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Note: I've left some blank pages at the end for corrections. Sorry this one isn't typeset.

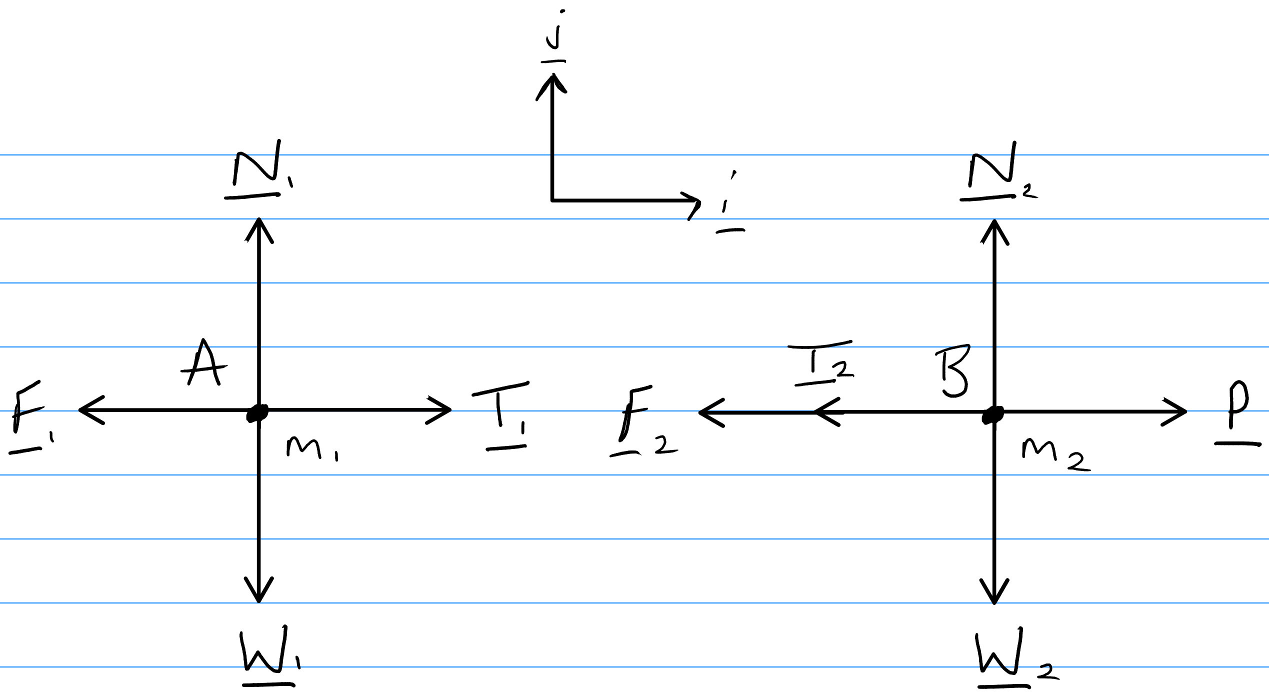
1) a) A 2-particle system has center of mass

$$\underline{r}_G = \frac{m_1 \underline{r}_1 + m_2 \underline{r}_2}{m_1 + m_2} \quad \text{(Handbook page 94)}$$

where $\underline{r}_1$ and $\underline{r}_2$ are the position vectors of the first and second particles. Substituting $\underline{r}_1 = x\underline{i}$ and $\underline{r}_2 = y\underline{i}$ gives the position of the center of mass of the two particles is

$$\underline{r}_G = \frac{m_1 x \underline{i} + m_2 y \underline{i}}{m_1 + m_2}$$

b)



$\underline{W}_1$ and $\underline{W}_2$ are the weights:

$\underline{W}_1 = -m_1 g \underline{j}$, $\underline{W}_2 = -m_2 g \underline{j}$

$\underline{N}_1$ and $\underline{N}_2$ are the normal reactions:

$\underline{N}_1 = |\underline{N}_1| \underline{j}$, $\underline{N}_2 = |\underline{N}_2| \underline{j}$

$\underline{F}_1$ and $\underline{F}_2$ are the forces due to friction:

$\underline{F}_1 = -\mu |\underline{N}_1| \underline{i}$, $\underline{F}_2 = -\mu |\underline{N}_2| \underline{i}$

$\underline{T}_1$ and $\underline{T}_2$ are the spring forces:

$\underline{T}_1 = 2k(y - x - L_0)\underline{i}$, $\underline{T}_2 = -\underline{T}_1$

$\underline{P}$ is the pushing force

$\underline{P} = P\underline{i}$

All the forces in the system are external, with the exception of $\underline{T}_1$ and $\underline{T}_2$

c) Applying Newton's second law to particle A gives

$$\underline{N}_1 + \underline{W}_1 + \underline{F}_1 + \underline{T}_1 = m_1 \ddot{x}_1 \underline{i}$$
$$(|\underline{N}_1| - m_1 g)\underline{j} + (2k(y - x - l_0) - \mu|\underline{N}_1|)\underline{i} = m_1 \ddot{x}_1 \underline{i} \quad (1)$$

Following the same procedure for particle B gives

$$\underline{N}_2 + \underline{W}_2 + \underline{F}_2 + \underline{T}_2 + \underline{P} = m_2 \ddot{x}_2 \underline{i}$$
$$(|\underline{N}_2| - m_2 g)\underline{j} + (P - \mu|\underline{N}_2| - 2k(y - x - l_0))\underline{i} = m_2 \ddot{\underline{x}}_2 \underline{i} \quad (2)$$

d) The motion of the center of mass of a system of n particles is described by $\underline{F}^{ext} = M\underline{r}_G$, where M is the total mass of the system, and $\underline{F}^{ext}$ is the sum of external forces (Handbook page 94).

