MST124

TMA 03 2021J

Covers Units 7 and 8 **Cut-off date** **9 March 2022**

You will find information about TMAs in the 'Assessment' area of the MST124 website. Please read that information before beginning work on this TMA.

If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

The work that you submit should include your working as well as your final answers.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not involve the use of any other mathematical software.

Your work should be written in a good mathematical style, as described in Section 6 of Unit 1, and as demonstrated by the example and activity solutions in the study units. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 10. You do not have to submit any work for Question 10.

Question 1 –   15 marks

You should be able to answer this question after studying Unit 7.

Differentiate the following functions, simplifying your answers as far as possible.

(a) $f(x) = \cos(x) \ln\left(\frac{1}{2}x\right)$ [3]

(b) $g(y) = \dfrac{y^{1/3}}{e^{2y+1}}$ [4]

(c) $h(z) = \sin(-z^2 + 2z)$ [3]

(d) $k(t) = \left(\cos(t)\, e^{7t} + 1\right)^5$ [5]

Question 2 –   10 marks

You should be able to answer this question after studying Unit 7.

Alisha wants to build a frame for a stained glass window. The shape of the window is a rectangle with a semicircle on top whose diameter is the same width as the rectangle, as shown below. Alisha has 4 m of framing material. She wants to maximise the total area A of the window.
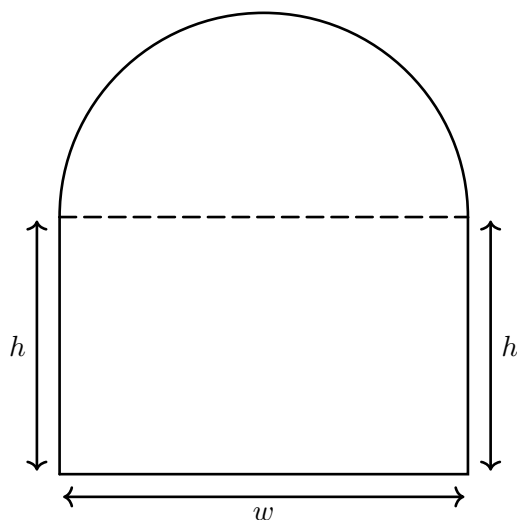
(a) Write down an equation for the total length of the framing material in terms of the height h and width w. Use this equation to express h in terms of w.   [3]

(b) Show that the total area A, in terms of w, is
$$A = 2w - \left(\frac{\pi + 4}{8}\right) w^2.$$ [2]

(c) Use differentiation to determine the width w that gives the maximum area A of the window, and show that this is a maximum. What is the maximum area of the window?   [5]

Question 3 – 5 marks

You should be able to answer this question after studying Unit 7.

Find the indefinite integral of the function

$$f(x) = \frac{1}{x}\left(2 + \sqrt{x(x+1)}\right)\left(-2 + \sqrt{x(x+1)}\right).$$

[5]

(Hint: it may be helpful to simplify $f(x)$ first.)

Question 4 – 10 marks

You should be able to answer this question after studying Unit 8.

Use integration by substitution to find the indefinite integral in part (a) and to evaluate the definite integral in part (b).

(a) $\displaystyle\int \frac{\sin(x)\cos(x) - e^x}{\cos^2(x) + 2e^x}\, dx$

[4]

(b) $\displaystyle\int_0^2 (2t - 4)^3(t + 4)\, dt$

[6]

Question 5 – 10 marks

You should be able to answer this question after studying Unit 8.

This question is about the function

$$f(\theta) = (\theta + 3)\sin(\theta/2).$$

(a) Explain why the graph of f lies on or above the x-axis for all values of θ in the interval $[4\pi, 6\pi]$.

[3]

(b) Write down an expression, involving a definite integral, that gives the area between the graph of f and the θ-axis, from $\theta = 4\pi$ to $\theta = 6\pi$.

[1]

(c) Use integration by parts to find the area described in part (b), giving both the exact answer and an approximation to three decimal places.

[6]

Question 6 – 10 marks

You should be able to answer this question after studying Unit 8.

(a) Use integration by substitution to find the indefinite integral

$$\int \left(x^2 + 2\right)(x + 1)^{42}\, dx.$$

[5]

(b) Use integration by parts to find the indefinite integral

$$\int x^2 e^{-3x}\, dx.$$

[5]

Question 7 – 10 marks

You should be able to answer this question after studying Unit 8.

Include a printout or screenshot of your Maxima worksheet for this question. You are not expected to annotate your Maxima worksheet with explanation. However, remember that for good mathematical communication you should present your answers clearly.

This question is about the function

$$f(x) = \frac{8\left(x^7 - 7x^4 + 3x\right)}{x^4 + 4}.$$

Use Maxima to do each of parts (a)–(d).

(In parts (c) and (d) you should round your answers to three decimal places yourself, instead of using Maxima to do this.)

(a) Plot the graph of f, choosing ranges of values on the x- and y-axes to make its two stationary points clearly visible. [3]

(b) Find the derivative of f. [1]

(c) Calculate the x- and y-coordinates of the local maximum of f, giving your answers to three decimal places. [3]

(d) The graph of f crosses the x-axis at $x = 0$. Calculate the value of x where the graph first crosses the x-axis to the right of $x = 0$. Find the area enclosed by the graph of f and the x-axis, between $x = 0$ and this value, giving your answer to three decimal places. [3]

Question 8 – 5 marks

You should be able to answer this question after studying Units 1 to 8.

