Law of conservation of mechanical energy

If the force on a particle depends only on the particle's position, then the total mechanical energy of the particle is constant. In other words,

$$E = \tfrac{1}{2}mv^2 + U(x), \tag{28}$$

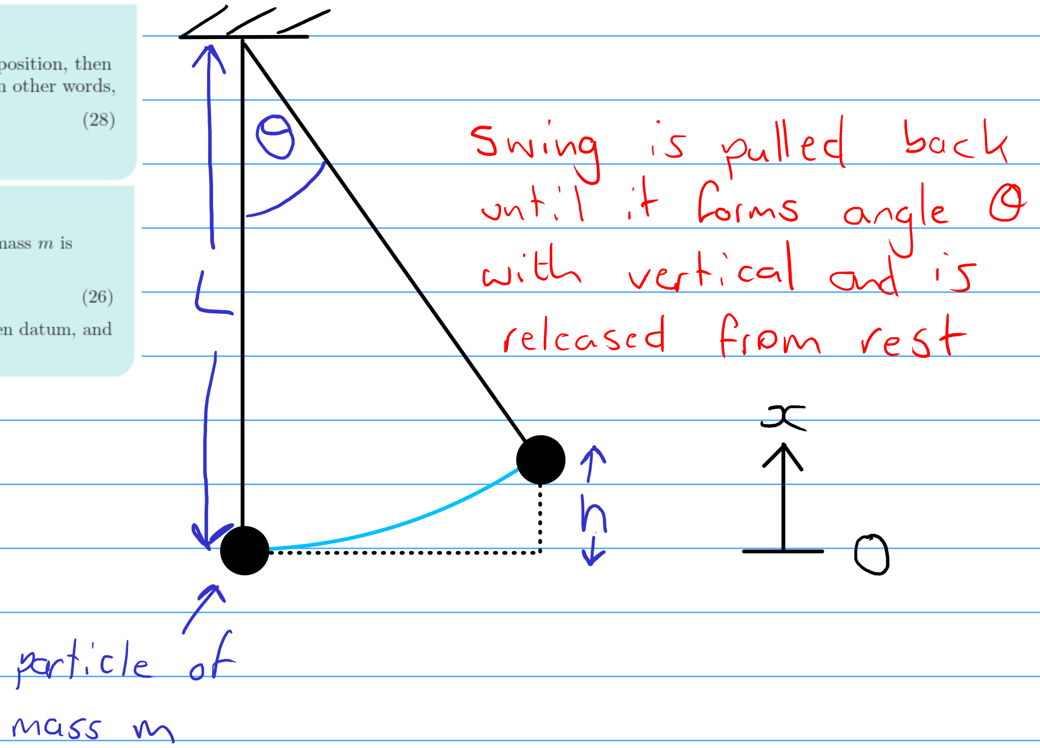
is a constant.

Gravitational potential energy

The **gravitational potential energy** of a particle of mass m is given by

$$U = mg \times \text{height above datum}, \tag{26}$$

where the height is measured vertically from some chosen datum, and g is the magnitude of the acceleration due to gravity.



At $t=0$, the swing has no kinetic energy and gravitational potential energy $U = mgh$, where m is the mass, g acceleration due to gravity, and h is the vertical height above the equilibrium position, given by

$$\begin{aligned} h &= L - L\cos\theta \\ &= L(1-\cos\theta) \end{aligned}$$

So the total energy E at $t=0$ is

$$\begin{aligned} E_1 &= U \\ &= mgh \\ &= mgL(1-\cos\theta) \end{aligned}$$

At the equilibrium position, the gravitational potential energy is 0 and the kinetic energy is $K = \frac{1}{2}mv^2$, where v is the velocity so at the equilibrium position the total energy is

$$E_2 = \frac{1}{2}mv^2$$

The law of conservation of energy tells us that

$$\Delta E = E_1 - E_2$$
$$= 0$$
$$E_1 = E_2$$
$$mgL(1 - \cos\Theta) = \frac{1}{2}mv^2$$

So now we have a model that can give us the swing's velocity as a function of its mass, the length of the chain, and the angle it makes with the vertical when released.

From here can we find the trajectories of projectiles released with velocity V at angles through the swing's cycle?