Hence
$$(|\underline{N}_1| + |\underline{N}_2| - m_1 g - m_2 g)\underline{j} + (P - \mu|\underline{N}_1| - \mu|\underline{N}_2|)\underline{i} = M\ddot{x}_G \underline{i}$$

Resolving equations (1) and (2) in the $\underline{j}$ direction gives $|\underline{N}_1| = m_1 g$ and $|\underline{N}_2| = m_2 g$

Therefore
$$(m_1 g + m_2 g - m_1 g - m_2 g)\underline{j} + (P - \mu m_1 g - \mu m_2 g)\underline{i} = M\ddot{x}_G \underline{i}$$
$$(P - \mu g(m_1 + m_2))\underline{i} = (m_1 + m_2)\ddot{x}_G \underline{i}$$

2) Define an x axis along the line of motion of the particles with positive direction in the direction of travel of the particle of mass m (before the collision). The velocities of the two particles before the collision can be written as

$\underline{\dot{r}}_1 = \dot{x}_1 \underline{i}$ and $\underline{\dot{r}}_2 = \dot{x}_2 \underline{i}$

The velocities of the two particles after the collision can be written as

$\underline{\dot{R}}_1 = \dot{X}_1 \underline{i}$ and $\underline{\dot{R}}_2 = \dot{X}_2 \underline{i}$

By the principle of conservation of momentum:

$$m_1 \dot{x}_1 \underline{i} + m_2 \dot{x}_2 \underline{i} = m_1 \dot{X}_1 \underline{i} + m_2 \dot{X}_2 \underline{i}$$

Substituting $m_1 = m$, $m_2 = 2m$, $\dot{x}_1 = 4$, $\dot{x}_2 = 0$ gives

$$4m\underline{i} = m\dot{X}_1 \underline{i} + 2m\dot{X}_2 \underline{i}$$

Dividing by m and resolving in the $\underline{i}$ direction:

$$4 = \dot{X}_1 + 2\dot{X}_2 \qquad (3)$$

The pre-impact kinetic energies of the particles are

$\frac{1}{2} m \dot{x}_1^2 = 8m$ and $\frac{1}{2}(2m\dot{x}_2^2) = 0$

The post-impact kinetic energies are

$\frac{1}{2} m \dot{X}_1^2$ and $\frac{1}{2}(2m\dot{X}_2^2)$

Since the collision is elastic, kinetic energy is conserved and

$$8m = \frac{1}{2} m \dot{X}_1^2 + \frac{1}{2}(2m\dot{X}_2^2)$$

$$16m = m\dot{X}_1^2 + 2m\dot{X}_2^2$$

$$16 = \dot{X}_1^2 + 2\dot{X}_2^2 \qquad (4)$$

From equation (3) we have

$4 = \dot{x}_1 + 2\dot{x}_2$

$\dot{x}_1 = 4 - 2\dot{x}_2$

Substituting into equation (4) gives

$16 = 16 - 16\dot{x}_2 + 4\dot{x}_2^2 + 2\dot{x}_2^2$

$0 = 3\dot{x}_2^2 - 8\dot{x}_2$

Therefore either $\dot{x}_2 = 0$ or $\dot{x}_2 = 8/3$. If $\dot{x}_2 = 0$ then $\dot{x}_1$ would be 4, which is impossible since the particle of mass 2m is in front, so $\dot{x}_2 = 8/3$

Substituting $\dot{x}_2 = 8/3$ into equation (4) gives

$16 = \dot{x}_1^2 + \frac{128}{9}$

So $\dot{x}_1 = 4/3$ (taking the positive square root).

Using the principle of conservation of momentum again, we find the velocity of the particle of mass m must act in the negative $\underline{i}$ direction

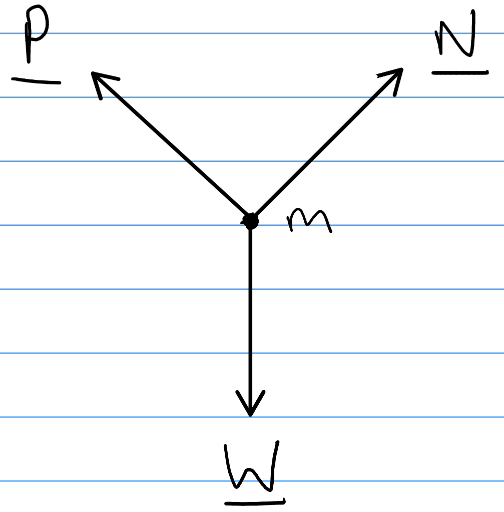
$m_1 \dot{x}_1 \underline{i} + m_2 \dot{x}_2 \underline{i} = m_1 \dot{X}_1 \underline{i} + m_2 \dot{X}_2 \underline{i}$

