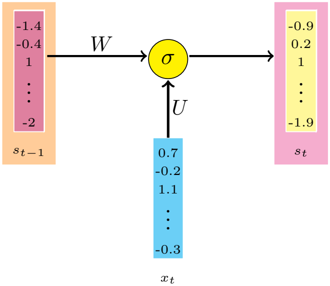
- Just to be clear, we have computed a state s_{t-1} at timestep $t-1$ and now we want to overload it with new information (x_t) and compute a new state (s_t)



- Just to be clear, we have computed a state s_{t-1} at timestep $t-1$ and now we want to overload it with new information (x_t) and compute a new state (s_t)
- While doing so we want to make sure that we use selective write, selective read and selective forget so that only important information is retained in s_t

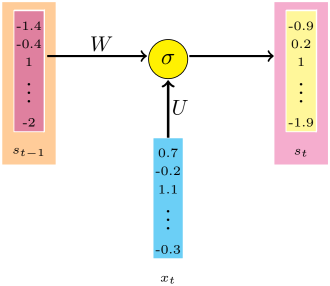


- Just to be clear, we have computed a state s_{t-1} at timestep $t-1$ and now we want to overload it with new information (x_t) and compute a new state (s_t)
- While doing so we want to make sure that we use selective write, selective read and selective forget so that only important information is retained in s_t
- We will now see how to implement these items from our wishlist



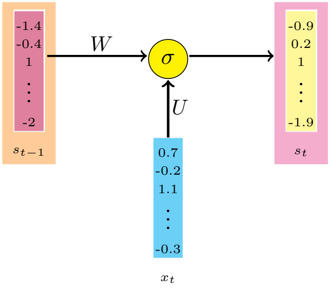
Selective Write

- Recall that in RNNs we use s_{t-1} to compute s_t



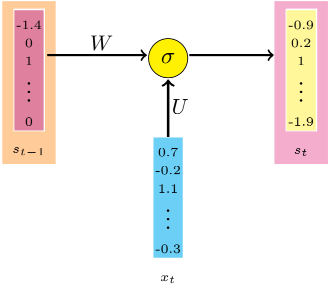
Selective Write

- Recall that in RNNs we use s_{t-1} to compute s_t
  $s_t = \sigma(Ws_{t-1} + Ux_t)$ *(ignoring bias)*



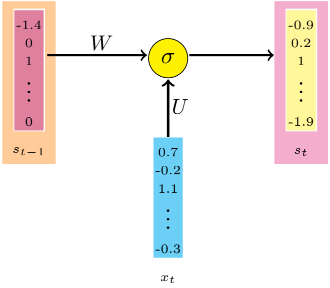
Selective Write

- Recall that in RNNs we use s_{t-1} to compute s_t
  $s_t = \sigma(W s_{t-1} + U x_t)$ *(ignoring bias)*
- But now instead of passing s_{t-1} as it is to s_t we want to pass (write) only some portions of it to the next state



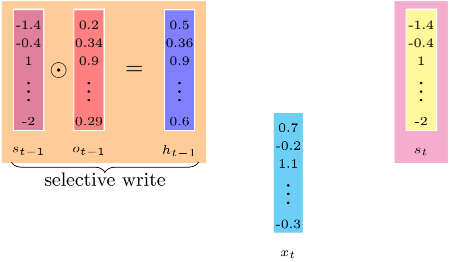
Selective Write

- Recall that in RNNs we use s_{t-1} to compute s_t
  $s_t = \sigma(Ws_{t-1} + Ux_t)$ *(ignoring bias)*
- But now instead of passing s_{t-1} as it is to s_t we want to pass (write) only some portions of it to the next state
- In the strictest case our decisions could be binary (for example, retain 1st and 3rd entries and delete the rest of the entries)



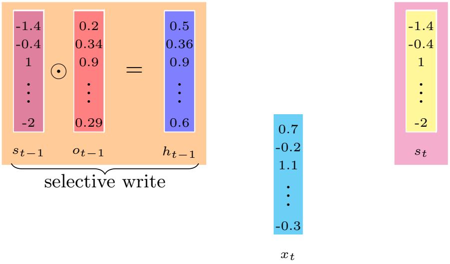
Selective Write

- Recall that in RNNs we use s_{t-1} to compute s_t
  $s_t = \sigma(W s_{t-1} + U x_t)$ *(ignoring bias)*
- But now instead of passing s_{t-1} as it is to s_t we want to pass (write) only some portions of it to the next state
- In the strictest case our decisions could be binary (for example, retain 1st and 3rd entries and delete the rest of the entries)
- But a more sensible way of doing this would be to assign a value between 0 and 1 which determines what fraction of the current state to pass on to the next state



