In [1]:

```
import matplotlib.pyplot as plt
import numpy as np
from qiskit import QuantumCircuit, Aer, transpile, assemble
from qiskit.visualization import plot_histogram
from math import gcd
from numpy.random import randint
import pandas as pd
from fractions import Fraction
print("Imports Successful")
```

```
Imports Successful
```

In [2]:

```
def c_amod15(a, power):
    """Controlled multiplication by a mod 15"""
    if a not in [2,7,8,11,13]:
        raise ValueError("'a' must be 2,7,8,11 or 13")
    U = QuantumCircuit(4)        p
    for iteration in range(power):
        if a in [2,13]:
            U.swap(0,1)
            U.swap(1,2)
            U.swap(2,3)
        if a in [7,8]:
            U.swap(2,3)
            U.swap(1,2)
            U.swap(0,1)
        if a == 11:
            U.swap(1,3)
            U.swap(0,2)
        if a in [7,11,13]:
            for q in range(4):
                U.x(q)
    U = U.to_gate()
    U.name = "%i^%i mod 15" % (a, power)
    c_U = U.control()
    return c_U
```

QFT

In [3]:

```
def qft_dagger(n):
    """n-qubit QFTdagger the first n qubits in circ"""
    qc = QuantumCircuit(n)
    # Don't forget the Swaps!
    for qubit in range(n//2):
        qc.swap(qubit, n-qubit-1)
    for j in range(n):
        for m in range(j):
            qc.cp(-np.pi/float(2**(j-m)), m, j)
        qc.h(j)
    qc.name = "QFT†"
    return qc
```

