Revise and refresh for MST124:

Welcome to Session 4

Thursday 16th September 2021

We'll start at 7.00pm and aim to finish by 9.00pm

Please check your Audio levels:

Speaker and Microphone Setup

This session will cover the topics in

Quadratic Expressions and Equations

Please feel free to use the chat box while waiting

Have paper, pen and your calculator to hand.

SESSION 4: QUADRATICS

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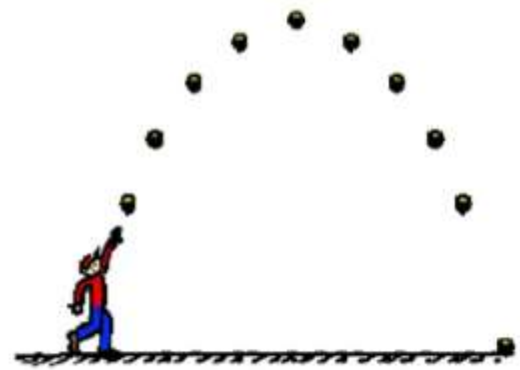
- Multiplying out pairs of brackets
- Quadratic graphs
- Factorising
- Solving quadratic equations
- How many solutions?
- Solving related equations

SESSION 4: QUADRATICS

Quadratics are the life blood of **mathematical modelling**.



$$h = 10t - t^2$$

$$s = \frac{u^2}{2a}$$

$$x^2 + x = 1$$

and many, many more things …

WHAT IS A QUADRATIC?

These are not quadratics – can you see why not?

$$y = \frac{1}{x^2}$$

$$t^2 + \frac{3}{t} = 0$$

$$2a^3 - 7a^2 + 4$$

$ax^2 + bx + c$

MULTIPLYING OUT PAIRS OF BRACKETS

First

F

$$(x+3)(x-2) = x^2$$

MULTIPLYING OUT PAIRS OF BRACKETS

First **O**uter **F** **O**

$$(x+3)(x-2) = x^2 - 2x$$

MULTIPLYING OUT PAIRS OF BRACKETS

First

Outer

F O I

$$(x+3)(x-2) = x^2 - 2x + 3x$$

Inner

MULTIPLYING OUT PAIRS OF BRACKETS

First | Outer
Inner | Last

F O I L

$$(x + 3)(x - 2) = x^2 - 2x + 3x - 6$$

MULTIPLYING OUT PAIRS OF BRACKETS

First **O**uter

$$(x+3)(x-2) = \overset{F}{x^2} \overset{O}{- 2x} \overset{I}{+ 3x} \overset{L}{- 6}$$

Inner **L**ast

MULTIPLYING OUT PAIRS OF BRACKETS

First Outer

F O I L

$$(x+3)(x-2) = x^2 - 2x + 3x - 6$$

Inner Last

$$= x^2 + x - 6$$

MULTIPLYING OUT PAIRS OF BRACKETS

a) $(x+2)(x+5)$

b) $(a+2)(2a+3)$

c) $(a+3)(a-3)$

d) $(x-2)^2$

e) $(3x+5y)(2x-y)$

MULTIPLYING OUT PAIRS OF BRACKETS

a) $(x+2)(x+5) = x^2 + 5x + 2x + 10 = x^2 + 7x + 10$

b) $(a+2)(2a+3) = 2a^2 + 7a + 6$

c) $(a+3)(a-3) = a^2 - 9$

d) $(x-2)^2 = x^2 - 4x + 4$

e) $(3x+5y)(2x-y) = 6x^2 + 7xy - 5y^2$

GRAPHS OF QUADRATICS

The general form of the quadratic is

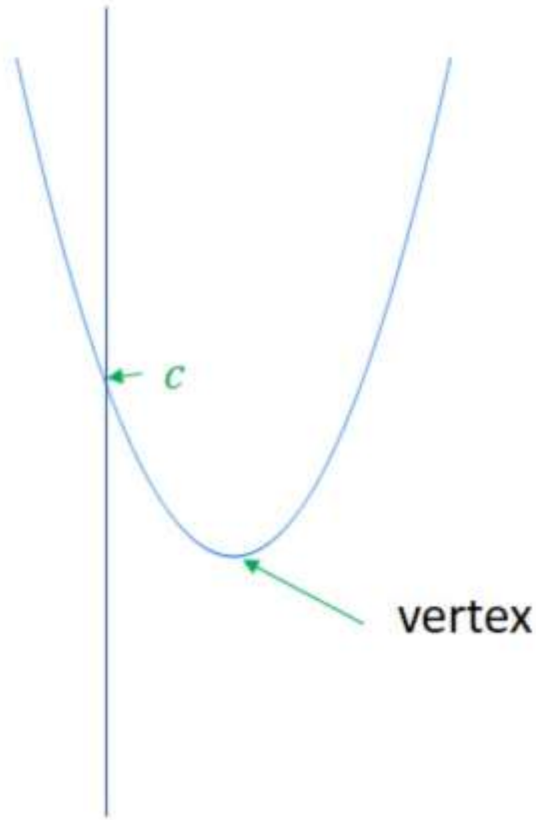
$$y = ax^2 + bx + c$$

Where a, b and c are constants and $a \neq 0$.

a is the coefficient of the x^2 term

b is the coefficient of the x term

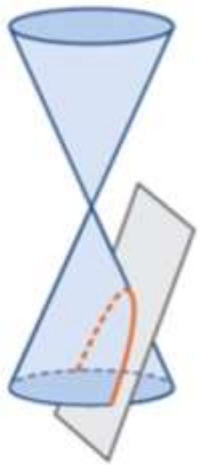
c is the constant term



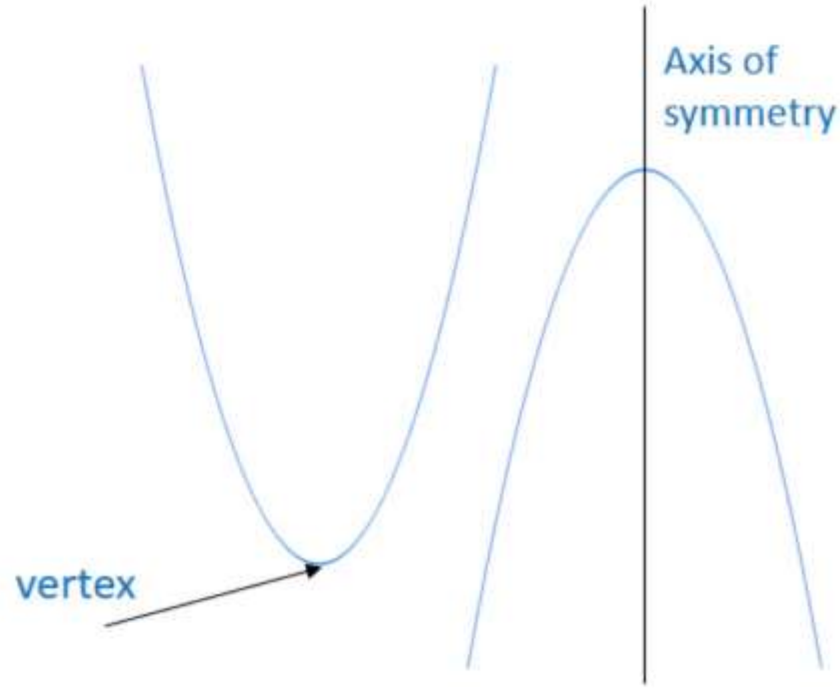
And the graph of $y = ax^2 + bx + c$ cuts the y-axis at c

GRAPHS OF QUADRATICS

The graphs of all quadratics are parabolas



They may be narrower or wider but always symmetrical



GRAPHS OF QUADRATICS

$$y = x^2 + 3x$$

$$y = 4 - x^2$$

GRAPHS OF QUADRATICS

$$y = x^2 + 3x$$

(u-shaped)

$$y = 4 - x^2$$

(n-shaped)

GRAPHS OF QUADRATICS

$$y = x^2 + 3x$$

(u-shaped)

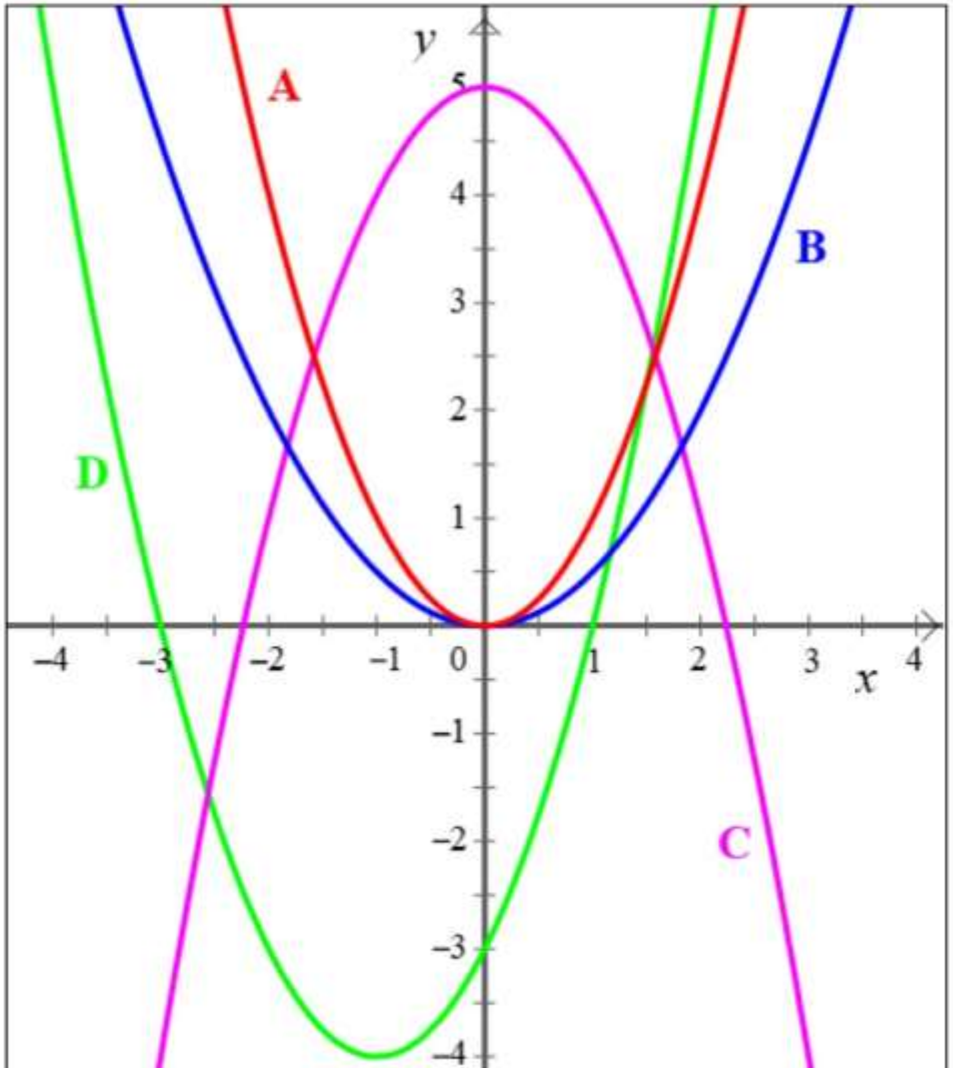
$$y = 4 - x^2$$

(n-shaped)

$$y = 5x^2$$

$$y = \frac{1}{3}x^2$$

GRAPHS OF QUADRATICS



1) $y = x^2$

2) $y = 2x^2$

3) $y = \frac{1}{2}x^2$

4) $y = x^2 + 5$

5) $y = -x^2 + 5$

6) $y = x^2 + 2x - 3$

QUADRATIC EXPRESSIONS

Three key ways of writing quadratic expressions:

1) $ax^2 + bx + c$ <u>Common form</u>

$x^2 - 5x + 2$

2) $a(x - r)(x - s)$ <u>Factorised Form</u>

$(x - 3)(x + 2)$

3) $a(x - h)^2 + k$ <u>Completed-Square Form</u>

$(x + 1)^2 - 3$

FACTORISING QUADRATIC EXPRESSIONS

Reverse process of multiplying out brackets

$3x^2 + 6x$ is factorised as $3x(x + 2)$

$x^2 - 3x - 10$ is factorised as $(x - 5)(x + 2)$

FACTORISING QUADRATIC EXPRESSIONS

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Expressions such as $ax^2 + bx$:

$6x^2 + 4x$

FACTORISING QUADRATIC EXPRESSIONS

Expressions such as $ax^2 + bx$:

$$6x^2 + 4x = 2x(3x + 2)$$

FACTORISING QUADRATIC EXPRESSIONS

Expressions such as $ax^2 + bx$:

$6x^2 + 4x = 2x(3x + 2)$

$5x - 15x^2$

FACTORISING QUADRATIC EXPRESSIONS

Expressions such as $ax^2 + bx$:

$$6x^2 + 4x = 2x(3x + 2)$$

$$5x - 15x^2 = 5x(1 - 3x)$$

FACTORISING QUADRATIC EXPRESSIONS

To factorise an expression like $x^2 - 5x + 6$ we need to think about where the terms come from when multiplying out the brackets:

$$(x + a)(x + b)$$

$$x^2 + 3x - 10$$

FACTORISING QUADRATIC EXPRESSIONS

(a) $x^2 - x - 12$

(b) $x^2 - 10x + 25$

(c) $x^2 - 81$

(Keep you answers for later too…)

FACTORISING QUADRATIC EXPRESSIONS

(a) $x^2 - x - 12 = (x + 3)(x - 4)$

(b) $x^2 - 10x + 25 = (x - 5)(x - 5) = (x - 5)^2$

(c) $x^2 - 81 = (x - 9)(x + 9)$

FACTORISING QUADRATIC EXPRESSIONS

(a) $x^2 - x - 12 = (x+3)(x-4)$

(b) $x^2 - 10x + 25 = (x-5)(x-5) = (x-5)^2$

(c) $x^2 - 81 = (x-9)(x+9)$

Alternative method?

$$x^2 - 81 = x^2 - 9^2 = (x-9)(x+9)$$

MULTIPLYING OUT PAIRS OF BRACKETS

$$(x - 9)(x + 9)$$

MULTIPLYING OUT PAIRS OF BRACKETS

$(x-9)(x+9)$

<u>Difference of two squares</u>

$$(x-p)(x+p)=x^2-p^2$$

MULTIPLYING OUT PAIRS OF BRACKETS

$(x-9)(x+9)$

Difference of two squares

$$(x-p)(x+p)=x^2-p^2$$

$$\begin{aligned}(x-9)(x+9)&=x^2-9^2\\&=x^2-81\end{aligned}$$

FACTORISING QUADRATIC EXPRESSIONS

Squared brackets

$$(x+p)^2 = x^2 + 2px + p^2$$

FACTORISING QUADRATIC EXPRESSIONS

Squared brackets

$$(x+p)^2 = x^2 + 2px + p^2$$

So

$$x^2 + 2px + p^2 = (x+p)^2$$

FACTORISING QUADRATIC EXPRESSIONS

Squared brackets

$$(x+p)^2 = x^2 + 2px + p^2$$

So

$$x^2 + 2px + p^2 = (x+p)^2$$

Hence

$$x^2 + 2px = (x+p)^2 - p^2$$

FACTORISING QUADRATIC EXPRESSIONS

$x^2 + 2px = (x + p)^2 - p^2$

$x^2 + 6x$

What will p be?

FACTORISING QUADRATIC EXPRESSIONS

$x^2 + 2px = (x + p)^2 - p^2$

$x^2 + 6x$

So $p = 3$.

FACTORISING QUADRATIC EXPRESSIONS

$x^2 + 2px = (x + p)^2 - p^2$

$x^2 + 6x$

So $p = 3$.

$$x^2 + 6x = (x + 3)^2 - 3^2$$

FACTORISING QUADRATIC EXPRESSIONS

$x^2 + 2px = (x + p)^2 - p^2$

$x^2 + 6x$

So $p = 3$.

$$\begin{aligned} x^2 + 6x &= (x + 3)^2 - 3^2 \\ &= (x + 3)^2 - 9 \end{aligned}$$

FACTORISING QUADRATIC EXPRESSIONS

$x^2 + 2px = (x + p)^2 - p^2$

$x^2 + 6x$

So $p = 3$.

$$\begin{aligned} x^2 + 6x &= (x + 3)^2 - 3^2 \\ &= (x + 3)^2 - 9 \end{aligned}$$

Hence, $x^2 + 6x = (x + 3)^2 - 9$.

FACTORISING QUADRATIC EXPRESSIONS

Check $x^2 + 6x = (x+3)^2 - 9$:

$$\begin{aligned}(x+3)^2 - 9 &= (x+3)(x+3) - 9\\ &= x^2 + 6x + 9 - 9\\ &= x^2 + 6x\end{aligned}$$

COMPLETING THE SQUARE

$$x^2 + 2px = (x + p)^2 - p^2$$

a) $x^2 + 18x$

b) $a^2 - 6a$

c) $2y^2 - 8y$

COMPLETING THE SQUARE

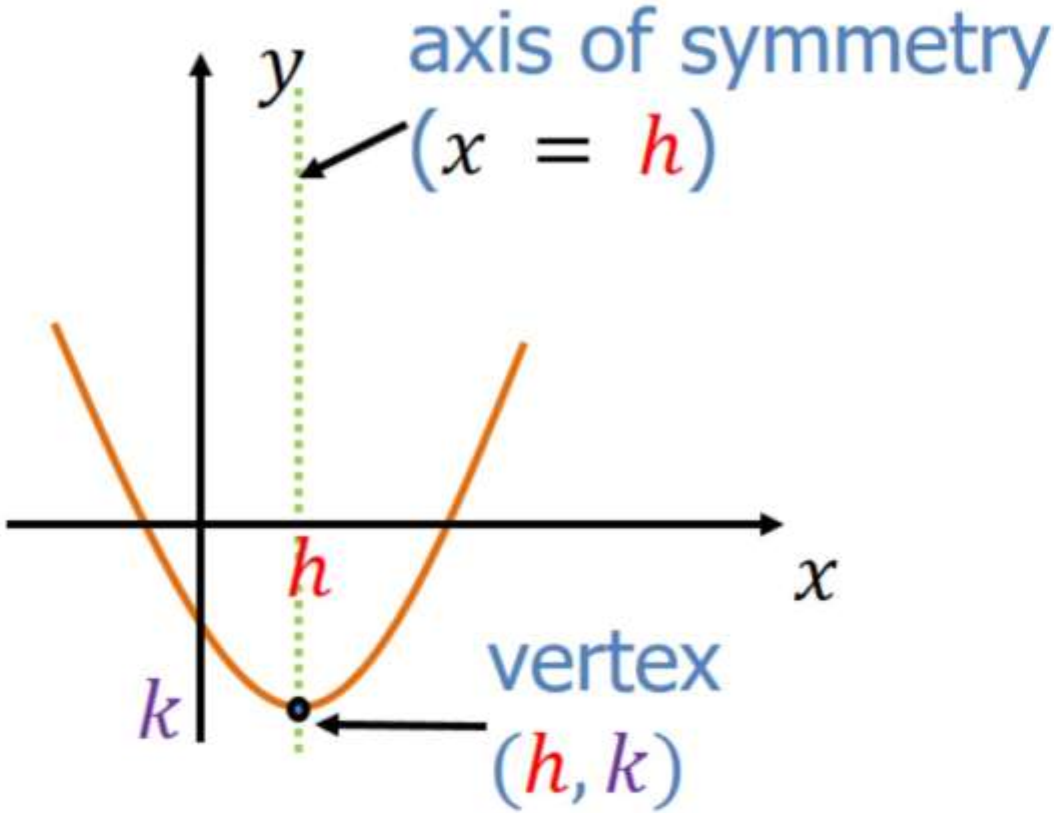
a) $$\begin{aligned} x^2 + 18x &= x^2 + 2 \times 9x \\ &= (x+9)^2 - 81 \end{aligned}$$

b) $$\begin{aligned} a^2 - 6a &= a^2 + 2 \times (-3a) \\ &= (a - 3)^2 - 9 \end{aligned}$$

c) $$\begin{aligned} 2y^2 - 8y &= 2(y^2 - 4y) \\ &= 2[(y-2)^2 - 4] \\ &= 2(y-2)^2 - 8 \end{aligned}$$

