1. Initialization

In [1]:

```
import matplotlib.pyplot as plt
import numpy as np
from qiskit import QuantumCircuit, Aer, transpile, assemble
from qiskit.visualization import plot_histogram
from math import gcd
from numpy.random import randint
import pandas as pd
from fractions import Fraction
print("Imports Successful")
```

```
Imports Successful
```

2. General Code

In this example we will solve the period finding problem for $a = 7$ and $N = 15$. We provide the circuits for U where:

$$U|y\rangle = |ay \bmod 15\rangle$$

without explanation. To create U^x, we will simply repeat the circuit x times. In the next section we will discuss a general method for creating these circuits efficiently. The function `c_amod15` returns the controlled-U gate for `a`, repeated `power` times.

Code for |ay mod N>

In [2]:

```
def c_amod15(a, power):
    """Controlled multiplication by a mod 15"""
    if a not in [2,7,8,11,13]:
        raise ValueError("'a' must be 2,7,8,11 or 13")
    U = QuantumCircuit(4)
    for iteration in range(power):
        if a in [2,13]:
            U.swap(0,1)
```

```
            U.swap(1,2)
            U.swap(2,3)
        if a in [7,8]:
            U.swap(2,3)
            U.swap(1,2)
            U.swap(0,1)
        if a == 11:
            U.swap(1,3)
            U.swap(0,2)
        if a in [7,11,13]:
            for q in range(4):
                U.x(q)
    U = U.to_gate()
    U.name = "%i^%i mod 15" % (a, power)
    c_U = U.control()
    return c_U
```

QFT

In [3]:

```
def qft_dagger(n):
    """n-qubit QFTdagger the first n qubits in circ"""
    qc = QuantumCircuit(n)
    # Don't forget the Swaps!
    for qubit in range(n//2):
        qc.swap(qubit, n-qubit-1)
    for j in range(n):
        for m in range(j):
            qc.cp(-np.pi/float(2**(j-m)), m, j)
        qc.h(j)
    qc.name = "QFT†"
    return qc
```

3. Circuit

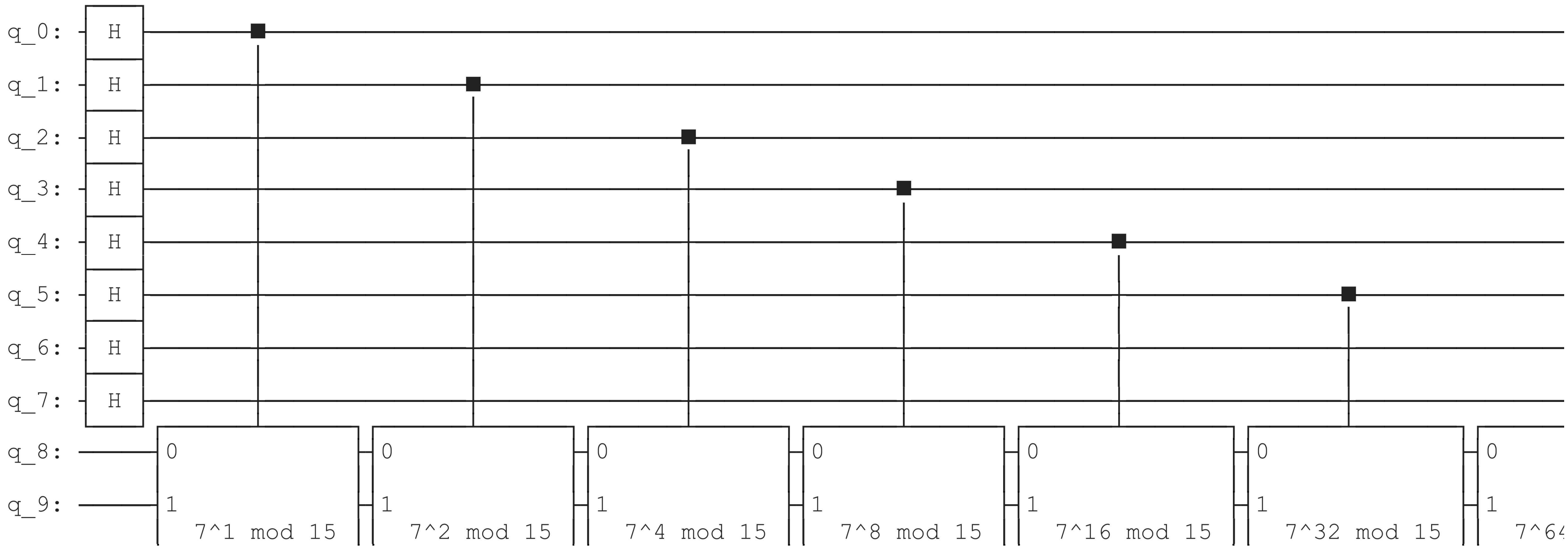
In [4]:

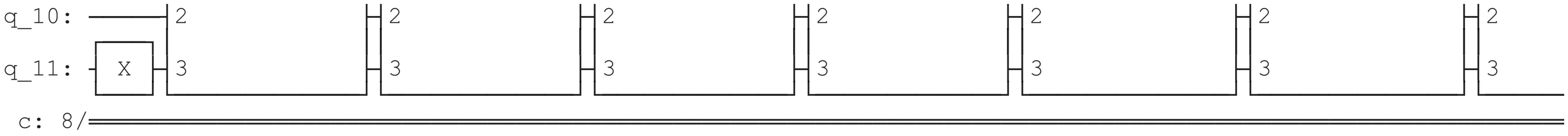
```
# Specify variables
n_count = 8  # number of counting qubits
a = 7
```

In [5]:

```python
# Create QuantumCircuit with n_count counting qubits
# plus 4 qubits for U to act on
qc = QuantumCircuit(n_count + 4, n_count)

# Initialize counting qubits
# in state |+>
for q in range(n_count):
    qc.h(q)

# And auxiliary register in state |1>
qc.x(3+n_count)

# Do controlled-U operations
for q in range(n_count):
    qc.append(c_amod15(a, 2**q), # second one is power of 2
             [q] + [i+n_count for i in range(4)]) # i+n_count will be 0+8, 1+8, 2+8, 3+8 where n_count = 8

# Do inverse-QFT
qc.append(qft_dagger(n_count), range(n_count))

# Measure circuit
qc.measure(range(n_count), range(n_count))
qc.draw(fold=-1)  # -1 means 'do not fold'
```

Out[5]:



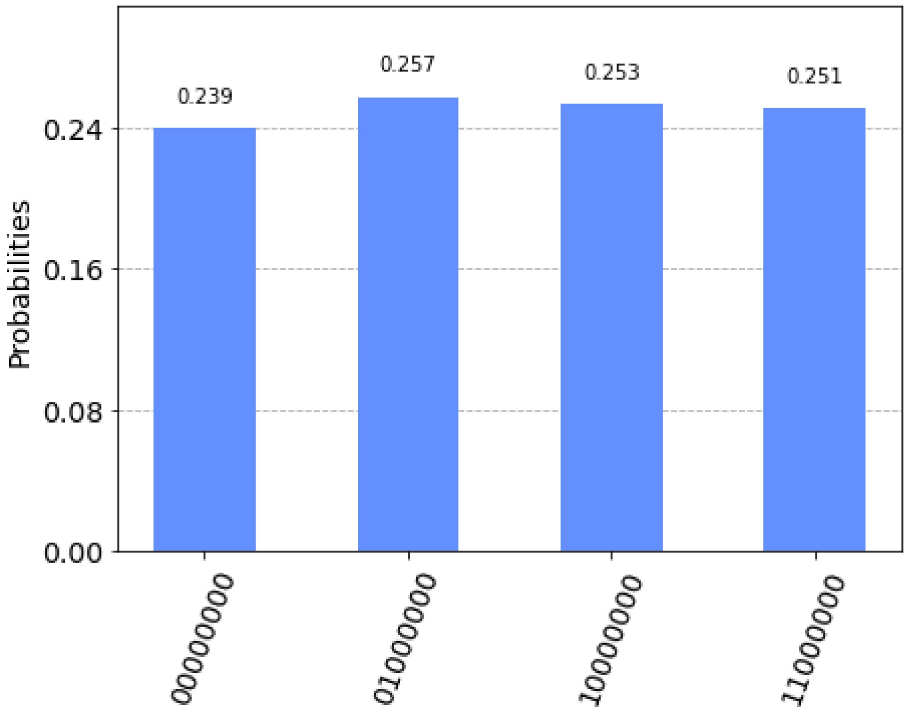


4. Result(Simulator)

In [6]:

```
aer_sim = Aer.get_backend('aer_simulator')
t_qc = transpile(qc, aer_sim)
qobj = assemble(t_qc)
results = aer_sim.run(qobj).result()
counts = results.get_counts()
plot_histogram(counts)
```

Out[6]:



Since we have 8 qubits, these results correspond to measured phases of:

In [7]:

```
rows, measured_phases = [], []
for output in counts:
    decimal = int(output, 2)  # Convert (base 2) string to decimal
    phase = decimal/(2**n_count)  # Find corresponding eigenvalue
    measured_phases.append(phase)
    # Add these values to the rows in our table:
    rows.append([f"{output}(bin) = {decimal:>3}(dec)",
                 f"{decimal}/{2**n_count} = {phase:.2f}"])
# Print the rows in a table
headers=["Register Output", "Phase"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
            Register Output           Phase
0  01000000(bin) =  64(dec)   64/256 = 0.25
1  00000000(bin) =   0(dec)    0/256 = 0.00
2  10000000(bin) = 128(dec)  128/256 = 0.50
3  11000000(bin) = 192(dec)  192/256 = 0.75
```

We can now use the continued fractions algorithm to attempt to find s and r. Python has this functionality built in: We can use the fractions module to turn a float into a Fraction object

The order (r) must be less than N, so we will set the maximum denominator to be 15:

In [8]:

```
rows = []
for phase in measured_phases:
    frac = Fraction(phase).limit_denominator(15)
    rows.append([phase, f"{frac.numerator}/{frac.denominator}", frac.denominator])
# Print as a table
headers=["Phase", "Fraction", "Guess for r"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
   Phase Fraction  Guess for r
0   0.25      1/4            4
1   0.00      0/1            1
2   0.50      1/2            2
3   0.75      3/4            4
```

We can see that two of the measured eigenvalues provided us with the correct result: $r = 4$, and we can see that Shor's algorithm has a chance of failing. These bad results are because $s = 0$, or because s and r are not coprime and instead of r we are given a factor of r. The easiest solution to this is to simply repeat the experiment until we get a satisfying result for r.

5. Factoring from period Finding

To see an example of factoring on a small number of qubits, we will factor 15, which we all know is the product of the not-so-large prime numbers 3 and 5.

In [9]:
```
N = 15
```

The first step is to choose a random number, a, between 1 and N-1.

In [10]:
```
np.random.seed(1) # This is to make sure we get reproduceable results
a = randint(2, 15)
print(a)
```

```
7
```

Next we quickly check it isn't already a non-trivial factor of N:

In [11]:
```
from math import gcd # greatest common divisor
gcd(a, N)
```

Out[11]:
```
1
```

Great. Next, we do Shor's order finding algorithm for `a = 7` and `N = 15`. Remember that the phase we measure will be s/r where:

$$a^r \bmod N = 1$$

and s is a random integer between 0 and $r - 1$.

In [18]:

```
def qpe_amod15(a):
    n_count = 8
    qc = QuantumCircuit(4+n_count, n_count)
    for q in range(n_count):
        qc.h(q)     # Initialize counting qubits in state |+>
    qc.x(3+n_count) # And auxiliary register in state |1>
    for q in range(n_count): # Do controlled-U operations
        qc.append(c_amod15(a, 2**q),
                 [q] + [i+n_count for i in range(4)])
    qc.append(qft_dagger(n_count), range(n_count)) # Do inverse-QFT
    qc.measure(range(n_count), range(n_count))
    # Simulate Results
    aer_sim = Aer.get_backend('aer_simulator')
    # Setting memory=True below allows us to see a list of each sequential reading
    t_qc = transpile(qc, aer_sim)
    qobj = assemble(t_qc, shots=1)
    result = aer_sim.run(qobj, memory=True).result()
    readings = result.get_memory()
    print("Register Reading: " + readings[0])
    phase = int(readings[0],2)/(2**n_count)
    print("Corresponding Phase: %f" % phase)
    return phase
```

In [19]:

```
phase = qpe_amod15(a) # Phase = s/r
Fraction(phase).limit_denominator(15) # Denominator should (hopefully!) tell us r
```

```
Register Reading: 01000000
Corresponding Phase: 0.250000
```

Out[19]:

```
Fraction(1, 4)
```

In [20]:

```
frac = Fraction(phase).limit_denominator(15)
s, r = frac.numerator, frac.denominator
print(r)
```

```
4
```

In [21]:

```
guesses = [gcd(a**(r//2)-1, N), gcd(a**(r//2)+1, N)]
print(guesses)
```

```
[3, 5]
```

The cell below repeats the algorithm until at least one factor of 15 is found. You should try re-running the cell a few times to see how it behaves.

In [23]:

```
a = 7
factor_found = False
attempt = 0
while not factor_found:
    attempt += 1
    print("\nAttempt %i:" % attempt)
    phase = qpe_amod15(a) # Phase = s/r
    frac = Fraction(phase).limit_denominator(N) # Denominator should (hopefully!) tell us r
    r = frac.denominator
    print("Result: r = %i" % r)
    if phase != 0:
        # Guesses for factors are gcd(x^{r/2} ±1 , 15)
        guesses = [gcd(a**(r//2)-1, N), gcd(a**(r//2)+1, N)]
        print("Guessed Factors: %i and %i" % (guesses[0], guesses[1]))
        for guess in guesses:
            if guess not in [1,N] and (N % guess) == 0: # Check to see if guess is a factor
                print("*** Non-trivial factor found: %i ***" % guess)
                factor_found = True
```

```
Attempt 1:
Register Reading: 00000000
Corresponding Phase: 0.000000
Result: r = 1

Attempt 2:
Register Reading: 01000000
Corresponding Phase: 0.250000
Result: r = 4
Guessed Factors: 3 and 5
```

```
*** Non-trivial factor found: 3 ***
*** Non-trivial factor found: 5 ***
```

In [ ]: