R&R for MST124: Welcome to Session 6

Friday 24th September 2021

We'll start at 7.00pm and finish by 9.00pm

This session will cover the basics of Exponentials and Logs

Have paper, pen and your calculator to hand.

This is the final session in the series of revision tutorials. The recordings will remain available for a few weeks if you need them. The R&R forum will close at the end of this month so you should post your questions and comments on the MST124 module forums.

While you are waiting....

We know that $2^1 = 2$ and that $2^2 = 4$. Think about what $2^{1.5}$ might mean, what sort of number is it?

Preparing for MST124

6 Exponentials and Logs

What’s in Session 6

- What is an exponential equation?
- Where do exponential relationships crop up?
- What are ‘logs’
- The algebra of logs
- Solving exponential equations using logs

Exponential sequences

Think about a situation – like cell division for example - when something is doubling in number in a sequence

So, starting with a single cell, imagine that it splits into two cells every hour.

Here is the resulting number of cells after the first hour:

After t hours	0	1
No of cells	1	2

Exponential sequences

Think about a situation – like cell division for example - when something is doubling in number in a sequence

So, starting with a single cell, imagine that it splits into two cells every hour.

Here is the resulting number of cells after the first two hours:

After t hours	0	1	2
No of cells	1	2	4

Exponential sequences

Think about a situation – like cell division for example - when something is doubling in number in a sequence

So, starting with a single cell, imagine that it splits into two cells every hour.

Here is the resulting number of cells for the first few hours:

After t hours	0	1	2	3
No of cells	1	2	4	8

Exponential sequences

Think about a situation – like cell division for example - when something is doubling in number in a sequence

So, starting with a single cell, imagine that it splits into two cells every hour.

Here is the resulting number of cells for the first few hours:

After t hours	0	1	2	3	4
No of cells	1	2	4	8	16

Exponential sequences

Think about a situation – like cell division for example - when something is doubling in number in a sequence

So, starting with a single cell, imagine that it splits into two cells every hour.

Here is the resulting number of cells for the first few hours:

After t hours	0	1	2	3	4	5
No of cells	1	2	4	8	16	

Exponential sequences

Think about a situation – like cell division for example - when something is doubling in number in a sequence

So, starting with a single cell, imagine that it splits into two cells every hour.

Here is the resulting number of cells for the first few hours:

After t hours	0	1	2	3	4	5	…	10
No of cells	1	2	4	8	16	32		

Exponential sequences

Think about a situation – like cell division for example - when something is doubling in number in a sequence

So, starting with a single cell, imagine that it splits into two cells every hour.

Here is the resulting number of cells for the first few hours:

After t hours	0	1	2	3	4	5	…	10	…	24
No of cells	1	2	4	8	16	32		1024		

1×2

$1 \times 2 \times 2$

$1 \times 2 \times 2 \times 2$

1×2^4

1×2^5

1×2^{10}

Exponential sequences

Think about a situation – like cell division for example - when something is doubling in number in a sequence

So, starting with a single cell, imagine that it splits into two cells every hour.

Here is the resulting number of cells for the first few hours:

After t hours	0	1	2	3	4	5	…	10	…	24	…	t
No of cells	1	2	4	8	16	32		1024		1.68×10^7		1×2^t

1×2^5

1×2^{10}

1×2^{24} (rounded to 3 s.f.)

Exponential equations

If we define t to be the number of hours after starting and N to be the number of cells at time t then we can write down the equation:

$$N = 1 \times 2^t$$

Exponential equations

If we define t to be the number of hours after starting and N to be the number of cells at time t then we can write down the equation:

$$N = 1 \times 2^t$$

This is an example of an exponential relationship. Notice here that unlike linear and quadratic equations, for example, the variable is the power (or exponent)

Exponential equations

If we define t to be the number of hours after starting and N to be the number of cells at time t then we can write down an equation:

$$N = 1 \times 2^t$$

This is an example of an exponential relationship. Notice here that unlike linear and quadratic equations, for example, the variable is the power (or exponent)

Note here that if we had started with 3 cells this would have been a 3 instead of 1. It is the <u>initial value</u>, try putting put t = 0

The 2 here comes from the situation where the cells were <u>doubling</u> every t (hr)

Exponential equations

These are examples of exponential relationships:

$$y = 2^x$$

$$y = 5 \times 3^x$$

$$y = 1.5^x$$

$$y = 0.7^x$$

Exponential equations

These are examples of exponential relationships:

Find the value of y when $x = 3$ for each equation

$$y = 2^x$$

$$y = 5 \times 3^x$$

$$y = 1.5^x$$

$$y = 0.7^x$$

Exponential equations

These are examples of exponential relationships:

Put $x = 3$

$$y = 2^{3} = 8$$

$$y = 5 \times 3^{3} = 135$$

$$y = 1.5^{3} = 3.375$$

$$y = 0.7^{3} = 0.343$$

How are these different from the last in the list?

Exponential equations

Find the values of $y = 2^x$ when x takes the following values:

$x = 0$

$x = -1$

$x = 1.5$

Exponential equations

Find the values of $y = 2^x$ when x takes the following values:

$x = 0$ $\quad y = 2^0 = 1$

Note that <u>any</u> number (but not 0) to the power 0 is 1

$x = -1$

$x = 1.5$

Exponential equations

Find the values of $y = 2^x$ when x takes the following values:

$x = 0$ $\qquad y = 2^0 = 1$

Note that <u>any</u> number (not 0) to the power 0 is 1

$x = -1$ $\qquad y = 2^{-1} = \frac{1}{2}$

Remember that <u>negative powers give reciprocals</u>

$x = 1.5$

Exponential equations

Find the values of $y = 2^x$ when x takes the following values:

$x = 0$ $y = 2^0 = 1$

Note that any number (not 0) to the power 0 is 1

$x = -1$ $y = 2^{-1} = \frac{1}{2}$

Remember that negative powers give reciprocals

$x = 1.5$ $y = 2^{1.5} = 2.828 \ldots$

Note that x can take any value

Exponential equations

Find the values of $y = 2^x$ when x takes the following values:

$x = 0$ $\quad$ $y = 2^0 = 1$

Note that any number (not 0) to the power 0 is 1

$x = -1$ $\quad$ $y = 2^{-1} = \frac{1}{2}$

Remember that negative powers give reciprocals

$x = 1.5$ $\quad$ $y = 2^{1.5} = 2.828 \ldots$

Note that x can take any value

This is an irrational number – why?

Exponential equations

Find the values of $y = 2^x$ when x takes the following values:

$x = 0$

$y = 2^0 = 1$

Note that any number (not 0) to the power 0 is 1

$x = -1$

$y = 2^{-1} = \frac{1}{2}$

Remember that negative powers give reciprocals

$x = 1.5$

$y = 2^{1.5} = 2.828 \ldots$

Note that x doesn't have to be a whole number

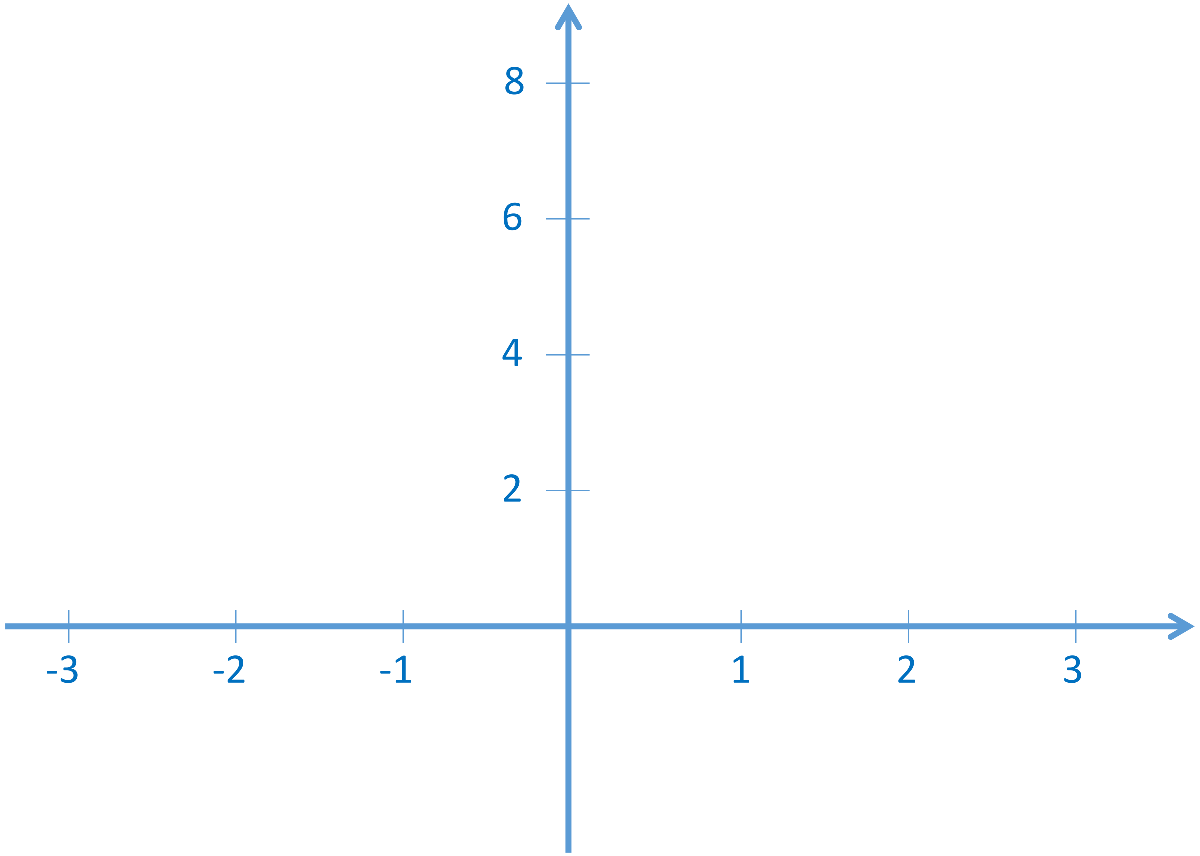
This is an irrational number – why?

$$2^{1.5} = 2^{\frac{3}{2}} = \left(2^{\frac{1}{2}}\right)^3 = \left(\sqrt{2}\right)^3 = 2\sqrt{2}$$

Graphs of Exponential equations

Let’s plot some values for the graph of $y = 2^x$

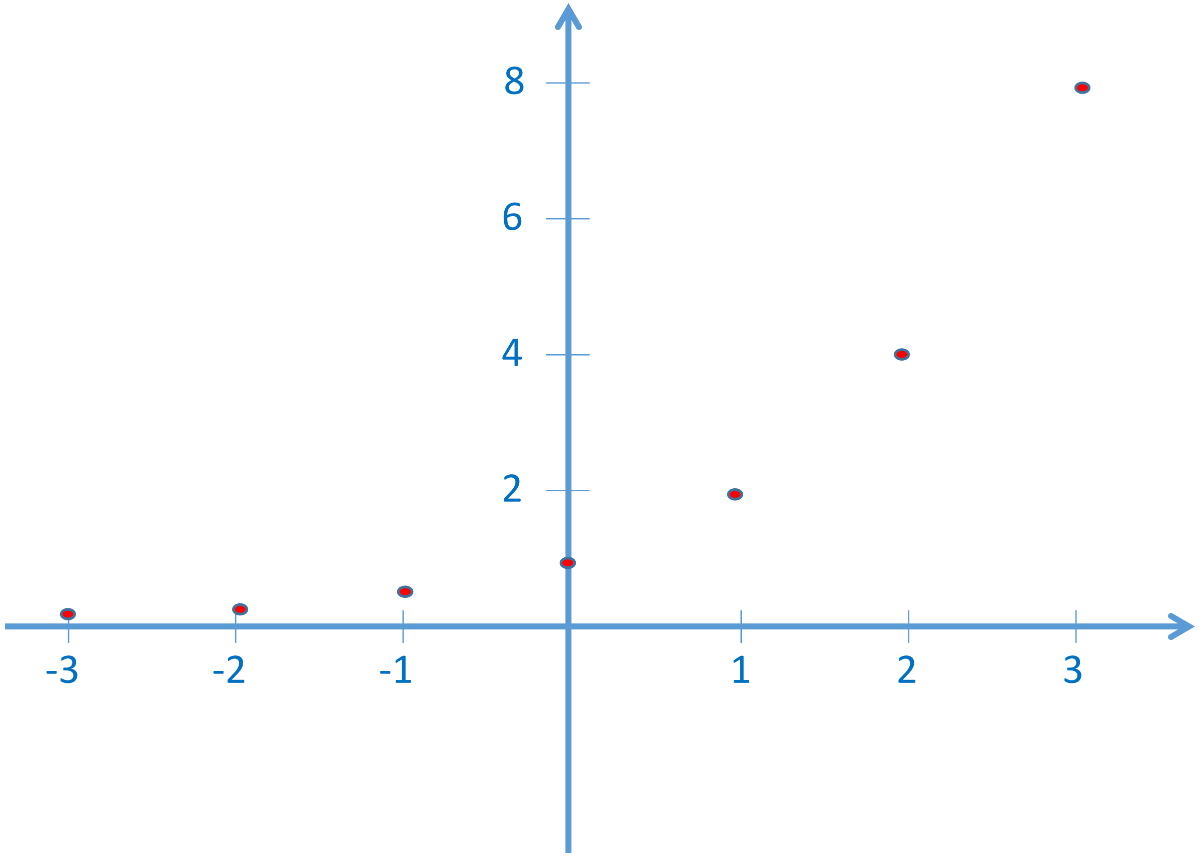
x	2^x
3	
2	
1	
0	
-1	
-2	
-3	



Graphs of Exponential equations

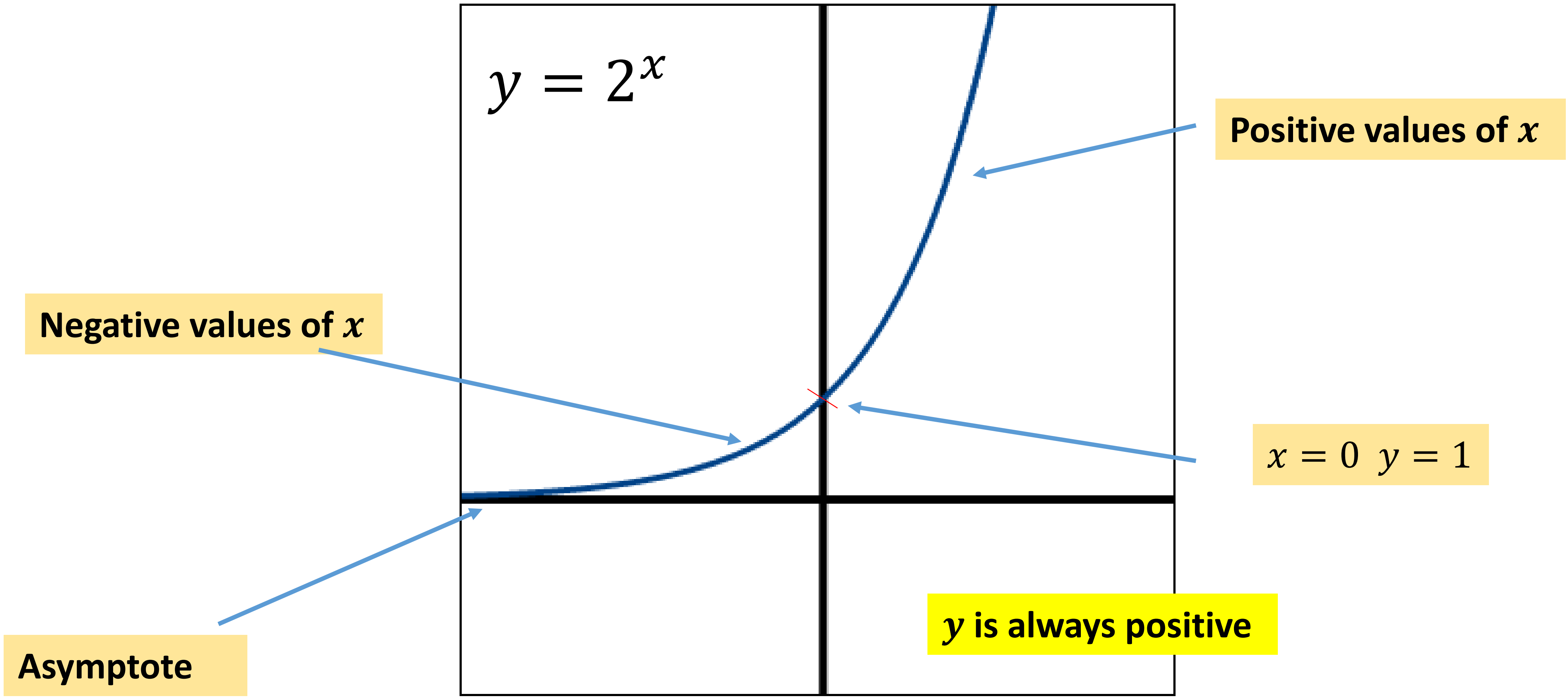
Let's plot some values for the graph of $y = 2^x$

x	2^x
3	8
2	4
1	2
0	1
-1	½ = 0.5
-2	¼ = 0.25
-3	1/8 =0.125



What features do you notice?

Exponential graphs

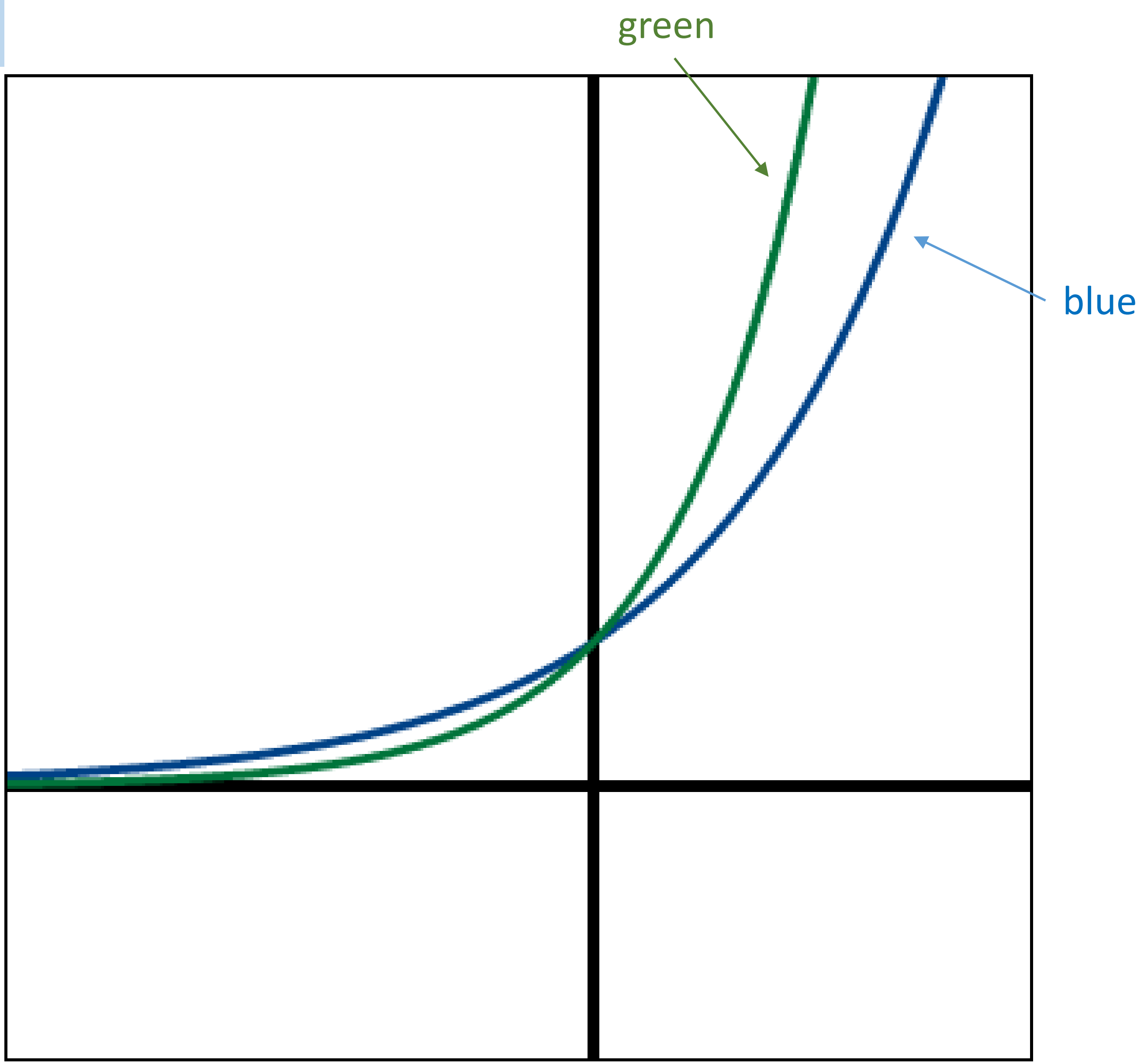


Exponential graphs

$$y = 2^x$$

$$y = 3^x$$

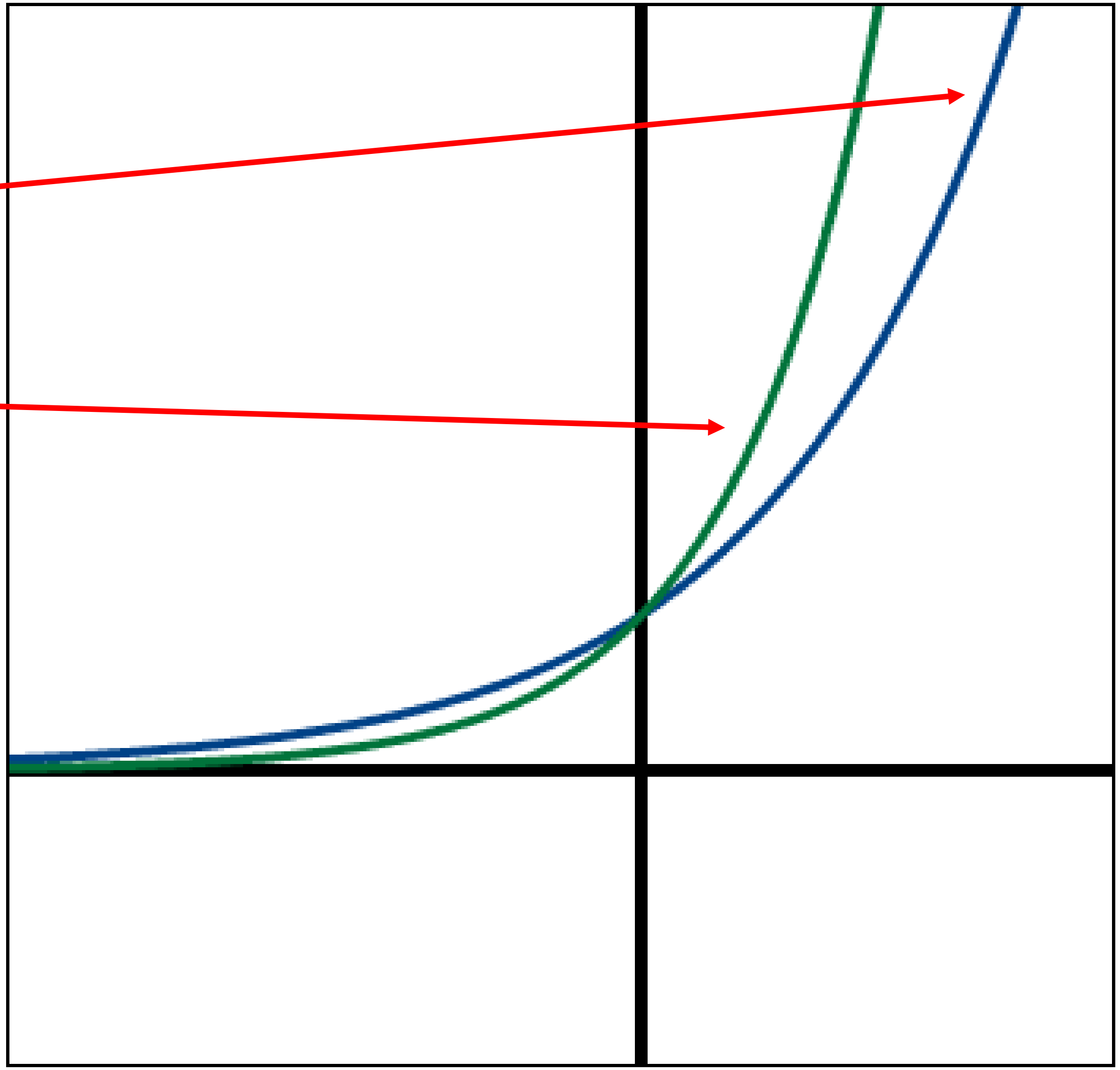
Which is which?



Exponential graphs

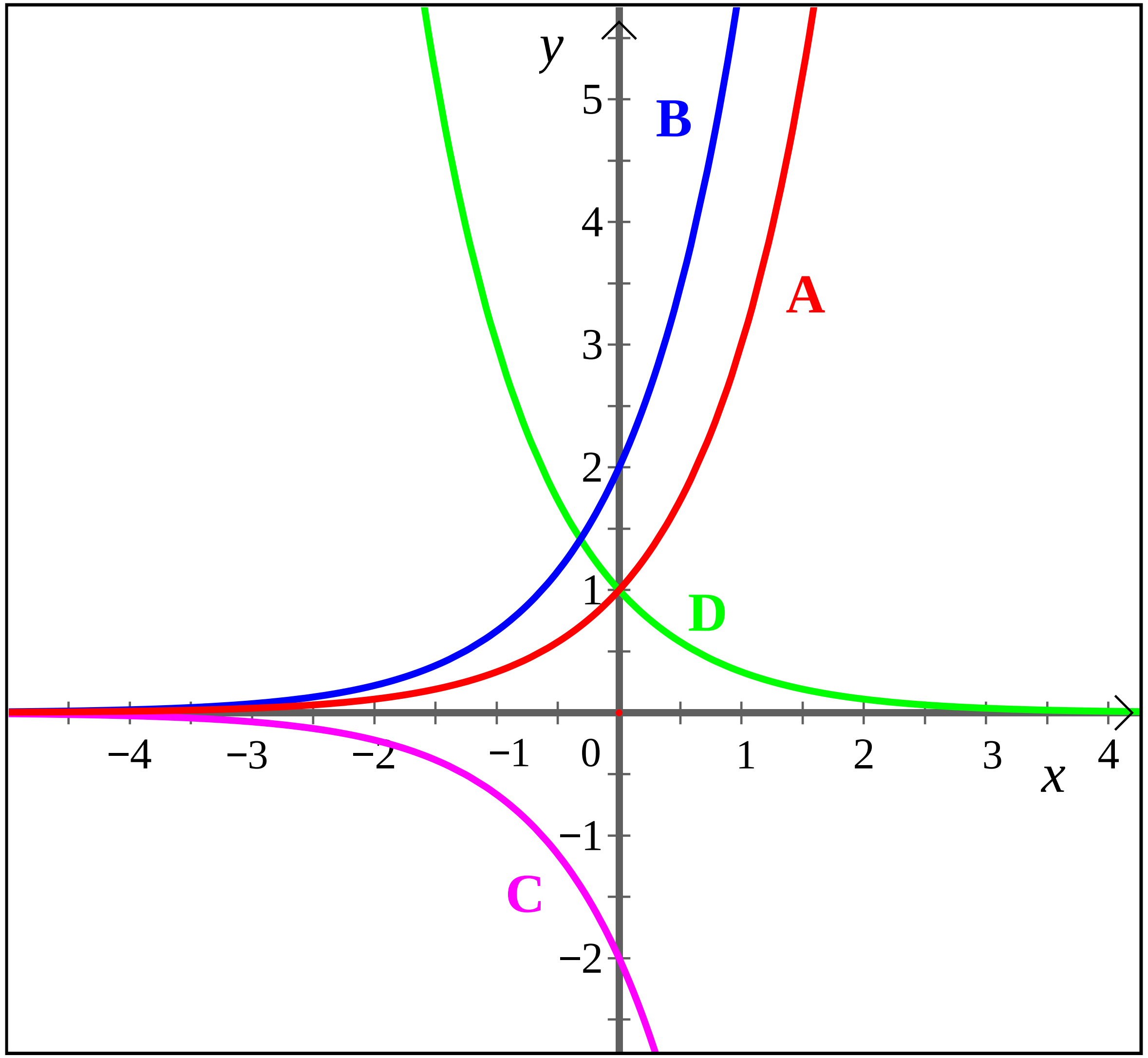
$$y = 2^x$$

$$y = 3^x$$



Which is which?

Exponential graphs



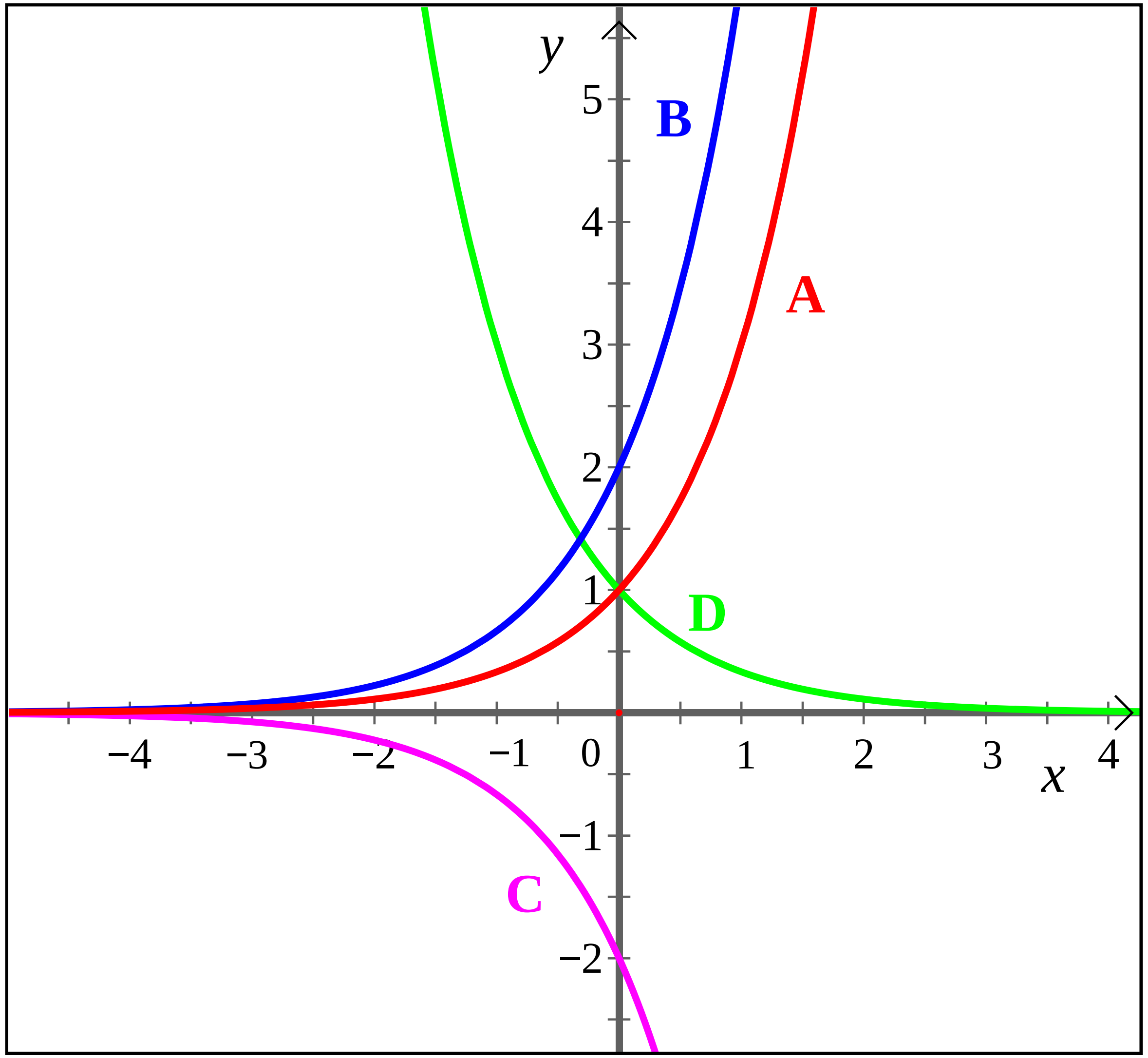
Match the graph to its equation

1. $y = 2 \times 3^{x}$

2. $y = 3^{-x}$

3. $y = 3^{x}$

Exponential graphs

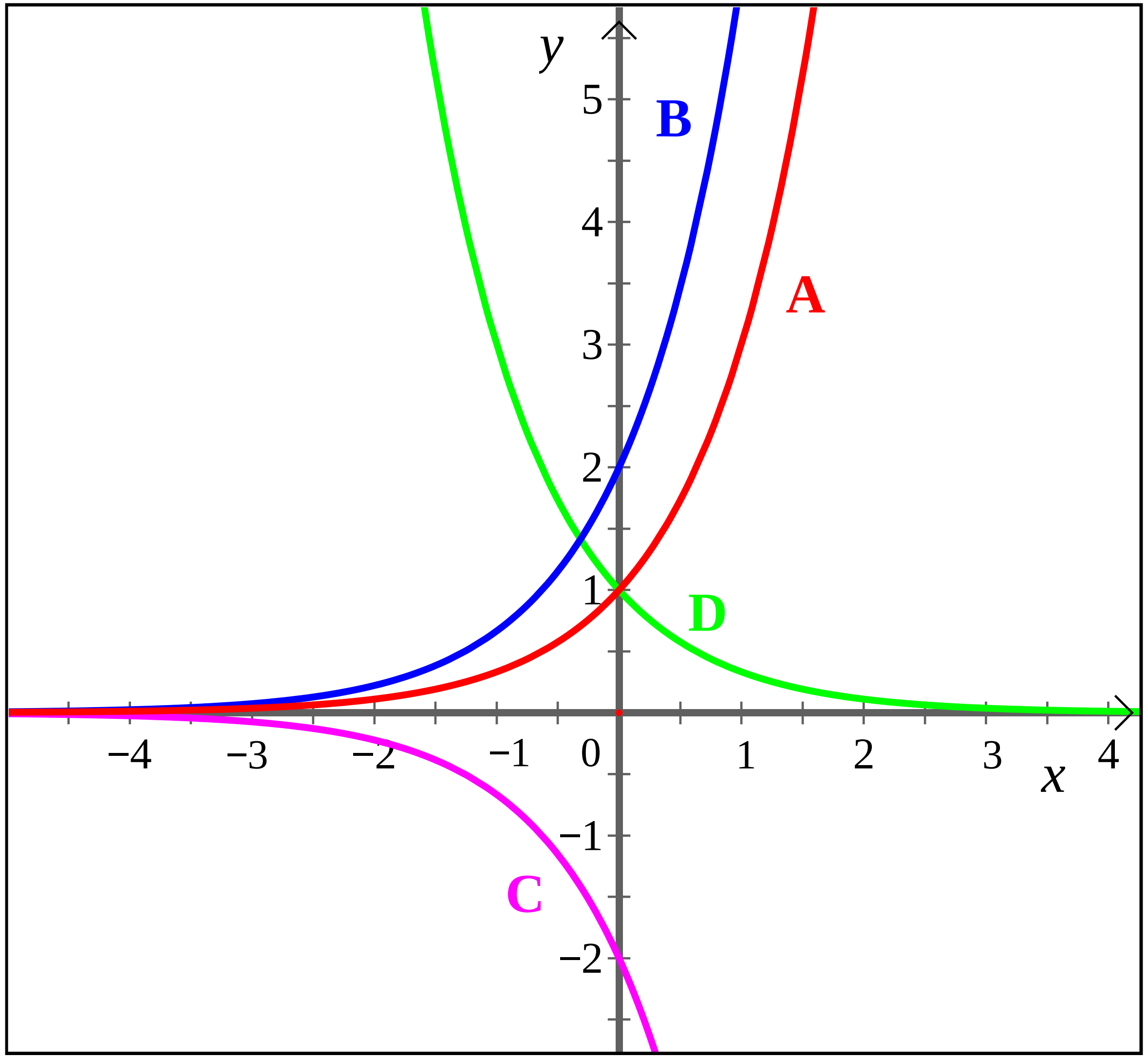


Match the graph to its equation

1. $y = 2 \times 3^x$ **B**
2. $y = 3^{-x}$ **D**
3. $y = 3^x$ **A**

Note that $3^{-x} = \left(\frac{1}{3}\right)^x$

Exponential graphs

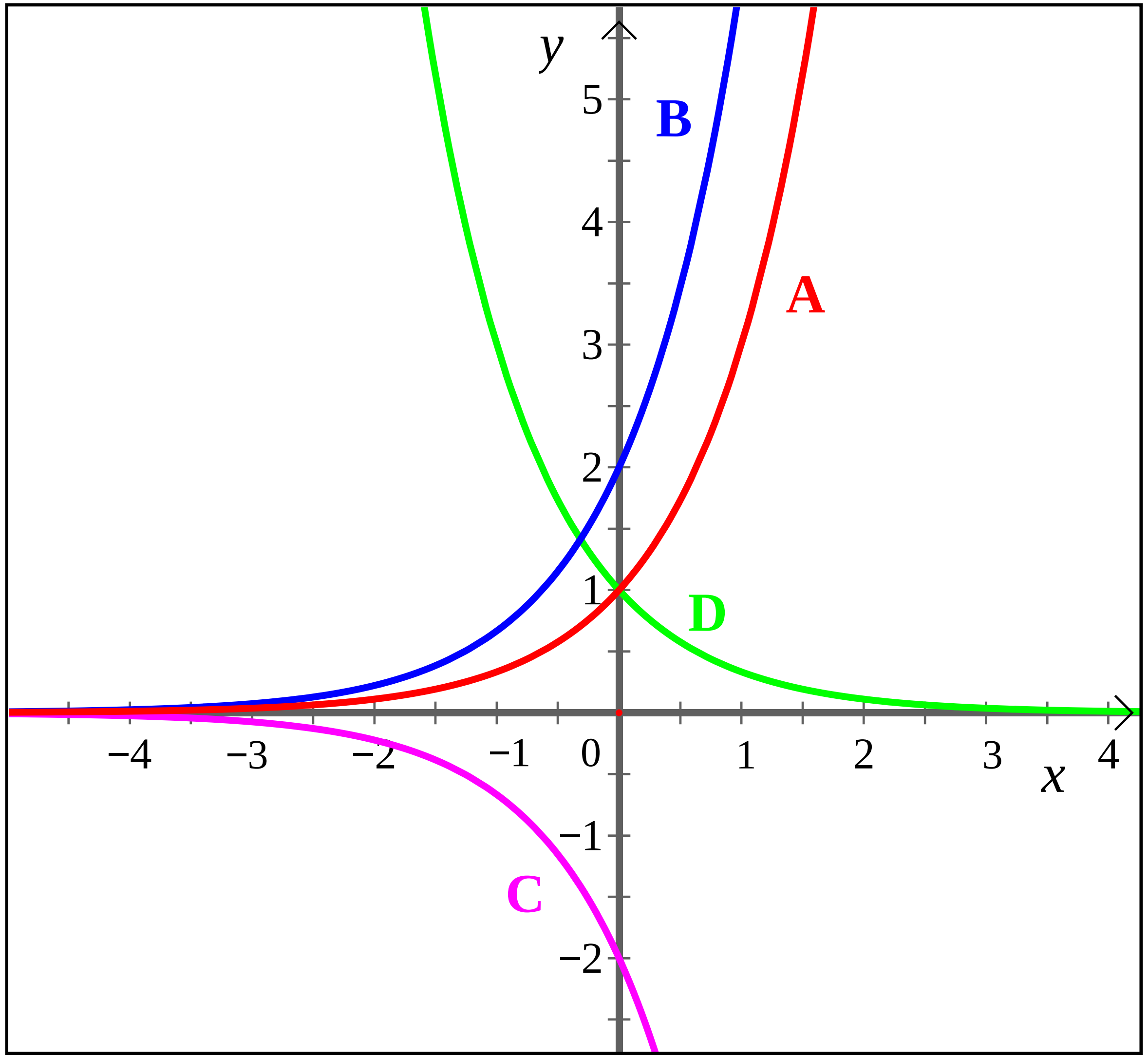


Match the graph to its equation

1. $y = 2 \times 3^{x}$ B
2. $y = 3^{-x}$ D
3. $y = 3^{x}$ A

What is the equation for graph C?

Exponential graphs



Match the graph to its equation

1. $y = 2 \times 3^x$ B

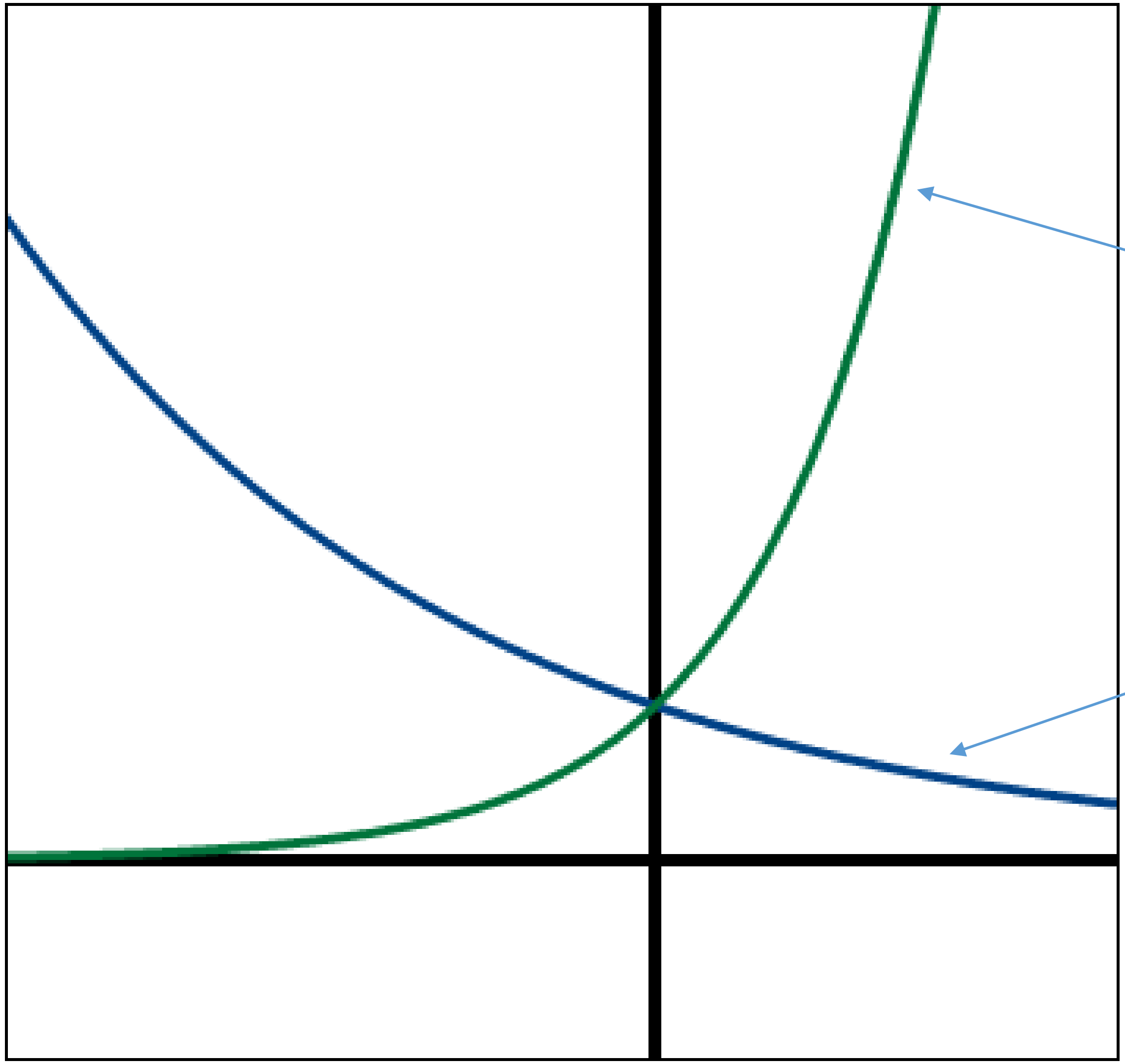
2. $y = 3^{-x}$ D

3. $y = 3^x$ A

C: $y = -2 \times 3^x$

The reflection of B in the *x*-axis

Exponential graphs



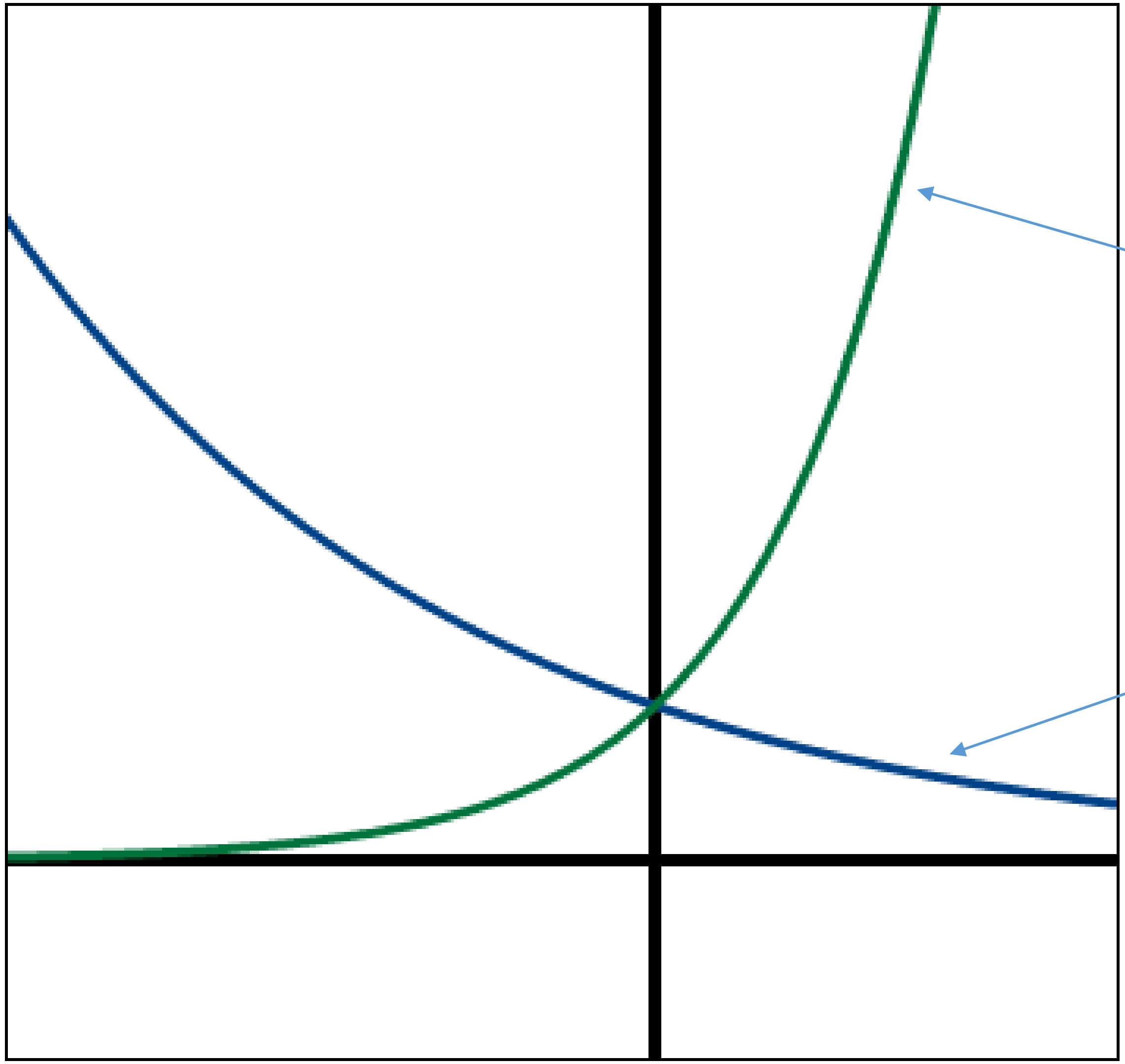
Growth

$y = 2.7^x$

Decay

$y = 0.5^x$

Exponential graphs

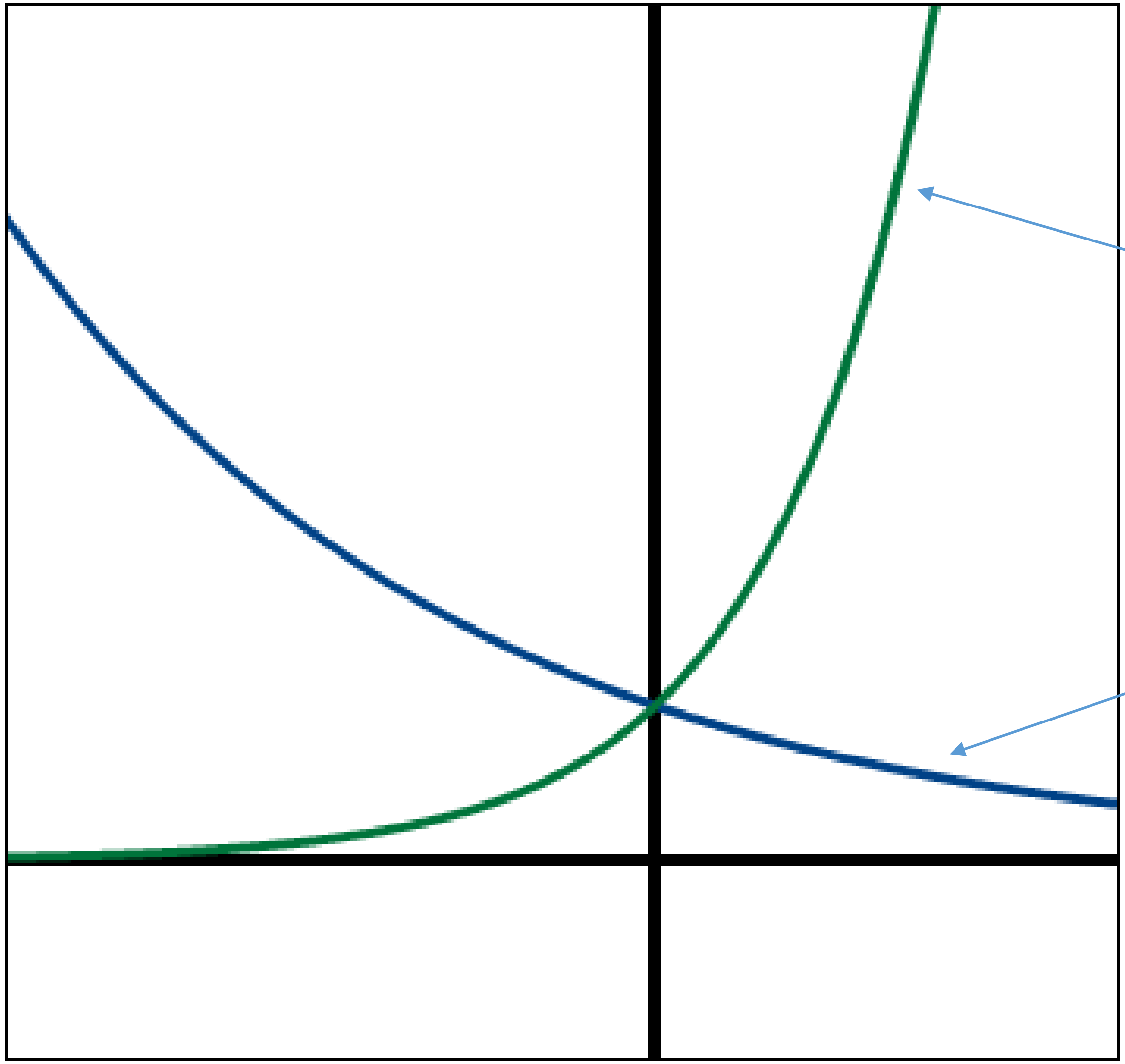


Growth
$y = 2.7^x$

Decay
$y = 0.5^x$

This is the same as $y = 2^{-x}$
Can you see why?

Exponential graphs



Growth

$y = 2.7^x$

Decay

$y = 0.5^x$

This is the same as $y = 2^{-x}$

$y = 0.5^x = \left(\frac{1}{2}\right)^x$

$= (2^{-1})^x = 2^{-x}$

Exponential model

$$y = ab^x$$

Initial value

Scale factor

Assuming $x > 0$

The model gives **growth** when $\boldsymbol{b}$ is?

Exponential model

$$y = ab^{x}$$

Initial value

Scale factor

Assuming $x > 0$

The model gives **growth** when $\boldsymbol{b > 1}$

The model gives **decay** when $\boldsymbol{b}$ is?

Exponential model

$$y = ab^x$$

Initial value

Scale factor

For $x > 0$, the model gives **growth** when $\boldsymbol{b > 1}$

For $x > 0$, the model gives **decay** when $\boldsymbol{0 < b < 1}$

What happens if $b = 1$?

Exponential model

$$y = ab^x$$

Initial value

Scale factor

For $x > 0$, the model gives **growth** when $\boldsymbol{b > 1}$

For $x > 0$, the model gives **decay** when $\boldsymbol{0 < b < 1}$

What happens if $b = 1$? $y = a$ (constant)

Exponential model

$$y = ab^x$$

Initial value

Scale factor

For $x > 0$, the model gives **growth** when $\boldsymbol{b > 1}$

For $x > 0$, the model gives **decay** when $\boldsymbol{0 < b < 1}$

How about when $b < 0$?

Important!!

Although it is possible to find values of, for example, $(-2)^x$ when x is an **integer**, it is meaningless (in real numbers) to write down something like $y = (-2)^x$ for values of x which are not whole numbers.

See what happens when you put $(-2)^{1.5}$ on your calculator…….

Where do exponential relationships crop up?

Exponential applications

"Rents increased annually by 10% ":

$$A = A_0 \times 1.1^t$$

An example of 'growth'

Why 1.1?

What do you think A, A_0 , and t represent?

Exponential applications

"Rents increased annually by 10% ":

$$A = A_0 \times 1.1^t$$

An example of 'growth'

1.1 is the growth factor and represents 110%

A represents the rent (e.g. in £)

A_0 is the starting rent (when $t = 0$)

t represents time (in years here)

What is the value of A when $t = 0$?

Exponential applications

"Rents increased annually by 10% ":

$$A = A_0 \times 1.1^t$$

An example of 'growth'

1.1 is the growth factor and represents 110%

A represents the rent (e.g. in £)

A_0 is the starting rent (when $t = 0$)

t represents time (in years here)

What is the value of A when $t = 0$?

$$A = A_0 \times 1.1^0 = A_0$$

Exponential applications

"Rents increased annually by 10%":

$$A = A_0 \times 1.1^t$$

Example

If the initial rent was £4000 p.a.,

after 10 years it would have risen to

$$A = 4000 \times 1.1^{10} \approx £10\,375$$

Exponential applications

"Rents increased annually by 10%":

$$A = A_0 \times 1.1^t$$

Example

If the initial rent was £4000 p.a.,

after 10 years it would have risen to

$$A = 4000 \times 1.1^{10} \approx £10\,375$$

What would the rent be after 20 years if it continued to increase at the same rate?

Exponential applications

"Rents increased annually by 10%":

$$A = A_0 \times 1.1^t$$

Example

If the initial rent was £4000 p.a.,

after 10 years it would have risen to

$$A = 4000 \times 1.1^{10} \approx £10\,375$$

What would the rent be after 20 years if it continued to increase at the same rate?

$$A = 4000 \times 1.1^{20} \approx £26\,910$$

Exponential applications

"The value of my car depreciated by 15% a year":

$$V = V_0 \times 0.85^t$$

An example of 'decay'

Why 0.85 ?

Exponential applications

"The value of my car depreciated by 15% a year":

$$V = V_0 \times 0.85^t$$

An example of 'decay'

0.85 is the growth factor and represents 85%

If the initial value of my car was £12 000 what would its value be after 5 years according to this model?

Exponential applications

"The value of my car depreciated by 15% a year":

$$V = V_0 \times 0.85^t$$

If the initial value was £12000 what would its value be after 5 years according to this model?

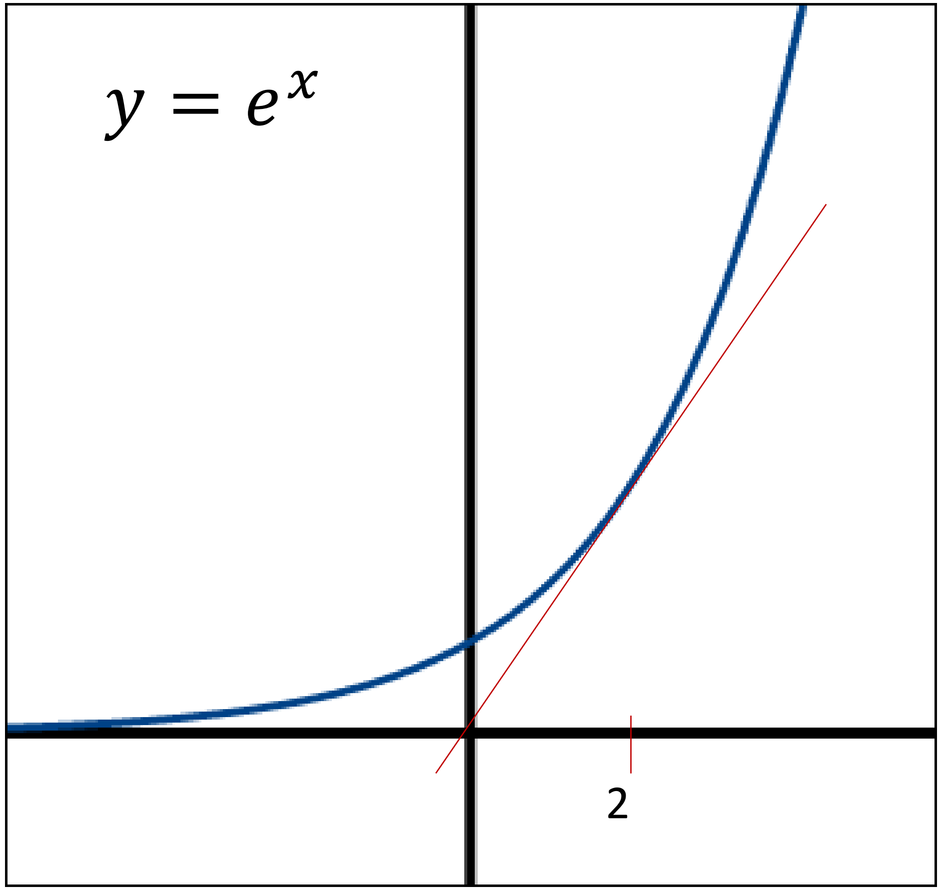
$V = 12000 \times 0.85^5 = 5324.46 \ldots = £5300$ (to 2 s.f.)

Why round to 2 s.f.?

A special exponential

The exponential function

$$y = ae^x$$



Euler's ("*oiler*") constant is a very important irrational number:

2.7182818...

Its special property is that the gradient of the graph of $y = e^x$ has the same value as y for any value of x

So, for example, when $x = 2,\ y = 7.389\ldots$ and so is the gradient of the graph at $x = 2$

The exponential function

Using your calculator

You will need to locate the e^x key. On most calculators it is the second function (SHIFT) of the 'ln' key.

To get e^2, for example, just key: SHIFT ln 2 =

You should get 7.389…

What is the value of e^{-1}?

The exponential function

Using your calculator

You will need to locate the e^x key. On most calculators it is the second function (SHIFT) of the 'ln' key.

To get e^2, for example, just key: SHIFT ln 2 =

You should get 7.389…

The value of $e^{-1} = 0.367\ \ldots$

The exponential function

This exponential equation crops up in many naturally occurring situations such as radioactive decay which is usually given as

$$N = N_0 e^{-\lambda t}$$

where N represents the number of radioactive nuclei present at time t seconds and λ is a (positive) constant called the decay constant.

“lambda”

Can you see why this will represent <u>decay</u> even though e is positive?

Logs

Logs – what are they?

You can read this as:
"log to the base 2 of 8"

$$\log_2 8 = 3$$

Means exactly the same thing as

$$8 = 2^3$$

'log' is short for *logarithm*

Logs

Examples

$$\log_3 9 = 2 \quad \text{means} \quad 9 = 3^2$$

$$\log_5 \frac{1}{5} = -1 \quad \text{means} \quad \frac{1}{5} = 5^{-1}$$

(without calculators)
What is the value of each of these?

$$\log_{10} 100 =$$

$$\log_2 16 =$$

$$\log_2 \frac{1}{4} =$$

$$\log_{10} 0.001 =$$

Logs

Examples

$\log_3 9 = 2$ means $9 = 3^2$

$\log_5 \frac{1}{5} = -1$ means $\frac{1}{5} = 5^{-1}$

What is the value of these?

$\log_{10} 100 = 2$

$\log_2 16 = \log_2 2^4 = 4$

$\log_2 \frac{1}{4} = \log_2 2^{-2} = -2$

$\log_{10} 0.001 = \log_{10} 10^{-3} = -3$

Logs

You can use your calculator to find logs with base 10 or base e (some calculators will give you logs to any base)

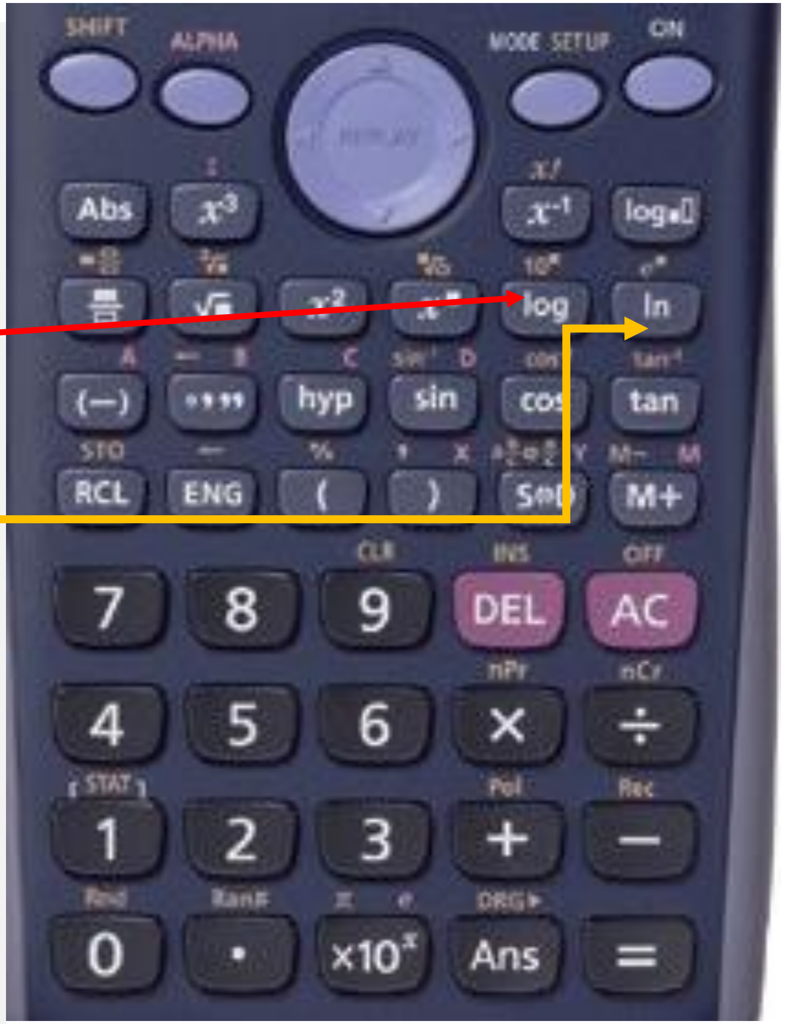
Logs to the base 10 are labelled 'log'

Logs to the base e are labelled 'ln' *(you can still say "log" although some say "lun")*

For example $\log_{10} 25 = 1.3979 \ldots$

(which means that $10^{1.3979\ldots} = 25$)

We normally omit the base when we are working with base 10, so $\log_{10} 25$ would be written simply as $\log 25$



Log to the base e (ln) is called the 'natural log'

Logs

Find, using your calculator, to 3 s.f.:

$\log 357 =$

$\log 0.3 =$

$\ln 12 =$

$\ln e =$

$\ln 1 =$

What happens when you try to find $\ln 0$ or $\log 0$?

Logs

Find, to 3 s.f.:

$\log 357 = 2.55$

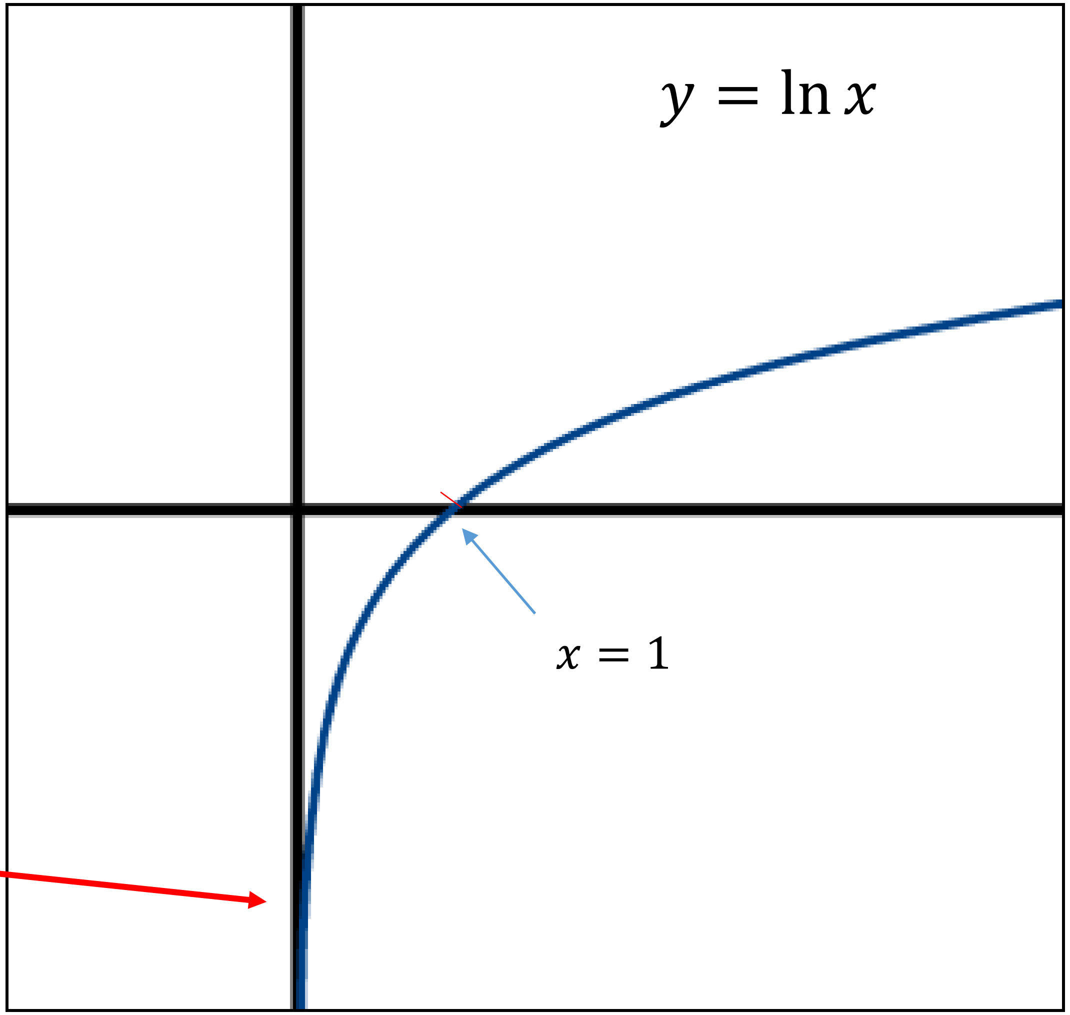
$\log 0.3 = -.523$

$\ln 12 = 2.48$

$\ln e = 1$

$\ln 1 = 0$

What happens when you try to find $\ln 0$ or $\log 0$?



Logs

Two more calculations to try:

1. $\frac{\log 23}{\log 15}$

2. $\frac{\ln 23}{\ln 15}$

Notice anything?

Logs

Two more calculations to try:

1. $\dfrac{\log 23}{\log 15} = 1.157\ldots$

2. $\dfrac{\ln 23}{\ln 15} = 1.157\ldots$

This result will be relevant when we solve equations.

Because logs are really just powers of the base numbers,

remember $8 = 2^3$ means $\log_2 8 = 3$

the rules for manipulating logs are similar to those for indices:

Examples:

$\log_2 3 + \log_2 5 = \log_2(3 \times 5) = \log_2 15$ (remember $a^m \times a^n = a^{m+n}$)

$\log 20 - \log 5 = \log\left(\frac{20}{5}\right) = \log 4$ (remember $a^m \div a^n = a^{m-n}$)

Because logs are really just powers of the base numbers,

remember $8 = 2^3$ means $\log_2 8 = 3$

the rules for manipulating logs are similar to those for indices:

Examples:

$\log_2 3 + \log_2 5 = \log_2(3 \times 5) = \log_2 15$ (remember $a^m \times a^n = a^{m+n}$)

$\log 20 - \log 5 = \log\left(\frac{20}{5}\right) = \log 4$ (remember $a^m \div a^n = a^{m-n}$)

$\ln 3^4 = 4 \ln 3$ (remember $(a^m)^n = a^{mn}$)

$\log 10 = 1$ (remember $a^1 = a$)

$\log 1 = 0$ (remember $a^0 = 1$)

Here are the rules for manipulating logs – you will find them all on page 6 in the MST124 Handbook:

$y = \log_b x$ is equivalent to $x = b^y$

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b x + \log_b y = \log_b (x\,y)$$

$$\log_b x - \log_b y = \log_b \left(\frac{x}{y}\right)$$

$$r \log_b x = \log_b x^r$$

Logs

Simplify the following expressions:

a) $\log 5 + \log 2$

b) $\log 2 + \log 6 - \log 4$

c) $\ln 32 - 3 \ln 2$

$y = \log_b x$ is equivalent to $x = b^y$

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b x + \log_b y = \log_b (x\,y)$$

$$\log_b x - \log_b y = \log_b \left(\frac{x}{y}\right)$$

$$r \log_b x = \log_b x^r$$

Logs

Simplify the following expressions:

a) $\log 5 + \log 2 \; = \log(5 \times 2) = \log 10 = 1$

b) $\log 2 + \log 6 - \log 4 \; = \log\left(\frac{2\times 6}{4}\right) = \log 3$

c) $\ln 32 - 3\ln 2 \; = \ln 2^5 - 3\ln 2 = 5\ln 2 - 3\ln 2 = 2\ln 2$

$y = \log_b x$ is equivalent to $x = b^y$

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b x + \log_b y = \log_b (x\, y)$$

$$\log_b x - \log_b y = \log_b \left(\frac{x}{y}\right)$$

$$r \log_b x = \log_b x^r$$

Logs

Simplify the following expressions:

a) $\log 5 + \log 2 \; = \log 10 = 1$

b) $\log 2 + \log 6 - \log 4 \; = \log\left(\frac{2\times 6}{4}\right) = \log 3$

c) $\ln 32 - 3\ln 2 \; = 5\ln 2 - 3\ln 2 = 2\ln 2$

d) $\log_3 4 + \log_3 6 - 3\log_3 2 =$

e) $\log(x+1) + \log(x+2) - \log(x^2-4) =$

f) $\ln(e^3) =$

$y = \log_b x$ is equivalent to $x = b^y$

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b x + \log_b y = \log_b(x\,y)$$

$$\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$$

$$r\log_b x = \log_b x^r$$

Logs

Simplify the following expressions:

a) $\log 5 + \log 2 \; = \log 10 = 1$

b) $\log 2 + \log 6 - \log 4 \; = \log\left(\frac{2\times 6}{4}\right) = \log 3$

c) $\ln 32 - 3\ln 2 \; = 5\ln 2 - 3\ln 2 = 2\ln 2$

d) $\log_3 4 + \log_3 6 - 3\log_3 2 = \log_3\left(\frac{4\times 6}{2^3}\right) = \log_3 3 = 1$

e) $\log(x+1) + \log(x+2) - \log(x^2-4) = \log\frac{(x+1)(x+2)}{(x-2)(x+2)} = \log\left(\frac{x+1}{x-2}\right)$

f) $\ln(e^3) = 3\ln e = 3$

$y = \log_b x$ is equivalent to $x = b^y$

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b x + \log_b y = \log_b(x\,y)$$

$$\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$$

$$r\log_b x = \log_b x^r$$

Important!!

Watch out that you <u>don't fall into the trap</u> of thinking that $\log(a+b)$ is $\log a + \log b$ because 'log' is an operator and <u>not</u> something you can multiply by.

To convince yourself try comparing $\log 5$ with $\log 2 + \log 3$

Solving exponential equations

Suppose we want to solve the exponential equation

$$2^x = 12$$

Suppose we want to solve the exponential equation

$$2^x = 12$$

Since $2^{3} = 8$ (too small) and $2^{4} = 16$ (too big), then x is not going to be a whole number and its value is somewhere <u>between</u> 3 and 4

So, how do we get x out of its position as a power of 2?

Solve the exponential equation

$$2^x = 12$$

Because we have an equation, we can 'take logs' of both sides:

$$\ln 2^x = \ln 12$$

We can use any base log here but since we will need to use a calculator choose either 'ln' or 'log'

Using the rule
$r \log_b x = \log_b x^r$

$$x \ln 2 = \ln 12$$

$$x = \frac{\ln 12}{\ln 2} = \mathbf{3.58} \text{ (to 3 s.f.)}$$

Check that $\frac{\log 12}{\log 2}$ gives the same answer

Example

Solve: $1.1^x = 2$

Take logs: $\ln 1.1^x = \ln 2$

So $x \ln 1.1 = \ln 2$

Therefore $x = \dfrac{\ln 2}{\ln 1.1} = 7.27\ldots = 7.3$ (to 1 d.p.)

check: $1.1^{7.27\ldots} = 1.999\ldots$

<u>Example</u>

Solve: $1.1^x = 2$

Take logs: $\ln 1.1^x = \ln 2$

So $x \ln 1.1 = \ln 2$

Therefore $x = \dfrac{\ln 2}{\ln 1.1} = 7.27\ldots = 7.3$ (to 1 d.p.) [check: $1.1^{7.27\ldots} = 1.999\ldots$]

Solve for x – give your answers to 2 d.p.:

a) $17^x = 56.9$

b) $5^{x+2} = 0.04$

c) $2.4 \times 0.85^x = 36$

a) $17^x = 56.9$

Take logs: $x \ln 17 = \ln 56.9$

so $x = \dfrac{\ln 56.9}{\ln 17} = 1.426\ldots = 1.43$ (to 2 d.p.)

a) $17^x = 56.9$

Take logs: $x \ln 17 = \ln 56.9$

so $x = \dfrac{\ln 56.9}{\ln 17} = 1.426\ldots = 1.43$ (to 2 d.p.)

b) $5^{x+2} = 0.04$

Take logs: $(x+2)\ln 5 = \ln 0.04$

so $x + 2 = \dfrac{\ln 0.04}{\ln 5} = -2,$ so $x = -4$

Logs – solving equations

a) $17^x = 56.9$

Take logs: $x \ln 17 = \ln 56.9$

so $x = \dfrac{\ln 56.9}{\ln 17} = 1.426 \ldots = 1.43$ (to 2 d.p.)

b) $5^{x+2} = 0.04$

Take logs: $(x+2)\ln 5 = \ln 0.04$

so $x + 2 = \dfrac{\ln 0.04}{\ln 5} = -2,$ so $x = -4$

c) $2.4 \times 0.85^x = 36$

Divide by 2.4: $0.85^x = 15$

Take logs: $x \ln 0.85 = \ln 15$

so $x = \dfrac{\ln 15}{\ln 0.85} = -16.662 \ldots = -16.66$ (to 2 d.p.)

What next

This session was the last of the revision tutorials

If you have found these sessions reasonably ok – perhaps that you're a bit rusty on some things but feel that most of it was within your grasp then you should be fine with MST124 provided you put the time in.

Some of what we have covered is revisited in Units 1 - 4 of MST124 so you should make a start on the units as soon as you can if you haven't yet done so.

The R&R forum will close at the end of the month and you should now post your questions and comments on the pre-start forum on the MST124 module website.

As you study MST124 we strongly recommend:

- At the end of studying each unit you do the practice quiz on-line
- Make use of the Exercise Booklets (they are on the website) – there is one for each unit.
- Try to attend tutorials – they can save you weeks of (lonely) study time.
- Ask for help, keep in touch with your tutor and read the feedback you get on your assignments carefully.

Good luck and enjoy the course…