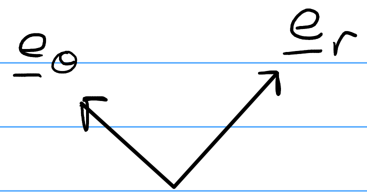
$4m\underline{i} = -4/3\, m\underline{i} + \frac{16}{3} m\underline{i}$

Therefore the velocities of the particles after collision are

$\underline{R}_1 = -4/3\,\underline{i}$ and $\underline{R}_2 = 16/3\,\underline{i}$

3) a)





Where $\underline{W}$ is the weight of the particle, $\underline{N}$ is the normal reaction of the cylinder on the particle, and $\underline{P}$ is the pulling force (as the string is a model string).

b) $\underline{P} = P\,\underline{e}_\theta$, $\underline{N} = |\underline{N}|\,\underline{e}_r$, and

$\underline{W} = -mg\sin\theta\,\underline{e}_r - mg\cos\theta\,\underline{e}_\theta$

c) A particle moving around a circle of radius R has acceleration (Handbook page 97)

$$\ddot{\underline{r}} = -R\dot{\theta}^2 \underline{e}_r + R\ddot{\theta}\,\underline{e}_\theta \qquad (5)$$

where $\dot{\theta}$ is the rate of rotation and $\omega = |\dot{\theta}|$ is the angular speed

Applying Newton's second law to the particle gives

$$m\ddot{\underline{r}} = \underline{W} + \underline{N} + \underline{P}$$
$$= (|\underline{N}| - mg\sin\theta)\underline{e}_r + (P - mg\cos\theta)\underline{e}_\theta$$

Using equation (5) and resolving in the $\underline{e}_r$ and $\underline{e}_\theta$ directions gives

$$-mR\dot{\theta}^2 = |\underline{N}| - mg\sin\theta$$
$$mR\ddot{\theta} = P - mg\cos\theta$$

Hence the tangential component is

$$\ddot{\theta} = \frac{P - mg\cos\theta}{mR}$$

As required.

d) The tangential component from part (c) can be written as $\ddot{\theta} = f(\theta)$ where

$$f(\theta) = \frac{P - mg\cos\theta}{mR}$$

Multiplying by $\dot{\theta}$ gives

$$\dot{\theta}\ddot{\theta} = \dot{\theta} f(\theta)$$

The chain rule tells us that

$$\frac{d}{dt}(\dot{\theta}^2) = \frac{d(\dot{\theta}^2)}{d\dot{\theta}} \frac{d\dot{\theta}}{dt} = 2\dot{\theta}\ddot{\theta}$$

Therefore

$$\frac{d}{dt}(\dot{\theta}^2) = 2\dot{\theta} f(\theta)$$

which can be integrated to give

$$\dot{\theta}^2 = 2\int f(\theta)\dot{\theta}\, dt$$

$$= 2\int f(\theta)\frac{d\theta}{dt}\, dt$$

$$= 2\int f(\theta)\, d\theta$$

$$= \frac{2}{mR}\int P - mg\cos\theta\, d\theta$$

$$= \frac{2(P\theta - mg\sin\theta)}{mR}$$

Substituting this expression for $\dot{\theta}^2$ into the radial component of the equation of motion gives

$$-2P\theta + 2mg\sin\theta = |\underline{N}| - mg\sin\theta$$

$$|\underline{N}| = 3mg\sin\theta - 2P\theta$$

As required.

e) The particle leaves the surface of the cylinder when $|\underline{N}| = 0$. Substituting $P = \frac{5}{4}mg$ into the expression for the normal reaction gives

$$3mg\sin\theta - \frac{5}{2}mg\theta = 0$$

$$3\sin\theta - \frac{5}{2}\theta = 0$$

as the condition for the angle at which the particle leaves the surface.

Solving this equation for θ using Maxima gives $\theta = 58.82...^\circ$. So the particle leaves the cylinder at an angle of 59° (to the nearest degree) to the horizontal.

Maxima printout on next page.

(%i1) n:3·sin(θ)−5/2·θ$

(%i2) np:find_root(n, θ, 0.001, %pi);

(%o2) 1.02673829137097

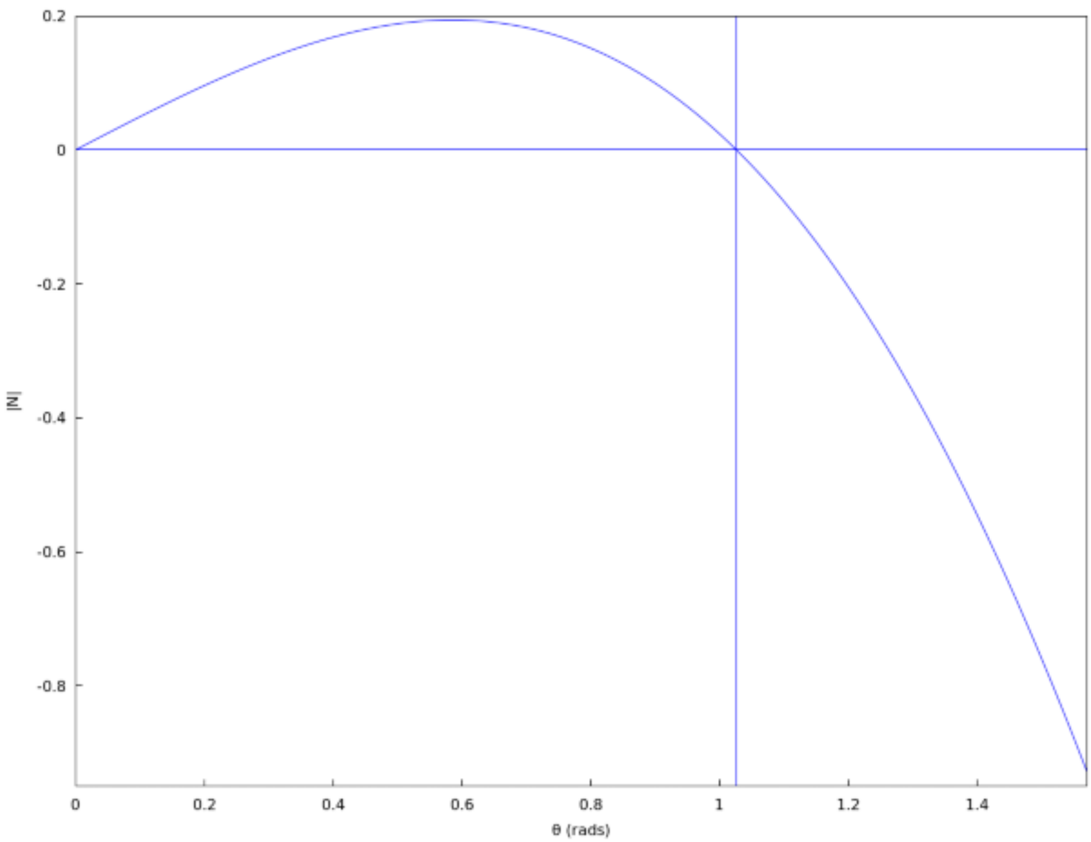
(%i3) float(np·180/%pi);

(%o3) 58.82777076002999

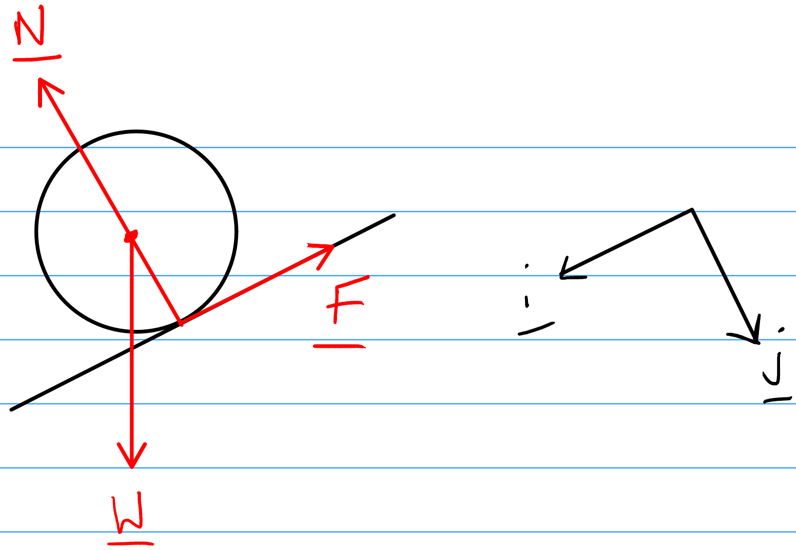
(%i5) load(draw)$

```
wxdraw2d(
   explicit(n, θ, 0, %pi/2),
   parametric(np, t, t, −0.95, 0.2),
   explicit(0, θ, 0, %pi/2),
  xlabel="θ (rads)", ylabel="|N|"
   );
```

(%t5)



4) a)



The normal reaction $|\underline{N}|$ and friction $|\underline{F}|$ forces act at the point of contact between the sphere and the slope. The weight $|\underline{W}|$ acts at the center of mass of the sphere.

b) The center of mass has acceleration $\ddot{x}\underline{i}$ (as the radius is constant). Applying Newton's second law to the center of mass gives

$$M\ddot{x}\underline{i} = \underline{W} + \underline{N} + \underline{F}$$
$$= Mg\sin\alpha\,\underline{i} + Mg\cos\alpha\,\underline{j} - |\underline{N}|\,\underline{j} - |\underline{F}|\,\underline{i}$$

Resolving in the $\underline{i}$ and $\underline{j}$ directions gives

$$M\ddot{x} = Mg\sin\alpha - |\underline{F}|$$
$$0 = Mg\cos\alpha - |\underline{N}|$$

c) If $\underline{r}_i^{rel}$ denotes the point of action of the ith force $\underline{F}_i$ relative to the center of mass, then the torque imparted by that force is

$$\Gamma_i = \underline{r}_i^{rel} \times \underline{F}_i$$ (Handbook page 101)

Applying this to each of the forces acting on the thin spherical shell gives

$$\begin{aligned}\Gamma_N &= \underline{r}_N^{rel} \times (-|\underline{N}|\,\underline{j}) \\ &= R\underline{j} \times (-|\underline{N}|\,\underline{j}) \\ &= \underline{0}\end{aligned}$$

$$\begin{aligned}\Gamma_W &= \underline{r}_W^{rel} \times (Mg\sin\alpha\,\underline{i} + Mg\cos\alpha\,\underline{j}) \\ &= \underline{0} \times (Mg\sin\alpha\,\underline{i} + Mg\cos\alpha\,\underline{j}) \\ &= \underline{0}\end{aligned}$$

$$\begin{aligned}\Gamma_F &= \underline{r}_F^{rel} \times (-|\underline{F}|\,\underline{i}) \\ &= R\underline{j} \times (-|\underline{F}|\,\underline{i}) \\ &= R|\underline{F}|\,\underline{k}\end{aligned}$$

d) The rotational acceleration of the spherical shell about its center of mass is

$$\ddot{x} = R\ddot{\theta} \qquad (6)$$

The moment of inertia of a spherical shell of mass M about an axis in the $\underline{K}$ direction through its center of mass is

$$I = \tfrac{2}{3}MR^2 \qquad \text{(Handbook page 100)}$$

The total torque relative to the center of mass is

$$\Gamma^{rel} = R|\underline{F}|\underline{K}$$
$$\Gamma^{rel}_{axis} = R|\underline{F}|$$

as calculated in part (c). The equation of relative rotational motion is

$$\Gamma^{rel}_{axis} = I\ddot{\theta} \qquad \text{(Handbook page 102)}$$

Therefore

$$R|\underline{F}| = \tfrac{2}{3}MR^2\ddot{\theta}$$
$$= \tfrac{2}{3}MR\ddot{x} \qquad \text{(using equation (6))}$$
$$|\underline{F}| = \tfrac{2}{3}M\ddot{x}$$

Substituting this into the equation of motion from part (b) gives

$$M\ddot{x} = Mg\sin\alpha - \tfrac{2}{3}M\ddot{x}$$
$$\ddot{x} = 3/5\, g\sin\alpha$$

As required.

e) To get the position of the sphere on the slope as a function of time, we integrate $\dot{x}(t)$ wrt. time

$$\dot{x}(t) = \tfrac{3}{5} g \int \sin\alpha \, dt$$

$$= \tfrac{3}{5} g \sin\alpha \, t$$

$$x(t) = \tfrac{3}{5} g \sin\alpha \int t \, dt$$

$$= \frac{3}{10} g \sin\alpha \, t^2$$

Therefore, the time for the sphere to roll a distance L down the slope without slipping is given by

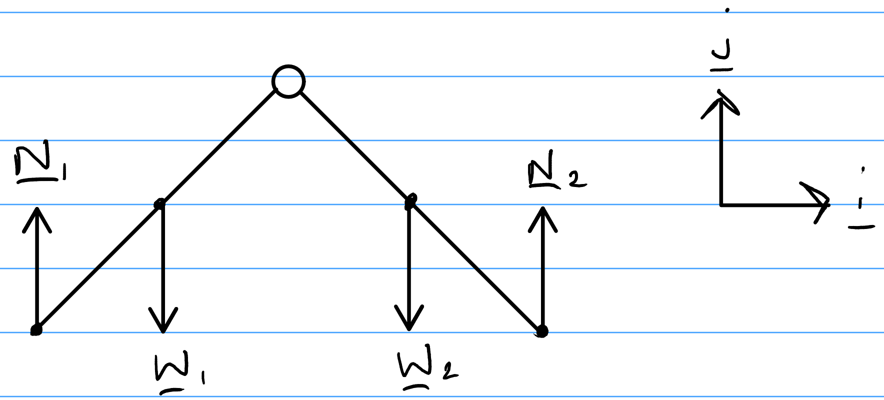
$$t^2 = \frac{10L}{3g\sin\alpha}$$

$$t = \sqrt{\frac{10L}{3g\sin\alpha}}$$ (taking the positive root)

5) If $\frac{d\theta}{dt}$ is the rate of change of excess temperature, then the differential equation is

$$\frac{d\theta}{dt} = \alpha - \beta\theta$$

6)
a)



Where $\underline{N}_1$ and $\underline{N}_2$ are the normal reactions acting at the points of contact with the floor and $\underline{W}_1$ and $\underline{W}_2$ are the weights acting at the center of mass of each step ladder.

$$\underline{N}_1 = |\underline{N}_1|\, \underline{j} \quad , \quad \underline{N}_2 = |\underline{N}_2|\, \underline{j}$$

$$\underline{W}_1 = -mg\,\underline{j} \quad , \quad \underline{W}_2 = -mg\,\underline{j}$$

As the ladders are at rest, $|\underline{N}_1| = |\underline{W}_1| = mg$ and $|\underline{N}_2| = |\underline{W}_2| = mg$. So the total normal reaction force acting on them is $2mg\underline{j}$.

b) The torque $\underline{\Gamma}$ of a force $\underline{F}$ about a fixed point O is $\underline{\Gamma} = \underline{r} \times \underline{F}$ (Handbook page 38) where $\underline{r}$ is the position vector, relative to O, of any point on the line of action of the force (in this case, the point of action).

