Dynamic Bottleneck Optimization for 2-Vertex and Strong Connectivity

Presented By –

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- Problem Statement-

  -On a complete weighted graph that changes dynamically by edge weight updates, we consider the problem of maintaining efficiently a minimum value b(bottleneck), such that the set of edges with weights less than b induces a strongly connected graph (in the directed case) on the same vertex set.

  -algorithm also re-evaluate b.

- Strong Connectivity –

  A graph is strongly connected if for every ordered pair u, v there is a directed path from u to v and v to u .

- This algorithm is a lazy dynamic version of static algorithm.
- Here we assume a complete weighted directed graph $\mathbf{K_n}$ with edge weight $w(a) \geq 0$
- For each edge $a = (u,v)$,

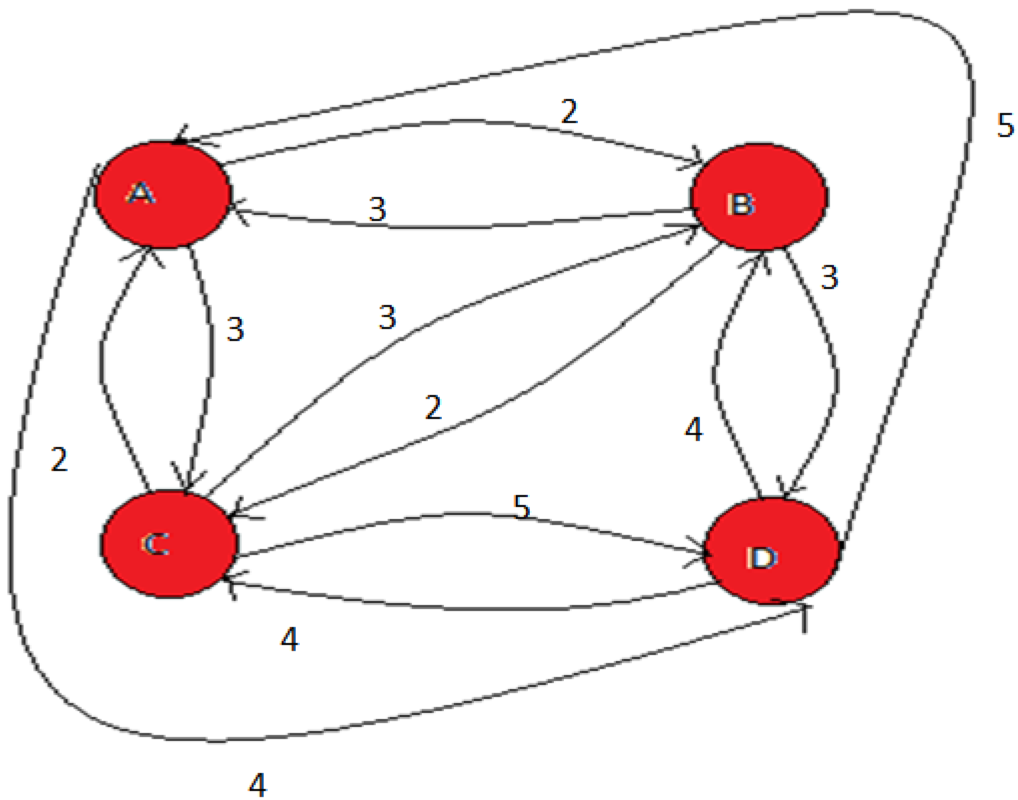
  $t(a)$ = tail vertex

  $h(a)$ = head vertex
- For a vertex subset S let $\delta(S) = \{a \in Kn \mid h(a) \in S, t(\notin \quad S\}$ (exactly the "in" cut-set of S).
- A contraction operation applies to a directed cycle of vertices and replaces the cycle by a single vertex (called the contraction vertex)

b-str connect algorithm

1. $H \leftarrow \emptyset$;
2. **while K_n is not a single vertex do:**
   (a) pick a vertex **v** with $\delta(v) \cap H = \emptyset$ and
   $\delta(v) \neq \emptyset$;
   (b) let $a^* \leftarrow \operatorname{argmin}_{a \varepsilon \delta(v)} w(a)$ and insert a^* in H
   (c) if a directed cycle has occurred, contract the
   cycle into a single vertex.
3. **return $\max_{a \varepsilon H} w(a)$**

Example-



increase(a, w'(a)) -

- **if a ε H and w'(a) > b do:**

  1. invalidate part of H relevant to **a**
  2. update w(a) to w'(a)
  3. update data structures
  4. execute b-str connect

decrease(a, w'(a)) -

if w'(a) < b and a replaces a' ε H do:

1. invalidate part of H relevant to a'
2. update w(a) to w'(a)
3. update data structures
4. execute b-str connect

Correctness proof -

- **Proof by contradiction-**

  Let us assume that there is an edge a in H which weight is greater then b.

  this condition is occurred only when i increase weight of any edge which is belongs to H, but after execute b-str connect algorithm my bottleneck value is updated to weight value of this edge(bottleneck b $= \max_{a \,\varepsilon\, H} w(a)$), so this condition is never happened.

- Complexity – $O(n^2)$

THANK YOU