Revise and refresh for MST124:

Welcome to Session 1

Wednesday September 8th 2021

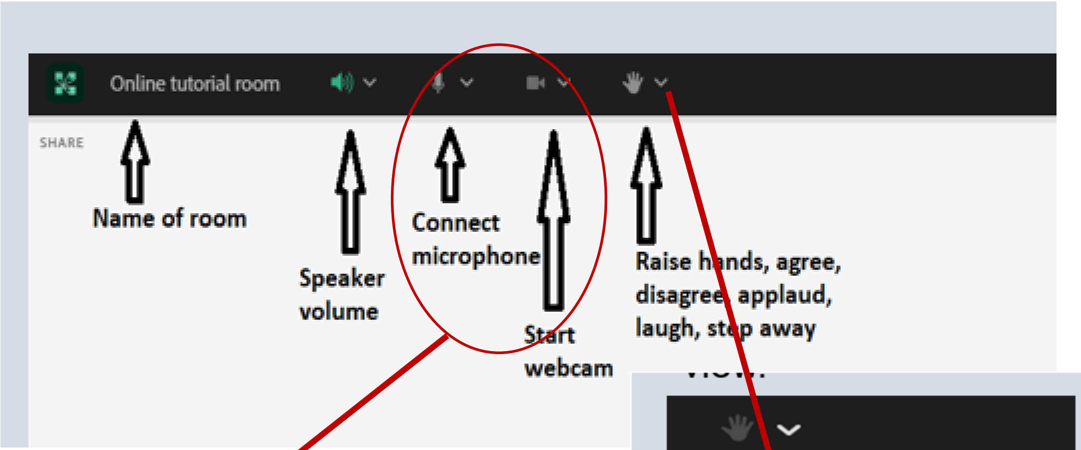
We'll start at 7.00pm and finish by 9.00pm

This session will cover the topics in Number

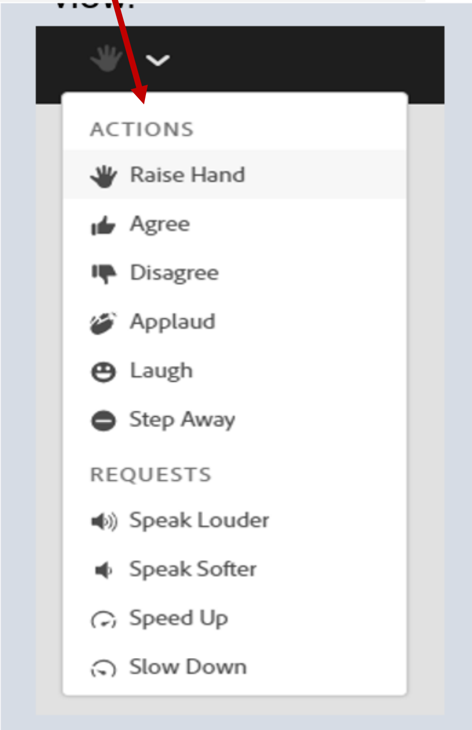
Have paper, pen and your calculator to hand.

This tutorial will be recorded – only first names will be shown.

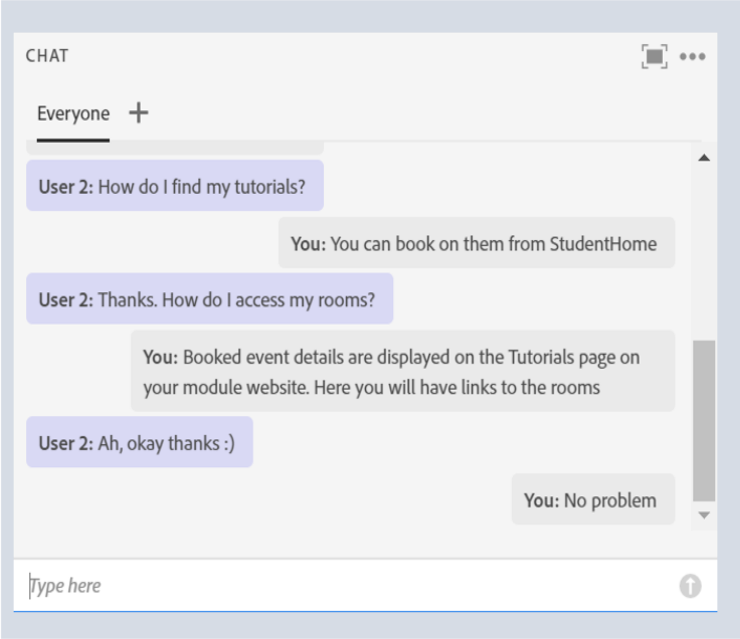
Adobe Connect –



You won't be using these today



We will be using a colour in chat so that our comments stand out – please keep yours neutral



Adobe Connect settings

It sometimes happens that I get a drop-out in my connection and I disappear. If that happens I can usually get back into the room within a minute or so.

This may also happen to you – just log back in.

If you have persistent problems with your connection it may be that others in your household are using up too much bandwidth (eg Netflix or gaming). This may also result in the sound breaking up.

Leaving the room and logging back in is worth trying in the first instance if you are having problems (other than maths!)

Revise and Refresh session protocols:

- Because these tutorials are attended by large numbers, it will not be practical to use microphones.

