Feature Selection and Engineering

SHALA2020.github.io

Learning objectives

- List advantages and disadvantages of feature selection and engineering
- List basic feature selection method types
- Write the formulae for a few feature selection metrics, e.g. t-test, AIC
- Write the steps for a few feature selection procedures, e.g. FS, BE
- Write the objective of LASSO and elastic-net
- Write the steps for PCA
- List some common features used for images, speech, and text

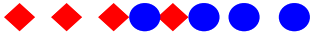
Why select features?

- Reduce overfitting
- Reduce confusion
- Reduce collinearity
- Reduce training time
- Simplify interpretation

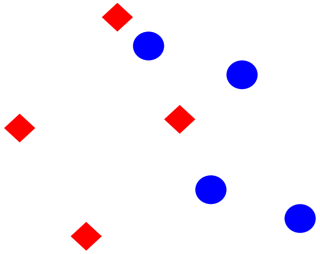
Disadvantages

- Inadvertently throw away useful information, because all selection methods have their own assumptions and biases

Compare two features



One of the
dimensions may
be useless



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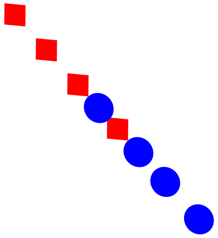
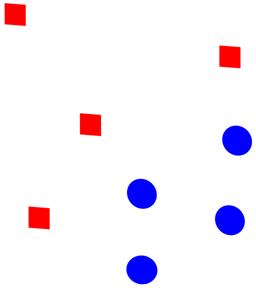


Rotating the
axes may be
useful



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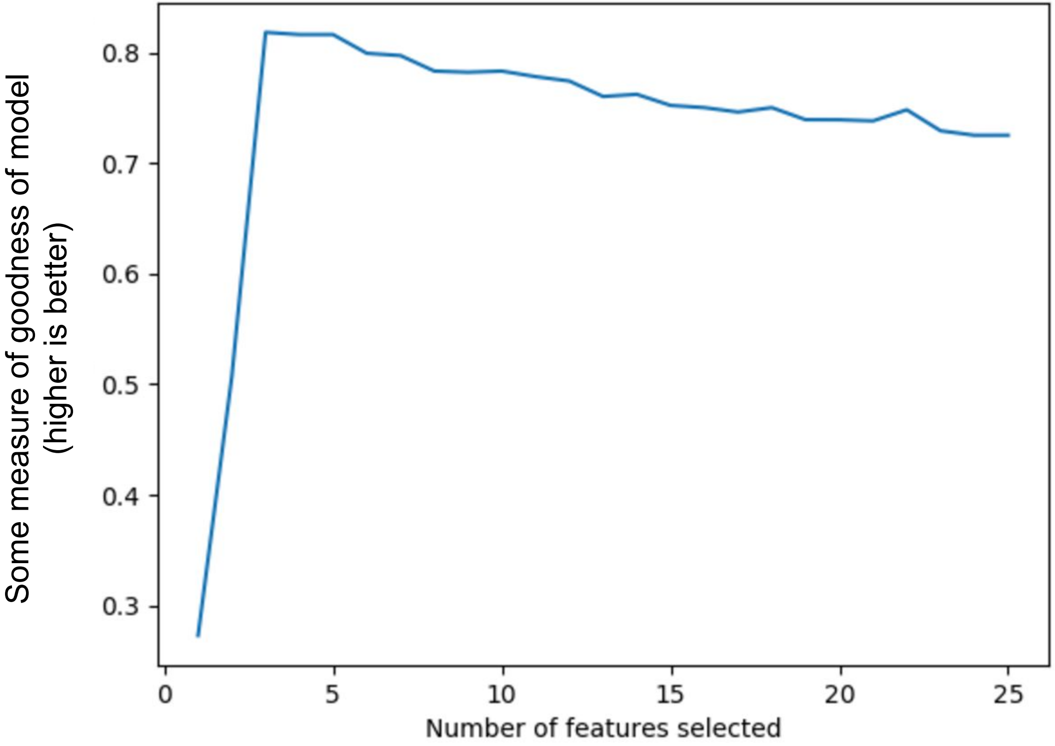
Usually, there is an optimal number (and subset) of features



Filtering

Basic idea: Remove “useless” or “redundant” features

- What makes a feature useful?
- What makes it non-redundant?

