Homework 1: Time Complexity, Basic Data Structures, Binary Search Trees, and Hashing

1. [2 + 3 = 5 points] What does the following algorithm do? Analyze its worst-case running time, and express it using "Big-Oh" notation.

   Algorithm Bar (*a*, *n*)
   Input: two integers, *a* and *n*
   Output: ?
   $k \leftarrow n$
   $b \leftarrow 1$
   $c \leftarrow a$
   while $k > 0$ **do**
       if $k \bmod 2 = 0$ **then**
           $k \leftarrow k/2$
           $c \leftarrow c * c$
       else
           $k \leftarrow k - 1$
           $b \leftarrow b * c$
   return *b*

2. [3 points] Algorithm A and B spend exactly $T_A(n) = 0.1n^2 \log_{10} n$ and $T_B(n) = 2.5n^2$ microseconds respectively, for a problem of size n. Choose the algorithm, which is better in the Big-O sense, and find out a problem size n_0 such that for any larger size $n > n_0$ the chosen algorithm outperforms the other. If your problems are of the size $n \leq 10^9$, which algorithm, will you recommend to use?

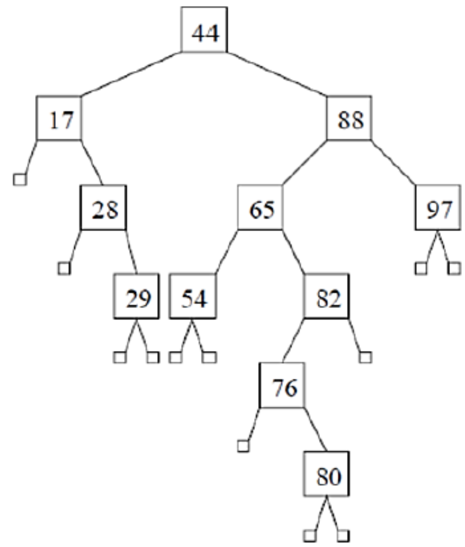
3. [3 points] Describe in pseudo-code a linear-time algorithm for reversing a queue Q. To access the queue, you are only allowed to use the methods of queue ADT. Hint: Consider using an auxiliary data structure.

4. [3 points] Draw a single binary tree T such that
   - each internal node of T stores a single character
   - a preorder traversal of T yields ABDGHEICFJ; and
   - an inorder traversal of T yields GDHBEIAFJC

5. [2 points] You are asked to implement a stack. You can use either a singly linked list or doubly linked list. Which one will you choose and why?

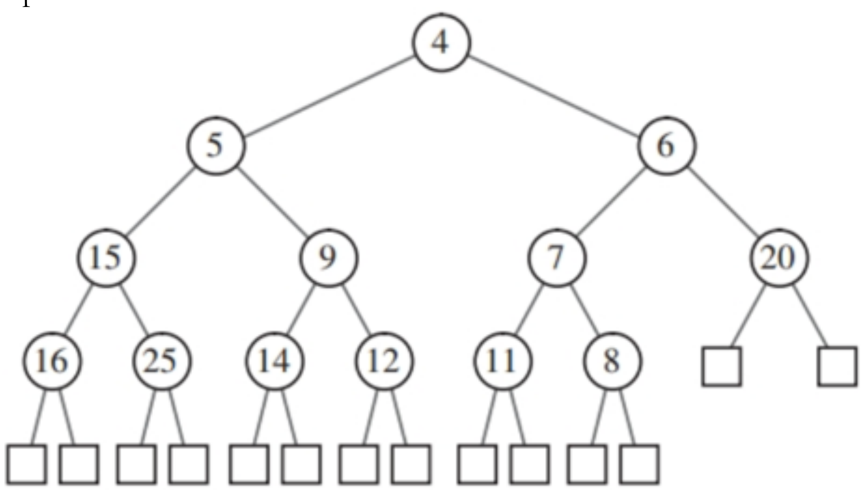
6. [3 + 1 = 4 points] Give pseudo-code of the algorithm least-common-ancestor that takes as input two nodes *v* and *w* in a tree T, and that gives as output the least common ancestor of *v* and *w* in T. What is the time complexity of your algorithm in terms of O?

7. [4 points] Remove from the given binary search tree the following keys (in this order): 65, 76, 88, 97. Draw the tree after each removal.



8. [5 points] Illustrate the execution of the bottom-up construction of a heap on the following sequence:(2, 5, 16, 4, 10, 23, 39, 18, 26, 15, 7, 9, 30, 31, 40).

9. [3 points] Let T be a heap storing n keys. Give an efficient algorithm for reporting all the keys in T that are smaller than or equal to a given query key x (which is not necessarily in T). For example, given the heap of below figure and query key x = 7, the algorithm should report 4, 5, 6, 7. Note that the keys do not need to be reported in sorted order. Your algorithm should run in $O(k)$ time, where k is the number of keys reported.



10. [3 points] Given two binary trees, write a function to check if they are the same or not. Two binary trees are considered the same if they are structurally identical and the nodes have the same value.

```
Input:     1         1      Input:     1         1      Input:     1         1
          / \       / \               /           \               / \       / \
         2   3     2   3             2             2             2   1     1   2
        [1,2,3], [1,2,3]            [1,2], [1, null,2]          [1,2,1], [1,1,2]
Output: true                Output: false               Output: false
```

11. [4 + 4 + 7 = 15 points] Draw 11-item hash table that results from hash function $h(i) = i \; mod \; 11$ to keys $22, 1, 13, 11, 24, 33, 18, 42,$ and 31 for each of the following methods. The items are inserted in order mentioned above in the hash table.
    - Separate chaining
    - Open addressing/linear probing
    - Double hashing;
        a) Use $d(i) = [i \; mod \; (m-1)] + 1$ for the secondary hash function. In both hash functions, the argument i is the value of the key being hashed. Here, m = 11.
        b) Use $d(i) = 7 - (i \; mod \; 7)$ for the secondary hash function. In both hash functions, the argument i is the value of the key being hashed.

Items	h(x)	(a) d(x)	(b) d(x)
22			
1			
13			
11			
24			
33			
18			
42			
31			

Index	Separate chaining	Open addressing	(a) Double Hashing	(b) Double Hashing
0				
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				