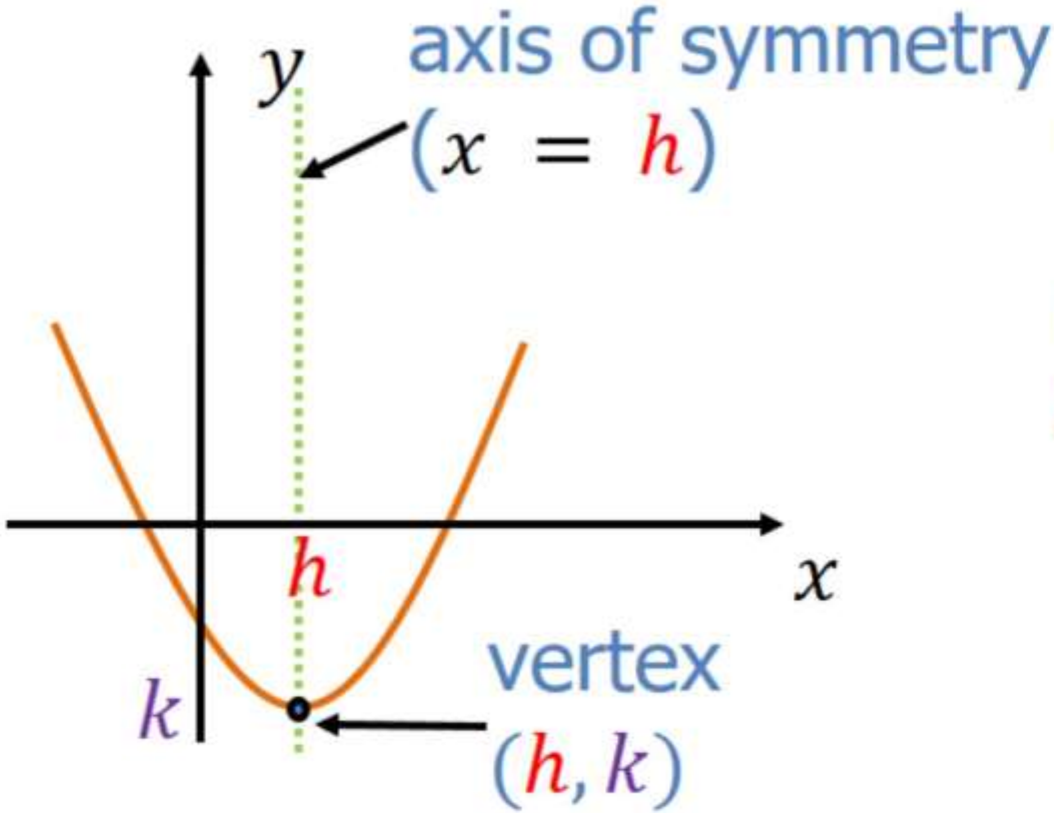
COMPLETING THE SQUARE

The form $a(x - h)^2 + k$ gives:



COMPLETING THE SQUARE

The form $a(x - h)^2 + k$ gives:



For example
$$y = (x - 2)^2 - 4$$
has vertex $(2, -4)$ as $h = 2$ and $k = -4$.

SOLVING QUADRATIC EQUATIONS

- One side of the equation must be a quadratic expression and the other side equal to 0
- Solve using a choice of methods.

SOLVING: FACTORISATION

$$x^2 - 5x + 6 = 0$$

SOLVING: FACTORISATION

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

SOLVING: FACTORISATION

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

So, either $x - 3 = 0$ or $x - 2 = 0$

SOLVING: FACTORISATION

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

So, either $x - 3 = 0$ or $x - 2 = 0$

That is $x = 3$ or $x = 2$

SOLVING: FACTORISATION

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

So, either $x - 3 = 0$ or $x - 2 = 0$

That is $x = 3$ or $x = 2$

Check: When $x = 3$, the LHS is $3^2 - 5(3) + 6 = 0$.

Also, when $x = 2$, the LHS is $2^2 - 5(2) + 6 = 0$.

SOLVING: FACTORISATION

(a) $x^2 - x - 12 = 0$

Hint: $x^2 - x - 12 = (x + 3)(x - 4)$

(b) $x^2 - 10x + 25 = 0$

Hint: $x^2 - 10x + 25 = (x - 5)(x - 5)$

(c) $x^2 - 81 = 0$

Hint: $x^2 - 81 = (x + 9)(x - 9)$

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SOLVING: FACTORISATION

(a) $x^2 - x - 12 = 0$
$(x + 3)(x - 4) = 0$
So, $x + 3 = 0$ or $x - 4 = 0$.
Hence, the solutions are $x = -3$ and $x = 4$.

(b) $x^2 - 10x + 25 = 0$
$(x - 5)(x - 5) = 0$
So, $x - 5 = 0$.
Hence, the solution is $x = 5$.

(c) $x^2 - 81 = 0$
$(x - 9)(x + 9) = 0$
So, $x - 9 = 0$ or $x + 9 = 0$.
Hence, the solutions are $x = 9$ and $x = -9$.

SOLVING: COMPLETING THE SQUARE

$$x^2 + 6x - 7 = 0$$

SOLVING: COMPLETING THE SQUARE

$$x^2 + 6x \qquad - 7 = 0$$

$$(x + 3)^2 - 9 - 7 = 0$$

Complete the square: $(x + 3)^2 - 9 - 7 = 0$

SOLVING: COMPLETING THE SQUARE

$$x^2 + 6x \qquad - 7 = 0$$

$$(x + 3)^2 - 9 - 7 = 0$$

Complete the square: $(x + 3)^2 - 9 - 7 = 0$

Rearrange: $(x + 3)^2 - 16 = 0$

Rearrange: $(x + 3)^2 = 16$

SOLVING: COMPLETING THE SQUARE

$$x^2 + 6x - 7 = 0$$

Complete the square: $(x+3)^2 - 9 - 7 = 0$

Rearrange: $(x+3)^2 - 16 = 0$

Rearrange: $(x+3)^2 = 16$

Take roots: $x + 3 = \pm 4$

SOLVING: COMPLETING THE SQUARE

$$x^2 + 6x - 7 = 0$$

Complete the square: $(x+3)^2 - 9 - 7 = 0$

Rearrange: $(x+3)^2 - 16 = 0$

Rearrange: $(x+3)^2 = 16$

Take roots: $x + 3 = \pm 4$

Rearrange: $x = -3 \pm 4$

SOLVING: COMPLETING THE SQUARE

$x^2 + 6x - 7 = 0$

Complete the square:	$(x+3)^2 - 9 - 7 = 0$
Rearrange:	$(x+3)^2 - 16 = 0$
Rearrange:	$(x+3)^2 = 16$
Take roots:	$x + 3 = \pm 4$
Rearrange:	$x = -3 \pm 4$
So the solutions are:	$x = -3 + 4 = 1$ and
	$x = -3 - 4 = -7$

SOLVING: COMPLETING THE SQUARE

a) $x^2 + 18x + 56 = 0$

Hint: $x^2 + 18x = (x + 9)^2 - 81$

b) $a^2 - 6a + 5 = 0$

Hint: $a^2 - 6a = (a - 3)^2 - 9$

SOLVING: COMPLETING THE SQUARE

a) $x^2 + 18x + 56 = 0$

$(x + 9)^2 - 81 + 56 = 0$

$(x + 9)^2 = 25$

$x + 9 = \pm 5$

Hence, $x = -14$ or $x = -4$

b) $a^2 - 6a + 5 = 0$

$(a - 3)^2 - 9 + 5 = 0$

$(a - 3)^2 = 4$

$a - 3 = \pm 2$

Hence, $a = 1$ or $a = 5$

SOLVING: QUADRATIC FORMULA

For the equation $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

SOLVING: QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x^2 - 5x + 6 = 0$

$a = 1, b = -5, c = 6$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}$$
$$= \frac{5 \pm \sqrt{25 - 24}}{2}$$
$$= \frac{5 \pm 1}{2}$$

Hence $x = \frac{5+1}{2} = 3$ and $x = \frac{5-1}{2} = 2$

SOLVING: QUADRATIC FORMULA

The Pros:

- Always works.
- The discriminant reveals additional information.

The Cons:

- Don’t forget to compute both terms (plus and minus).
- It’s a “magnet” for algebraic errors.
- If the answers turn out to be rational (no surds) then there was a simple factorisation that wasn’t spotted!

SOLVING: COMPARING METHODS

Factorisation:

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

So $x = 3$ or $x = 2$.

Completed-square form:

$x^2 - 5x + 6 = 0$

$$\left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6 = 0$$
$$\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 6 = 0$$
$$\left(x - \frac{5}{2}\right)^2 = \frac{1}{4}$$
$$x - \frac{5}{2} = \pm\frac{1}{2}$$

So $x = \frac{5}{2} + \frac{1}{2} = 3$
or $x = \frac{5}{2} - \frac{1}{2} = 2$

Quadratic formula:

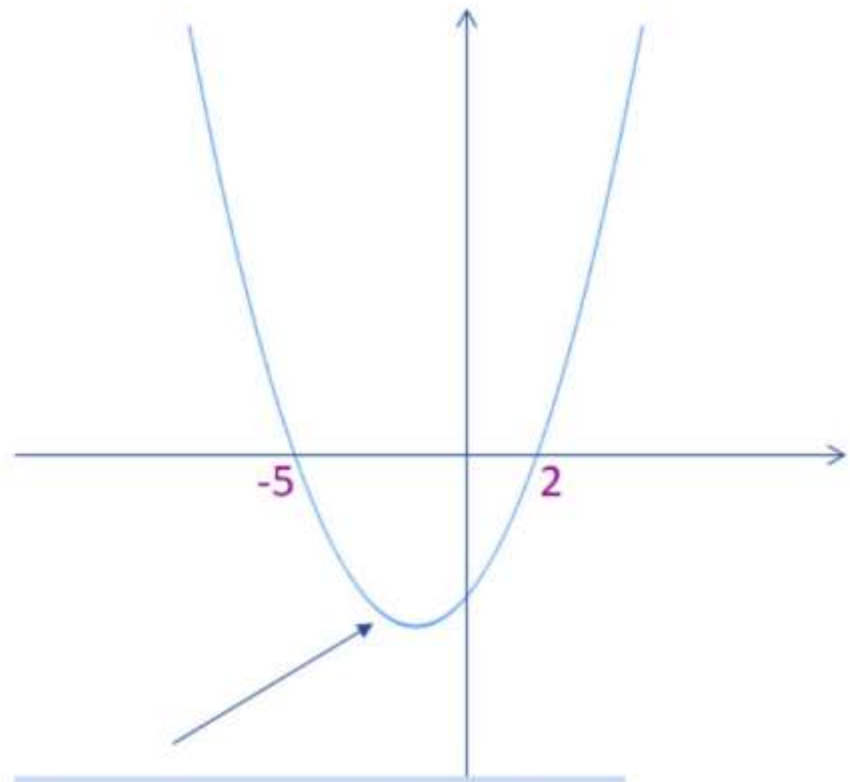
$x^2 - 5x + 6 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}$$
$$= \frac{5 \pm \sqrt{25 - 24}}{2}$$
$$= \frac{5 \pm 1}{2}$$

So $x = \frac{5+1}{2} = 3$
or $x = \frac{5-1}{2} = 2$

SOLUTIONS OF A QUADRATIC EQUATION

The solutions of a quadratic equation are the points where the graph of the quadratic intercepts the x-axis.