Algorithm:

- Loop through features x_i
  - Compute measure of utility and non-redundancy $m(x_i)$
- Sort features based on measure m
- Pick the top-k features

Filtering measures for regression

- Pearson correlation

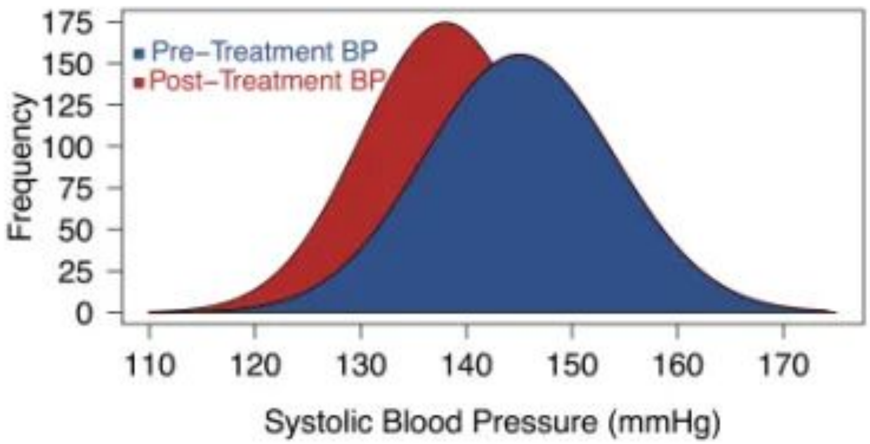
$$\frac{Cov(x,y)}{\sqrt{Var(x)}\sqrt{Var(y)}}$$

- Spearman correlation

$$\frac{Cov(rank(x),rank(y))}{\sqrt{Var(rank(x))}\sqrt{Var(rank(y))}}$$

Filtering measures for classification

- Hypothesis testing such as t-test, Wilcoxon rank test etc.



