1. Initialization

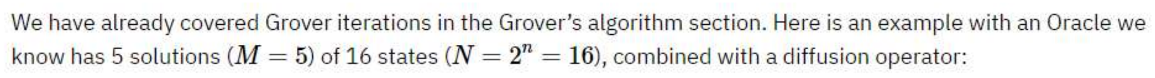
In [1]:
```
import matplotlib.pyplot as plt
import numpy as np
import math

# importing Qiskit
import qiskit
from qiskit import QuantumCircuit, transpile, assemble, Aer

# import basic plot tools
from qiskit.visualization import plot_histogram
```

In this guide will choose to ‘count’ on the first 4 qubits on our circuit (we call the number of counting qubits t, so t = 4), and to 'search' through the last 4 qubits (n = 4). With this in mind, we can start creating the building blocks of our circuit.

2. The Controlled-Grover Iteration


We have already covered Grover iterations in the Grover’s algorithm section. Here is an example with an Oracle we know has 5 solutions ($M = 5$) of 16 states ($N = 2^n = 16$), combined with a diffusion operator:

In [2]:
```
def example_grover_iteration():
    """Small circuit with 5/16 solutions"""
    # Do circuit
    qc = QuantumCircuit(4)
    # Oracle
    qc.h([2,3])
    qc.ccx(0,1,2)
    qc.h(2)
    qc.x(2)
    qc.ccx(0,2,3)
    qc.x(2)
    qc.h(3)
    qc.x([1,3])
    qc.h(2)
    qc.mct([0,1,3],2)
    qc.x([1,3])
    qc.h(2)
    # Diffuser
    qc.h(range(3))
    qc.x(range(3))
    qc.z(3)
    qc.mct([0,1,2],3)
    qc.x(range(3))
    qc.h(range(3))
    qc.z(3)
    return qc
```

We can use **.to_gate()** and **.control()** to create a controlled gate from a circuit. We will call our Grover iterator **grit** and the controlled Grover iterator **cgrit**:

In [3]:
```
# Create controlled-Grover
grit = example_grover_iteration().to_gate()
```

```
grit.label = "Grover"
cgrit = grit.control()
```

3. Inverse QFT

In [4]:

```
#This code implements the QFT on n qubits:
def qft(n):
    """Creates an n-qubit QFT circuit"""
    circuit = QuantumCircuit(4)
    def swap_registers(circuit, n):
        for qubit in range(n//2):
            circuit.swap(qubit, n-qubit-1)
        return circuit
    def qft_rotations(circuit, n):
        """Performs qft on the first n qubits in circuit (without swaps)"""
        if n == 0:
            return circuit
        n -= 1
        circuit.h(n)
        for qubit in range(n):
            circuit.cp(np.pi/2**(n-qubit), qubit, n)
        qft_rotations(circuit, n)

    qft_rotations(circuit, n)
    swap_registers(circuit, n)
    return circuit
```

We create the gate with t = 4 qubits as this is the number of counting qubits we have chosen in this guide:

In [5]:

```
# inversing the circuit
qft_dagger = qft(4).to_gate().inverse()
qft_dagger.label = "QFT†"
```

4. Algorithm

First we need to put all qubits in the $|+\rangle$ state:

In [7]:

```
# Create QuantumCircuit
t = 4   # no. of counting qubits
n = 4   # no. of searching qubits
qc = QuantumCircuit(n+t, t) # Circuit with n+t qubits and t classical bits

# Initialize all qubits to |+>
for qubit in range(t+n):
    qc.h(qubit)

# Begin controlled Grover iterations
iterations = 1
for qubit in range(t): # qubits in range of counting quibts
    for i in range(iterations):
        qc.append(cgrit, [qubit] + [*range(t, n+t)]) #controlled grover oracle
    iterations *= 2     # iteration in power of 2

# Do inverse QFT on counting qubits
qc.append(qft_dagger, range(t))
```

```python
# Measure counting qubits
qc.measure(range(t), range(t))

# Display the circuit
qc.draw()
```