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Please keep your questions to the topic in hand – we will set aside time at the end for general questions about the structure and assessment of MST124 if there is anything you want to ask.

Revise and refresh for MST124

1. Number

(and a tiny bit of algebra)

What’s in this session

- Number systems
- Working with negative numbers
- Fractions – 4 rules
- A note about rounding
- Powers and index laws
- Roots and surds
- A word about setting out your work

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Units 1 and 2 of MST124 are intended to be 'revision' units and will include some of what we will cover today

Number systems

This is mainly about knowing the names for different types of numbers and what they mean

Number fields

Natural numbers are whole and positive

1 2 3 4 5 …

Natural Numbers $\mathbb{N}$

Natural Numbers $\mathbb{N}$

Number fields

Integers are whole numbers and the set includes 0

... -4 -3 -2 -1 0 1 2 3 4 5 ...

Integers $\mathbb{Z}$

The letter $\mathbb{Z}$ stands for *zahlen,* the German word for number

Natural Numbers $\mathbb{N}$

Integers $\mathbb{Z}$

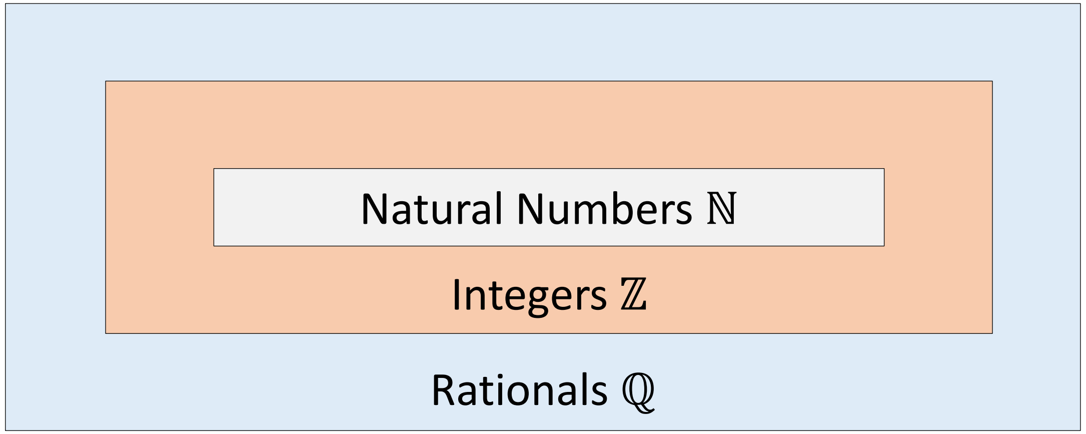
Number fields

Rational numbers can be written as <u>fractions</u>

$$-4 \ldots -3 \ldots -2 \ldots -\frac{8}{7} \ldots -1 \ldots -\frac{1}{2} \ldots 0 \ldots \frac{1}{3} \ldots \frac{1}{2} \ldots 1 \ldots \frac{5}{3} \ldots 2 \ldots \frac{57}{23} \ldots 3 \ldots$$

Rationals $\mathbb{Q}$

This integer is also rational because you could think of it as, say, $\frac{3}{1}$ or $\frac{6}{2}$ ...



A word about rational numbers and decimals

When you convert a rational number to a decimal it will either:

terminate, like $\frac{1}{2} = 0.5$ and $\frac{1}{8} = 0.125$

or

recur, like $\frac{1}{3} = 0.333\ldots$ and $\frac{7}{54} = 0.12962962\ldots$

This (three dots) is an ellipsis and indicates that figures continue

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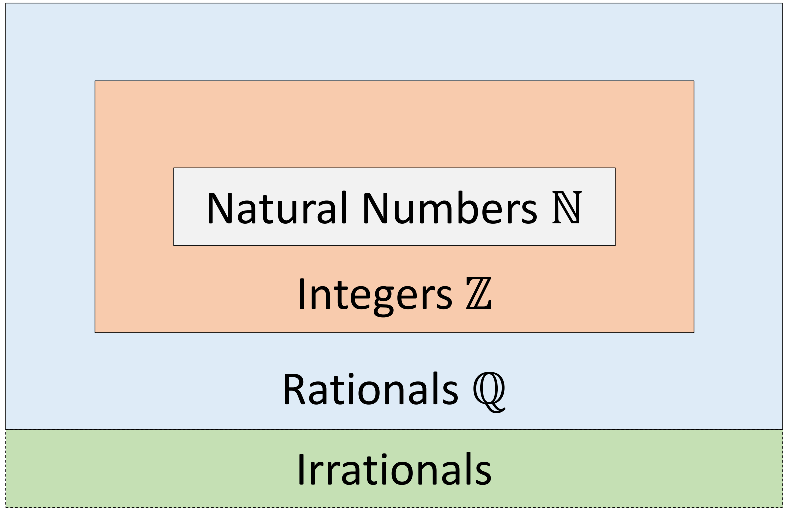
This (three dots) is an ellipsis and indicates that figures continue

So this begs the question:

"What about the non-recurring decimals?"

The gaps between the rationals are filled with the **irrationals**

$$-4 \ldots -3 \ldots -2 \ldots -\sqrt{2} \ldots -\frac{8}{7} \ldots -1 \ldots -\frac{1}{2} \ldots 0 \ldots \frac{\sqrt{5}}{3} \ldots \frac{1}{3} \ldots \frac{1}{2} \ldots 1 \ldots \sqrt{3} \ldots \frac{5}{3} \ldots 2 \ldots e \ldots \frac{57}{23} \ldots 3 \ldots \pi \ldots$$



Number fields

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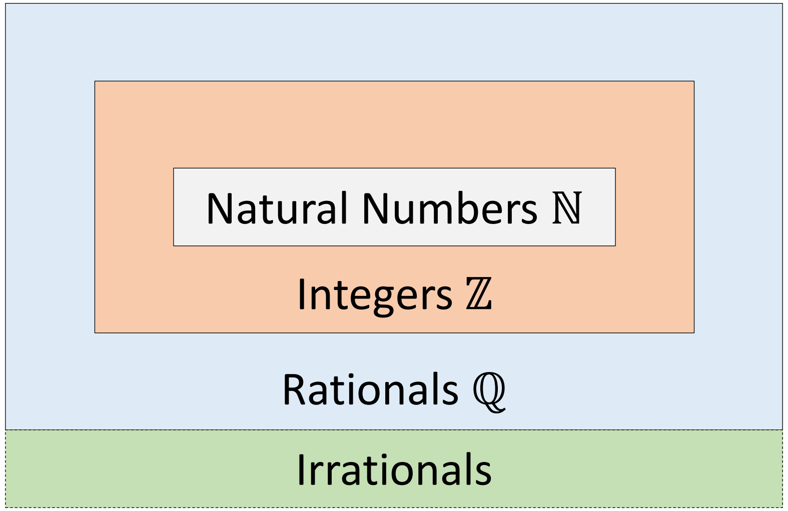
When you convert an irrational number to a decimal it will never terminate or recur

$$\sqrt{2} = 1.414213 \ldots$$

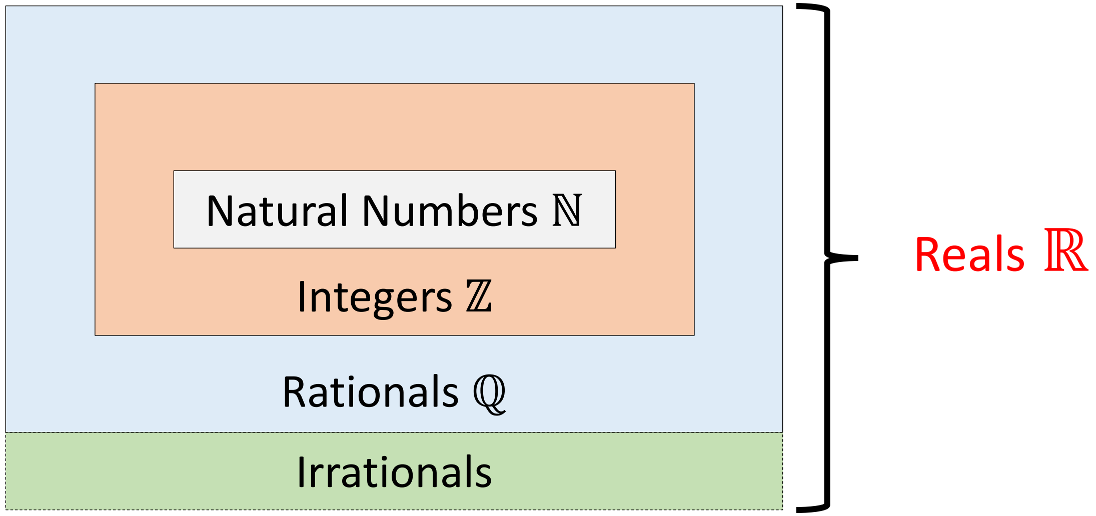
$$\pi = 3.141592 \ldots$$

That's why we use symbols like π and leave values in surd form (like $\sqrt{2}$) when possible.



Number fields

We call the whole set the ***real*** numbers



Number fields

Finally the **imaginary** numbers

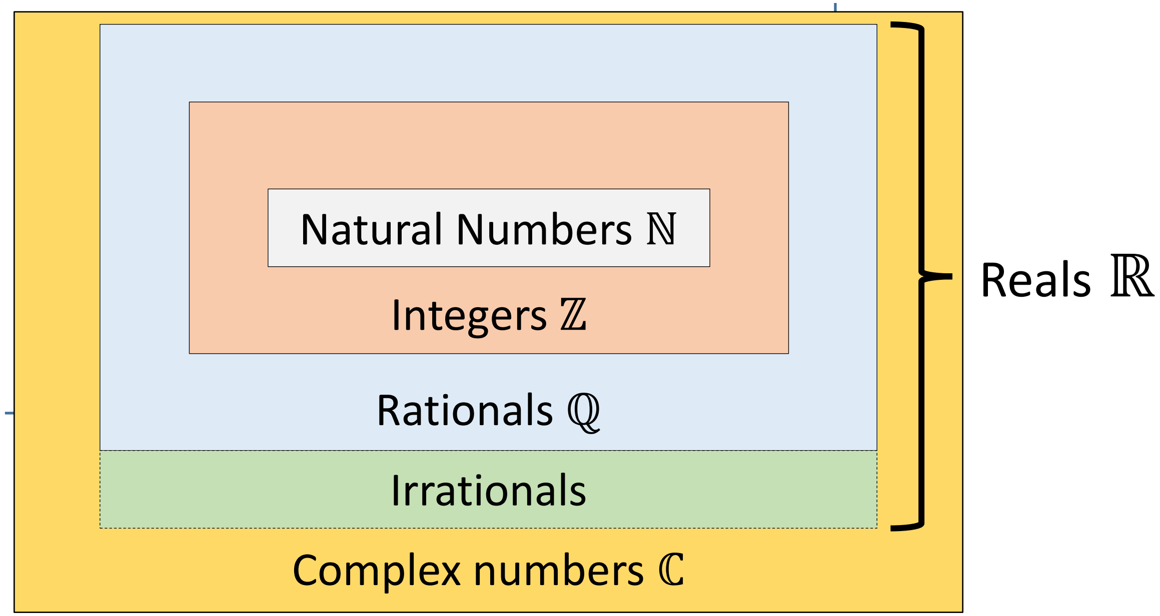
$$3i \;\vdots\; 2i \;\vdots\; i \;\vdots$$