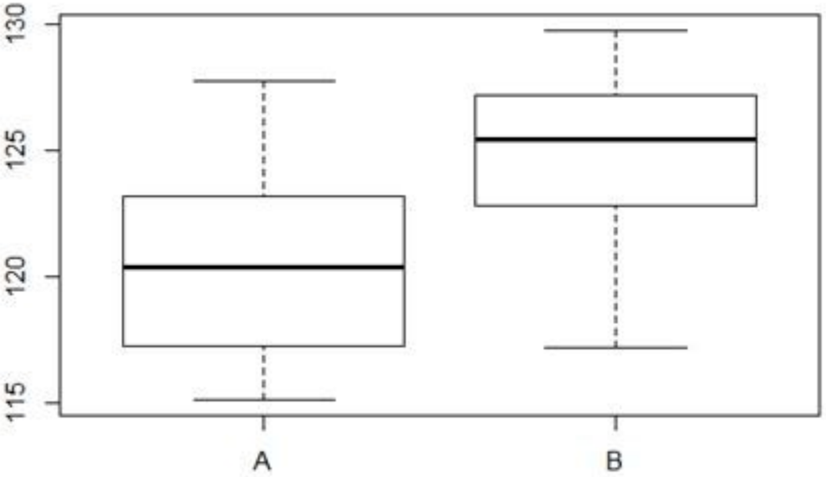
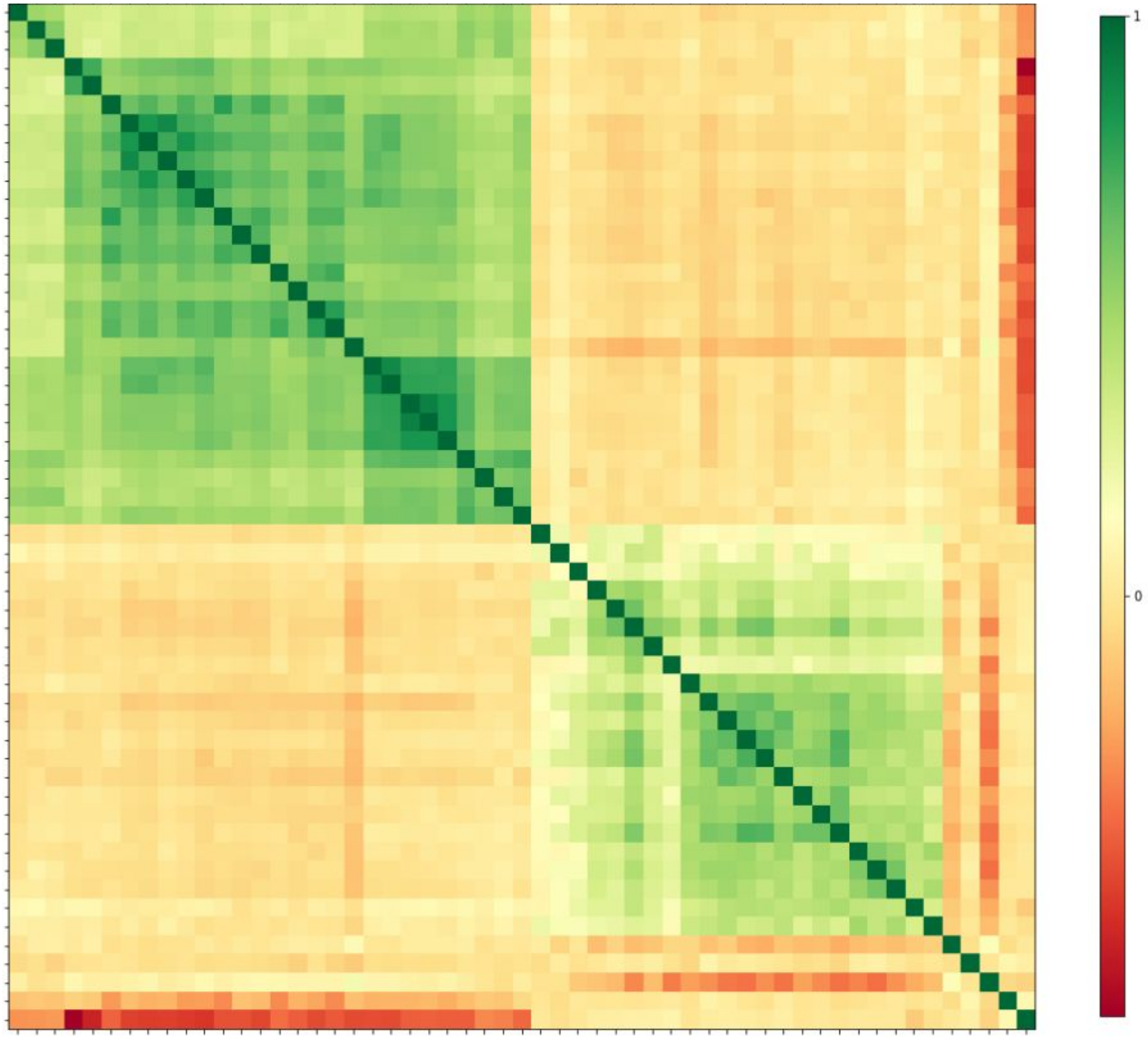


Image sources: https://www.r-bloggers.com/t-tests/, https://data.library.virginia.edu/the-wilcoxon-rank-sum-test/

Correlation-based clustering removes redundant features

One can choose a representative variable of each cluster

The selected variable can be based on interpretability



Wrapper methods

Basic idea:

- Add or remove features
- And measure model accuracy

Model selection criteria:

- K-fold CV
- AIC
- BIC

K-fold cross-validation

Held-out	Training

Result 1	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
Result 2	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
Result 3	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
Result 4	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5
Result 5	Fold 1	Fold 2	Fold 3	Fold 4	Fold 5

Overall result

AIC and BIC

AIC - Akaike Information Criterion

- $2k - 2\ln(L)$; where k is number of parameters, L is likelihood

BIC - Bayesian Information Criterion

- $k\ln(n) - 2\ln(L)$; where k is number of parameters, n is number of samples, L is likelihood

Forward selection

- Start with no variables in set S and all variables in set A
- For i = 1 to d
  - for j = 1 to d-i+1
    - Compute a measure m of adding variable $x_{A(j)}$ from A to the model
    - Select the variable with the best measure
      - If the change in measure m meets some criteria
        - Remove it from A and put it in S
      - Else exit

Backward elimination (also RFE)

- Start with all variables in set S and no variables in set A
- For i = 1 to d
  - for j = 1 to d
    - Compute a measure m of removing variable $x_{S(j)}$ from S to the model
    - Select the variable with the best change in measure
      - If the change in measure m meets some criteria
        - Remove it from S and put it in A
      - Else exit