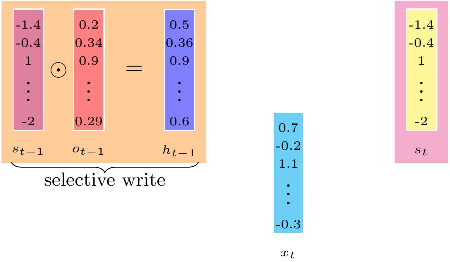
Selective Write

- We introduce a vector o_{t-1} which decides what fraction of each element of s_{t-1} should be passed to the next state



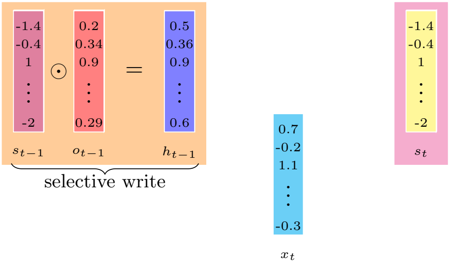
Selective Write

- We introduce a vector o_{t-1} which decides what fraction of each element of s_{t-1} should be passed to the next state
- Each element of o_{t-1} gets multiplied with the corresponding element of s_{t-1}



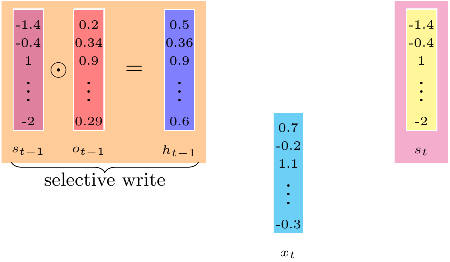
Selective Write

- We introduce a vector o_{t-1} which decides what fraction of each element of s_{t-1} should be passed to the next state
- Each element of o_{t-1} gets multiplied with the corresponding element of s_{t-1}
- Each element of o_{t-1} is restricted to be between 0 and 1



Selective Write

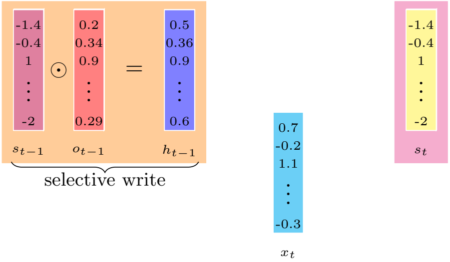
- We introduce a vector o_{t-1} which decides what fraction of each element of s_{t-1} should be passed to the next state
- Each element of o_{t-1} gets multiplied with the corresponding element of s_{t-1}
- Each element of o_{t-1} is restricted to be between 0 and 1
- But how do we compute o_{t-1}? How does the RNN know what fraction of the state to pass on?



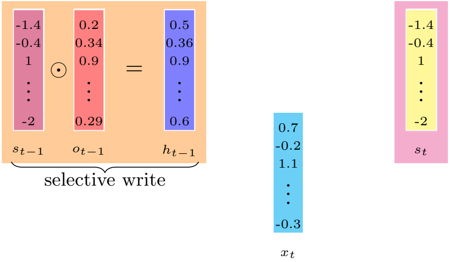
Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)

Selective Write



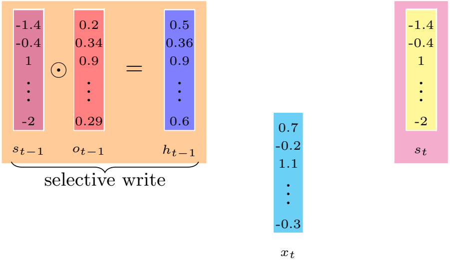
- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)
- We compute o_{t-1} and h_{t-1} as



Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)
- We compute o_{t-1} and h_{t-1} as

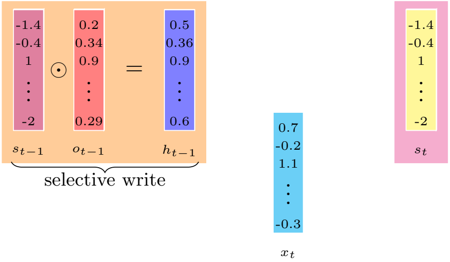
$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$



Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)
- We compute o_{t-1} and h_{t-1} as

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$
$$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$$

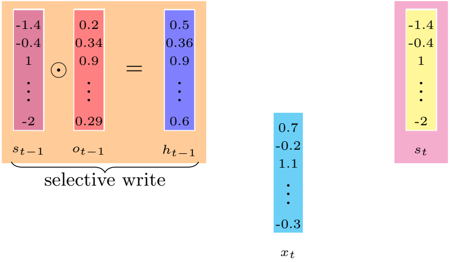


Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)
- We compute o_{t-1} and h_{t-1} as

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$
$$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$$

- The parameters W_o, U_o, b_o need to be learned along with the existing parameters W, U, V