The torque applied by $\underline{W}_1$ is

$$\underline{\Gamma}_W = (a\cos\theta\,\underline{i} + a\sin\theta\,\underline{j}) \times -mg\underline{j}$$
$$= -amg\cos\theta\,\underline{K}$$

The torque applied by $\underline{N}_1$ is

$$\underline{\Gamma}_N = (2a\cos\theta\,\underline{i} + 2a\sin\theta\,\underline{j}) \times mg\underline{j}$$
$$= 2amg\cos\theta\,\underline{K}$$

As the step ladder is in equilibrium, the forces and torques must sum to $\underline{0}$. Let $\underline{\Gamma}_S$ be the torque applied by the string:

$$\underline{\Gamma}_S = (amg\cos\theta\,\underline{i} + amg\sin\theta\,\underline{j}) \times (|\underline{S}|\,\underline{i})$$
$$= -|\underline{S}|amg\sin\theta\,\underline{K}$$

Then we must have

$$\underline{0} = \underline{\Gamma}_W + \underline{\Gamma}_N + \underline{\Gamma}_S$$
$$= 2amg\cos\theta\,\underline{K} - amg\cos\theta\,\underline{K} - |\underline{S}|amg\sin\theta\,\underline{K}$$
$$= amg\cos\theta\,\underline{K} - |\underline{S}|amg\sin\theta\,\underline{K}$$

Resolving in the $\underline{K}$ direction and rearranging

$$|\underline{S}| = \frac{amg\cos\theta}{amg\sin\theta}$$

giving the magnitude of the tension in the string in terms of the given parameters.

7) a) An eigenvector of a square matrix $\underline{A}$ is a non-zero vector $\underline{v}$ such that

$$\underline{A}\underline{v} = \lambda \underline{v}$$

for some scalar λ (Handbook page 51).

Hence

$$\begin{pmatrix} 3 & -5 & 4 \\ -1 & -1 & 4 \\ -1 & 5 & -2 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix} = -6 \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

So $\lambda = -6$ is the eigenvalue corresponding to the eigenvector.

b) The eigenvector equation of $\underline{A}$ for $\lambda = 2$ is

$$\begin{pmatrix} 3-2 & -5 & 4 \\ -1 & -1-2 & 4 \\ -1 & 5 & -2-2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Therefore, we have

$$x - 5y + 4z = 0 \qquad (6)$$
$$-x - 3y + 4z = 0 \qquad (7)$$
$$-x + 5y - 4z = 0 \qquad (8)$$

Adding (7) and (8) gives

$$-2x + 2y = 0$$
$$x = y$$

Adding (6) and (7) gives

$$-8y + 8z = 0$$
$$y = z$$

Therefore any eigenvector where $x = y = z$ is an eigenvector of $\underline{A}$ corresponding to $\lambda = 2$ e.g. $(1, 1, 1)^T$.

8) a) A stationary point of a function $f(x,y)$ is a point (a,b) at which
$$f_x(a,b) = f_y(a,b) = 0$$

For the given function
$$f_x(x,y) = -12x + 3y^2 - 12y - 36$$
$$f_y(x,y) = 6xy - 12x$$

Thus, we have the pair of simultaneous equations
$$-12x + 3y^2 - 12y - 36 = 0 \quad (9)$$
$$6xy - 12x = 0 \quad (10)$$

Equation (10) is satisfied when $x = 0$ or $y = 2$.

Substituting $y = 2$ into (9) gives
$$-12x + 12 - 24 - 36 = 0$$
$$-12x = 48$$
$$x = -4$$

Therefore $(-4, 2)$ is a stationary point.
Substituting $x = 0$ into (9) gives
$$3y^2 - 12y - 36 = 0$$
$$y^2 - 4y - 12 = 0$$
$$(y+2)(y-6) = 0$$

So $(0,-2)$ and $(0,6)$ are also stationary points.

b) We apply the $AC - B^2$ test for classifying the stationary point $(-4, 2)$. (Handbook page 62)

Let

$$A = f_{xx}(-4,2) = -12 + 3y^2 - 12y$$
$$= -12$$
$$B = f_{xy}(-4,2) = 6y - 12$$
$$= 0$$
$$C = f_{yy}(-4,2) = 6x$$
$$= -24$$

Then as

$$AC - B^2 = -12(-24) - 0$$
$$= 288$$

is positive and A is negative, $(-4,2)$ is a local maximum.