Forward selection and backward elimination compared

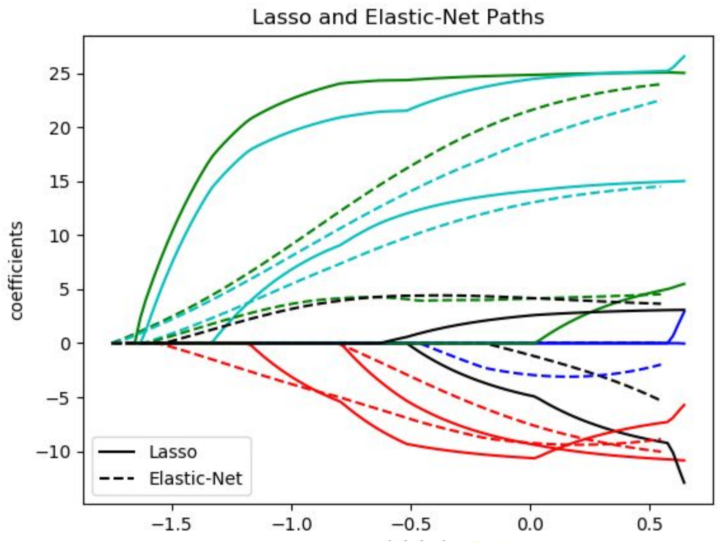
- Both are greedy methods
- In FS two variables individually may be uninformative and thus not considered, but together may be informative
- In BE we may start with a very complicated model initially itself

Regularization for feature selection

- LASSO (L1 penalty over weights)

$$L_p \text{ norm is } \left(\sum_i |w_i|^p \right)^{1/p}$$

- Elastic-net (L1+L2 penalty over weights)



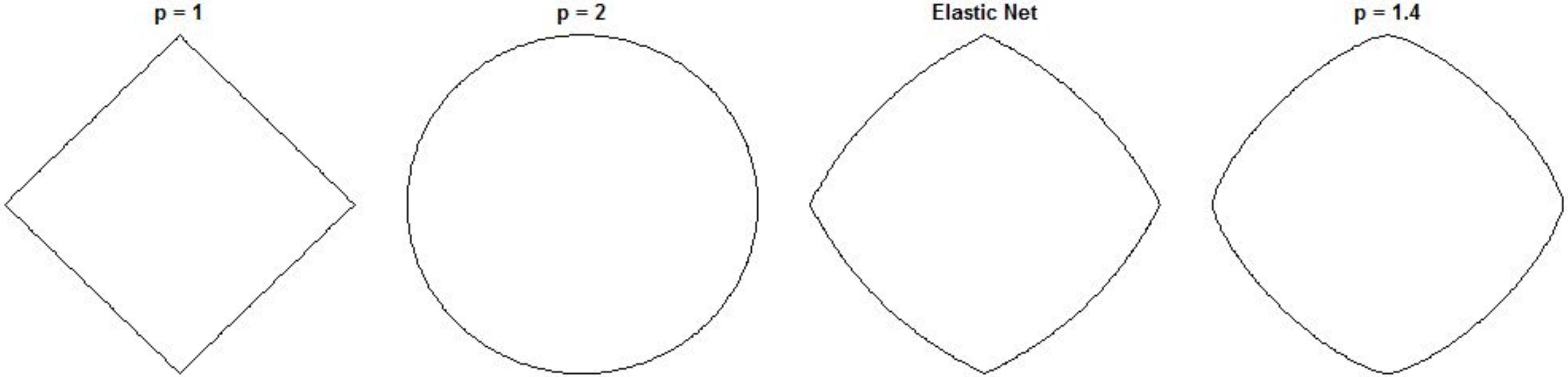


Image source: https://stats.stackexchange.com/questions/224531/bridge-penalty-vs-elastic-net-regularization; https://scikit-learn.org

Unsupervised feature reduction using PCA

Principal component analysis

- Start with *D*-dimensional data
- Compute covariance matrix Σ
- Eigen decompose $\Sigma = U \Lambda U^{\top}$
  - *U* contains eigenvectors (principal directions) and *Λ* is a diagonal matrix of eigenvalues
- Select *d*<*D* directions (eigenvalues) with the highest eigenvalues
- Project data to those *d* dim.s

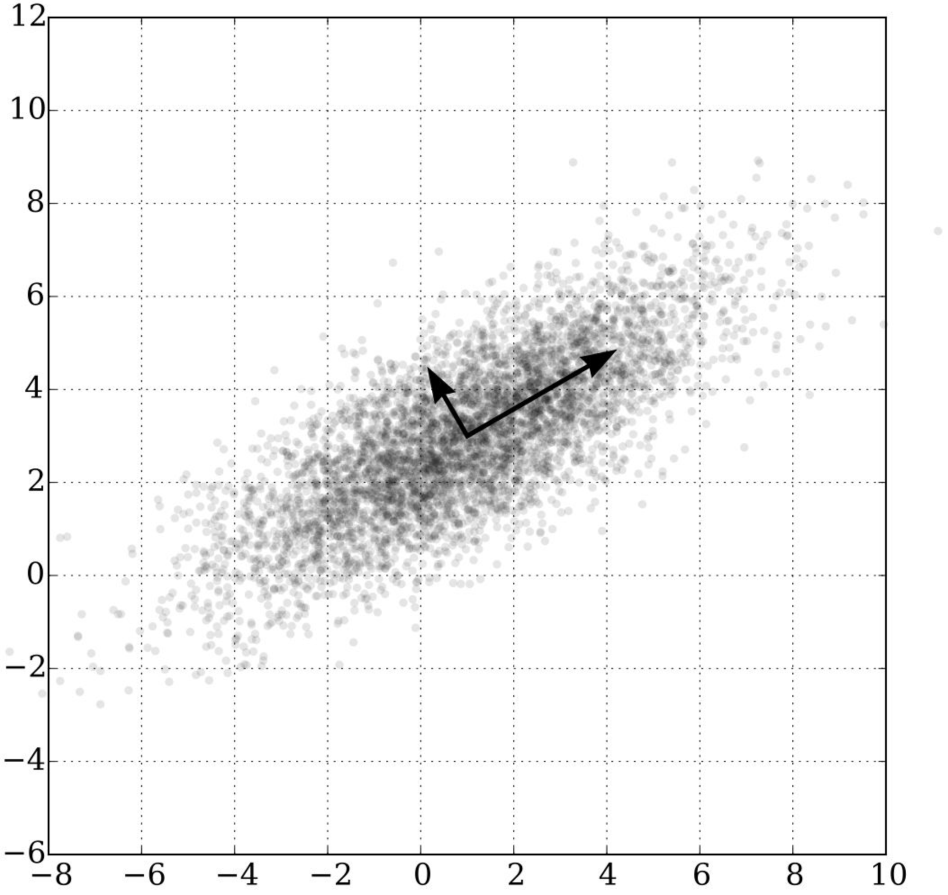


Image source: wikipedia.org

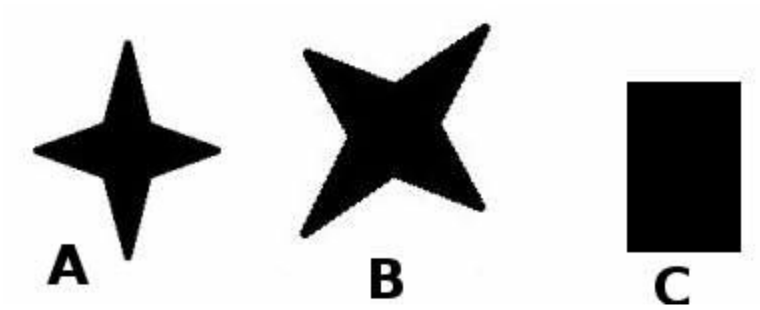
How to engineer features?

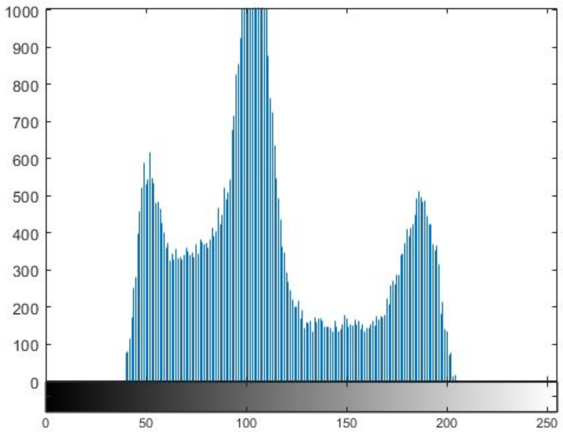
Features based on domain knowledge and statistics for:

- Images
- Audio
- Text

Features for images

- Intensity histogram
  - Its mean, median, mode, skew, kurtosis
- Color histogram
  - Bivariate histogram
- Hu invariant moments





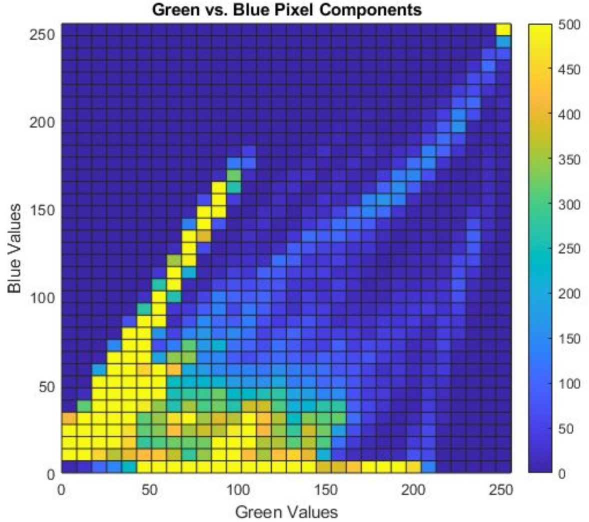
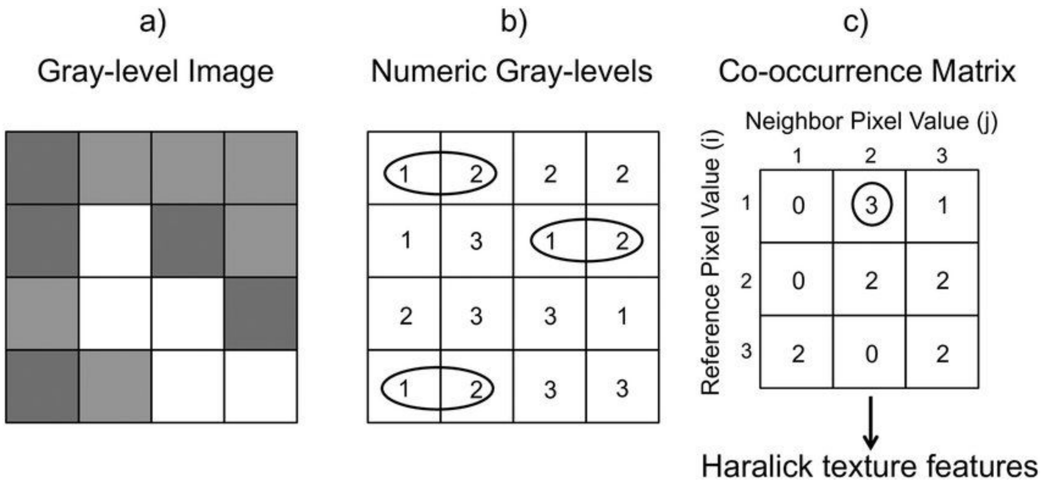


Image source: mathworks.com, opencv.org

Features for images

GLCM



Haar features

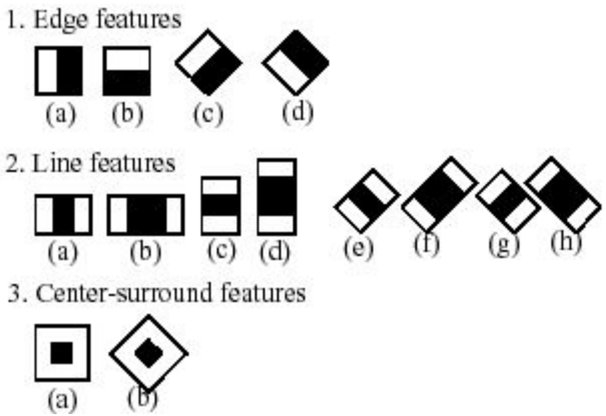


Image source: Do, Quyen et al. Texture analysis of magnetic resonance images of the human placenta throughout gestation: A feasibility study. PLOS ONE’19; opencv.org