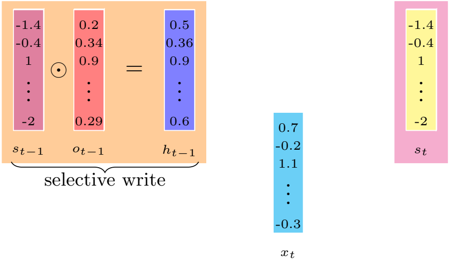


Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)
- We compute o_{t-1} and h_{t-1} as

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$
$$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$$

- The parameters W_o, U_o, b_o need to be learned along with the existing parameters W, U, V
- The sigmoid (logistic) function ensures that the values are between 0 and 1

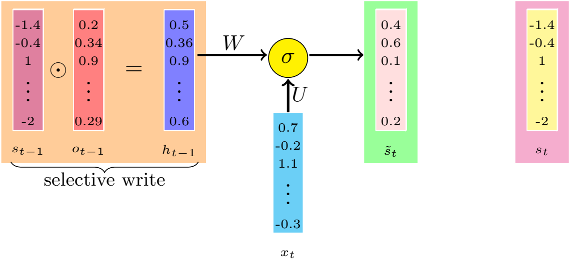


Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)
- We compute o_{t-1} and h_{t-1} as

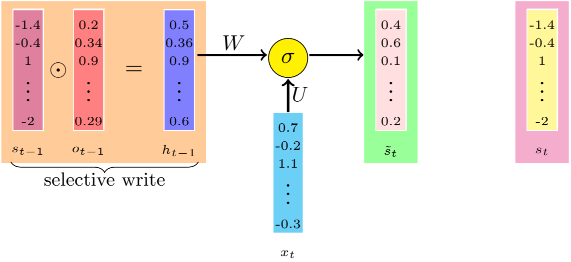
$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$
$$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$$

- The parameters W_o, U_o, b_o need to be learned along with the existing parameters W, U, V
- The sigmoid (logistic) function ensures that the values are between 0 and 1
- o_t is called the output gate as it decides how much to pass (write) to the next time step



Selective Read

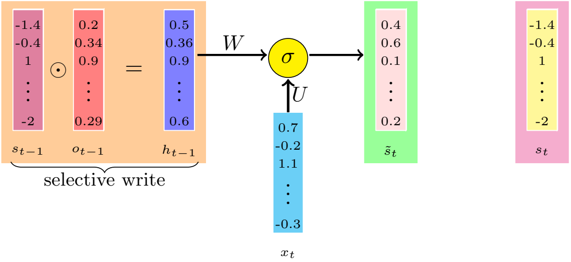
- We will now use h_{t-1} to compute the new state at the next time step



Selective Read

- We will now use h_{t-1} to compute the new state at the next time step
- We will also use x_t which is the new input at time step t

$$\tilde{s}_t = \sigma(Wh_{t-1} + Ux_t + b)$$

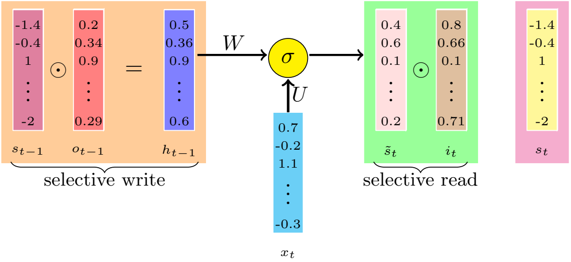


Selective Read

- We will now use h_{t-1} to compute the new state at the next time step
- We will also use x_t which is the new input at time step t

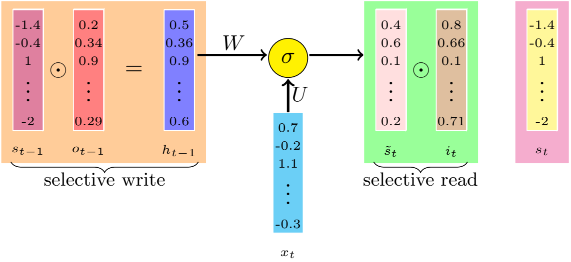
$$\tilde{s}_t = \sigma(Wh_{t-1} + Ux_t + b)$$

- Note that W, U and b are similar to the parameters that we used in RNN (for simplicity we have not shown the bias b in the figure)



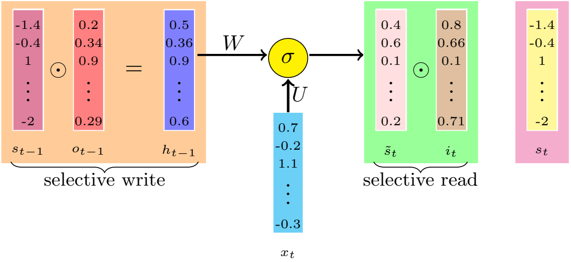
Selective Read

- $\tilde{s}_t$ thus captures all the information from the previous state (h_{t-1}) and the current input x_t