9) a) Let $u = -xy$ and $v = x + 2y - 6$, then the Jacobian matrix is given by

$$\underline{J}(x,y) = \begin{pmatrix} u_x(x,y) & u_y(x,y) \\ v_x(x,y) & v_y(x,y) \end{pmatrix}$$

$$= \begin{pmatrix} -y & -x \\ 1 & 2 \end{pmatrix}$$

Thus

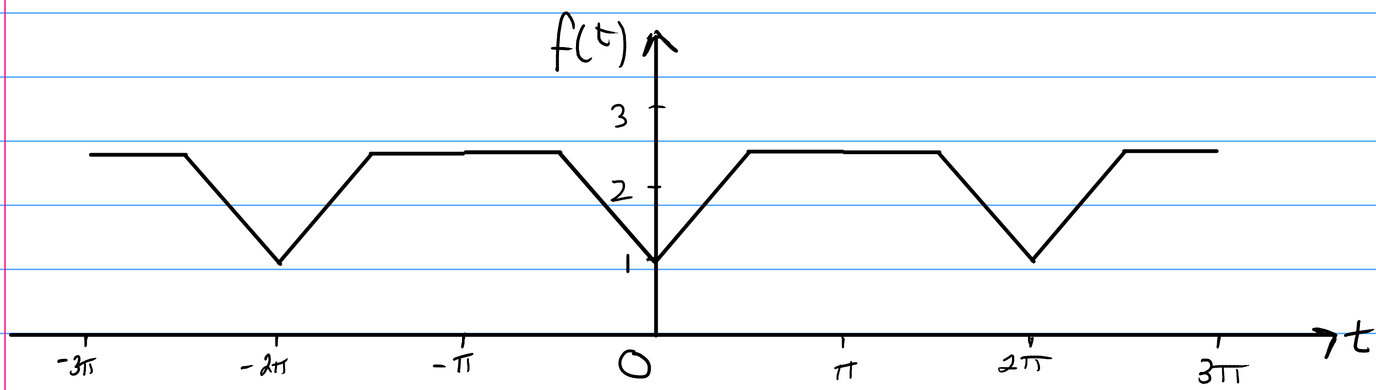
$$\underline{J}(0,3) = \begin{pmatrix} -3 & 0 \\ 1 & 2 \end{pmatrix}$$

b) As $\underline{J}(0,3)$ is a triangular matrix, its eigenvalues are its diagonal entries, -3 and 2. Following the decision tree in Handbook page 76, as $\underline{J}(0,3)$ has one positive and one negative eigenvalue, the equilibrium point is a saddle.

10) a) The even periodic extension of $f(t)$ defined on the interval $0 \leq t \leq T$ is given by

$$f_{even}(t) = \begin{cases} f(t) & \text{for } 0 \leq t < T \\ f(-t) & \text{for } -T \leq t < 0 \end{cases}$$

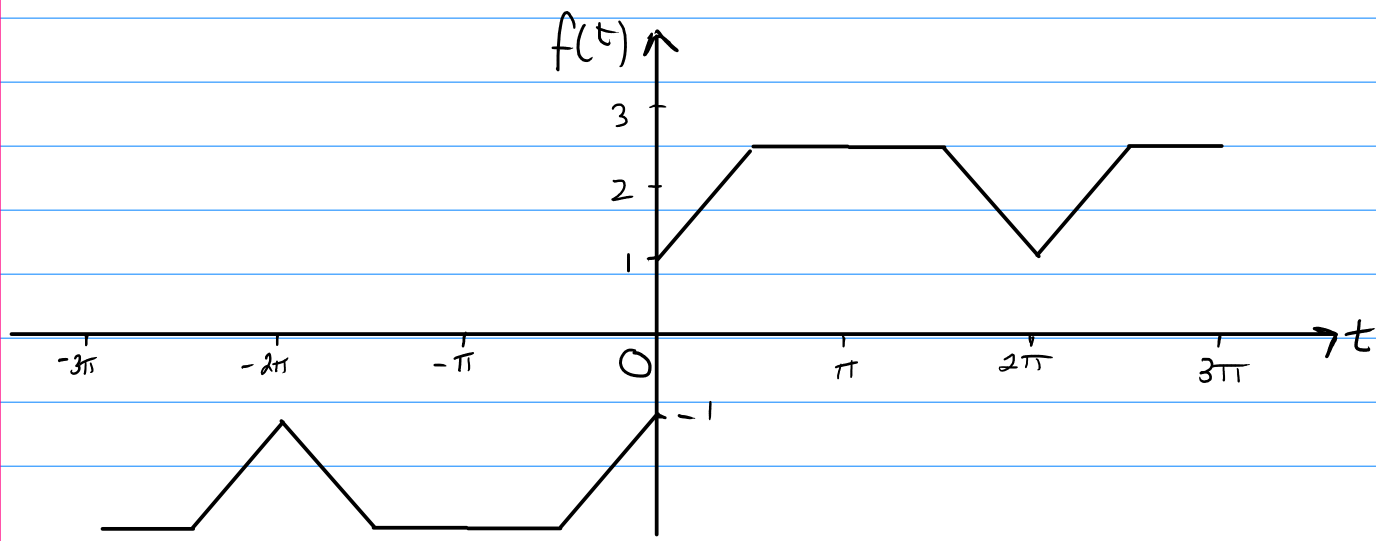
$$f_{even}(t + 2T) = f_{even}(t)$$ (Handbook page 80)



The odd periodic extension is given by

$$f_{odd}(t) = \begin{cases} f(t) & \text{for } 0 \leq t < T \\ -f(-t) & \text{for } -T \leq t < 0 \\ 0 & \text{for } t = 0 \text{ or } t = T \end{cases}$$

$$f_{odd}(t + 2T) = f_{odd}(t)$$



b) The Fourier series of the even periodic extension is

$$F(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(nt)$$

where the coefficients A_n are given by

$$A_n = \frac{2}{2\pi} \int_{-\pi}^{\pi} f(t) \cos\left(\frac{2n\pi t}{2\pi}\right) dt \quad (n = 1, 2, \ldots)$$

I don't know how to proceed from here sorry!