Graph of $y = x^2 + 3x - 10$

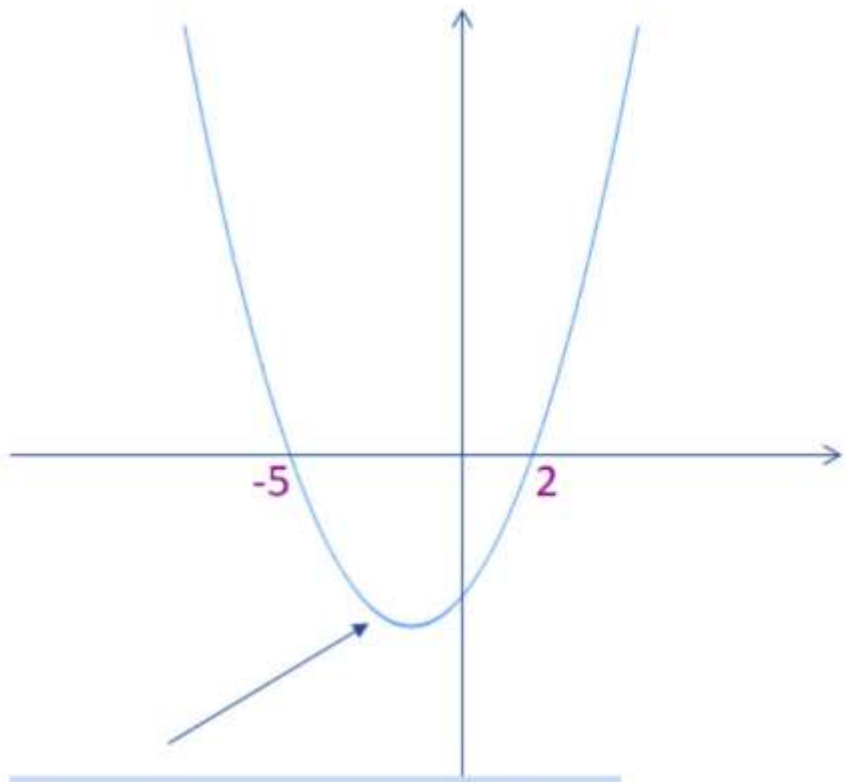
SOLUTIONS OF A QUADRATIC EQUATION

The solutions of a quadratic equation are the points where the graph of the quadratic intercepts the x-axis.

The graph of $y = x^2 + 3x - 10$ cuts the x-axis twice because

$$x^2 + 3x - 10 = 0$$

has two solutions ($x = -5$ and $x = 2$).



Graph of $y = x^2 + 3x - 10$

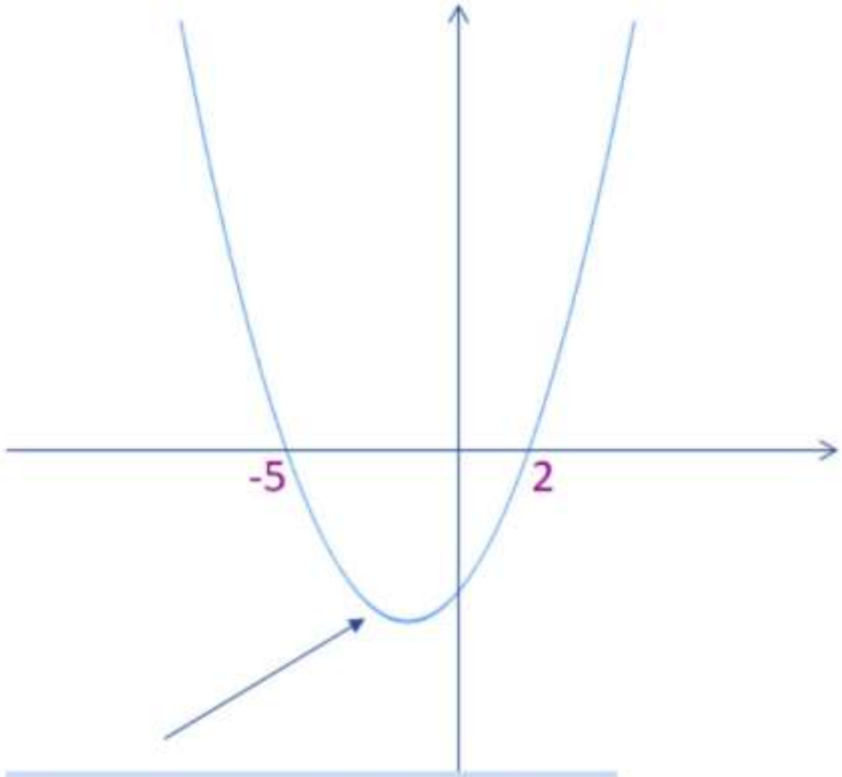
SOLUTIONS OF A QUADRATIC EQUATION

Now consider

$$0 = x^2 + 3x - 10$$

And the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Graph of $y = x^2 + 3x - 10$

SOLUTIONS OF A QUADRATIC EQUATION

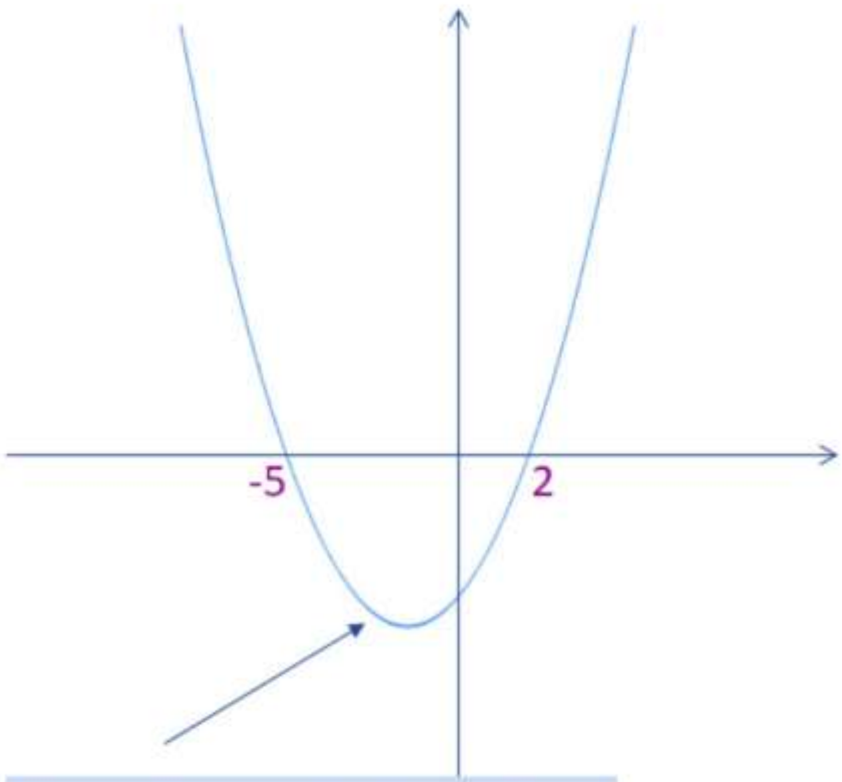
Now consider

$$0 = x^2 + 3x - 10$$

And the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ is known as the discriminant. Here it is equal to $3^2 - 4 \times 1 \times (-10) = 49$



Graph of $y = x^2 + 3x - 10$

SOLUTIONS OF A QUADRATIC EQUATION

Now consider

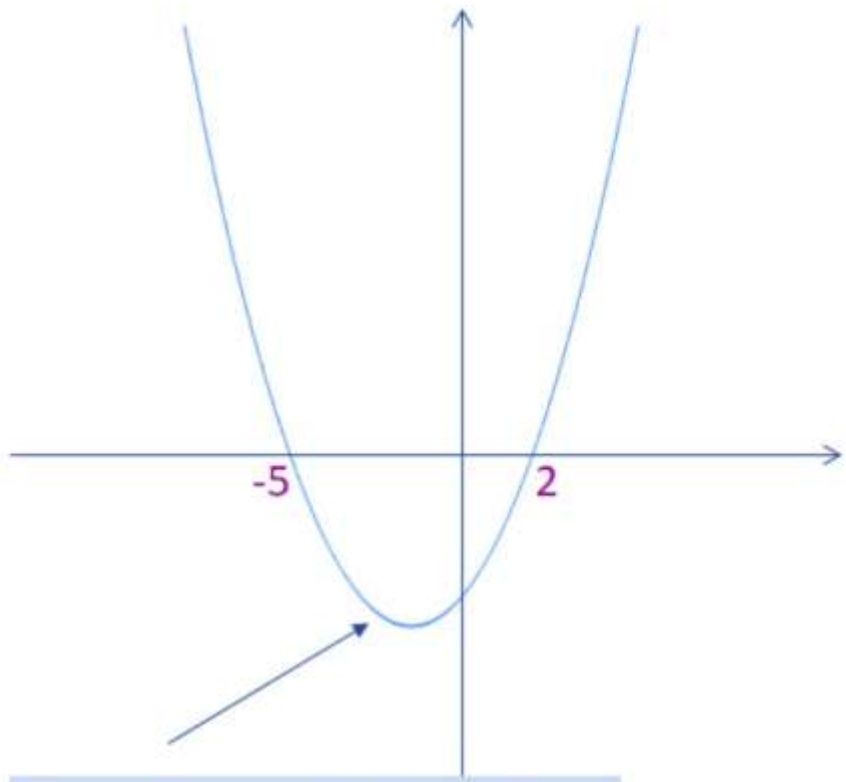
$$0 = x^2 + 3x - 10$$

And the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ is known as the discriminant. Here it is equal to $3^2 - 4 \times 1 \times (-10) = 49$

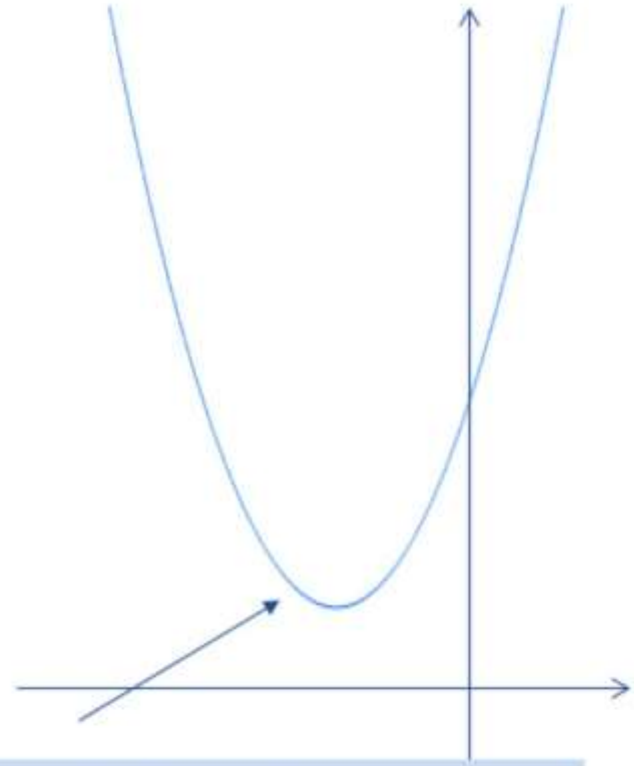
So the discriminant is positive.



Graph of $y = x^2 + 3x - 10$

SOLUTIONS OF A QUADRATIC EQUATION

Now consider $0 = x^2 + x + 1$.



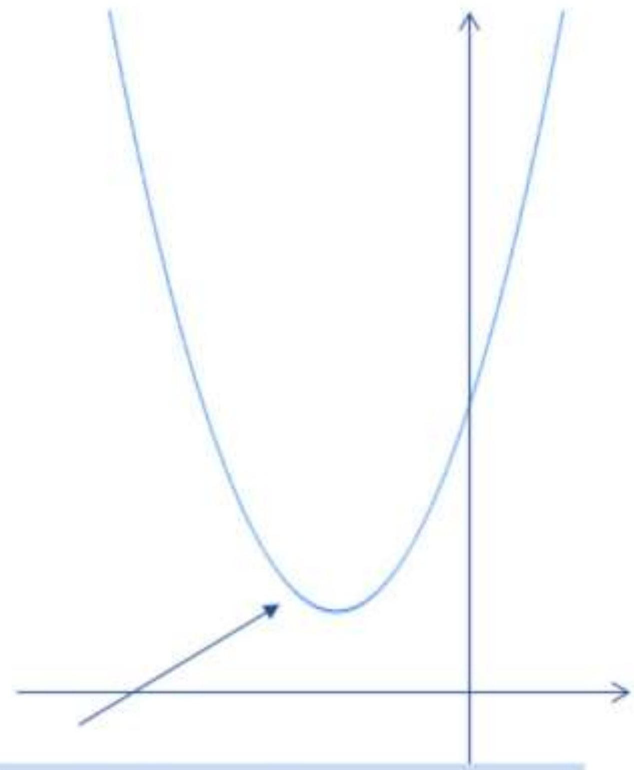
Graph of $y = x^2 + x + 1$

SOLUTIONS OF A QUADRATIC EQUATION

Now consider $0 = x^2 + x + 1$.

Here the discriminant is

$$b^2 - 4ac = 1 - 4 \times 1 \times 1 = -3$$



Graph of $y = x^2 + x + 1$

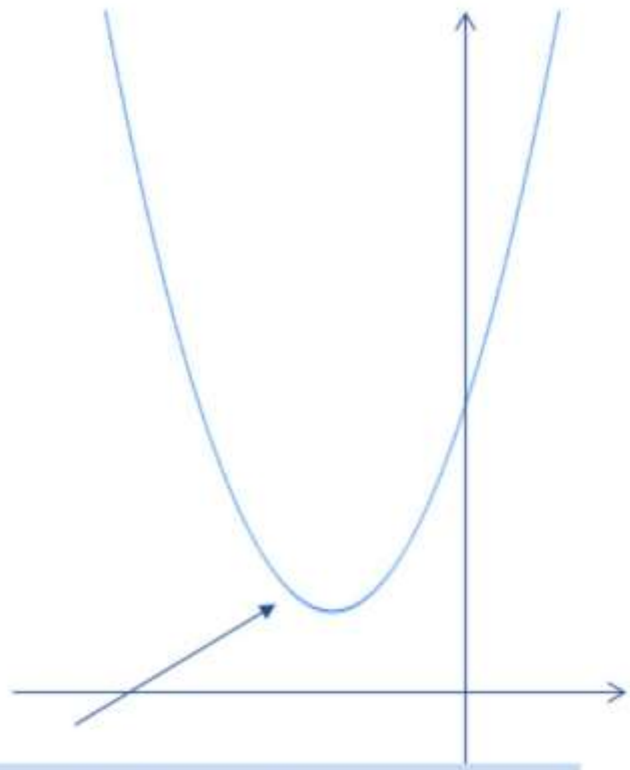
SOLUTIONS OF A QUADRATIC EQUATION

Now consider $0 = x^2 + x + 1$.

Here the discriminant is

$$b^2 - 4ac = 1 - 4 \times 1 \times 1 = -3$$

Hence the discriminant is negative.



Graph of $y = x^2 + x + 1$

SOLUTIONS OF A QUADRATIC EQUATION

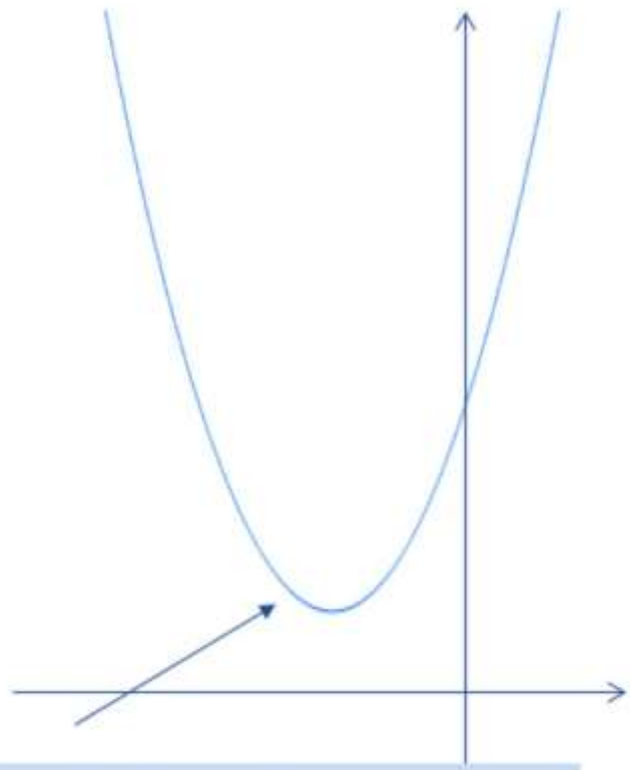
Now consider $0 = x^2 + x + 1$.

Here the discriminant is

$$b^2 - 4ac = 1 - 4 \times 1 \times 1 = -3$$

Hence the discriminant is negative.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Graph of $y = x^2 + x + 1$

SOLUTIONS OF A QUADRATIC EQUATION

Now consider $0 = x^2 + x + 1$.

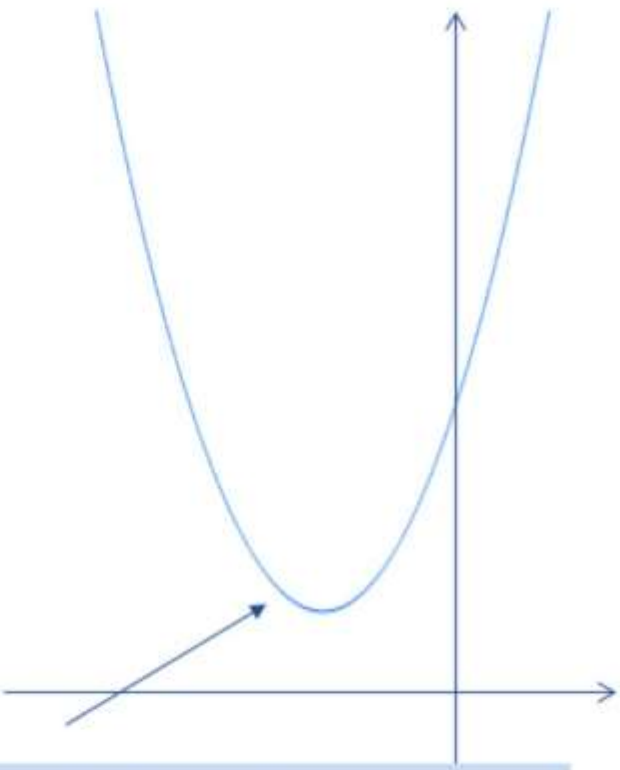
Here the discriminant is

$$b^2 - 4ac = 1 - 4 \times 1 \times 1 = -3$$

Hence the discriminant is negative.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

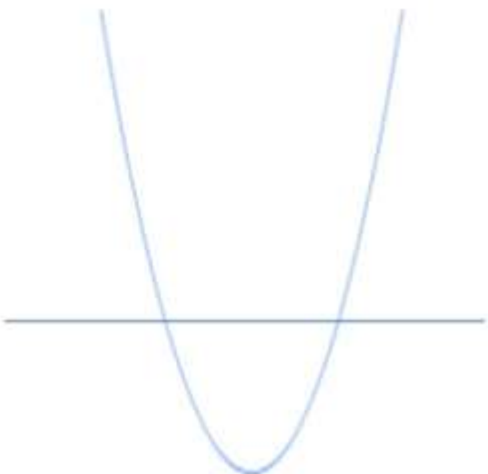
$$= \frac{-1 \pm \sqrt{-3}}{2}$$



Graph of $y = x^2 + x + 1$

SOLUTIONS OF A QUADRATIC EQUATION

$b^2 - 4ac > 0$

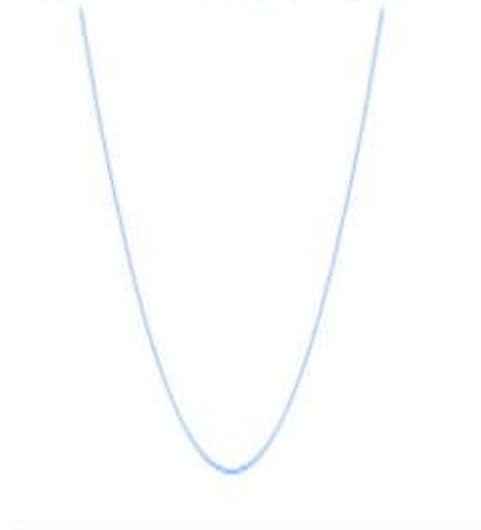


2 solutions

$b^2 - 4ac = 0$

1 solution

$b^2 - 4ac < 0$



no solutions

A bit extra!