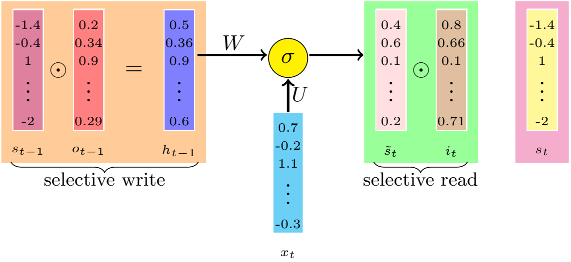
Selective Read

- $\tilde{s}_t$ thus captures all the information from the previous state (h_{t-1}) and the current input x_t
- However, we may not want to use all this new information and only selectively **read** from it before constructing the new cell state s_t



Selective Read

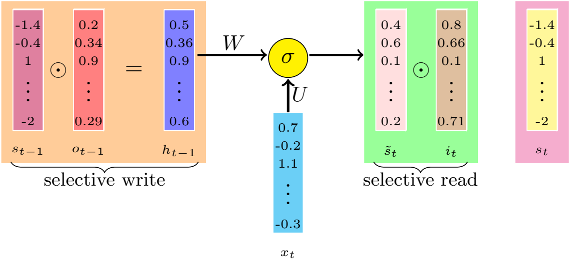
- $\tilde{s}_t$ thus captures all the information from the previous state (h_{t-1}) and the current input x_t
- However, we may not want to use all this new information and only selectively **read** from it before constructing the new cell state s_t
- To do this we introduce another gate called the input gate



Selective Read

- $\tilde{s}_t$ thus captures all the information from the previous state (h_{t-1}) and the current input x_t
- However, we may not want to use all this new information and only selectively **read** from it before constructing the new cell state s_t
- To do this we introduce another gate called the input gate

$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$

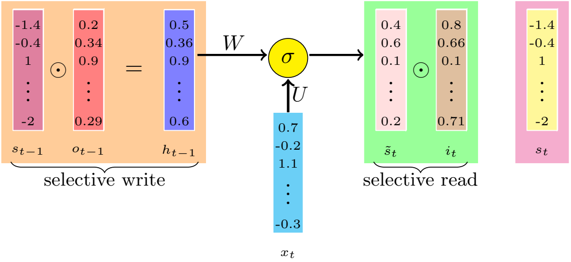


Selective Read

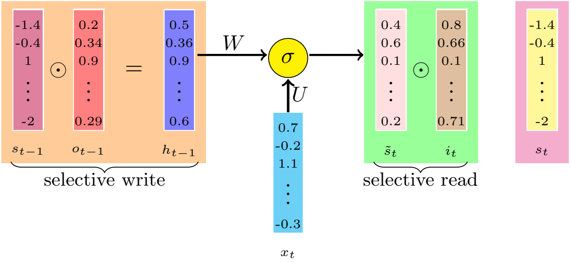
- $\tilde{s}_t$ thus captures all the information from the previous state (h_{t-1}) and the current input x_t
- However, we may not want to use all this new information and only selectively **read** from it before constructing the new cell state s_t
- To do this we introduce another gate called the input gate

$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$

- and use $i_t \odot \tilde{s}_t$ as the selectively read state information



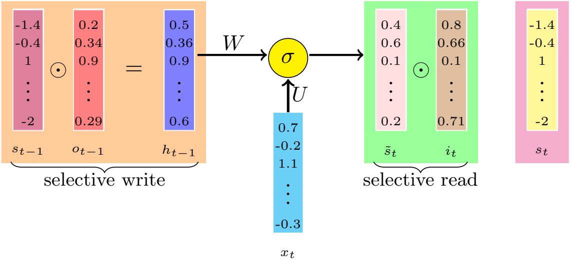
- So far we have the following



- So far we have the following

Previous state:

s_{t-1}



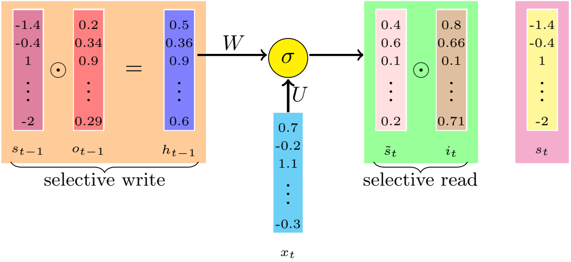
- So far we have the following

Previous state:

s_{t-1}

Output gate:

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$



- So far we have the following

Previous state:

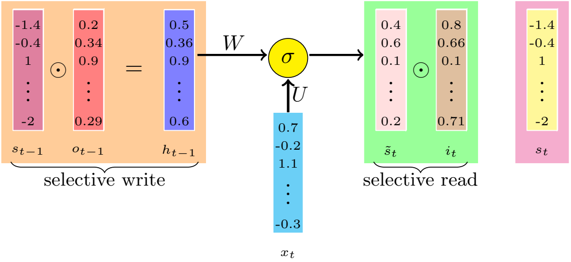
s_{t-1}

Output gate:

$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$

Selectively Write:

$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$



- So far we have the following

Previous state:

s_{t-1}

Output gate:

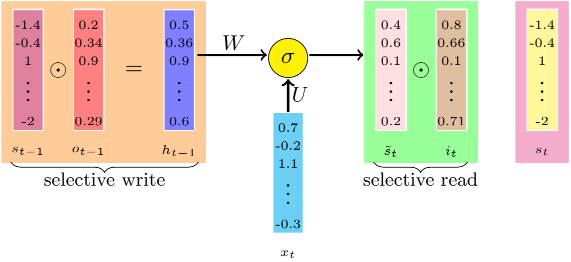
$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$

Selectively Write:

$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$

Current (temporary) state:

$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$



- So far we have the following

Previous state:

s_{t-1}

Output gate:

$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$

Selectively Write:

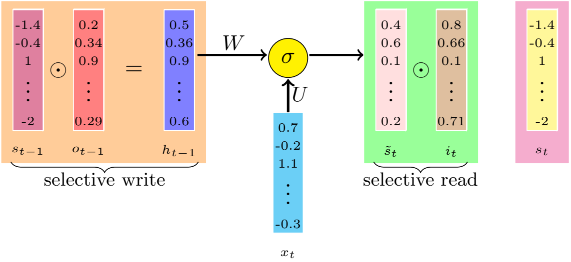
$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$

Current (temporary) state:

$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$

Input gate:

$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$



- So far we have the following

Previous state:

s_{t-1}

Output gate:

$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$

Selectively Write:

$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$

Current (temporary) state:

$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$

Input gate:

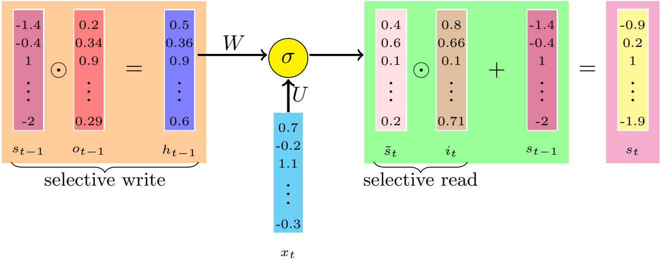
$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$

Selectively Read:

$i_t \odot \tilde{s}_t$

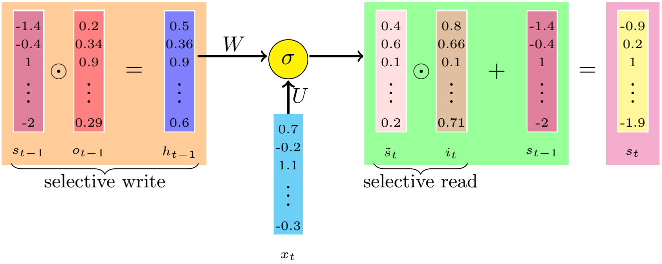
Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state



Selective Forget

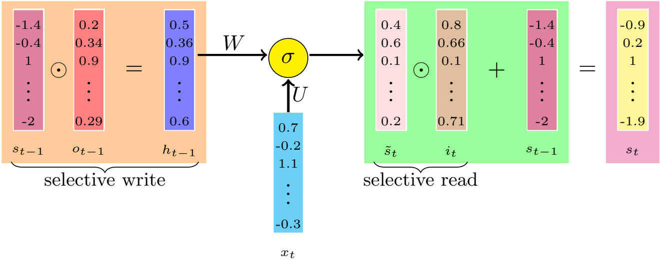
- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state
- Here is one simple (but effective) way of doing this:



Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state
- Here is one simple (but effective) way of doing this:

$$s_t = s_{t-1} + i_t \odot \tilde{s}_t$$

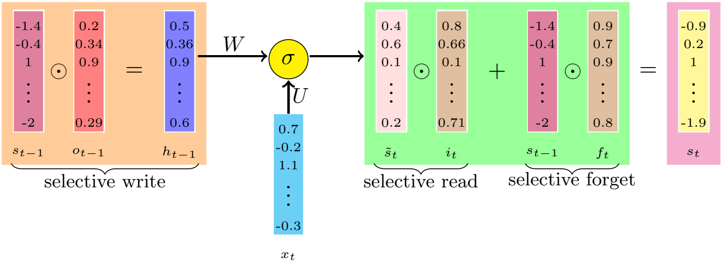


Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state
- Here is one simple (but effective) way of doing this:

$$s_t = s_{t-1} + i_t \odot \tilde{s}_t$$

- But we may not want to use the whole of s_{t-1} but forget some parts of it

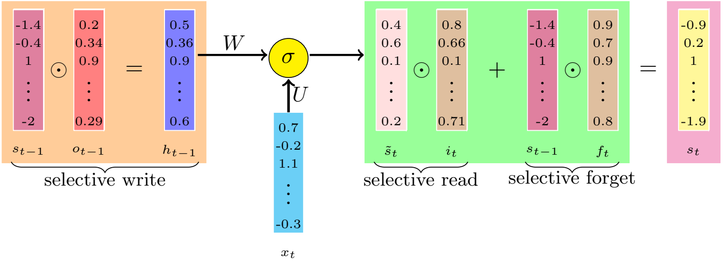


Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state
- Here is one simple (but effective) way of doing this:

$$s_t = s_{t-1} + i_t \odot \tilde{s}_t$$

- But we may not want to use the whole of s_{t-1} but forget some parts of it
- To do this we introduce the forget gate



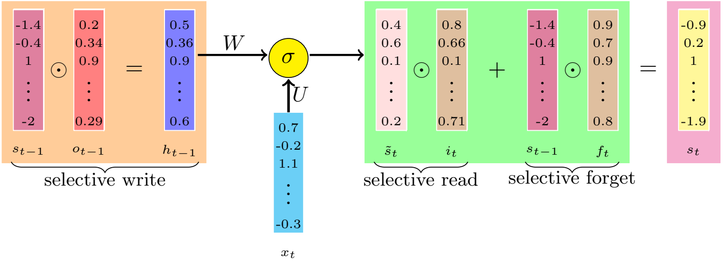
Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state
- Here is one simple (but effective) way of doing this:

$$s_t = s_{t-1} + i_t \odot \tilde{s}_t$$

- But we may not want to use the whole of s_{t-1} but forget some parts of it
- To do this we introduce the forget gate

$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$



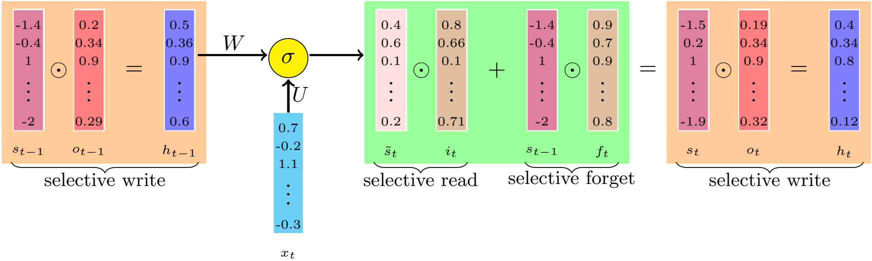
Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state
- Here is one simple (but effective) way of doing this:

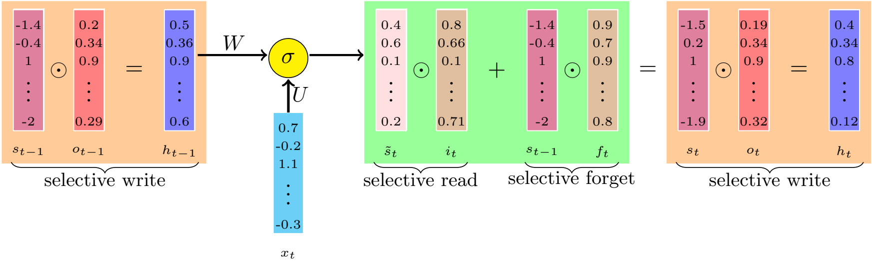
$$s_t = s_{t-1} + i_t \odot \tilde{s}_t$$

- But we may not want to use the whole of s_{t-1} but forget some parts of it
- To do this we introduce the forget gate

$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$
$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$



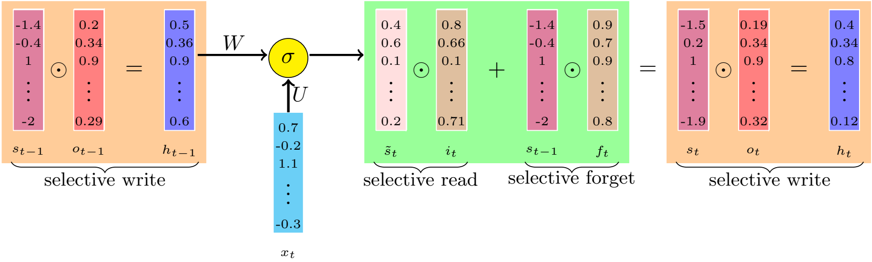
- We now have the full set of equations for LSTMs



- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

States:

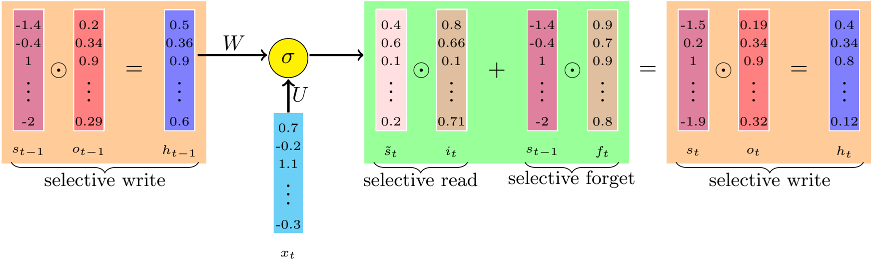


- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

States:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$



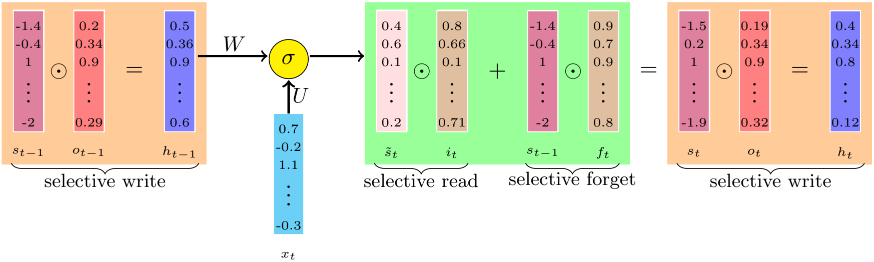
- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

States:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$

$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$

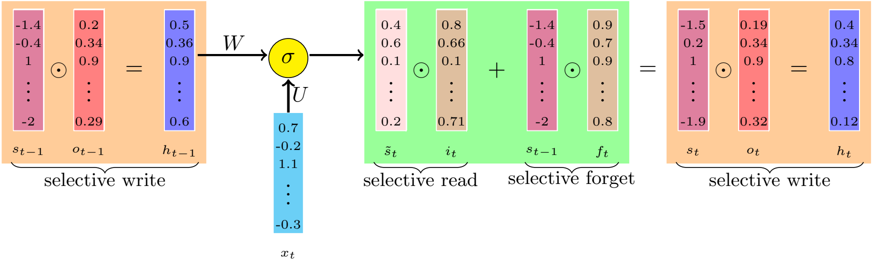


- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$
$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:



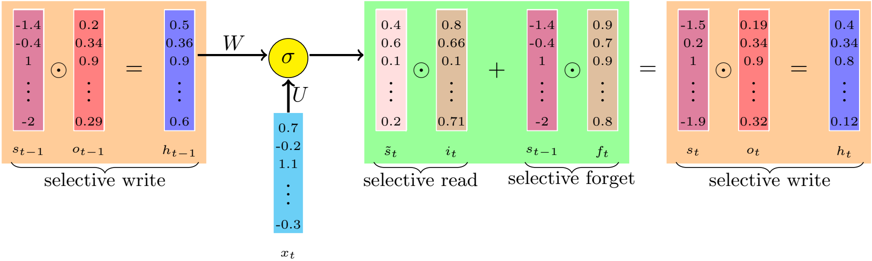
- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$
$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:

$$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$$



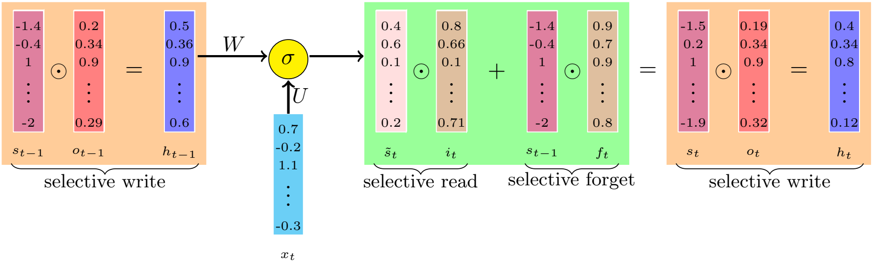
- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$
$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:

$$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$$
$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$



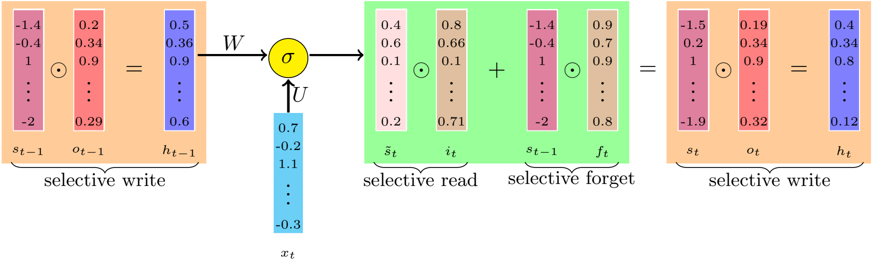
- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$
$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:

$$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$$
$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$
$$h_t = o_t \odot \sigma(s_t)$$



- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$
$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:

$$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$$
$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$
$$h_t = o_t \odot \sigma(s_t) \text{ and } rnn_{out} = h_t$$

Note

- LSTM has many variants which include different number of gates and also different arrangement of gates
- The one which we just saw is one of the most popular variants of LSTM
- Another equally popular variant of LSTM is Gated Recurrent Unit which we will see next

$$\begin{bmatrix} -1.4 \\ -0.4 \\ 1 \\ \vdots \\ -2 \end{bmatrix}$$

s_{t-1}

The full set of equations for GRUs

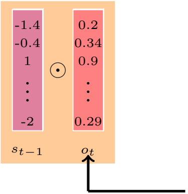
$$\begin{bmatrix} -1.4 \\ -0.4 \\ 1 \\ \vdots \\ -2 \end{bmatrix}$$

s_{t-1}

The full set of equations for GRUs

Gates:

States:

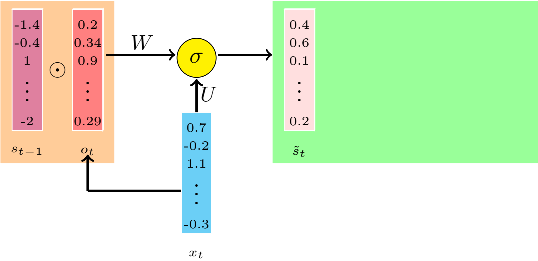


The full set of equations for GRUs

Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

States:



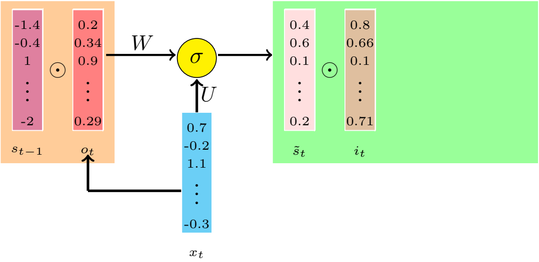
The full set of equations for GRUs

Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

States:

$$\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + U x_t + b)$$



The full set of equations for GRUs

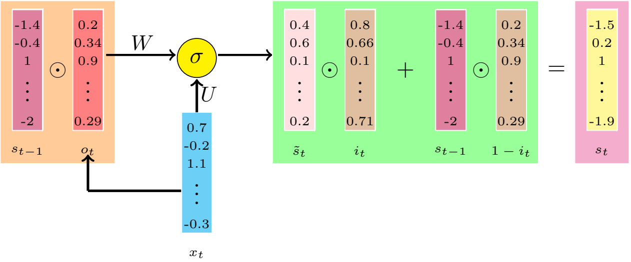
Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

$$\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + U x_t + b)$$



The full set of equations for GRUs

Gates:

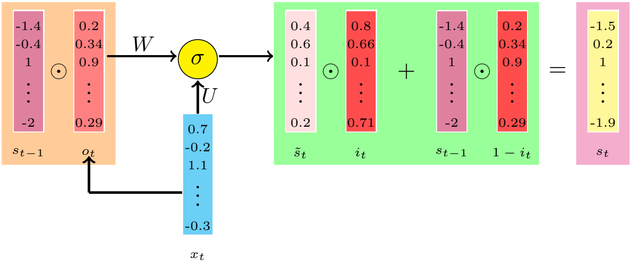
$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

$$\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + U x_t + b)$$

$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s}_t$$



The full set of equations for GRUs

Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

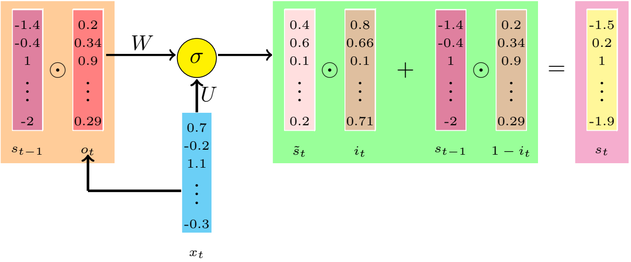
$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

$$\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$

$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s}_t$$

- No explicit forget gate (the forget gate and input gates are tied)



The full set of equations for GRUs

Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

$$\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + U x_t + b)$$

$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s}_t$$

- No explicit forget gate (the forget gate and input gates are tied)
- The gates depend directly on s_{t-1} and not the intermediate h_{t-1} as in the case of LSTMs