Complete Solutions to the Physics GRE

Exam #8677

Taylor Faucett

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Chapter 1

Physics GRE Solutions

1.1 PGRE8677 #1

1. A rock is thrown vertically upward with initial speed v_0. Assume a friction force proportional to $-\mathbf{v}$, where $\mathbf{v}$ is the velocity of the rock, and neglect the buoyant force exerted by air. Which of the following is correct?

   (A) The acceleration of the rock is always equal to $\mathbf{g}$.
   (B) The acceleration of the rock is equal to $\mathbf{g}$ only at the top of the flight.
   (C) The acceleration of the rock is always less than $\mathbf{g}$.
   (D) The speed of the rock upon return to its starting point is v_0.
   (E) The rock can attain a terminal speed greater than v_0 before it returns to its starting point.

Recommended Solution

(A) In a frictionless environment, this would be true. However, note that the problem tells us that friction is not negligible (in fact it is proportional to the rocks velocity $-v$) so it isn't possible for the acceleration to always be g.

(B) Since the frictional force is always fighting the rocks motion proportional to its speed, the only point at which frictional forces disappear will be when the rock stops moving ($v = 0$) at the top of its flight. At this point, the only acceleration on the rock will then be the acceleration due to gravity.

(C) As we just demonstrated in (B), the acceleration can be g at the top of the rocks flight

(D) In a frictionless environment, kinetic energy at the bottom of the rocks path will be converted to potential at the peak of its flight. The rock will then head back to its initial point converting all potential back to kinetic energy. However, in an environment with non-negligible friction, some of that initial kinetic energy is lost to the surroundings through heat, sound, etc and so the initial energy can never be recovered at the end of the rocks flight.

(E) Even in an ideal (i.e. frictionless) environment, this can't be true. This could only be true if there was some force applied to the rock on the way down which is not the case.

Correct Answer
(B)

Alternate Solution

Sum the forces in the vertical direction for the rocks motion,

$$F_{tot} = -F_{fric} - F_G \tag{1.1}$$
$$F_{tot} = -kv - mg \tag{1.2}$$

(A) From the equation, we can see that it is not generally true that the total acceleration on the rock is simply g

(B) Plug in $v = 0$ and g becomes the only acceleration and so this is correct

(C) As demonstrated in (B), acceleration is not less than g when $v = 0$

(D) This can't be true because energy is dissipated from friction so the final velocity can't be equal to its initial velocity

(E) Conservation of energy won't allow you to have more velocity in the end than you started out with and we would expect to have a lower final velocity than initial velocity based on dissipation of energy from friction

Correct Answer
(B)

1.2 PGRE8677 #2

2. A satellite orbits the Earth in a circular orbit. An astronaut on board perturbs the orbit slightly by briefly firing a control jet aimed toward the Earth's center. Afterward, which of the following is true of the satellite's path?

(A) It is an ellipse.
(B) It is a hyperbola.
(C) It is a circle with larger radius.
(D) It is a spiral with increasing radius.
(E) It exhibits many radial oscillations per revolution.

Recommended Solution

We can first eliminate (D) because we would need a continuous force pushing the satellite away from the Earth to get an orbit spiraling outward. We can then eliminate (E) because oscillations arent going to occur unless the potential energy of the satellite remains the same after the force is applied, which isnt the case. Next, eliminate (C) because, as a general rule, when we perturb an orbit with a brief bit of thrust, the satellite will return to the same point on the next pass but the rest of its orbit will be altered, so it cant be circular again. Finally, when choosing between (A) and (B), a small energy change from a circular orbit is more likely to become an elliptical orbit and, in general, elliptical orbits are more common for lower energy orbits, like a satellite orbiting earth.

Correct Answer
(A)

1.3 PGRE8677 #3

3. For blue light, a transparent material has a relative permittivity (dielectric constant) of 2.1 and a relative permeability of 1.0. If the speed of light in a vacuum is c, the phase velocity of blue light in an unbounded medium of this material is

(A) $\sqrt{3.1}\, c$

(B) $\sqrt{2.1}\, c$

(C) $\frac{c}{\sqrt{1.1}}$

(D) $\frac{c}{\sqrt{2.1}}$

(E) $\frac{c}{\sqrt{3.1}}$

Recommended Solution

Recall from quantum mechanics (specifically from the relativistic de Broglie relations) that wave speed is

$$\begin{aligned} v_\text{p} &= \frac{E}{p} && (1.3)\\ &= \frac{\gamma m c^2}{\gamma m v} && (1.4)\\ &= \frac{c^2}{v} && (1.5)\end{aligned}$$

The velocity of light through a medium is related to its relative permittivity and relative permeability by

$$v = \frac{1}{\epsilon\mu} \tag{1.6}$$

Plug this in and get

$$v_\text{p} = \frac{c^2}{\epsilon\mu} = \frac{c}{\sqrt{2.1}} \tag{1.7}$$

Correct Answer
(D)

Alternate Solution

It is generally true that only particles with mass (i.e. not a photon) will have phase velocities faster than the speed of light. Checking the possibilities

(A) $\sqrt{3.1}c \approx 1.7c$

(B) $\sqrt{2.1}c \approx 1.4c$

(C) $c/\sqrt{1.1} \approx 0.95c$

(D) $c/\sqrt{2.1} \approx 0.7c$

(E) $c/\sqrt{3.1} \approx 0.56c$

From this you can comfortably eliminate (A) and (B) and cautiously eliminate (C) because it is nearly the speed of light. With (D) and (E), ask yourself whether it makes more sense for the permittivity and permeability to be additive (i.e. $1.0 + 2.1$) or multiplicative (i.e. $(1.0)(2.1)$). Even if you can't recall that the velocity of an electromagnetic wave is the inverse product of the two values, you could at least make an educated guess by recalling that every time you have ever seen the permittivity of free space (ϵ_0), it has always acted as a scaling value. For example in the Gravitational Potential equation

$$U_G = \frac{1}{4\pi\epsilon_0}\frac{mM}{r} \tag{1.8}$$

Correct Answer
(D)

1.4 PGRE8677 #4

4. The equation $y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$, where A, T, and λ are positive constants, represents a wave whose

(A) amplitude is $2A$

(B) velocity is in the negative x-direction

(C) period is $\frac{T}{\lambda}$

(D) speed is $\frac{x}{t}$

(E) speed is $\frac{\lambda}{T}$

Recommended Solution

The general equation of a traveling wave is

$$y(x,t) = A\sin\left(\frac{2\pi}{\lambda}\right)(x \pm vt) \tag{1.9}$$

Where A is the amplitude, T is the period, λ is wavelength, t is time and x is position.

(A) The amplitude in this instance is A. In order to get an amplitude of $2A$, we would need our equation to have the amplitdue, A, multiplied by 2

(B) From the general traveling wave equation, if the $\pm$ ends up being a negative then the wave propagates in the positive direction and the converse is also true.

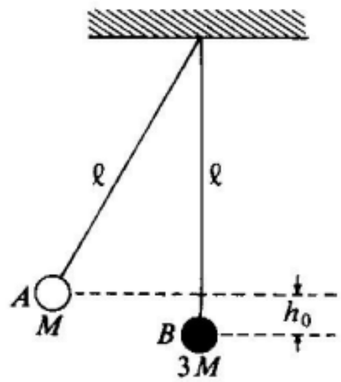
(C) The period is $T = 1/f$ and propagation velocity is $v = f\lambda$ so $T = \lambda/v$ not $T = T/\lambda$. Also, T/λ would give the wrong units for the period.

(D) Propagation speed is $v = f\lambda$ and the period $T = 1/f$ so $v = \lambda/T$, not x/t.

(E) Propagation speed is $v = f\lambda$ and the period $T = 1/f$ so $v = \lambda/T$

Correct Answer
(E)

1.5 PGRE8677 #5



5. Two small spheres of putty, *A* and *B*, of mass *M* and 3*M*, respectively, hang from the ceiling on strings of equal length ℓ. Sphere *A* is drawn aside so that it is raised to a height h_0 as shown above and then released. Sphere *A* collides with sphere *B*; they stick together and swing to a maximum height *h* equal to

(A) $\frac{1}{16}h_0$ (B) $\frac{1}{8}h_0$ (C) $\frac{1}{4}h_0$
(D) $\frac{1}{3}h_0$ (E) $\frac{1}{2}h_0$

Recommended Solution

This problem asks us about an inelastic collision so we can't assume that energy is conserved between the initial and final stages of the swinging spheres. Break the problem up into 3 separate time frames with the first phase involving just the falling of sphere A. In this phase, the initial potential energy is equal to the kinetic energy immediately before the two spheres collide

$$Mgh_0 = \frac{1}{2}Mv_0^2 \tag{1.10}$$

so the velocity before impact is

$$v_0 = \sqrt{2gh_0} \tag{1.11}$$

In the second phase, consider the collision of the ball, which should conserve momentum

$$Mv_o = (M + 3M)v_f = 4Mv_f \tag{1.12}$$

combine Equation 1.11 and Equation 1.12 to get

$$M\left(\sqrt{2gh_0}\right) = 4Mv_f \tag{1.13}$$

so the final velocity of the spheres after they've combined is

$$v_f = \sqrt{\frac{2gh_0}{16}} = \sqrt{\frac{gh_0}{8}} \tag{1.14}$$

finally, in the third phase, we know that the kinetic energy of the combined spheres immediately after collision will be equal to the potential energy at their peak

$$\frac{1}{2}(4M)v_f^2 = (4M)gh_f \tag{1.15}$$

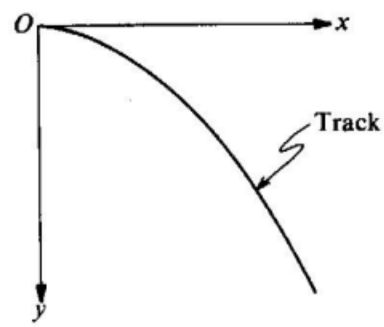
Combine Equation 1.14 and 1.15 to get

$$\frac{1}{2}\left(\sqrt{\frac{gh_0}{8}}\right)^2 = gh_f \tag{1.16}$$

$$h_f = \frac{1}{16}h_0 \tag{1.17}$$

Correct Answer
(A)

1.6 PGRE8677 #6



6. A particle is initially at rest at the top of a curved frictionless track. The x- and y-coordinates of the track are related in dimensionless units by $y = \frac{x^2}{4}$, where the positive y-axis is in the vertical downward direction. As the particle slides down the track, what is its tangential acceleration?

(A) 0

(B) g

(C) $\frac{gx}{2}$

(D) $\frac{gx}{\sqrt{x^2+4}}$

(E) $\frac{gx^2}{\sqrt{x^2+16}}$

Recommended Solution

Note that as $x \to \infty$, the track becomes infinitely steep and the particle should free fall at an acceleration of g. Only (D) approaches g in this limit.

Correct Answer
(D)

Alternate Solution

Consider the particle at some point on the curve and break it up into its components (Figure 1.1)

The tangential acceleration is going to be the component of the acceleration due to gravity multiplied by a unit vector in the same direction as the tangent

$$a_t = \vec{g} \cdot \hat{t} \tag{1.18}$$

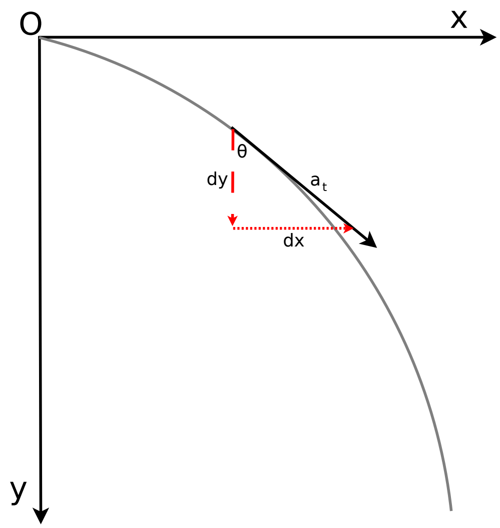


Figure 1.1: Velocity components on a particle as it moves through an arc

since we are talking about a simple unit vector, take the dot product of the two and get

$$a_t = g\cos(\theta) \tag{1.19}$$

and with the angle θ, we get

$$a_t = g\cos(\theta) = g\frac{dy}{a_t} = g\frac{dy}{\sqrt{dx^2 + dy^2}} \tag{1.20}$$

recall that

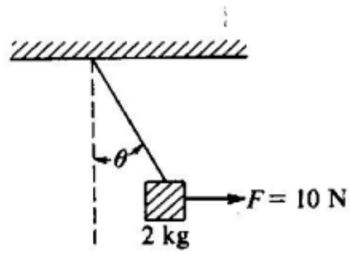
$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2}{4}\right) = \frac{x}{2} \tag{1.21}$$

so Equation 1.20 becomes

$$a_t = \frac{gy'}{\sqrt{1 + (y')^2}} = \frac{gx}{\sqrt{4 + x^2}} \tag{1.22}$$

Correct Answer
(D)

1.7 PGRE8677 #7



7. A 2-kilogram box hangs by a massless rope from a ceiling. A force slowly pulls the box horizontally to the side until the horizontal force is 10 newtons. The box is then in equilibrium as shown above. The angle that the rope makes with the vertical is closest to

(A) arctan 0.5
(B) arcsin 0.5
(C) arctan 2.0
(D) arcsin 2.0
(E) 45°

Recommended Solution

Draw out your force diagram with the applied force shown pulling with $10N$ to the right and a tension force with an x component in the opposite direction of the applied force and a y component along the dotted line (pointing up). When the box is in equilibrium, acceleration will be 0 so we are concerned with the point at which the applied force of $10N$ is equal to the opposing tension force

$$F_{T-x} = 10N \tag{1.23}$$

Take $\tan(\theta)$ to get

$$\tan(\theta) = \frac{F_{T-x}}{F_{T-y}} \tag{1.24}$$

and re-arrange it to get

$$F_{T-x} = F_{T-y}\ \tan(\theta) \tag{1.25}$$

The force in the vertical direction will only be due to gravity so we can use $F_{T-y} = 20N$ as an approximation. Plug this into Equation 1.25 and then combine F_{T-x} with Equation 1.23 to get

$$10N = (20N)\tan(\theta) \tag{1.26}$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) \tag{1.27}$$

Correct Answer
(A)

1.8 PGRE8677 #8

8. A 5-kilogram stone is dropped on a nail and drives the nail 0.025 meter into a piece of wood. If the stone is moving at 10 meters per second when it hits the nail, the average force exerted on the nail by the stone while the nail is going into the wood is most nearly

(A) 10 N
(B) 100 N
(C) 1000 N
(D) 10,000 N
(E) 100,000 N

Recommended Solution

Using our kinematic equation

$$v^2 = v_0^2 + 2a\Delta x \tag{1.28}$$

and assuming the initial velocity is zero, solve for the acceleration

$$(10 \text{ m/s})^2 = 2a(0.025 \text{ m}) \tag{1.29}$$
$$a = 2000 \text{ m/s}^2 \tag{1.30}$$

With a mass of 5 kg, use Newton's second law to get

$$F = (5 \text{ kg})(2000 \text{ m/s}^2) = 10,000 \text{ N} \tag{1.31}$$

Correct Answer
(D)

Alternate Solution

The kinetic energy of the stone right at impact will be

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}(5\ kg)(10\ m/s)^2 = 250\ J \tag{1.32}$$

This calculation doesn't account for the small potential energy the stone will have once it has hit the nail and it also makes the assumption (which the problem doesn't necessarily give us) that energy is being conserved. However, as an approximation, let's assume that all energy is accounted for and conserved so we can let all the kinetic energy translate into the work done when moving the nail

$$W = F \cdot \Delta x = 250\ J \tag{1.33}$$
$$F = \frac{250 \text{ J}}{0.025 \text{ m}} = 10,000 \text{ N} \tag{1.34}$$

Correct Answer
(D)

1.9 PGRE8677 #9

9. A wire of diameter 0.02 meter contains 1×10^{28} free electrons per cubic meter. For an electric current of 100 amperes, the drift velocity for free electrons in the wire is most nearly

(A) 0.6×10^{-29} m/s

(B) 1×10^{-19} m/s

(C) 5×10^{-10} m/s

(D) 2×10^{-4} m/s

(E) 8×10^{3} m/s

Recommended Solution

The drift velocity of a charged particle due to an electric field is

$$v_d = \frac{i}{nqA} \tag{1.35}$$

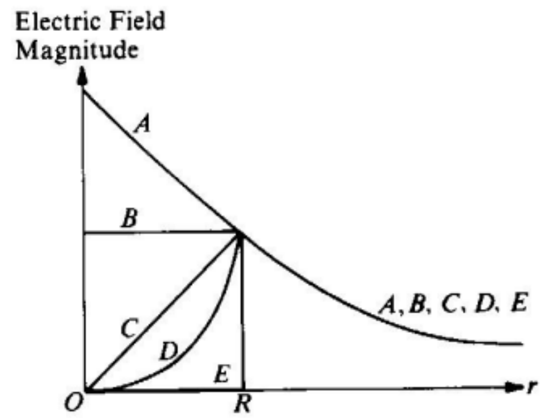
where i is the current, n is the number of particles, q is the charge of the particle and A is the cross sectional area. We could calculate the value exactly but it is generally quicker and sufficient, especially with the wide range of choices, to do a quick approximation.

$$v_d = \frac{100\ A}{(1 \times 10^{28}\ e^-/m^3)(2 \times 10^{-19}\ C)(3 \times 10^{-3}\ m^2)} \approx 2 \times 10^{-5} \tag{1.36}$$

which is closest to (D)

Correct Answer
(D)

1.10 PGRE8677 #10



10. An isolated sphere of radius R contains a uniform volume distribution of positive charge. Which of the curves on the graph above correctly illustrates the dependence of the magnitude of the electric field of the sphere as a function of the distance r from its center?

(A) A (B) B (C) C (D) D (E) E

Recommended Solution

From Gauss's law, we have

$$\oint_S E \cdot dA = \frac{Q}{\epsilon_0} \tag{1.37}$$

For a sphere, the surface area dA will be $4\pi r^2$ and the charge volume Q will be the volume of a sphere, $Q = 4/3\ \pi r^3$. Plug things in to get

$$E\left(4\pi r^2\right) = \left(\frac{4}{3\epsilon_0}\pi r^3\right) \tag{1.38}$$

$$E = \frac{r}{3\epsilon_0} \tag{1.39}$$

so the curve must be linear, i.e. (C)

Correct Answer
(C)

1.11 PGRE8677 #11

11. **Which of the following equations is a consequence of the equation $\nabla \times \mathbf{H} = \dot{\mathbf{D}} + \mathbf{J}$?**

(A) $\nabla \cdot (\dot{\mathbf{D}} + \mathbf{J}) = 0$

(B) $\nabla \times (\dot{\mathbf{D}} + \mathbf{J}) = 0$

(C) $\nabla (\dot{\mathbf{D}} \cdot \mathbf{J}) = 0$

(D) $\dot{\mathbf{D}} + \mathbf{J} = 0$

(E) $\dot{\mathbf{D}} \cdot \mathbf{J} = 0$

Recommended Solution

You should recall from quantum mechanics (especially if you were forced to do the rigorous proof from Griffith's canonical quantum mechanics textbook) that the divergence of the curl of any vector field is always 0. Take the divergence of each side to get

$$\nabla \cdot \left(\dot{D} + J\right) = \nabla \cdot (\nabla \times H) \tag{1.40}$$

$$\nabla \cdot \left(\dot{D} + J\right) = 0 \tag{1.41}$$

Correct Answer
(A)

1.12 PGRE8677 #12

12. A source of 1-kilohertz sound is moving straight toward you at a speed 0.9 times the speed of sound. The frequency you receive is

(A) 0.1 kHz
(B) 0.5 kHz
(C) 1.1 kHz
(D) 1.9 kHz
(E) 10 kHz

Recommended Solution

Without knowing much about the physics involved, we can solve this problem with a bit of musical knowledge. From the Doppler effect, we know that pitch (frequency) goes up as a source of sound moves toward you, eliminating (A) and (B). Next, recall that the next octave of any tone is twice that of the original, meaning that (C) and (D) would both be suggesting that even at near the speed of light the source hasn't even exceeded an octave in sound. (E) is the most reasonable solution.

Correct Answer
(E)

Alternate Solution

If you recall the equation for the frequency of a moving sound wave, you can calculate the exact solution

$$f = \left[\frac{1}{1 \pm \frac{v_{source}}{v_{wave}}}\right] f_0 \tag{1.42}$$

Plug everything into Equation 1.42 to get

$$f = \left[\frac{1}{1 - 0.9}\right] f_0 = 10 f_0 \tag{1.43}$$

Correct Answer
(E)

1.13 PGRE8677 #13

13. Two coherent sources of visible monochromatic light form an interference pattern on a screen. If the relative phase of the sources is varied from 0 to 2π at a frequency of 500 hertz, which of the following best describes the effect, if any, on the interference pattern?

(A) It is unaffected because the frequency of the phase change is very small compared to the frequency of visible light.
(B) It is unaffected because the frequency of the phase change is an integral multiple of π.
(C) It is destroyed except when the phase difference is 0 or π.
(D) It is destroyed for all phase differences because the monochromaticity of the sources is destroyed.
(E) It is not destroyed but simply shifts positions at a rate too rapid to be detected by the eye.

Recommended Solution

We can eliminate (A) and (B) because we know that the interference pattern should be affected, at the very least we know this is the case for the interference extrema as the relative phase changes. Eliminate (C) because the interference pattern certainly wont be destroyed for all but two points. Finally, eliminate (D) because the monochromacity is not destroyed and, even if it stopped being monochromatic, that wouldnt stop an interference pattern. Alternatively, instead of doing process of elimination you can directly figure that (E) is correct because the Flicker-Fusion Threshold (The frequency at which an objects movement starts to become imperceptible to the human eye) occurs at roughly 60 Hz, which is a far cry from the 500 Hz given in the problem.

Correct Answer
(E)

1.14 PGRE8677 #14

14. For an ideal gas, the specific heat at constant pressure C_p is greater than the specific heat at constant volume C_v because the
(A) gas does work on its environment when its pressure remains constant while its temperature is increased
(B) heat input per degree increase in temperature is the same in processes for which either the pressure or the volume is kept constant
(C) pressure of the gas remains constant when its temperature remains constant
(D) increase in the gas's internal energy is greater when the pressure remains constant than when the volume remains constant
(E) heat needed is greater when the volume remains constant than when the pressure remains constant

Recommended Solution

When you hear ideal gas law, always go straight to the ideal gas law equation (the 'Piv-nert' equation)

$$PV = nRT \tag{1.44}$$

(A) Recall from the first law of thermodynamics that work done on the system is, $W = PdV$. Combining this with the ideal gas law shows that (A) is true.

(B) Recall that $C_v = \left|\frac{dQ}{dT}\right|_v$ and $C_p = \left|\frac{dQ}{dT}\right|_p$. If (B) was correct, then $\left|\frac{dQ}{dT}\right|_v = \left|\frac{dQ}{dT}\right|_p = C_p = C_v$ which the initial question clearly tells us is not true.

(C) This isn't necessarily true, especially with changes in volume.

(D) Recall that $C_v = \left|\frac{dU}{dT}\right|$ which tells us that the energy of an ideal gas is only dependent on temperature.

(E) Presumably the correct answer will never be an ill-formed question (or it won't be graded if it is) so exclude (E)

Correct Answer
~~(E)~~

1.15 PGRE8677 #15

15. A sample of N atoms of helium gas is confined in a 1.0 cubic meter volume. The probability that none of the helium atoms is in a 1.0×10^{-6} cubic meter volume of the container is

(A) 0 (B) $(10^{-6})^N$ (C) $(1 - 10^{-6})^N$
(D) $1 - (10^{-6})^N$ (E) 1

Recommended Solution

If we had no atoms in the box ($N = 0$) then the probability of not finding an atom would be 1. From this, eliminate (A) and (D). When we have only 1 atom in the box ($N = 1$) it would be very unlikely that the single atom would occupy the specific volume of 1.0×10^{-6} cm^3 so the probability of not finding it there should be close to 1 but not exactly 1. From this, eliminate (B) and (E).

Correct Answer
(C)

Alternate Solution

If we were to break the box up into small cubes, each with a volume of 1.0×10^{-6} cm^3, we would see that the total numbers of cubes in the container is

$$\frac{\text{cubes}}{\text{container}} = \frac{1.0 \times 10^{-6}\ \text{cm}^3}{1\ \text{cm}^3} = 10^6\ \text{cubes} \tag{1.45}$$

Assuming that the probability of being in any given cube in the container is equally likely we would expect the probability of finding 1 atom in a single cube to be

$$P_{1\ \text{atom}} = \frac{1}{10^6} \tag{1.46}$$

Keep in mind, however, that this is the probability of finding the atom inside a single small cube. What we want is the probability that we DON'T find the atom inside that cube. From this, we know that the probability of the atom being outside the cube will be

$$P_{\text{out}} = 1 - P_{\text{in}} \tag{1.47}$$

$$P_{\text{out}} = 1 - \left(10^{-6}\right) \tag{1.48}$$

Extrapolating this to multiple atoms, we use the multiplication rule to get

$$P_{\text{out}} = \left(1 - 10^{-6}\right)^n \tag{1.49}$$

Correct Answer
(C)

1.16 PGRE8677 #16

16. Except for mass, the properties of the muon most closely resemble the properties of the

(A) electron (B) graviton (C) photon (D) pion (E) proton

Recommended Solution

The muon is a lepton/fermion (spin-1/2) and has charge of -1

(A) electron Electrons are leptons/fermions (spin-1/2) and have a charge of -1. They differ from the muon in that their mass is about ($1/200^{\text{th}}$) that of a muon. Muons are are frequently referred to as heavy electrons because of their similar properties.

(B) graviton You could probably immediately throw this option out because we aren't even particularly sure that these exist. However, if it exists, the graviton should be massless, have no charge, have spin-2 and is a force particle (boson).

(C) photon Massless, no charge (at least for our purposes), spin-1 and a force particle (boson).

(D) pion Not an elementary particle, depending on which $\pi -$ meson you get spin (+1, -1 or 0), depending on which $\pi -$ meson you get charge (0, +e, -e).

(E) proton Not an elementary particle (hadron) and has a charge of +1.

Correct Answer
(A)

1.17 PGRE8677 #17

17. Suppose that ${}^{A}_{Z}X$ decays by natural radioactivity in two stages to ${}^{A-4}_{Z-1}Y$. The two stages would most likely be which of the following?

	First Stage	Second Stage
(A)	β^- emission with an antineutrino	α emission
(B)	β^- emission	α emission with a neutrino
(C)	β^- emission	γ emission
(D)	Emission of a deuteron	Emission of two neutrons
(E)	α emission	γ emission

Recommended Solution

The problem tells us that we need to get from ${}^{A}_{Z}X$ to ${}^{A-4}_{Z-1}Y$ in two steps,

$$ {}^{A}_{Z}X \rightarrow^{?}_{?} X' \rightarrow^{?}_{?} X'' \rightarrow^{A-4}_{Z-1} Y \tag{1.50}$$

in β^- decay, we have

$$ {}^{A}_{Z}X \rightarrow^{A}_{Z+1} X' + e^- + \bar{v}_e \tag{1.51}$$

for α decay,

$$ {}^{A}_{Z}X \rightarrow^{A-4}_{Z-2} X' + \alpha \tag{1.52}$$

and for γ decay,

$$ {}^{A}_{Z}X \rightarrow^{A}_{Z+1} X' + \gamma \tag{1.53}$$

From these, it should be clear that we can get our solution by the combination

$$ {}^{A}_{Z}X \rightarrow \beta^- \text{ decay} \rightarrow \alpha \text{ decay} \rightarrow^{A-4}_{Z-1} Y $$

$$ {}^{A}_{Z}X \rightarrow^{A}_{Z+1} X' \rightarrow^{A-4}_{Z-2} X'' \rightarrow^{A-4}_{Z-1} Y $$

From this, it should be clear that the only two step process which will give us

Correct Answer
(A)

1.18 PGRE8677 #18

18. The wave function $\psi(x) = A \exp\left\{-\frac{b^2x^2}{2}\right\}$, where A and b are real constants, is a normalized eigenfunction of the Schrödinger equation for a particle of mass M and energy E in a one dimensional potential $V(x)$ such that $V(x) = 0$ at $x = 0$. Which of the following is correct?

(A) $V = \frac{\hbar^2 b^4}{2M}$

(B) $V = \frac{\hbar^2 b^4 x^2}{2M}$

(C) $V = \frac{\hbar^2 b^6 x^4}{2M}$

(D) $E = \hbar^2 b^2(1 - b^2x^2)$

(E) $E = \frac{\hbar^2 b^4}{2M}$

Recommended Solution

Recall the time-independent Schrӧdinger equation (Equation 1.54)

$$E\psi(x) = -\frac{\hbar^2}{2m}\nabla^2\psi(x) + V(x)\psi(x) \tag{1.54}$$

take the second derivative of $\psi(x)$ to get

$$\begin{aligned} \psi'(x) &= -b^2x\psi(x) && (1.55)\\ \psi''(x) &= b^4x^2\psi(x) && (1.56) \end{aligned}$$

plug everything in to get

$$E\psi(x) = -\frac{\hbar^2}{2M}\left[b^4x^2\psi(x)\right] + V(x)\psi(x) \tag{1.57}$$

and simplify Equation 1.57 to get

$$E = -\frac{\hbar^2 b^4 x^2}{2M} + V(x) \tag{1.58}$$

Plug $x = 0$ into Equation 1.58 to use the condition, $V(0) = 0$,

$$E = -\frac{\hbar^2 b^4 (0)^2}{2M} + V(0) = 0 \tag{1.59}$$

Which tells us that all of the energy we found previously is accounted for entirely by $V(x)$,

$$V(x) = \frac{\hbar^2 b^4 x^2}{2M} \tag{1.60}$$

Additionally, you can also get to this conclusion by realizing that E shouldn't have any dependence on x but that $V(x)$ should, so the only term we have must be accounted for by $V(x)$.

Correct Answer
(B)

Alternate Solution

We can immediately eliminate (A) because it lacks the necessary dependence on x. Next, we can eliminate (C) because the wavefunction (ψ) given makes our potential a quantum harmonic oscillator which must have a dependence on x^2 rather than x^4. Next, eliminate (D) because the energy, E, must have some dependence on the mass of the particle. This will get you to the point at which you can either guess between the two solutions or do enough of the time independent Schrdinger equation to see some dependence on x emerge.

Correct Answer
(B)

1.19 PGRE8677 #19

19. The energy levels of the hydrogen atom are given in terms of the principal quantum number n and a positive constant A by the expression

(A) $A\left(n+\frac{1}{2}\right)$

(B) $A(1-n^2)$

(C) $A\left(-\frac{1}{4}+\frac{1}{n^2}\right)$

(D) $A\,n^2$

(E) $-\frac{A}{n^2}$

Recommended Solution

Recall from the Bohr Model of the Hydrogen atom that the total energy of the atom in relation to n is

$$E_{tot} = \frac{-13.6 \text{ eV}}{n^2} \tag{1.61}$$

which agrees with (E) when $A = -13.6$ eV.

Correct Answer
(E)

1.20 PGRE8677 #20

20. A positive kaon (K^+) has a rest mass of 494 MeV/c^2, whereas a proton has a rest mass of 938 MeV/c^2. If a kaon has a total energy that is equal to the proton rest energy, the speed of the kaon is most nearly

(A) 0.25 *c*
(B) 0.40 *c*
(C) 0.55 *c*
(D) 0.70 *c*
(E) 0.85 *c*

Recommended Solution

Recall Einstein's famous equation for energy with a resting mass

$$E = mc^2 \tag{1.62}$$

and his lesser known equation of relativistic momentum

$$E = P_0 c^4 + m_0 c^2 \tag{1.63}$$

In the problem, the proton will need to use the resting mass energy equation and the kaon uses the relativistic momentum equation, giving

$$\begin{aligned} m_p c^2 &= P_k c^4 + m_k c^2 && (1.64)\\ m_p &= P_k c^2 + m_k && (1.65) \end{aligned}$$

We could plug in the exact values given, however we can simplify the calculations by using masses $K^+ = 500 MeV/c^2$ and $P^+ = 1000 MeV/c^2$, giving

$$\begin{aligned} (1000\ MeV/c^2)^2 &= P_k c^2 + (500\ MeV/c^2)^2 && (1.66)\\ P_k &= 750,000\ MeV/c^2 && (1.67) \end{aligned}$$

Recall that the relativistic momentum is

$$P_k = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \tag{1.68}$$

substituting that in to our previous equation gives

$$\frac{m_0 v}{\sqrt{1 - v^2/c^2}} = 750,000\ MeV/c^2 \tag{1.69}$$

you can get the velocity by itself by moving the denominator on the LHS to the RHS, squaring both sides and then grouping terms to get

$$V_k^2 = \frac{3}{4}c^2 \tag{1.70}$$

$$V_k = \sqrt{\frac{3}{4}}c \tag{1.71}$$

$$V_k \approx 0.86c \tag{1.72}$$

Correct Answer
(E)

1.21 PGRE8677 #21

21. Two observers O and O' observe two events, A and B. The observers have a constant relative speed of 0.8 c. In units such that the speed of light is 1, observer O obtained the following coordinates:

Event A: $x = 3,\ y = 3,\ z = 3,\ t = 3$

Event B: $x = 5,\ y = 3,\ z = 1,\ t = 5$

What is the length of the space-time interval between these two events, as measured by O'?

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) 3 (E) $2\sqrt{3}$

Recommended Solution

The equation for the space time interval is

$$\Delta s^2 = \Delta r^2 - c^2 \Delta t^2 \tag{1.73}$$

but since the problem specifies that $c = 1$, Equation 1.73 becomes

$$\Delta s^2 = \Delta r^2 - \Delta t^2 \tag{1.74}$$

Recall the fantastically cool fact that the Pythagorean theorem works the same in any number of dimensions, so the length Δr is

$$\begin{aligned} \Delta r^2 &= \Delta x^2 + \Delta y^2 + \Delta z^2 && (1.75) \\ &= 2^2 + 0^2 + 2^2 && (1.76) \\ &= 8 && (1.77) \end{aligned}$$

The time difference is $\Delta t = 2$ so

$$\begin{aligned} \Delta s^2 &= 8 - 4 && (1.78) \\ \Delta s &= 2 && (1.79) \end{aligned}$$

Correct Answer
(C)

1.22 PGRE8677 #22

22. Which of the following statements most accurately describes how an electromagnetic field behaves under a Lorentz transformation?

(A) The electric field transforms completely into a magnetic field.
(B) If initially there is only an electric field, after the transformation there may be both an electric and a magnetic field.
(C) The electric field is unaltered.
(D) The magnetic field is unaltered.
(E) It cannot be determined unless a gauge transformation is also specified.

Recommended Solution

Recall the electromagnetic field tensors

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix}$$

Which clearly demonstrates that both an electric field and/or a magnetic field may exist. This also happens to contradict (A), (C), and (D).

Correct Answer
(B)

1.23 PGRE8677 #23

23. Which of the following statements concerning the electrical conductivities at room temperature of a pure copper sample and a pure silicon sample is NOT true?

(A) The conductivity of the copper sample is many orders of magnitude greater than that of the silicon sample.
(B) If the temperature of the copper sample is increased, its conductivity will decrease.
(C) If the temperature of the silicon sample is increased, its conductivity will increase.
(D) The addition of an impurity in the copper sample always decreases its conductivity.
(E) The addition of an impurity in the silicon sample always decreases its conductivity.

Recommended Solution

Similar: GR0177 Q53 for the resistivity of undoped semicomductors

Most of these options can be analyzed by knowing that copper is a good conductor and silicon is generally used as a semi-conductor

(A) It is generally true that the conductivity is higher in magnitude for a conductor than a semi-conductor.

(B) Conductivity and temperature are inversely related for a conductor. In this specific option as the temperature goes up we would expect the conductivity to go down.

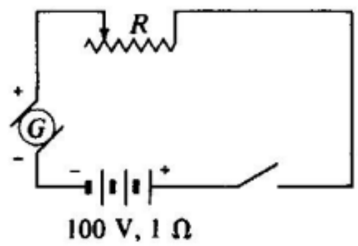
(C) For semi-conductors, as temperature increases, conductivity increases and vice versa.

(D) It is generally true that impurities in a conductor will decrease conductivity.

(E) The addition of impurities to a semi-conductor, commonly called doping, is done to increase conductivity rather than decrease conductivity. This is the only false statement of the bunch.

Correct Answer
(E)

1.24 PGRE8677 #24



24. The battery in the diagram above is to be charged by the generator *G*. The generator has a terminal voltage of 120 volts when the charging current is 10 amperes. The battery has an emf of 100 volts and an internal resistance of 1 ohm. In order to charge the battery at 10 amperes charging current, the resistance *R* should be set at

(A) 0.1 Ω (B) 0.5 Ω (C) 1.0 Ω (D) 5.0 Ω (E) 10.0 Ω

Recommended Solution

Recall Kirchhoff's voltage loop law which tells us that the sum of all voltages about a complete circuit must sum to 0

$$\sum V_k = 0 \tag{1.80}$$

Recalling that $V = IR$, sum the voltage of the battery, generator and resistor

$$G - V - V_R = 0 \tag{1.81}$$

However, since there is an internal resistance of 1 Ω, $V_R = (R + 1\ \Omega)I$

$$\begin{aligned} G - V - (R + 1\ \Omega)I &= 0 && (1.82)\\ 120 \text{ volts} - 100 \text{ volts} - R(10 \text{ amps}) - 10 \text{ volts} &= 0 && (1.83)\\ 10 \text{ volts} - R(10 \text{ amps}) &= 0 && (1.84)\\ R &= 1\ \Omega && (1.85) \end{aligned}$$

Correct Answer
(C)

1.25 PGRE8677 #25

25. A charged particle is released from rest in a region where there is a constant electric field and a constant magnetic field. If the two fields are parallel to each other, the path of the particle is a

(A) circle
(B) parabola
(C) helix
(D) cycloid
(E) straight line

Recommended Solution

From the lorentz force, we know that the magnetic field vector maximizes force when it is orthogonal to the electric field

$$
\begin{aligned}
F &= q\vec{E} + \left(qv \times \vec{B}\right) && (1.86)\\
&= q\vec{E} + qv\vec{B}\sin(\theta) && (1.87)
\end{aligned}
$$

Initially, the particle is at rest so the initial velocity is $v = 0$ and only the electric field will provide any force. Once the particle is in motion, however, the velocity is non-zero and will be moving in the direction of the electric field. Since this problem explicitly states that the electric field and magnetic field are parallel, we know that v and B are also parallel and thus, our angle is $\theta = 0$

$$F = q\vec{E} + qv\vec{B}\sin(0) = q\vec{E} \tag{1.88}$$

so we would only expect the electric field to influence the particle and for it to provide force solely in the direction of the field.

Correct Answer
(E)

1.26 PGRE8677 #26

26. A nickel target ($Z = 28$) is bombarded with fast electrons. The minimum electron kinetic energy needed to produce x-rays in the K series is most nearly

(A) 10 eV
(B) 100 eV
(C) 1000 eV
(D) 10,000 eV
(E) 100,000 eV

Recommended Solution

Henry Moseley discovered, from the Bohr model, that when an external electron strikes and knocks loose one of the innermost electrons, a gap is created which allows an electron from the next Bohr level ($n = 2$) to fall into its place. This transition is known as a K series transition. In general, all transitions from some Bohr level $n \geq 2$ down to $n = 1$ is one of the K series transitions (Figure 1.2)

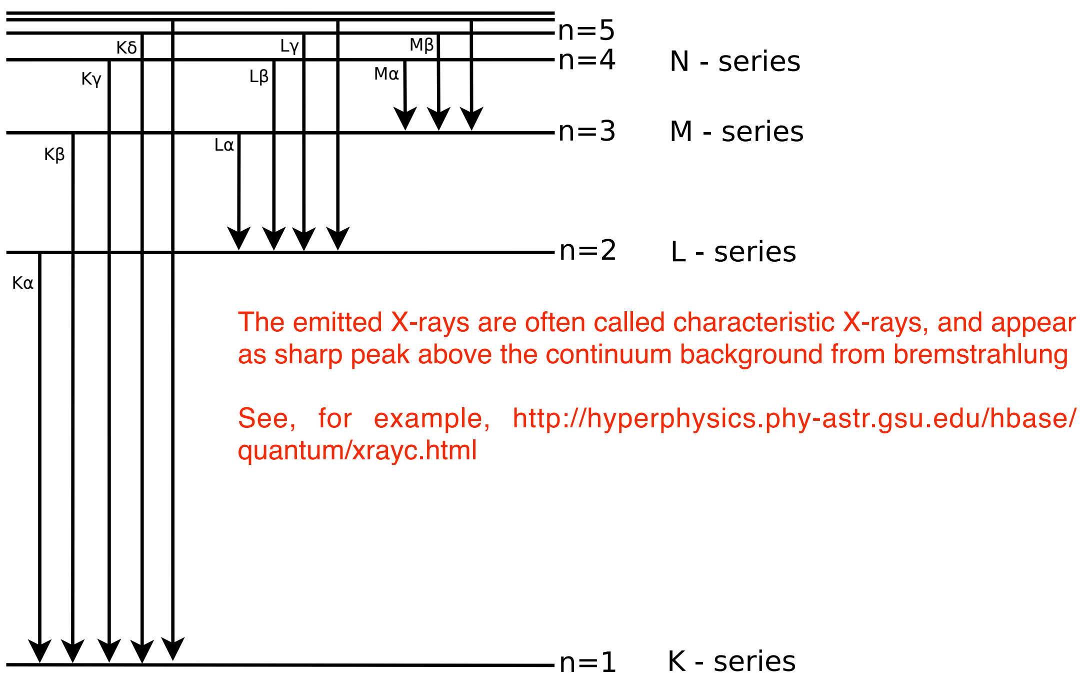


Figure 1.2: K, L and M series electron transitions

The specific transition we are concerned with (i.e. the minimum energy transition) is that of K_α so we can use Moseley's law

$$E = R_e \left(Z - \beta\right)^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{1.89}$$

where R_e is the Rydberg energy at $R_e = 13.6\ eV$ and β corresponds to the transition type (K, L, M, N, ...). To make things quicker, round all of your numbers off to get

$$E = (15\ eV)\,(30)^2\left(\frac{1}{1}-\frac{1}{4}\right) \tag{1.90}$$

$$E = 10,125 \tag{1.91}$$

$$E \approx 10,000 \tag{1.92}$$

Correct Answer
(D)

1.27 PGRE8677 #27

27. The hypothesis that an electron possesses spin is qualitatively significant for the explanation of all of the following topics EXCEPT the
    (A) structure of the periodic table
    (B) specific heat of metals
    (C) anomalous Zeeman effect
    (D) deflection of a moving electron by a uniform magnetic field
    (E) fine structure of atomic spectra

Recommended Solution

The solution to this answer should stick out like a sore thumb. Without knowing any information about any of the other possible answers, you should immediately know that any particle with a charge (like an electron) will experience deflection under a uniform magnetic field regardless of whether it has spin or not. The Lorentz force exhibits precisely this fact,

$$F = q\vec{E} + q\left(\vec{v} \times \vec{B}\right) \tag{1.93}$$

where q is the particles charge. In fact, it wasn't until we realized that neutral particles could experience deflection (I'm speaking of the Stern-Gerlach experiment)that we realized we had to incorporate particle spin into the mix.

Correct Answer
(D)

Alternate Solution

(A) Recall that electron spin, along with the Pauli exclusion principle, determined the arrangement of electrons in energy level diagrams and, from that, influences our understanding of the structure of the periodic table.

(B) Einstein and Debye models of specific heat both eventually incorporated electron contribution into their descriptions which necessitates a discussion on spin.

(C) The Zeeman effect is a method of distinguishing between electrons with the same energy level by instituting a magnetic field to influence them to different energies. The "anomalous Zeeman effect" appeared in all instances when the net spin of a grouping of electrons wasn't 0. This effect wasn't understood until electron spin was understood, hence why it was called "anomalous".

(D) Electron deflection could be explained using only the Lorentz force and its dependence on the electrons charge.

(E) The "Gross" atomic structure describes the splitting of line spectra without factoring in the effects of electron spin. "Fine" atomic structure was a correction to this model which factored in spin and relativistic effects.

Correct Answer
(D)

1.28 PGRE8677 #28

28. Eigenfunctions for a rigid dumbbell rotating about its center have a ϕ dependence of the form $\psi(\phi) = Ae^{im\phi}$, where m is a quantum number and A is a constant. Which of the following values of A will properly normalize the eigenfunction?

(A) $\sqrt{2\pi}$

(B) 2π

(C) $(2\pi)^2$

(D) $\dfrac{1}{\sqrt{2\pi}}$

(E) $\dfrac{1}{2\pi}$

Recommended Solution

The condition for normalizing a wavefunction is

$$\int \left|\psi^2(x)\right| = 1 \tag{1.94}$$

for this specific scenario, our wavefunction is for a rigid rotator with dependence on ϕ in which the dumbbell rotates from $\phi = -\pi$ to $\phi = \pi$

$$\int_{-\pi}^{\pi} \left|\psi^2(\phi)\right| = \int_{-\pi}^{\pi} A^2 e^{2im\phi} = 1 \tag{1.95}$$

Recall from doing Fourier series approximations that the RHS of Equation 1.95 is equivalent to $2\pi A^2$, giving us

$$2\pi A^2 = 1 \tag{1.96}$$

$$A = \frac{1}{\sqrt{2\pi}} \tag{1.97}$$

Correct Answer
(D)

1.29 PGRE8677 #29

29. A negative test charge is moving near a long straight wire in which there is a current. A force will act on the test charge in a direction parallel to the direction of the current if the motion of the charge is in a direction

(A) toward the wire
(B) away from the wire
(C) the same as that of the current
(D) opposite to that of the current
(E) perpendicular to both the direction of the current and the direction toward the wire

Recommended Solution

Use Fleming's left hand rule (Figure 1.3) to figure out the direction of force relative to a moving charge

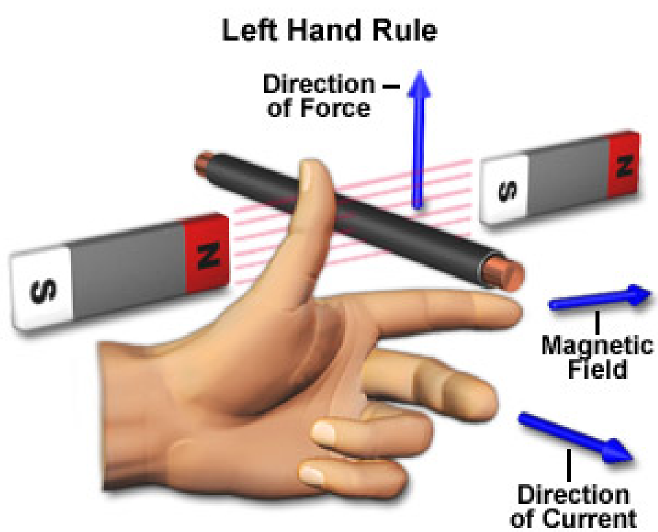


Figure 1.3: Fleming's left hand rule

Using the wire in the image as the wire in the problem, you should see that we want to rotate the disembodied left hand such that the force vector (thumb) is pointing in the same direction as the wire. Doing so results in the current vector (middle finger) pointing towards the wire.

Correct Answer
(A)

1.30 PGRE8677 #30

30. The configuration of the potassium atom in its ground state is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^1$. Which of the following statements about potassium is true?

(A) Its $n = 3$ shell is completely filled.
(B) Its $4s$ subshell is completely filled.
(C) Its least tightly bound electron has $\ell = 4$.
(D) Its atomic number is 17.
(E) Its electron charge distribution is spherically symmetrical.

Recommended Solution

(A) This could only be true if all subshells (S, P, D) were completely full, which isn't the case.

(B) Since the valence shell is $4s1$, it should be clear that this single electron doesn't fill out the subshell.

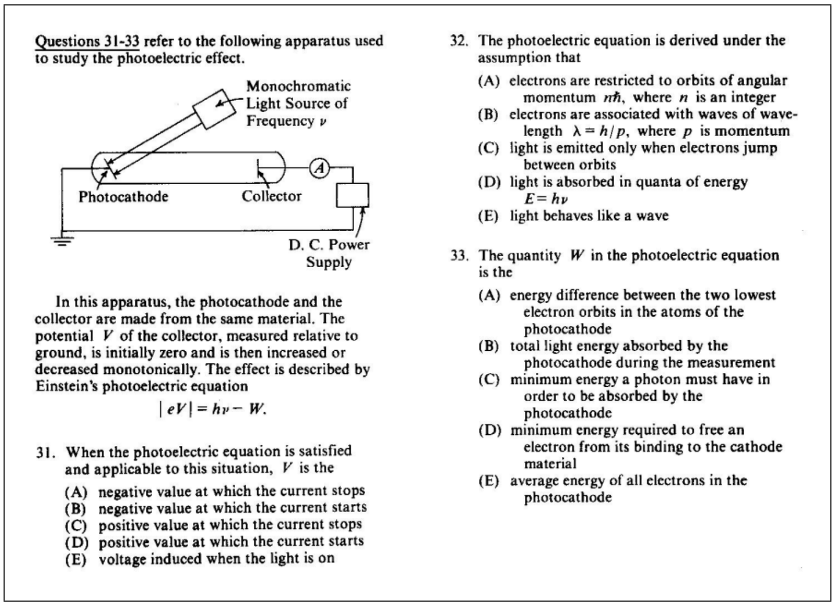
(C) We are only concerned with angular quantum number $l = 2$ which corresponds to s, not $l = 4$ which corresponds to g. Why does ETS keep asking about the wrong quantum numbers?

(D) You can figure that this isn't correct by having the periodic table memorized, in which case you would either know that Potassium has atomic number 19 or that the element with atomic number 17 is chlorine. Alternatively, because the electron configuration is described for its ground state, just add up all of the electrons (superscripts) to get a total of 19 electrons and, therefore, 19 protons.

(E) Atoms with a single electron in its outer shell can be thought of as analogous to a hydrogen atom. From this (i.e. without doing any painful calculations) you can convince yourself that the charge distribution of Potassium will be similar to that of the hydrogen atom, in fact spherical.

Correct Answer
(E)

1.31 PGRE8677 #31



Questions 31-33 refer to the following apparatus used to study the photoelectric effect.

In this apparatus, the photocathode and the collector are made from the same material. The potential V of the collector, measured relative to ground, is initially zero and is then increased or decreased monotonically. The effect is described by Einstein's photoelectric equation

$$|eV| = h\nu - W.$$

31. When the photoelectric equation is satisfied and applicable to this situation, V is the
    (A) negative value at which the current stops
    (B) negative value at which the current starts
    (C) positive value at which the current stops
    (D) positive value at which the current starts
    (E) voltage induced when the light is on

32. The photoelectric equation is derived under the assumption that
    (A) electrons are restricted to orbits of angular momentum $n\hbar$, where n is an integer
    (B) electrons are associated with waves of wavelength $\lambda = h/p$, where p is momentum
    (C) light is emitted only when electrons jump between orbits
    (D) light is absorbed in quanta of energy $E = h\nu$
    (E) light behaves like a wave

33. The quantity W in the photoelectric equation is the
    (A) energy difference between the two lowest electron orbits in the atoms of the photocathode
    (B) total light energy absorbed by the photocathode during the measurement
    (C) minimum energy a photon must have in order to be absorbed by the photocathode
    (D) minimum energy required to free an electron from its binding to the cathode material
    (E) average energy of all electrons in the photocathode

Recommended Solution

This problem is testing your knowledge of physics history. Phillip Lennard experimented with essentially the same setup (Figure ??) in the early stages of the development of the photoelectric effect. Lennard fired different frequencies of light at a photocell and collected the photoelectrons emitted. attached to his photocell was a variable power supply, a voltmeter and a micro-ammeter

As the light knocked electrons from the photocell, it would gain a positive charge. The second plate also gained a positive charge because it was connected in the same circuit as the photocell. ~~The recently freed electrons would then be attracted to the now positively charged second plate and would flow to it.~~ Finally, the electrons would move through the circuit back to the photocell and start the process over, generating a current. The variable power supply was placed in this circuit such that it would fight the flow of electrons from moving back to the photocell (which occurs because the negative end of the potential is attached to the end of the circuit not receiving light).

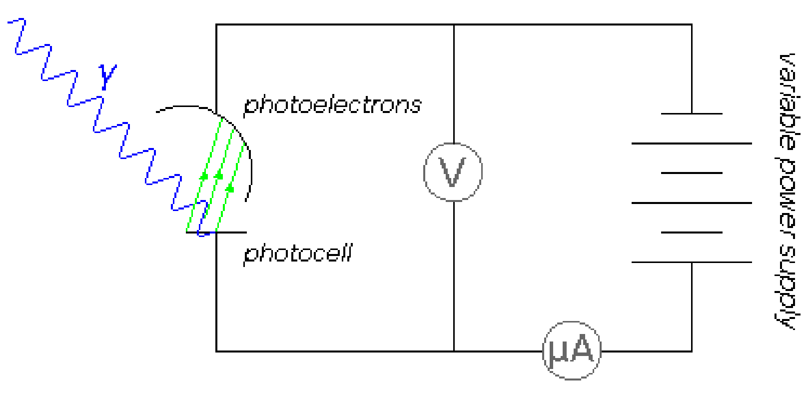


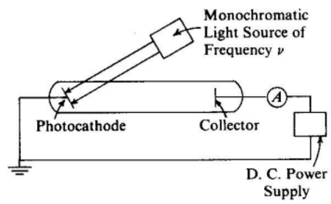
Figure 1.4: Phillip Lennard's experimental photocell arrangment

At low potentials, low energy electrons would be caught and pushed back while higher electrons could flow through, meaning the micro-ammeter would get a decreased reading. Lennard discovered that he could keep increasing the potential of the power supply until he reached a potential that would completely cease electron movement. At this point, that is when the micro-ammeter first reads 0, we've found the potential V on the collector. From this, eliminate all choices but (A) and (C). Lastly, you can determine that the potential must be negative because the potential of the power supply is fighting the electron movement so it must be negative valued.

Correct Answer
(A)

1.32 PGRE8677 #32

Questions 31-33 refer to the following apparatus used to study the photoelectric effect.



In this apparatus, the photocathode and the collector are made from the same material. The potential V of the collector, measured relative to ground, is initially zero and is then increased or decreased monotonically. The effect is described by Einstein's photoelectric equation

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    (C) minimum energy a photon must have in order to be absorbed by the photocathode
    (D) minimum energy required to free an electron from its binding to the cathode material
    (E) average energy of all electrons in the photocathode

Recommended Solution

(A) The photoelectric equation was derived well before quantum mechanics came around.

(B) The important aspect of the photoelectric effect was the discrete quantization of energy. Determination of an electrons wavelength won't get you there.

(C) Eliminate this choice because, aside from not being relevant, it isn't even true.

(D) This is the answer we've been looking for. The important distinction that the photoelectric effect gave us was that Energy is discrete and that it is dependent on the frequency of a light source, not the intensity.

(E) The photoelectric effect necessitates that we think of light as particles rather than waves.

Correct Answer
(D)

1.33 PGRE8677 #33

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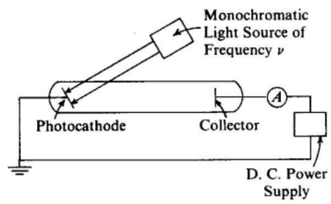

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    (D) minimum energy required to free an electron from its binding to the cathode material
    (E) average energy of all electrons in the photocathode

Recommended Solution

I think it's worth noting that this question does some strong hinting, whether intentional or unintentional, that the previous 2 questions can be solved without knowing much if anything about the equation they gave you. In the photoelectric equation, W represents the work function (in my experience, ϕ is more commonly used) and it represents the minimum amount of work (energy) necessary to pop an electron from a material. Thus, you can read the photoelectric equation from left to right as "The total kinetic energy of an ejected electron ($|eV|$) is equal to the energy of the photon that hit it ($h\nu$) minus the energy it required to pop the electron out in the first place (W)". Reading it like this should convince you that really the photoelectric equation is nothing more than a conservation of energy argument.

Correct Answer
(D)

1.34 PGRE8677 #34

Questions 34-36

The potential energy of a body constrained to move on a straight line is kx^4 where k is a constant. The position of the body is x, its speed v, its linear momentum p, and its mass m.

34. The force on the body is

(A) $\frac{1}{2}mv^2$ (B) $-4kx^3$ (C) kx^4

(D) $-\frac{kx^5}{5}$ (E) mg

35. The Hamiltonian function for this system is

(A) $\frac{p^2}{2m} + kx^4$ (B) $\frac{p^2}{2m} - kx^4$ (C) kx^4

(D) $\frac{1}{2}mv^2 - kx^4$ (E) $\frac{1}{2}mv^2$

36. The body moves from x_1 at time t_1 to x_2 at time t_2. Which of the following quantities is an extremum for the $x - t$ curve corresponding to this motion, if end points are fixed?

(A) $\int_{t_1}^{t_2} \left(\frac{1}{2}mv^2 - kx^4\right) dt$

(B) $\int_{t_1}^{t_2} \left(\frac{1}{2}mv^2\right) dt$

(C) $\int_{t_1}^{t_2} (mxv) dt$

(D) $\int_{x_1}^{x_2} \left(\frac{1}{2}mv^2 + kx^4\right) dx$

(E) $\int_{x_1}^{x_2} (mv) dx$

Recommended Solution

Recall that Work is force integrated over the area in which the force is applied and that work has the opposite sign of the potential energy ($-W = U$),

$$-W = U = -\int F \cdot ds \tag{1.98}$$

Since we are given the energy and asked to find force, differentiate your work equation to get the force equation

$$\frac{d}{dx}(U) = \frac{d}{dx}\left(-Kx^4\right) \tag{1.99}$$

$$F = -4kx^3 \tag{1.100}$$

Correct Answer
(B)

1.35 PGRE8677 #35

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(C) $\int_{t_1}^{t_2} (mxv) dt$

(D) $\int_{x_1}^{x_2} \left(\frac{1}{2}mv^2 + kx^4\right) dx$

(E) $\int_{x_1}^{x_2} (mv) dx$

Recommended Solution

You should remember that the Hamiltonian is the sum of kinetic energy and potential energy (as opposed to the Lagrangian which is the difference between the two). From this you should get

$$H = T + V \tag{1.101}$$

$$H = \frac{p^2}{2m} + kx^4 \tag{1.102}$$

In case you immediately think of $T = \frac{1}{2}mv^2$ for the kinetic energy, take a second to remind/convince yourself that

$$\frac{1}{2}mv^2 = \frac{p^2}{2m} \tag{1.103}$$

Correct Answer
(A)

1.36 PGRE8677 #36

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(C) $\int_{t_1}^{t_2} (mxv) dt$

(D) $\int_{x_1}^{x_2} \left(\frac{1}{2}mv^2 + kx^4\right) dx$

(E) $\int_{x_1}^{x_2} (mv) dx$

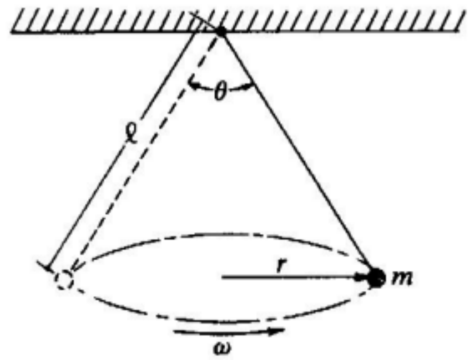
Recommended Solution

You should recall from any course that was heavy into mechanics, that the equations of motion for a system come out as a result of applying a minimization on the actions of the system, which is known as the principle of least action. From this, you should recall that our equations of motion are generally always time dependent, not position dependent and you can eliminate (D) and (E). As for the other three, you will just have to remember that the principle of least action is the integral with respect to time of the Lagrangian,

$$\int_{t_1}^{t_2} \frac{1}{2}mv^2 - kx^4 \tag{1.104}$$

Correct Answer
(A)

1.37 PGRE8677 #37



37. The figure above represents a point mass m attached to the ceiling by a cord of fixed length ℓ . If the point mass moves in a horizontal circle of radius r with uniform angular velocity ω, the tension in the cord is

(A) $mg\left(\frac{r}{\ell}\right)$

(B) $mg\cos\left(\frac{\theta}{2}\right)$

(C) $\frac{m\omega r}{\sin\left(\frac{\theta}{2}\right)}$

(D) $m(\omega^2 r^2 + g^2)^{\frac{1}{2}}$

(E) $m(\omega^4 r^2 + g^2)^{\frac{1}{2}}$

Recommended Solution

Consider that when the radius, r, goes to 0, the tension force should be just the force as a result of an object hanging from a string, i.e. $F_T = mg$. In this limit, only (D) and (E) are remaining. Next, compare the units in both to see that in (D) they are asking you to add a term with units of m^2/s^2 with another term with units of m^2/s^4 and that ain't happening.

Correct Answer
(E)

Alternate Solution

Draw out your force diagram (Figure 1.5),

It should be clear that the tension force is

$$F_T^2 = (F_{T-x})^2 + (F_{T-y})^2 \tag{1.105}$$

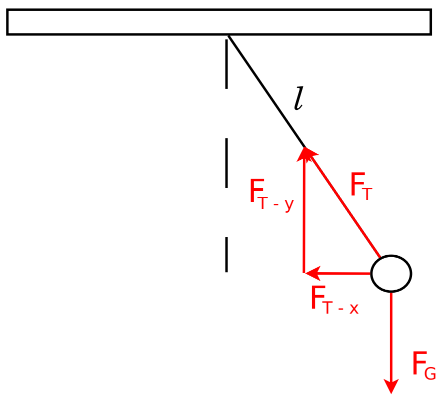


Figure 1.5: Component forces on a mass rotating on a cord

The vertical component of tension will be equal and opposite that of the force from gravity because it won't be accelerating in that direction,

$$F_{T-y} = F_G = mg \tag{1.106}$$

The tension in the horizontal will just be F_{T-x} and it will be equal to the centripetal acceleration due to its rotation,

$$F_{T-x} = ma_c = m\omega^2 r \tag{1.107}$$

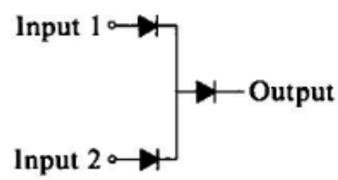
Plug Equations 1.106 and 1.107 into Equation 1.105 to get

$$F_T^2 = \left(m\omega^2 r\right)^2 + (mg)^2 \tag{1.108}$$

$$F_T = m\sqrt{\omega^4 r^2 + g^2} \tag{1.109}$$

Correct Answer
(E)

1.38 PGRE8677 #38



38. If logical 0 is 0 volts and logical 1 is +1 volt, the circuit shown above is a logic circuit commonly known as
    (A) an OR gate
    (B) an AND gate
    (C) a 2-bit adder
    (D) a flip-flop
    (E) a fanout

Recommended Solution

Construct a truth table based on the conditions of the problem

Input 1	Input 2	Output
0	0	0
1	0	1
0	1	1
1	1	1

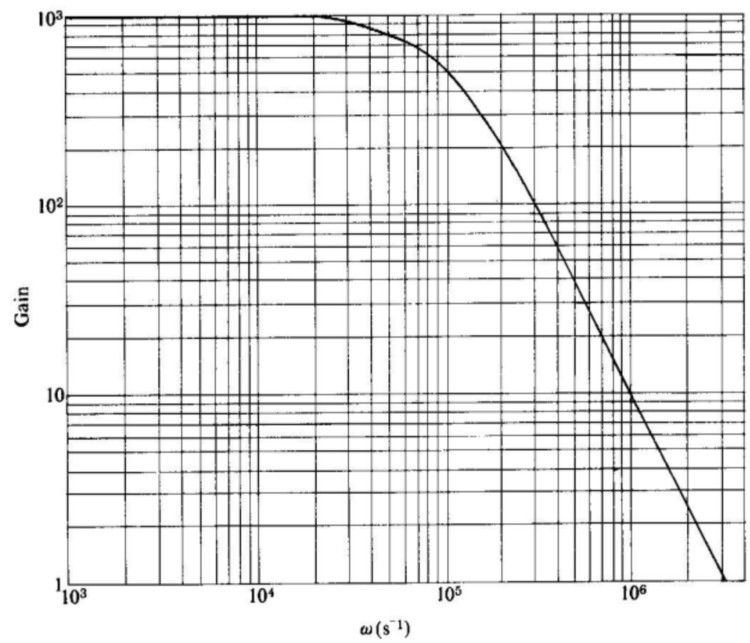
which is the truth table of an OR gate.
Recall that the truth table for an AND gate is

Input 1	Input 2	Output
0	0	0
1	0	0
0	1	0
1	1	1

To recall these tables, simply remember that for an OR gate, it must be true that either Input 1 OR Input 2 must be true for the output to be true. For an AND gate, however, it must be true that input 1 AND input 2 are true for the output to be true.

Correct Answer
(A)

1.39 PGRE8677 #39



39. The gain of an amplifier is plotted *versus* angular frequency ω in the diagram above. If K and a are positive constants, the frequency dependence of the gain near $\omega = 3 \times 10^5$ second^{-1} is most accurately expressed by

(A) $Ke^{-a\omega}$ (B) $K\omega^2$ (C) $K\omega$ (D) $K\omega^{-1}$ (E) $K\omega^{-2}$

Recommended Solution

In the area of $\omega = 3 \times 10^5$ second^{-1}, the line is roughly linear and decreasing. From this, we can eliminate any solution which shows an increase in gain with an increase in angular frequency, i.e. eliminate (B) and (C). Next, consider that we are talking about a log-log graph and (A) won't give us a linearly decreasing function with this type of scaling. Finally, between (D) and (E), we need to look at the slope near $\omega = 3 \times 10^5$ second^{-1}. The horizontal and vertical axes are scaled with the same step size so we can approximate the slope as moving down 2 steps and to the right roughly 4 steps, giving a slope of roughly 2/4 or 1/2. From this, we see that the gain is dependent on the squared value of the angular frequency

Correct Answer
(E)

1.40 PGRE8677 #40

40. An experimenter measures 9934 counts during one hour from a radioactive sample. From this number the counting rate of the sample can be estimated with a standard deviation of most nearly

(A) 100 (B) 200 (C) 300
(D) 400 (E) 500

Recommended Solution

The standard deviation for radioactive emission is described by the Poisson noise,

$$\sigma = \sqrt{\lambda} \tag{1.110}$$

where λ is the average number of radioactive samples. To simplify things, let $\lambda \approx 10,000$ so

$$\sigma = \sqrt{10,000} = 100 \tag{1.111}$$

Correct Answer
(A)

1.41 PGRE8677 #41

41. Which of the following nuclei has the largest binding energy per nucleon? (Consider the most abundant isotope of each element.)

(A) Helium
(B) Carbon
(C) Iron
(D) Uranium
(E) Plutonium

Recommended Solution

The binding energy for an atom is the amount of energy required to strip an electron from the atom. We can immediately eliminate (D) and (E) because the binding energy of these atoms is so low that it decomposes on its own (i.e. they are both radioactive). Next, eliminate (B) because it doesn't have a full valence shell and will generally be more willing to give up an electron than atoms with completed shells. Finally, choose (C) because Iron has a much stronger nucleus binding electrons in their shells and it is generally true that as you move down the periodic table, binding energy increases.

Correct Answer
(B)

1.42 PGRE8677 #42

42. A proton beam is incident on a scatterer 0.1 centimeter thick. The scatterer contains 10^{20} target nuclei per cubic centimeter. In passing through the scatterer, one proton per incident million is scattered. The scattering cross section is

(A) 10^{-29} cm^2
(B) 10^{-27} cm^2
(C) 10^{-25} cm^2
(D) 10^{-23} cm^2
(E) 10^{-21} cm^2

Recommended Solution

If you happen to recall the mean free path equation for the probability of stopping a particle moving through a medium, then this problem is a relatively straightforward plug-n-chug problem

$$P(x) = n\sigma dx \tag{1.112}$$

Where $P(x)$ is the probability of stopping the particle in the distance dx, n is the number of nuclei, σ is the scattering cross section and dx is the thickness of the scatterer. Plug everything in to get

$$\begin{aligned} \sigma &= \frac{P}{ndx} & (1.113)\\ &= \frac{(1\times 10^{-6}\ \text{nuclei})}{\left(10^{20}\frac{\text{nuclei}}{\text{cm}^3}\right)(0.1\text{cm})} & (1.114)\\ &= 1\times 10^{-25}\text{cm}^2 & (1.115)\end{aligned}$$

Correct Answer
(C)

Alternate Solution

In case you don't recall the relevant equation, examine the units of all the information given to you to see if you can derive it on the spot. You are given,

Scatterer Thickness 0.1 cm

Nuclei 10^{20} nuclei/cm^3

Quantity making it a given distance $\frac{1 \text{ nuclei}}{1\times 10^6}$

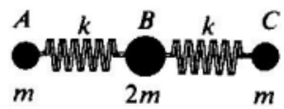
Since we ultimately want to get a result with units of cm^2, we will have to combine our 3 known values as

$$\frac{\text{Quantity making it a given distance}}{\text{(Nuclei)(Scatterer Thickness)}} = \frac{\text{nuclei}}{\left(\frac{\text{nuclei}}{\text{cm}^3}\right)(\text{cm})} \tag{1.116}$$

Plug in your values to get (C).

Correct Answer
(C)

1.43 PGRE8677 #43



43. Three masses are connected by two springs as shown above. A longitudinal normal mode with frequency $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ is exhibited by

(A) A, B, C all moving in the same direction with equal amplitude
(B) A and C moving in opposite directions with equal amplitude, and B at rest
(C) A and C moving in the same direction with equal amplitude, and B moving in the opposite direction with the same amplitude
(D) A and C moving in the same direction with equal amplitude, and B moving in the opposite direction with twice the amplitude
(E) none of the above

Recommended Solution

Ideally, you should recognize that the frequency given in the problem as being that of a Simple Harmonic Oscilattor (SHO)

$$\omega_{\text{SHO}} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1.117}$$

The simple harmonic oscillator involves a mass oscillating about a fixed point. Based on this fact, we should expect the frequency for this linearly oscillating system of springs and masses to exhibit the SHO frequency when they are also moving about a fixed point. Of the potential solutions, only (B) describes masses A and C as moving about a fixed point, specifically mass B.

Correct Answer
(B)

Alternate Solution

In case you don't recall the frequency of a SHO, you can calculate it by first considering all of the forces on one of the masses,

$$F_{\text{net}} = m\ddot{x} = -kx \tag{1.118}$$

for a SHO, the position equation is

$$x(t) = A\sin(2\pi\omega t + \phi) \tag{1.119}$$

plug this into the force equation to get the differential equation

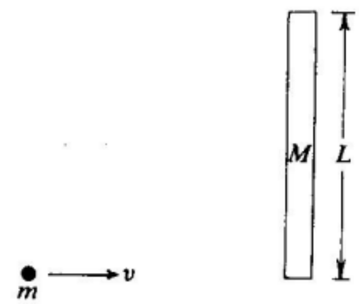
$$m\frac{d}{dt}\left(A\sin(2\pi\omega t+\phi)\right)=-k\left(A\sin(2\pi\omega t+\phi)\right) \tag{1.120}$$

and when you find the general solution to the diffeq, you should get

$$\omega=\frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1.121}$$

Correct Answer
(B)

1.44 PGRE8677 #44



View from Above

44. A uniform stick of length L and mass M lies on a frictionless horizontal surface. A point particle of mass m approaches the stick with speed v on a straight line perpendicular to the stick that intersects the stick at one end, as shown above. After the collision, which is elastic, the particle is at rest. The speed V of the center of mass of the stick after the collision is

(A) $\frac{m}{M}v$ (B) $\frac{m}{M+m}v$ (C) $\sqrt{\frac{m}{M}}v$

(D) $\sqrt{\frac{m}{M+m}}v$ (E) $\frac{3m}{M}v$

Recommended Solution

The most difficult part of this problem isn't performing the calculation but convincing yourself of the correct solution. When the particle strikes the stick, the particle stops which tells us that all of the momentum that the particle originally had has been transferred to the stick. Because the particle doesn't strike the stick at its center of mass, the stick will start to spin in addition to moving to the right. This may convince you that if the momentum is conserved, that a fraction of that momentum results in a linear motion for the stick and the other fraction of that momentum goes into the rotation. However, if we consider all of the momentum on the stick, we get the following diagram.

Note that at any instant of time, the momentum of the ends of the stick (and every point between the end and the COM) has a horizontal and vertical component of momentum exactly opposite that of the opposite end of the stick (Figure 1.6). Since momentum must be conserved, we sum the momentum components to get

$$P_{\text{net - x}} = P_{\text{COM}} + P_x + (-P_x) = P_{\text{COM}} \quad (1.122)$$

$$P_{\text{net - y}} = P_y + (-P_y) = 0 \quad (1.123)$$

From this, we get that the net momentum transferred from the particle to the stick is all contained within the center of mass. Finally, compare the initial and final momentum to get

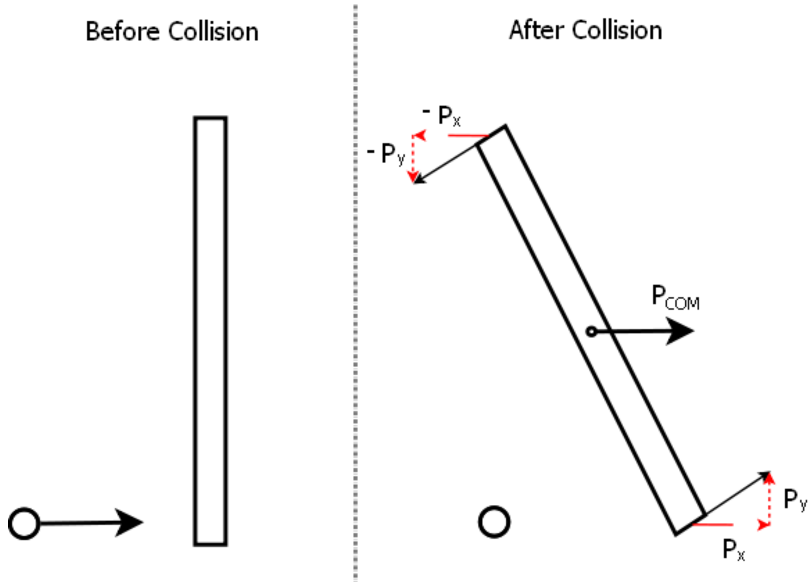


Figure 1.6: Conservation of momentum in a rotating system

$$P_i = P_f \tag{1.124}$$
$$mv = MV \tag{1.125}$$
$$V = \frac{mv}{M} \tag{1.126}$$

Correct Answer
(A)

1.45 PGRE8677 #45

45. Photons of wavelength λ scatter elastically on free protons initially at rest. The wavelength of the photons scattered at 90° is increased by

(A) $\lambda/137$
(B) $\lambda/1836$
(C) $h/m_e c$, where h is Planck's constant, m_e the rest mass of an electron, and c the speed of light
(D) $h/m_p c$, where h is Planck's constant, m_p the rest mass of a proton, and c the speed of light
(E) zero

Recommended Solution

This problem is making reference to Compton scattering. Compton scattering is the phenomena by which photons collide with a particle, impart some of their kinetic energy to the particle and then scatters off at a lower energy (Figure 1.7).

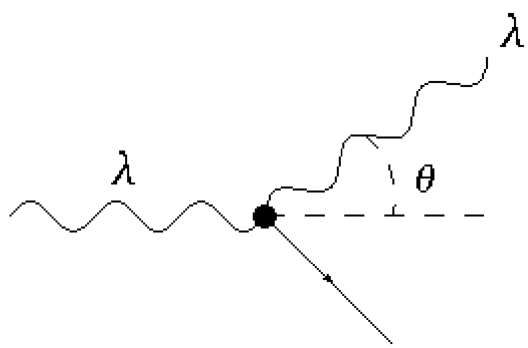


Figure 1.7: Photon wavelength change due to compton scattering

The Compton equation derived for these interactions is

$$\lambda' - \lambda = \frac{h}{m_p c}(1 - \cos(\theta)) \tag{1.127}$$

If we plug in all the values provided in the problem, we get

$$\lambda' - \lambda = \frac{h}{m_p c}\left(1 - \cos\left(\frac{\pi}{2}\right)\right) \tag{1.128}$$

$$\lambda' = \frac{h}{m_p c} + \lambda \tag{1.129}$$

which tells us that the new wavelength, λ' has an extra $\frac{h}{m_p c}$ added to it.

Correct Answer
(D)

1.46 PGRE8677 #46

46. A blackbody at temperature T_1 radiates energy at a power level of 10 milliwatts (mW). The same blackbody, when at a temperature $2T_1$, radiates energy at a power level of

(A) 10 mW (B) 20 mW (C) 40 mW (D) 80 mW (E) 160 mW

Recommended Solution

According to Stefan's Law (Stefan-Boltzmann's Law), power radiation of a blackbody is only dependent on temperature according to

$$j^* = \sigma T^4 \tag{1.130}$$

where σ is a constant equal to

$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67 \times 10^{-8} \frac{\text{J}}{\text{sm}^2\text{K}^4} \tag{1.131}$$

For our problem, we are doubling the temperature so our power is

$$\begin{aligned} j^* &= \sigma(2T)^4 && (1.132) \\ &= 16\sigma T^4 && (1.133) \\ &= 160 \text{ mW} && (1.134) \end{aligned}$$

Correct Answer
(E)

1.47 PGRE8677 #47

47. The Franck-Hertz experiment and related scattering experiments show that

(A) electrons are always scattered elastically from atoms
(B) electrons are never scattered elastically from atoms
(C) electrons of a certain energy range can be scattered inelastically, and the energy lost by electrons is discrete
(D) electrons always lose the same energy when they are scattered inelastically
(E) there is no energy range in which the energy lost by electrons varies continuously

Recommended Solution

Without knowing anything about the Frank-Hertz experiment, you can immediately eliminate options (A) and (B) from the list because it is neither true that electron collisions are always elastic, nor is it true that they are always inelastic. By a similar line of reasoning, since we've already argued that some electron scattering can be elastic, it wouldn't make ~~since~~ to then say that electrons always lose some specific amount of energy, so we eliminate (D). Finally, when choosing between (C) and (E), you will need to know that the Nobel Prize winning Frank-Hertz experiment demonstrated that at a specific energy range (4.9 volts to be specific) electrons begin to experience inelastic collisions and that the energies lost in the collision came in discrete amounts.

Correct Answer
(C)

1.48 PGRE8677 #48

48. A transition in which one photon is radiated by the electron in a hydrogen atom when the electron's wave function changes from ψ_1 to ψ_2 is forbidden if ψ_1 and ψ_2

(A) have opposite parity
(B) are orthogonal to each other
(C) are zero at the center of the atomic nucleus
(D) are both spherically symmetrical
(E) are associated with different angular momenta

Recommended Solution

From our quantum mechanical selection rules, we know that any change of state must be accompanied by a change in the quantum angular momentum number

$$\Delta l = \pm 1 \tag{1.135}$$

This tells us that if the wave function is spherically symmetric, i.e. $l = 0$, then after a change to a new wave function, $l \neq 0$ and so they can't both be spherically symmetric.

Correct Answer
(D)

1.49 PGRE8677 #49

49. The Hamiltonian operator in the Schrödinger equation can be formed from the classical Hamiltonian by substituting

(A) wavelength and frequency for momentum and energy
(B) a differential operator for momentum
(C) transition probability for potential energy
(D) sums over discrete eigenvalues for integrals over continuous variables
(E) Gaussian distributions of observables for exact values

Recommended Solution

The classical Hamiltonian equation is

$$\begin{aligned} H &= T + V && (1.136) \\ &= \frac{p^2}{2m} + V && (1.137) \end{aligned}$$

The quantum mechanical Hamiltonian is

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V \tag{1.138}$$

examine both equations to see that we can move from the classical Hamiltonian to the quantum mechanical one if we allow $p = i\hbar\nabla$.

Correct Answer
(B)

1.50 PGRE8677 #50

50. The Hall effect is used in solid-state physics to measure
    (A) ratio of charge to mass
    (B) magnetic susceptibility
    (C) the sign of the charge carriers
    (D) the width of the gap between the conduction and valence bands
    (E) Fermi energy

Recommended Solution

If you place a flat conducting bar with a current passing through it into a magnetic field with field vectors pointing perpendicularly to the direction of current, the Lorentz force will cause the stream of electrons to curve to one face of the bar while the remaining positive charges are left on the opposite face of the bar (Figure 1.8). This results in a potential difference in the conductor known as the "Hall Voltage" which can be calculated using

$$V_H = -\frac{IB}{dne} \tag{1.139}$$

where I is the current, B is the magnetic flux density, d is the plate depth, e is the electron charge and n is the charge carrier density. One of the benefits the Hall effect has to experimental physics relates to the direction in which a current will curve depending on the direction of the magnetic field, current and charge.

Effectively, the direction in which the electrons curve determines the sign of the Hall Voltage and provided the direction of the current and magnetic field are known, it is trivial to determine the sign of the charge carriers.

Correct Answer
(C)

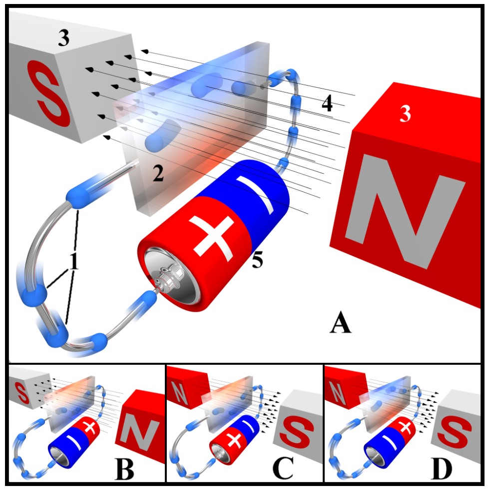


Figure 1.8: Moving charges tend to one side of a conducting bar from the Hall effect

1.51 PGRE8677 #51

51. One feature common to both the Debye theory and the Einstein theory of the specific heat of a crystal composed of N identical atoms is that the

(A) average energy of each atom is $3kT$
(B) vibrational energy of the crystal is equivalent to the energy of $3N$ independent harmonic oscillators
(C) crystal is assumed to be continuous for all elastic waves
(D) speed of the longitudinal elastic waves is less than the speed of the transverse elastic waves
(E) upper cutoff frequency of the elastic waves is the same

Recommended Solution

The Debye and Einstein models of specific heat were essentially identical except for the way in which they were scaled. Debye scaled his model around a value he called the Debye temperature (T_D) that was dependent on a number of properties of the material. Einstein, on the other hand, scaled his model based on a single frequency value ($h\nu$) and constant k. Although they were identical in terms of their predictions that vibrational energies were dependent on 3N independent harmonic oscillators, Debye's scaling proved to be more accurate in the low temperature range and so it prevailed.

Correct Answer
(B)

1.52 PGRE8677 #52

52. A cube has a constant electric potential V on its surface. If there are no charges inside the cube, the potential at the center of the cube is

(A) zero (B) $V/8$ (C) $V/6$
(D) $V/2$ (E) V

Recommended Solution

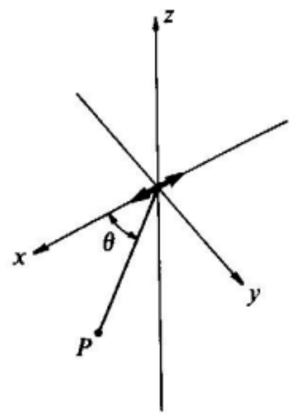
From what you learned about electrostatics, you should recall that in a region where there are no charges and with a uniform potential on the surface, electric potential is described by Laplace's equation

$$\nabla^2 V = 0 \tag{1.140}$$

It is also true that within the boundaries of any region which is satisfied by Laplace's equations, there can be no local minima or maxima for V (courtesy of the Divergence Theorem). From this condition, we know that whatever value of potential we have on the surface, the value inside must be exactly the same, else we've found a maximum or minimum. Therefore, the potential at the center of the cube must be V, just as it is on the surface of the cube.

Correct Answer
(E)

1.53 PGRE8677 #53



53. A charged particle oscillates harmonically along the x-axis as shown above. The radiation from the particle is detected at a distant point P, which lies in the xy-plane. The electric field at P is in the

(A) $\pm z$ direction and has a maximum amplitude at $\theta = 90°$
(B) $\pm z$ direction and has a minimum amplitude at $\theta = 90°$
(C) xy-plane and has a maximum amplitude at $\theta = 90°$
(D) xy-plane and has a minimum amplitude at $\theta = 90°$
(E) xy-plane and has a maximum amplitude at $\theta = 45°$

Recommended Solution

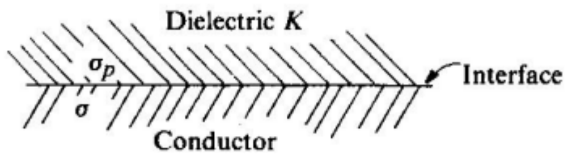
When a charged particle oscillates in 1-dimension it will radiate electromagnetic radiation in a direction perpendicular to to the motion of the particle. For an imperfect but useful analogy, think along the lines of the Lorentz Force and the orthogonality of its electric and magnetic fields. Expanding on this Lorentz Force analogy a bit more, recall that the orthogonality condition comes as a sine function from a cross product

$$\begin{aligned} F &= q\left(\vec{E} + \vec{v} \times \vec{B}\right) && (1.141)\\ &= qE + vB\sin(\theta) && (1.142) \end{aligned}$$

The dipole oscillation, much like the Lorentz Force, has this same sine dependence. Knowing only these two facts we can conclude that the point P must be in the xy-plane, because the problem has already specified that it lies in this plane, and that the amplitude is maximized when $\theta = 90°$

Correct Answer
(C)

1.54 PGRE8677 #54



54. A dielectric of dielectric constant K is placed in contact with a conductor having surface charge density σ, as shown above. What is the polarization (bound) charge density σ_p on the surface of the dielectric at the interface between the two materials?

(A) $\sigma \frac{K}{1-K}$ (B) $\sigma \frac{K}{1+K}$ (C) σK

(D) $\sigma \frac{1+K}{K}$ (E) $\sigma \frac{1-K}{K}$

Recommended Solution

This problem is extremely quick and easy if we can simply recall the definition of the dielectric constant, K. The dielectric constant is a scaling factor which describes the concentration of electrostatic flux in any given material. It is calculated by taking the ratio between the amount of electrical energy stored by the material when some voltage is applied to it (absolute permittivity of the material, ϵ_s) relative to the permittivity of a vacuum, ϵ_0,

$$K = \frac{\epsilon_s}{\epsilon_0} \tag{1.143}$$

from the previous equation, it should be clear that $K = 1$ when the absolute permittivity of the material is equal to that of a vacuum. Since the polarization charge density, σ_p, is

$$\begin{aligned} \sigma_p &= P \cdot \hat{n} & (1.144) \\ &= \chi_e \vec{E} \hat{n} & (1.145) \end{aligned}$$

where P is the electric polarization, χ_e is the electric susceptibility and $\vec{E}$ is the electric field. Since the electric susceptibility is

$$\chi_e = \frac{P}{\epsilon_0 \vec{E}} = \frac{\epsilon_s}{\epsilon_0} - 1 \tag{1.146}$$

We see that when $\epsilon_s = \epsilon_0$ (i.e. when $K = 1$), the bound charge density should equal 0. This limit is only satisfied by (E)

Correct Answer
(E)

1.55 PGRE8677 #55

55. **The mean kinetic energy of electrons in metals at room temperature is usually many times the thermal energy kT. Which of the following can best be used to explain this fact?**

(A) The energy-time uncertainty relation
(B) The Pauli exclusion principle
(C) The degeneracy of the energy levels
(D) The Born approximation
(E) The wave-particle duality

Recommended Solution

You should recall the thermal energy referenced in the problem from Boltzmann's constant, which effectively tied together the microscopic ~~affects~~ of fermions to the macroscopic view of thermal energy,

$$PV = NkT \tag{1.147}$$

When dealing with individual fermions, in our case electrons, we are primarily concerned with the Fermi Energy which describes the highest energy level occupied by an electron. The highest occupied energy level can be found using energy level diagrams and continuously appending electrons to the lowest unoccupied levels. At the point that we've run out of electrons to assign to energy levels, the highest level electron determines the Fermi Energy and, additionally, the Fermi Velocity. If the Pauli exclusion principle weren't true, then electrons would have no reason to build into higher energy levels (in fact they would likely all bunch into the ground state energy level) and so, without the Pauli exclusion principle, the mean kinetic energy of electrons wouldn't be dependent on the thermal energy of the system.

Correct Answer
(B)

1.56 PGRE8677 #56

56. If ψ is a normalized solution of the Schrödinger equation and Q is the operator corresponding to a physical observable x, the quantity $\psi^* Q \psi$ may be integrated in order to obtain the

(A) normalization constant for ψ
(B) spatial overlap of Q with ψ
(C) mean value of x
(D) uncertainty in x
(E) time derivative of x

Recommended Solution

From quantum mechanics, we know that the expectation value or the mean value of an operator is found by

$$\langle \psi^* | Q | \psi \rangle = \int_{-\infty}^{\infty} \psi^* Q \psi \; dx \tag{1.148}$$

where Q is some operator. In this problem they tell us that Q is the operator corresponding to observable x and traditionally, the operator Q is used such that

$$(Q\psi)(x) = (x\psi)(x) \tag{1.149}$$

so finding the expectation value of Q is effectively the same thing as finding the mean value of x and so the correct answer is (C). Be aware, however, that an operator which "corresponds to a physical observable x" doesn't necessarily imply that you are finding the mean value of x, even though it does mean you are finding the mean value of the observable itself.

Correct Answer
(C)

1.57 PGRE8677 #57

57. Which of the following is an eigenfunction of the linear momentum operator $-i\hbar \frac{\partial}{\partial x}$ with a positive eigenvalue $\hbar k$; i.e., an eigenfunction that describes a particle that is moving in free space in the direction of positive x with a precise value of linear momentum?

(A) $\cos kx$ (B) $\sin kx$ (C) e^{-ikx}
(D) e^{ikx} (E) e^{-kx}

Recommended Solution

The eigenfunction of a linear operator is any non-zero function, f, that satisfies the condition

$$Af = \lambda f \tag{1.150}$$

where λ is the eigenvalue. From this, we will want to plug in all of the given functions, f, into

$$\hbar k f = -i\hbar \frac{\partial f}{\partial x} \tag{1.151}$$

without doing any work, it should be obvious that both (A) and (B) won't work because only the RHS of Equation 1.151 contains a derivative and so the trig functions will be different on each side. However, for the purposes of rigor, we can compute each of the 5 possible solutions

(A) $\hbar k \cos(kx) \neq i\hbar k \sin(kx)$

(B) $\hbar k \sin(kx) \neq -i\hbar k \cos(kx)$

(C) $\hbar k e^{-ikx} = i^2 \hbar k e^{-ikx} \neq -\hbar k e^{ikx}$

(D) $\hbar k e^{ikx} = -i^2 \hbar k e^{ikx} = \hbar k e^{ikx}$

(E) $\hbar k e^{-kx} \neq i\hbar k e^{-kx}$

Correct Answer
(D)

1.58 PGRE8677 #58

58. In an ordinary hologram, coherent monochromatic light produces a 3-dimensional picture because wave information is recorded for which of the following?

    I. Amplitude
    II. Phase
    III. Wave-front angular frequency

    (A) I only
    (B) I and II only
    (C) I and III only
    (D) II and III only
    (E) I, II, and III

Recommended Solution

Looking through the three options provided, we see that III, Wave-front angular frequency, i.e. the color of the object, is one of the properties which the problem claims a hologram might "record". If you've ever seen a real hologram (i.e. not one from star wars) then you know that color is something that is not conserved (Figure 1.9). In fact, the question gives this away when it tells you that the resulting hologram is monochromatic.

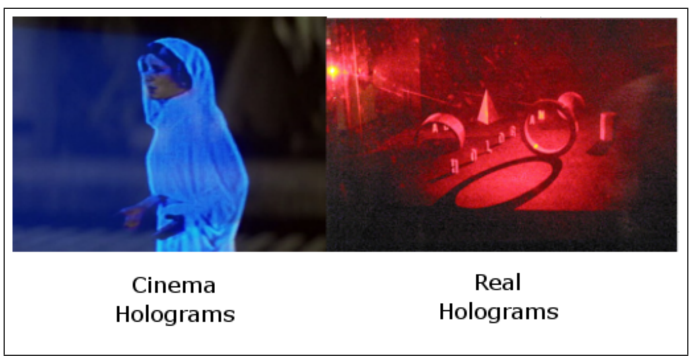


Figure 1.9: Comparison of realistic holograms to cinematic holograms

From this, we can eliminate any solutions which claim to record Wave-front angular frequency, i.e. (C), (D) and (E). When choosing between (A) and (B), we simply need to determine whether any phase information is saved by the hologram. Since a hologram is generated as the result of interference between two light beams and wave interference is dependent on wave phase, choose (B).

Correct Answer
(B)

1.59 PGRE8677 #59

59. The dispersion law for a certain type of wave motion is $\omega = (c^2k^2 + m^2)^{\frac{1}{2}}$, where ω is the angular frequency, k is the magnitude of the propagation vector, and c and m are constants. The group velocity of these waves approaches

(A) infinity as $k \rightarrow 0$ and zero as $k \rightarrow \infty$
(B) infinity as $k \rightarrow 0$ and c as $k \rightarrow \infty$
(C) c as $k \rightarrow 0$ and zero as $k \rightarrow \infty$
(D) zero as $k \rightarrow 0$ and infinity as $k \rightarrow \infty$
(E) zero as $k \rightarrow 0$ and c as $k \rightarrow \infty$

Recommended Solution

The group velocity, v_g, is related to the angular frequency, ω, by

$$v_g = \frac{\partial \omega}{\partial k} \tag{1.152}$$

ω is given in the problem so we take the derivative with respect to k to get

$$\begin{aligned} \frac{\partial \omega}{\partial k} &= \frac{\partial}{\partial k}\sqrt{c^2k^2 + m^2} && (1.153) \\ &= \frac{c^2k}{\sqrt{c^2k^2 + m^2}} && (1.154) \end{aligned}$$

and plugging in the limit $k \rightarrow 0$, you should get that $v_g = 0$ which eliminates all possible solutions but (D) and (E). Next, letting $k \rightarrow \infty$, both terms with c and k will blow up and m will effectively be 0, giving us

$$\begin{aligned} \frac{c^2k}{\sqrt{c^2k^2}} &= \frac{c^2k}{ck} && (1.155) \\ &= c && (1.156) \end{aligned}$$

Correct Answer
(E)

1.60 PGRE8677 #60

60. A particle of mass m that moves along the x-axis has potential energy $V(x) = a + bx^2$, where a and b are positive constants. Its initial velocity is v_0 at $x = 0$. It will execute simple harmonic motion with a frequency determined by the value of

(A) b alone
(B) b and a alone
(C) b and m alone
(D) b, a, and m alone
(E) b, a, m, and v_0

Recommended Solution

For a simple harmonic oscillator the frequency is typically given as

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1.157}$$

this gives you a mass dependence and allows you to eliminate (A) and (B). Additionally, it also demonstrates that there is no initial velocity dependence and we can eliminate (E). Finally, when choosing between (C) and (D), ask yourself where we get the potential energy equation in the first place. Recall that a mass which exhibits simple harmonic motion due to a spring has a spring force of

$$F_s = -kx \tag{1.158}$$

Additionally, recall that

$$V = \int F_s \, dx \tag{1.159}$$

which means we can solve for the potential energy of the SHO by integrating the spring force equation,

$$\begin{aligned} V &= \int F_s \, dx && (1.160) \\ &= kx^2 + C && (1.161) \end{aligned}$$

where C is just some constant. This expression is identical to the potential energy described in the problem and so we conclude, by Hooke's law, that the force and, by extension, the potential energy of the system is dependent on the constant of proportionality for position, b, but not dependent on the arbitrary constant, a.

Correct Answer
(C)

Alternate Solution

Recall that the potential energy of a system is related to the force on that system by

$$\frac{dV}{dx} = -F(x) \tag{1.162}$$

so our potential energy, V, becomes

$$\begin{aligned} F_s &= -\frac{d}{dx}\left(a + bx^2\right) &(1.163)\\ &= -2bx &(1.164)\end{aligned}$$

By Newton's second law,

$$\begin{aligned} F_s &= m\ddot{x} &(1.165)\\ -2bx &= m\ddot{x} &(1.166)\end{aligned}$$

but since $\ddot{x} = \omega^2 x$, we get

$$\begin{aligned} 2b &= m\omega^2 &(1.167)\\ \omega &= \sqrt{\frac{2b}{m}} &(1.168)\end{aligned}$$

finally, since the angular frequency, ω, is related to frequency, f, by $\omega = 2\pi f$

$$f = \frac{1}{2\pi}\sqrt{\frac{2b}{m}} \tag{1.169}$$

Correct Answer
(C)

1.61 PGRE8677 #61

Questions 61-62

The equation of motion of a rocket in free space can be written

$$m\frac{dv}{dt} + u\frac{dm}{dt} = 0$$

where m is the rocket's mass, v is its velocity, t is time, and u is a constant.

61. The constant u represents the speed of the
   (A) rocket at $t = 0$
   (B) rocket after its fuel is spent
   (C) rocket in its instantaneous rest frame
   (D) rocket's exhaust in a stationary frame
   (E) rocket's exhaust relative to the rocket

62. The equation can be solved to give v as a function of m. If the rocket has $m = m_0$ and $v = 0$ when it starts, what is the solution?
   (A) $u\, m_0/m$
   (B) $u \exp(m_0/m)$
   (C) $u \sin(m_0/m)$
   (D) $u \tan(m_0/m)$
   (E) None of the above.

Recommended Solution

The problem specifies that the rocket has velocity v and that the motion of the rocket is determined by the equation

$$m\frac{dv}{dt} + u\frac{dm}{dt} = 0 \tag{1.170}$$

we will consider the rocket moving to the right and let this direction of motion be positive. In order for the net momentum of the system to be 0, there must be another velocity, in our case u, moving in a direction opposite to that of the rocket. For options (A), (B) and (C), each of the velocities mentioned should be positive as they are all instances of the rocket in its motion in the positive direction (to the right). Now, when choosing between (D) and (E), realize that our original equation of motion describes a stationary frame viewing a moving frame. From this, we know that u couldn't be the exhaust speed in a stationary frame, else we wouldn't be taking into account the varying reference frames.

Correct Answer
(E)

1.62 PGRE8677 #62

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    - (E) rocket's exhaust relative to the rocket

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    - (B) $u \exp(m_0/m)$
    - (C) $u \sin(m_0/m)$
    - (D) $u \tan(m_0/m)$
    - (E) None of the above.

Recommended Solution

The question gives you the initial conditions that when $m = m_0$ we get $v = 0$. Substitute $m = m_0$ into each to see which, if any, goes to 0.

(A) $u\left(\frac{m}{m}\right) = u \neq 0$

(B) $ue^{\frac{m}{m}} = ue \neq 0$

(C) $u\sin\left(\frac{m}{m}\right) = u\sin(1) \neq 0$

(D) $u\tan\left(\frac{m}{m}\right) = u\tan(1) \neq 0$

(E) (A), (B), (C) and (D) were all wrong, therefore we choose (E)

Correct Answer
(E)

Alternate Solution

The quick method to solve this problem involves using some very irresponsible math that we physicists (and to a larger extent, chemists) like to use to simplify differentials. multiply the dt out of our equation of motion to get

$$mdv + udm = 0 \tag{1.171}$$
$$mdv = -udm \tag{1.172}$$

get dv on its own and integrate both sides

$$\int dv = -u \int \frac{dm}{m} \tag{1.173}$$

and we will just generalize the results as

$$\Delta v = -u \left[\ln(m)\right]_{m_0}^{m} \tag{1.174}$$
$$v_f - v_0 = -u \ln\left(\frac{m}{m_0}\right) \tag{1.175}$$

since the question tells us that $v_0 = 0$, we make the substitutions to get,

$$v_f = -u \ln\left(\frac{m}{m_0}\right) \tag{1.176}$$

which doesn't match any of the options given as possible solutions.

Correct Answer
(E)

1.63 PGRE8677 #63

63. A point charge $-q$ coulombs is placed at a distance d from a large grounded conducting plane. The surface charge density on the plane a distance D from the point charge is

(A) $\frac{q}{4\pi D}$

(B) $\frac{qD^2}{2\pi}$

(C) $\frac{qd}{2\pi D^2}$

(D) $\frac{qd}{2\pi D^3}$

(E) $\frac{qd}{4\pi\epsilon_0 D^2}$

Recommended Solution

Anytime you see different arrangements of variables in which all or most of the options will have different units than one another, check the units. Recall that we are looking for units of charger per area

(A) $\frac{q}{4\pi D} \equiv \frac{\mathrm{C}}{\mathrm{m}}$

(B) $\frac{qD^2}{2\pi} \equiv \mathrm{C} \cdot \mathrm{m}^2$

(C) $\frac{qd}{2\pi D^2} \equiv \frac{\mathrm{C}}{\mathrm{m}}$

(D) $\frac{qd}{2\pi D^3} \equiv \frac{\mathrm{C}}{\mathrm{m}^2}$

(E) $\frac{qd}{4\pi\epsilon_0 D^2} \equiv \frac{\mathrm{C}}{\mathrm{m}}$

Correct Answer
(D)

1.64 PGRE8677 #64

64. An alternating current electrical generator has a fixed internal impedance $R_g + jX_g$ and is used to supply power to a passive load that has an impedance $R_g + jX_\ell$, where $j = \sqrt{-1}$, $R_g \neq 0$, and $X_g \neq 0$. For maximum power transfer between the generator and the load, X_ℓ should be equal to

(A) 0 (B) X_g (C) $-X_g$

(D) R_g (E) $-R_g$

Recommended Solution

In electronics, when we want to maximize power transfer we need to do some impedance matching in which the load impedance and complex source impedance is

$$Z_S = Z_L^* \tag{1.177}$$

plugging in given impedance values,

$$\begin{aligned} Z_S &= Z_L^* && (1.178) \\ R_g + jX_g &= R_g + jX_l && (1.179) \end{aligned}$$

In order for the RHS to be equal to the LHS,

$$X_l = -X_g \tag{1.180}$$

Correct Answer
(C)

1.65 PGRE8677 #65

65. A current i in a circular loop of radius b produces a magnetic field. At a fixed point far from the loop, the strength of the magnetic field is proportional to which of the following combinations of i and b?

(A) ib (B) ib^2 (C) i^2b (D) $\frac{i}{b}$ (E) $\frac{i}{b^2}$

Recommended Solution

A magnetic dipole occurs in any instance that you have a closed circulation of electrical current. In the case of our problem, we have an enclosed loop of wire with a current i. Any given dipole can be described by its dipole moment and in the case of a magnetic dipole, we get the magnetic dipole moment

$$\mu = \int i \; dA \tag{1.181}$$

where dA is a differential piece of the area about which the current circulates. In our case, our area is simply that of a circle with radius b and so we get a magnetic moment of

$$\begin{aligned} \mu &= i\pi b^2 && (1.182) \\ &\propto ib^2 && (1.183) \end{aligned}$$

Correct Answer
(B)

1.66 PGRE8677 #66

66. For a system in which the number of particles is fixed, the reciprocal of the Kelvin temperature T is given by which of the following derivatives? (Let P = pressure, V = volume, S = entropy, and U = internal energy.)

(A) $\left(\frac{\partial P}{\partial V}\right)_S$

(B) $\left(\frac{\partial P}{\partial S}\right)_V$

(C) $\left(\frac{\partial S}{\partial P}\right)_U$

(D) $\left(\frac{\partial V}{\partial P}\right)_U$

(E) $\left(\frac{\partial S}{\partial U}\right)_V$

Recommended Solution

Without knowing much about thermodynamics, we can figure that our final solution must contain some units of temperature, specifically we expect to get something like inverse Kelvins. Looking at the units of the 4 different variables, we get

$P = \text{atm}$
$V = \text{m}^3$
$S = \text{J/K}$
$U = \text{J}$

Note that only entropy, S, has any temperature units and the only way to isolate the temperature unit is to do

$$\left(\frac{\partial S}{\partial U}\right)_V \tag{1.184}$$

Correct Answer
(E)

Alternate Solution

Recall, from thermodynamics, the equation

$$dU = T\ dS - PdV \tag{1.185}$$

you should never forget this equation because chemistry, more than any other subject, is T dS (T dS = tedious..... GET IT!). Temperature is only well-defined for a system in equilibrium, so we let $dV = 0$ and we get,

$$dU = T\,dS \tag{1.186}$$

$$\frac{1}{T} = \left(\frac{dS}{dU}\right)_V \tag{1.187}$$

Correct Answer
(E)

1.67 PGRE8677 #67

67. A large isolated system of N weakly interacting particles is in thermal equilibrium. Each particle has only 3 possible nondegenerate states of energies 0, ϵ, and 3ϵ. When the system is at an absolute temperature $T >> \epsilon/k$, where k is Boltzmann's constant, the average energy of each particle is

(A) 0 (B) ϵ (C) $\frac{4}{3}\epsilon$ (D) 2ϵ (E) 3ϵ

Recommended Solution

Recall from thermodynamics that with excessive amounts of energy at the disposal of a system, particles will tend to equally populate all energy levels. Since the question tells us that temperature (i.e. thermal energy) is significantly larger than the energy levels, we know that every energy level must have an equal likelihood of being populated. From this, calculate the average of the 3 energy levels as

$$\frac{0+\epsilon+3\epsilon}{3} = \frac{4}{3}\epsilon \tag{1.188}$$

Correct Answer
(C)

1.68 PGRE8677 #68

68. If a newly discovered particle X moves with a speed equal to the speed of light in vacuum, then which of the following must be true?

(A) The rest mass of X is zero.
(B) The spin of X equals the spin of a photon.
(C) The charge of X is carried on its surface.
(D) X does not spin.
(E) X cannot be detected.

Recommended Solution

This problem and the solution should be one of those ideas you know by heart. If you don't, stop what ever you are doing and commit this to your memory. As an object with mass moves faster and approaches extremely high velocities, the mass of the object begins to increase. As you approach the speed of light, the mass approaches infinity. The only way a particle can achieve a velocity at the speed of light is if the particle has a rest mass of 0. In case the rule of thumb isn't enough to convince you, recall

$$E = \sqrt{p^2c^2 + m_0^2c^4} \tag{1.189}$$

let $m \to 0$, giving

$$\begin{aligned} E &= \sqrt{p^2c^2} && (1.190)\\ &= pc && (1.191) \end{aligned}$$

then, by the Planck relationship,

$$\begin{aligned} pc &= h\nu && (1.192)\\ p &= \frac{h\nu}{c} && (1.193)\\ &= \frac{h}{\lambda} && (1.194) \end{aligned}$$

which we know, experimentally, is the momentum of the photon.

Correct Answer
(A)

1.69 PGRE8677 #69

Questions 69-71

A car of rest length 5 meters passes through a garage of rest length 4 meters. Due to the relativistic Lorentz contraction, the car is only 3 meters long in the garage's rest frame. There are doors on both ends of the garage, which open automatically when the front of the car reaches them and close automatically when the rear passes them. The opening or closing of each door requires a negligible amount of time.

69. The velocity of the car in the garage's rest frame is

(A) 0.4 c
(B) 0.6 c
(C) 0.8 c
(D) greater than c
(E) not determinable from the data given

70. The length of the garage in the car's rest frame is

(A) 2.4 m
(B) 4.0 m
(C) 5.0 m
(D) 8.3 m
(E) not determinable from the data given

71. Which of the following statements is the best response to the question:

"Was the car ever inside a closed garage?"

(A) No, because the car is longer than the garage in all reference frames.
(B) No, because the Lorentz contraction is not a "real" effect.
(C) Yes, because the car is shorter than the garage in all reference frames.
(D) Yes, because the answer to the question in the garage's rest frame must apply in all reference frames.
(E) There is no unique answer to the question, as the order of door openings and closings depends on the reference frame.

Recommended Solution

In this problem, you should immediately eliminate (D) because we know the car can't move faster than the speed of light. Next, eliminate (E) because there is sufficient information in this problem to answer the question explicitly. Lastly, look for the speed which justifies our use of relativistic equations to account for the contraction of a 2 meters, which would be $0.8c$.

Correct Answer
(C)

Alternate Solution

Length contraction from relativistic effects is related to the Lorentz factor by,

$$L' = \frac{L}{\gamma} = L\sqrt{1 - \frac{v^2}{c^2}} \tag{1.195}$$

where L' is the length of a contracted object in the rest frame and L is the rest length. Plug in the given values to solve

$$\begin{aligned} 3 \text{ m} &= 5 \text{ m}\sqrt{1 - \frac{v^2}{c^2}} && (1.196)\\ \frac{9}{25} &= 1 - \frac{v^2}{c^2} && (1.197)\\ \frac{16}{9} &= \frac{v^2}{c^2} && (1.198)\\ v &= \frac{4}{5}c && (1.199) \end{aligned}$$

Correct Answer
(C)

1.70 PGRE8677 #70

Questions 69-71

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(E) There is no unique answer to the question, as the order of door openings and closings depends on the reference frame.

Recommended Solution

Now, we are talking about the moving car being the rest frame and the garage being the contracted frame. We use the same contraction equation used in the previous problem, however we let L' be the contracted garage reference frame and let L be the rest frame for the garage. In the previous equation, we calculated the speed of the car as $v = 0.8c$, so we plug everything into our contraction equation to get

$$L' = 4\text{ m}\sqrt{1-\frac{v^2}{c^2}} \tag{1.200}$$

$$= 4\text{ m}\sqrt{0.36} \tag{1.201}$$

$$= 2.4\text{ m} \tag{1.202}$$

Correct Answer
(A)

1.71 PGRE8677 #71

Questions 69-71

A car of rest length 5 meters passes through a garage of rest length 4 meters. Due to the relativistic Lorentz contraction, the car is only 3 meters long in the garage's rest frame. There are doors on both ends of the garage, which open automatically when the front of the car reaches them and close automatically when the rear passes them. The opening or closing of each door requires a negligible amount of time.

69. The velocity of the car in the garage's rest frame is

(A) 0.4 c
(B) 0.6 c
(C) 0.8 c
(D) greater than c
(E) not determinable from the data given

70. The length of the garage in the car's rest frame is

(A) 2.4 m
(B) 4.0 m
(C) 5.0 m
(D) 8.3 m
(E) not determinable from the data given

71. Which of the following statements is the best response to the question:

"Was the car ever inside a closed garage?"

(A) No, because the car is longer than the garage in all reference frames.
(B) No, because the Lorentz contraction is not a "real" effect.
(C) Yes, because the car is shorter than the garage in all reference frames.
(D) Yes, because the answer to the question in the garage's rest frame must apply in all reference frames.
(E) There is no unique answer to the question, as the order of door openings and closings depends on the reference frame.

Recommended Solution

I'm generally not a big fan of these multi-part GRE questions, however I really like this one because it illustrates one of the classic paradoxes, or at least apparent paradoxes, in relativity. In the vehicles reference frame, the garage is too short to contain the vehicle. However, in the garage reference frame the car should have no problem fitting in the garage. The primary qualitative lesson we learned from Einstein is that we can't prefer one reference frame to another. Based on this, we aren't justified in saying that one solution is more "true" than the other. The only solution which alludes to this inherent duality of solutions is (E), which describes another classical qualitative result from Einstein. Specifically, I'm referring to our inability to achieve truly instantaneous events between different reference frames.

Correct Answer
(E)

1.72 PGRE8677 #72

72. The measured index of refraction of x-rays in rock salt is less than one. This is consistent with the theory of relativity because

(A) relativity deals with light waves traveling in a vacuum only
(B) x-rays cannot transmit signals
(C) x-ray photons have imaginary mass
(D) the theory of relativity predates the development of solid-state physics
(E) the phase velocity and group velocity are different

Recommended Solution

This problem confuses me because it has 1 correct choice and 4 absurd choices. This means you don't need to know why the correct answer is correct, only that the other 4 are ridiculous.

(A) What? Since when? No it doesn't!

(B) What? Of course they can transmit signals! How else would we get any useful information out of x-rays in, for example, medical applications.

(C) WHAT? Imaginary mass? When? Where? NO!

(D) And? Why would the order in which physical theories were discovered determine the accuracy of Relativity?

(E) The phase velocity and group velocities indeed are different. This is important because the refractive index of light through a medium is the ratio of the phase speed of light through the medium to that of light through a vacuum. It is, therefore, significant that the phase speed never be faster than the speed of light, which is frequently true of the group velocity. If they weren't different, we might get $n > 1$. However, you don't really need to know any of that to solve this problem, do you!

Correct Answer
(E)

1.73 PGRE8677 #73

73. It is necessary to coat a glass lens with a non-reflecting layer. If the wavelength of the light in the coating is λ, the best choice is a layer of material having an index of refraction between those of glass and air and a thickness of

(A) $\frac{\lambda}{4}$ (B) $\frac{\lambda}{2}$ (C) $\frac{\lambda}{\sqrt{2}}$ (D) λ (E) 1.5λ

Recommended Solution

If you've ever seen the rainbow patterns that form on water bubbles and oil slicks, then you've seen thin film interference. This phenomena occurs as a result of light waves reflecting off of the top surface of a medium with a higher index of refraction than that from which the light came from and, additionally, light waves reflecting off of the bottom of that surface as another transition to a higher index of refraction occurs (Figure 1.10)

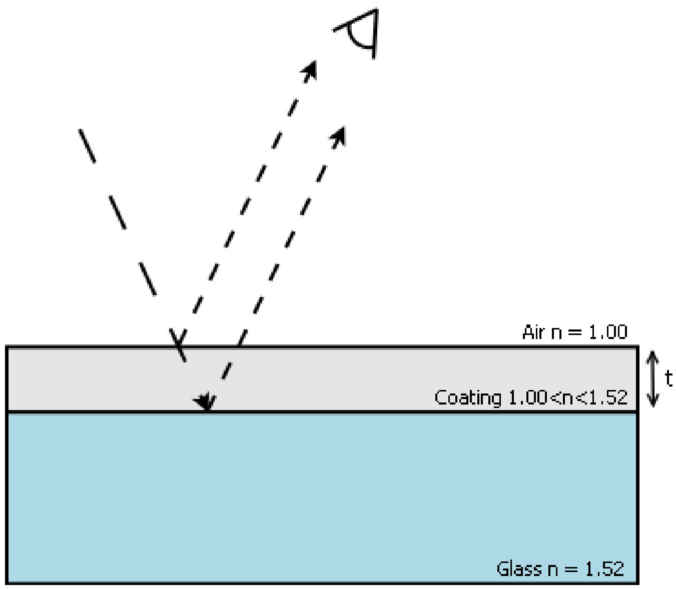


Figure 1.10: Thin film interference to create a non-reflecting layer

The same phenomena occurs whether the light is incident on the top surface at an angle or orthogonal to it, however I've intentionally drawn it at an angle to make it easier to observe the phase shift between the two reflected rays. For every reflection of a light wave off a medium of higher index of refraction, we get a shift of half the wavelength of the light,

$$\Delta_{\text{air}} = \frac{\lambda}{2} \tag{1.203}$$

$$\Delta_{\text{coating}} = \left(\frac{\lambda}{2} + 2t\right) \tag{1.204}$$

In the previous equations, note that Δ_{coating} has an additional $2t$ of phase shift because it must also pass both ways through the thickness of the coating. In order for the coating to eliminate all reflections, we need the outgoing waves to experience destructive interference, so we find the relative shift of the system

$$\begin{aligned}
\Delta &= \left(\Delta_{\text{coating}}\right) - \left(\Delta_{\text{air}}\right) && (1.205)\\
&= \left(2t + \frac{\lambda}{2}\right) - \left(\frac{\lambda}{2}\right) && (1.206)\\
&= 2t && (1.207)
\end{aligned}$$

since we only get destructive interference with waves of differing phase shift at half-odd integer values , set the relative shift to

$$\begin{aligned}
2t &= \left(m + \frac{1}{2}\right)\lambda && (1.208)\\
t &= \frac{\lambda m}{2} + \frac{\lambda}{4} && (1.209)
\end{aligned}$$

where $m = 0, 1, 2, 3, \ldots$. Since we are only concerned with the thinnest possible coating which will give us deconstructive intereference, we choose $m = 0$ and $t = \lambda/4$.

Correct Answer
(A)

1.74 PGRE8677 #74

74. Unpolarized light is incident on two ideal polarizers in series. The polarizers are oriented so that no light emerges through the second polarizer. A third polarizer is now inserted between the first two and its orientation direction is continuously rotated through 180°. The maximum fraction of the incident power transmitted through all three polarizers is

(A) zero (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$ (E) 1

Recommended Solution

Consider a beam of unpolarized light headed towards you and you have three ideal polarizers in hand

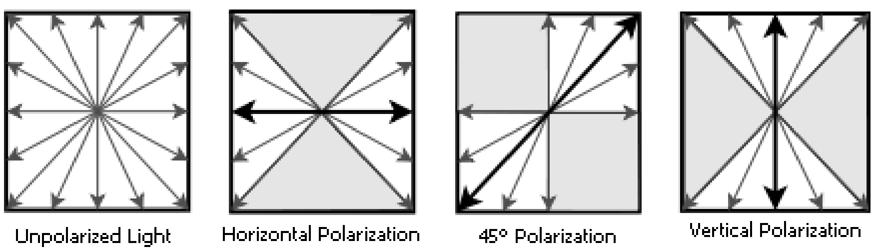


Figure 1.11: Different orientations of a polarizing filter

In Figure 1.11, grey vectors indicate individual directions of oscillation, black vectors indicate the net polarization, greyed out areas indicate areas of absorbed polarization and white areas indicate unaffected polarization. If we wanted to completely eliminate all light using two of the polarizers, we could place them in series with a rotation of exactly $\pi/2$ between them, for example using the horizontal and vertical polarization. In this arrangment, anything that survived through the horizontal polarizer would be caught by the vertical polarizer and no light would be transmitted on the other side. However, imagine we were to place another polarizer between the horizontal and vertical polarizers such that this third polarizer is rotated $\pi/4$ or 45 with respect to the other two. As the light first passes through the horizontal polarizer, only half of the photons that hit the polarizer will pass through. These photons will continue to the 45 polarizer where, again, half of the remaining photons get absorbed and the other half pass through. After the photons pass through the 45 polarizer, the remaining photons will spread out from -22.5 to +67.5. This occurs because linearly polarized light will always completely fill a full 90 of angular spread, which I didn't mention previously because the previous instances came out with a 90 spread. Finally, the remaining photons will pass through the vertical polarizer giving a final total of 1/8 the original number of photons (Figure 1.12)

Correct Answer
(B)

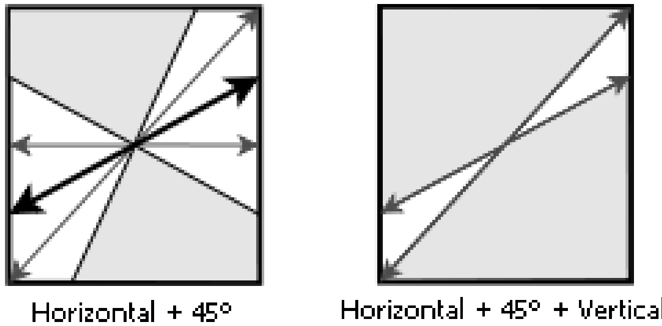
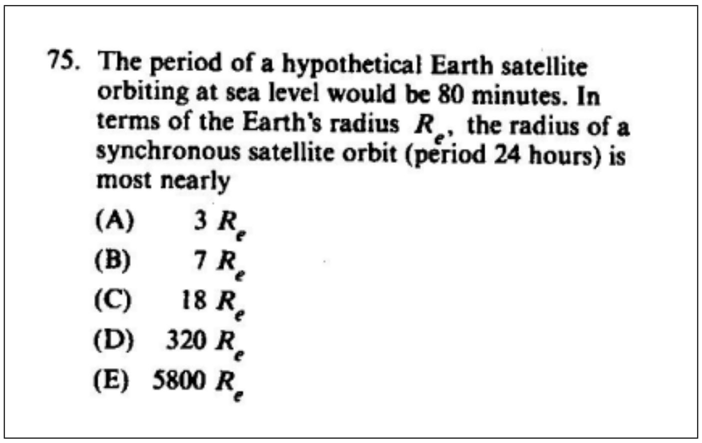


Figure 1.12: Remaining light after passing through the three polarizer system

1.75 PGRE8677 #75



75. The period of a hypothetical Earth satellite orbiting at sea level would be 80 minutes. In terms of the Earth's radius R_e, the radius of a synchronous satellite orbit (period 24 hours) is most nearly

(A) $3 R_e$
(B) $7 R_e$
(C) $18 R_e$
(D) $320 R_e$
(E) $5800 R_e$

Recommended Solution

Recall Kepler's third law of motion

> *"The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit"*

so we have

$$T^2 = \beta r^3 \tag{1.210}$$

The period is given as $T = 80$ minutes, so

$$(80 \text{ min})^2 = \beta R_e^3 \tag{1.211}$$

$$\beta = \frac{6400 \text{ min}^2}{R_e^3} \tag{1.212}$$

using the same proportionality constant, β, but for 24 hours (i.e. 1440 minutes), we get

$$
\begin{aligned}
T^2 &= \beta r^3 && (1.213)\\
(1440 \text{ min})^2 &= \left(\frac{6400 \text{ min}^2}{R_e^3}\right) r^3 && (1.214)\\
r^3 &= \frac{1440 \text{ min}^2 \ R_e^3}{6400 \text{ min}^2} && (1.215)
\end{aligned}
$$

For a quick and easy approximation, pretend that $1440 = 1500$ and $6400 = 6000$, so then we get

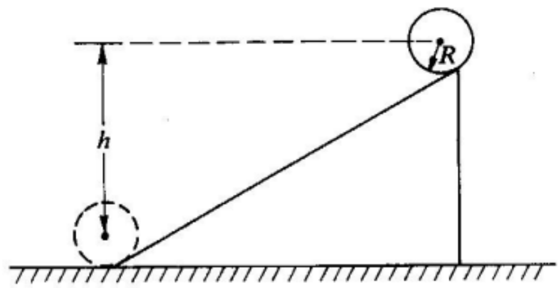
$$
\begin{aligned}
r^3 &= \frac{(1440 \cdot 1440) R_e^3}{6400} && (1.216)\\
&= \frac{1440}{4} \ R_e^3 && (1.217)\\
&= 360 && (1.218)
\end{aligned}
$$

Finally, take the cube root of both sides to get

$$
\begin{aligned}
r &= \sqrt[3]{360} && (1.219)\\
&\approx 7 && (1.220)
\end{aligned}
$$

Correct Answer
(B)

1.76 PGRE8677 #76



76. A hoop of mass M and radius R is at rest at the top of an inclined plane as shown above. The hoop rolls down the plane without slipping. When the hoop reaches the bottom, its angular momentum around its center of mass is

(A) $MR\sqrt{gh}$

(B) $\frac{1}{2}MR\sqrt{gh}$

(C) $M\sqrt{2gh}$

(D) Mgh

(E) $\frac{1}{2}Mgh$

Recommended Solution

Recall the angular momentum equation

$$L = I\omega \tag{1.221}$$

in which the moment of inertia, I, of a ring is

$$I_{\text{ring}} = MR^2 \tag{1.222}$$

which gives us

$$L = MR^2\omega \tag{1.223}$$

To solve for ω sum the total energy of the rolling ring

$$U_{\text{tot}} = U_{\text{roll}} + U_{\text{trans}} \tag{1.224}$$

$$Mgh = \left(\frac{1}{2}I\omega^2\right) + \left(\frac{1}{2}Mv^2\right) \tag{1.225}$$

but since $\omega = v/R$, we get

$$
\begin{aligned}
Mgh &= \left(\frac{1}{2}MR^2\omega^2\right) + \left(\frac{1}{2}MR^2\omega^2\right) && (1.226)\\
gh &= R^2\omega^2 && (1.227)\\
\omega &= \sqrt{\frac{gh}{R^2}} && (1.228)
\end{aligned}
$$

Then combining ω into the angular momentum equation, we get the solution

$$
\begin{aligned}
L &= MR^2\omega && (1.229)\\
&= MR^2\sqrt{\frac{gh}{R^2}} && (1.230)\\
&= MR\sqrt{gh} && (1.231)
\end{aligned}
$$

Correct Answer
(B)

1.77 PGRE8677 #77

77. A particle is constrained to move along the x-axis under the influence of the net force $\mathbf{F} = -k\mathbf{x}$ with amplitude A and frequency f, where k is a positive constant. When $x = A/2$, the particle's speed is

(A) $2\pi f A$ (B) $\sqrt{3}\pi f A$ (C) $\sqrt{2}\pi f A$

(D) $\pi f A$ (E) $\frac{1}{3}\pi f A$

Recommended Solution

Any object with a net force of $F = -kx$ is a simple harmonic oscillator (via Hooke's Law). The position equation for a SHO is

$$x = A\cos(\omega t) \tag{1.232}$$

take the derivative of position to get the velocity equation

$$\frac{dx}{dt} = -A\sin(\omega t)\omega \tag{1.233}$$

If we square both equations and get their trig functions by themselves, we can apply the identity $\cos^2(\theta) + \sin^2(\theta) = 1$,

$$\frac{x^2}{A^2} = \cos^2(\omega t) \tag{1.234}$$

$$\frac{v^2}{A^2\omega^2} = \sin^2(\omega t) \tag{1.235}$$

using our trig identity and letting our position be $x = A/2$,

$$\cos^2(\omega t) + \sin^2(\omega t) = 1 \tag{1.236}$$

$$\frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1 \tag{1.237}$$

$$\frac{1}{4} + \frac{v^2}{A^2\omega^2} = 1 \tag{1.238}$$

Now solve for v

$$\frac{v^2}{A^2\omega^2} = \frac{3}{4} \tag{1.239}$$

$$v^2 = \frac{3}{4}A^2\omega^2 \tag{1.240}$$

Finally, since $\omega = 2\pi f$,

$$v = \sqrt{\frac{3}{4}A^2 4\pi^2 f^2} \tag{1.241}$$
$$= \sqrt{3}\pi f A \tag{1.242}$$

Correct Answer
(B)

1.78 PGRE8677 #78

78. A system consists of two charged particles of equal mass. Initially the particles are far apart, have zero potential energy, and one particle has nonzero speed. If radiation is neglected, which of the following is true of the total energy of the system?

(A) It is zero and remains zero.
(B) It is negative and constant.
(C) It is positive and constant.
(D) It is constant, but the sign cannot be determined unless the initial velocities of both particles are known.
(E) It cannot be a constant of the motion because the particles exert force on each other.

Recommended Solution

Total energy of the system must be the sum of the kinetic and potential energies

$$U_{\text{net}} = T + V \tag{1.243}$$

We are told that the initial potential is 0 and one of the particles has an initial velocity and, therefore, an initial kinetic energy

$$U_0 = \frac{1}{2}mv_0^2 \tag{1.244}$$

Since the problem tells us to neglect radiation, we assume energy is conserved so the final energy needs to equal the initial energy

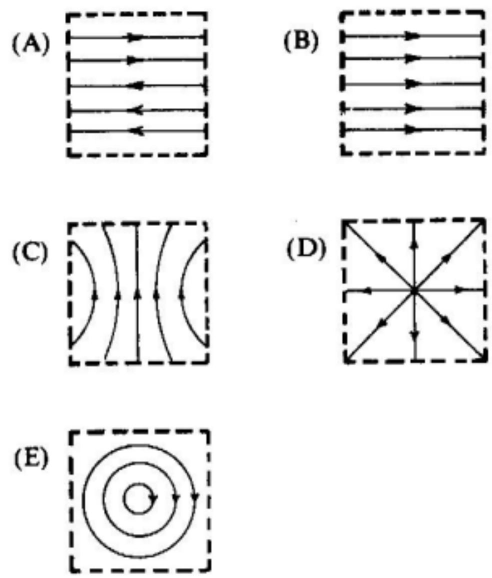
$$U_f = U_0 = T + V = \frac{1}{2}mv_0^2 \tag{1.245}$$

which tells us that the total energy of the system is constant over time and also that the energy is positive (assuming neither particle has negative mass).

Correct Answer
(C)

1.79 PGRE8677 #79

79. One of Maxwell's equations is $\nabla \cdot \mathbf{B} = 0$. Which of the following sketches shows magnetic field lines that clearly violate this equation within the region bounded by the dashed lines?



Recommended Solution

Recall that Maxwell's eqution $\nabla \cdot \vec{B} = 0$ tells us that divergence of the magnetic field must always be 0 and that divergence is defined as the amount of outward flux of a vector field. With this definition, we know that we need to look for any vector field that has some non-zero flux leaving the area.

(A) 5 flux lines come in and 5 flux lines go out. Net flux leaving the area is 0.

(B) 5 flux lines come in and 5 flux lines go out. Net flux leaving the area is 0.

(C) 5 flux lines come in and 5 flux lines go out. Net flux leaving the area is 0.

(D) 0 flux lines come in and 8 flux lines go out. Net flux leaving the area is NOT 0.

(E) 0 flux lines come in and 0 flux lines go out. Net flux leaving the area is 0.

Correct Answer
(D)

1.80 PGRE8677 #80

80. Which of the following electric fields could exist in a finite region of space that contains no charges? (In these expressions, A is a constant, and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors pointing in the x, y, and z directions, respectively.)

(A) $A(2xy\mathbf{i} - xz\mathbf{k})$
(B) $A(-xy\mathbf{j} + xz\mathbf{k})$
(C) $A(xz\mathbf{i} + xz\mathbf{j})$
(D) $Axyz(\mathbf{i} + \mathbf{j})$
(E) $Axyz\mathbf{i}$

Recommended Solution

From Gauss's law, we get

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \tag{1.246}$$

with no charges in the region, $\rho = 0$ and

$$\nabla \cdot \vec{E} = 0 \tag{1.247}$$

take the derivative of each term with respect to its appropriate variable (i.e. $x \to \hat{\imath}$, $y \to \hat{\jmath}$ and $z \to \hat{k}$)

(A) $A(2y - x) \neq 0$

(B) $A(-x + x) = 0$

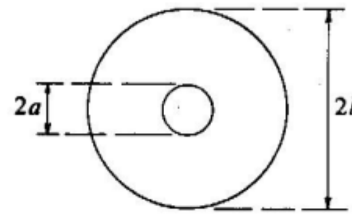
(C) $A(z + 0) \neq 0$

(D) $Ayz + Axz \neq 0$

(E) Ayz

Correct Answer
(B)

1.81 PGRE8677 #81



81. A small circular wire loop of radius a is located at the center of a much larger circular wire loop of radius b as shown above. The larger loop carries an alternating current $I = I_0 \cos \omega t$, where I_0 and ω are constants. The magnetic field generated by the current in the large loop induces in the small loop an emf that is approximately equal to which of the following? (Either use mks units and let μ_0 be the permeability of free space, or use Gaussian units and let μ_0 be $4\pi/c^2$.)

(A) $\left(\frac{\pi\mu_0 I_0}{2}\right)\frac{a^2}{b}\omega \cos \omega t$

(B) $\left(\frac{\pi\mu_0 I_0}{2}\right)\frac{a^2}{b}\omega \sin \omega t$

(C) $\left(\frac{\pi\mu_0 I_0}{2}\right)\frac{a}{b^2}\omega \sin \omega t$

(D) $\left(\frac{\pi\mu_0 I_0}{2}\right)\frac{a}{b^2} \cos \omega t$

(E) $\left(\frac{\pi\mu_0 I_0}{2}\right)\frac{a}{b} \sin \omega t$

Recommended Solution

Recall from Faraday's law of induction,

$$\varepsilon = -\frac{d\phi_{\vec{B}}}{dt} \tag{1.248}$$

since the magnetic field is proportional to the current, $\vec{B} \propto I$, we know we will eventually have to take the derivative of I and this means we should end up with $\sin(\omega t)$ rather than $\cos(\omega t)$ so we can eliminate (A) and (D). Next, since we know we want to end up with a units of volts for our EMF, check the units of (B), (C) and (E). μ_0 is given in the front of the test booklet and as long as you recall that weber = volt · sec,

(B) $\left(\frac{volt \cdot sec}{Amp \cdot m}\right)(Amp)\left(\frac{m^2}{m}\right)\left(\frac{1}{sec}\right) = volt$

(C) $\left(\frac{volt \cdot sec}{Amp \cdot m}\right)(Amp)\left(\frac{m}{m^2}\right)\left(\frac{1}{sec}\right) = \frac{volt}{m^2}$

(E) $\left(\frac{volt \cdot sec}{Amp \cdot m}\right)(Amp)\left(\frac{m}{m}\right)\left(\frac{1}{sec}\right) = \frac{volt}{m}$

Correct Answer
(B)

Alternate Solution

From Faraday's law of induction,

$$\varepsilon = -\frac{d\phi_{\vec{B}}}{dt} \tag{1.249}$$

where the magnetic flux, $\phi_{\vec{B}}$, is

$$\phi_{\vec{B}} = \int \vec{B} \cdot dA = \vec{B}\pi a^2 \tag{1.250}$$

From the Biot-Savart law, we get the magnetic field from a current passing through a loop of wire as

$$\begin{aligned} \vec{B} &= \int \frac{\mu_0 I dl \times \hat{r}}{4\pi R^2} && (1.251) \\ &= \int \frac{\mu_0 I dl \sin(\theta)}{4\pi R^2} && (1.252) \end{aligned}$$

since all of the potential solutions have no θ dependence like they should, we'll assume that the approximate solution the problem refers to is simply finding the resulting EMF at the optimized angle, $\theta = \pi/2$, with radius b,

$$\begin{aligned} \vec{B} &= \int \frac{\mu_0 I dl \sin(\pi/2)}{4\pi b^2} && (1.253) \\ &= \frac{\mu_0 I}{4\pi b^2} \int dl && (1.254) \\ &= \frac{\mu_0 I}{4\pi b^2}(2\pi b) && (1.255) \\ &= \frac{\mu_0 I}{2b} && (1.256) \end{aligned}$$

Plug this all back into the magnetic flux to get

$$\begin{aligned} \phi_{\vec{B}} &= \frac{\mu_0 I \pi a^2}{2b} && (1.257) \\ &= \frac{\mu_0 \pi}{2} \frac{a^2}{b} (I_0 \cos(\omega t)) && (1.258) \end{aligned}$$

Finally, take the derivative of $\phi_{\vec{B}}$ with respect to time and get

$$\begin{aligned} V &= \frac{\mu_0 \pi}{2} \frac{a^2}{b} I_0 \frac{d}{dt}(\cos(\omega t)) && (1.259) \\ &= \frac{\mu_0 \pi}{2} \frac{a^2}{b} I_0 \omega \sin(\omega t) && (1.260) \end{aligned}$$

Correct Answer
(B)

1.82 PGRE8677 #82

82. The emission spectrum of an atomic gas in a magnetic field differs from that of the gas in the absence of a magnetic field. Which of the following is true of the phenomenon?
    (A) It is called the Stern-Gerlach effect.
    (B) It is called the Stark effect.
    (C) It is due primarily to the nuclear magnetic moment of the atoms.
    (D) The number of emission lines observed for the gas in a magnetic field is always twice the number observed in the absence of a magnetic field.
    (E) The number of emission lines observed for the gas in a magnetic field is either greater than or equal to the number observed in the absence of a magnetic field.

Recommended Solution

This problem is describing a phenomena called the Zeeman effect, in which an atom with electrons in a degenerate state (i.e. electrons with equivalent energies but differing electron configurations) is passed through a magnetic field. Since the affect of a magnetic field is related to the electron configuration, the degenerate electrons will have their energies altered by the magnetic field in different ways and the single emission line which was initially composed of multiple degenerate electrons becomes visible as different emission lines (Figure 1.13),

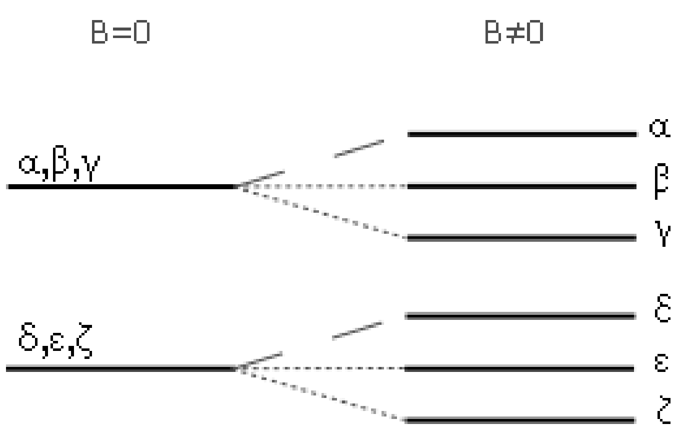


Figure 1.13: The Zeeman effect for degenerate energy states

This tells us that the possible solutions is either (D) or (E) but since there is no specific restriction which says the emission lines must always be doubled, we choose (E).

Correct Answer
(E)

1.83 PGRE8677 #83

83. A spectral line is produced by a gas that is sufficiently dense that the mean time between atomic collisions is much shorter than the mean lives of the atomic states responsible for the line. Compared with the same line produced by a low-density gas, the line produced by the higher-density gas will appear

(A) the same
(B) more highly polarized
(C) broader
(D) shifted toward the blue end of the spectrum
(E) split into a doublet

Recommended Solution

The phrasing of this problem tells us everything we need to know in order to figure out the difference between a low-density and high-density state of the same gas. For a high density gas the mean life of an atomic state is significantly longer than the amount of time it takes for atomic collisions to occur, so by the time the atomic life is up, multiple collisions have occurred and energy has been swapped around a bunch of times. In a less dense state, not as many of these energy exchanges have occurred so there are fewer distinct energies and the ones that are there are more well-defined than the high-density gas. This is all you need to know to tell you that the high-density gas will have a broader spectral line than the low-density gas. If you want something a bit more rigorous, recall Heisenberg's Energy-Time uncertainty principle

$$\Delta E \ \Delta t \geq \frac{\hbar}{2} \tag{1.261}$$

For the higher-density gas, as compared to a low-density gas, the decrease in time uncertainty requires an increase in energy uncertainty so we get a broader (i.e. less precise) spectral line. Alternatively, the increase in time uncertainty for the less-dense gas will give require that the energy uncertainty be less than the high-density gas and so we expect a more well-defined (i.e. thinner) spectral line.

Correct Answer
(C)

1.84 PGRE8677 #84

84. Sodium has eleven electrons and the sequence in which energy levels fill in atoms is 1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 4*s*, 3*d*, etc. What is the ground state of sodium in the usual notation $^{2S+1}L_J$?

(A) $^1S_0$ (B) $^2S_{\frac{1}{2}}$ (C) $^1P_0$

(D) $^2P_{\frac{1}{2}}$ (E) $^3P_{\frac{1}{2}}$

Recommended Solution

Sodium has 11 electrons, just as the problem so kindly gives us, so we construct our energy level diagram (Figure 1.14)

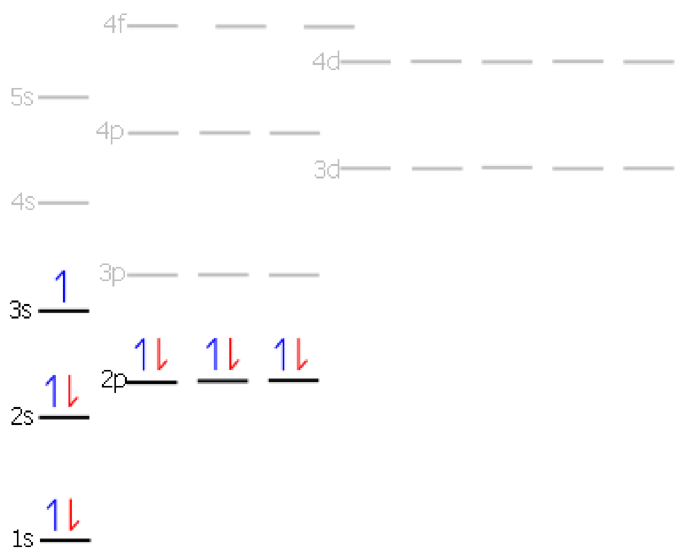


Figure 1.14: Energy level diagram of Sodium in its ground state

With our one lone electron, we expect to get a spin of $S = 1/2$. This tells us that our term symbol component of $2S+1$ will become $2(1/2)+1 = 2$, so our term symbol will be of the form

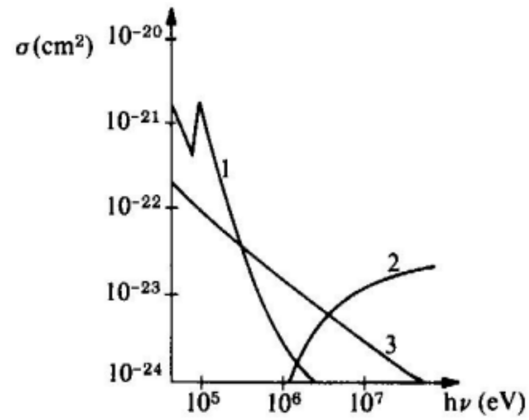
$$^2L_J \tag{1.262}$$

which allows us to eliminate all but (B) and (D). Since the final electron is in the 3s shell, $L = 0$ and since $J = L + S$, $J = 1/2$. From this, our term symbol should be

$$^2S_{\frac{1}{2}} \tag{1.263}$$

Correct Answer
(B)

1.85 PGRE8677 #85



85. The figure above shows the photon interaction cross sections for lead in the energy range where the Compton, photoelectric, and pair production processes all play a role. What is the correct identification of these cross sections?

(A) 1 = photoelectric, 2 = Compton, 3 = pair production
(B) 1 = photoelectric, 2 = pair production, 3 = Compton
(C) 1 = Compton, 2 = pair production, 3 = photoelectric
(D) 1 = Compton, 2 = photoelectric, 3 = pair production
(E) 1 = pair production, 2 = photoelectric, 3 = Compton

Recommended Solution

As a general rule,

Photoelectric effect: Low Energy (eV Range)

Compton Scattering: Mid Energy (KeV Range)

Pair Production: High Energy (> 1 MeV)

Based on this general rule, we know that line #2 should correspond to pair production because it only exists at higher energy ranges, which eliminates (A), (D) and (E). Between (B) and (C), it is more likely that line #1 corresponds to the Photoelectric effect because the interaction peaks at a lower energy range.

Correct Answer
(B)

1.86 PGRE8677 #86

86. The exponent in Coulomb's inverse square law has been found to differ from two by less than one part in a billion by measuring which of the following?

(A) The charge on an oil drop in the Millikan experiment
(B) The deflection of an electron beam in an electric field
(C) The neutrality of charge of an atom
(D) The electric force between two charged objects
(E) The electric field inside a charged conducting shell

Recommended Solution

In Faraday's Ice Pail experiment, Michael Faraday took an insulated pail which he connected to an electrometer. A brass ball with a charge on its surface was lowered down into the pail. Because the brass ball surface has a net negative charge, the pails positive charges were attracted to the inner walls of the pail and negative charges were pushed to the outside of the pail and also pushed down the wire to the electrometer. As the charged brass ball was lowered enough to touch the inner floor of the pail, the charge was completely discharged to the pail and the ball could be removed with the electrometer reading a complete transfer in charge. From this experiment, Faraday was able to determine that the inverse square law was accurate to at least one part in a billion.

Correct Answer
(E)

1.87 PGRE8677 #87

87. In a gas of N diatomic molecules, two possible models for a classical description of a diatomic molecule are:

Which of the following statements about this gas is true?

(A) Model I has a specific heat $c_v = \frac{3}{2}Nk$.
(B) Model II has a smaller specific heat than Model I.
(C) Model I is always correct.
(D) Model II is always correct.
(E) The choice between Models I and II depends on the temperature.

Recommended Solution

Recall that the specific heat for any atom/molecule is proportional to the degrees of freedom of the molecule. In other words, the more degrees of freedom a molecule has the higher the specific heat.

(A) a specific heat of 3/2 Nk corresponds to a monotomic atom with only translational degrees of freedom, so we know that these diatomic molecules must be larger than that.

(B) Model I has translational and rotational degrees of freedom while Model II has tranlational, rotational and vibrational degrees of freedom. We would expect the specific heat to be larger in Model II than Model I, not the other way around.

(C) At high energies (i.e. high temperatures) vibrational effects must be considered, so we can't always treat a molecule as a rigid rotor.

(D) At low energies (i.e. low temperatures) vibrational effects are minimal, so we don't always have to treat a molecule as a springy dumbbell.

(E) At low energies (i.e. low temperatures) there is minimal vibrational effect so we can treat the molecule as if it is a rigid rotor. However, at higher energies (i.e. higher temperatures) vibrational effects are involved so we want to treat it as a springy dumbbell. In other words, the model we use on a molecule is dependent on its temperature.

Correct Answer
(E)

1.88 PGRE8677 #88

88. Consider a system of N noninteracting particles confined in a volume V at a temperature such that the particles obey classical Boltzmann statistics. If the temperature is lowered to the point at which quantum effects become important, the pressure of the gas may differ depending on whether the particles are fermions or bosons. Let P_F be the pressure exerted by the particles if they are fermions, P_B be the pressure if they are bosons, and P_C be the pressure the particles would exert if quantum effects are ignored. Which of the following is true?

(A) $P_F = P_B = P_C$

(B) $P_F > P_C > P_B$

(C) $P_F > P_B > P_C$

(D) $P_F < P_B < P_C$

(E) $P_F < P_C < P_B$

Recommended Solution

Recall that fermions must obey the Pauli Exclusion principle which restricts the particles from occupying the same energy levels. Bosons, on the other hand have no such restriction. This tells us that in its lowest energy state, N bosons will all be able to occupy the minimal energy state while N fermions will be forced to occupy higher energy levels after the ground level has been filled. This tells us that the pressure, which is proportional to temperature (energy), should be greater for the fermions than the bosons, which eliminates all but (B) and (C). Since the boson pressure is the low pressure extreme and fermion pressure is the high pressure extreme, it is only logical to conclude that the classical result (think $PV = nRT$) will be somewhere in between.

Correct Answer
(B)

1.89 PGRE8677 #89

89. A system containing two identical particles is described by a wave function of the form

$$\psi = \frac{1}{\sqrt{2}} [\psi_\alpha(x_1)\,\psi_\beta(x_2) + \psi_\beta(x_1)\,\psi_\alpha(x_2)]$$

where x_1 and x_2 represent the spatial coordinates of the particles and α and β represent all the quantum numbers, including spin, of the states that they occupy. The particles might be

(A) electrons
(B) positrons
(C) protons
(D) neutrons
(E) deuterons

Recommended Solution

For two identical particles, the wave function for those particles can only be either symmetric or anti-symmetric under interchange of x_1 and x_2 so our two options are

$$\psi(x_1, x_2, t) = \psi(x_2, x_1, t) \tag{1.264}$$

or

$$\psi(x_1, x_2, t) = -\psi(x_2, x_1, t) \tag{1.265}$$

As it turn out, whether or not the wave function is symmetric or anti-symmetric is determined by the type of particle. Specifically, symmetric wave functions obey Bose-Einstein statistics and correspond to bosons. Anti-symmetric wave functions obey Fermi-Dirac statistics and correspond to fermions. From this, our two potential wave functions are

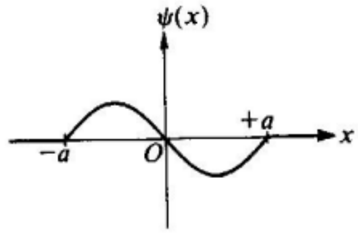
$$\psi_{boson} = \frac{1}{\sqrt{2}} [\psi_\alpha(x_1)\psi_\beta(x_2) + \psi_\beta(x_1)\psi_\alpha(x_2)] \tag{1.266}$$

$$\psi_{fermion} = \frac{1}{\sqrt{2}} [\psi_\alpha(x_1)\psi_\beta(x_2) - \psi_\beta(x_1)\psi_\alpha(x_2)] \tag{1.267}$$

Clearly, the wave function referred to in the problem corresponds to a boson and not a fermion. Electrons, positrons, protons and neutrons are all fermions while the deuteron is a boson, so we choose (E).

Correct Answer
(E)

1.90 PGRE8677 #90



90. The figure above shows one of the possible energy eigenfunctions $\psi(x)$ for a particle bouncing freely back and forth along the x-axis between impenetrable walls located at $x = -a$ and $x = +a$. The potential energy equals zero for $|x| < a$. If the energy of the particle is 2 electron volts when it is in the quantum state associated with this eigenfunction, what is its energy when it is in the quantum state of lowest possible energy?

(A) 0 eV (B) $\frac{1}{\sqrt{2}}$ eV (C) $\frac{1}{2}$ eV

(D) 1 eV (E) 2 eV

Recommended Solution

Without solving for anything explicitly, you can be sure that the ground state energy of any system will never be 0 eV and that the ground state energy, which is the lowest energy state, will be less than the total energy of the particle at 2 eV, so eliminate (A) and (E). Next, recall that the energy of the infinite square well is proportional to the nodes of its wave function by

$$E_n = \frac{n^2\hbar^2\pi^2}{2ML^2} \tag{1.268}$$

where the zero point energy, or equivalently the minimum energy value E_1, is

$$E_1 = \frac{\hbar^2\pi^2}{2ML^2} \tag{1.269}$$

so the zero point energy is related to the rest of the energy levels by

$$E_n = n^2 E_1 \tag{1.270}$$

The problem tells us that the energy, $E_n = 2$ eV so with $n = 2$, we get

$$E_1 = \frac{E_n}{n^2} \tag{1.271}$$

$$E_1 = \frac{2 \text{ eV}}{2^2} \tag{1.272}$$

$$= \frac{1}{2} \text{ eV} \tag{1.273}$$

Correct Answer
(C)

1.91 PGRE8677 #91

91. When a narrow beam of monoenergetic electrons impinges on the surface of a single metal crystal at an angle of 30 degrees with the plane of the surface, first-order reflection is observed. If the spacing of the reflecting crystal planes is known from x-ray measurements to be 3 ångstroms, the speed of the electrons is most nearly

(A) 1.4×10^{-4} m/s
(B) 2.4 m/s
(C) 5.0×10^{3} m/s
(D) 2.4×10^{6} m/s
(E) 4.5×10^{9} m/s

Recommended Solution

Without doing much work, you can eliminate (A) and (B) because they are much too slow to be the velocity of an electron. We can also eliminate (E) because the electrons shouldn't be moving faster than light. As a general rule, expect electron velocities around 10^5 m/s which, if you are feeling lucky and a bit adventurous, then you could risk it and guess (D). To be more rigorous, you will have to recall the Bragg condition from Bragg diffraction optics

$$2d\sin(\theta) = n\lambda \tag{1.274}$$

and recall the de Broglie relation

$$\lambda = \frac{h}{p} \tag{1.275}$$

combining the two gives you

$$2d\sin(\theta) = n\frac{h}{mv} \tag{1.276}$$

Since we are told that first-order reflection occurs, $n = 1$. Using approximate values for the remaining constants, we can solve for v,

$$\begin{aligned} v &= \frac{h}{md} && (1.277)\\ &\approx \frac{6 \times 10^{-34}\ \text{m}^2\text{kg/s}}{(10 \times 10^{-31}\ \text{kg})(3 \times 10^{-10}\ \text{m})} && (1.278)\\ &\approx 5 \times 10^{7}\ \text{m/s} && (1.279) \end{aligned}$$

which is closest to (D).

Correct Answer
(D)

1.92 PGRE8677 #92

92. Which of the following is NOT compatible with the selection rule that controls electric dipole emission of photons by excited states of atoms?

(A) Δn **may have any negative integral value.**

(B) $\Delta \ell = \pm 1$

(C) $\Delta m_\ell = 0, \pm 1$

(D) $\Delta s = \pm 1$

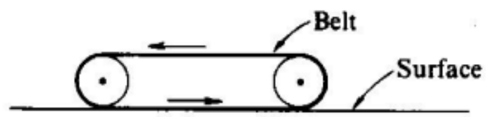
(E) $\Delta j = \pm 1$

Recommended Solution

Keep in mind that we are looking for a selection rule which is NOT compatible with electric dipole emission. You don't need to recall what any of the specific selection rules are as long as you recall what type of selection rules we have. There most definitely are selection rules for Δj, Δl, Δm_l and Δn but there are no selection rules for spin, Δs. Thus, without knowing if any of the other selection rules are correct, we can be sure that the proposed spin rule is NOT correct.

Correct Answer
(D)

1.93 PGRE8677 #93



93. An electric sander has a continuous belt that rubs against a wood surface as shown schematically above. The sander is 100 percent efficient and draws a current of 9 amperes from a 120-volt line. The belt speed is 10 meters per second. If the sander is pushing against the wood with a normal force of 100 newtons, the coefficient of friction is most nearly

(A) 0.02 (B) 0.2 (C) 0.4
(D) 1.1 (E) 10

Recommended Solution

Recall that the frictional force f, is equal to the product of the coefficient of friction, μ and the normal force $F_n = 100$ newtons

$$f = \mu(100 \text{ newtons}) \tag{1.280}$$
$$\mu = \frac{f}{(100 \text{ newtons})} \tag{1.281}$$

Next, recall that power, P, is

$$P = IV = Fv \tag{1.282}$$

where I is current, V is voltage, F is force and v is velocity. Solve for F with the problems given values for velocity, $v = 10$ m/s and current and voltage $I = 9$ Amps and $V = 120$ volts

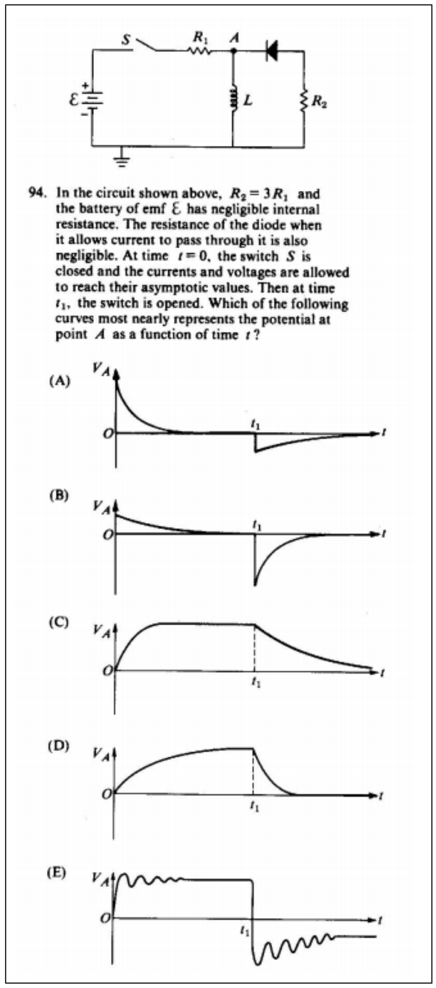
$$F = \frac{IV}{v} \tag{1.283}$$
$$= \frac{(9 \text{ Amps})(120 \text{ volts})}{(10 \text{ m/s})} \tag{1.284}$$
$$= 108 \text{ N} \tag{1.285}$$

This force is the applied force of the sander which we need for the coefficient of friction equation, so make the substitution to get

$$\mu = \frac{108 \text{ N}}{100 \text{ N}} \tag{1.286}$$
$$= 1.1 \tag{1.287}$$

Correct Answer
(D)

1.94 PGRE8677 #94



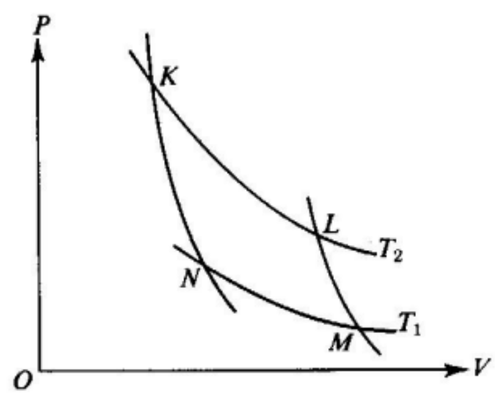
94. In the circuit shown above, $R_2 = 3R_1$ and the battery of emf $\mathcal{E}$ has negligible internal resistance. The resistance of the diode when it allows current to pass through it is also negligible. At time $t = 0$, the switch S is closed and the currents and voltages are allowed to reach their asymptotic values. Then at time t_1, the switch is opened. Which of the following curves most nearly represents the potential at point A as a function of time t?

Recommended Solution

Before switch S is closed, the voltage through the circuit should be 0. As soon as we throw the switch at time $t = 0$, the maximum voltage is reached and it continually decreases as it reaches equilibrium while passing through resistors R_2 and R_1. From this, we know that our solution should start at a maximum V_A and approach 0 which is not true of (C), (D) and (E). Once the switch is opened again at time t_1, the current is able to flow again and since the potential collected by the inductor only has to pass current through one of the resistors, R_2 to be specific, we expect the initial potential at t_1 to be greater than at t_0 so (B) is correct.

Correct Answer
(B)

1.95 PGRE8677 #95



95. In the cycle shown above, KL and NM represent isotherms, while KN and LM represent reversible adiabats. A system is carried through the Carnot cycle $KLMN$, taking in heat Q_2 from the hot reservoir T_2 and releasing heat Q_1 to the cold reservoir T_1. All of the following statements are true EXCEPT:

(A) $Q_1/T_1 = Q_2/T_2$.
(B) The entropy of the hot reservoir decreases.
(C) The entropy of the system increases.
(D) The work W done is equal to the net heat absorbed, $Q_2 - Q_1$.
(E) The efficiency of the cycle is independent of the working substance.

Recommended Solution

The Carnot cycle is, theoretically, perfectly efficient and so we should expect in a Carnot engine, the entropy of the system should be perfectly conserved, $dS = 0$. (C) is in disagreement with this aspect of the Carnot cycle so it is incorrect.

Correct Answer
(C)

1.96 PGRE8677 #96

96. A particle of mass M is in an infinitely deep square well potential V where

$V = 0$ for $-a \leqq x \leqq a$, and

$V = \infty$ for $x < -a$, $a < x$.

A very small perturbing potential V' is superimposed on V such that

$V' = \epsilon\left(\frac{a}{2} - |x|\right)$ for $\frac{-a}{2} \leqq x \leqq \frac{a}{2}$, and

$V' = 0$ for $x < \frac{-a}{2}$, $\frac{a}{2} < x$.

If ψ_0, ψ_1, ψ_2, ψ_3, ... are the energy eigenfunctions for a particle in the infinitely deep square well potential, with ψ_0 being the ground state, which of the following statements is correct about the eigenfunction ψ_0' of a particle in the perturbed potential $V + V'$?

(A) $\psi_0' = a_{00}\psi_0$, $a_{00} \neq 0$

(B) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all odd values of n

(C) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all even values of n

(D) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} \neq 0$ for all values of n

(E) None of the above

Recommended Solution

The perturbation described in this problem should look a little something like Figure 1.15

From this, we see that the maximum impact of the perturbation will be at the center, $x = 0$ and so wave functions with a node at that location will be relatively unaffected (i.e. wave functions with odd-values for n). However, because all wave functions with odd-valued n are also odd functions, we know that the total area under the curve will sum to 0 (which is something a perturbation could have corrupted but, in our case, didn't). So, for all odd values of n, the perturbed wavefunction will still have its coefficient $a_{0,n} = 0$ and we choose (B).

Correct Answer
(B)

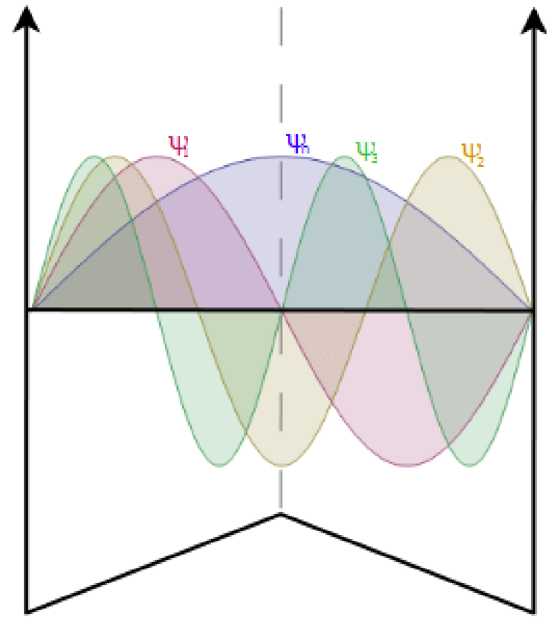


Figure 1.15: Infinite square well subject to a perturbation

1.97 PGRE8677 #97

Recommended Solution

First off, eliminate (E) because it should have no dependence on time. Next, eliminate (B) and (D) because they are the same answer and so if they are both correct, we have a problem. Now, when choosing between (A) and (D), consider that the total angular momentum will be the sum of the translational component and the rotational component

$$L = L_{trans} + L_{rot} \tag{1.288}$$

Recall that angular momentum is

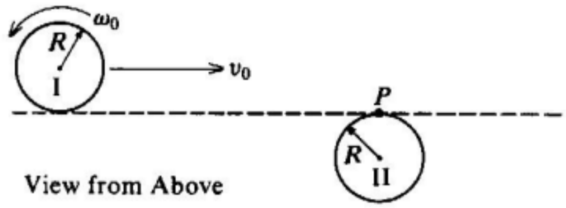
$$L = I\omega_0 = \vec{R} \times \vec{p} = \vec{R} \times M\vec{v} \tag{1.289}$$

For the translational component of angular momentum, we use the fact that $L = \vec{R} \times M\vec{v}$ with our initial velocity $\vec{v}_0$

$$L_{trans} = \vec{R} \times M\vec{v}_0 \tag{1.290}$$
$$= RM\left(\frac{1}{2}\omega_0 R\right) \tag{1.291}$$
$$= \frac{1}{2}MR^2\omega_0 \tag{1.292}$$

For the rotational component, we use the moment of inertia given in the problem

$$L_{rot} = I\omega_0 \tag{1.293}$$
$$= \frac{1}{2}MR^2\omega_0 \tag{1.294}$$



97. Two uniform cylindrical disks of identical mass M, radius R, and moment of inertia $\frac{1}{2}MR^2$, as shown above, collide on a frictionless, horizontal surface. Disk I, having an initial counterclockwise angular velocity ω_0 and a center-of-mass velocity $v_0 = \frac{1}{2}\omega_0 R$ to the right, makes a grazing collision with disk II initially at rest. If after the collision the two disks stick together, the magnitude of the total angular momentum about the point P is

(A) zero

(B) $\frac{1}{2}MR^2\omega_0$

(C) $\frac{1}{2}MRv_0$

(D) MRv_0

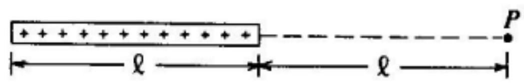
(E) dependent on the time of the collision

However, because the rotational component is rotating in the opposite direction to that of the translational component, we must take the difference of the components to get

$$
\begin{aligned}
L &= L_{trans} - L_{rot} && (1.295)\\
&= \frac{1}{2}MR^2\omega_0 - \frac{1}{2}MR^2\omega_0 && (1.296)\\
&= 0 && (1.297)
\end{aligned}
$$

Correct Answer
(A)

1.98 PGRE8677 #98



98. The long thin cylindrical glass rod shown above has length ℓ and is insulated from its surroundings. The rod has an excess charge Q uniformly distributed along its length. Assume the electric potential to be zero at infinite distances from the rod. If k is the constant in Coulomb's law, the electric potential at a point P along the axis of the rod and a distance ℓ from one end is $\frac{kQ}{\ell}$ multiplied by

(A) $\frac{4}{9}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) ln 2

(E) 1

Recommended Solution

Start with Coulomb's Law for a point charge

$$V = \frac{kQ}{l} \tag{1.298}$$

from the perspective of point P, each differential piece of the glass rod will apply a force and it will do so proportional to the charge density $\rho = Q/l$. Thus, we integrate the differential pieces of glass rod,

$$\begin{aligned} V &= \int_l^{2l} \frac{k\rho}{l}\, dl && (1.299)\\ &= k\rho \int_l^{2l} \frac{dl}{l} && (1.300)\\ &= k\rho\left[\ln(2l) - \ln(l)\right] && (1.301)\\ &= k\rho \ln(2) && (1.302)\\ &= \frac{kQ}{l}\, \ln(2) && (1.303) \end{aligned}$$

Correct Answer
(D)

1.99 PGRE8677 #99

99. The positronium "atom" consists of an electron and a positron bound together by their mutual Coulomb attraction and moving about their center of mass, which is located halfway between them. Thus the positronium "atom" is somewhat analogous to a hydrogen atom. The ground-state binding energy of hydrogen is 13.6 electron volts. What is the ground-state binding energy of positronium?

(A) $\left(\frac{1}{2}\right)^2 \times 13.6$ eV

(B) $\frac{1}{2} \times 13.6$ eV

(C) 13.6 eV

(D) 2×13.6 eV

(E) $(2)^2 \times 13.6$ eV

Recommended Solution

Recall that the ground state energy of Hydrogen is equal to 1 Rydberg, $-13.6\ eV$. Positronium involves an electron-positron pair while hydrogen involves a proton-electron pair. There is no difference between the two in terms of charge but there is a significant difference in mass. Since Rydberg's constant is mass dependent, we have to alter the original Rydberg constant

$$R_{hydrogen} = \frac{m_e m_p}{m_e + m_p} \frac{e^4}{8c\epsilon_0^2 h^3} \tag{1.304}$$

Which becomes,

$$R_{positronium} = \frac{m_e m_e}{m_e + m_e} \frac{e^4}{8c\epsilon_0^2 h^3} \tag{1.305}$$

$$\frac{m_e}{2} \frac{e^4}{8c\epsilon_0^2 h^3} \tag{1.306}$$

To convince yourself that this makes the Rydberg constant half as large, consider that the ratio of the proton mass to electron is approximately 1836:1. Calculating the original effective mass with this, you get

$$\frac{m_p m_e}{m_p + m_e} \Longrightarrow \frac{1 \times 1836}{1 + 1836} \approx 1 \tag{1.307}$$

Using the same values for our new effective mass

$$\frac{m_e}{2} \Longrightarrow \frac{1}{2} \tag{1.308}$$

Since the energy is proportional to the Rydberg constant, the ground state energy of positronium must be half of the hydrogen ground state energy

$$E_{positronium} = \frac{E_{hydrogen}}{2} = -6.8eV \tag{1.309}$$

again, if you aren't convinced, consider the Rydberg equation for hydrogen

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{1.310}$$

and since $E = h\nu = \frac{hc}{\lambda}$

$$E = \frac{hc}{\lambda} = hc\frac{R}{2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{1.311}$$

Correct Answer
(B)

1.100 PGRE8677 #100

100. The screen of a pinhole camera is at a distance D from the pinhole, which has a diameter d. The light has an effective wavelength λ. ($\lambda << D$) For which of the following values of d will the image be sharpest?

(A) $\sqrt{\lambda D}$ (B) λ (C) $\frac{\lambda}{10}$

(D) $\frac{\lambda^2}{D}$ (E) $\frac{D^2}{\lambda}$

Recommended Solution

The quickest solution to this problem (which you probably want because it is the last problem and you are likely running out of time) is to consider which of these solutions would actually make for a reasonable "pinhole" camera. For some sample values, let's pick $\lambda = 400$ nm and $D = 50$ cm $= 5.0 \times 10^8$ nm to satisfy our condition that $\lambda << D$

(A) $\sqrt{(400 \text{ nm})(5.0 \times 10^8 \text{ nm})} \approx 450,000 \text{ nm} = 0.045 \text{ cm}$: This doesn't seem unreasonable to me.

(B) λ: The pinhole is the size of our wavelength, in our case 400 nm, and it is very unlikely you are going to pull off a pinhole this small.

(C) $\frac{\lambda}{10} = \frac{400 \text{ nm}}{10} = 40$ nm: This is even smaller than the pinhole in (B) and is even less likely to be useful.

(D) $\frac{\lambda^2}{D} = \frac{(400 \text{ nm})^2}{5.0 \times 10^8 \text{ nm}} = 0.00032$ nm: Ha! Good luck with that.

(E) $\frac{D^2}{\lambda} = \frac{(5.0 \times 10^8 \text{ nm})^2}{400 \text{ nm}} = 6.25 \times 10^{14} \text{ nm} = 625,000$ m: That's a HUGE PIN!

Correct Answer
(A)