Faculty of Science, Technology, Engineering and Mathematics
MST124 Essential mathematics 1

MST124

TMA 02 2021J

Covers Units 3, 4, 5 and 6 **Cut-off date 19 January 2022**

You will find information about TMAs in the 'Assessment' area of the MST124 website. Please read that information before beginning work on this TMA.

If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

The work that you submit should include your working as well as your final answers.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not involve the use of any other mathematical software.

Your work should be written in a good mathematical style, as described in Section 6 of Unit 1, and as demonstrated by the example and activity solutions in the study units. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 10. You do not have to submit any work for Question 10.

PLAGIARISM WARNING – the use of assessment help services and websites

The **work that you submit for any assessment/examination on any module should be your own**. Submitting work produced by or with another person, or a web service or an automated system, **as if it is your own** is cheating. It is **strictly forbidden** by the University.

You should not:

- provide any assessment question to a website, online service, social media platform or any individual or organisation, as this is an infringement of copyright.
- request answers or solutions to an assessment question on any website, via an online service or social media platform, or from any individual or organisation.
- use an automated system (other than one prescribed by the module) to obtain answers or solutions to an assessment question and submit the output as your own work.
- discuss examination questions with any other person, including your tutor.

The University actively monitors websites, online services and social media platforms for answers and solutions to assessment questions, and for assessment questions posted by students. Work submitted by students for assessment is also monitored for plagiarism.

A student who is found to have posted a question or answer to a website, online service or social media platform and/or to have used any resulting, or otherwise obtained, output as if it is their own work has committed a disciplinary offence under **Section SD 1.2** of our Code of Practice for Student Discipline. **This means the academic reputation and integrity of the University has been undermined.**

The Open University's Plagiarism policy defines plagiarism in part as:

- using text obtained from assignment writing sites, organisations or private individuals.
- obtaining work from other sources and submitting it as your own.

If it is found that you have used the services of a website, online service or social media platform, or that you have otherwise obtained the work you submit from another person, this is considered serious academic misconduct and you will be referred to the Central Disciplinary Committee for investigation.

Question 1 – 5 marks

You should be able to answer this question after studying Unit 3.

Use a table of signs to solve the inequality

$$\frac{2x-5}{4x+10} \leq 0.$$

Give your answer in interval notation. [*5*]

Question 2 – 5 marks

You should be able to answer this question after studying Unit 3.

When animals or plants are introduced to a new environment where they have no predators or competition, their numbers can increase extremely rapidly. The crown-of-thorns starfish eats coral and has caused widespread destruction on the Great Barrier Reef, for example. Their population growth can in some circumstances be modelled by an exponential growth function.

Suppose that $s(t)$ is the number of starfish per hectare on a particular reef at time t (in years), where $t = 0$ represents the time when starfish were first observed on the reef. Assume that the population is modelled by the exponential growth function

$$s(t) = Ae^{kt} \quad (t \geq 0),$$

where A and k are constants. After 2 years the population was 50 per hectare, and after 5 years the population was recorded as 1200 per hectare.

(a) Show that the values of the constants A and k are 6.01 and 1.06, respectively, to three significant figures. [*4*]

(b) What is the expected population per hectare after 10 years if no action is taken and the population continues to follow the exponential growth model? Give your answer to the nearest whole number.

(Hint: make sure to use the accurate values of A and k, if you found these in part (a). You may use rounded values if you do not have accurate values, but you will lose marks for this.) [*1*]

Question 3 – 15 marks

You should be able to answer this question after studying Unit 3.

(a) This part of the question concerns the graph of the function

$$f(x) = (x+3)^2 - 1.$$

(i) Explain how the graph of f can be obtained from the graph of $y = x^2$ by using appropriate translations.
(You are not asked to sketch any graphs in this part, but you may find it helpful to do so.) [*2*]

(ii) Write down the image set of the function f, in interval notation. [*1*]

(b) This part of the question concerns the function

$$g(x) = (x+3)^2 - 1 \quad (-3 \leq x \leq 0).$$

The function g has the same rule as the function f in part (a), but a smaller domain.

(i) Sketch the graph of g, using equal scales on the axes. (You should draw this by hand, rather than using any software.) Mark the coordinates of the endpoints of the graph. [*2*]

(ii) Give the image set of g, in interval notation. [*1*]

(iii) Show that the inverse function g^{-1} has the rule

$$g^{-1}(x) = -3 + \sqrt{x+1},$$

justifying each step clearly, and give its domain and image set. [*6*]

(iv) Add a sketch of $y = g^{-1}(x)$ to the graph that you produced in part (b)(i). Mark the coordinates of the endpoints of the graph of $g^{-1}(x)$. [*3*]

Question 4 – 10 marks

You should be able to answer this question after studying Unit 4.

In triangle ABC (with the usual notation, as used in Figure 44 of Unit 4), angle $A = 27°$, side $b = 30$ cm and side $c = 20$ cm.