11) The curl of a vector field in spherical coordinates is

$$\underline{\Delta} \times \underline{V} = \left(\frac{1}{r}\frac{\partial V_\phi}{\partial \theta} - \frac{1}{r\sin\theta}\frac{\partial V_\theta}{\partial \phi} + \frac{\cot\theta}{r} V_\phi\right)\underline{e}_r + \left(-\frac{\partial V_\phi}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial V_r}{\partial \phi} - \frac{1}{r}V_\phi\right)\underline{e}_\theta + \left(\frac{\partial V_\theta}{\partial r} - \frac{1}{r}\frac{\partial V_r}{\partial \theta} + \frac{1}{r}V_\theta\right)\underline{e}_\phi$$

Calculating each component separately:

$$(\underline{\Delta} \times \underline{V})_r =$$

$$\frac{2\sin\phi}{r\sin^2\theta} - \frac{2\sin\phi\sin\theta}{r\sin\theta} - \frac{2\sin\phi\cos^2\theta}{r\sin^2\theta} = 0$$

$$(\underline{\Delta} \times \underline{V})_\theta =$$

$$-\frac{2\sin\phi\cos\theta}{r\sin\theta} + \frac{2\sin\phi\cos\theta}{r\sin\theta} = 0$$

$$(\underline{\Delta} \times \underline{V})_\phi =$$

$$\frac{2\cos\phi\sin\theta}{r} - \frac{2\sin\theta\cos\phi}{r} = 0$$

As $\underline{\Delta} \times \underline{V} = \underline{0}$, $\underline{V}$ is a conservative vector field.

12) The scalar line integral of the vector field $\underline{F}(\underline{r})$ along the path C given by $\underline{r}(t)$ from $\underline{r}(0)$ to $\underline{r}(1)$ is given by

$$\int_C \underline{F}(\underline{r}) \cdot d\underline{r} = \int_0^1 \underline{F}(t) \cdot \frac{d\underline{r}}{dt}\, dt$$

(Handbook page 88)

Differentiating the components of $\underline{r}$ gives

$$\frac{dx}{dt} = 3t^2, \quad \frac{dy}{dt} = 2t$$

The components of $\underline{F}$ in terms of t are

$$F_1 = x^2 + 2y^3 = t^6 + 2t^6 = 3t^6$$
$$F_2 = xy^2 = t^3 t^4 = t^7$$

Thus

$$\int_C \underline{F}(\underline{r}) \cdot d\underline{r} = \int_0^1 \left(F_1 \frac{dx}{dt} + F_2 \frac{dy}{dt} \right) dt$$

$$= \int_0^1 \left(3t^6(3t^2) + t^7(2t) \right) dt$$

$$= \left[9t^8 + 2t^8 \right]_0^1$$

$$= 11$$

Therefore, the scalar line integral is 11.

13) The volume integral in spherical coordinates is given by

$$\int_D f \, dV = \int_D f(r, \theta, \phi) \, r^2 \sin\theta \, d\phi \, d\theta \, dr$$

As the volume is a hemisphere, we integrate over the ranges $0 < r \leq R$, $0 < \theta \leq \frac{\pi}{2}$, and $-\frac{\pi}{2} < \phi \leq \frac{\pi}{2}$

Substituting the function $f(r, \theta, \phi)$ we have

$$\int_{r=0}^{r=R} \left(\int_{\theta=0}^{\theta=\pi/2} \left(\int_{\phi=-\pi}^{\phi=\pi} (r + \sin\phi) \, r^2 \sin\theta \, d\phi \right) d\theta \right) dr$$

$$= \int_{r=0}^{r=R} \left(\int_{\theta=0}^{\theta=\pi/2} \left(\Big[\phi r^3 \sin\theta - r^2 \cos\phi \sin\theta \Big]_{\phi=-\pi}^{\phi=\pi} \right) d\theta \right) dr$$

$$= \int_{r=0}^{r=R} \left(\int_{\theta=0}^{\theta=\pi/2} 2\pi r^3 \sin\theta \, d\theta \right) dr$$

$$= \int_{r=0}^{r=R} \left(\Big[-2\pi r^3 \cos\theta \Big]_{\theta=0}^{\theta=\pi/2} \right) dr$$

$$= \int_{r=0}^{r=R} 2\pi r^3 \, dr$$

$$= \left[\frac{\pi}{2} r^4 \right]_0^R$$

$$= \frac{\pi}{2} R^4$$

Therefore, the volume integral is

$$I = \frac{\pi}{2} R^4$$