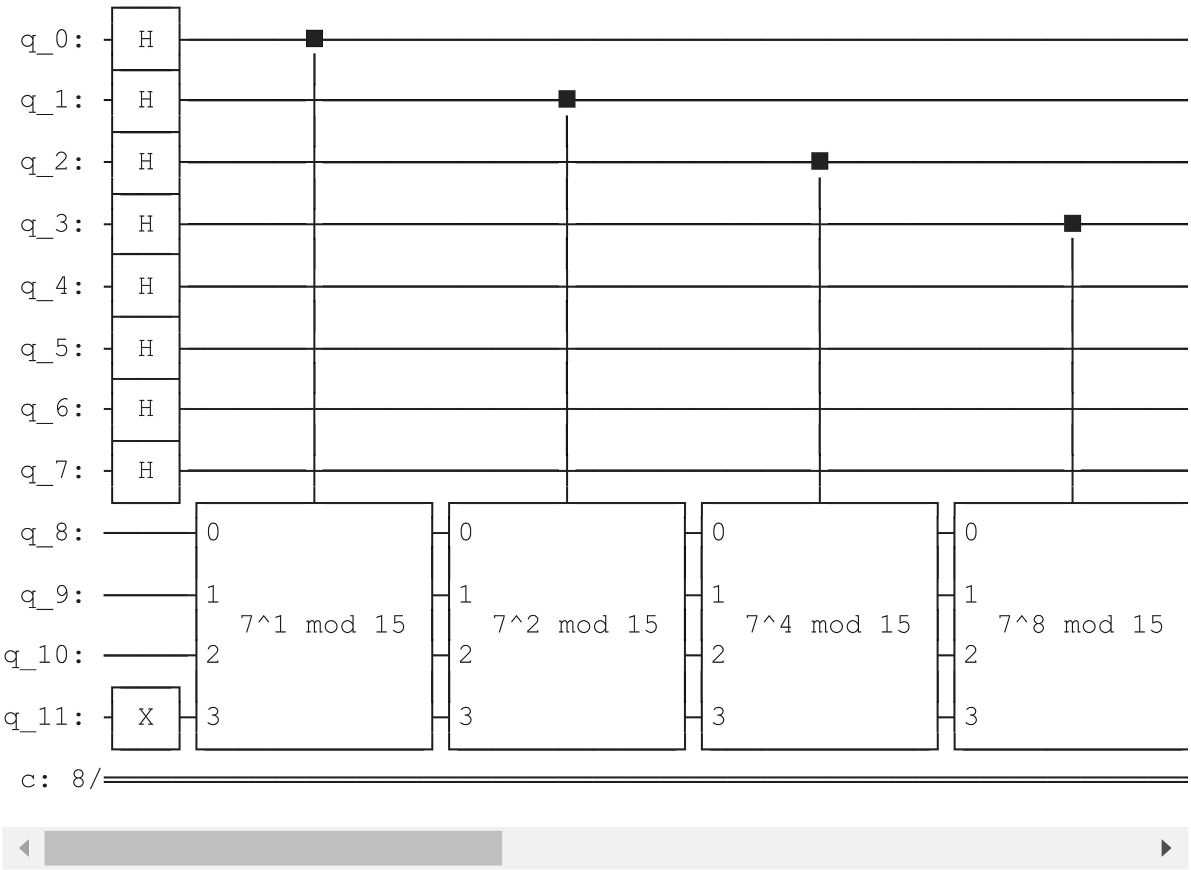
Circuit

In [4]:

```
# Specify variables
n_count = 8  # number of counting qubits
```

```
a = 7

# Create QuantumCircuit with n_count counting qubits
# plus 4 qubits for U to act on
qc = QuantumCircuit(n_count + 4, n_count)

# Initialize counting qubits
# in state |+>
for q in range(n_count):
    qc.h(q)

# And auxiliary register in state |1>
qc.x(3+n_count)

# Do controlled-U operations
for q in range(n_count):
    qc.append(c_amod15(a, 2**q), # second one is power of 2
             [q] + [i+n_count for i in range(4)]) # i+n_count will be 0+8, 1+8, 2+8,

# Do inverse-QFT
qc.append(qft_dagger(n_count), range(n_count))

# Measure circuit
qc.measure(range(n_count), range(n_count))
qc.draw(fold=-1)  # -1 means 'do not fold'
```

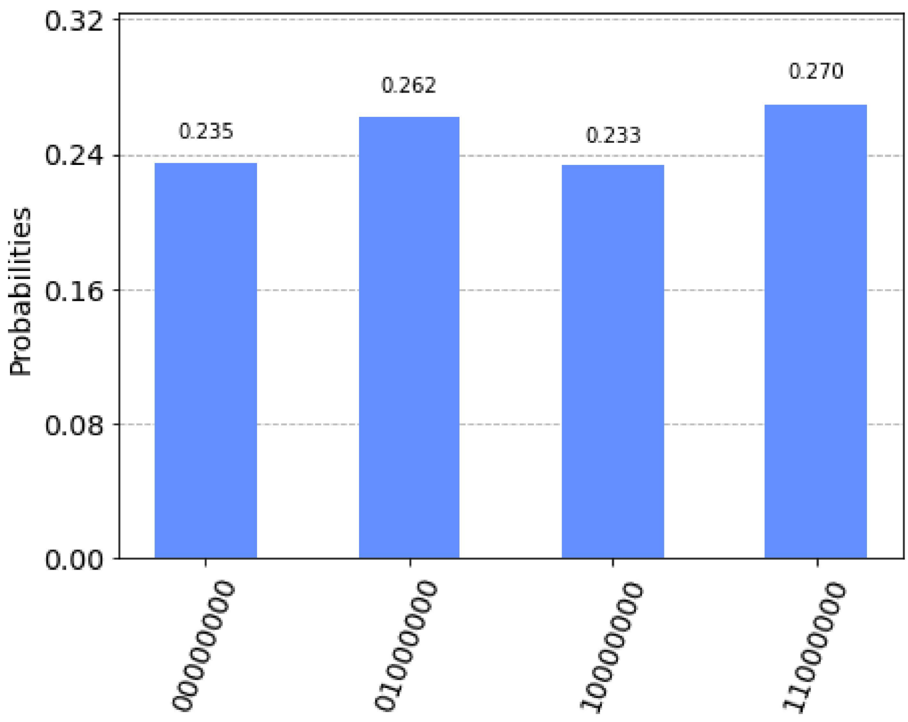
Out[4]:



In [5]:

```
aer_sim = Aer.get_backend('aer_simulator')
t_qc = transpile(qc, aer_sim)
qobj = assemble(t_qc)
results = aer_sim.run(qobj).result()
counts = results.get_counts()
plot_histogram(counts)
```

Out[5]:



In [6]:

```
rows, measured_phases = [], []
for output in counts:
    decimal = int(output, 2)  # Convert (base 2) string to decimal
    phase = decimal/(2**n_count)  # Find corresponding eigenvalue
    measured_phases.append(phase)
    # Add these values to the rows in our table:
    rows.append([f"{output}(bin) = {decimal:>3}(dec)", 
                 f"{decimal}/{2**n_count} = {phase:.2f}"])
# Print the rows in a table
headers=["Register Output", "Phase"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
            Register Output           Phase
0  11000000(bin) = 192(dec)  192/256 = 0.75
1  01000000(bin) =  64(dec)   64/256 = 0.25
2  00000000(bin) =   0(dec)    0/256 = 0.00
3  10000000(bin) = 128(dec)  128/256 = 0.50
```

In [7]:

```
rows = []
for phase in measured_phases:
    frac = Fraction(phase).limit_denominator(15)
    rows.append([phase, f"{frac.numerator}/{frac.denominator}", frac.denominator])
# Print as a table
headers=["Phase", "Fraction", "Guess for r"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
   Phase Fraction  Guess for r
0   0.75      3/4            4
1   0.25      1/4            4
2   0.00      0/1            1
3   0.50      1/2            2
```

In [ ]:

Main Program

In [8]:

```
def qpe_amod15(a):
    n_count = 8
    qc = QuantumCircuit(4+n_count, n_count)
    for q in range(n_count):
        qc.h(q)     # Initialize counting qubits in state |+>
    qc.x(3+n_count) # And auxiliary register in state |1>
    for q in range(n_count): # Do controlled-U operations
        qc.append(c_amod15(a, 2**q),
                 [q] + [i+n_count for i in range(4)])
    qc.append(qft_dagger(n_count), range(n_count)) # Do inverse-QFT
    qc.measure(range(n_count), range(n_count))
    # Simulate Results
    aer_sim = Aer.get_backend('aer_simulator')
    # Setting memory=True below allows us to see a list of each sequential reading
    t_qc = transpile(qc, aer_sim)
    qobj = assemble(t_qc, shots=1)
    result = aer_sim.run(qobj, memory=True).result()
    readings = result.get_memory()
    print("Register Reading: " + readings[0])
    phase = int(readings[0],2)/(2**n_count)
    print("Corresponding Phase: %f" % phase)
    return phase
```

In [9]:

```
N = 15
```

In [13]:

```
a = 7
factor_found = False
attempt = 0
while not factor_found:
    attempt += 1
    print("\nAttempt %i:" % attempt)
    phase = qpe_amod15(a) # Phase = s/r
    frac = Fraction(phase).limit_denominator(N) # Denominator should (hopefully!) te
    r = frac.denominator
    print("Result: r = %i" % r)
    if phase != 0:
        # Guesses for factors are gcd(x^{r/2} ±1 , 15)
        guesses = [gcd(a**(r//2)-1, N), gcd(a**(r//2)+1, N)]
        print("Guessed Factors: %i and %i" % (guesses[0], guesses[1]))
        for guess in guesses:
            if guess not in [1,N] and (N % guess) == 0: # Check to see if guess is a
                print("*** Non-trivial factor found: %i ***" % guess)
                factor_found = True
```

```
Attempt 1:
Register Reading: 00000000
Corresponding Phase: 0.000000
Result: r = 1

Attempt 2:
Register Reading: 10000000
Corresponding Phase: 0.500000
Result: r = 2
Guessed Factors: 3 and 1
*** Non-trivial factor found: 3 ***
```

In [ ]: