The Open University

MST124

Essential mathematics 1

Exercise Booklet 1

1 Numbers

Exercise 1

Evaluate the following expressions without using a calculator.

(a) $2 + 4 \times 3 - (10 - 3^2) + 1$

(b) $(-3)^2 - (-2) \times 4 \times (-1)$

(c) $-4^2 - \dfrac{18}{-3}$

(d) $5 + (-8) - (-2)^3 \times 3$

(e) $(4 + 8) \div 2 \times 3$

Exercise 2

Round the following numbers to two significant figures.

(a) 4.235 (b) 0.097 76 (c) 102

(d) 17.49

Exercise 3

Evaluate the following expressions when $x = -5$ and $y = -2$, without using a calculator.

(a) $2y - 3x + 7$

(b) $(-2xy - x^2)(x - y)$

Exercise 4

Find the lowest common multiple and highest common factor of 360 and 588.

2 Algebraic expressions

Exercise 5

Simplify the following expressions.

(a) $(-5a) \times 3a^2b$

(b) $(-2x) \times (3xy) \times \left(-\frac{2}{3}xy^4\right)$

(c) $3ab - ba - a + 7b + 5a$

(d) $(2a) \times (-7ab) + (-b)^2 + (3a)^2 \times 5b$

(e) $6t \times \frac{1}{3}t + (5t) \times (-2) - (-5t) \times 2 - (2t)^2$

Exercise 6

Multiply out the brackets in the following expressions, and simplify as far as possible.

(a) $4 - (2 - x)$

(b) $2a(3 - a) + a^2(4 - a)$

(c) $-6x^2(3 + \frac{2}{3}x - \frac{1}{4}y)$

(d) $b(1 - b) - (b - 1)b$

(e) $-2K^2H(H - K) + H^2(HK + 2K^2)$

Exercise 7

Multiply out the brackets in the following expressions, and simplify as far as possible.

(a) $(a + 7)(a - 1)$ (b) $(c + 4)(4c - 3)$

(c) $(3x - 4y)(3x + 4y)$ (d) $(2A + B)^2$

(e) $(9 - 2t)(5t - 1)$ (f) $(3a - \frac{2}{3})^2$

(g) $\left(x + \dfrac{1}{x}\right)^2$

(h) $2(x + 3)(x - 4) - (x - 6)^2$

(i) $(\sqrt{x} + 2\sqrt{y})(\sqrt{x} - 3\sqrt{y})$

(j) $(a + 2b)(a - 2b + c)$

(k) $(a^2 + 3a - 1)(a^3 - a + 2)$

3 Algebraic factors, multiples and fractions

Exercise 8

Factorise the following expressions.

(a) $2a^2b - 7ab^2$

(b) $24tw + 18wt^2 - 20t$

(c) $34\sqrt{x} - 4y\sqrt{x}$

(d) $15a^2 - 30a^3$

(e) $\frac{1}{4}k + \frac{3}{2}k^2 - \frac{3}{4}hk$

Exercise 9

Simplify the following algebraic fractions.

(a) $\dfrac{2xy^3}{6y^2}$ (b) $\dfrac{15x(x-2)^2}{3x^2(x-2)}$

(c) $\dfrac{4x - x^2}{2x^3}$ (d) $\dfrac{2a - 10a^4}{7a^2 + 5a}$

(e) $\dfrac{6d^2 - 2de}{12d - 4e}$

Exercise 10

Write each of the following expressions as a single algebraic fraction, simplifying your answer where possible.

(a) $\dfrac{2}{a} - \dfrac{5}{a}$ (b) $\dfrac{b}{4} + \dfrac{5b}{6}$

(c) $\dfrac{2}{\sqrt{x}} + \dfrac{y}{\sqrt{x}} - \dfrac{1}{2\sqrt{x}}$ (d) $\dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3}$

(e) $\dfrac{a-b}{2a} - \dfrac{2a+b}{2b}$ (f) $25 - \dfrac{10}{a^2} + \dfrac{1}{a}$

Exercise 11

Simplify the following expressions.

(a) $\dfrac{6a}{b} \times \dfrac{5b^2}{3a^2}$ (b) $\dfrac{4}{x^2} \div \dfrac{20}{x^3y}$

(c) $\dfrac{1/d}{1/d^2}$ (d) $\dfrac{x^2+x}{2} \Big/ \dfrac{1+x}{12x}$

4 Roots and powers

Exercise 12

Simplify the following surds.

(a) $\sqrt{3}\sqrt{7}$ (b) $\sqrt{3}\sqrt{21}$ (c) $\dfrac{\sqrt{7}}{\sqrt{21}}$

(d) $\dfrac{\sqrt{2}\sqrt{3}\sqrt{3}}{\sqrt{27}}$ (e) $\sqrt{80}$ (f) $2\sqrt{3} + \sqrt{3}$

(g) $\sqrt{125} - \sqrt{45} - \sqrt{75}$ (h) $(\sqrt{7})^3 - \sqrt{28}$

Exercise 13

Multiply out the brackets in the following expressions, simplifying your answers where possible.

(a) $\sqrt{3}(2\sqrt{3} - \sqrt{2})$

(b) $(1 + \sqrt{5})(1 - 3\sqrt{5})$

(c) $(\sqrt{7} - 1)(\sqrt{7} + 1)$

(d) $(2 + \sqrt{3})^2 - 2(1 + \sqrt{12})$

(e) $(2\sqrt{5} - 5\sqrt{2})(3\sqrt{2} + \sqrt{5})$

Exercise 14

Write each of the following expressions as a single fraction, simplifying your answer where possible.

(a) $\dfrac{\sqrt{2}}{3} - \dfrac{1-\sqrt{2}}{2}$ (b) $\dfrac{1}{\sqrt{2}} + \dfrac{3}{\sqrt{7}}$

(c) $4 - \dfrac{1}{\sqrt{12}} + \dfrac{10}{\sqrt{3}}$

Exercise 15

Rationalise the denominators in the following surds.

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{33}{\sqrt{11}}$ (c) $\dfrac{\sqrt{3}}{\sqrt{12}}$

(d) $\dfrac{7}{1+\sqrt{5}}$ (e) $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

(f) $\dfrac{\sqrt{15}}{\sqrt{5}-\sqrt{3}}$

Exercise 16

Evaluate the following expressions without using your calculator.

(a) -5^2 (b) $(-5)^2$ (c) $(-2)^3$

(d) $\left(\frac{1}{5}\right)^2$ (e) $\frac{1}{5^2}$ (f) $-\left(\frac{1}{2}\right)^3$

(g) 2^{-3} (h) 13^0 (i) $\left(\frac{3}{2}\right)^{-1}$

(j) $\left(\frac{2}{5}\right)^{-2}$

Exercise 17

Simplify the following expressions.

(a) $\frac{b^2b^4}{b}$ (b) $\left(\frac{a}{a^2}\right)^3$

(c) $a^2(a^2)^2$ (d) $(hk^2k)^5$

Exercise 18

Multiply out the brackets or expand the fraction, as appropriate, simplifying your answers.

(a) $(a^2-3)(a^5+7)$ (b) $(x^2+x^{-1})^2$

(c) $\frac{m^2+3m+5}{m}$

Exercise 19

Simplify the following expressions, ensuring that the simplified versions contain no negative indices. (In parts (g) and (h), you are not expected to multiply out the brackets.)

(a) $3x^{-2}$ (b) $(7b^{-1})^2$ (c) $\frac{p^3}{r^{-2}p^4}$

(d) $\frac{12y}{y^{-7}}$ (e) $\left(\frac{x}{y^{-1}}\right)^{-1}$

(f) $\left(\frac{3b^{-2}c}{6(bc)^{-3}}\right)^{-2}$ (g) $\frac{12(x+3y)^5}{3(x+3y)^{-2}}$

(h) $\frac{a^{-2}(3+b)^3}{a^{-6}(3+b)^{-3}}$

Exercise 20

Rewrite the following expressions so that they contain root signs instead of fractional indices.

(a) $4k^{1/2}$ (b) $(4k)^{3/2}$ (c) $A^{-1/3}$

(d) $(3t-w)^{1/5}$

Exercise 21

Simplify the following expressions, replacing any root signs by fractional indices.

(a) $\sqrt[3]{x^2}$ (b) $y^2\sqrt{y}$ (c) $\frac{x^{5/2}}{\sqrt{x}}$

(d) $\frac{\sqrt{b^3}}{b^{1/2}}$ (e) $\frac{(2x)^{5/4}x^{3/4}}{\sqrt[4]{2}\,x}$ (f) $\frac{35t^{2/3}}{7t^{-3/2}}$

Exercise 22

Express the following numbers in scientific notation.

(a) 919 090 000 (b) 10 (c) 9.951

(d) 0.0005

Exercise 23

Express the following numbers in ordinary notation.

(a) 5.6×10^3 (b) 2×10^7

(c) 3.987×10^{-1} (d) 7.707×10^{-7}

5 Equations

Exercise 24

(a) Show that $x = 4$ satisfies the equation $3x + 1 = 17 - x$.

(b) Determine whether the equation $9x - y = 5$ is satisfied by $x = 2$ and $y = 14$.

(c) Determine whether the equation $a^2 - 2a - 3 = 0$ is satisfied by $a = -1$.

Exercise 25

Solve the following equations.

(a) $3(a+4) = 7a - 16$

(b) $2x - 7(3-x) = 15$

(c) $\dfrac{12y}{5} - 3 = 2(5-y)$

(d) $x - 1 - (3-x) - (x-7) = \dfrac{1}{2}$

(e) $\dfrac{7}{t} - \dfrac{4}{5t} = 31$

(f) $\dfrac{6}{k+1} = \dfrac{1}{1-k}$

(g) $\dfrac{1}{13X-5} = \dfrac{2}{3}$

(h) $\dfrac{1}{b+2} - \dfrac{3}{b-1} = \dfrac{5}{2+b}$

(i) $\dfrac{x}{x+1} - \dfrac{2}{x-1} = 1$

Exercise 26

Rearrange the following equations to make a the subject.

(a) $4b = 7 - 22a$

(b) $6a = 5b - 3ab + c$

(c) $21k + ah = \dfrac{3-a}{4}$

(d) $\dfrac{2}{m} + \dfrac{3}{5a} = \dfrac{6}{7m}$

(e) $\dfrac{3}{x} - \dfrac{1}{x+1} - \dfrac{x}{4a} = 0$

(f) $\dfrac{1}{a} - \dfrac{b^2}{1+a} = 0$

Exercise 27

Rearrange the following equations to make y the subject. Assume that all variables take only positive values and in part (b) that $y > \frac{1}{2}$.

(a) $x - 2y^2 = 9$ (b) $(2y-1)^2 = 4a$

(c) $\sqrt{\dfrac{2}{y}} = 3gh$ (d) $\dfrac{3p+A}{y^3} = 7 + A$

(e) $(y^2-7)^{1/3} = pq$ (f) $y^{2/3} = 4 - x$

6 Writing mathematics

Exercise 28

Consider the following question and poorly-written solution.

Question

The wheels of a bicycle each have diameter 63.5 cm.

(i) Find the circumference of one of the wheels, giving your answer correct to three significant figures.

(ii) Each time the front wheel makes a complete revolution, the bicycle moves forward by the distance given by the circumference of the wheel. Work out how far the bicycle travels when the wheel makes 375 revolutions, giving your answer to the nearest metre.

Poorly-written solution

(i) $c = 2\pi r$
So $2 \times 3.14 \times 31.75 = 199$ cm

(ii) Distance $= 199 \times 375$
$= 74625$
$= 746$ m

(a) Criticise the poorly-written solution. How could it be improved?

(b) Write out a better solution.

Exercise 29 is overleaf.

Exercise 29

Consider the following question and poorly-written solution.

Question

Evaluate the expression $\dfrac{h^2+\sqrt{2k}}{3+hk}$ when $h=-2$ and $k=0.5$.

Poorly-written solution

$-2^2+\sqrt{2}\times .5/3+-2\times .5=$
$4+\sqrt{1}/3-1=5/2\Rightarrow 2.5$

(a) Criticise the poorly-written solution. How could it be improved?

(b) Write out a better solution.

Solutions to exercises

Solution to Exercise 1

(a) $2+4\times 3-(10-3^2)+1$
$=2+12-(10-9)+1$
$=14-1+1$
$=14$

(b) $(-3)^2-(-2)\times 4\times(-1)=9-8=1$

(c) $-4^2-\dfrac{18}{-3}=-16+6=-10$

(d) $5+(-8)-(-2)^3\times 3=5-8-(-8)\times 3$
$=5-8+24$
$=21$

(e) $(4+8)\div 2\times 3=12\div 2\times 3=6\times 3=18$

(When operations have the same precedence, work from left to right.)

Solution to Exercise 2

(a) $4.235=4.2$ (to 2 s.f.)

(b) $0.097\,76=0.098$ (to 2 s.f.)

(c) $102=100$ (to 2 s.f.)

(d) $17.49=17$ (to 2 s.f.)

Solution to Exercise 3

(a) When $x=-5$ and $y=-2$,

$2y-3x+7=2\times(-2)-3\times(-5)+7$
$=-4-(-15)+7$
$=-4+15+7$
$=18.$

(b) When $x=-5$ and $y=-2$,

$(-2xy-x^2)(x-y)$
$=(-2\times(-5)\times(-2)-(-5)^2)\times(-5-(-2))$
$=(-20-25)\times(-5+2)$
$=(-45)\times(-3)$
$=135.$

Solution to Exercise 4

$360=2^3\times 3^2\times 5$, and
$588=2^2\times 3\times 7^2$.

So the LCM of 360 and 588 is
$2^3\times 3^2\times 5\times 7^2=17\,640$.

The HCF is $2^2\times 3=12$.

Solution to Exercise 5

(a) $(-5a) \times 3a^2b = -15a^3b$

(b) $(-2x) \times (3xy) \times (-\frac{2}{3}xy^4)$
$= (-2 \times 3 \times (-\frac{2}{3}))x^3y^5$
$= 4x^3y^5$

(c) $3ab - ba - a + 7b + 5a = 2ab + 4a + 7b$

(d) $(2a) \times (-7ab) + (-b)^2 + (3a)^2 \times 5b$
$= -14a^2b + b^2 + (9a^2) \times 5b$
$= -14a^2b + b^2 + 45a^2b$
$= 31a^2b + b^2$

(e) $6t \times \frac{1}{3}t + (5t) \times (-2) - (-5t) \times 2 - (2t)^2$
$= 2t^2 - 10t + 10t - 4t^2$
$= -2t^2$

Solution to Exercise 6

(a) $4 - (2 - x) = 4 - 2 + x$
$= 2 + x$

(b) $2a(3 - a) + a^2(4 - a) = 6a - 2a^2 + 4a^2 - a^3$
$= 6a + 2a^2 - a^3$

(c) $-6x^2(3 + \frac{2}{3}x - \frac{1}{4}y) = -18x^2 - 4x^3 + \frac{3}{2}x^2y$

(d) $b(1 - b) - (b - 1)b = b - b^2 - b^2 + b$
$= 2b - 2b^2$

(e) $-2K^2H(H - K) + H^2(HK + 2K^2)$
$= -2K^2H^2 + 2K^3H + H^3K + 2H^2K^2$
$= 2K^3H + H^3K$

Solution to Exercise 7

(a) $(a + 7)(a - 1) = a^2 - a + 7a - 7$
$= a^2 + 6a - 7$

(b) $(c + 4)(4c - 3) = 4c^2 - 3c + 16c - 12$
$= 4c^2 + 13c - 12$

(c) $(3x - 4y)(3x + 4y)$
$= 9x^2 + 12xy - 12xy - 16y^2$
$= 9x^2 - 16y^2$

(d) $(2A + B)^2 = (2A + B)(2A + B)$
$= 4A^2 + 2AB + 2AB + B^2$
$= 4A^2 + 4AB + B^2$

(e) $(9 - 2t)(5t - 1) = 45t - 9 - 10t^2 + 2t$
$= -10t^2 + 47t - 9$

(f) $(3a - \frac{2}{3})^2 = (3a - \frac{2}{3})(3a - \frac{2}{3})$
$= 9a^2 - 2a - 2a + \frac{4}{9}$
$= 9a^2 - 4a + \frac{4}{9}$

(g) $\left(x + \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$
$= x^2 + 1 + 1 + \left(\frac{1}{x}\right)^2$
$= x^2 + 2 + \frac{1}{x^2}$

(h) $2(x + 3)(x - 4) - (x - 6)^2$
$= 2(x^2 - 4x + 3x - 12) - (x^2 - 12x + 36)$
$= 2x^2 - 2x - 24 - x^2 + 12x - 36$
$= x^2 + 10x - 60$

(i) $(\sqrt{x} + 2\sqrt{y})(\sqrt{x} - 3\sqrt{y})$
$= x - 3\sqrt{x}\sqrt{y} + 2\sqrt{x}\sqrt{y} - 6y$
$= x - \sqrt{xy} - 6y$

(j) $(a + 2b)(a - 2b + c)$
$= a(a - 2b + c) + 2b(a - 2b + c)$
$= a^2 - 2ab + ac + 2ab - 4b^2 + 2bc$
$= a^2 - 4b^2 + ac + 2bc$

(k) $(a^2 + 3a - 1)(a^3 - a + 2)$
$= a^2(a^3 - a + 2) + 3a(a^3 - a + 2)$
$\quad - 1(a^3 - a + 2)$
$= a^5 - a^3 + 2a^2 + 3a^4 - 3a^2 + 6a$
$\quad - a^3 + a - 2$
$= a^5 + 3a^4 - 2a^3 - a^2 + 7a - 2$

Solution to Exercise 8

(a) $2a^2b - 7ab^2 = ab(2a - 7b)$

(b) $24tw + 18wt^2 - 20t = 2t(12w + 9wt - 10)$

(c) $34\sqrt{x} - 4y\sqrt{x} = 2\sqrt{x}(17 - 2y)$

(d) $15a^2 - 30a^3 = 15a^2(1 - 2a)$

(e) $\frac{1}{4}k + \frac{3}{2}k^2 - \frac{3}{4}hk = \frac{1}{4}k(1 + 6k - 3h)$

Solution to Exercise 9

(a) $\dfrac{2xy^3}{6y^2} = \dfrac{xy}{3}$

(b) $\dfrac{15x(x-2)^2}{3x^2(x-2)} = \dfrac{5(x-2)}{x}$

(c) $\dfrac{4x-x^2}{2x^3} = \dfrac{x(4-x)}{2x^3} = \dfrac{4-x}{2x^2}$

(d) $\dfrac{2a-10a^4}{7a^2+5a} = \dfrac{2a(1-5a^3)}{a(7a+5)} = \dfrac{2(1-5a^3)}{7a+5}$

(e) $\dfrac{6d^2-2de}{12d-4e} = \dfrac{2d(3d-e)}{4(3d-e)} = \dfrac{d}{2}$

Solution to Exercise 10

(a) $\dfrac{2}{a} - \dfrac{5}{a} = \dfrac{2-5}{a} = -\dfrac{3}{a}$

(b) $\dfrac{b}{4} + \dfrac{5b}{6} = \dfrac{3b+10b}{12} = \dfrac{13b}{12}$

(c) $\dfrac{2}{\sqrt{x}} + \dfrac{y}{\sqrt{x}} - \dfrac{1}{2\sqrt{x}} = \dfrac{4+2y-1}{2\sqrt{x}}$
$= \dfrac{3+2y}{2\sqrt{x}}$

(d) $\dfrac{1}{x} + \dfrac{1}{x^2} + \dfrac{1}{x^3} = \dfrac{x^2+x+1}{x^3}$

(e) $\dfrac{a-b}{2a} - \dfrac{2a+b}{2b} = \dfrac{b(a-b)-a(2a+b)}{2ab}$
$= \dfrac{ab-b^2-2a^2-ab}{2ab}$
$= \dfrac{-b^2-2a^2}{2ab}$

(f) $25 - \dfrac{10}{a^2} + \dfrac{1}{a} = \dfrac{25a^2-10+a}{a^2}$

Solution to Exercise 11

(a) $\dfrac{6a}{b} \times \dfrac{5b^2}{3a^2} = \dfrac{10b}{a}$

(b) $\dfrac{4}{x^2} \div \dfrac{20}{x^3y} = \dfrac{4}{x^2} \times \dfrac{x^3y}{20} = \dfrac{xy}{5}$

(c) $\dfrac{1/d}{1/d^2} = \dfrac{1}{d} \times \dfrac{d^2}{1} = d$

(d) $\dfrac{x^2+x}{2} \Big/ \dfrac{1+x}{12x} = \dfrac{x^2+x}{2} \times \dfrac{12x}{1+x}$
$= \dfrac{x(x+1)\times 12x}{2(1+x)}$
$= 6x^2$

Solution to Exercise 12

(a) $\sqrt{3}\sqrt{7} = \sqrt{3\times 7} = \sqrt{21}$

(b) $\sqrt{3}\sqrt{21} = \sqrt{3}\sqrt{3\times 7} = \sqrt{3}\sqrt{3}\sqrt{7} = 3\sqrt{7}$

(c) $\dfrac{\sqrt{7}}{\sqrt{21}} = \sqrt{\dfrac{7}{21}} = \sqrt{\dfrac{1}{3}} = \dfrac{1}{\sqrt{3}}$

(d) $\dfrac{\sqrt{2}\sqrt{3}\sqrt{3}}{\sqrt{27}} = \dfrac{3\sqrt{2}}{\sqrt{9\times 3}} = \dfrac{3\sqrt{2}}{3\sqrt{3}} = \sqrt{\dfrac{2}{3}}$

(e) $\sqrt{80} = \sqrt{16\times 5} = \sqrt{16}\times\sqrt{5} = 4\sqrt{5}$

(f) $2\sqrt{3} + \sqrt{3} = 3\sqrt{3}$

(g) $\sqrt{125} - \sqrt{45} - \sqrt{75}$
$= \sqrt{25\times 5} - \sqrt{9\times 5} - \sqrt{25\times 3}$
$= 5\sqrt{5} - 3\sqrt{5} - 5\sqrt{3}$
$= 2\sqrt{5} - 5\sqrt{3}$

(h) $(\sqrt{7})^3 - \sqrt{28} = \sqrt{7}\times\sqrt{7}\times\sqrt{7} - \sqrt{4\times 7}$
$= 7\sqrt{7} - 2\sqrt{7}$
$= 5\sqrt{7}$

Solution to Exercise 13

(a) $\sqrt{3}(2\sqrt{3}-\sqrt{2}) = 2\sqrt{3}\sqrt{3} - \sqrt{3}\sqrt{2}$
$= 2\times 3 - \sqrt{6}$
$= 6 - \sqrt{6}$

(b) $(1+\sqrt{5})(1-3\sqrt{5}) = 1 - 3\sqrt{5} + \sqrt{5} - 3\sqrt{5}\sqrt{5}$
$= 1 - 2\sqrt{5} - 3\times 5$
$= -14 - 2\sqrt{5}$

(c) $(\sqrt{7}-1)(\sqrt{7}+1) = \sqrt{7}\times\sqrt{7} + \sqrt{7} - \sqrt{7} - 1$
$= 7 - 1$
$= 6$

(d) $(2+\sqrt{3})^2 - 2(1+\sqrt{12})$
$= (2+\sqrt{3})(2+\sqrt{3}) - 2 - 2\sqrt{12}$
$= 4 + 2\sqrt{3} + 2\sqrt{3} + \sqrt{3}\sqrt{3} - 2 - 2\sqrt{4\times 3}$
$= 4 + 4\sqrt{3} + 3 - 2 - 2\times 2\sqrt{3}$
$= 5.$

(e) $(2\sqrt{5} - 5\sqrt{2})(3\sqrt{2} + \sqrt{5})$
$= 6\sqrt{5}\sqrt{2} + 2\sqrt{5}\sqrt{5} - 15\sqrt{2}\sqrt{2} - 5\sqrt{2}\sqrt{5}$
$= 6\sqrt{10} + 10 - 30 - 5\sqrt{10}$
$= \sqrt{10} - 20$

Solution to Exercise 14

(a) $\dfrac{\sqrt{2}}{3} - \dfrac{1-\sqrt{2}}{2} = \dfrac{2\sqrt{2} - 3(1-\sqrt{2})}{6}$

$= \dfrac{2\sqrt{2} - 3 + 3\sqrt{2}}{6}$

$= \dfrac{5\sqrt{2} - 3}{6}$

(b) $\dfrac{1}{\sqrt{2}} + \dfrac{3}{\sqrt{7}} = \dfrac{\sqrt{7} + 3\sqrt{2}}{\sqrt{2}\sqrt{7}} = \dfrac{\sqrt{7} + 3\sqrt{2}}{\sqrt{14}}$

(c) $4 - \dfrac{1}{\sqrt{12}} + \dfrac{10}{\sqrt{3}} = 4 - \dfrac{1}{\sqrt{4 \times 3}} + \dfrac{10}{\sqrt{3}}$

$= 4 - \dfrac{1}{2\sqrt{3}} + \dfrac{10}{\sqrt{3}}$

$= \dfrac{8\sqrt{3} - 1 + 20}{2\sqrt{3}}$

$= \dfrac{8\sqrt{3} + 19}{2\sqrt{3}}$

Solution to Exercise 15

(a) $\dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

(b) $\dfrac{33}{\sqrt{11}} = \dfrac{33}{\sqrt{11}} \times \dfrac{\sqrt{11}}{\sqrt{11}} = \dfrac{33\sqrt{11}}{11} = 3\sqrt{11}$

(c) $\dfrac{\sqrt{3}}{\sqrt{12}} = \dfrac{\sqrt{3}}{\sqrt{12}} \times \dfrac{\sqrt{12}}{\sqrt{12}} = \dfrac{\sqrt{36}}{12} = \dfrac{6}{12} = \dfrac{1}{2}$

(d) $\dfrac{7}{1+\sqrt{5}} = \dfrac{7}{1+\sqrt{5}} \times \dfrac{1-\sqrt{5}}{1-\sqrt{5}}$

$= \dfrac{7(1-\sqrt{5})}{1-(\sqrt{5})^2}$

$= \dfrac{7-7\sqrt{5}}{-4}$

$= \dfrac{7\sqrt{5}-7}{4}$

(e) $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}} = \dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \dfrac{\sqrt{3}(\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \dfrac{3+\sqrt{6}}{3-2}$

$= 3 + \sqrt{6}$

(f) $\dfrac{\sqrt{15}}{\sqrt{5}-\sqrt{3}} = \dfrac{\sqrt{15}}{\sqrt{5}-\sqrt{3}} \times \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \dfrac{\sqrt{15}(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \dfrac{\sqrt{5\times3\times5}+\sqrt{5\times3\times3}}{5-3}$

$= \dfrac{5\sqrt{3}+3\sqrt{5}}{2}$

Solution to Exercise 16

(a) $-5^2 = -(5 \times 5) = -25$

(b) $(-5)^2 = (-5) \times (-5) = 25$

(c) $(-2)^3 = (-2) \times (-2) \times (-2) = -8$

(d) $\left(\dfrac{1}{5}\right)^2 = \dfrac{1}{5} \times \dfrac{1}{5} = \dfrac{1}{25}$

(e) $\dfrac{1}{5^2} = \dfrac{1}{5 \times 5} = \dfrac{1}{25}$

(f) $-\left(\dfrac{1}{2}\right)^3 = -\left(\dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2}\right) = -\dfrac{1}{8}$

(g) $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

(h) $13^0 = 1$

(i) $\left(\dfrac{3}{2}\right)^{-1} = \dfrac{2}{3}$

(j) $\left(\dfrac{2}{5}\right)^{-2} = \left(\left(\dfrac{2}{5}\right)^{-1}\right)^2 = \left(\dfrac{5}{2}\right)^2 = \dfrac{25}{4}$

or

$\left(\dfrac{2}{5}\right)^{-2} = \dfrac{1}{\left(\frac{2}{5}\right)^2} = \dfrac{1}{\frac{4}{25}} = \dfrac{25}{4}$

Solution to Exercise 17

(a) $\dfrac{b^2 b^4}{b} = \dfrac{b^6}{b} = b^5$

(b) $\left(\dfrac{a}{a^2}\right)^3 = \left(\dfrac{1}{a}\right)^3 = \dfrac{1}{a^3}$

(c) $a^2(a^2)^2 = a^2 a^4 = a^6$

(d) $(hk^2k)^5 = (hk^3)^5 = h^5 k^{15}$

Solution to Exercise 18

(a) $(a^2-3)(a^5+7) = a^2a^5 + 7a^2 - 3a^5 - 21$
$= a^7 - 3a^5 + 7a^2 - 21$

(b) $(x^2+x^{-1})^2 = (x^2+x^{-1})(x^2+x^{-1})$
$= x^2x^2 + x^2x^{-1} + x^{-1}x^2 + x^{-1}x^{-1}$
$= x^4 + x + x + x^{-2}$
$= x^4 + 2x + \dfrac{1}{x^2}$

(c) $\dfrac{m^2+3m+5}{m} = \dfrac{m^2}{m} + \dfrac{3m}{m} + \dfrac{5}{m}$
$= m + 3 + \dfrac{5}{m}$

Solution to Exercise 19

(a) $3x^{-2} = \dfrac{3}{x^2}$

(b) $(7b^{-1})^2 = \left(\dfrac{7}{b}\right)^2 = \dfrac{49}{b^2}$
or $(7b^{-1})^2 = 49b^{-2} = \dfrac{49}{b^2}$

(c) $\dfrac{p^3}{r^{-2}p^4} = \dfrac{r^2}{p}$

(d) $\dfrac{12y}{y^{-7}} = 12y \times y^7 = 12y^8$

(e) $\left(\dfrac{x}{y^{-1}}\right)^{-1} = (xy)^{-1} = \dfrac{1}{xy}$

(f) $\left(\dfrac{3b^{-2}c}{6(bc)^{-3}}\right)^{-2} = \left(\dfrac{c(bc)^3}{2b^2}\right)^{-2}$
$= \left(\dfrac{cb^3c^3}{2b^2}\right)^{-2}$
$= \left(\dfrac{bc^4}{2}\right)^{-2}$
$= \left(\left(\dfrac{bc^4}{2}\right)^{-1}\right)^2$
$= \left(\dfrac{2}{bc^4}\right)^2$
$= \dfrac{4}{b^2c^8}$

(g) $\dfrac{12(x+3y)^5}{3(x+3y)^{-2}} = 4(x+3y)^5(x+3y)^2$
$= 4(x+3y)^7$

(h) $\dfrac{a^{-2}(3+b)^3}{a^{-6}(3+b)^{-3}} = \dfrac{a^6(3+b)^3(3+b)^3}{a^2}$
$= a^4(3+b)^6$

Solution to Exercise 20

(a) $4k^{1/2} = 4\sqrt{k}$

(b) $(4k)^{3/2} = \sqrt{(4k)^3}$
$= (\sqrt{4k})^3$
$= 4k \times \sqrt{4k}$
$= 8k\sqrt{k}$

(c) $A^{-1/3} = \dfrac{1}{A^{1/3}} = \dfrac{1}{\sqrt[3]{A}}$

(d) $(3t-w)^{1/5} = \sqrt[5]{3t-w}$

Solution to Exercise 21

(a) $\sqrt[3]{x^2} = x^{2/3}$

(b) $y^2\sqrt{y} = y^2y^{1/2} = y^{5/2}$

(c) $\dfrac{x^{5/2}}{\sqrt{x}} = \dfrac{x^{5/2}}{x^{1/2}} = x^2$

(d) $\dfrac{\sqrt{b^3}}{b^{1/2}} = \dfrac{b^{3/2}}{b^{1/2}} = b$

(e) $\dfrac{(2x)^{5/4}x^{3/4}}{\sqrt[4]{2}\,x} = \dfrac{2^{5/4}x^{5/4}x^{3/4}}{2^{1/4}x} = \dfrac{2x^2}{x} = 2x$

(f) $\dfrac{35t^{2/3}}{7t^{-3/2}} = 5t^{2/3}t^{3/2} = 5t^{2/3+3/2} = 5t^{13/6}$

Solution to Exercise 22

(a) $919\,090\,000 = 9.1909 \times 10^8$

(b) $10 = 1 \times 10^1$

(c) $9.951 = 9.951 \times 10^0$

(d) $0.0005 = 5 \times 10^{-4}$

Solution to Exercise 23

(a) $5.6 \times 10^3 = 5600$

(b) $2 \times 10^7 = 20\,000\,000$

(c) $3.987 \times 10^{-1} = 0.3987$

(d) $7.707 \times 10^{-7} = 0.000\,000\,770\,7$

Solution to Exercise 24

(a) The equation is

$$3x + 1 = 17 - x.$$

When $x = 4$,

$$\text{LHS} = 3 \times 4 + 1 = 13,$$

and

$$\text{RHS} = 17 - 4 = 13.$$

So $x = 4$ satisfies the equation.

(b) The equation is

$$9x - y = 5.$$

When $x = 2$ and $y = 14$,

$$\text{LHS} = 9 \times 2 - 14 = 4 \neq \text{RHS}.$$

So $x = 2$ and $y = 14$ do not satisfy the equation.

(c) The equation is

$$a^2 - 2a - 3 = 0.$$

When $a = -1$,

$$\begin{aligned}\text{LHS} &= (-1)^2 - 2 \times (-1) - 3 \\ &= 1 + 2 - 3 \\ &= 0 \\ &= \text{RHS}.\end{aligned}$$

So $a = -1$ does satisfy the equation.

Solution to Exercise 25

(a) $3(a + 4) = 7a - 16$

$3a + 12 = 7a - 16$

$12 + 16 = 7a - 3a$

$4a = 28$

$a = 7$

The solution is $a = 7$.

(b) $2x - 7(3 - x) = 15$

$2x - 21 + 7x = 15$

$9x = 15 + 21$

$9x = 36$

$x = 4$

The solution is $x = 4$.

(c) $\dfrac{12y}{5} - 3 = 2(5 - y)$

$12y - 15 = 10(5 - y)$

$12y - 15 = 50 - 10y$

$12y + 10y = 50 + 15$

$22y = 65$

$y = \dfrac{65}{22}$

The solution is $y = \dfrac{65}{22}$.

(d) $x - 1 - (3 - x) - (x - 7) = \dfrac{1}{2}$

$x - 1 - 3 + x - x + 7 = \dfrac{1}{2}$

$x + 3 = \dfrac{1}{2}$

$x = -\dfrac{5}{2}$

The solution is $x = -\dfrac{5}{2}$.

(e) $\dfrac{7}{t} - \dfrac{4}{5t} = 31$

$5t\left(\dfrac{7}{t} - \dfrac{4}{5t}\right) = 31 \times 5t$

$35 - 4 = 155t$

$155t = 31$

$t = \dfrac{31}{155} = \dfrac{1}{5}$

The solution is $t = \dfrac{1}{5}$.

(f) $\dfrac{6}{k + 1} = \dfrac{1}{1 - k}$

$6(1 - k) = k + 1$

$6 - 6k = k + 1$

$7k = 5$

$k = \dfrac{5}{7}$

The solution is $k = \dfrac{5}{7}$.

(g) $\dfrac{1}{13X-5}=\dfrac{2}{3}$

$3=2(13X-5)$

$3=26X-10$

$26X=13$

$X=\dfrac{1}{2}$

The solution is $X=\dfrac{1}{2}$.

(h) $\dfrac{1}{b+2}-\dfrac{3}{b-1}=\dfrac{5}{2+b}$

$(b+2)(b-1)\left(\dfrac{1}{b+2}-\dfrac{3}{b-1}\right)$

$=\dfrac{5(b+2)(b-1)}{2+b}$

$b-1-3(b+2)=5(b-1)$

$b-1-3b-6=5b-5$

$7b=-2$

$b=-\dfrac{2}{7}$

The solution is $b=-\dfrac{2}{7}$.

(An alternative method:

$\dfrac{1}{b+2}-\dfrac{3}{b-1}=\dfrac{5}{2+b}$

$-\dfrac{3}{b-1}=\dfrac{5}{2+b}-\dfrac{1}{b+2}$

$-\dfrac{3}{b-1}=\dfrac{4}{2+b}$

$-3(2+b)=4(b-1)$

$-6-3b=4b-4$

$7b=-2$

$b=-\dfrac{2}{7}$

The solution is $b=-\dfrac{2}{7}$.)

(i) $\dfrac{x}{x+1}-\dfrac{2}{x-1}=1$

$x(x-1)-2(x+1)=(x+1)(x-1)$

$x^2-x-2x-2=x^2-1$

$3x=-1$

$x=-\dfrac{1}{3}$

The solution is $x=-\dfrac{1}{3}$.

Solution to Exercise 26

(a) $4b=7-22a$

$22a=7-4b$

$a=\dfrac{7-4b}{22}$

(b) $6a=5b-3ab+c$

$6a+3ab=5b+c$

$a(6+3b)=5b+c$

$a=\dfrac{5b+c}{6+3b}$

(assuming $6+3b\neq 0$; that is, $b\neq -2$)

(c) $21k+ah=\dfrac{3-a}{4}$

$84k+4ah=3-a$

$4ah+a=3-84k$

$a(4h+1)=3-84k$

$a=\dfrac{3-84k}{4h+1}$

(assuming $4h+1\neq 0$; that is, $h\neq -\frac{1}{4}$)

(d) $\dfrac{2}{m}+\dfrac{3}{5a}=\dfrac{6}{7m}$

$\dfrac{3}{5a}=\dfrac{6}{7m}-\dfrac{14}{7m}$

$\dfrac{3}{5a}=\dfrac{-8}{7m}$

$-40a=21m$

$a=-\dfrac{21m}{40}$

(e) $\dfrac{3}{x}-\dfrac{1}{x+1}-\dfrac{x}{4a}=0$

$\dfrac{3}{x}-\dfrac{1}{x+1}=\dfrac{x}{4a}$

$\dfrac{3(x+1)-x}{x(x+1)}=\dfrac{x}{4a}$

$\dfrac{2x+3}{x(x+1)}=\dfrac{x}{4a}$

$4a=\dfrac{x^2(x+1)}{2x+3}$

$a=\dfrac{x^2(x+1)}{4(2x+3)}$

(assuming $2x+3\neq 0$; that is, $x\neq -\frac{3}{2}$)