Features for audio

MFCC features

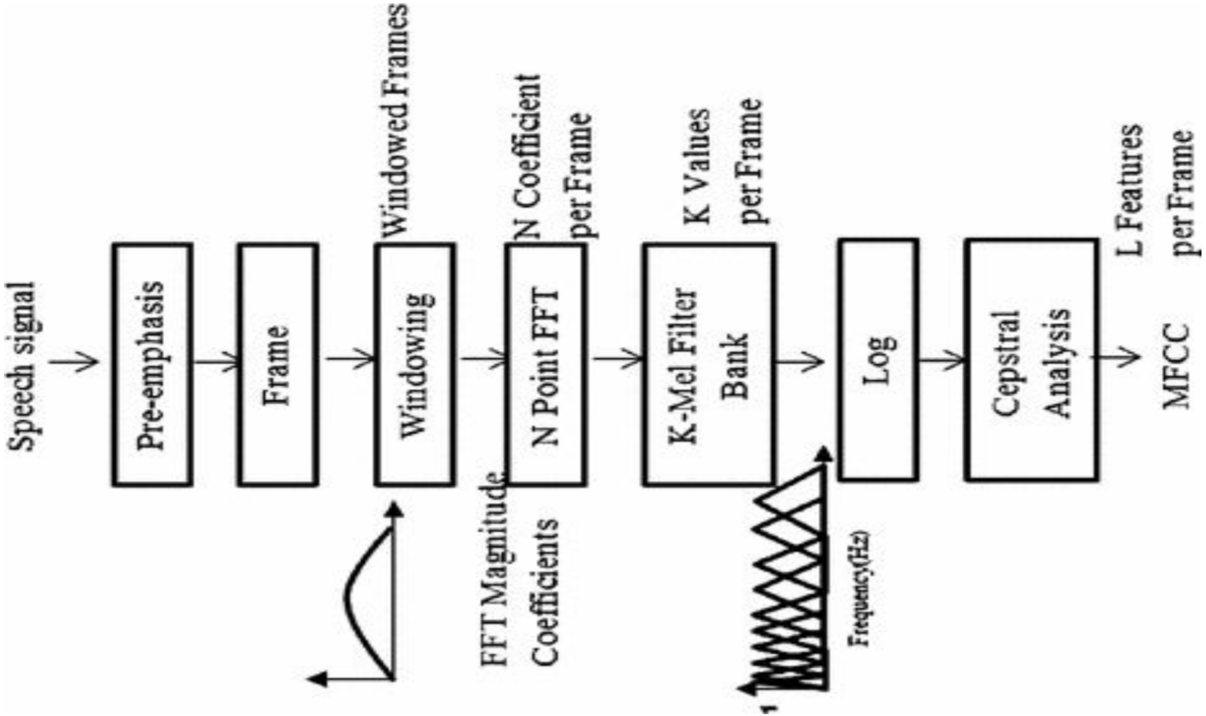


Image source: Srivastava S, Bhardwaj S, Kiran P. Gaussian Membership Function-Based Speaker Identification Using Score Level Fusion of MFCC and GFCC. InProceedings of the International Congress on Information and Communication Technology 2016 (pp. 283-291). Springer, Singapore.

Features for text

- Bag-of-words and Frequency of *n*-grams
  - "India posted a score of 256/8 in their allotted 50 overs in the third and deciding ODI of the series. Virat Kohli was the top-scorer for men in blue with a classy 71, while Adil Rashid and David Willey picked up three wickets each"
  - Counts
    - $\begin{array}{c} India \\ Posted \\ Score \end{array} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$
  - The counts can be normalized
  - The words can be standardized
    - Score
    - Scorer
  - What about uninformative words?
- TF-IDF
  - Term frequency – inverse document frequency
  - TF $f_{t,d}$ is the count of term *t* in document *d*
    - Usually normalized in some sense
    - $\mathrm{tf}(t,d) = \frac{f_{t,d}}{\sum_{t' \in d} f_{t',d}}$
  - IDF penalizes terms that occur often in all documents, e.g. "the"
    - $\mathrm{idf}(t,D) = \log \frac{|D|}{1+|\{d \in D : t \in d\}|}$
  - TF-IDF is $\mathrm{tf}(t,d) \times \mathrm{idf}(t,D)$
  - Form a vector of TF-IDF for various terms
    - Which terms?