Out[7]:

```
     ┌───┐
q_0: ┤ H ├──────■────────────────────────────────────────────────────────────────
     ├───┤      │
q_1: ┤ H ├──────┼────────────■────────────■──────────────────────────────────────
     ├───┤      │            │            │
q_2: ┤ H ├──────┼────────────┼────────────┼────────────■────────────■────────────■
     ├───┤      │            │            │            │            │            │
q_3: ┤ H ├──────┼────────────┼────────────┼────────────┼────────────┼────────────┼
     ├───┤ ┌────┴─────┐ ┌────┴─────┐ ┌────┴─────┐ ┌────┴─────┐ ┌────┴─────┐ ┌────┴
q_4: ┤ H ├─┤0         ├─┤0         ├─┤0         ├─┤0         ├─┤0         ├─┤0
     ├───┤ │          │ │          │ │          │ │          │ │          │ │
q_5: ┤ H ├─┤1         ├─┤1         ├─┤1         ├─┤1         ├─┤1         ├─┤1
     ├───┤ │  Grover  │ │  Grover  │ │  Grover  │ │  Grover  │ │  Grover  │ │  Gro
q_6: ┤ H ├─┤2         ├─┤2         ├─┤2         ├─┤2         ├─┤2         ├─┤2
     ├───┤ │          │ │          │ │          │ │          │ │          │ │
q_7: ┤ H ├─┤3         ├─┤3         ├─┤3         ├─┤3         ├─┤3         ├─┤3
     └───┘ └──────────┘ └──────────┘ └──────────┘ └──────────┘ └──────────┘ └──────
c: 4/══════════════════════════════════════════════════════════════════════════════

«
«q_0: ────────────────────────────────────────────────────────────────────────────
«
«q_1: ────────────────────────────────────────────────────────────────────────────
«
«q_2: ──────■─────────────────────────────────────────────────────────────────────
«           │
«q_3: ──────┼────────────■────────────■────────────■────────────■────────────■────
«      ┌────┴─────┐ ┌────┴─────┐ ┌────┴─────┐ ┌────┴─────┐ ┌────┴─────┐ ┌────┴────
«q_4: ─┤0         ├─┤0         ├─┤0         ├─┤0         ├─┤0         ├─┤0
«      │          │ │          │ │          │ │          │ │          │ │
«q_5: ─┤1         ├─┤1         ├─┤1         ├─┤1         ├─┤1         ├─┤1
«      │  Grover  │ │  Grover  │ │  Grover  │ │  Grover  │ │  Grover  │ │  Grover
«q_6: ─┤2         ├─┤2         ├─┤2         ├─┤2         ├─┤2         ├─┤2
«      │          │ │          │ │          │ │          │ │          │ │
«q_7: ─┤3         ├─┤3         ├─┤3         ├─┤3         ├─┤3         ├─┤3
«      └──────────┘ └──────────┘ └──────────┘ └──────────┘ └──────────┘ └─────────
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«
«                                             ┌───────┐ ┌─┐
«q_0: ─────────────────────────────────────────┤0      ├─┤M├──────────
«                                             │       │ └╥┘┌─┐
«q_1: ─────────────────────────────────────────┤1      ├──╫─┤M├───────
«                                             │  QFT† │  ║ └╥┘┌─┐
«q_2: ─────────────────────────────────────────┤2      ├──╫──╫─┤M├────
«                                             │       │  ║  ║ └╥┘┌─┐
«q_3: ──────■────────────■────────────■────────┤3      ├──╫──╫──╫─┤M├─
«      ┌────┴─────┐ ┌────┴─────┐ ┌────┴─────┐ └───────┘  ║  ║  ║ └╥┘
«q_4: ─┤0         ├─┤0         ├─┤0         ├────────────╫──╫──╫──╫──
«      │          │ │          │ │          │            ║  ║  ║  ║
«q_5: ─┤1         ├─┤1         ├─┤1         ├────────────╫──╫──╫──╫──
«      │  Grover  │ │  Grover  │ │  Grover  │            ║  ║  ║  ║
«q_6: ─┤2         ├─┤2         ├─┤2         ├────────────╫──╫──╫──╫──
«      │          │ │          │ │          │            ║  ║  ║  ║
«q_7: ─┤3         ├─┤3         ├─┤3         ├────────────╫──╫──╫──╫──
«      └──────────┘ └──────────┘ └──────────┘            ║  ║  ║  ║
«c: 4/═════════════════════════════════════════════════════╩══╩══╩══╩══
«                                                         0  1  2  3
```
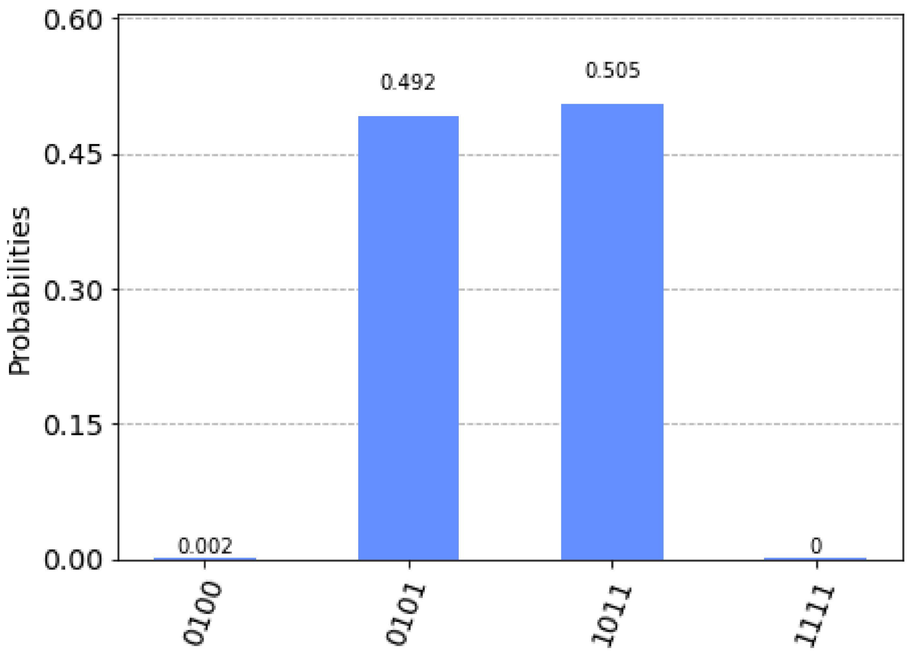