(f) $\dfrac{1}{a} - \dfrac{b^2}{1+a} = 0$

$\dfrac{1}{a} = \dfrac{b^2}{1+a}$

$1 + a = ab^2$

$ab^2 - a = 1$

$a(b^2 - 1) = 1$

$a = \dfrac{1}{b^2 - 1}$

(assuming $b^2 - 1 \neq 0$; that is, $b \neq \pm 1$)

Solution to Exercise 27

(a) $x - 2y^2 = 9$

$2y^2 = x - 9$

$y^2 = \dfrac{x-9}{2}$

$y = \sqrt{\dfrac{x-9}{2}}$

(Note that the equation is valid only for values of x such that $x > 9$, because if $x < 9$ then the square root above is undefined, and if $x = 9$ then $y = 0$, but we're told that y is positive.)

(b) $(2y - 1)^2 = 4a$

$2y - 1 = 2\sqrt{a}$

$2y = 2\sqrt{a} + 1$

$y = \dfrac{2\sqrt{a} + 1}{2}$

(c) $\sqrt{\dfrac{2}{y}} = 3gh$

$\dfrac{2}{y} = 9g^2h^2$

$y = \dfrac{2}{9g^2h^2}$

(d) $\dfrac{3p + A}{y^3} = 7 + A$

$y^3 = \dfrac{3p + A}{7 + A}$

$y = \left(\dfrac{3p + A}{7 + A}\right)^{1/3}$

(e) $(y^2 - 7)^{1/3} = pq$

$y^2 - 7 = p^3q^3$

$y^2 = 7 + p^3q^3$

$y = \sqrt{7 + p^3q^3}$

(Note that the original equation is valid only for values of y such that $y^2 > 7$.)

(f) $y^{2/3} = 4 - x$

$y^2 = (4 - x)^3$

$y = (4 - x)^{3/2}$

(Note that the equation is valid only for values of x such that $x < 4$, because if $x > 4$ then the expression $(4 - x)^{3/2}$ is undefined, and if $x = 4$ then $y = 0$, but we're told that y is positive.)

Solution to Exercise 28

(a) Here are some ways in which the solution could be improved.

For part (i):

- The formula for the circumference of a circle is quoted correctly, but the solution should explain what the symbols c and r represent.
- The calculation on the second line needs some words of explanation.
- The solution should not use the number 3.14 in place of the calculator value for π.
- The solution should explain where the value 31.75 comes from.
- The solution should indicate the rounding level (here it is to 3 significant figures). As written, the quantities on either side of the final equals sign are not equal.
- There should be a conclusion.

For part (ii):

- The reasoning underlying the calculation should be included.
- The working should not use the rounded answer from part (i); this has introduced a rounding error.
- 74 625 is not equal to 746 m. The conversion from cm to m should be

given correctly and the rounding level should be included.

- There should be a conclusion.

(b) Here is a better solution.

(i) The formula for the circumference c of a circle is $c = 2\pi r$, where r is the radius.

The diameter of the wheel is 63.5 cm, so the radius is half of this, 31.75 cm.

Substituting $r = 31.75$ into the formula above gives

$$c = 2 \times \pi \times 31.75 = 199.49\ldots.$$

The circumference of the wheel is 199 cm (to 3 s.f.).

(ii) One revolution of the wheel moves the bicycle forward by the circumference.

So the distance moved in 375 revolutions will be

$$\begin{aligned} &375 \times \text{ the circumference} \\ &= 375 \times 199.49\ldots \text{ cm} \\ &= 74\,809.1\ldots \text{ cm} \\ &= 748.091\ldots \text{ m}. \end{aligned}$$

The bicycle travels 748 m (to the nearest m) in 375 revolutions.

Solution to Exercise 29

(a) Here are some ways in which the solution could be improved.

- It is good practice to indicate the values that are to be substituted for the variables, for example by starting,

  'When $h = -2$ and $k = 0.5$'.
- The calculation should begin with the expression to be evaluated, $\dfrac{h^2 + \sqrt{2k}}{3 + hk}$.
- (-2) should be in brackets, particularly for $(-2)^2$, which is not the same as -2^2.
- The square root sign should extend across all of 2×0.5. Alternatively, 2×0.5 could be written in brackets.
- The fractions should be written correctly. For example, by the BIDMAS rules, $4 + \sqrt{1}/3 - 1$ means

  $$4 + \frac{\sqrt{1}}{3} - 1,$$

  which is not what is intended. To avoid confusion, you should write fractions in the form $\dfrac{x}{y}$ wherever possible.
- The number .5 should be written as 0.5.
- The $=$ sign should be used instead of the $\Rightarrow$ sign between 5/2 and 2.5.
- Putting each step of the calculation on a new line would make the calculation easier to follow and the conclusion clearer.

(b) Here is a better solution.

When $h = -2$ and $k = 0.5$,

$$\begin{aligned} \frac{h^2 + \sqrt{2k}}{3 + hk} &= \frac{(-2)^2 + \sqrt{2 \times 0.5}}{3 + (-2) \times 0.5} \\ &= \frac{4 + \sqrt{1}}{3 - 1} \\ &= \frac{5}{2} \\ &= 2.5. \end{aligned}$$