$$-4 \ldots -3 \ldots -2 \ldots -\sqrt{2} \ldots -\frac{8}{7} \ldots -1 \ldots -\frac{1}{2} \ldots 0 \ldots \frac{\sqrt{5}}{3} \ldots \frac{1}{3} \ldots \frac{1}{2} \ldots 1 \ldots \sqrt{3} \ldots \frac{5}{3} \ldots 2 \ldots e \ldots \frac{57}{23} \ldots 3 \ldots \pi \ldots$$

$$\vdots\; -i \;\vdots\; -2i \;\vdots\; -3i \;\vdots$$

$2 + 3i$

Complex numbers will be covered in the final unit of MST124



Working with **negative numbers**

What's the difference between -3^2 and $(-3)^2$?

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$$-3^2 = -9$$

$$(-3)^2 = 9$$

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$$-3^2 = -9$$

$$(-3)^2 = 9$$

This crops up a lot and often leads to errors in calculations.

For example, if you're evaluating the expression $x^2 - 2x + 3$ when $x = -3$, it's important to use brackets:

When $x = -3$,

$$x^2 - 2x + 3 = (-3)^2 - 2 \times (-3) + 3$$

$$= 9 + 6 + 3 = 18$$

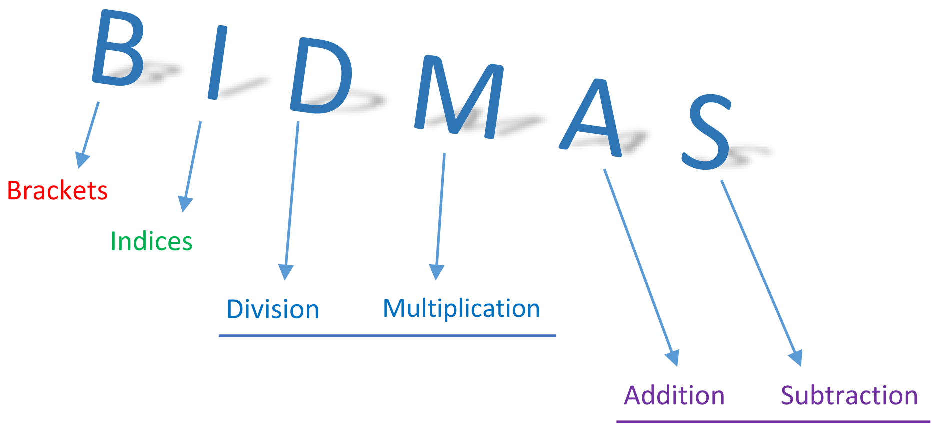
Compare this with the <u>incorrect</u>

$$x^2 - 2x + 3 = -3^2 - 2 \times -3 + 3$$

$$= -9 + 6 + 3 = 0$$

This is poor notation - avoid having two operators next to one another

A word about BIDMAS



Example, find the value of

$$9 - \frac{(5-2)^2}{4} \times 8$$

$$= 9 - \frac{3^2}{4} \times 8 \qquad \text{(brackets)}$$

$$= 9 - \frac{9}{4} \times 8 \qquad \text{(indices)}$$

$$= 9 - 9 \times 2 \qquad \text{(multiplication, division)}$$

$$= 9 - 18 = -9 \qquad \text{(adding, subtracting)}$$

Without your calculator, find the value of

$$12 - (-2)^2 - \frac{6}{-3}$$

Without your calculator, find the value of

$$12 - (-2)^2 - \frac{6}{-3} = 10$$

Notation

What can you say about these three fractions?

$$\frac{-2}{3} \qquad \frac{2}{-3} \qquad -\frac{2}{3}$$

Notation

They all have the same value

$$\frac{-2}{3} \qquad \frac{2}{-3} \qquad -\frac{2}{3}$$

These are calculations

This is an 'answer'

A note about rounding

Make sure you know how to round to decimal places and significant figures

$0.001746 = 0.00$ (to 2 decimal places)

$0.001756 = 0.0018$ (to 2 significant figures)

A note about rounding

Suppose we are calculating the solution to an equation related to a 'real world' situation with solutions to <u>2 decimal places</u> and we get to a stage that looks like this:

$$x = \frac{3 + \sqrt{9 + 8}}{2}$$

You will be using your calculator and this is how you should show the result:

$$x = \frac{3+\sqrt{9+8}}{2} = 3.5615\ldots$$

This symbol is called an 'ellipsis' and indicates that the figures continue

$$= 3.56 \text{ (to 2 decimal places)}$$

Always state the accuracy level you are using. (You can use the abbreviations 's.f.' or 'd.p.')

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$$= 3.56 \text{ (to 2 decimal places)}$$

Can you see why to would be incorrect to write

$$x = \frac{3 + \sqrt{9 + 8}}{2} = 3.561552813$$

$$= 3.56 \text{ (to 2 decimal places)}$$

A note about rounding

Suppose we are calculating the solution to an equation related to a 'real world' situation with solutions to <u>2 decimal places</u> and we get to a stage that looks like this:

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$$x = \frac{3+\sqrt{9+8}}{2} = 3.5615\ldots$$

$$= 3.56 \text{ (to 2 decimal places)}$$

- The level of rounding will depend on the context and is often given in the question.
- 'Exact' solutions should be left as <u>fractions</u> or <u>surds</u>

Fractions

Although you will probably use your calculator to work with numerical fractions, you do need to know how they work in order to deal with algebraic fractions

Cancelling

$$\frac{18}{27} = \frac{2\times 9}{3\times 9} = \frac{2}{3}$$

"cancelling" is dividing common factors

The equivalent process algebraically is:

$$\frac{ab}{cb} = \frac{a \times b}{c \times b} = \frac{a}{c}$$

Fractions

Adding, subtracting

when adding/subtracting, fractions must have the same denominator

$$\frac{2}{3} + \frac{3}{4} = \frac{2\times 4}{3\times 4} + \frac{3\times 3}{4\times 3} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

The equivalent process algebraically is:

$$\frac{a}{b} + \frac{c}{d} = \frac{a\times d}{b\times d} + \frac{c\times b}{d\times b} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

Fractions

Multiplying

$$\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$$

multiply tops and multiply bottoms

The equivalent process algebraically is:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Fractions

Dividing

This is the <u>reciprocal</u> of $\frac{5}{7}$

$$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$$

"Turn the second fraction over and multiply"

The equivalent process algebraically is:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Powers and indices

Powers of numbers

3^4 means $3 \times 3 \times 3 \times 3 = 81$ (not 12!)

base

power, index or exponent

What are:

$(-3)^4 =$ $\left(\frac{2}{3}\right)^3 =$ $(-1)^6 =$ $10^5 =$

Powers of numbers

3^4 means $3 \times 3 \times 3 \times 3 = 81$ (not 12!)

base

power, index or exponent

What are:

$(-3)^4 = 81$

$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$(-1)^6 = 1$

$10^5 = 100\,000$

Powers of numbers – rules for indices

When multiplying numbers with the same base we add the powers

$$3^4 \times 3^2 = 3^{4+2} = 3^6$$

Generalised rules

$$a^m \times a^n = a^{m+n}$$

Powers of numbers – rules for indices

When multiplying numbers with the same base we add the powers

$$3^{4} \times 3^{2} = 3^{6}$$

When dividing numbers with the same base we subtract the powers

$$\frac{3^{5}}{3^{2}} = 3^{5-2} = 3^{3}$$

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$$\frac{a^{m}}{a^{n}} = a^{m-n}$$

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When dividing numbers with the same base we subtract the powers

$$\frac{3^{5}}{3^{2}} = 3^{3}$$

When raising a power to another power, we multiply the powers

$$(3^{4})^{3} = 3^{4\times 3} = 3^{12}$$

Generalised rules

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

Powers of numbers

Simplify:

1. $10^4 \times 10^3 \times 10 =$

2. $\frac{2^5 \times 2^3}{2 \times 2^4} =$

3. $(\pi^5)^3 =$

4. $\frac{5^3}{5^3} =$

5. $\frac{3^2}{3^5} =$

Generalised rules

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

Powers of numbers

Simplify:

1. $10^4 \times 10^3 \times 10 = 10^{4+3+1} = 10^8$

2. $\frac{2^5 \times 2^3}{2 \times 2^4} = \frac{2^8}{2^5} = 2^{8-5} = 2^3$

3. $(\pi^5)^3 = \pi^{5\times 3} = \pi^{15}$

4. $\frac{5^3}{5^3} = 5^0 = 1$

5. $\frac{3^2}{3^5} = 3^{-3} = \frac{1}{3^3}$

Generalised rules

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

Powers of numbers

$$a^{-1} = \frac{1}{a}$$

this is called the reciprocal of a

It follows that $\left(\frac{1}{a}\right)^{-1} = a$ like $\left(\frac{1}{3}\right)^{-1} = 3$

and $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$

Notice also that $(ab)^n = a^n b^n$

And $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

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Notice also that $(ab)^n = a^n b^n$

And $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Find the values of the following, without using your calculator (leave as fractions)

$$\left(\frac{3}{4}\right)^{-1}$$

$$\left(\frac{3}{4}\right)^{3}$$

$$\left(\frac{3}{4}\right)^{-2}$$

Powers of numbers

So $a^{-1} = \frac{1}{a}$

which is called the reciprocal of a

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And $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

The values of the following,

$$\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$$

$$\left(\frac{3}{4}\right)^{3} = \frac{3^3}{4^3} = \frac{27}{64}$$

$$\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2} = \frac{16}{9}$$

More on powers

Fractional powers indicate roots

$\sqrt{3} = 3^{\frac{1}{2}}$ because $\left(3^{\frac{1}{2}}\right)^2 = 3^{\frac{1}{2}\times 2} = 3^1 = 3$

$\sqrt[3]{2} = 2^{\frac{1}{3}}$

$\sqrt[5]{7} = 7^{\frac{1}{5}}$

$$\boxed{a^{\frac{1}{n}} = \sqrt[n]{a}}$$

All the index rules apply in the same way:

$$2^{\frac{1}{3}} \times 2^{\frac{1}{2}} = 2^{\frac{1}{3}+\frac{1}{2}} = 2^{\frac{5}{6}}$$

$$\left(5^{\frac{1}{2}}\right)^3 = 5^{\frac{3}{2}}$$

$$3^{-\frac{2}{3}} = \frac{1}{3^{\frac{2}{3}}}$$

Powers of numbers

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Simplify:

$$\frac{2}{2^{\frac{1}{2}}} =$$

$$\frac{5^{\frac{2}{3}} \times 5^{\frac{4}{3}}}{5^{\frac{1}{3}} \times 5^{-\frac{4}{3}}} =$$

Simplify:

$$\left(7^{-\frac{2}{5}}\right)^{-5} =$$

$$\left(\sqrt[3]{13^2}\right)^{\frac{3}{2}} =$$

Powers of numbers

Simplify:

$$\frac{2}{2^{\frac{1}{2}}} = 2^{1-\frac{1}{2}} = 2^{\frac{1}{2}}$$

$$\frac{5^{\frac{2}{3}} \times 5^{\frac{4}{3}}}{5^{\frac{1}{3}} \times 5^{-\frac{4}{3}}} = \frac{5^2}{5^{-1}} = 5^3$$

Simplify:

$$\left(7^{-\frac{2}{5}}\right)^{-5} = 7^2$$

$$\left(\sqrt[3]{13^2}\right)^{\frac{3}{2}} = 13$$

Roots and surds

The square root is the inverse of squaring, so the square root of 9 is 3 or -3 since 3^2 and $(-3)^2$ are both 9.

So therefore, if $a^2 = 9$ then $a = \pm\sqrt{9} = \pm 3$

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Similarly the cube root is the inverse of taking the cube of a number so $\sqrt[3]{8} = 2$ because $2^3 = 8$. Notice this time there is only one (real) cube root and so the option of a negative version doesn't crop up.

Roots and surds

Find the values of the following:

a) $\sqrt[4]{16}$ b) $\sqrt[5]{32}$ c) $\sqrt[3]{-8}$ d) $\sqrt[3]{125}$ e) $\sqrt{-9}$

What is:

$\sqrt{1}$

$\sqrt{0}$

Roots and surds

Find the values of the following:

a) $\sqrt[4]{16} = 2$ b) $\sqrt[5]{32} = 2$ c) $\sqrt[3]{-8} = -2$ d) $\sqrt[3]{125} = 5$ e) $\sqrt{-9} = !?$

Later in MST124 you will study Complex Numbers and you'll then be able to give an answer to this

What is:

$\sqrt{1} = 1$

$\sqrt{0} = 0$

Roots and surds

Square roots of numbers which are not square numbers (like 4, 9, 16) are irrational

So instead of converting them to approximate decimals we try to work with them in exact form which we call surds

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These numbers are all in surd form:

$\sqrt{2}$ $\quad\quad$ $\frac{\sqrt{7}}{3}$ $\quad\quad$ $1+\sqrt{3}$ $\quad\quad$ $5\sqrt{5}$ $\quad\quad$ $\sqrt{2}+\sqrt{3}$

You will need to be able to manipulate surds without turning them into approximate decimals:

Examples

$$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$$

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$$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$$

$$\sqrt{5} \times \sqrt{3} = \sqrt{15}$$

[this is just $5^{\frac{1}{2}} \times 3^{\frac{1}{2}} = (5 \times 3)^{\frac{1}{2}}$]

You will need to be able to manipulate surds without turning them into approximate decimals:

Examples

$$2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$$

$$\sqrt{5} \times \sqrt{3} = \sqrt{15}$$

[this is just $5^{\frac{1}{2}} \times 3^{\frac{1}{2}} = (5 \times 3)^{\frac{1}{2}}$]

$$\frac{\sqrt{27}}{\sqrt{18}} = \sqrt{\frac{27}{18}} = \sqrt{\frac{3}{2}}$$

[this is $\frac{27^{\frac{1}{2}}}{18^{\frac{1}{2}}} = \left(\frac{27}{18}\right)^{\frac{1}{2}}$]

Roots and surds

You will need to be able to manipulate surds without turning them into approximate decimals:

For example $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$

and $\sqrt{5} \times \sqrt{3} = \sqrt{15}$

also $\dfrac{\sqrt{27}}{\sqrt{18}} = \sqrt{\dfrac{27}{18}} = \sqrt{\dfrac{3}{2}}$

