Preface

Hi，我是xination。

這是我趁晚上顧 nuclear accelerator 時（night shift or grave yard shift）

時寫的，有可能腦袋不清楚，而寫不好的地方，

請多見諒。

另外，我光學很弱...

18，20，22，69，91 我不知道要怎麼寫

My background：

目前剛結束 Ph.D. 第一年

正踏入真正的研究途中

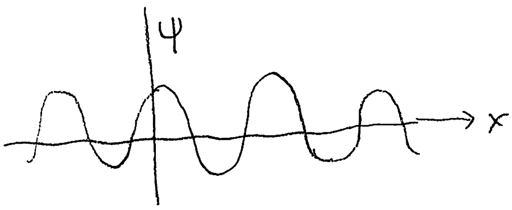
是個 nuclear physics experimentalist ~

#1

$$\Psi(x,t) = e^{i(kx-wt)}$$

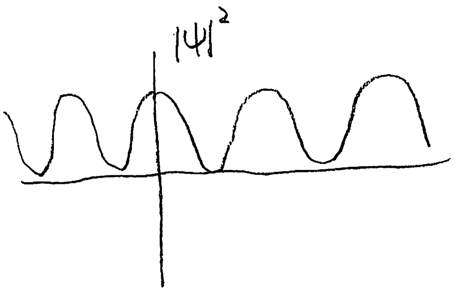
這個 wave function 是個像是 sin, cos 的形式

at certain moment ($t=t_0$)



分佈於整個 x - axis

ie.



we don't know where is the wave

but we can know the momentum exactly

在 quantum 中，要求的東西得用 operator

$\hat{P}_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$ ⟵… 在 position space (ie x - space)

P_x 的形式

(PS: 在 momentum space $\hat{P}_x = P_x$)

then $$\hat{P}_x \Psi = \frac{\hbar}{i}\frac{\partial}{\partial x}\left[e^{i(kx-wt)}\right]$$

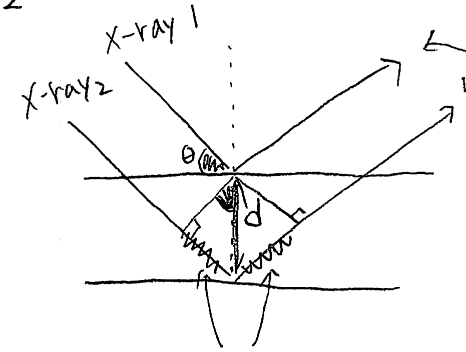
$$= \frac{\hbar}{i}(ik)\left[e^{i(kx-wt)}\right]$$

$$\hat{P}_x \Psi = \underline{\underline{\hbar k}}\ \Psi$$

Therefore

we should choose (C)

#2



之間會有波程差 $= 2d\sin\theta$

so. $2d\sin\theta = m\lambda$ 為產生 construction interferrence 的條件

$m = 1, 2, 3, \ldots$

題目要 longest wavelength

and $\lambda = \frac{2d\sin\theta}{m}$. choose $m = 1$

$\lambda = 2d\sin\theta$

$\theta = 90°$, λ 有 max 值

so $\underline{\lambda = 2d}$

↓

we choose (D)

3

K characteristic x-ray

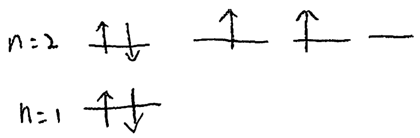
是指到 $n=1$ shell 的 x-ray

ex. K_α mean $n=2 \to n=1$

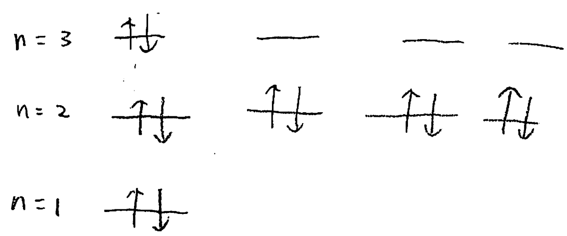
K_β mean $n=3 \to n=1$

PS. 我們題目應該是設定 neutral atom

so



$^{12}_{6}C$



$^{24}_{12}C$

主要相似之處, 都有 2 個 外層电子

若由 hydrogen-like atom 去思考的話 ⇐ 資格考很常出現這類 problem

$E_n \propto \dfrac{Z^2}{a_0^2}$ where $a_0 \equiv$ Bohr radius

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{me^2} \quad \text{SI}$$

reduce mass, but ≃ 电子的 mass

在 CGS. 中 a_0 比較 easy

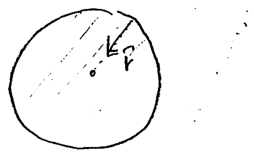
a_0 不變 但 $\dfrac{(Z=6)^2}{(Z=12)^2} = \dfrac{1}{4}$

$$a_0 = \frac{\hbar^2}{me^2}$$

↳ 我猜是 (A)

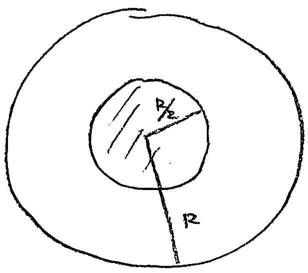
#4.

$$\vec{F} = \frac{GMm}{r^2}(-\hat{r})$$



所以 $\frac{F(R)}{F(2R)} = \frac{1/R^2}{1/(2R)^2} = 4$ — 選 (c)

#5.



别忘了，小球的 mass 比較小

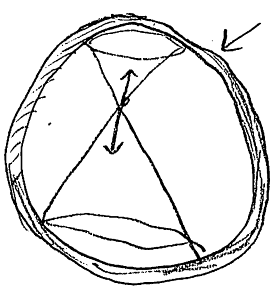
Set 大球的 mass $= M_{big}$

小球的 mass $= M_{small}$

$$\frac{M_{small}}{M_{big}} = \frac{1}{8}$$

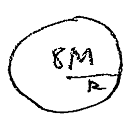
所以 $\frac{F(R)}{F(R/2)} = \frac{M_{big}/R^2}{M_{small}/(R/2)^2} = \frac{8\ \frac{M_{small}}{R^2}}{4\ \frac{M_{small}}{R^2}} = 2$ — 選 (c)

PS



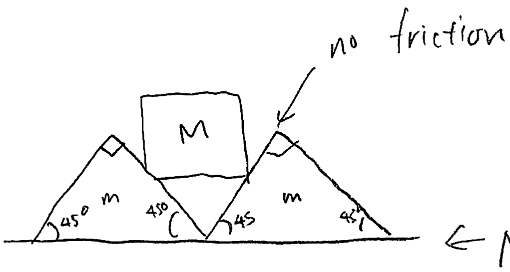
in mass uniformly distributed shell,
the shell doesn't create
any gravational force inside,
since it would be canceled.

so. we just need to consider:

$F(R)$ " 

$F(\frac{R}{2})$ = 

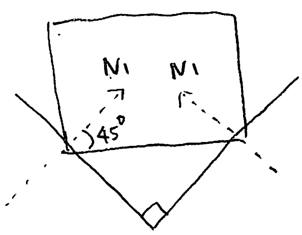
#6.



$\leftarrow \mu_s < 1$ ($\mu_s \equiv$ static friction coeffiction)

and friction $= \mu_s N$

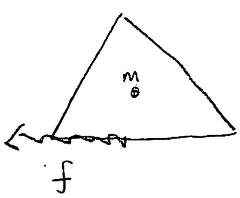
find the largest M



(1) $$2N_1 \cdot \sin 45^\circ = M \cdot g$$ (支撐 M)

$$\sqrt{2} N_1 = Mg$$

$$N_1 = \frac{Mg}{\sqrt{2}}$$



(2) $$f = \mu_s \cdot (mg + N_1 \cdot \sin 45^\circ)$$

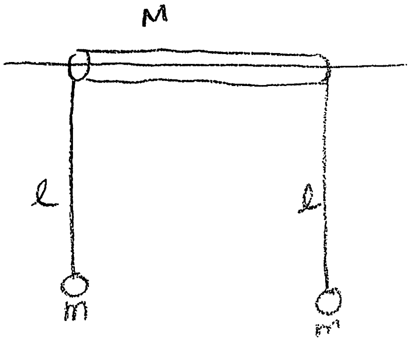
and $f \geq N_1 \cos 45^\circ$ …… for not moving

$$\mu_s \left(mg + \frac{Mg}{\sqrt{2}} \frac{\sqrt{2}}{2}\right) \geq \frac{Mg}{\sqrt{2}} \frac{\sqrt{2}}{2}$$

$\mu_s \left(m + \frac{1}{2} M\right) \geq \frac{1}{2} M$ ，之後只是簡單的移項

$\Rightarrow \frac{2\mu m}{1-\mu} \geq M$ …… 選 (D)

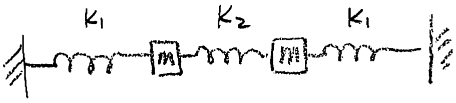
#7



· small oscillation

這題可以不用計算，用concept即可

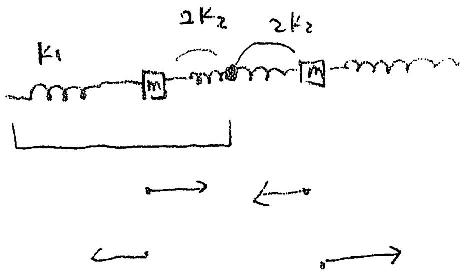
· 先 review



的問題

它有2種 normal mode (ie. oscillation frequency)

(1) 以



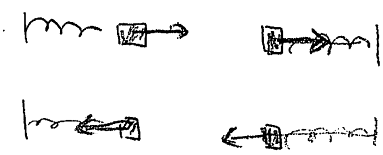
PS. spring 越短 k

spring const 越大

所以 cut 一半，k 值 up↑

$$w = \sqrt{\frac{k_1 + 2k_2}{M}}$$

(2)



這種 case，類似中間的 spring 沒作用.

$$w = \sqrt{\frac{k_1}{M}}$$

題目說到 frequency 有3個

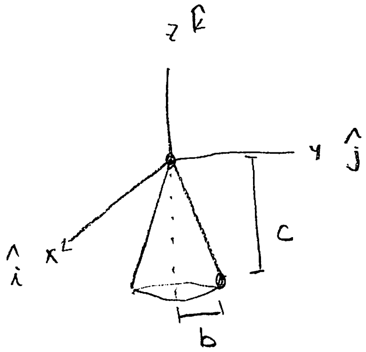
①. $\sqrt{\frac{g}{l}\frac{M+2m}{M}}$

↑
類似 case(1)

so. 最後個，類推的話 $\sqrt{\frac{g}{l}}$

↑
選(A)

#8



這題我們把 $\hat{i}$ $\hat{j}$ $\hat{k}$ 換成 $\hat{x}, \hat{y}, \hat{z}$ 會更有感覺

題目要找的是 torque $\vec{\tau} = (\tau_x, \tau_y, \tau_z)$

↑ which τ_z is negative

而 $\vec{\tau} = \vec{r} \times \vec{F}$

↑ $\vec{r}$ 為 pivot 到施力點的 vector

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$= \cdots \hat{z}(r_x F_y - r_y F_x)$ ⇐ 其 $\hat{z}$ 分量

then, 我們代入選項看看

(a) $\vec{r} = (0, b, -c)$

$\vec{F} = (0, 0, a)$ $\quad \tau_z = 0$

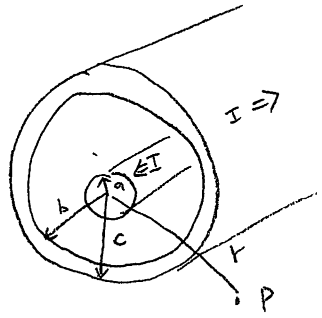
⋮

(c) $\vec{r} = (-b, 0, -c)$

$\vec{F} = (0, a, 0)$ $\quad \underline{\tau_z = -ab}$

選(C)

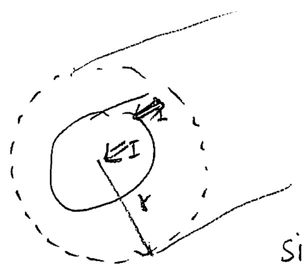
#9



求 outside 之處；P 點的磁場

by Ampere's law ⇒ $\oint \vec{B} \cdot d\vec{l} = \mu_0 \underbrace{I_{enclosed}}$

enclosed 指 Ampere's loop 的 current



since $I_{enclosed} = 0$

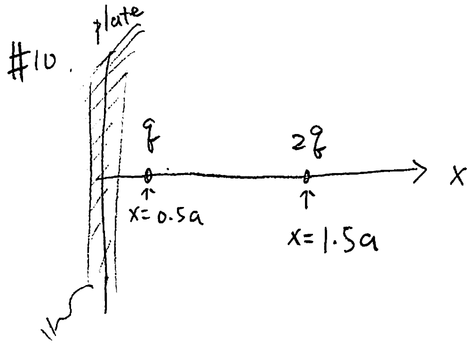
so $\underline{\vec{B} = 0}$ ⇐ 選(A)

PS. $\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J}$

(C.G.S 制 很好用的)

or $\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c} \vec{J}$

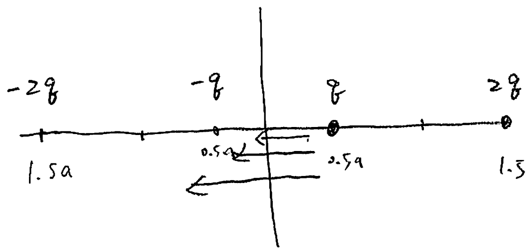
#10.



Φ : electric potential

原本只用於解 $\nabla^2 \Phi = 0$,的 unique solution 之討論

這題只是 "mirror method" 的 concept



$$\vec{F} = \frac{2q^2}{(2a)^2}(-\hat{x}) + \frac{q^2}{a^2}(-\hat{x}) + \frac{2q^2}{a^2}(-\hat{x})$$

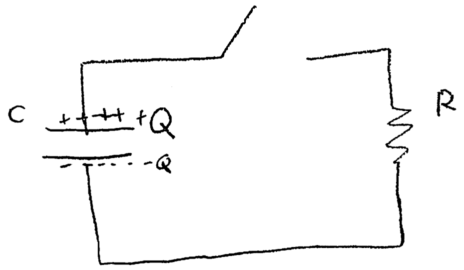
$$= \frac{q^2}{a^2}\left[\frac{1}{2} + 1 + 2\right](-\hat{x})$$

$$= \frac{q^2}{a^2}\left[\frac{7}{2}\right](-\hat{x})$$

C.G.S

for S.I = $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a^2}\frac{7}{2}$ ⋯ 選 (E)

#11



假設原本在 capacitor 上的 charge = Q coulomb

then the potential difference ΔV, by $C \equiv \frac{Q}{\Delta V}$

we get $\Delta V = \frac{Q}{C}$

The energy stored, $U = \frac{1}{2} C (\Delta V)^2$

So, if we want $U \to \frac{1}{2} U$

then $\Delta V \to \frac{1}{\sqrt{2}} \Delta V \Rightarrow Q \to \frac{Q}{\sqrt{2}}$

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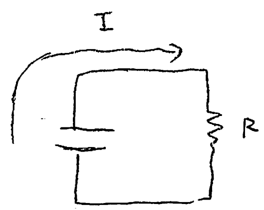
by RC circit



$$\frac{Q}{C} + RI = 0$$

$$\frac{Q}{C} + R\frac{dQ}{dt} = 0 \quad , \text{ by } I \equiv \frac{dQ}{dt}$$

then, $-\frac{1}{RC} dt = \frac{1}{Q} dQ$

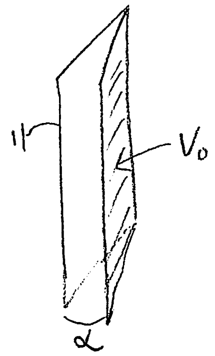
$$Q(t) = Q e^{-t/RC}$$

$$\frac{Q}{\sqrt{2}} = Q e^{-t/RC}$$

選 (E)

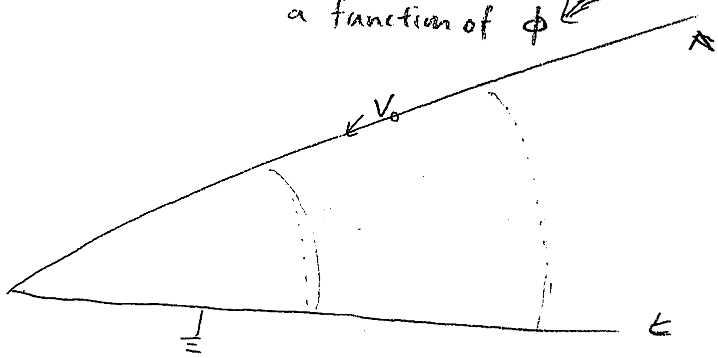
$$\left.\begin{aligned} \text{so } \frac{1}{\sqrt{2}} &= e^{-t/RC} \\ \frac{1}{2}\ln 2 &= t/RC \end{aligned}\right\} \Rightarrow \boxed{t = \frac{RC\ln 2}{2}}$$

12



potential inside is just a function of ϕ

it muse satisfy the boundary condition



$V(\phi=\alpha) = V_0$

$V(\phi=0) = 0$

So. The Dnly choice is (B)

$$V(\phi) = \frac{V_0 \phi}{\alpha}$$

#13

if magnetic monopoles exist:

I $\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$ ← 在 material 之中的 maxwell equation

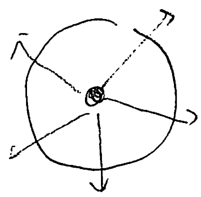
其 $\vec{B} = \mu\vec{H}$; $\mu = \mu_0(1+\chi_m)$

$\vec{D} = \varepsilon\vec{E}$; $\varepsilon = \varepsilon_0(1+\chi_e)$

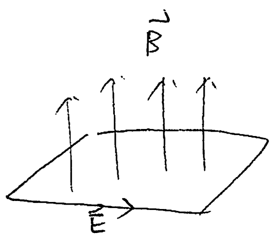
II $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ ← Faraday's law

III $\nabla\cdot\vec{D} = \rho$ ← Gauss's law

IV $\nabla\cdot\vec{B} = 0$ ← 如果有 monopole，就得修正了



$\nabla\cdot\vec{B} \neq 0$



$$\oint\vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\underbrace{\int\vec{B}\cdot d\vec{a}}$$

不過仔細看 Farady law 的話 這邊有個積分

if $\oint\vec{B}\cdot d\vec{a}$ 的話

有 monopole, $\oint\vec{E}\cdot d\vec{l} \neq 0$

没有 $\oint\vec{E}\cdot d\vec{l} = 0$

so... choose II and IV

#14

water temperature $20^\circ C \rightarrow 20.5^\circ$, $\Delta T = 0.5$

if temperature of blackbody is double

by Stefan - Boltzmann law:

emissive power $\propto T^4$

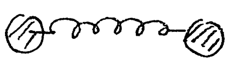
So. actually, it increase to $2^4 = 16$ times

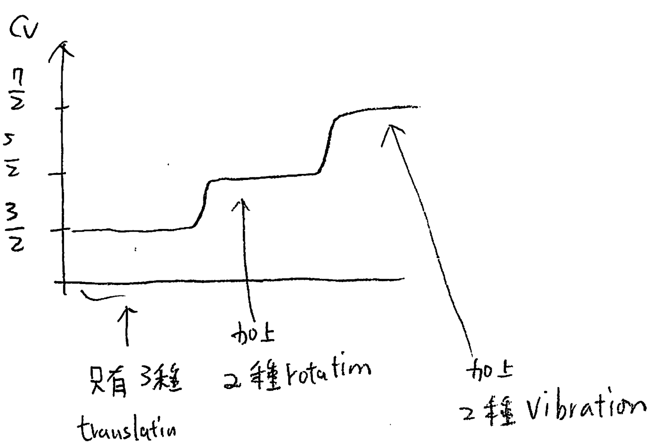
$$0.5 \times 16 = 8$$

So we expect $20^\circ C \longrightarrow 28^\circ C$

選 (c)

#15.





選 (c)

這種是 equi - partition of energy 的 concept

簡單的說 1 degree of freedom 就有 $\frac{1}{2} k_B T = E/N$

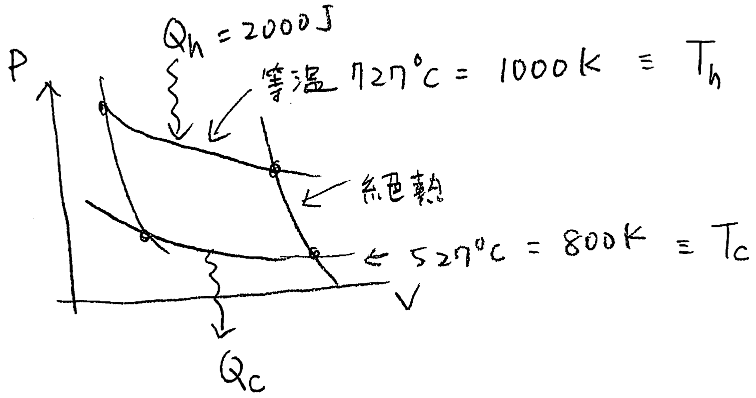
then for very high Temperature

$$E/N = \frac{1}{2} k_B T \times 7$$

$$C_v = \frac{\partial}{\partial T}(E)\Big|_{\text{vol-fixed}} = \frac{7}{2} N k_B = \frac{7}{2} R$$

#16.

Carrot engine 的 problem:



by efficiency $\eta = 1 - \frac{T_c}{T_h} = 1 - \frac{800}{1000} = 0.2$

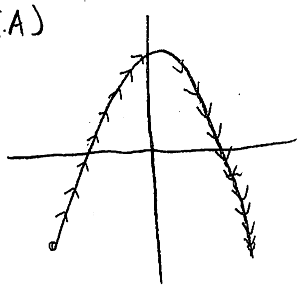
$$\eta = \frac{\text{Work output}}{\text{Work input}} = \frac{\text{Work output}}{2000} = 0.2$$

So the output = 400J 選(A)

#17

我們來畫畫看

(A)

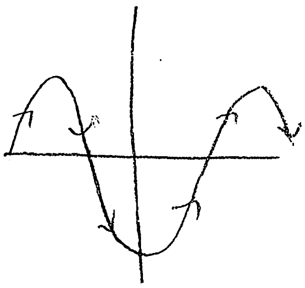


$y_{min} \to y_{max} \to y_{min}$

$x_{min} \to x_{max}$

Yes ⇙ 選 (A)

(B)

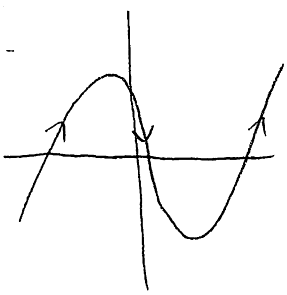


$0 \to y_{max} \to 0 \to y_{min} \to 0 \to y_{max} \to 0$

$x_{min} \to x_{max}$

No

(C)

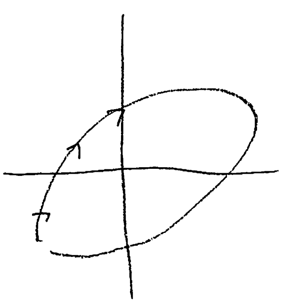


$y_{min} \to y_{max} \to y_{min} \to y_{max}$

$x_{min} \to x_{max}$

No

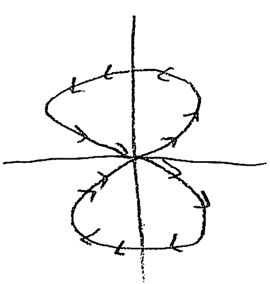
(D)



$y_{min} \to y_{max}$

$x_{min} \to \cancel{x}_{max}$

(E)



$0 \to y_{max} \to 0 \to y_{min} \to 0$

.... x is twice of y

#18.

這題… 我不會…

#19.

$V = \frac{4}{3}\pi r^3$ if $r \doteq 6.4 \times 10^6$

$\doteq 1.33 \times 3.14 \times (6.4 \times 10^6)^3$

$\doteq 10^{20}$

Fe 的密度 $\doteq 56 \ g/cm^3$

$\Rightarrow 56 \ \dfrac{\frac{1}{1000} kg}{(\frac{1}{10})^3 m^3} = 56 \ kg/m^3$

mass $\doteq 56 \times 10^{20}$

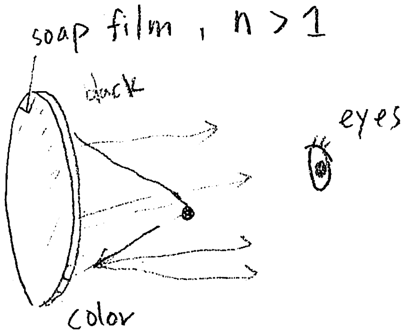
我猜 (A) : 6×10^{24} kg

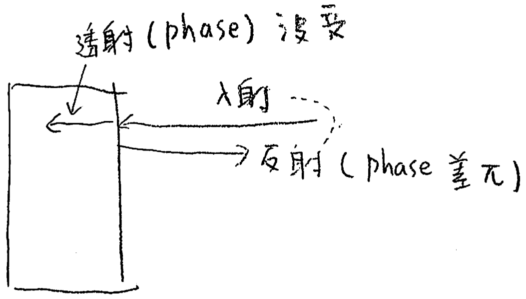
(結果是對的 Ya!)

#20

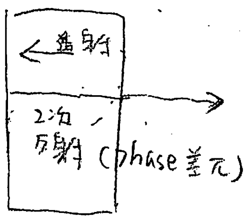
不知道要怎麼處理… 我光學很弱

#21.



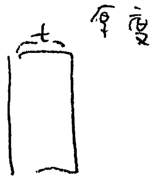


所以 (III) 是正確的



所以 (IV) 是正確的

一次反射 和二次反射的 light ray 之間差 phase = π

所以 if 

$2t = m\lambda$

then 二次反射的光 正好 cancel 一次反射的光

同理 if $2t = (m+\frac{1}{2})\lambda$,則是 constructive

However if t 太小了,

則會使完全沒反射 (none 一次 and 二次)

∴ (II) 是對的

#22. 不太知道怎麼算

#23.

Fermi - temperature T_F ，其定義為

$$\underbrace{\varepsilon_F}_{\uparrow} = K_B T_F$$

Fermi energy

而 conducting electron 是約在 Fermi-energy 附近的

so $\frac{1}{2} m V^2 = K_B T_F$

$$V = \left[\frac{2 K_B T_F}{m} \right]^{1/2}$$

$$= \left[\frac{2 \cdot (1.38 \times 10^{-23}) \cdot 8000}{9.1 \times 10^{-31}} \right]^{1/2}$$

$$\simeq \left[\frac{2 \cdot 2 \cdot 8 \cdot 10^{-20}}{10 \cdot 10^{-31}} \right]^{1/2}$$

$$\simeq \left(2 \times 10^{11} \right)^{1/2}$$

... 我猜是(E)

24

Ar …有18個電子

電子組態 $\underbrace{[Ne]}_{10}3s^2 3p^6$

因為都填滿滿的，所以沒能和其它的 Ar atom 共享電子

(所以 ionic ×

covalent ×)

Metallic bond …我不熟

但應該是錯的

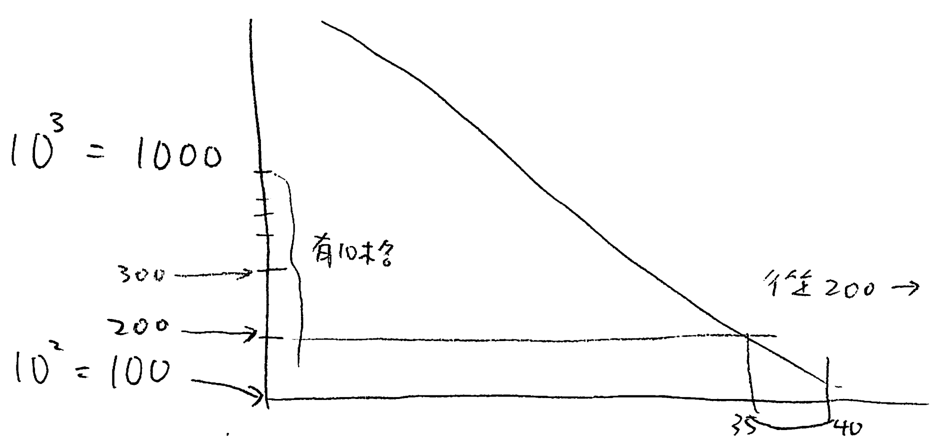
So. 選 (E)

25

(D) … 有玩過 Geiger counter 的人都應該知道

ps. neutrio 可以穿過整個地球～

26



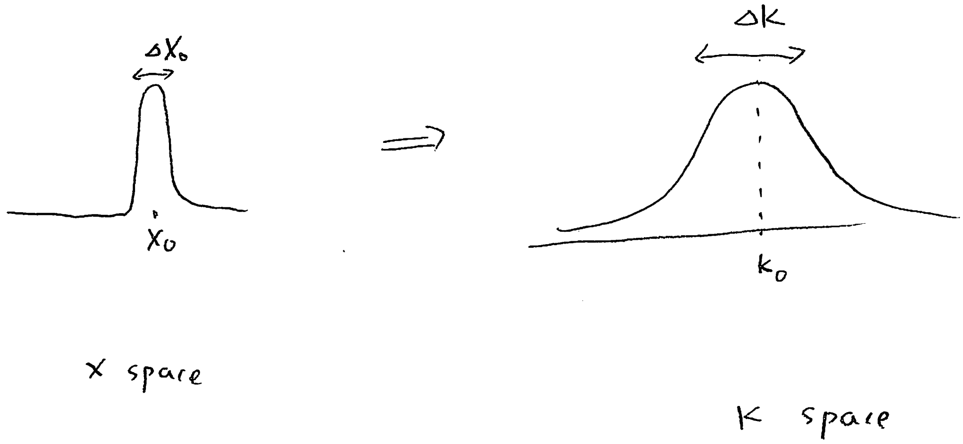
從 200 → 100 的時間 約 5 min

選(B)

#27

$$\psi(x,t) = \int_{-\infty}^{\infty} e^{i(kx-\omega t)} f(k)\, dk$$

k = wave number



by $\Delta x \Delta p \geq \frac{\hbar}{2}$

$\Delta x \Delta k \geq \frac{1}{2}$

So 我會選 (B) $\Delta k = \frac{1}{\Delta x}$

28

$$\psi(\theta,\phi) = \frac{1}{\sqrt{30}}\left(5\ Y_{\ell=4,m=3} + Y_{6,3} - 2\ Y_{6,0}\right)$$

PS. $Y_{\ell,m}$ 是互相 orthogonal 的

find the probability

in this state

$$\text{Probability} = \left|\langle Y_{4,3} | \psi(\theta,\phi)\rangle\right|^2 + \left|\langle Y_{6,3} | \psi\rangle\right|^2$$

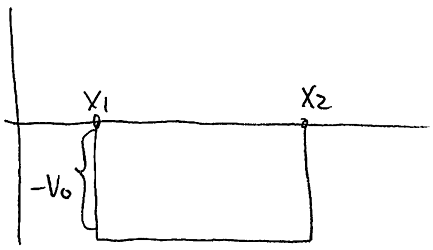
$$= \left|\frac{5}{\sqrt{30}}\right|^2 + \frac{1}{30}$$

$$= \frac{25}{30} + \frac{1}{30}$$

$$= \frac{13}{15}$$

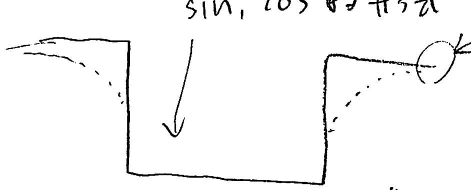
$\therefore$ 選 (E)

#29



bound state means $E < |V_0|$

wave function 會長成



Probability $\propto |\psi|^2$

and at boundary x_1, x_2

因為 V_0 不是 ∞，所以 ψ is continuous.

所以選 (B)

$\Rightarrow$ ie. expontially decay

(不會有 ∿∿∿)

oscillation

and $E_n \propto \frac{Z^2}{m a_0^2}$

#30

PS. a_0 = Bohr radius ; $a_0 \propto \frac{1}{m}$

$E_n \propto m Z^2$; m 為 reduced mass

之前 $\frac{1}{m_p} + \frac{1}{m_e} = \frac{1}{m}$

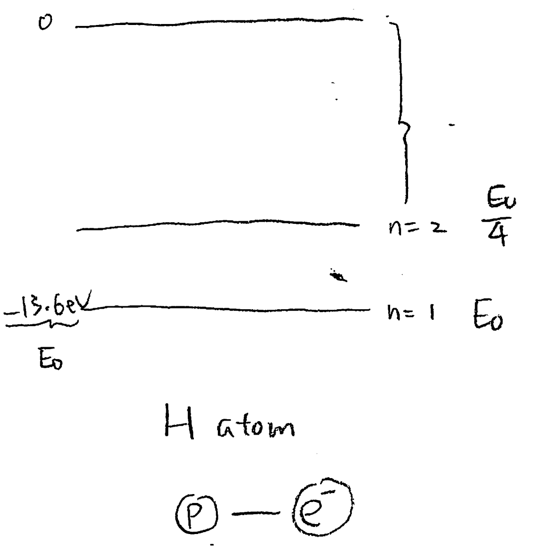
$\Rightarrow E_n \propto Z^2 \cdot m$ (這題中 Z 不變，但 m 變了) $m \doteq m_e$

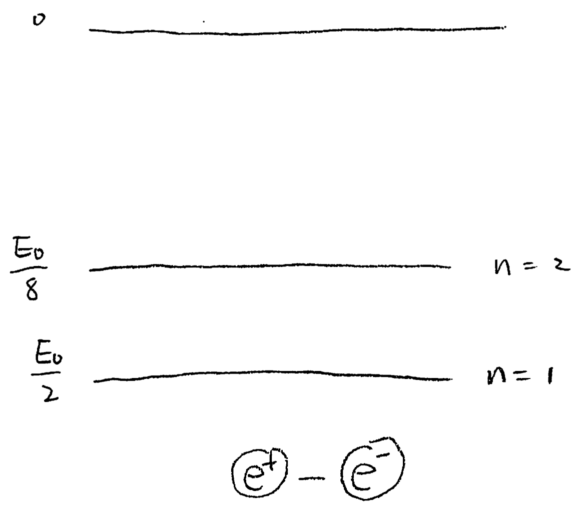
now positronium $e^+ - e^-$ $\frac{1}{m} = \frac{1}{m_{e^-}} + \frac{1}{m_{e^+}}$ and $m_{e^-} = m_{e^+}$

so $m = \frac{m_e}{2}$

所以 positronium 的 E_n 為 Hydrogen atom E_n 的 $\frac{1}{2}$

30 (continue)





so 選 (E)

31

^3S mean $^{2S+1}\boxed{l=0}$ (for Helium)

$2S+1$ ← $S=1$

有2個electron

$l=0$, $S=1$

so $j = l+S = 1$

選 (B)

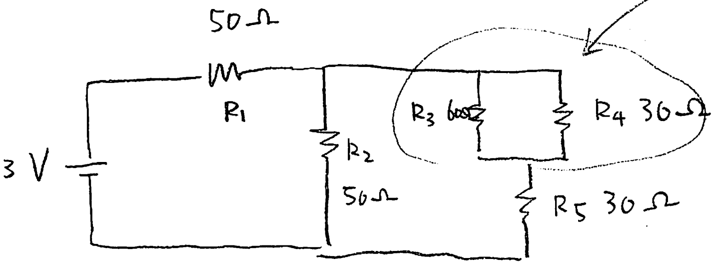
PS. 一般的 rule

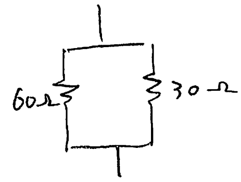
$$|l-s| \le j \le l+s$$

triangle rule ~.

32

先算這個

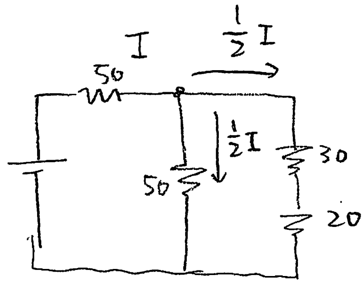




$= 20\Omega$

$$\frac{1}{R} = \frac{1}{60} + \frac{1}{30}$$

$$R = 20\Omega$$

$\Rightarrow$ 

因為 $30+20=50$
所以 兩個 branch 都是 $\frac{1}{2}I$

alway true

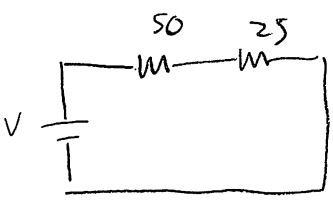
power $P = IV$ by $V = IR$ (只有在 Ohm's law 成立才行)

$= I^2R$

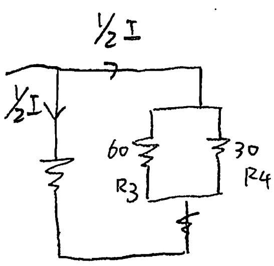
So. very obvious 選 R_1 有最大的 power — (A)

33

$$\frac{1}{R} = \frac{1}{50} + \frac{1}{50} \Rightarrow R = 25$$



$I = \frac{3}{75}$ ← total current

then 

← 由於 $R_4 : R_3 = 30 : 60 = 1 : 2$

thus 通過 R_4 的current $= \left(\frac{1}{2}I\right) \times \frac{2}{3} = I_4$

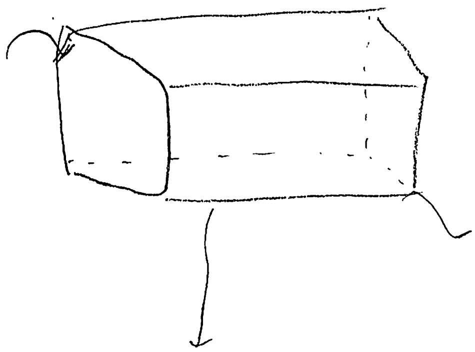
so voltage $= I_4 R_4 = \frac{1}{2} \cdot \frac{3}{75} \cdot \frac{2}{3} \cdot 30$

$= \frac{30}{75} = \frac{10}{25}$ ← 選 (A)

#34

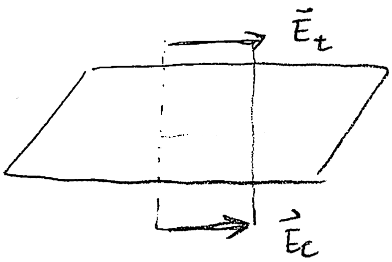
一個 perfect 的 conducting cavity

接上 reasonator



我們以這面作說明 by $\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \xRightarrow{\text{imply}} \hat{n} \times (\vec{E}_{out} - \vec{E}_{in})\Big|_s = 0$

at surface

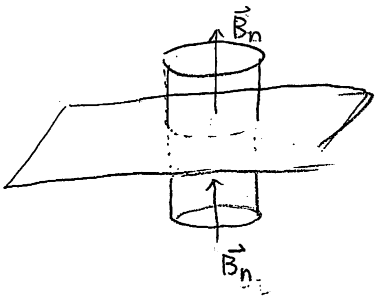


we can assume 外面 $\vec{E}_t = 0$

所以 $\vec{E}_t = 0$ across bundary

另外 $\nabla \bullet \vec{B} = 0 \rightarrow$ 代表

裡面的 $\vec{B}_n$ = 外面的 $\vec{B}_n$



而 we can assume 外面 $\vec{B}_n = 0$

所以 $\vec{B}_n = 0$

or 用我的 notation:

故選 (D) $\qquad \hat{n} \cdot (\vec{B}_{out} - \vec{B}_{in})\Big|_s = 0$

(CGS)

PS.

$$\nabla \cdot \vec{E} = 4\pi\rho \xrightarrow{\text{boundary}} \hat{n} \cdot (\vec{E}_{out} - \vec{E}_{in})\Big|_s = 4\pi\sigma$$

surface charge dencity

$$\nabla \cdot \vec{B} = 0 \longrightarrow \hat{n} \cdot (\vec{B}_{out} - \vec{B}_{in})\Big|_s = 0$$

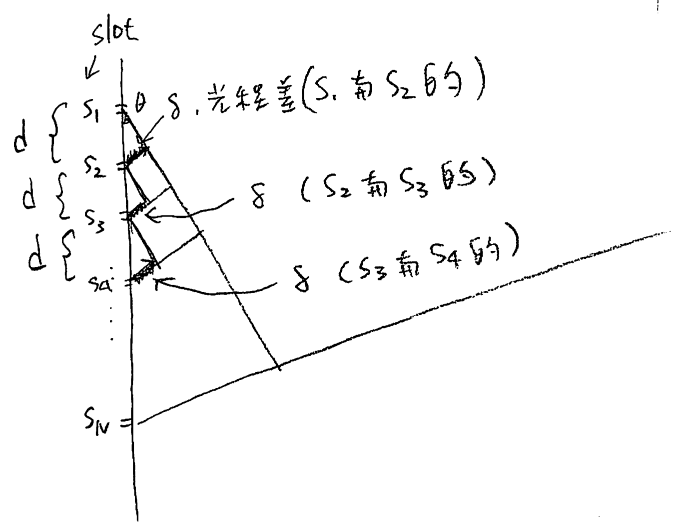
$$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \longrightarrow \hat{n} \times (\vec{E}_{out} - \vec{E}_{in})\Big|_s = 0$$

surface current density

$$\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J} \longrightarrow \hat{n} \times (\vec{B}_{out} - \vec{B}_{in})\Big|_s = \frac{4\pi}{c}\vec{K}$$

35 ⋯ (XD 我光学很弱)

$\lambda = 5200 \text{ Å}$



all $\delta = d\sin\theta$

if $\delta = m\lambda$ 時

S_1 和 S_2 是 constructive interference

S_2 和 S_3 也是

⋮

So $d\sin\theta = m\lambda$ 為 constructive interferace 的條件.

$$d = \frac{1 \text{ cm}}{2000 \text{ line}} = \frac{10^{-2}}{2 \cdot 10^{3}} = 5 \times 10^{-6} \text{ m}$$

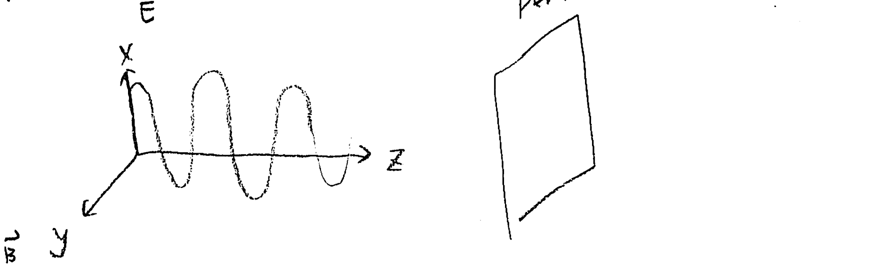
$m = 1$, first-order max

So $\sin\theta = \frac{1}{5\times10^{-6}} \cdot 5200 \times 10^{-10}$

$\sin\theta \simeq 0.104$

by my calculator $\theta \simeq 5.96$ ⋯ so 選 (B)

#36



這就如同 wave 撞到牆 反射一樣

but 要小心一點.

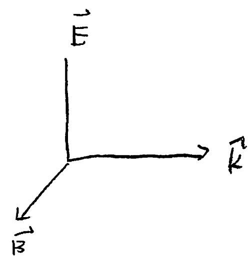
我們設原先的 polarization (電場 方向 的方向) 在 $+\hat{x}$

而 wave 以 $+\hat{z}$ 的方向前進

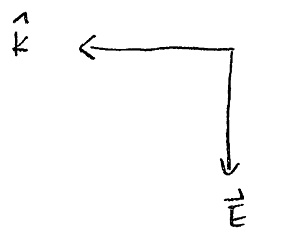
ie. $\hat{k} = \hat{z}$

wave number 的 vector

且 $\hat{E} \times \hat{B} = \hat{k}$ 方向上有這層關係



now 反射後



由於 $\vec{E} \times \vec{B} = \vec{k}$

所以 $\vec{B}$ 仍然在 $\hat{y}$ 方向 ... $\vec{B}$ 沒有 reversed.

所以 選 (c)

ps. 入射的 $\vec{E} = E_0 \hat{x}\, e^{i(kz - \omega t)}$

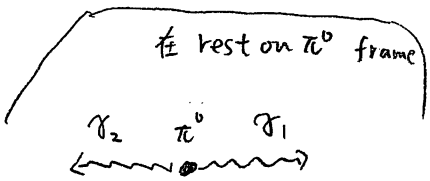
差了 π 的 phase

反射的 $\vec{E} = (-E_0 \hat{x})\, e^{i(-kz - \omega t)}$

37

π^0，其 mass = 135 MeV

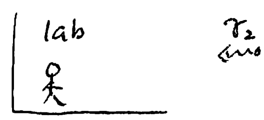
with $\beta = 0.8\ \hat{k}$ (ps. $\beta = v/c$)



這題，只用到非常簡單的相對論

γ-ray 也就是一種 EM wave

它的速度 = c in any frame



而 γ_2 是往後發射的，

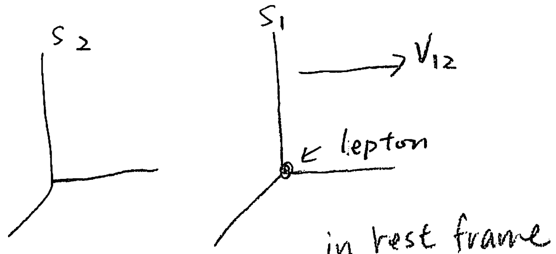
then 它的 velocity = $-c\ \hat{k}$

選 (A)

#38

average half-life $t_{1/2} = \Delta t_1$ in S_1 frame (rest frame)

↙ 我換個方式表達



in S_2

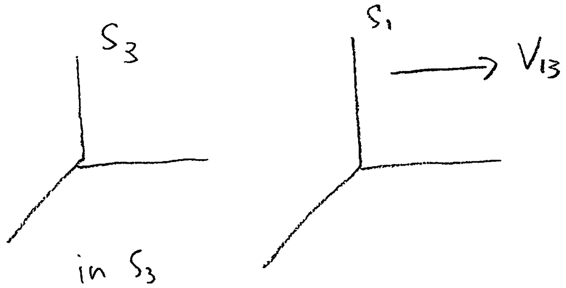
$t_{1/2} = \Delta t_2$

in rest frame

$t_{1/2} = \Delta t_1 \equiv \tau$

by $\Delta t_2 = \gamma_2 \underbrace{\tau}_{\tau}$ proper time

$$\gamma_2 = \frac{1}{\sqrt{1-\left(\frac{V_{12}}{c}\right)^2}}$$



in S_3

$t_{1/2} = \Delta t_3$

by $\Delta t_3 = \gamma_3 \underbrace{\tau}$ proper time

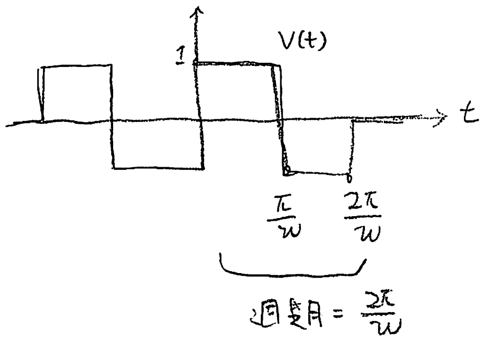
$$\gamma_3 = \frac{1}{\sqrt{1-\left(\frac{V_{13}}{c}\right)^2}}$$

Therefore, … 一下子就知道該選 (B)

$$(B) = \frac{1}{\sqrt{1-\left(\frac{V_{13}}{c}\right)^2}}\tau = \Delta t_3$$

#39

$n = 1.2.3 \cdots \infty$



首先，V(t) 是 odd function

只能有 sin 的 term

只剩下 (A).(B) 可以考慮

(A) ... $\sin(n w t)$

(B) .... $\sin\left[(2n+1) w t\right]$

. ps.

$$f(t) = C_0 + \sum_{n=1}^{\infty}\left[a_n \cos(n w t) + b_n \sin(n w t)\right]$$

$C_0 = 0$, $a_n = 0$, by odd function

$$b_n = \frac{w}{\pi}\int_{-\frac{\pi}{w}}^{\frac{\pi}{w}} \underset{\substack{\uparrow \\ \text{step function}}}{f(t)} \cdot \sin(n w t)$$

$$= \frac{w}{\pi}\left[\int_{-\frac{\pi}{w}}^{0} (-1)\sin(n w t)\,dt + \int_{0}^{\frac{\pi}{w}} (+1)\sin(n w t)\,dt\right]$$

$$= \frac{2w}{\pi}\int_{0}^{\frac{\pi}{w}} \sin(n w t)\,dt = \frac{2w}{\pi}\left(-\frac{n}{w}\right)\cos(n w t)\Big|_{t=0}^{\frac{\pi}{w}}$$

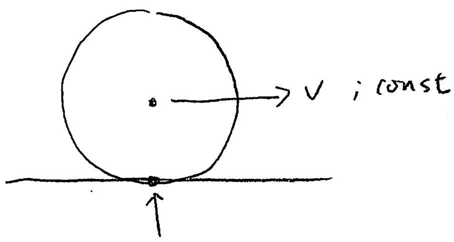
$$= \frac{4}{\pi}\left[1 - \cos(n\pi)\right]$$

$n = 1.3.5$ $b_n = \frac{4}{n\pi}$

$n = 2.4.6$ $b_n = 0$

選 (B)

40

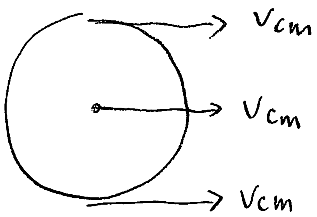


問這一點的加速度，它是向上的，選 (C)

整個這個 motion (i.e rolling = rotation + translation)

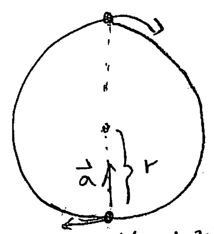
看這邊

translatation



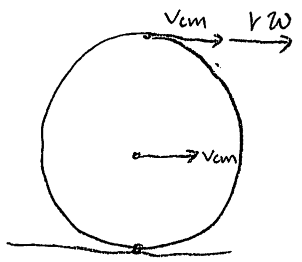
+

rotation



$V = r\omega$, and $a = \dfrac{V^2}{r} = r\omega^2$

⇓ 相加之後

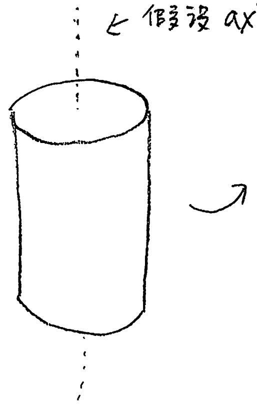


since no slipping means $V_{cm} = r\omega$

#41

moment of inertia $I = 4$ [kg.m²]

← 假設 axis 在这四斜]



$w_i = 80$ [rad/s]

$w_f = 40$ [rad/s]

由於 動能 $T = \frac{1}{2} I w^2$

$$T_i - T_f = \frac{1}{2} I \left[w_i^2 - w_f^2 \right]$$

$$= \frac{1}{2} \cdot 4 \cdot \left[6400 - 1600 \right]$$

$$= 2 \cdot 4800 = 9600$$ 選(D)

#42

$$\underset{\text{torque}}{\tau} = I \underset{\frac{dw}{dt}}{\alpha}$$

$$= 4 \cdot \frac{80-40}{10}$$

$$= 16$$ …… 選(D)

43

$$\frac{\partial \mathcal{L}}{\partial q_n} = 0 \quad \leftarrow \text{題目給的}$$

求 P_n .

由於 $\dfrac{\partial \mathcal{L}}{\partial q_n} = \dfrac{d}{dt}\left(\dfrac{\partial \mathcal{L}}{\partial \dot{q}_n}\right) = 0$

and canonical momentum $\equiv \dfrac{\partial \mathcal{L}}{\partial \dot{q}_n} = P_n$

(or call generalized momentum)

(or conjugate momentum)

因為 $\dfrac{d}{dt}(P_n) = 0$

so $P_n =$ const ... 選(B)

這就是 Noether's theorem

\# 44

$\mathcal{L} \equiv T - V$

$T =$ 動能 $= \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$

$V =$ 位能 $= mgy$ …… 設 $y=0$ 為 $V=0$

而題目說 $y = ax^2$

so. $\mathcal{L} = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) \underbrace{- mgy}$

由最後一項先刪除 (C) (D) (E)

then by $x = \left(\frac{y}{a}\right)^{1/2}$

ps $\dot{x} \equiv \frac{dx}{dt}$ ，是對時間微分

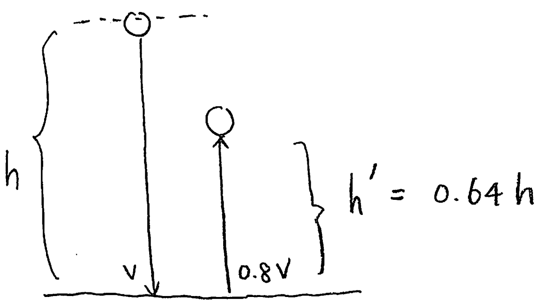
$\dot{x} = \frac{1}{\sqrt{a}} \frac{1}{2} y^{-1/2} \dot{y}$

$\dot{x}^2 = \frac{1}{4ay} \dot{y}^2$

so $\mathcal{L} = \frac{1}{2} m \dot{y}^2 \left(\frac{1}{4ay} + 1 \right) - mgy$

選 (A)

#45



「簡單推導一下公式

$$h = 0 + \frac{1}{2}gt^2$$

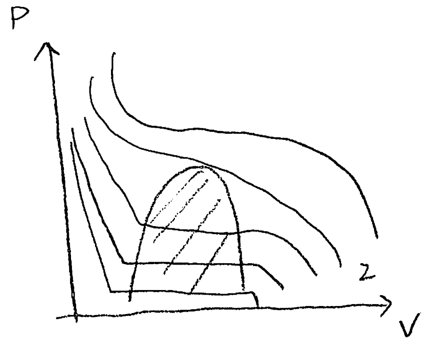
$$V = gt$$

$$\Rightarrow \quad h = \frac{V^2}{2g} \quad ; \quad h \propto V^2$$

所以 $\dfrac{h'}{h} = \left(\dfrac{0.8}{1}\right)^2 = 0.64$

選(D)

46



雖然還搞不清楚題目想達答什麼

但是只有 curve 長的有 "critiral" curve 的感覺

所以我選 (B)

47

這題，我也是用直覺的

vapor 是氣態，要 volume大，P小

liquid 是液態 要 volumn 較小，P較大

所以夕它是居中的区域

(A) P小，V小

(D) P大，V大

都不可能

接著想到 equilibrium，

應該會有不一樣的 curve

so，我猜 (B)

…結果對了

48

這題考 作實驗的技巧

$$\underset{\uparrow\ \sigma_F}{F} = \overset{\text{standard deviation} = \sigma_m}{m}\ \overset{\text{standard deviation (error)} = \sigma_a}{a}$$

求 $\frac{\sigma_F}{F}$

這題答案是(C) $\frac{\sigma_F}{F} = \left[\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_a}{a}\right)^2\right]^{1/2}$

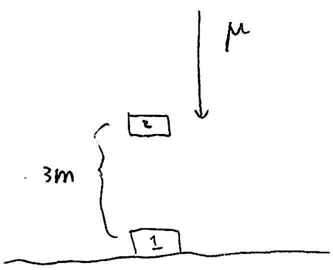
....雖我當助教的時候，剛好有看到....

（我已經忘了是怎麼推導的了，sorry）

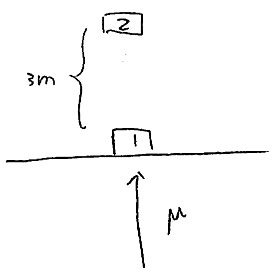
49

Scintillation counter = scintillator

在 nuclear physics or high energy physics 中的實驗很常用到

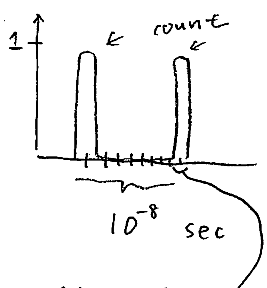


or



設 μ 以光速移動

$\Delta t = \frac{3}{3\times10^8} = 10^{-8}$



所以我們的解析度要 10^{-9} sec or 更小才夠

.... 選 (B)

#50

這題目的意思是

$$|\psi\rangle = \sum_i C_i \underbrace{|\phi_i\rangle}_{\text{basis}}$$

而 $\hat{A}|\phi_i\rangle = \alpha_i |\phi_i\rangle$

$\hat{B}|\phi_i\rangle = \beta_i |\phi_i\rangle$

$\hat{A}$. $\hat{B}$ 能有共同的 eigenstate $|\phi_i\rangle$

是因為 $[\hat{A}, \hat{B}] = 0$ → you can try

$\hat{A}\hat{B}|\phi_i\rangle = \hat{A}(\beta_i|\phi_i\rangle)$

$\hat{B}\hat{A}|\phi_i\rangle = \hat{B}(\alpha_i|\phi_i\rangle)$

obviously

若 share $|\phi_i\rangle$

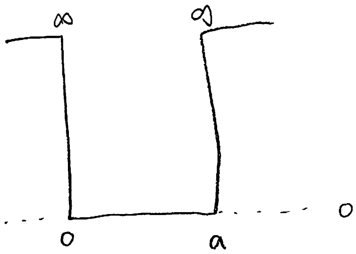
$\Leftrightarrow [\hat{A}, \hat{B}] = 0$

ps. $\hat{X}$, $\hat{P}$ 就無法有相同的 $|\phi_i\rangle$

因為 $[\hat{X}, \hat{P}] \neq 0$

故選 (B)

#51



$$|\psi_n\rangle = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{n\pi}{a}x\right)$$

by $\langle p\rangle = \langle \psi_n | \hat{p} | \psi_n\rangle$

$$= \langle \psi_n | \frac{\hbar}{i}\frac{\partial}{\partial x} | \psi_n\rangle$$

$$= \frac{\hbar}{i}\frac{2}{a}\int_0^a \sin\left(\frac{n\pi}{a}x\right)\cdot\left(\frac{n\pi}{a}\right)\cdot\cos\left(\frac{n\pi}{a}x\right)\cdot dx$$

$$= \frac{\hbar}{i} n\pi \int_0^a \sin\left(2\frac{n\pi}{a}x\right)\cdot dx$$

$$= \frac{\hbar}{i} n\pi\cdot\frac{a}{2n\pi}\underbrace{\cos\left(\frac{2n\pi}{a}x\right)\Big|_0^a}$$

$$= 0$$

PS. 其實這題用不算

因為在 infinity wall 的的 any stationary state $|\psi_n\rangle$

都是由 P_n 和 $-P_n$ 所組成的駐波

ie $|\psi_n\rangle = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right) = \sqrt{\frac{2}{a}}\cdot\frac{1}{2}\left(e^{i\left(\frac{n\pi}{a}\right)x} - e^{-i\left(\frac{n\pi}{a}\right)x}\right)$

with $P_n = \frac{n\pi}{a}$, $P_n = -\frac{n\pi}{a}$

所以 expectation value $= 0$

#52

$$\int_0^a \psi_n^* \psi_\ell(x)\,dx = \delta_{n\ell}$$

⋯→ 我習慣寫成 $\langle \psi_n | \psi_\ell \rangle = \delta_{n\ell}$

這表是 $\{|\psi_n\rangle\}$ 是 basis，因為它們具有 orthogonal and normal

= orthonormal

選 (B)

#53

這題中

$\hat{H} = \frac{\hat{p}^2}{2m} + V$ 由於 $V=0$ inside the wall

所以 $E_n = \langle \psi_n | \hat{H} | \psi_n \rangle$

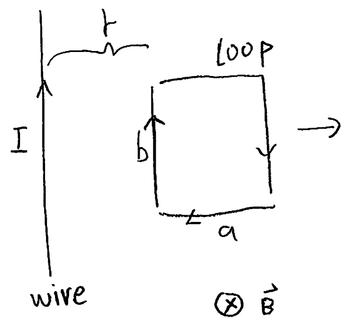
$= \langle \psi_n | \frac{\hat{p}^2}{2m} | \psi_n \rangle$

.... or by $p = \hbar k = \hbar\left(\frac{n\pi}{a}\right)$ ⋯ by $|\psi_n\rangle = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right)$

$$E = \frac{p^2}{2m} = \frac{\hbar^2 \pi^2}{2ma^2} n^2 \qquad n = 1.2.3.\cdots$$

so 選 (B) $E \geq \frac{\pi^2 \hbar^2}{2ma^2}$

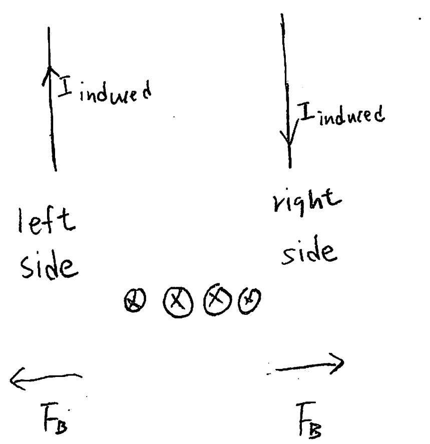
#54



wire 產生於 loop 的 $\vec{B}$ 是 into paper 的方向

當 loop 遠離 其 $\vec{B}$ 變小

所以 induce current (of loop) 會是 clockwise 的



由於 $\vec{F} = L\,\vec{I} \times \vec{B}$

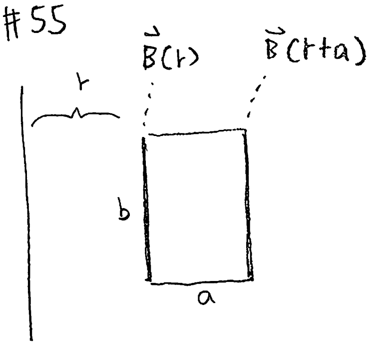
length

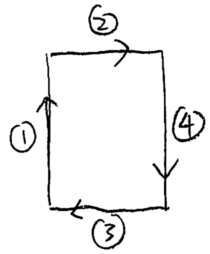
current and its direction

B field

所以選(E)

#55





我們將loop分成4個部份討論

②.③ 很明顯，其 Force 的 sum = 0

因為 $\vec{B}$ 於② = $\vec{B}$ 於③

但 ①.④ 不能互相 cancel

因為於①，$\vec{B} = \vec{B}(r)$

於④，$\vec{B} = \vec{B}(r+a)$

by Ampere law

$\therefore B \cdot 2\pi r = \mu_0 I$

$$B = B(r) = \frac{\mu_0 I}{2\pi} \frac{1}{r}$$

induced current

so 於①，$B = \dfrac{\mu_0 I}{2\pi} \dfrac{1}{r}$，$F_{①} = b \cdot i \cdot \dfrac{\mu_0 I}{2\pi} \dfrac{1}{r}$ 方向 $\rightarrow$

於④ $B = \dfrac{\mu_0 I}{2\pi} \dfrac{1}{r+a}$，$F_{④} = b \cdot i \cdot \dfrac{\mu_0 I}{2\pi} \dfrac{1}{r+a}$ $\leftarrow$

the net force : $b \dfrac{\mu_0 i I}{2\pi} \left(\dfrac{1}{r} - \dfrac{1}{r+a} \right)$

$= \dfrac{\mu_0 i I}{2\pi} \dfrac{ab}{r(r+a)}$ …選 (D)

#56

$$\hat{H}\Psi = \left(\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2\right)\Psi = E_n\Psi$$

frenquency $= \nu$

for harmonic oscillator

by $E_n = \hbar\omega(n+\frac{1}{2})$, $n = 0, 1, 2, 3, \ldots$

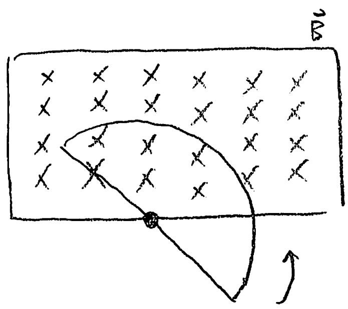
start from zero

$= h\nu(n+\frac{1}{2})$

$E_0 = \frac{1}{2}h\nu$ ... this guy is zero point energy

選 (c)

#57



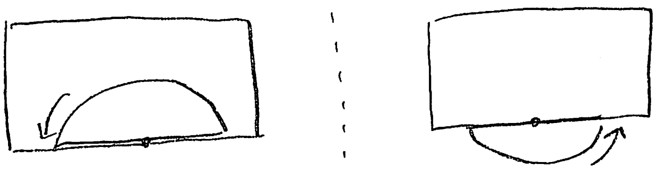
$$\varepsilon = -\frac{\partial \Phi_B}{\partial t}$$

where $\Phi_B(t) = \int \vec{B} \cdot d\vec{a}$ =磁通量.

因為 $\vec{B}$ 是 fixed 的

所以 $\varepsilon \propto \frac{\partial \text{Area}}{\partial t}$ (the change rate of area

#57 (continue)



這兩者有不同的 sign for ε

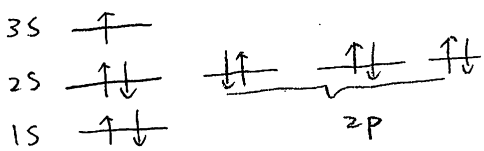
而在期間
因為 uniformed rotate
所以 ε 保持 <u>固定的大小</u> for Both two cases

so 選 (A)

#58

Sodium 有11個电子,

so.



$1S^2\ 2S^2\ 2p^6\ 3S^1$, 選 (C)

#59

$_2$He 有 2 個 電子, 其 ground state

$$\Psi = \psi \cdot \chi$$

space part → ψ, spin part → χ

在 space 上, 1S state 因 $l=0$

其為 symmatric

1S $\uparrow\downarrow$

so, 在 spin 上, 必須是 anti symmetric

才能滿足電子的 fermion 性質 (ie. anti-symmetric under exchange)

PS. two electrons 的 spin

thus 得選 singlet ....(A)

electron 1   electron 2

基本上, 它有 4 個可能

$$|\uparrow\rangle_{①}|\uparrow\rangle_{②} \equiv |\uparrow\uparrow\rangle$$

$$|\uparrow\rangle_{①}|\downarrow\rangle_{②} \equiv |\uparrow\downarrow\rangle$$

$$|\downarrow\rangle_{①}|\uparrow\rangle_{②} \equiv |\downarrow\uparrow\rangle$$

$$|\downarrow\rangle_{①}|\downarrow\rangle_{②} \equiv |\downarrow\downarrow\rangle$$

$\Rightarrow$

They form a complete basis

now, we change basis:

新的 basis, 可設計成

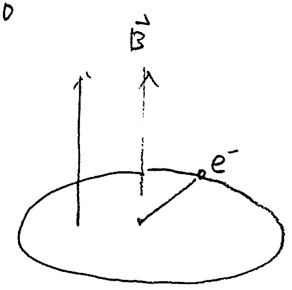
$$\left.\begin{array}{c} |\uparrow\uparrow\rangle \\ \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \\ |\downarrow\downarrow\rangle \end{array}\right\}$$

它們仍舊 orthogonal and form a complete basis

we call these three triplet

$$\left.\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\uparrow\downarrow\rangle\right)\right\}$$ singlet (anti symmetric)

60



我們先推一下公式好了

by 向心力由 F_B 提供 and $\vec{F}_B = q\vec{v} \times \vec{B}$

設定 $\vec{v} \perp \vec{B}$, then $F_B = evB$

then $F_B = mr\omega^2$ by $r\omega = v$

$= mv\omega$

$= evB$

so $\omega = \frac{e}{m}B$

so $B = 1\ T$

$m = 0.1\ m_e = 9.1 \times 10^{-32}\ kg$

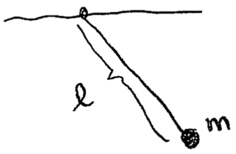
$e = 1.6 \times 10^{-19}\ C$

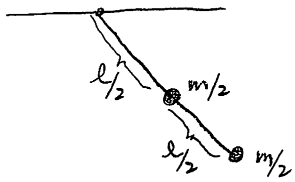
then $\omega = \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-32}}$ by my calculator

$\doteqdot 1.758 \times 10^{12}$

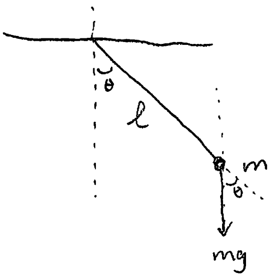
選(D)

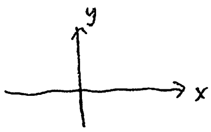
#61





PS. 先來推導一下公式好了





← 最原始的 case
(for small oscillation)

X 方向 $F_x = m(l\ddot{\theta}) \doteq -mg\sin\theta$

from $\dfrac{\partial^2 弧長}{\partial t^2}$, then $\sin\theta \doteq \theta$ when θ is small

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

so $\omega_1^2 = \dfrac{g}{l}$

or 利用 torque 的方式

$$\tau = I\alpha$$

$$-mg\sin\theta \cdot l = ml^2 \cdot \ddot{\theta}$$

$\vec{\tau} = \vec{r} \times \vec{F}$, $|\vec{\tau}| = mg\sin\theta \cdot l$

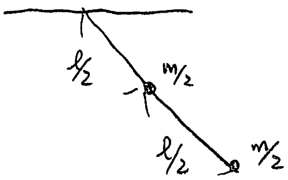
$I = ml^2$

$\alpha = \ddot{\theta}$

$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

#61 (continue)

for



$$I = \frac{m}{2} \cdot \left(\frac{l}{2}\right)^2 + \frac{m}{2}(l)^2$$

$$= \frac{1}{2} m l^2 \left[\frac{1}{4} + 1\right]$$

$$= \frac{5}{8} m l^2$$

$$\tau = -\frac{1}{2} mg\sin\theta \cdot \frac{1}{2} l - \frac{1}{2} mg\sin\theta \cdot l$$

$$= -mg\sin\theta \left[\frac{1}{4} + \frac{1}{2}\right]$$

$$= -mg\sin\theta \left[\frac{3}{4}\right] \cdot l$$

so $$-mg\sin\theta\, l \frac{3}{4} = \frac{5}{8} m l^2 \cdot \ddot{\theta}$$

$$\ddot{\theta} + \frac{6}{5}\frac{g}{l}\theta = 0$$

$$w_2^2 = \frac{6}{5}\frac{g}{l}$$

so $$\frac{w_2}{w_1} = \left[\frac{6}{5}\right]^{1/2}$$ .... 選 (A)

62

$n = 1$ (mole 數)

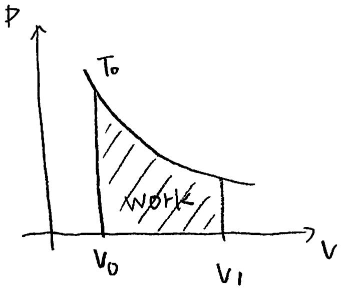
T_0

V_0

reversible isothermal expansion

T_0

$V_0 \rightarrow V_1$



ie $\int_1^2 đw \neq w|_1^2$

ps. đ 表示 not exact, depend on path

by $đw = PdV$

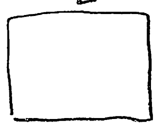
and $P = \dfrac{nRT}{V}$

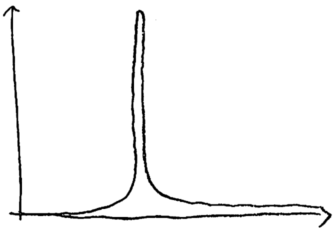
so $\int đw = \int_{V_0}^{V_1} \dfrac{nRT_0}{V} dV$

$= nRT_0 \ln(V_1/V_0)$

所以 選 E

63





ex. 1000個 coins

出現 500 head, 500 tail 時

有最多的 possible microstate

(so entroy is max

by $S = k_B \ln \Omega$)

then 這個系統應會停在有最多 microstates

之下的 configuratin

因為其机率遠大大於其它的 configuration

Thus 選 (D)

#64

$$(E_x, E_y, E_z) = (0, 0, kz)$$

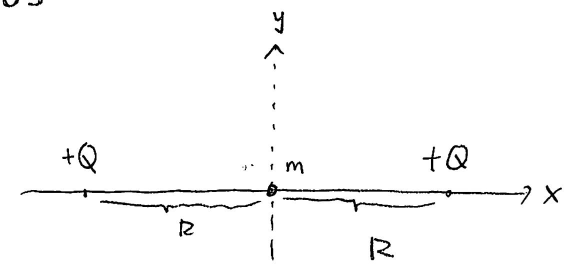
for (B)

by $\vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = k = 4\pi\rho$

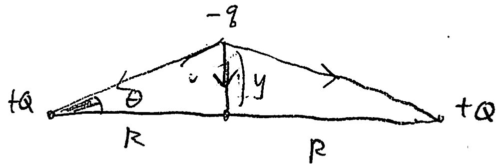
so. we have ρ inside Yes.