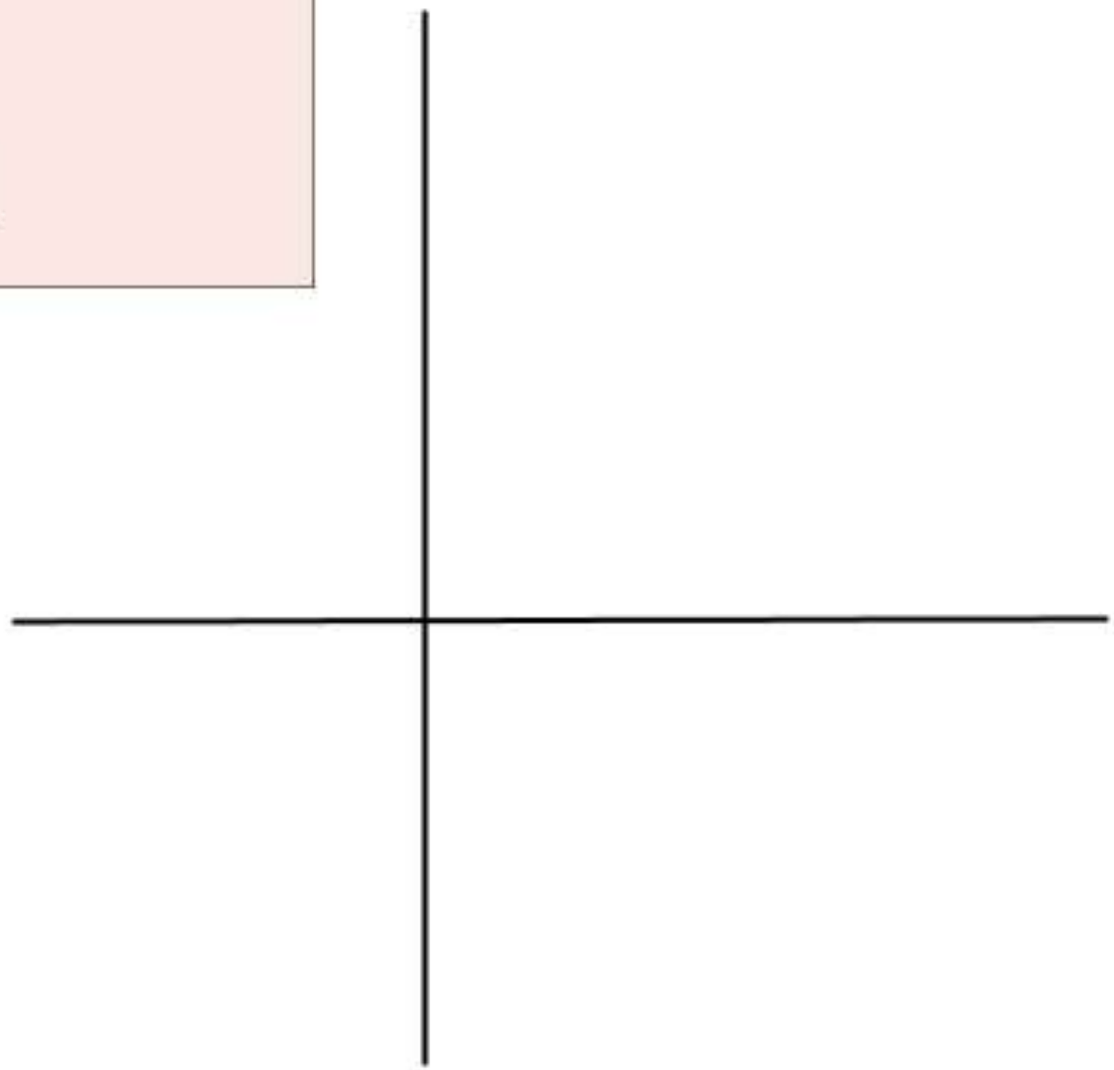
The quadratic formula

The solutions of the quadratic equation

$$ax^2 + bx + c = 0$$

are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$



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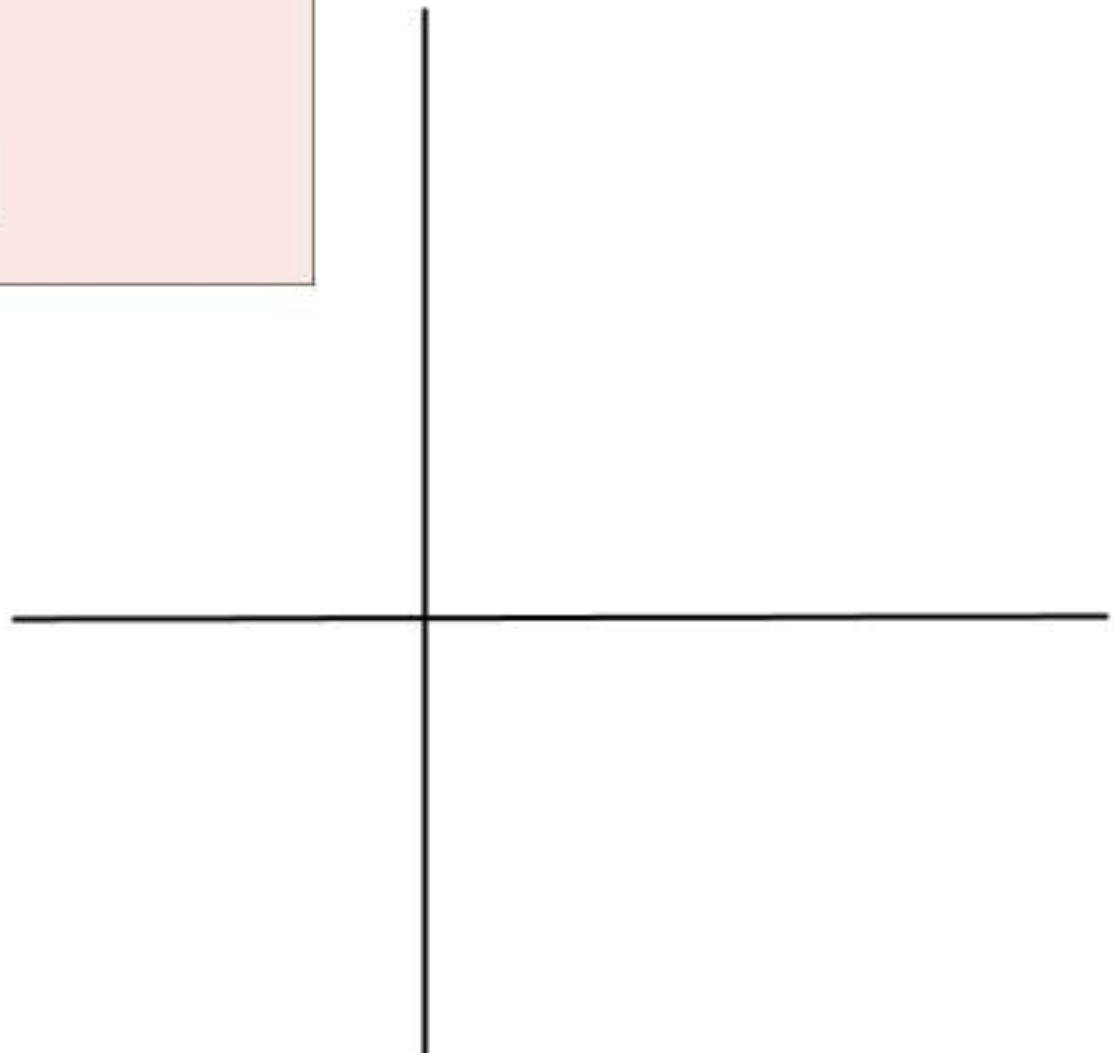


SOLUTIONS OF A QUADRATIC EQUATION

Find the number of solutions to the following quadratic equations (the discriminant is $b^2 - 4ac$).

a. $3x^2 + 5x - 52 = 0$

b. $3x^2 - 5x + 52 = 0$

c. $4x^2 + 20x = -25$

SOLUTIONS OF A QUADRATIC EQUATION

a. $3x^2 + 5x - 52 = 0$

$b^2 - 4ac = 5^2 - 4 \times 3 \times (-52) = 649.$

The discriminant is positive so the equation has two solutions.

b. $3x^2 - 5x + 52 = 0$

$b^2 - 4ac = (-5)^2 - 4 \times 3 \times 52 = -599.$

The discriminant is negative so the equation has no solutions.