Generally, you should leave numerical answers in <u>exact form</u> such as surds (or fractions). Sometimes you will be asked to give answers to, for example, 3 s.f. or 2 d.p.– usually when the context is ‘real world’ such as velocity, distance, area…

Try simplifying these

a) $2\sqrt{3} \times 3\sqrt{3}$

b) $7\sqrt{5} - 11\sqrt{5} + 3\sqrt{5}$

c) $\sqrt{2}\left(2\sqrt{3} + 3\sqrt{2}\right)$

d) $\sqrt[3]{5} \times \sqrt[3]{25}$

e) $\frac{\sqrt{28}}{\sqrt{7}}$

Try simplifying these

a) $2\sqrt{3} \times 3\sqrt{3} = 2 \times 3 \times \left(\sqrt{3}\right)^2 = 6 \times 3 = 18$

b) $7\sqrt{5} - 11\sqrt{5} + 3\sqrt{5} = -\sqrt{5}$

c) $\sqrt{2}\left(2\sqrt{3} + 3\sqrt{2}\right) = 2\sqrt{6} + 3\left(\sqrt{2}\right)^2 = 2\sqrt{6} + 6$

d) $\sqrt[3]{5} \times \sqrt[3]{25} = \sqrt[3]{5 \times 25} = \sqrt[3]{125} = 5$

e) $\frac{\sqrt{28}}{\sqrt{7}} = \sqrt{\frac{28}{7}} = \sqrt{4} = 2$

Remember that although you can combine surds when multiplying or dividing you <u>cannot</u> do that <u>when adding or subtracting</u>

So $\sqrt{6} = \sqrt{2} \times \sqrt{3}$

or $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}}$

The same is not true for $\sqrt{4+9}$ as this does not equal $\sqrt{4} + \sqrt{9}$

Roots and surds

You should also be able to simplify surds when possible:

For example $\sqrt{200} = \sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

$\sqrt{63} = \sqrt{9 \times 7} = \sqrt{9}\sqrt{7} = 3\sqrt{7}$

Notice that we found factors that are <u>square</u> numbers

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Notice that we found factors that are <u>square</u> numbers

Simplify: $\sqrt{80} =$

$3\sqrt{90} =$

$3\sqrt{12} + 2\sqrt{27} =$

Simplify:

$$\sqrt{80} = \sqrt{16 \times 5} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$$

$$3\sqrt{90} = 3\sqrt{9 \times 10} = 3\sqrt{9}\sqrt{10} = 3 \times 3\sqrt{10} = 9\sqrt{10}$$

$$3\sqrt{12} + 2\sqrt{27} = 3\sqrt{4 \times 3} + 2\sqrt{9 \times 3}$$

$$= 3 \times \sqrt{4}\sqrt{3} + 2 \times \sqrt{9}\sqrt{3}$$

$$= 3 \times 2\sqrt{3} + 2 \times 3\sqrt{3}$$

$$= 6\sqrt{3} + 6\sqrt{3} = 12\sqrt{3}$$

It usually makes sense not to divide by an irrational number. So fractions that have a surd in the denominator can be re-written so that the denominator is rational:

Example:

$\frac{1}{\sqrt{2}}$ can be written as $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ which has a rational denominator

This is equivalent to 1 so we aren't changing the value of the original number

It usually makes sense not to divide by an irrational number. So fractions that have a surd in the denominator can be adjusted so that the denominator is rational:

Example: $\frac{1}{\sqrt{2}}$ can be written as $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ which has a rational denominator

Rationalise the denominator:

$$\frac{2}{\sqrt{3}} =$$

$$\frac{\sqrt{3}}{\sqrt{5}} =$$

Surds – rationalising denominators

It usually makes sense not to divide by an irrational number. So fractions that have a surd in the denominator can be adjusted:

Example: $\frac{1}{\sqrt{2}}$ can be written as $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ which has a rational denominator

Rationalise the denominator:

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$$

Surds – rationalising denominators

It usually makes sense not to divide by an irrational number. So fractions that have a surd in the denominator can be adjusted:

For example: $\frac{1}{\sqrt{2}}$ can be written as $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ which has a rational denominator

Rationalise the denominator:

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$$

There is a bit more work involved when rationalising the denominator of fractions such as $\frac{3}{2\sqrt{3}+1}$.

This is covered in Unit 1 of the module.

That’s the work on Number that you need to be familiar with.

GMC

If you've studied MU123 then you'll know that we are keen to develop good academic habits in presenting and communicating maths.

It is important that others can follow your reasoning when you present solutions to problems. Marks are allocated to GMC (Good Mathematical Communication) on each of your TMAs.

Setting out your work

Find the value of: $\sqrt{(-2)^2 - 4 \times (-3) \times 2}$ giving your answer correct to 1 decimal place.

Here is someone's solution – the final value is numerically correct.
Can you see why this solution is not easy to make sense of?

$$4 * -3 * 2 = -24$$

$$-2^2 = 4$$

$$\sqrt{} = 4 - -24 = 28 = 5.3$$

Setting out your work

Find the value of: $\sqrt{(-2)^2 - 4 \times (-3) \times 2}$ giving your answer correct to 1 decimal place.