選 (B)

65



這題，我們得先求出 restoring force



$$F_y = -2 \cdot \frac{1}{4\pi\varepsilon_0} \frac{Qq}{R^2+y^2} \sin\theta$$

我討厭用 SI unit

and $\sin\theta \simeq \tan\theta \simeq \frac{y}{R}$

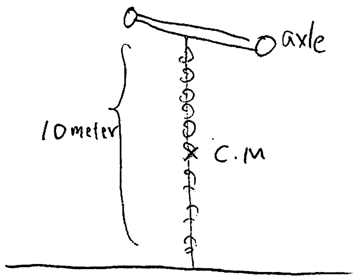
by θ is small

so $$m\ddot{y} = -\frac{Qq}{2\pi\varepsilon_0} \frac{1}{R^2+y^2} \frac{y}{R}$$, and 設 $R \gg y$

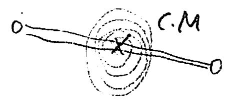
$$\ddot{y} + \underbrace{\frac{Qq}{2\pi m R^3}}_{=\omega^2} y = 0$$

so $$\omega = \left[\frac{Qq}{2\pi m R^3}\right]^{1/2}$$ …… 選 (E)

#66



$\Longrightarrow$



total mass of chain

$= 10 \times 2$

$= 20$ kg

而 CM上升了 5 m (PS. C.M可代表整個系統)

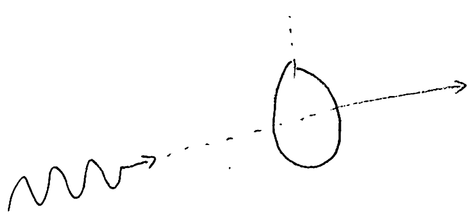
所需的 work

$= mgh = 20 \times 10 \times 5$

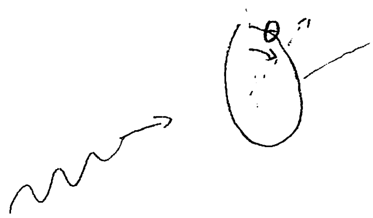
$= 1000$ J

選 (C)

#67



then



intensity

$= A + B\cos 2\theta$

$A > B > 0$

(1) $I \doteq A + B\cos\theta$

for any θ, Intensity > 0 且為 const A

說明原本的光一定有 unpolarized light

67 (continue)

PS.

circular polarized 的光為

$$\vec{E} = A\,\hat{e} \cdot e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

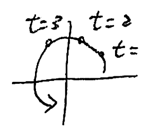
A ↑ real　　$\hat{e}$ ↑ polarization

$$\hat{e} \Rightarrow \begin{cases} \hat{e}_1 = -\frac{1}{\sqrt{2}}(\hat{x}+i\hat{y}) & \text{Right-handed} \\ \hat{e}_2 = +\frac{1}{\sqrt{2}}(\hat{x}-i\hat{y}) & \text{Left-handed} \end{cases}$$

以 $\hat{e}_1$ 而說

若我們取 $\vec{E}$ 的 Real part and set $\vec{k} = k\hat{z}$, and 取 $z=0$ 的 plane

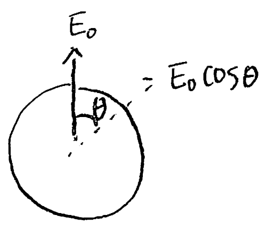
we will get : $\vec{E} = E_0\{\cos(\omega t)\,\hat{x} + \sin(\omega t)\,\hat{y}\}$



↖ that's why we call righ-handed.

它的 intensity after going through polaroid is depend on time

而 plane polarized 的 light



intensity $\propto (E_0\cos\theta)^2$

and by $\cos 2\theta = \cos(\theta+\theta)$

$= \cos^2\theta - \sin^2\theta$

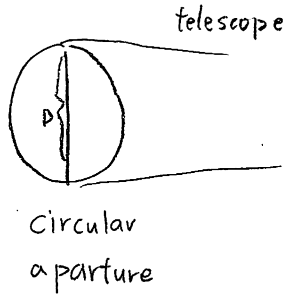
$= 1 - 2\cos^2\theta$

這解釋了 $B\cos 2\theta$ 的來源

so 選(C) :

68

利用 $\theta_{min} \doteq 1.22 \dfrac{\lambda}{直徑D}$



$\theta_{min} = 8 \times 10^{-6}$ rad

$\lambda = 5500 \times 10^{-10}$ m

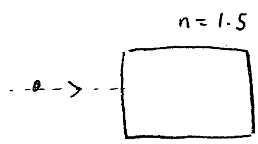
$= 5.5 \times 10^{-7}$

於是 直徑 $D \doteq 1.22 \dfrac{5.5 \times 10^{-7}}{8 \times 10^{-6}} = 8.38 \times 10^{-2}$ m

$= 8.38$ cm

$\doteq 10$ cm 選 (C)

69



這題我不懂.... Orz

#70

$$E = \gamma mc^2 = \underbrace{T}_{\text{動能}} + \underbrace{mc^2}_{\text{rest energy}}$$

ps. so $\gamma = 100$

$$E = \sqrt{p^2c^2 + (mc^2)^2}$$

$$p^2c^2 = E^2 - (mc^2)^2$$

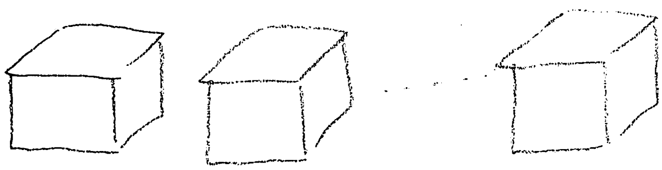
$$= (100^2 - 1)(mc^2)^2$$

so $p \doteq 100\ mc$ ⋯ 選(D)

#71

我們有2個 state E_1 and E_2 for each subsystem

有 N_0 個 subsystem



這題是 canonical ensemble 的概念

其中 $P_r \propto e^{-\beta E_r}$, index r 表示 第幾個 energy state

$\beta = \frac{1}{k_B T}$

P_r 為 r-th energy state 出現的機率

71 (continue)

由於只有兩個 state (ie. only P_1, P_2

thus $P_1 = \dfrac{e^{-\beta E_1}}{e^{-\beta E_1}+e^{-\beta E_2}}$ ← 底下這個是 normalization 用的

since $P_r \propto e^{-\beta E_r}$

hence $P_r = \dfrac{e^{-\beta E_r}}{\sum_r e^{-\beta E_r}}$

$$P_2 = \frac{e^{-\beta E_1}}{e^{-\beta E_1}+e^{-\beta E_2}}$$

這題問 $N_0 \cdot P_1 = N_0 \dfrac{e^{-\beta E_1}}{e^{-\beta E_1}+e^{-\beta E_2}}$

$$= N_0 \frac{1}{1+e^{-\beta(E_2-E_1)}}$$

$E_2 - E_1 \equiv \epsilon$

$$= \frac{N_0}{1+e^{-\beta\varepsilon}}$$

$\therefore$ 選 (B)

72

$$\text{internal energy} = E_1 N_0 + \frac{N_0 \varepsilon}{1+e^{\beta\varepsilon}} = E$$

$$C_V = \left.\frac{\partial E}{\partial T}\right|_V$$

$$= \frac{\partial E}{\partial \beta}\frac{\partial \beta}{\partial T} \qquad \beta = \frac{1}{k_B T}, \quad \frac{\partial \beta}{\partial T} = -\frac{1}{k_B T^2} = -k_B \beta^2$$

$$= -\frac{N_0 \varepsilon}{(1+e^{\beta\varepsilon})^2} e^{\beta\varepsilon} \cdot \varepsilon \cdot (-k_B \beta^2)$$

$$= N_0 k_B \cdot (\varepsilon\beta)^2 \frac{e^{\beta\varepsilon}}{(1+e^{\beta\varepsilon})^2}$$

so 選 (A)

73

(A) is wrong, E_2 ~~oooo~~ $N_0/2$, E_1 ~~oooo~~ $N_0/2$ entropy有max值

(B) wrong

(C) Yes.

這是個很多 textbook 都有的 two energy levels 的 example

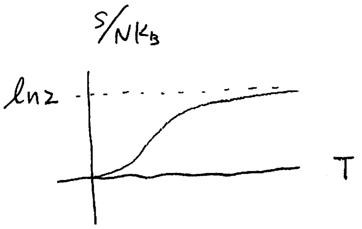
要求 entropy S, 先求出 cannonical partition function Z

$$Z = \sum_r e^{-\beta E_r} = e^{-\beta E_1} + e^{-\beta E_2}$$

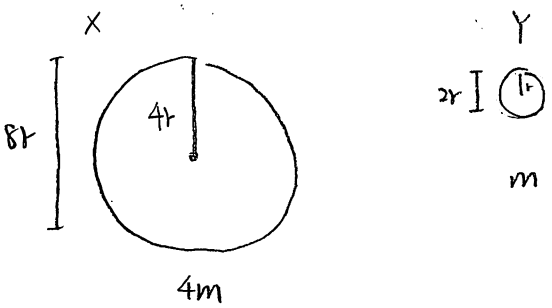
then by $F \equiv -k_B T \ln Z$, $S = -\left(\frac{\partial F}{\partial T}\right)_{N,V}$

73 (continue)

therefore, the result is:

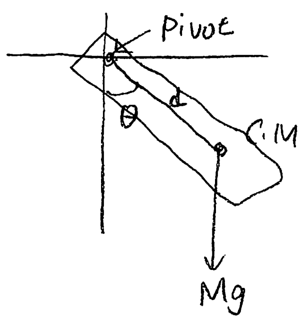


74



這題是在玩 physcial pendulum 的概念.

先來推導一下公式



$$\tau_p = Mgd\sin\theta = -I_p\,\alpha \qquad \alpha = \ddot{\theta}$$

τ_p ↑ 指對pivot

I_p: moment of inertia

subscript p
代表對 pivot

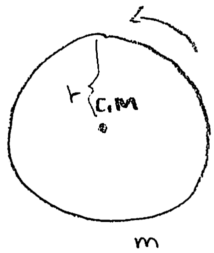
$$\text{so} \quad \ddot{\theta} + \underbrace{\frac{Mgd}{I_p}}_{=\omega^2}\theta = 0$$

$\omega = \frac{2\pi}{T}$ ，由這個，可以求出 period

74 (continue)

由 parallel theorem

$I_p = I_{C.M} + Md^2$



circle 的 $I_{c.m} = mr^2$

for X, $I_p = (4m)(4r)^2 + (4m)(4r)^2$

$= 2 \cdot (4m)(4r)^2$

for Y, $I_p = mr^2 + mr^2$

$= 2mr^2$

$w_x^2 = \dfrac{4m \cdot g \cdot 4r}{2 \cdot 4m \cdot (4r)^2} = \dfrac{1}{2} \dfrac{g}{4r}$

$w_y^2 = \dfrac{m \cdot g \cdot r}{2mr^2} = \dfrac{1}{2} \dfrac{g}{r}$

then $\dfrac{w_x^2}{w_y^x} = \dfrac{1}{4}$

$\dfrac{w_x}{w_y} = \dfrac{1}{2} \Rightarrow \dfrac{T_x}{T_y} = 2$

so $T_x = T$

$T_y = T/2 \rightarrow$ 選 (B)

#75

$$^{235}_{92}U \rightarrow ^{231}_{90}Th + ^{4}_{2}He$$

(A) is wrong ，要看 α particle 的動能

(B) wrong

由動量守恆

$$0 = m_{Th} V_{Th} + m_{He} V_{He}$$

They will have the same magnitude of momentum but diffrent sign.

$\Rightarrow$ so (D) is wrong

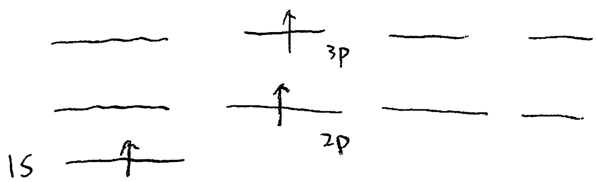
by $\left|\frac{V_{He}}{V_{Th}}\right| = \frac{m_{Th}}{m_{He}}$

$$\frac{T_{He}}{T_{Th}} = \frac{\frac{1}{2} m_{He} V_{He}^2}{\frac{1}{2} m_{Th} V_{Th}^2} = \frac{m_{Th}}{m_{He}}$$

so He 的動能 >> Th 的動能

選 (E)

#76



orbital part
分别是 $\ell = 0$
$\ell = 1$
$\ell = 1$ } so the max is 2

spin part $\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \Rightarrow$ max $= \frac{3}{2}$

max $= 2 + \frac{3}{2} = \frac{7}{2}$

選 (A)

77

在 atom 中, Bohr magneton $\equiv \mu_B = \frac{e\hbar}{2m_e}$

电子因為 spin 而產生的 intrinsic magnetic momentum μ

$$\mu = \underbrace{g_s}_{\text{這邊的 g 為 g-factor, 电子的為} \doteq 2.0023} \mu_B \underbrace{S}_{\text{spin 的量子數} (\frac{1}{2} \text{ or } -\frac{1}{2})}$$

在 nucleus 中, 相同的 concept, 但 call nuclear magneton μ_N

$$\mu_N \equiv \frac{e\hbar}{2m_p}$$

改用質子的 mass

其 $\mu_N \ll \mu_B$

then nucleus 的 magnetic momentum μ

$$\mu = g \, \mu_N \, S$$

for proton $g = 5.58$
neutron $g = -3.82$

\# 77 (continue)

題目問 $\frac{\mu_{nucleus}}{\mu_{electron}} = \frac{g\,\mu_N\,S}{g_s\,\mu_B\,S}$ S 都為 $\frac{1}{2}$

but $\mu_B >> \mu_B$

所以 ratio < 1

選 (E)

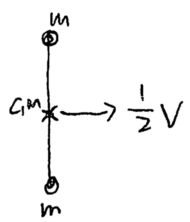
\# 78

首先來看質心的運動 : only in x direction

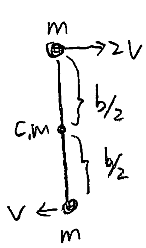
$M V = m(+2v) + m(-v)$; and $M = 2m$

$V = \frac{1}{2} v$

ie.



then 我們再看 angular momentum to C.M

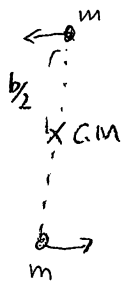


$\Rightarrow \vec{L} = m\,\vec{r} \times \vec{v}$

so the net $L = m\left(\frac{b}{2}\right)\cdot 2v + m\left(\frac{b}{2}\right)(v)$

$= \frac{3}{2} m b v$

78 (continue)

so 

moment of inertia

再由 $L = I\omega$

$= 2m\left(\frac{b}{2}\right)^2 \omega = \frac{3}{2} m b v$

$\Rightarrow \omega = \frac{3v}{b}$

上面的 skater

會相對 C.M 作 circular motion

其 x軸 $\frac{b}{2}\sin(\omega t)$ ... ps. $t=0$

y軸 $\frac{b}{2}\cos(\omega t)$ $y(0) = \frac{b}{2}$

$x(0) = 0$

Total motion will be

$X = X_{cm}(t) + x(t)$

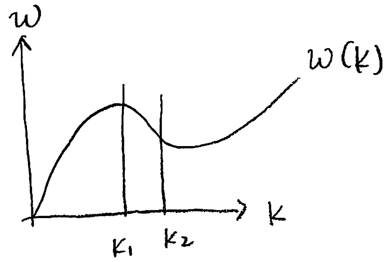
$= \frac{1}{2}vt + \frac{b}{2}\sin\left(\frac{3v}{b}t\right)$

$Y = y_{cm}(t) + y(t)$

$= 0 + \frac{b}{2}\cos\left(\frac{3v}{b}t\right)$

so 選 (C)

#79



when $k_1 < k < k_2$

ps. 我們先來簡單的談一下 dispersion relation

由在 vaccum 中

$\nabla \cdot \vec{E} = 0$

$\nabla \cdot \vec{B} = 0$

$\nabla \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$

$\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = 0$

可以導出 $\left[\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \cdot\right]\left\{\begin{matrix}\vec{E}\\ \vec{B}\end{matrix}\right. = 0$

and $\vec{E}$ 為 plane wave 的話

$\vec{E} = \vec{E}(\vec{r},t) = \vec{E}_0 \exp[i(\vec{k}\cdot\vec{r} - wt)]$

代入可得

$\left[-\frac{1}{c^2}w^2 + k^2\right]\left\{\begin{matrix}\vec{E}\\ \vec{B}\end{matrix}\right. = 0$

$\Rightarrow \quad w = kc$

79 (continue)

若是在物質中

$\nabla \cdot \vec{D} = 0$ and $\vec{D} = \epsilon \vec{E}$

$\nabla \cdot \vec{B} = 0$ $\vec{H} = \frac{1}{\mu} \vec{B}$

$\nabla \times \vec{E} + \frac{1}{c} \frac{\partial \vec{B}}{\partial t} = 0$

$\nabla \times \vec{H} - \frac{1}{c} \frac{\partial \vec{D}}{\partial t} = 0$

可得 $\left[-\frac{1}{\tilde{c}^2} w^2 + k^2 \right] \begin{Bmatrix} \vec{E} \\ \vec{B} \end{Bmatrix} = 0$

where $\tilde{c}$, under my notation, is $\tilde{c} = \frac{c}{\sqrt{\mu \epsilon}} = \frac{c}{n}$ ← 折射率

在物質中的 speed of light

so $w = \tilde{c}\, k$

$w = \frac{c}{\sqrt{\mu \epsilon}}\, k$

but! ϵ 是 w 的 function

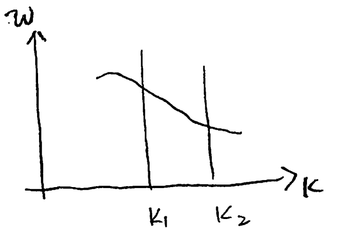
and $\mu \simeq 1$

so. actually $w(k) = \frac{c}{\sqrt{\epsilon(w)}}\, k$

基本上: $\frac{w}{k} \equiv$ phase velocity

$\frac{dw}{dk} \equiv$ group velocity

\# 79 (continue)

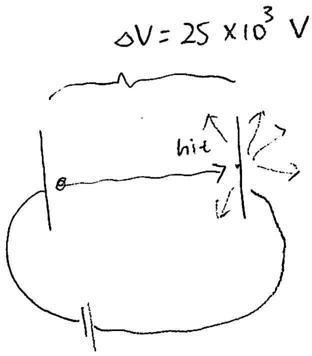


$V_p = \frac{w}{k}$, positive

$V_g = \frac{dw}{dk}$ = slop , and it is negative

they are not in the same direction

\# 80



电子的動能 = $e(\Delta V)$

而 X-ray 最大的 energy $= e(\Delta V)$

$= h\nu$

$= \frac{hc}{\lambda}$ ⇐ 最小 λ

所以 $\lambda = \frac{hc}{e(\Delta V)}$

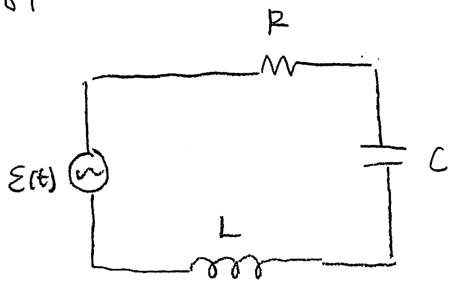
$= \frac{(6.62\times10^{-34})\cdot(3\times10^{8})}{(1.6\times10^{-19})(25\times10^{3})}$

$= 4.965\times10^{-11}$ from my calculator

$\simeq 0.5$ Å

選 (B)

81

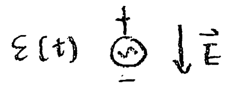


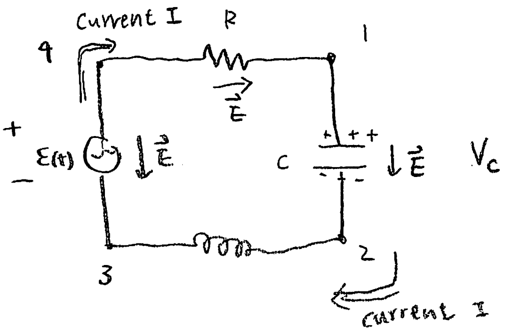
$$\mathcal{E}(t) = \mathcal{E}_m \cdot \cos(\omega t)$$

我們先來推導一下公式 吧~

by Faraday law: $\oint \vec{E} \cdot d\vec{l} = \mathcal{E} = -L\frac{dI}{dt}$

我們假設在此 moment





So $\oint \vec{E} \cdot d\vec{l} = \underset{1\to 2}{V_c} - \underset{3\to 4}{\mathcal{E}_m \cos(\omega t)} + \underset{4\to 1}{IR} = -L\frac{dI}{dt}$

而 $I = \frac{dQ}{dt}$

$V_c = \frac{Q}{C}$

So $\frac{Q}{C} - \mathcal{E}_m \cos(\omega t) + R\frac{dQ}{dt} = -L\frac{d^2Q}{dt^2}$

rewrite : $\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{1}{LC}Q = \mathcal{E}_m/L \cos(\omega t)$

#81 (continue)

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{1}{LC}Q = \frac{\mathcal{E}_m}{L}\cos(\omega t)$$

要怎麼解呢:

recall $y'' + Ay' + By = f(x)$

其解為 $y = \underbrace{y_h}_{\text{homogenous solution}} + \underbrace{y_p}_{\text{particular solution}}$

ie $y'' + Ay' + By = 0$
的解

基本的解法為 設 $y = e^{rx}$

then $r^2 + Ar + B = 0$

....

but 在我們的題目中

會得到 $\sim e^{-\square t}$, ie 這是 transient term

as $t \to \infty$ $y_h \to 0$

至於 y_p.

我們解 $y'' + Ay' + By = e^{i\omega t}$ ← 最後取 real part 即可

因為 steady for y

we can set $y\ (\text{or } Q) = K \cdot e^{i(\omega t + \delta)}$

與 driving term 相同 ← phase term

代入一下，就可解出

$$I(t) = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \cdot \cos(\omega t - \phi)$$

and $\tan\phi = \dfrac{\omega L - \frac{1}{\omega C}}{R}$

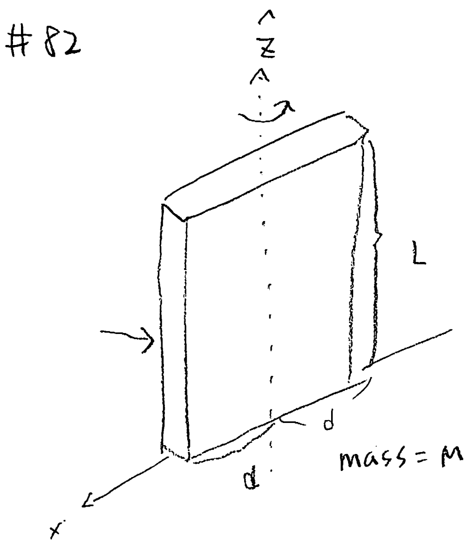
#81 (continue)

所以 when $\omega L - \frac{1}{\omega C} = 0$ 時, 有最大的 steady current

$$\Rightarrow \omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} \quad \cdots \text{選 (C)}$$

#82

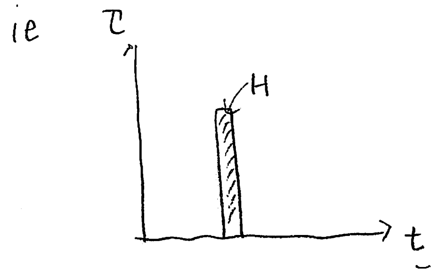


angular impulse $\vec{H} = \int \overset{\text{const}}{\vec{\tau}}\, dt = \int (I \overset{\text{const}}{\vec{\alpha}})\, dt$

$$= I\alpha t$$

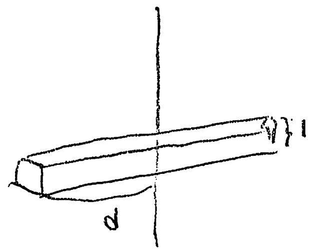
$$= I\omega$$

ie



這題,基本上,是在算 moment of inertia I

定義 $\sigma = \frac{M}{2dL}$



而 $I = 2\int_0^L \int_0^d x^2 \sigma \, dx\, dy$ (y, x)

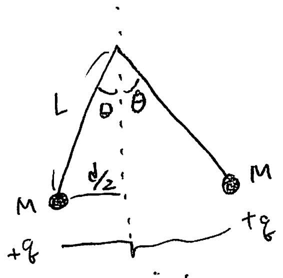
$$= \frac{2M}{2dL} \int_{y=0}^{y=L} \left(\frac{d^3}{3} \right) dy$$

$$= \frac{1}{3} M d^2$$

而 $H = I\omega$

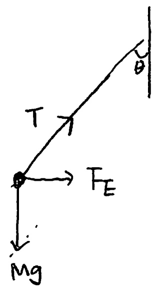
$$\omega = \frac{3H}{Md^2} \quad \cdots \text{選 (D)}$$

83

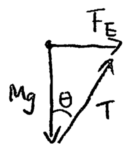


求 d

先作 force analysis



So



$$\Rightarrow \tan\theta = \frac{F_E}{Mg}$$

$$F_E = Mg\tan\theta = K\frac{q^2}{d^2}$$

by $\tan\theta \simeq \sin\theta = \frac{d/2}{L}$

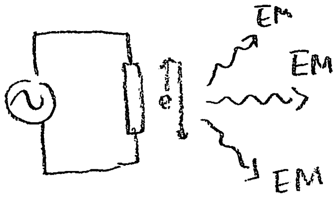
when θ is small

So. $$Mg\frac{d}{2L} = k\frac{q^2}{d^2}$$

$$d^3 = \frac{2kq^2L}{Mg}$$

$$d = \left(\frac{2kq^2L}{Mg}\right)^{1/3}$$ ⋯⋯ 選 (A)

84



(A) yes , for a moving charge

$$\underset{\substack{\uparrow \\ \text{solid angle}}}{\frac{d\overset{\text{power}}{P(t)}}{d\Omega}} = \frac{q^2}{4\pi c} \frac{\left(\hat{n} \times [(\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}]\right)^2}{(1 - \vec{\beta} \cdot \vec{n})^6}$$

$$\vec{\beta} = \frac{\vec{v}}{c}$$

(B) yes

(C) far from electron, mean radiation zone

.... 而 $\frac{\text{energy}}{4\pi R^2}$ = power , so $\propto \frac{1}{R^2}$ yes

(D) no, when $\hat{n} \parallel \dot{\vec{\beta}}$

then $\frac{dP}{d\Omega} = 0$

(E) I think yes.

85

$$E = \gamma mc = 1.5\,[\mathrm{MeV}]$$

$$= \sqrt{(pc)^2 + (mc^2)^2}$$

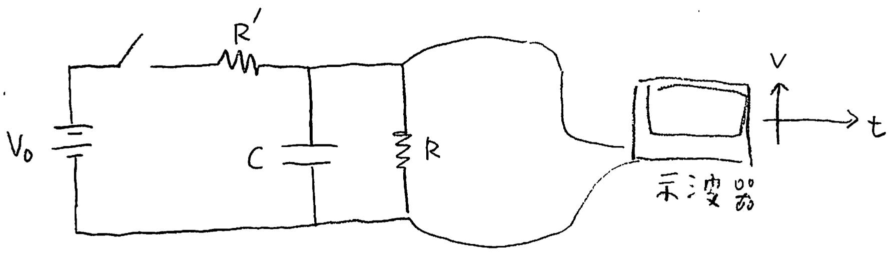
so $$p = \frac{1}{c}\left[\, E^2 - (mc)^2 \,\right]^{1/2}$$

$$= \frac{1}{c}\left[\, 1.5^2 - 0.5^2 \,\right]^{1/2}$$

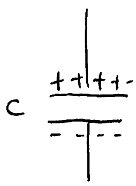
$$= \frac{1}{c}\sqrt{2}$$

$$\simeq 1.4\ [\mathrm{MeV}/c] \quad \cdots \text{選 (C)}$$

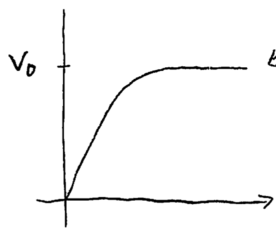
86



當 switch close 來時



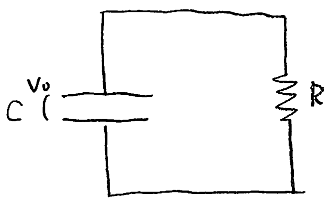
會讓 capacitor 充電

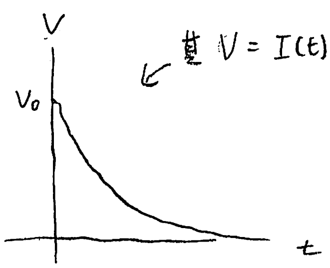


← 完全充好了

---

then, 當 open switch ,

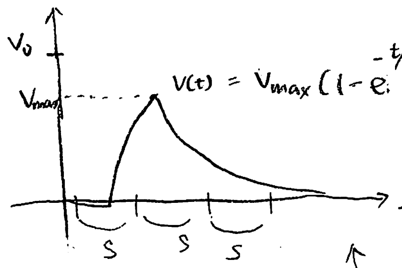




← 其 $V = I(t)\cdot R = V_0\left(1 - e^{-\frac{t}{Rc}}\right)$

$$\frac{Q}{C} + R\frac{dQ}{dt} = 0$$

#86 (continue)



$V(t) = V_{max}(1-e^{-t/RC})$ ← 示波器的圖形

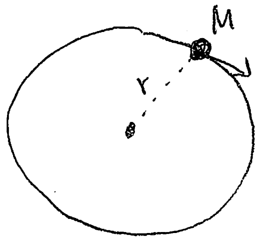
← 以 const speed s 在掃描

我們可以 measure $\Delta t = s$ 時,

$V(t)$ 下降的程度, then 可以求出 c

選 (B) s 和 R

#87

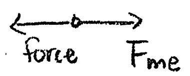


circular motion = $F_{向心力} = m\frac{v^2}{r}$

hence $\frac{k}{r^3} = m\frac{v^2}{r}$

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}k\frac{1}{r^2}$ = 動能 T

這個 force 之下



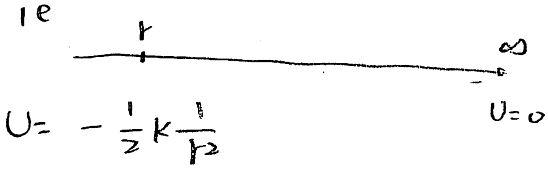
$\rightarrow \int_r^\infty \frac{k}{r^3}\,dr = -\frac{k}{2r^2}\Big|_r^\infty$