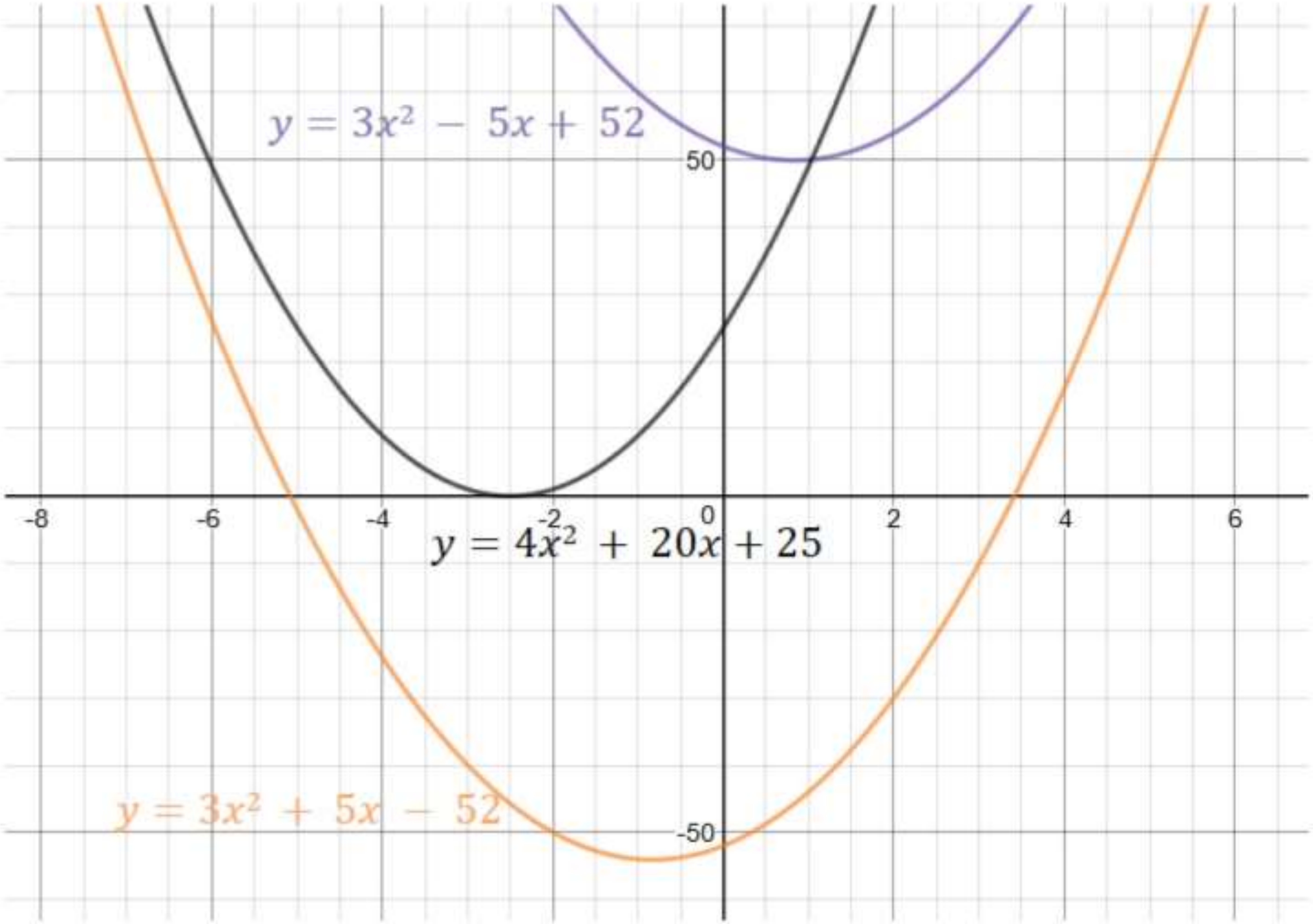
c. $4x^2 + 20x = -25$

rearrange to $4x^2 + 20x + 25 = 0.$

$b^2 - 4ac = 20^2 - 4 \times 4 \times 25 = 0.$

The discriminant so the equation has one solution.

SOLUTIONS OF A QUADRATIC EQUATION



REVIEW: TRUE OR FALSE?

1) $(x + 3)(x + 1) = x^2 + 4x + 3$

2) $(x - 3)(x - 1) = x^2 - 4x - 3$

3) The equation of the axis of symmetry of the parabola $y = x^2 - 6x - 9$ is $x = 3$.

4) The parabola $y = 5 - x^2$ is u-shaped.

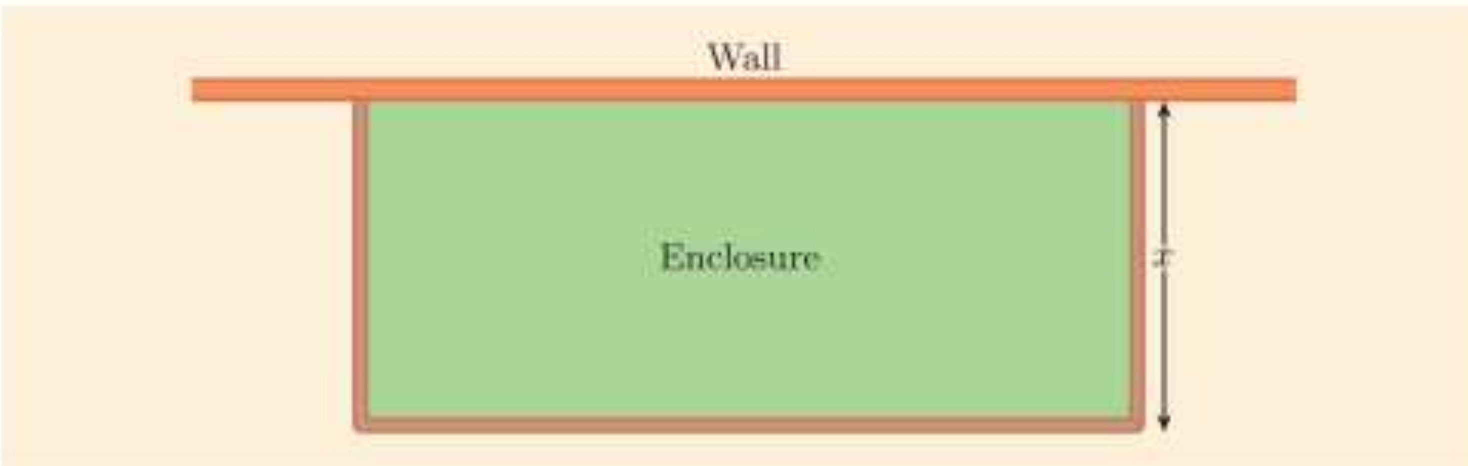
5) The vertex of the parabola $y = (x + 2)^2 + 4$ is $(2, 4)$.

6) The equation $x^2 + 2x + 6 = 0$ has two solutions.

Would you like a Bonus Question??

A Bonus Question!

A farmer wants to make a rectangular enclosure next to an existing wall, using 120 m of fencing. Let the area of the enclosure be $A\,\mathrm{m}^2$, and let its width, as shown in the diagram below, be $x\,\mathrm{m}$.



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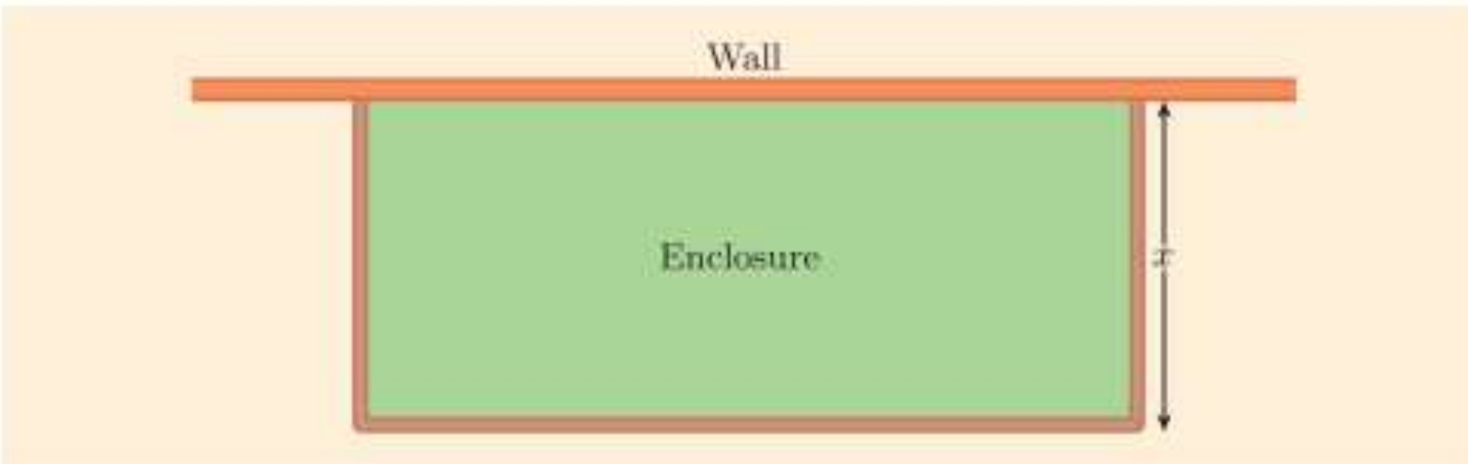

(a) Find an expression for the length of the enclosure in terms of x.

(b) Find a formula for A in terms of x.

(c) Hence find the maximum area of the enclosure, and the length and width that give this maximum area.

A Bonus Question!

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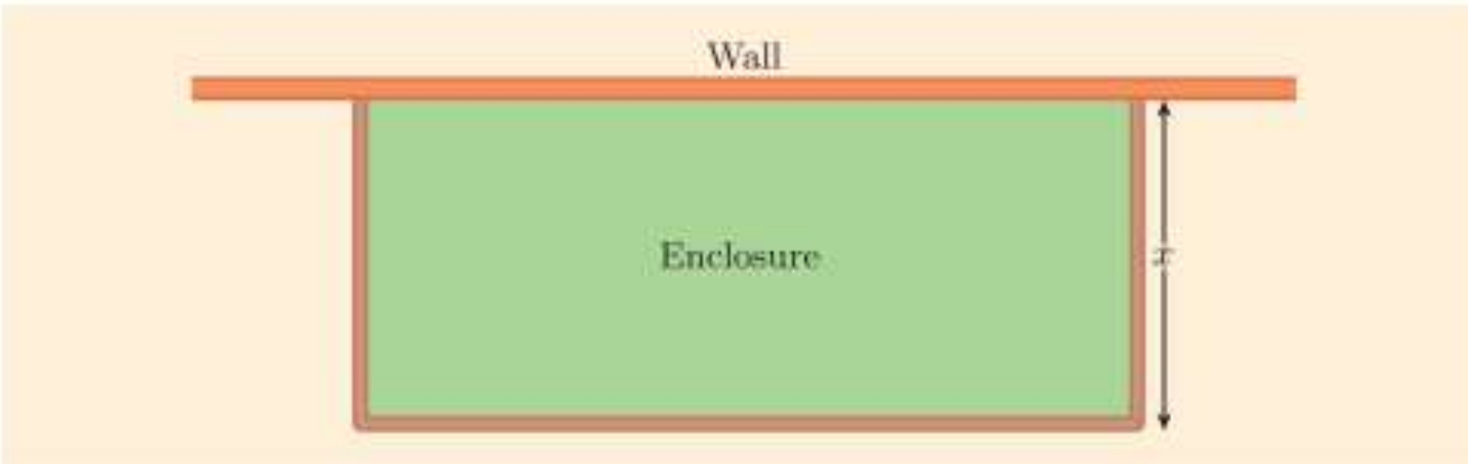
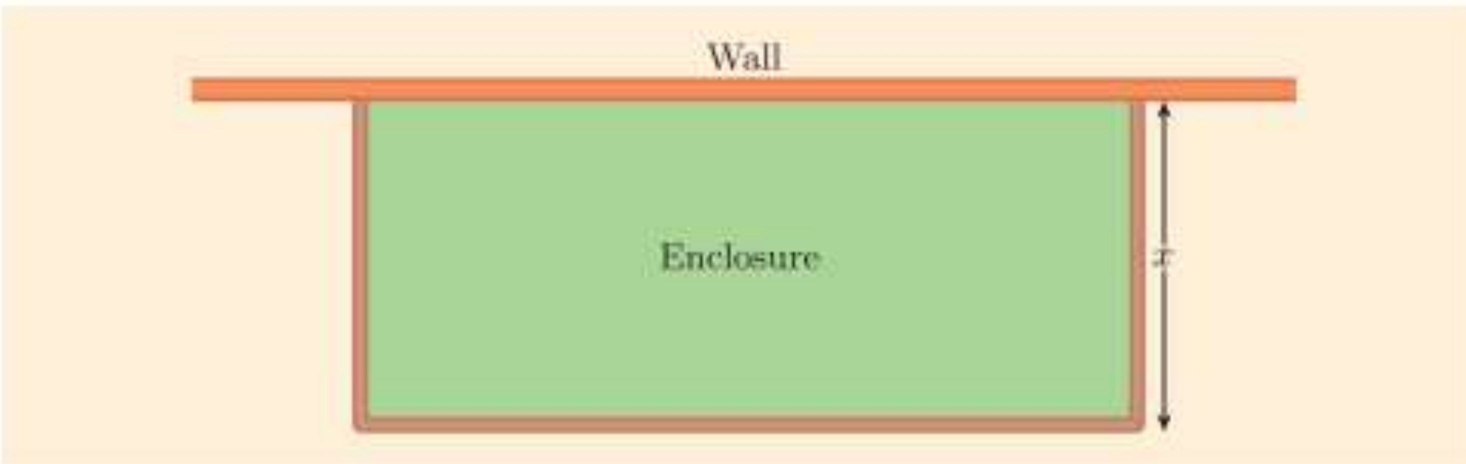


(a) Find an expression for the length of the enclosure in terms of x.

(a) The total length of fencing is 120 m, and two of the three sides of the enclosure are x m long. So the length of the third side is $(120 - 2x)$ m, and this is the length of the enclosure.

A Bonus Question!

A farmer wants to make a rectangular enclosure next to an existing wall, using 120 m of fencing. Let the area of the enclosure be $A\,\mathrm{m}^2$, and let its width, as shown in the diagram below, be x m.



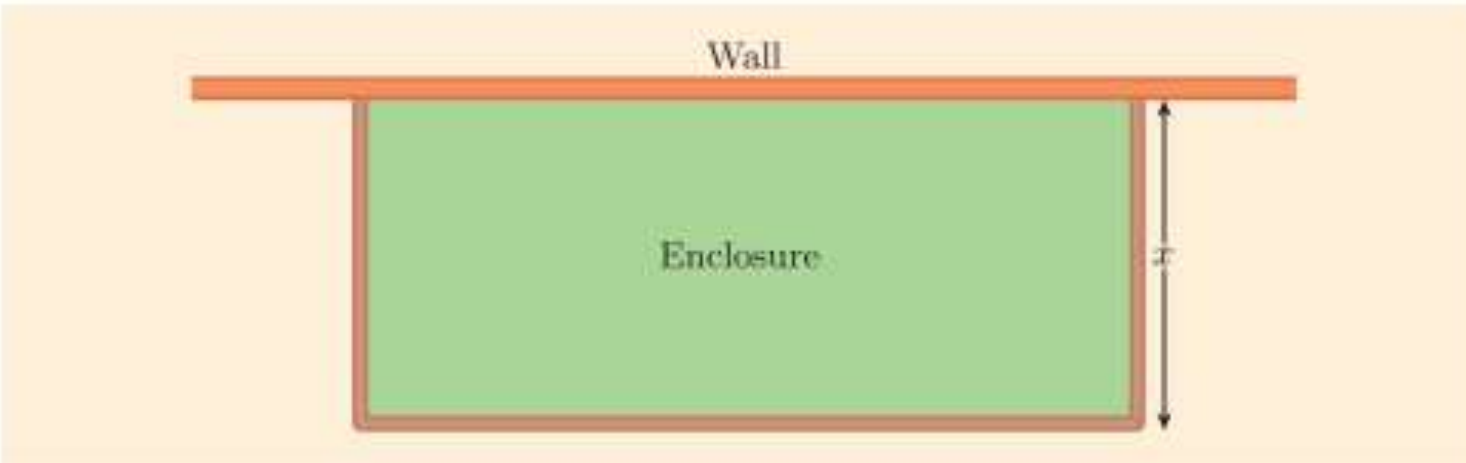
(b) Find a formula for A in terms of x.

(b) The area of the enclosure is given by

$A = x(120 - 2x)$.

A Bonus Question!

A farmer wants to make a rectangular enclosure next to an existing wall, using 120 m of fencing. Let the area of the enclosure be $A\,\text{m}^2$, and let its width, as shown in the diagram below, be x m.



(c) Hence find the maximum area of the enclosure, and the length and width that give this maximum area.

$$A = x(120 - 2x).$$

(c) Hence find the maximum area of the enclosure, and the length and width that give this maximum area.

(c) The formula found in part (b) is already factorised, so the quickest way to find the vertex is to find the x-intercepts first.

Putting $A = 0$ gives

$$x(120 - 2x) = 0,$$

so

$$x = 0 \quad \text{or} \quad x = \frac{120}{2} = 60.$$

So the x-intercepts are 0 and 60.

The value halfway between the x-intercepts is 30.

Substituting $x = 30$ into the equation of the parabola gives

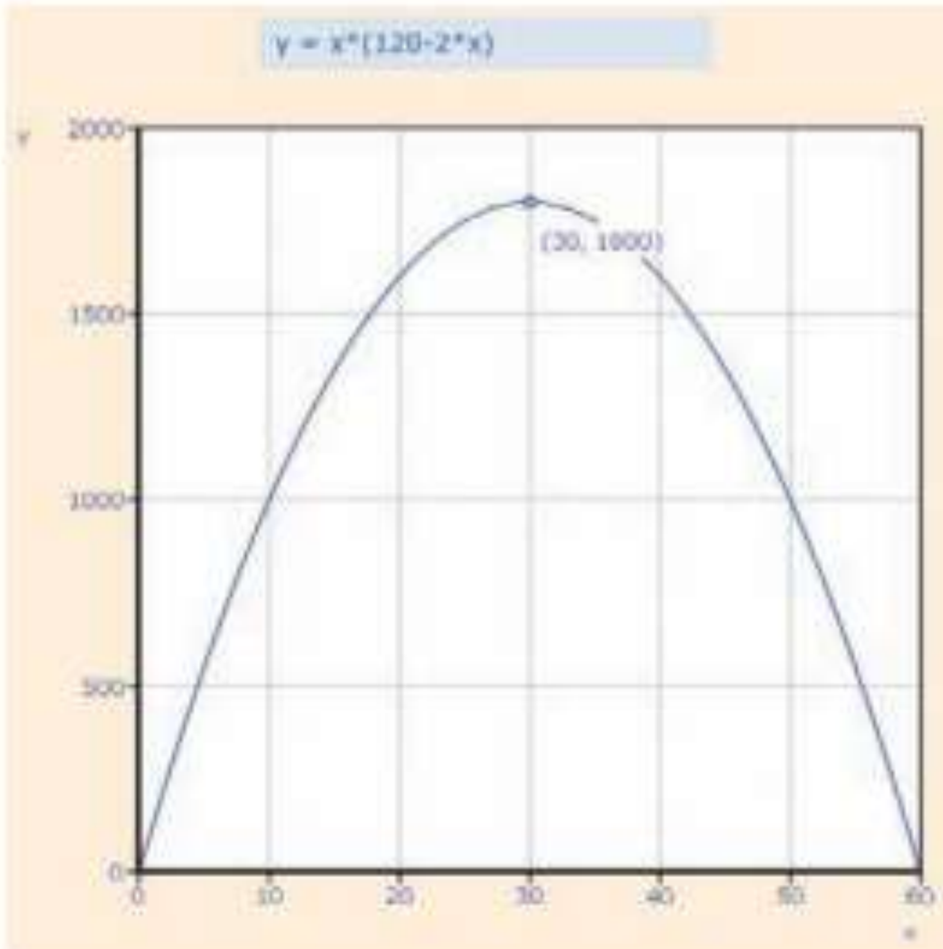
$$\begin{aligned} A = 30(120 - 2 \times 30) &= 30(120 - 60) \\ &= 30 \times 60 = 1800. \end{aligned}$$

So the vertex is $(30, 1800)$.

Hence the maximum area of the enclosure is $1800\,\text{m}^2$, and this is achieved when the width is $30\,\text{m}$ and the length is $(120 - 2 \times 30)\,\text{m} = 60\,\text{m}$.

(b) The area of the enclosure is given by

$$A = x(120 - 2x).$$



WHERE NEXT?

The next session is on Wed 22th September and covers the topic of Trigonometry.

If you are finding this level of work quite difficult then you may not be ready to start MST124 and you should speak to your tutor or the Student Support Team (link on the module home page) as soon as you can.

We would strongly advise you to do the 'Getting ready for MST124' quiz that is on the module website before starting the units. It will help you to identify areas you need to focus on.