These are fragments and there is nothing to explain why they are here. Try to keep everything together

$$4 * -3 * 2 = -24$$

These are not mathematical symbols, never use them when writing maths!

$$-2^2 = 4$$

$$\sqrt{\quad} = 4 - -24 = 28 = 5.3$$

This answer has been rounded but there is nothing to indicate what level of accuracy has been applied

This is a meaningless symbol on its own

Avoid this

This equals sign isn't correct because 28 does not equal 5.3

Setting out your work

This is a better approach

Find the value of : $\sqrt{(-2)^2 - 4 \times (-3) \times 2}$

$$\begin{aligned}\sqrt{(-2)^2 - 4 \times (-3) \times 2} &= \sqrt{4 - (-24)} \\ &= \sqrt{28} \\ &= 5.291\ldots \\ &= 5.3 \quad \text{(to 1 d.p.)}\end{aligned}$$

Show the expression you are working with

Give unrounded value first using three dots to indicate figures continue

Align = signs

State rounding level

Handwriting is fine too

Find the value of : $\sqrt{(-2)^2 - 4 \times (-3) \times 2}$

$$\sqrt{(-2)^2 - 4 \times (-3) \times 2} = \sqrt{4 - (-24)}$$

$$= \sqrt{28}$$

$$= 5.291\ldots$$

$$= 5.3 \text{ (1 dp)}$$

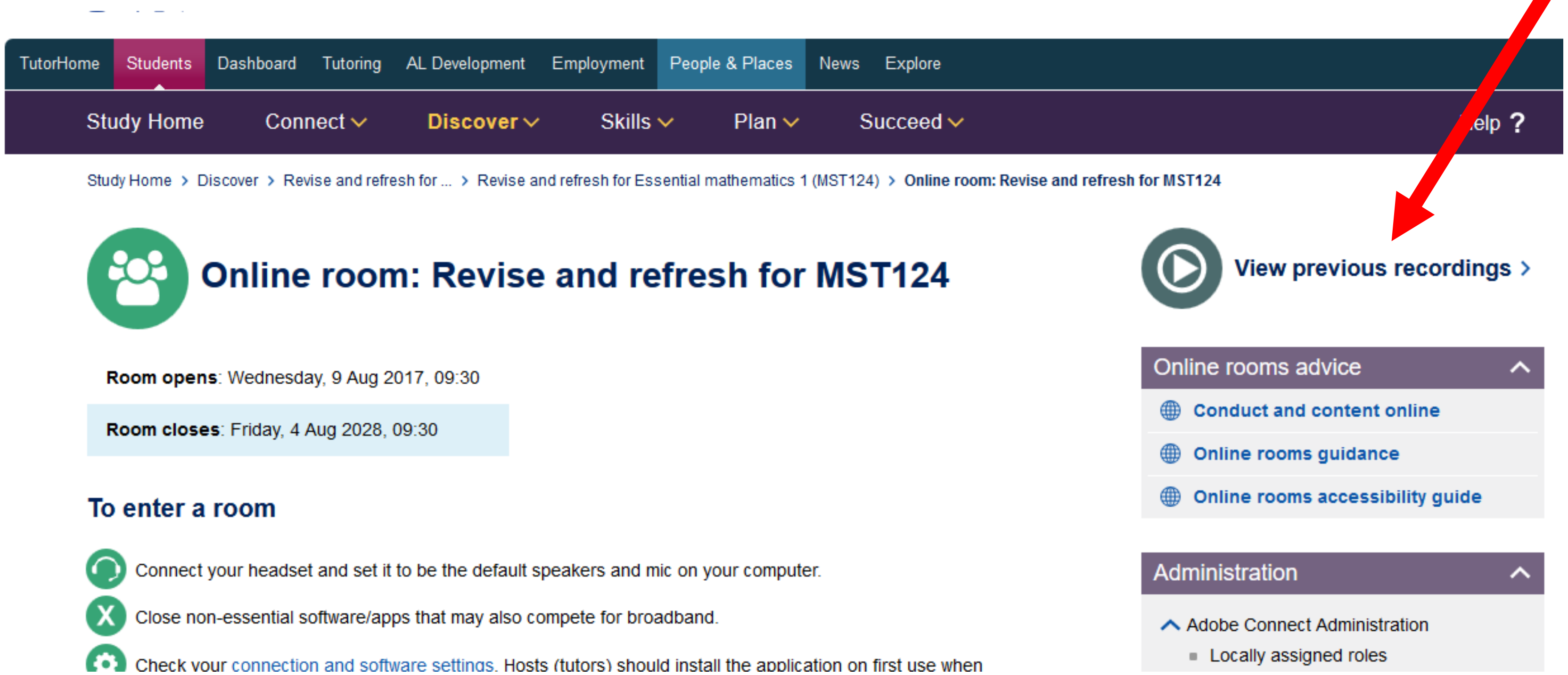
Setting out your work

You don't have to explain in words what you are doing but each line must follow logically from the previous one.

What next

The next session on Friday 10th is on algebra.

You can watch/listen to the recording of today's (and all) sessions by following the link on the R&R online room page.



Finally….

Today's session was intended to be a revision of ideas and skills that you have already but may not have used for a while.

If you are finding this level of work quite difficult then you may not be ready to start MST124 and you should speak to your tutor or the Student Support Team (link on the module home page) as soon as you can.

We would strongly advise you to do the '[Are you ready for MST124]' quiz that is on the module website before starting the units. It will help you to identify areas you need to focus on.