$= \frac{1}{2}k\frac{1}{r^2}$

在處於平衡, 而我得付出

$W = \int_r^\infty F_{me}\cdot dr$ 的功 才能 將它 從 r 搬到 ∞ 處。

ie



; $U = -\frac{1}{2}k\frac{1}{r^2}$, $T = \frac{1}{2}k\frac{1}{r^2}$ 〉 $E = 0$

選 (C)

ie 若 object 從 ∞ 到 r, 將釋放出 $\frac{1}{2}k\frac{1}{r^2}$ 的 energy

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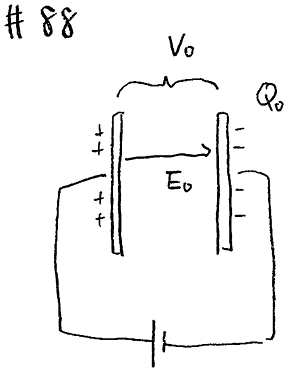
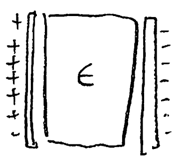


in vacoum

$E_0 = D_0$

then



$D_f = \epsilon E_f$

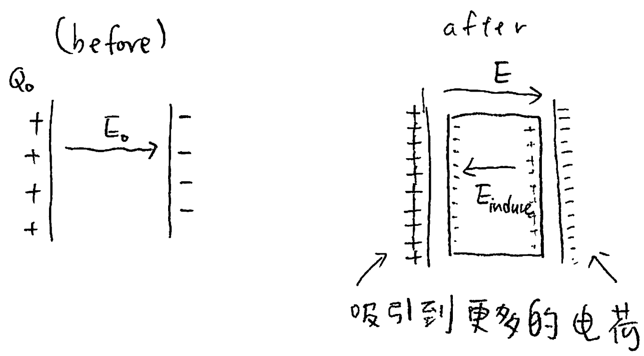
(A) wrong $V_f = V_0$

(B)

(C) wrong $Q_f > Q_0$

因為加入 dielectric 先會使內部的电場變小

而使得在同樣的 voltage 之下，可以累積更多的 charge



一直到 $\vec{E}_f = \vec{E} + \vec{E}_{ind} = \vec{E}_0$ 為止

(D) $E_f = E_0$

(E) by $D = \epsilon E$ ← total field

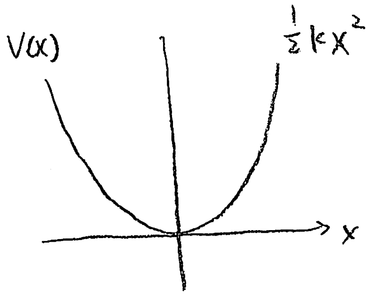
在 vaccum 下 $D_0 = 1 \cdot E_0$

在有 dielectric 下 $D_f = \epsilon E_f$

因為 $E_0 = E_f$ 所以

$\epsilon > 1$ $D_f > D_0$ 選 (E)

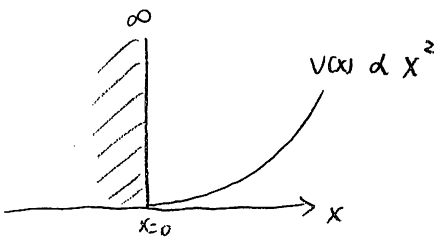
89



其 $E_n = \hbar\omega(n+\frac{1}{2})$

and $\omega^2 = \frac{k}{m}$

now



也就是說必須 require：

$$\underbrace{\psi(x=0)=0}_{\text{因為 } x=0 \ \ V=\infty} ,$$

原本的 wave function

$$\psi_0 = A\, e^{-\alpha x^2} \quad \text{where } A = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}, \ \alpha = \frac{m\omega}{2\hbar}$$

而 $\psi_n = A\dfrac{1}{\sqrt{2^n \cdot n!}}\underbrace{H_n(\xi)}_{\text{Hermite polynomial}}\, e^{-\alpha x^2}$, $\xi = \sqrt{2\alpha}\, x$

only for $n = 1, 3, 5, 7 \cdots$ odd 的時候

$x=0$, $H_n(0)=0$

(PS. 因為 $e^{-\alpha x^2} \neq 0$，唯一能產生 0 的，只有 $H_n(\xi)$)

所以 選 (E)

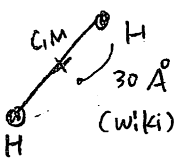
#90

由 $\hat{H}_{rot} = \frac{\hat{L}^2}{2I}$

so $\hat{H}_{rot}\psi = E_{rot}\psi$

we will get $\frac{\hbar^2 \ell(\ell+1)}{2I}$

so $\Delta E \simeq \frac{2\hbar^2}{I}$ $\quad (\ell=2)-(\ell=1) = 4$ 最小的 case



$$I = 2 \cdot m_p r^2$$
$$= 2 \times (1.67 \times 10^{-27}) \times (30 \times 10^{-10})^2$$
$$= 3 \times 10^{-44}$$

so $\Delta E = \frac{2\hbar^2}{I} = \frac{2(6.63 \times 10^{-34})^2}{3 \times 10^{-44}} \underbrace{\frac{1}{1.6 \times 10^{-19}}}_{\text{轉成eV}}$

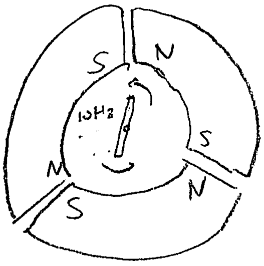
$$\simeq 1.82 \times 10^{-4}$$

所以 選(B) 10^{-3} eV order

91

目前我還不太懂…

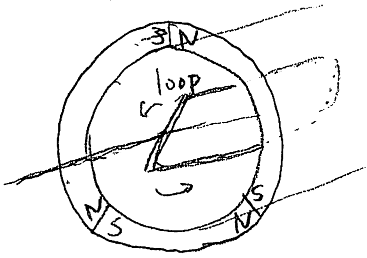
92



這題好像 3 phase motor 喔…

中間的小 loop 以 10Hz 在轉動

問所產生的 voltage (在小 loop 上) 的 frequency



因 loop 在轉一圈的過程中

會有3個 gap (N→S)

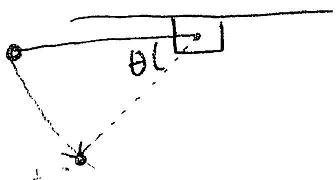
所以 voltage 會改變的 frequency 是 rotation 的3倍

so

$10Hz \times 3 = 30Hz$

選 (D)

93

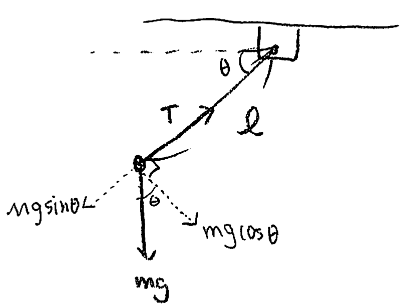


求在左边時的 $|\vec{a}|$

小技巧：在答案中, test θ=0，哪個的值=g

…只有(E)

force analysis



由 $\tau = mg\cos\theta \cdot l = I\alpha$

$= ml^2\alpha$

$\alpha = \frac{g}{l}\cos\theta$

而切線方向的加速度 $= a_c = l\alpha = g\cos\theta$

now, 我們向心的加速度

$$a_c = \frac{v^2}{\ell}$$

$$\underbrace{mg\ell\sin\theta}_{\text{所減少的位能}} = \underbrace{\frac{1}{2}mv^2}_{\text{now, 擁有的動能}}$$

hence $a_c = 2g\sin\theta$

finally $\vec{a} = \vec{a_c} + \vec{a_t}$

and $|\vec{a}| = \sqrt{a_c^2 + a_t^2}$

$$= \left[4g^2\sin^2\theta + g^2\cos^2\theta\right]^{1/2}$$

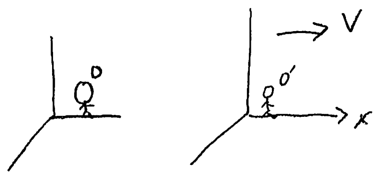
$$= g\left[3\sin^2\theta + 1\right]^{1/2}$$

⋯⋯選 (E)

94.

Lorentz transform

and set $c=1$



以 O 看的 event 為 (ct, x, y, z)
$\updownarrow$ 同一個
O' event 為 (ct', x', y', z')

其 relation : 以 x 方向有相對速度 v

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} \quad \text{or} \quad X'^{\mu} = \Lambda^{\mu}{}_{\nu} X^{\nu}$$

ps.
and $(cdt')^2 - (dx')^2 - (dy')^2 - (dz')^2 = (cdt)^2 - (x)^2 - (y)^2 - (z)^2$

這叫 Lorentz invariance.

也寫成 $dX'_{\mu} dX'^{\mu} = dX_{\mu} dX^{\mu} = (ds)^2$

選項中，都是 $y=y'$，$z=z'$，表示只有在 x方向有相對運動

我們 assume (by $c=1$)

event 1 時 $\begin{bmatrix} t' \\ x' \end{bmatrix} = \begin{bmatrix} t \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

event 2 $\quad O' \Rightarrow \begin{bmatrix} t' \\ x' \end{bmatrix}$

$O \Rightarrow \begin{bmatrix} t \\ x \end{bmatrix}$

而它們滿足 $(t'-0)^2 - (x'-0)^2 = (t-0)^2 - (x-0)^2$

ie $\quad t'^2 - x'^2 = t^2 - x^2$

(A) $x' = 4x$ $\quad$ then $\quad t'^2 - x'^2 = \left(\frac{1}{4}t\right)^2 - (4x)^2 \neq t^2 - x^2$
$t' = \frac{1}{4}t$

it is wrong..

94 (continue)

... then we try (C)

$$x' = \frac{5}{4}x - \frac{3}{4}t$$

$$t' = \frac{5}{4}t - \frac{3}{4}x$$

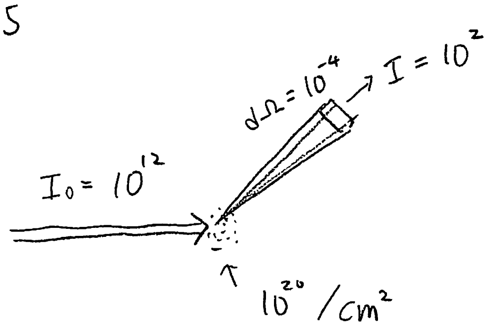
$$\Rightarrow (t')^2 - (x')^2 = \left(\frac{5}{4}t - \frac{3}{4}x\right)^2 - \left(\frac{5}{4}x - \frac{3}{4}t\right)^2$$

$$= \left[\left(\frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]t^2 - \left[\left(\frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]x^2$$

$$= t^2 - x^2$$

沒錯，就選 (c)

95



so. differential cross section

$$\frac{1}{I_0}\frac{I}{d\Omega} = \frac{1}{10^{12}}\frac{10^2}{10^{-4}} = 10^{-6}$$

為了要使 independent of incident beam, 所以要除以 I_0

then per $cm^2 \longrightarrow \frac{10^{-6}}{10^{20}} \Rightarrow 10^{-26}$ ... (c)

PS. 這題我也不太懂，
不過直覺是對的

96

這個… 我光学很弱不懂…

97

先 try dimensional tesk

$E = \frac{\hbar^2 k^2}{2m}$ ie $[E] = \frac{[\hbar]^2[k]^2}{[m]}$

(A) $\frac{1}{2}\hbar^2 k \left(\frac{dk}{dE}\right) \Rightarrow \frac{[\hbar]^2[k]^2}{[E]} = [m]$ ✓

(B) $\frac{\hbar^2 k}{\left(\frac{dk}{dE}\right)} \neq [m]$

(C) $[\hbar]^2 [k]^{4/3} [E]^{2/3} \neq [m]$

(D) $\frac{[\hbar]^2[k]^2}{[E]} = [m]$ ✓

(E) $\frac{[\hbar]^2[m]^2[k]^2}{(E)^2} \neq [m]$

…. 可選的有 A or D ，二選一吧 ~

這類似的題目剛好我 homework 有作過 (但我還不知道是怎麼導的… Orz , sorry)

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E(k)}{dk^2}$$

if $E = \frac{\hbar^2 k^2}{2m}$, then $m^* = m$

(free election)

\# 98

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

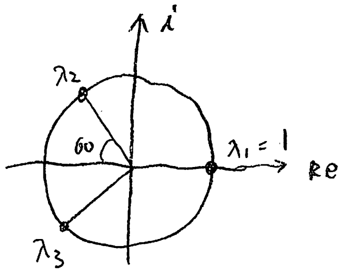
$$A v_i = \lambda_i v_i$$

by $\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$

then $\therefore$ $-\lambda \begin{vmatrix} -\lambda & 0 \\ 0 & -\lambda \end{vmatrix} - \begin{vmatrix} 0 & 1 \\ 1 & -\lambda \end{vmatrix} = 0$

$$-\lambda^3 + 1 = 0$$

$$\lambda = e^{\frac{2\pi i}{3} \cdot n} \qquad n = 0. 1. 2$$



$\lambda_1 = 1$

$\lambda_2 = -\cos 60 + i \sin 60 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

$\lambda_3 = -\cos 60 - i \sin 60 = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

所以 (B) 是錯的

99

$$E_n = \underbrace{E_n^{(0)}}_{\text{沒有 pertubation 時的解}} + \underbrace{E_n^{(1)}}_{\text{first order correction}} + E_n^{(2)} + \cdots$$

沒有 pertubation
時的解:
ie $\hat{H}_0|\psi_n^{(0)}\rangle = E_n^{(0)}|\psi_n^{(0)}\rangle$

經過一番推導

$$E_n^{(1)} = \langle \underbrace{\psi_n^{(0)}}_{\text{原本的 wave functin}} | \underbrace{\hat{H}'}_{\text{perturbation term}} | \psi_n^{(0)} \rangle$$

原本的
wave functin

perturbation term

$\hat{H}' = -eEr$ ps. $-\dfrac{\partial U}{\partial r} = F$

而 ground state $|\psi_0^{(0)}\rangle = A e^{-r/a}$

even function

there fore $A^2 \int_{-\infty}^{\infty} r\, e^{-2r/a} \cdot r^2 \sin\theta \, d\theta \, d\phi \, dr$

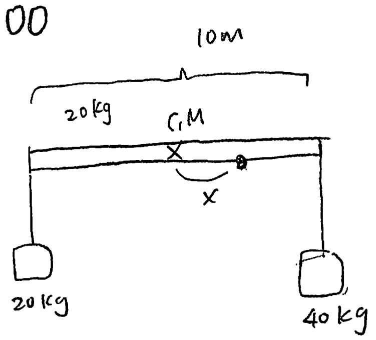
r^3 為 odd

$\Rightarrow$ 所以 積分 $= 0$

ie $E_n^{(1)} = 0$

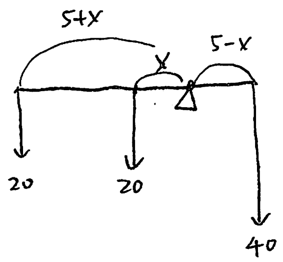
選 (A)

100



求 x

靜力平衡 $\Rightarrow \sum \vec{\tau}_i = 0$



hence:

$$20(5+x) + 20x = 40(5-x)$$

$$80x = 100$$

$$x = 1.25 \quad \cdots \text{選 (C)}$$