TMA01

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Question 1

a) Simplify the expression:

$$\frac{6b^2(a+2b^2)(a-2b^2)}{32ab^5-8a^3b}, \text{ where } a, b \neq 0 \text{ and } a \neq 2b^2$$

Solution:

$$\begin{aligned}
\frac{6b^2(a+2b^2)(a-2b^2)}{32ab^5-8a^3b} &= \frac{3b^2(a+2b^2)(a-2b^2)}{16ab^5-4a^3b} \\
&= \frac{3b^2(a^2-4b^4)}{4ab(4b^4-a^2)} \\
&= \frac{3b^2}{-4ab} \times \frac{a^2-4b^4}{a^2-4b^4} \\
&= -\frac{3b}{4a}
\end{aligned}$$

b) Multiply out the brackets in the following expression, simplifying your answer as far as possible:

$$(5\sqrt{7} - \sqrt{3})(\sqrt{7} + 4\sqrt{3})$$

Solution:

$$\begin{aligned}
(5\sqrt{7} - \sqrt{3})(\sqrt{7} + 4\sqrt{3}) &= 5 \times 7 + 20\sqrt{3}\sqrt{7} - \sqrt{3}\sqrt{7} - 4 \times 3 \\
&= 35 + 19\sqrt{3}\sqrt{7} - 12 \\
&= 23 + 19\sqrt{21}
\end{aligned}$$

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c) Simplify the expression:

$$\frac{\sqrt[6]{(64x^3)}x^{2/3}}{(4x)^{1/4}}, \text{ where } x \neq 0$$

Solution:

$$\begin{aligned}
\frac{\sqrt[6]{(64x^3)}x^{2/3}}{(4x)^{1/4}} &= \frac{64^{1/6}x^{1/2}x^{2/3}}{4^{1/4}x^{1/4}} \\
&= \frac{2x^{7/6}}{\sqrt{2} \times x^{1/4}} \\
&= \frac{2x^{11/12}}{\sqrt{2}} \\
&= \frac{2\sqrt{2}x^{11/12}}{2} \\
&= \sqrt{2}x^{11/12}
\end{aligned}$$

d) Solve the following equation, stating any assumptions that you have made about x:

$$\frac{4}{2x+1} + \frac{3}{5-3x} = 0$$

Solution:

$$\frac{4}{2x+1} + \frac{3}{5-3x} = 0$$

$$\frac{4}{2x+1} = -\frac{3}{5-3x}$$

$$4 = \frac{-6x-3}{5-3x}$$

$$20 - 12x = -6x - 3$$

$$23 = 6x$$

$$x = \frac{23}{6}, \text{ assuming that } x \neq -\frac{1}{2} \text{ and } x \neq \frac{5}{3}$$

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e) Rearrange the following equation to make g the subject:

$$f = \pi r - \frac{gt}{2r}, \text{ where } r, t \neq 0$$

Solution:

$$f = \pi r - \frac{gt}{2r}$$

$$f + \frac{gt}{2r} = \pi r$$

$$\frac{gt}{2r} = \pi r - f$$

$$gt = 2r(\pi r - f)$$

$$g = \frac{2\pi r^2 - 2fr}{t}$$

Question 2

a) Suggest three ways in which the well-written solution is better than the poorly-written solution (and so is likely to be awarded considerably higher marks).

- The well-written solution states the formulas used and defines each of the variables used in them. The poorly-written solution states just the formula to the area of the round cake, and does not define its variables.
- In the well-written solution, the final solution to the problem is clearly summarized as a sentence. The poorly-written solution gives its answer as a line with little descriptive text and two equals signs.
- The precision and units of the final answer are given in the well-written solution. The poorly-written solution gives no units and no description of the rounding.

b) Explain where the students went wrong in their calculations, and give the correct answer.

In the formula for the area of the iced surface of the round cake, both students have incorrectly swapped the r^2 exponent between the two terms, using equation (1), instead of the correct equation (2).

$$\begin{aligned}\text{Area} &= \pi r + 2\pi r^2 h \\ &= \pi \times 10 + 2\pi \times 100 \times 8 \\ &= 5057.964... \text{ cm}^2\end{aligned} \tag{1}$$

$$\begin{aligned}\text{Area} &= \pi r^2 + 2\pi r h \\ &= \pi \times 100 + 2\pi \times 10 \times 8 \\ &= 816.814... \text{ cm}^2\end{aligned} \tag{2}$$

This led to the wrong area for the round cake being substituted into subsequent calculations. Substituting the correct value gives the following solution (where s is the side length of the cubic cake):

$$\begin{aligned}\text{Area} &= s^2 + 4s^2 \\ 816.814... \text{ cm}^2 &= 5s^2 \\ s &= \pm 12.781... \text{cm}\end{aligned}$$

So choosing the positive square root, the cubic cake can have a side length of at most 12 cm to the nearest cm. The value of s to the nearest cm is 13cm, but there is not enough icing to cover a cubic cake with a 13cm side length.

c) Identify two ways in which this solution differs from Well-written solution 1, and for one of the differences, give a reason why this could be useful.

- This solution includes the units on variables substituted into formulas. This is useful because it helps prevent errors due to use of variables on different measurement scales.
- This solution doesn't explicitly state that a choice is made between the positive and negative square root.

Question 3

a) Show that the point (5,4) lies on the line with equation $5y = 8x - 20$.

To determine if the point (5,4) lies on the line of $5y = 8x - 20$, I substitute the values of $x = 5$ and $y = 4$ and check if the left-hand side (LHS) of the equation equals the right-hand side (RHS).

$$\begin{aligned} \text{LHS} &= 5y \\ &= 5 \times 4 \\ &= 20 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 8x - 20 \\ &= 8 \times 5 - 20 \\ &= 20 \end{aligned}$$

As the LHS = the RHS, the point (5,4) satisfies, and lies on the line of, the equation.

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b) Find the equation of the line through the points (1,0) and (3,-3).

A straight line graph has an equation in the form $y = mx + c$, where x and y are the variables, m is the slope of the graph, and c is the y intercept.

When two points that satisfy the equation are known, the slope can be found using:

$$\frac{y_2 - y_1}{x_2 - x_2}$$

Substituting the (x, y) coordinate pairs (1,0) and (3,-3) gives:

$$\frac{0 - (-3)}{1 - 3} = -\frac{3}{2}$$

b) (continued)

By substituting the slope and the x and y values of one of the points into the equation of a straight line, c can be found by making it the subject:

$$y = mx + c$$

$$0 = -\frac{3}{2} \times 1 + c$$

$$c = \frac{3}{2}$$

Therefore, the equation of the straight line that passes through the points (1,0) and (3,-3) is $y = -\frac{3}{2}x + \frac{3}{2}$

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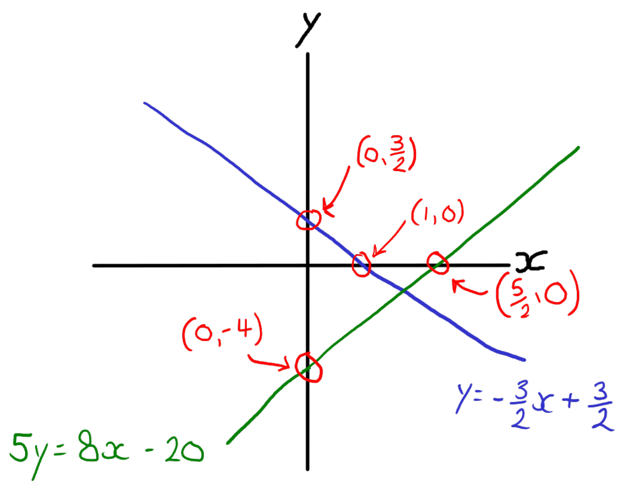
c) Draw the line that you found in part (b) and the line with equation $5y = 8x - 20$ on the same set of axes.



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d) Use your graph to estimate the coordinates of the point of intersection of the two lines, giving them to the nearest integer.

My estimate of the point of intersection between the two lines from their graphs is (2,-1), to the nearest integer.

e) By solving the equations of the two lines simultaneously, find the exact coordinates of this point of intersection.

The pair of simultaneous equations to be solved are shown below.

$$y = -\frac{3}{2}x + \frac{3}{2} \tag{3}$$
$$5y = 8x - 20 \tag{4}$$

Using the elimination method, I start by multiplying equation (3) by 5 to make the coefficients of y the same:

$$5y = -\frac{15}{2}x + \frac{15}{2} \tag{5}$$
$$5y = 8x - 20$$

I then subtract equation(5) from equation (4) and solve for x:

$$0 = \frac{31}{2}x - \frac{55}{2}$$

$$\frac{55}{2} = \frac{31}{2}x$$

$$x = \frac{55}{31}$$

To find y I substitute the value of x into equation (4):

$$5y = 8\left(\frac{55}{31}\right) - 20$$

$$5y = \frac{440}{31} - \frac{620}{31}$$

$$5y = -\frac{180}{31}$$

$$y = -\frac{36}{31}$$

So the exact coordinate of intersection between these lines is $\left(\frac{55}{31}, -\frac{36}{31}\right)$.

Question 4

a) i) Write the quadratic expression $4x^2 - 16x - 33$ in completed square form.

The completed square form of a quadratic expression is $p(x-q)^2 + r$, where p is a common factor of the coefficients of x^2 and x (in $ax^2 + bx + c$ form), q is half the coefficient of x (after the factor of p has been taken out), and r is the sum of the original c term, and $-(q^2)$.

Therefore, the completed square form of $4x^2 - 16x - 33$ is:

$$4(x-2)^2 - 49$$

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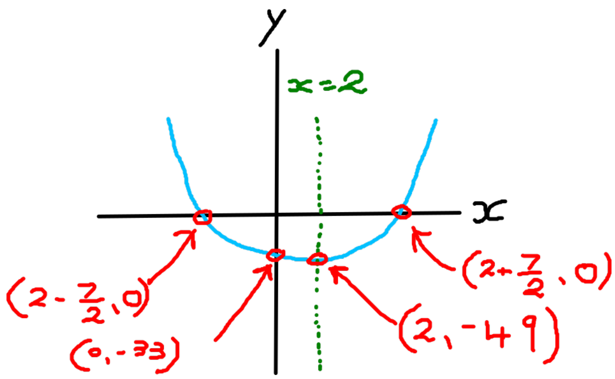
a) ii) Hence solve the equation $4x^2 - 16x - 33 = 0$.

To solve the equation for x when $y = 0$, I set the completed square form of the equation equal to zero, and solve for possible values of x:

$$\begin{aligned} 4(x-2)^2 - 49 &= 0 \\ 4(x-2)^2 &= 49 \\ (x-2)^2 &= \frac{49}{4} \\ \\ x - 2 &= \pm\frac{7}{2} \\ \\ x &= 2 \pm \frac{7}{2} \end{aligned}$$

Therefore, the solutions to the equation $4x^2 - 16x - 33 = 0$ are $x = 2 + \frac{7}{2}$ and $x = 2 - \frac{7}{2}$.

a) iii) Sketch the graph of $y = 4x^2 - 16x - 33 = 0$



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b) i) What will be the exact height of the ball when it passes directly over the fence?

As x is the horizontal displacement of the ball from Billy, the ball will be directly over the fence when $x = 3$. The height of the ball above ground over the fence can be found by substituting $x = 3$ into the equation:

$$y = -0.15x^2 + 1.2x + 0.6$$
$$y = -0.15(3)^2 + 1.2 \times 3 + 0.6$$
$$y = 2.85$$

So the exact height of the ball directly over the fence is 2.85 metres.

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b) ii) How far away from the fence does the ball hit the ground? Give your solution to one decimal place.

To find the distance from the fence the ball hits the ground, I start by solving the equation for x when $y = 0$, using the quadratic formula shown in equation (6):

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{6}$$

Where a, b, and c are the coefficients of x^2, x, and the constant term, respectively. Substituting the values for a, b, and c into equation (6) gives:

$$x = \frac{-1.2 \pm \sqrt{1.2^2 - 4 \times (-0.15) \times 0.6}}{2(-0.15)}$$

$$x = \frac{-1.2 \pm \sqrt{1.8}}{-0.3}$$

As x represents the displacement from Billy's position, the value of x must be positive and is therefore $x = 8.472...$ As the fence is at position $x = 3$, the ball lands $8.472... - 3 = 5.472...$ or 5.5 metres from the fence, to one decimal place.

Question 5

a) Find the exact solutions of the equation 2x^5 + 9x^4 - 8x^3 - 49x^2 + 6x + 40 = 0

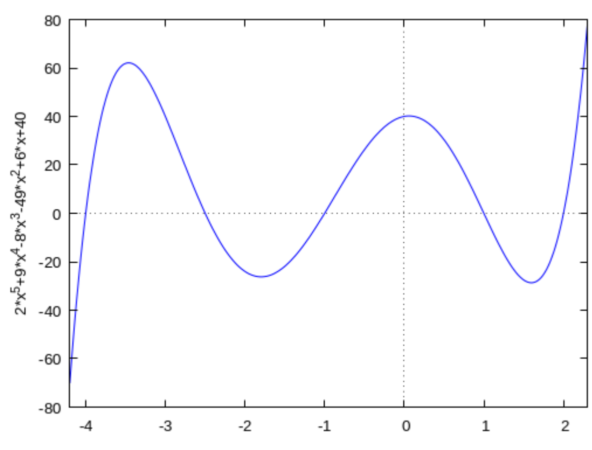
(%i1) solve(2·x^5 + 9·x^4 − 8·x^3 − 49·x^2 + 6·x + 40 = 0);

(%o1) $[x=-4, x=-1, x=-\frac{5}{2}, x=1, x=2]$

b) Plot the graph of the equation y = 2x^5 + 9x^4 - 8x^3 - 49x^2 + 6x +40

(%i2) wxplot2d(2·x^5 + 9·x^4 − 8·x^3 − 49·x^2 + 6·x +40, [x, −4.2, 2.3]);

(%t2)



(%o2)