Factorising quadratics

- If you take any two linear expressions and multiply them, you get a quadratic

- Factorising is the reverse of this process
- It means writing the quadratic as a product of two sets of brackets

- When x has coefficient 1:
  - ↳ If possible to factorise with integer coefficients then it must be in the form:

$$(x \quad)(x \quad)$$

where the gaps are filled with constant terms

- The product of the constant terms must give the constant term c in the quadratic
- The sum of the constant terms must give b, the coefficient of x in the quadratic

$$(x+3)(x-5) = x^2 - 2x - 15$$

- When x^2 doesn't have coefficient 1
  - ↳ Start by taking out common factors

$$4x^2 + 2x - 2 = 2(2x^2 + x - 1)$$

  - ↳ If the coefficient of x^2 is negative then take out a factor of -1

$$-3x^2 + x + 2 = -(3x^2 - x - 2)$$

↳ Once you've done these things you can just factorise the simpler quadratic and place the common factor in front

- There may be multiple pairs of common factors for the coefficient of x^2 so you must try each

$6x^2 + 11x - 35$

$(6x \quad)(x \quad)$ OR $(3x \quad)(2x \quad)$

- If none of the factor pairs of x^2 give a solution, the quadratic cannot be factorised using integers

- Quadratics with no constant term:

↳ Take out x as a common factor

$3x^2 - 6x = 3x(x-2)$

↳ If the quadratic is a difference of two squares

$A^2 - B^2 = (A+B)(A-B)$

then you can factorise immediately

- Quadratics when $a \neq 1$

↳ Factorise out the coefficient of x^2

$2x^2 - 12x + 20 = 2(x^2 - 6) + 20$

↳ It doesn't matter if the coefficient of x^2 is not a factor of x

$-2x^2 + x + 1 = -2(x^2 + \frac{1}{2}x) + 1$

↳ The quadratic in brackets is now in the form

$x^2 + bx$

↳ You can now complete the square of the quadratic and finally simplify to get the completed square form of the original quadratic

- Completing the square
  - Works on any quadratic with solutions incl. those that can't be factorised with integers

$$2x^2 - 8x + 5 = 0$$
$$2(x^2 - 4x) + 5 = 0$$
$$2((x-2)^2 - 4) + 5 = 0$$
$$2(x-2)^2 - 3 = 0$$
$$(x-2)^2 = 3/2$$
$$x - 2 = \pm\sqrt{3/2}$$
$$\therefore \quad x = \sqrt{3/2} + 2 \quad \text{OR} \quad x = 2 - \sqrt{3/2}$$