(a) Use the cosine rule to show that the length of side a is 15.19, rounded to two decimal places. [*2*]

(b) Without using the cosine rule again, find the remaining angles B and C, giving your answers to the nearest degree. [*4*]

(c) (i) Find the area of this triangle, giving your answer in cm^2 to two decimal places. [*2*]

(ii) By what factor would the area of the triangle increase if the lengths of side b and side c were doubled, and also the angle A were doubled (so now angle $A = 54°$)? Give your answer to two decimal places. [*2*]

Question 5 – 10 marks

You should be able to answer this question after studying Unit 4.

(a) Using the exact values of the tangents of $3\pi/4$ and $\pi/3$, and the angle difference identity for tangent, show that an exact value of $\tan(5\pi/12)$ is

$$2+\sqrt{3}.$$ [*4*]

(b) Use the double-angle identity for tangent, and the exact value of $\tan(5\pi/6)$, for a different way to find an exact value of $\tan(5\pi/12)$.

(Hint: you will need to solve a quadratic equation.) [*4*]

(c) Use an appropriate trigonometric identity involving $\tan^2\theta$ to show that an exact value of $\cos(5\pi/12)$ is

$$\frac{1}{2\sqrt{2+\sqrt{3}}}.$$ [*2*]

Question 6 – 10 marks

You should be able to answer this question after studying Unit 5 and also Section 7 of the Computer Algebra Guide.

Use Maxima to plot the parabola

$$y = 2x^2 - 16x + 28$$

and the ellipse

$$4x^2 + 25y^2 - 32x - 100y + 64 = 0$$

on the same graph. Plotting an ellipse in Maxima is done in the same way as plotting a circle.

Use Maxima to find the coordinates of the points of intersection between the parabola and the ellipse. State the values of the coordinates rounded to two decimal places, but do not attempt to use Maxima for rounding.

Include a printout or screenshot of your Maxima worksheet with your solutions. You are not expected to annotate your Maxima worksheet with explanations. [*10*]

Question 7 – 17 marks

You should be able to answer this question after studying Unit 5.

An anti-whaling boat, the *BlueYonder*, is travelling in perfectly calm seas at a speed (relative to the surrounding sea) of $40\,\mathrm{km\,h^{-1}}$ on a bearing of $160°$. It is aiming to cross the path of a whaling ship, the *Narwhal*, which is sailing in perfectly calm seas at a speed (relative to the surrounding sea) of $30\,\mathrm{km\,h^{-1}}$ on a bearing of $70°$ (so that their paths should cross at right angles).

Take unit vectors $\mathbf{i}$ to point east and $\mathbf{j}$ to point north.

(a) Express the velocity $\mathbf{b}$ of the *BlueYonder* and the velocity $\mathbf{n}$ of the *Narwhal* in component form, both relative to the sea, giving the numerical values in $\mathrm{km\,h^{-1}}$ to one decimal place. *[6]*

(b) Unknown to both vessels, a strong current arises and this has the effect of applying an additional velocity $\mathbf{c}$ to both vessels, of $15\,\mathrm{km\,h^{-1}}$ in the direction from west to east. Write the velocity $\mathbf{c}$ in component form, and hence express the resultant velocities of the *BlueYonder* and the *Narwhal* in component form, giving numerical values to one decimal place. *[3]*

(c) Find the magnitude of the resultant velocity of each vessel. Give magnitudes in $\mathrm{km\,h^{-1}}$ to one decimal place. *[4]*

(d) Hence find the actual angle between the courses of the two vessels, taking into account the effect of the current. Give your answer to the nearest degree.

(Hint: use the dot product of the resultant velocity vectors.) *[4]*

Question 8 – 18 marks

You should be able to answer this question after studying Unit 6.

This question concerns the function

$$f(x) = -x^3 - \tfrac{3}{2}x^2 + 6x + 6.$$

(a) Find the stationary points of f, and give their exact x- and y-coordinates. *[5]*

(b) Use the first derivative test to classify the stationary points that you found in part (a). *[5]*

(c) Sketch the graph of f, indicating the y-intercept and the points that you found in part (a). (You should draw this by hand, rather than using any software, and you can use different scales on the two axes if appropriate.) *[5]*

(d) Find the greatest and least values taken by f on the interval $[-3, 3]$. *[3]*

Question 9 – 5 marks

You should be able to answer this question after studying Unit 6.

An object moves along a straight line. Its displacement s (in metres) from a reference point at time t (in seconds) is given by

$$s = 4t^3 - 21t^2 + 18t \quad (t \geq 0).$$

Answer the following questions using calculus and algebra. You may find it helpful to sketch or plot graphs, but no marks will be awarded for graphical arguments or solutions.

(a) Find expressions for the velocity v and the acceleration a of the object at time t. *[2]*

(b) Find any times at which the velocity of the object is zero. *[2]*

(c) The minimum velocity occurs when the acceleration is zero. Show that this happens at time $t = \frac{7}{4}$ seconds, and find the value of this minimum velocity. *[1]*

Question 10 – 5 marks

A score out of 5 marks for good mathematical communication throughout TMA 02 will be recorded under Question 10. *[5]*