TMA02

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Question 1

Use a table of signs to solve the inequality

$$\frac{2x-5}{4x+10} \leq 0.$$

Give your answer in interval notation.

Solution:
If the numerator $2x - 5 = 0$ then $x = \frac{5}{2}$, and if the denominator $4x + 10 = 0$ then $x = -\frac{5}{5}$. Using these values of x to construct a table of signs:

x	$\left(-\infty, -\frac{5}{2}\right)$	$-\frac{5}{2}$	$\left(-\frac{5}{2}, \frac{5}{2}\right)$	$\frac{5}{2}$	$\left(\frac{5}{2}, \infty\right)$
$2x - 5$	-	-	-	0	$+$
$4x + 10$	-	0	$+$	$+$	$+$
$\frac{2x-5}{4x+10}$	$+$	$*$	-	0	$+$

where $+$, -, 0, and $*$ indicate that the expressions are positive, negative, zero, or undefined for each interval of x, respectively.

The solution set of $\frac{2x-5}{4x+10} \leq 0$ is therefore the interval $\left(-\frac{5}{2}, \frac{5}{2}\right]$.

Question 2

a) Show that the values of the constants A and k are 6.01 and 1.06, respectively, to three significant figures.

Solution:
Given that $s(2) = 50$ and $s(5) = 1200$, we can find the value of k by taking out a factor of A:

$$\begin{aligned} \frac{Ae^{5k}}{Ae^{2k}} &= \frac{1200}{50} \\ e^{3k} &= 24 \end{aligned}$$

Taking the natural logarithm of both sides and solving for k gives

$$\begin{aligned} 3k &= \ln 24 \\ k &= \frac{\ln 24}{3} \\ &= 1.059... \\ &= 1.06 \text{ (3 s.f.)} \end{aligned}$$

Substituting 1.059... for k and 5 for t in $s(t) = Ae^{kt}$ and solving for A gives

$$\begin{aligned} Ae^{5\times 1.059...} &= 1200 \\ A &= \frac{1200}{e^{(5\times 1.059...)}} \\ &= 6.009... \\ &= 6.01 \text{ (3 s.f.)} \end{aligned}$$

Therefore, constants A and k are 6.01 and 1.06, respectively (to 3 s.f.).

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b) What is the expected population per hectare after 10 years if no action is taken and the population continues to follow the exponential growth model? Give your answer to the nearest whole number.

Solution:
Substituting 10 for t and the values calculated in the previous question for k and A into the exponential model gives:

$$6.01...e^{(10\times 1.059...)} = 239625.656...$$

So the the expected population per hectare after 10 years is 239626 (to the nearest whole number).

Question 3

a)
i) Explain how the graph of f can be obtained from the graph of y = x 2 by using appropriate translations.

Solution:
The graph of f can be obtained by translating the graph of $y = x^2$ left (towards the negative x direction) by 3 units and down (towards the negative y direction) by 1 unit.

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ii) Write down the image set of the function f , in interval notation.

Solution:
The image set of f is $[-1, \infty)$, as the minimum output value is -1 and there is no greatest output value.

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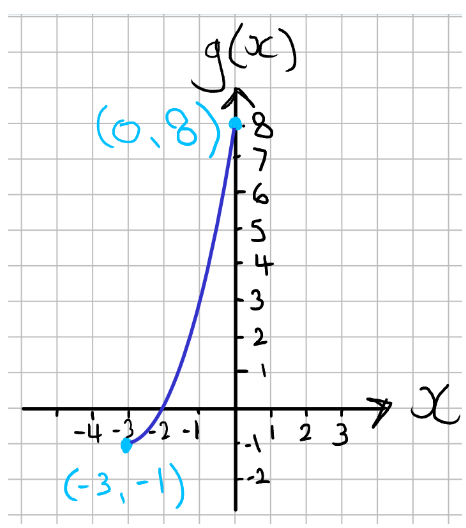
b)
i) Sketch the graph of g, using equal scales on the axes. (You should draw this by hand, rather than using any software.) Mark the coordinates of the endpoints of the graph.

Solution:



The graph of g is shown with its endpoints marked.

ii) Give the image set of g, in interval notation.

Solution:
The image set of g is $[-1, 8]$.

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iii) Show that the inverse function g^{-1} has the rule

$$g^{-1} = -3 + \sqrt{x+1}$$

Solution:
To find the inverse function of g, let y represent the image of x under g, and rearrange the rule of g to make x the subject.

$$\begin{aligned} y &= (x+3)^2 - 1 && (1) \\ y + 1 &= (x+3)^2 \\ \sqrt{y+1} &= x + 3 \\ x &= \pm\sqrt{y+1} - 3 && (2) \end{aligned}$$

Whereas equation (1) expresses y as a function of x, equation (2) expresses x as a function of y, so substituting $g^{-1}(x)$ for y gives either

$$g^{-1}(x) = -3 + \sqrt{x+1}$$

or

$$g^{-1}(x) = -3 - \sqrt{x+1}$$

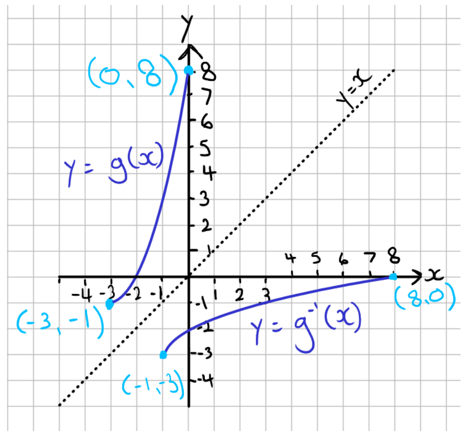
as the inverse of g. As g has domain $[-3, 0]$, the inverse of g is

$$g^{-1}(x) = -3 + \sqrt{x+1}$$

and has domain $[-1, 8]$, and image set $[-3, 0]$.

iv) Add a sketch of $y = g^{-1}(x)$ to the graph that you produced in part b)i). Mark the coordinates of the endpoints of the graph of $g^{-1}(x)$

Solution:



The graphs of $y = g(x)$ and $y = g^{-1}(x)$ with endpoints marked. The dotted line is added to indicate the two graphs are mirror images of each other in the line $y = x$.

Question 4

a) Use the cosine rule to show that the length of side a is 15.19, rounded to two decimal places.

Solution:
The cosine rule can be used to find a side length when the angle opposite that side is known, and the two other lengths are known or can be calculated. The cosine rule is

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Substituting 30cm for length b, 20cm for length c, and 27° for angle A gives

$$\begin{aligned} a^2 &= 30^2 + 20^2 - 2 \times 30 \times 20 \cos 27 \\ &= 230.792... \\ a &= \sqrt{230.792...} \\ &= 15.191... \end{aligned}$$

So the length of side a is 15.19cm, rounded to two decimal places.

b) Without using the cosine rule again, find the remaining angles B and C, giving your answers to the nearest degree.

Solution:
When a side length and an opposite angle are known, the sine rule can be used to find unknown angles if their opposite side lengths are known. The sine rule is

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

To find angle B, I substitute 30cm for length b, 15.191... cm for length a, and 27° for angle A

$$\frac{\sin 27}{15.191...} = \frac{\sin B}{30}$$

$$\sin B = \frac{30 \sin 27}{15.191...}$$

$$= 0.896...$$
$$B = 63.703...°$$

So $B = 64°$ (to the nearest degree) or $B = 180 - 63.703... = 116°$ (to the nearest degree). Knowing the final angle C will help narrow down the correct angle for B.

To find angle C, I substitute 20cm for length c, 15.191... cm for length a, and 27° for angle A

$$\frac{\sin 27}{15.191...} = \frac{\sin C}{20}$$

$$\sin C = \frac{20 \sin 27}{15.191...}$$

$$= 0.597...$$
$$C = 36.703...°$$

So $C = 37°$ (to the nearest degree) or $C = 180 - 36.703... = 143°$ (to the nearest degree).

Given these options for the angles A and B, the only triangle that satisfies the sine rule has angles $A = 27°$, $B = 116°$, $C = 37°$ (all to the nearest degree).

c)
i) Find the area of this triangle, giving your answer in cm^2 to two decimal places.

Solution:
The area of any triangle is given by

$$\frac{1}{2}bc\sin A$$

where b and c are side lengths and A is the angle between them. Substituting 30cm for side length b, 20cm for side length c, and $27°$ for A gives

$$\text{area} = \frac{1}{2}30 \times 20 \sin 27$$

$$= 136.197...$$

So the area of the triangle is 136.20cm^2 (2 d.p.).

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ii) By what factor would the area of the triangle increase if the lengths of side b and side c were doubled, and also the angle A were doubled (so now angle $A = 54°$)? Give your answer to two decimal places.

Solution:
The scale factor for the change in area is given by

$$\text{scale factor} = \frac{\text{area of larger triangle}}{\text{area of smaller triangle}}$$

Substituting the side lengths 30cm, 20cm, and angle $27°$ for the smaller triangle, and the side lengths 60cm, 40cm, and angle $54°$ for the larger triangle gives

$$\text{scale factor} = \frac{0.5 \times 60 \times 40 \sin 54}{0.5 \times 30 \times 20 \sin 27}$$

$$= 7.128...$$

So the area of the triangle with doubled side lengths b and c and doubled angle A, is increased by a factor of 7.13 (2 d.p.) over the original triangle.

Question 5

a) Using the exact values of the tangents of $3\pi/4$ and $\pi/3$, and the angle difference identity for tangent, show that an exact value of $\tan(5\pi/12)$ is $2+\sqrt{3}$.

Solution:
The angle different identity for tangent is

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

Substituting $3\pi/4$ for A and $\pi/3$ for B gives

$$\tan(3\pi/4 - \pi/3) = \frac{\tan(3\pi/4) - \tan(\pi/3)}{1+\tan(3\pi/4)\tan(\pi/3)} \tag{3}$$

The exact value of $\pi/3$ is $\sqrt{3}$, and the exact value of $3\pi/4$ is -1 (because $\tan \pi/4 = 1$ and $3\pi/4 = \pi - \pi/4$ and lies in the second quadrant of the ASTC diagram). Substituting these exact values into equation (3) gives:

$$\begin{aligned}
\tan(3\pi/4 - \pi/3) &= \frac{-1-\sqrt{3}}{1+(-1)\sqrt{3}} \\
&= \frac{-1-\sqrt{3}}{1-\sqrt{3}} \\
&= \frac{-1-\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \\
&= \frac{-2\sqrt{3}-4}{-2} \\
&= 2+\sqrt{3}
\end{aligned}$$

b) Use the double-angle identity for tangent, and the exact value of $\tan(5\pi/6)$, for a different way to find an exact value of $\tan(5\pi/12)$.

Solution:
The double angle identity for tangent is

$$\tan 2\theta = \frac{2\tan\theta}{1 - tan^2\theta}$$

Let $\theta = 5\pi/12$ and let $t = \tan\theta$. The exact value of $\tan 5\pi/6$ is $-\frac{1}{\sqrt{3}}$ (because $\tan\pi/6 = \frac{1}{\sqrt{3}}$ and $5\pi/6 = \pi - \pi/6$ and lies in the second quadrant of the ASTC diagram). Substituting t and the exact value of $\tan 5\pi/6$ into the double angle identity gives

$$-\frac{1}{\sqrt{3}} = \frac{2t}{1-t^2}$$

$$-\frac{1-t^2}{\sqrt{3}} = 2t$$

$$-\frac{1-t^2}{\sqrt{3}} - 2t = 0$$

$$-\frac{1-t^2}{\sqrt{3}} - \frac{2t\sqrt{3}}{\sqrt{3}} = 0$$

$$\frac{t^2 - 2\sqrt{3}t - 1}{\sqrt{3}} = 0 \tag{4}$$

Equation (4) is satisfied when the numerator on the left hand side is equal to 0. Since the numerator is a quadratic expression in the form $ax^2 + bx + c$ (where $a = 1$, $b = -2\sqrt{3}$, and $c = -1$), I use the quadratic formula to find the values of t that satisfy equation (4). The quadratic formula is

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting 1 for a, $-2\sqrt{3}$ for b, and -1 for c gives

$$t = \frac{-(-2\sqrt{3}) \pm \sqrt{(-2\sqrt{3})^2 - 4 \times 1 \times (-1)}}{2} \tag{5}$$

Therefore, $t = \sqrt{3} - 2$ or $t = \sqrt{3} + 2$, but since $5\pi/12$ is acute, $\tan(5\pi/12)$ must be $\sqrt{3} + 2$.

c) Use an appropriate trigonometric identity involving $\tan^2\theta$ to show that an exact value of $\cos(5\pi/12)$ is

$$\frac{1}{2\sqrt{2+\sqrt{3}}}$$

Solution:
The Pythagorean identity $1+\tan^2\theta = \sec^2\theta$ can be rearranged using the fact that $\sec\theta = \frac{1}{\cos\theta}$ into

$$\frac{1}{1+\tan^2\theta} = \cos^2\theta$$

Substituting $\sqrt{3}+2$ for $\tan\theta$ and $5\pi/12$ for θ gives

$$\frac{1}{1+(\sqrt{3}+2)^2} = \cos^2(5\pi/12)$$

$$\frac{1}{8+4\sqrt{3}} = \cos^2(5\pi/12)$$

$$\frac{1}{4(2+\sqrt{3})} = \cos^2(5\pi/12)$$

$$\frac{1}{2\sqrt{2+\sqrt{3}}} = \cos(5\pi/12)$$

Therefore, the exact value of $\cos(5\pi/12)$ is

$$\frac{1}{2\sqrt{2+\sqrt{3}}}$$

Question 6

Solution:

```
(%i1) load(implicit_plot);
(%o1) /usr/share/maxima/5.43.2/share/contrib/implicit_plot.lisp

(%i2) parabola: y = 2·x^2 − 16·x + 28;
```

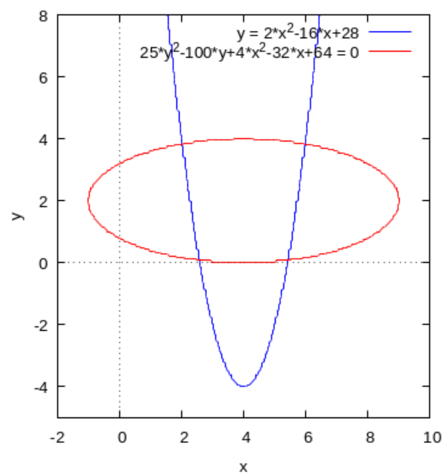
(%o2) $y=2\,x^{2}-16\,x+28$

```
(%i3) ellipse: 4·x^2 + 25·y^2 − 32·x − 100·y + 64 = 0;
```

(%o3) $25\,y^{2}-100\,y+4\,x^{2}-32\,x+64=0$

```
(%i4) wximplicit_plot([parabola, ellipse], [x, −2, 10], [y, −5, 8], same_xy);
```

(%t4)



(%o4)

```
(%i5) solve([parabola, ellipse]);
(%o5) [[x=2.020531400966183,y=3.836592785745328],[x=2.57111801242236
,y=0.08340734759790068],[x=5.428881650380021,y=0.08340734759790068],
[x=5.979468599033816,y=3.836592785745328]]
```

The points of intersection (rounded to 2 d.p.) between the parabola and the ellipse are (2.02, 3.84), (2.57, 0.08), (5.43, 0.08), and (5.98, 3.84).

Question 7

a) Express the velocity **b** of the BlueYonder and the velocity **n** of the Narwhal in component form, both relative to the sea, giving the numerical values in km h^{-1} to one decimal place.

Solution:
If we consider the vector **b** to be the hypotenuse of a right-angled triangle whose other sides are the components a (parallel to unit vector **i**) and b (parallel to unit vector **j**), then we can use the sine rule to calculate the lengths of a and b and hence represent **b** in component form.

As the bearing of **b** is 160°, the angle it forms with a is 160 - 90 = 70°, and the angle it forms with b is therefore 180 - 90 - 70 = 20°. The sine rule is

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Substituting 40km h^{-1} for c, 90° for C (the right angle opposite c), and 20° for A gives

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{40}{\sin 90} = \frac{a}{\sin 20}$$

$$\begin{aligned} a &= 40 \sin 20 \\ &= 13.680 ... \text{km h}^{-1} \end{aligned}$$

To find side length b, I substitute 40km h^{-1} for c, 90° for C, and 70° for B:

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{40}{\sin 90} = \frac{b}{\sin 70}$$

$$\begin{aligned} b &= 40 \sin 70 \\ &= 37.587 ... \text{km h}^{-1} \end{aligned}$$

So velocity **b** in component form is $\mathbf{b} = 13.7\text{km h}^{-1}\mathbf{i} - 37.6\text{km h}^{-1}\mathbf{j}$ (components to 1 d.p.). The **j** component is negative because **b** points towards the south-east.

If we also consider the vector **n** to be the hypotenuse of a right-angled triangle whose other sides are the components a (parallel to unit vector **i**) and b (parallel to unit vector **j**), we can calculate the sides of a and b and thus express n in component form in the same way we did for **b**.

As the bearing of **n** is 70°, the angle it forms with a is 90 - 70 = 20°, and the angle it forms with b is therefore 180 - 90 - 20 = 70°. Substituting 30km h^{-1} for c, 90° for C, and 70° for A gives

$$\begin{aligned} a &= 30 \sin 70 \\ &= 28.190...\text{km h}^{-1} \end{aligned}$$

Substituting 30km h^{-1} for c, 90° for C, and 20° for B gives

$$\begin{aligned} b &= 30 \sin 20 \\ &= 10.260...\text{km h}^{-1} \end{aligned}$$

So velocity **n** in component form is $\mathbf{n} = 28.2\text{km h}^{-1}\mathbf{i} \ + \ 10.3\text{km h}^{-1}\mathbf{j}$ (components to 1 d.p.). Both components are positive because **n** points towards the north-east.

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b) Write the velocity **c** in component form, and hence express the resultant velocities of the BlueYonder and the Narwhal in component form, giving numerical values to one decimal place.

Solution:
As **c** points from west to east and has magnitude 15km h^{-1}, its component form is $\mathbf{c} = 15\text{km h}^{-1}\mathbf{i} \ + \ 0\text{km h}^{-1}\mathbf{j}$.

The resultant velocities of the BlueYonder and Narwhal can be found using vector addition, where the addition of two 2-dimensional vectors **u** and **w** is given by

$$\mathbf{u} + \mathbf{w} = (u_1 + w_1)\mathbf{i} + (u_2 + w_2)\mathbf{j}$$

where u_1, u_2, w_1, and w_2 and components 1 and 2 of **u**, and components 1 and 2 of **w**, respectively. Therefore, the resultant vector of **b** and **c** is

$$\begin{aligned} \mathbf{b} + \mathbf{c} &= (40 \sin 20 + 15) + (-40 \sin 70 + 0) \\ &= 28.680...\text{km h}^{-1}\mathbf{i} - 37.587...\text{km h}^{-1}\mathbf{j} \end{aligned}$$

and the resultant vector of $\mathbf{n}$ and $\mathbf{c}$ is

$$\begin{aligned}\mathbf{n}+\mathbf{c} &= (30\sin 70 + 15) + (30\sin 20 + 0)\\ &= 43.190...\text{km h}^{-1}\mathbf{i} + 10.260...\text{km h}^{-1}\mathbf{j}\end{aligned}$$

So the resultant velocities of the BlueYonder and the Narwhal in component form (to 1 d.p.) are

$$\begin{aligned}\text{BlueYonder velocity} &= 28.7\text{km h}^{-1}\mathbf{i} - 37.6\text{km h}^{-1}\mathbf{j}\\ \text{Narwhal velocity} &= 43.2\text{km h}^{-1}\mathbf{i} + 10.3\text{km h}^{-1}\mathbf{j}\end{aligned}$$

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c) Find the magnitude of the resultant velocity of each vessel. Give magnitudes in km h^{-1} to one decimal place.

Solution:
The magnitude of an n-dimensional vector $\mathbf{u}$ is given by

$$|\mathbf{u}| = \sqrt{{u_1}^2 + {u_2}^2 + ... + {u_n}^2}$$

where u_n is the nth component of $\mathbf{u}$. Substituting the components of the resultant vector of $\mathbf{b}$ and $\mathbf{c}$ gives

$$\begin{aligned}|\mathbf{b}+\mathbf{c}| &= \sqrt{(40\sin 20 + 15)^2 + (-40\sin 70)^2}\\ &= 47.280...\text{km h}^{-1}\end{aligned}$$

Substituting the components of the resultant vector of $\mathbf{n}$ and $\mathbf{c}$ gives

$$\begin{aligned}|\mathbf{n}+\mathbf{c}| &= \sqrt{(30\sin 70 + 15)^2 + (30\sin 20)^2}\\ &= 44.392...\text{km h}^{-1}\end{aligned}$$

So the magnitudes of the resultant velocities for BlueYonder and Narwhal are 47.3km h^{-1} and 44.4km h^{-1}, respectively (1 d.p.)

d) Hence find the actual angle between the courses of the two vessels, taking into account the effect of the current. Give your answer to the nearest degree.

Solution:
The dot product of two n-dimensional vectors $\mathbf{u}$ and $\mathbf{w}$ is

$$\mathbf{u} \cdot \mathbf{w} = (u_1 w_1) + (u_2 w_2) + ... + (u_n w_n)$$

where u_n and w_n are the nth components of $\mathbf{u}$ and $\mathbf{w}$, respectively. Substituting the components of $\mathbf{b} + \mathbf{n}$ into this formula gives

$$\begin{aligned} \mathbf{b} \cdot \mathbf{n} &= (40 \sin 20 + 15)(30 \sin 70 + 15) + (-40 \sin 70)(30 \sin 20) \\ &= 853.073... \end{aligned}$$

An alternative formula for the dot product between two vectors $\mathbf{u}$ and $\mathbf{w}$ is

$$\mathbf{u} \cdot \mathbf{w} = |\mathbf{u}||\mathbf{w}| \cos \theta$$

where $|\mathbf{b}|$ and $|\mathbf{n}|$ are the magnitudes of $\mathbf{u}$ and $\mathbf{w}$, respectively, and θ is the angle between between them. Substituting the dot product of $\mathbf{b}$ and $\mathbf{n}$ and the magnitudes of $\mathbf{b}$ and $\mathbf{n}$ into this formula and rearranging for θ gives

$$\begin{aligned} 853.073... &= 47.280... \times 44.392 \cos \theta \\ \cos \theta &= \frac{853.073...}{2098.905...} \\ \cos \theta &= 0.406... \\ \theta &= 66.018... \end{aligned}$$

Therefore the angle between the courses of the two vessels, taking into account the effect of the current, is 66° to the nearest degree.

Question 8

a) Find the stationary points of f , and give their exact x- and y-coordinates.

Solution:
To find the stationary points of $f(x) = -x^3 - \frac{3}{2}x^2 + 6x + 6$, I find the first derivative of f and find the values of x for which $f'(x) = 0$.
The first derivative of f is

$$f'(x) = -3x^2 - 3x + 6$$

Setting the derivative equal to 0 and solving for x gives

$$-3(x^2 + x - 2) = 0$$
$$-3(x - 1)(x + 2) = 0$$

So $f(x)$ is stationary at points (1, $\frac{19}{2}$) and (-2, -4).

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b) Use the first derivative test to classify the stationary points that you found in part a).

Solution:
I start by constructing a table of signs to indicate whether the first derivative is positive or negative either side of each stationary point.

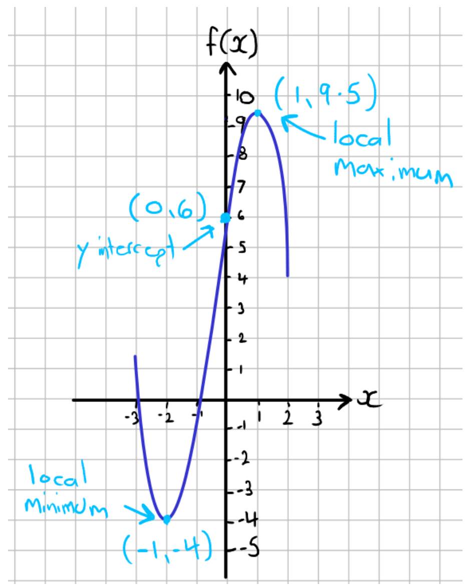
x	$(-\infty, -2)$	-2	$(-2, 1)$	1	$(1, \infty)$
$x + 2$	-	0	+	+	+
$x - 1$	-	-	-	0	+
$-3(x - 1)(x + 2)$	-	0	+	0	-

where +, -, and 0 indicate that the expressions are positive, negative, or zero for each interval of x, respectively.

The point $x = -2$ is a local minimum as the gradient is negative to its left and positive to its right. The point $x = 1$ is a local maximum as the gradient is positive to its left and negative to its right.

c) Sketch the graph of f , indicating the y-intercept and the points that you found in part a).

Solution:



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d) Find the greatest and least values taken by f on the interval [-3, 3].

Solution:
The greatest and least values of a function on an interval will be a local maximum/minimum or at an end point of the interval. Calculating $f(x)$ at the local maximum, local minimum, and endpoints of the interval gives

$$\begin{aligned} f(-3) &= 1.5 \\ f(-2) &= -4 \\ f(1) &= 9.5 \\ f(3) &= -16.5 \end{aligned}$$

Therefore, the greatest value of f on the interval [-3, 3] is 9.5 (when $x = 1$), and the least value of f on the same interval is -16.5 (when $x = 3$).

Question 9

a) Find expressions for the velocity v and the acceleration a of the object at time t.

Solution:
As displacement s is a function of time t, the first derivative of S gives the velocity v. In Leibniz notation:

$$\frac{\mathrm{d}s}{\mathrm{d}t} = 12t^2 - 42t + 18$$

So $v = 12t^2 - 42t + 18$.

As acceleration is the second derivative of displacement, it can be calculated by finding the first derivative of velocity:

$$\begin{aligned}\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} &= \frac{\mathrm{d}v}{\mathrm{d}t} \\ &= 24t - 42\end{aligned}$$

So $a = 24t - 42$.

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b) Find any times at which the velocity of the object is zero.

Solution:
As velocity is given by a quadratic equation, I start by factorising it:

$$v = 6(2t^2 - 7t + 3)$$

and substituting the values of $a = 2$, $b = -7$, and $c = 3$ into the quadratic formula to find values of t for which $v = 0$:

$$\begin{aligned}t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 2 \times 3}}{2 \times 2} \\ &= \frac{7 \pm 5}{4}\end{aligned}$$

So the velocity of the object is 0 at $t = 3$ seconds and $t = \frac{1}{2}$ seconds.

c) The minimum velocity occurs when the acceleration is zero. Show that this happens at time $t = \frac{4}{7}$ seconds, and find the value of this minimum velocity.

Solution:
The acceleration a of the object is given by $a = 24t - 42$. By setting a to 0, I can solve for t to find the time at which the acceleration was 0:

$$\begin{aligned} 0 &= 24t - 42 \\ &= 2(12t - 21) \\ &= 12t - 21 \\ 21 &= 12t \\ t &= \frac{21}{12} \\ &= \frac{7}{4} \end{aligned}$$

So acceleration is zero at $t = \frac{7}{4}$ seconds. I find the velocity at this time by substituting this value of t into the equation for velocity:

$$\begin{aligned} v &= 12t^2 - 42t + 18 \\ &= 12\left(\frac{7}{4}\right)^2 - 42\left(\frac{7}{4}\right) + 18 \\ &= -\frac{75}{4} \end{aligned}$$

So the minimum velocity is -18.75m s^{-1}.