In this question, you are asked to consider how you will prepare for the exam and how you will manage your time while writing the exam.

Before you attempt this question, we advise you to look at the revision resources available in the Help Centre *and the specific resources available from the* Mathematics and Statistics subject website.

(a) Write a brief explanation of how you will prepare for your exam. For example, this could include:

   - where and when you will revise

   - revision techniques that you will use

   - the tricky topics within this module

   - resources that you will use.   [2]

(b) *This part of the question requires you to consider how you will manage your time during the exam.*

   The MST124 exam consists of 42 multiple choice questions, comprising 34 questions each worth 2% of the total exam mark, and 8 questions each worth 4% of the total exam mark. The examination is scheduled to take 3 hours to complete.

   (i) Assuming that the time should be allocated in proportion to the marks for each question:

   - Approximately how long should you spend on answering a question that is worth 2% of the total marks?

   - Approximately how long should you spend on answering a question that is worth 4% of the total marks?   [2]

   (ii) List any other factors that you may need to take into account when estimating how long you should spend on each question.   [1]

Question 9 – 20 marks

You should be able to answer this question after studying Units 1 to 8.

In this question, you are asked to attempt the mini examination paper on pages 8–10 and enter your answers on the question paper.

You must submit your completed answer form as part of this TMA. You can either print and complete the form, and scan or photograph it to produce a PDF file, or complete it electronically by using annotation software. In either case, you must then incorporate it into your TMA.

You are not required to submit any working for the mini examination paper.

Complete the mini examination paper on pages 8–10, writing your answers on a printed copy of the question paper or entering them electronically on a digital copy of the question paper. [20]

Question 10 – 5 marks

Your score out of 5 marks for good mathematical communication in Questions 1 to 7 will be recorded under Question 10.

You do not need to submit any work for this question. [5]

MST124 Mini examination paper

This paper has **TWO** sections. You should attempt **ALL** questions in each section.

Section A has 6 questions, each worth 2 marks.
Section B has 2 question, each worth 4 marks.

Each question in Section A is multiple-choice, with **ONE** correct answer from five options. Answer each question by making a mark within the circle to the left-hand side of one of the options given. No marks will be deducted for incorrectly answered questions.

For both questions in Section B, write your answers in the boxes provided. Do not include any working; full marks will be given for a completely correct answer only. No marks will be deducted for incorrectly answered questions.

SECTION A

Question 1

Which of the following is equivalent to $(2 - x)(3x + 1) - 3(1 - x^2)$?

    ○   **A** $-x - 1$

    ○   **B** -1

    ○   **C** $6x - 3$

    ○   **D** $5x - 1$

    ○   **E** $-2x^2 - 5x - 1$

Question 2

Which of the following is equivalent to the inequality $\dfrac{1}{x + 2} > x - 1$?

    ○   **A** $-x^2 - x - 3 > 0$

    ○   **B** $\dfrac{-x^2 - x + 3}{x + 2} < 0$

    ○   **C** $-x^2 - x - 3 < 0$

    ○   **D** $\dfrac{-x^2 - x + 3}{x + 2} > 0$

    ○   **E** $\dfrac{-x^2 - x - 1}{x + 2} > 0$

Question 3

How many values of θ in radians between $-\pi/2$ and $3\pi/2$ satisfy $\cos(\theta) = -1/2$?

○   **A**   1

○   **B**   2

○   **C**   3

○   **D**   4

○   **E**   5

Question 4

What is the derivative of $e^x \cos(x)$?

○   **A** $e^x \cos(x) + e^x \sin(x)$

○   **B** $-e^x \sin(x)$

○   **C** $e^x \cos(x) - e^x \sin(x)$

○   **D** $\dfrac{e^x \cos(x) + e^x \sin(x)}{\cos^2(x)}$

○   **E** $e^x - \sin(x)$

Question 5

On which interval is the function $f(x) = -x^2 + 3x - 2$ increasing?

○   **A** $(-\infty, 3/2)$

○   **B** $(1, 2)$

○   **C** $[1, 2]$

○   **D** $(3/2, \infty)$

○   **E** $[1, 3/2)$

Question 6

Which of the following is equal to $\int_0^1 \left(\sqrt{x} - 2x \right) dx$?

○ **A** $-\frac{2}{3}$

○ **B** $\frac{1}{2}$

○ **C** $\frac{1}{3}$

○ **D** $\frac{2}{3}$

○ **E** $-\frac{1}{3}$

SECTION B

Question 7

A function f has derivative $f'(x) = 2x^2 + x$. What are the stationary points of f, and their natures?

The x-coordinate of one stationary point is at $x =$ ☐ .

It is a ☐ (options: local maximum, local minimum or horizontal point of inflection).

The x-coordinate of the other stationary point is at $x =$ ☐ .

It is a ☐ (options: local maximum, local minimum or horizontal point of inflection).

Question 8

The velocity v (in metres per second) of an object in terms of the time t (in seconds) is given by

$$v = 4t^2 + 1.$$

The displacement of the object at time $t = 1$ is $3\,\mathrm{m}$. What is the displacement of the object (in metres) at time $t = 3$?

The displacement is ☐ metres (to 2 d.p.) at $t = 3$.

[END OF QUESTION PAPER]