5. Simulator

In [9]:
```
# Execute and see results
aer_sim = Aer.get_backend('aer_simulator')
transpiled_qc = transpile(qc, aer_sim)
qobj = assemble(transpiled_qc)
job = aer_sim.run(qobj)
hist = job.result().get_counts()
plot_histogram(hist)
```

Out[9]:



We can see two values stand out, having a much higher probability of measurement than the rest. These two values correspond to $e^{i\theta}$ and $e^{-i\theta}$, but we can't see the number of solutions yet. We need to little more processing to get this information, so first let us get our output into something we can work with (an `int`).

We will get the string of the most probable result from our output data:

In [12]:
```
measured_str = max(hist, key=hist.get)
```

In [13]:
```
measured_int = int(measured_str,2)
print("Register Output = %i" % measured_int)
```

```
Register Output = 11
```

6. Finding the number of solutions(M)

We will create a function, `calculate_M()` that takes as input the decimal integer output of our register, the number of counting qubits (t) and the number of searching qubits (n).

First we want to get θ from `measured_int`. You will remember that QPE gives us a measured $\text{value} = 2^n\phi$ from the eigenvalue $e^{2\pi i\phi}$, so to get θ we need to do:

$$\theta = \text{value} \times \frac{2\pi}{2^t}$$

Or, in code:

In [14]:

```
theta = (measured_int/(2**t))*math.pi*2
print("Theta = %.5f" % theta)
```

```
Theta = 4.31969
```

In [15]:

```
N = 2**n
M = N * (math.sin(theta/2)**2)
print("No. of Solutions = %.1f" % (N-M))
```

```
No. of Solutions = 4.9
```

In [16]:

```
m = t - 1 # Upper bound: Will be less than this
err = (math.sqrt(2*M*N) + N/(2**(m+1)))*(2**(-m))
print("Error < %.2f" % err)
```

```
Error < 2.48
```

In [ ]: