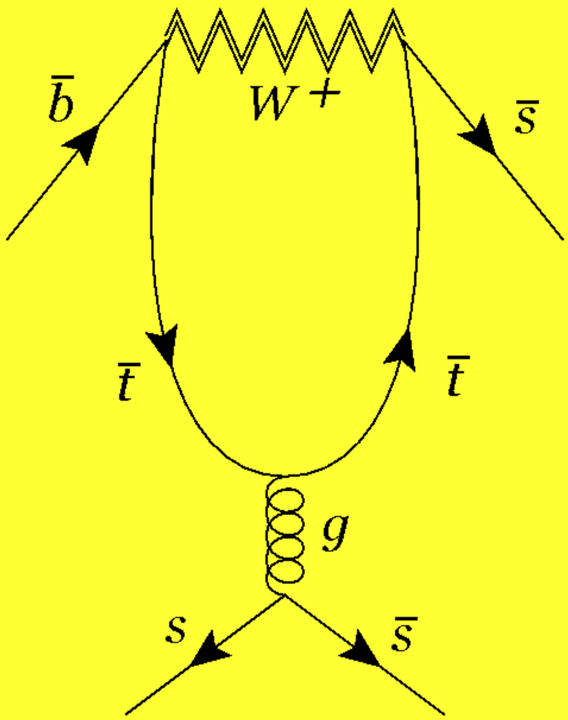
Complete Solutions to the Physics GRE



Exam #9677

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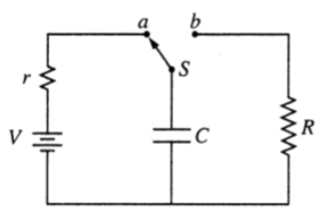


Figure 1

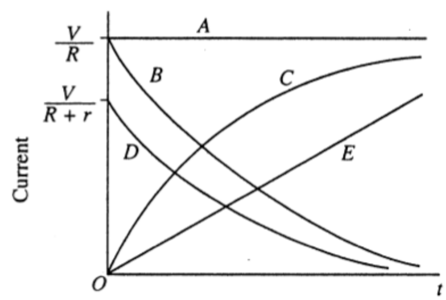


Figure 2

1. The capacitor shown in Figure 1 above is charged by connecting switch S to contact a. If switch S is thrown to contact b at time $t = 0$, which of the curves in Figure 2 above represents the magnitude of the current through the resistor R as a function of time?

(A) A
(B) B
(C) C
(D) D
(E) E

Recommended Solution

First, consider the state of this circuit before the switch, S, is changed from point a to point b. When connected to the potential, V, a constant current is passed through the capacitor. The capacitor will continue to gain a potential until the potential difference on the capacitor is equal to that of the potential. Once the switch is moved over to b, that potential is gone and what is left is the potential stored on the capacitor, which will proceed through the resistor, R. Unlike the potential, the capacitor won't be able to maintain a constant current so we would expect the current to decline as the resistor "resists" the current and energy dissipates. At this point you can cross off any curve on the plot which isn't decreasing over time.

Now, all you have to do is decide whether the initial potential energy provided by V will be V/R or $V/(R+r)$. Considering that resistor r is isolated from resistor R, it is reasonable to conclude that only resistor R will have an influence on the current.

Correct Answer
(B)

Alternate Solution

Using the same reasoning as in the "Recommended Solution", eliminate choices (A),(C) and (E) due to the fact that a current supported only by a charged capacitor must decline as it is forced to pass through a resistor. Again, we need to figure out what the initial current must be for the charged capacitor-resistor circuit. Recall Kirchhoff's first rule (also commonly known as the Loop Rule)

Kirchhoff's First Rule The sum of the changes in potential energy encountered in a complete traversal of any loop of a circuit must equal zero

Applying the Loop Rule to the second stage of our circuit (that is to say, once the capacitor has reached an equal potential to that of V and the switch, S, has been thrown to point b) we must account for two potentials summing to 0

$$V_{cap} + V_{res} = 0 \tag{1.1}$$
$$iR + q/C = 0 \tag{1.2}$$

R and C are both constants but i and q are functions of time and so with 1 equation and 2 variables, we are stuck. And yet, we are too industrious to leave things at that. We can relate the charge, q, to the current, i, because current is merely the change in current.

$$i = \frac{dq}{dt} \tag{1.3}$$

which gives us

$$\frac{dq}{dt}R - q/C = 0 \tag{1.4}$$

Now that the only variable is q, we can solve this ordinary homogenous differential equation. Doing so, we get

$$q = q_0 e^{-t/RC} \tag{1.5}$$

Where e is Euler's number, not an elemental charge number. Taking the derivative of q then gives us the current as

$$\frac{dq}{dt} = i = -\left(\frac{q_0}{RC}\right) e^{-t/RC} \tag{1.6}$$

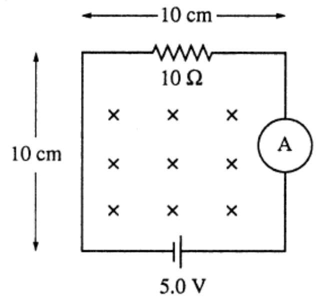
Setting $t = 0$ for the initial time and utilizing our equation for capacitance ($q_0 = CV_0$) we can find the initial current as

$$i_0 = -\left(\frac{CV_0}{RC}\right) e^{-0/RC} = -V_0/R \tag{1.7}$$

Which states that the initial current of the final stage of the circuit is V/R. We can ignore the negative on the initial current as it is merely indicating that the capacitors charge is decreasing.

Correct Answer
(B)

1.2 PGRE9677 #2



2. The circuit shown above is in a uniform magnetic field that is into the page and is decreasing in magnitude at the rate of 150 tesla/second. The ammeter reads

(A) 0.15 A
(B) 0.35 A
(C) 0.50 A
(D) 0.65 A
(E) 0.80 A

Recommended Solution

In this problem we are asked to determine the reading of an ammeter attached to a circuit with a potential (V) of 5.0 V, a resistor of 10 Ω while sitting in a uniform but changing magnetic field of 150 tesla/second. In this scenario we have to account for two different phenomena which are generating a current. The first phenomena generating a current is the 5.0 V potential which will create a current according to Ohm's Law

$$I = \frac{V}{R} \tag{1.8}$$

$$I = \frac{5.0 \text{ volts}}{10\ \Omega} = 0.5\ A \tag{1.9}$$

This current will be moving in a counterclockwise direction because the "positive flow of current" moves in the opposite direction of the electron flow. In other words, the current will be moving from the positive terminal to negative terminal (The larger bar of the potential V to the smaller bar of the same potential V).

The second source of current comes from the changing magnetic field being directed "into the page". We know that a changing magnetic field generates a current thanks to Maxwell's/Faraday's laws, and in particular we know from Faraday's law of induction that a changing magnetic flux will induce a potential (emf) ε

$$\varepsilon = \left|\frac{d\phi_B}{dt}\right| \tag{1.10}$$

where ϕ_B is

$$\phi_B = \int\int_S \vec{B}(\vec{r}, t) \cdot d\vec{A} \tag{1.11}$$

which simply states that the magnetic flux (ϕ_B) is equal to the magnetic field (B) through some total surface area (i.e. the surface integral over differential pieces of area dA)

Taking the surface integral for this simple loop of wire is the same as just calculating the area of the square with sides of 10 cm × 10 cm, or in more standard units, .10 m × .10 m which gives a total surface area of $A = .01$ m^2. This makes the flux equation

$$\phi_B = \vec{B}(\vec{r}, t) \cdot \vec{A} \tag{1.12}$$

Since the vector $\vec{A}$ is the vector perpendicular to the surface area and it is in the same direction as the magnetic field lines,

$$\vec{B} \cdot \vec{A} = B\ A \tag{1.13}$$

substituting that in for our equation calculating the emf (ε)

$$\varepsilon = \left|\frac{d(\phi_B)}{dt}\right| = \left|\frac{dB}{dt}\right| A \tag{1.14}$$

$$\varepsilon = \left(150\ \frac{\text{tesla}}{\text{sec}}\right)\left(0.01\ m^2\right) = 0.15\ A \tag{1.15}$$

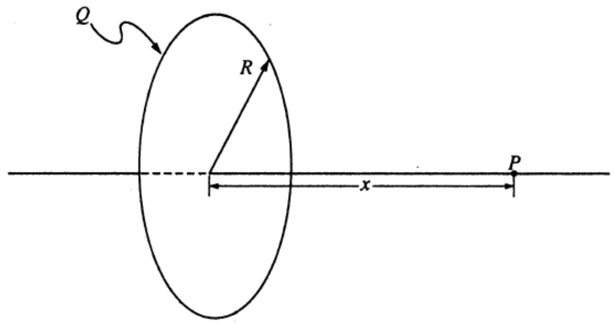
Finally, calculate the difference between the 2 potentials (Equations 1.9 and 1.15) we found and you have

$$0.5\ A - 0.15\ A = 0.35\ A \tag{1.16}$$

Correct Answer
(B)

1.3 PGRE9677 #3

Questions 3-4 refer to a thin, nonconducting ring of radius R, as shown below, which has a charge Q uniformly spread out on it.



3. The electric potential at a point P, which is located on the axis of symmetry a distance x from the center of the ring, is given by

(A) $\frac{Q}{4\pi\varepsilon_0 x}$

(B) $\frac{Q}{4\pi\varepsilon_0\sqrt{R^2 + x^2}}$

(C) $\frac{Qx}{4\pi\varepsilon_0(R^2 + x^2)}$

(D) $\frac{Qx}{4\pi\varepsilon_0(R^2 + x^2)^{3/2}}$

(E) $\frac{QR}{4\pi\varepsilon_0(R^2 + x^2)}$

Figure 1.1: Electric potential at point P in relation to ring of radius R

Recommended Solution

Point p in Figure 1.1 represents a "test charge" location and we want to know what the electric potential is at that point due to a uniformly charged ring with charge Q. Using Coulombs Law in Gaussian units, we have

$$U = \int_s \frac{1}{4\pi\epsilon_0}\frac{dQ}{r} \tag{1.17}$$

where r is the distance between any point on the ring and point P, and dQ is a differential piece of the ring. because the distance from every point on the ring to our point P is the same, r is a constant. We can pull all constants out from the integral, giving

$$U = \frac{1}{4r\pi\epsilon_0}\int_s dQ \tag{1.18}$$

$$U = \frac{Q}{4r\pi\epsilon_0} \tag{1.19}$$

Lastly, we need to solve for r. Drawing a line between the ring and P gives us a right triangle which allows us to use the Pythagorean theorem (Figure 1.2)

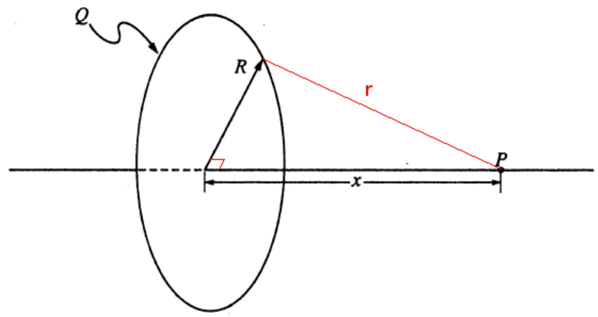


Figure 1.2: Constructing a right triangle between points P and R

$$r = \sqrt{R^2 + x^2} \tag{1.20}$$

and substituting Equation 1.20 and 1.19 gives us

$$U = \frac{Q}{4\pi\epsilon_0\sqrt{R^2 + x^2}} \tag{1.21}$$

Correct Answer
(B)

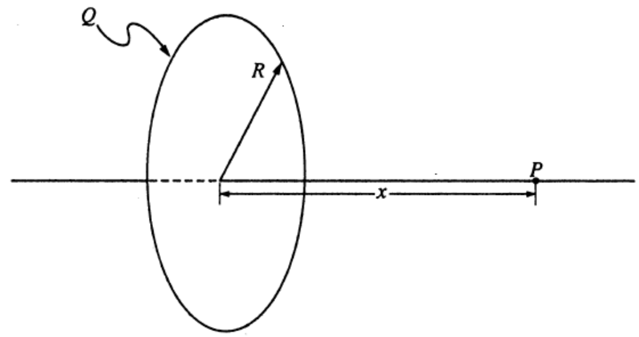
Alternate Solution

The trick to this alternate solution involves manipulating the variables involved. No specific R or x values are given, so the correct solution must work for any choice of R and x. For example, letting x go to infinity, we would expect the electric potential to go to 0. For solutions (C) and (D), letting x go to infinity would make the potential go to infinity. Similarly, if we let R go to infinity, again potential should go to zero. This isn't the case for solution (E), so cross it off. Finally, you just have to decide whether or not the distance between the ring (or rather differential pieces of the ring, dQ) and point P is in agreement with (A), in which $r = x$, or if it is in agreement with (B), in which $r = \sqrt{R^2 + x^2}$. By the Pythagorean theorem, the hypotenuse can't be the same length as one of its sides so $r \neq x$.

Correct Answer
(B)

1.4 PGRE9677 #4

Questions 3-4 refer to a thin, nonconducting ring of radius R, as shown below, which has a charge Q uniformly spread out on it.



4. A small particle of mass m and charge $-q$ is placed at point P and released. If $R >> x$, the particle will undergo oscillations along the axis of symmetry with an angular frequency that is equal to

(A) $\sqrt{\frac{qQ}{4\pi\varepsilon_0 mR^3}}$

(B) $\sqrt{\frac{qQx}{4\pi\varepsilon_0 mR^4}}$

(C) $\frac{qQ}{4\pi\varepsilon_0 mR^3}$

(D) $\frac{qQx}{4\pi\varepsilon_0 mR^4}$

(E) $\sqrt{\frac{qQx}{4\pi\varepsilon_0 m}\frac{1}{R^2+x^2}}$

Recommended Solution

To solve this problem we will need Coulomb's law, Hooke's Law and the general equation for angular frequency

Coulomb's Law: $F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$

Hooke's Law: $F = -kx$

Angular Frequency: $\omega = \sqrt{k/m}$

In our situation we have a charged surface, specifically a ring, so we will need to replace one of our point charges with this value, which can be found by integrating a differential piece of charge over the entire ring.

$$F = \frac{-q}{4\pi\epsilon_0 r^2}\int_S dQ \tag{1.22}$$

$$F = \frac{-qQ}{4\pi\epsilon_0 r^2} \tag{1.23}$$

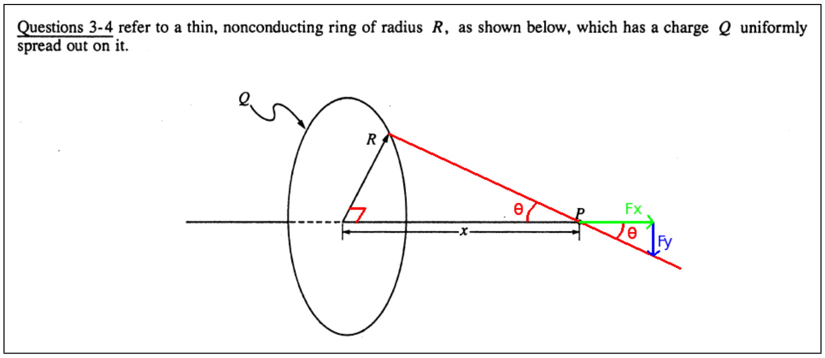


Figure 1.3: Component forces at point P

Equation 1.23 represents the equation for the net force. However, we will need to decompose the net force into its horizontal and vertical components. Fortunately, for every force in the vertical direction from one piece of the ring, the piece on the opposite side of the ring will cause an equal and opposite force.

The vertical forces cancel each other out and we are left with the horizontal forces. Hopefully it is clear from Figure 1.3 that the horizontal force component should be

$$F_x = Fcos(\theta) = F(x/R) \tag{1.24}$$

substitute 1.23 with 1.24 to get

$$F = \frac{-qQ}{4\pi\epsilon_0 r^2}\frac{x}{R} \tag{1.25}$$

Finally, we can calculate the distance between P and any point on the ring using the pythagorean theorem

$$r^2 = R^2 + x^2 \tag{1.26}$$

combining 1.26 with 1.25 gives us

$$F = \frac{-qQx}{4\pi\epsilon_0(R^2+x^2)R} \tag{1.27}$$

Now taking Hooke's law, solve for k to get

$$k = -F/x \tag{1.28}$$

and substitute our force equation into it. After simplifying, you should get

$$k = \frac{qQ}{4\pi\epsilon_0(R^2+x^2)R} \tag{1.29}$$

and then substitute this in for the k in our angular frequency equation and simplify to get

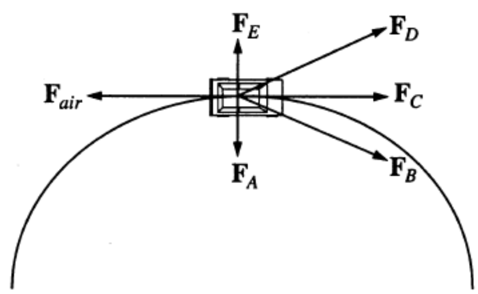
$$\omega = \sqrt{\frac{qQ}{4\pi\epsilon_0(R^2+x^2)Rm}} \tag{1.30}$$

Finally, take into account that the problem asked you to consider when $R >> x$, meaning x effectively goes to 0, giving

$$\omega = \sqrt{\frac{qQ}{4\pi\epsilon_0 mR^3}} \tag{1.31}$$

Correct Answer
(A)

1.5 PGRE9677 #5



5. A car travels with constant speed on a circular road on level ground. In the diagram above, $\mathbf{F}_{air}$ is the force of air resistance on the car. Which of the other forces shown best represents the horizontal force of the road on the car's tires?

(A) $\mathbf{F}_A$
(B) $\mathbf{F}_B$
(C) $\mathbf{F}_C$
(D) $\mathbf{F}_D$
(E) $\mathbf{F}_E$

Recommended Solution

To start off with, figure out every force that will be on this car as it travels through the arc. The problem identifies for us the force from air resistance, F_{air}. Additionally, we can identify that there will be a net centripetal force pointing towards the center of the arc. Keep in mind that the centripetal force is not one of the component forces but the sum total of all the forces. drawing out the force diagram gives us Figure 1.4

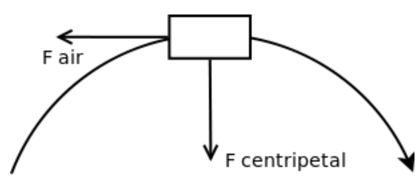


Figure 1.4: Net force diagram on a turning vehicle

Now we must ask ourselves which direction the horizontal force must point such that when it is added to F_{air}, we will get a net centripetal force pointing down. Check each of the 5 choices in Figure 1.5 and you will quickly see that only (B) can be correct.

Correct Answer
(B)

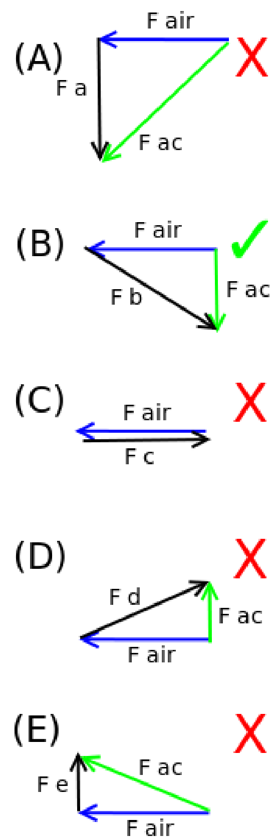


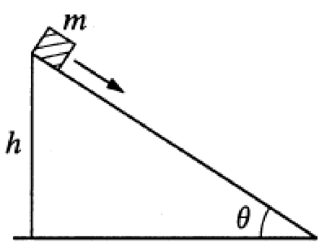
Figure 1.5: Potential force combinations

Alternate Solution

A quick and qualitative way of figuring out the solution is to consider why the car is turning at all. The force from air resistance cant be responsible for the motion and the centripetal force is merely a description of the net force (i.e. the sum of the component forces) acting on the car. As such, the force that the road is applying to our car tires must have some vertical component of force pointing down. You can eliminate choices (C), (D), and (E) based on this. Next, consider that the air resistance force has a horizontal component of force (technically it only has this horizontal force) and so for our net centripetal force to have no horizontal force, the allusive force we are finding must have an equal and opposite horizontal component. (A) has no horizontal component so cross it off.

Correct Answer
(B)

1.6 PGRE9677 #6



6. A block of mass m sliding down an incline at constant speed is initially at a height h above the ground, as shown in the figure above. The coefficient of kinetic friction between the mass and the incline is μ. If the mass continues to slide down the incline at a constant speed, how much energy is dissipated by friction by the time the mass reaches the bottom of the incline?

(A) mgh/μ
(B) mgh
(C) $\mu mgh/\sin\theta$
(D) $mgh\sin\theta$
(E) 0

Recommended Solution

This problem can be solved quickly by utilizing conservation of energy laws. At the top of the incline, the only energy for the system is gravitational potential energy

$$U_G = mgh \tag{1.32}$$

According to the description, the block slides down the incline at a constant speed. This means that the kinetic energy $U_k = \frac{1}{2}mv^2$, is the same at the beginning of the blocks motion as it is at the end and thus none of the potential energy we started with changed to kinetic energy. However, because energy must be conserved, all of the energy that didn't become kinetic energy (i.e. all mgh of it) had to have dissipated from friction between the incline and the block.

Correct Answer
(B)

Alternate Solution

Start by drawing out a force diagram for the block at the top of the slide (Figure 1.6)

take the sum of all of the forces in the x and y direction. Note that the acceleration in the x direction is 0 because the speed is constant, therefore the net force is 0.

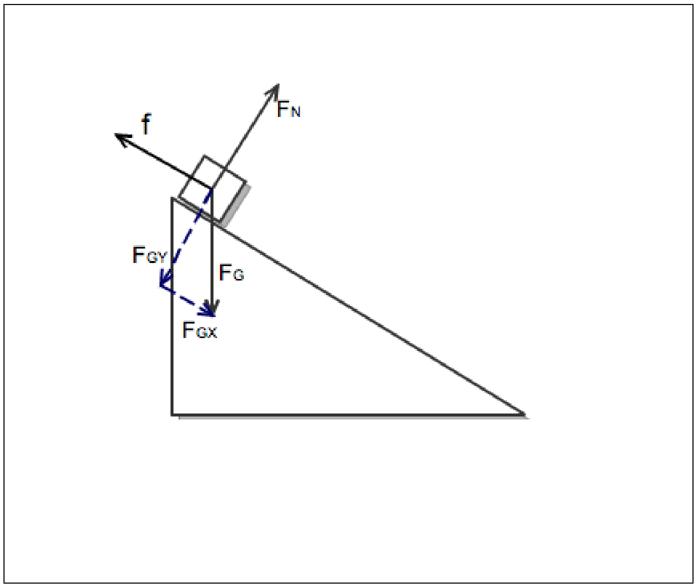


Figure 1.6: Force diagram of a block/incline system

$$\sum F_{net,x} = F_{GX} - f = 0 \tag{1.33}$$

$$\sum F_{net,y} = F_N - F_{GY} = 0 \tag{1.34}$$

Using trigonometry, you should see that $F_{GX} = F_G \sin(\theta)$ and $F_{GY} = F_G \cos(\theta)$. Since $F = mg$, $F_{GX} = mg \sin(\theta)$ and $F_{GY} = mg \cos(\theta)$. Additionally, $f = \mu F_N$ so we get

$$f = F_{GX} = \mu F_N = mg \sin(\theta) \tag{1.35}$$

and

$$F_N = mg \cos(\theta) \tag{1.36}$$

Combine Equations 1.35 and 1.36 and get

$$\mu mg \cos(\theta) = mg \sin(\theta) \tag{1.37}$$

$$\mu = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta) \tag{1.38}$$

Now, recall that work, which is equivalent in magnitude to energy, is

$$W = F(\Delta X) \tag{1.39}$$

and since we are concerned with the energy (work) generated from friction, the force in 1.39 must be f. Make the substitution to get

$$W_f = f(\Delta X) \tag{1.40}$$

$$W_f = mg \sin(\theta)(\Delta X) \tag{1.41}$$

where ΔX is the length of the ramp. Since $\sin(\theta) = h/\Delta X$, we know that

$$\Delta X = h/\sin(\theta) \tag{1.42}$$

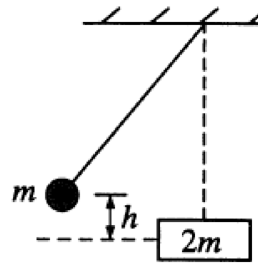
finally, combine 1.41 and 1.42

$$W_f = \frac{mg\sin(\theta)h}{\sin(\theta)} \tag{1.43}$$

$$W_f = mgh \tag{1.44}$$

Correct Answer
(B)

1.7 PGRE9677 #7



7. As shown above, a ball of mass m, suspended on the end of a wire, is released from height h and collides elastically, when it is at its lowest point, with a block of mass $2m$ at rest on a frictionless surface. After the collision, the ball rises to a final height equal to

(A) 1/9 h
(B) 1/8 h
(C) 1/3 h
(D) 1/2 h
(E) 2/3 h

Recommended Solution

Because the collision is elastic, we know that energy and momentum are conserved. From this, we know that the initial potential at height h will equal the kinetic energy immediately before the ball strikes the brick. Additionally, we know that the momentum of the ball/brick system must be the same before collision as after. This gives us the equations

$$\frac{1}{2}mV_{b,0}^2 = \frac{1}{2}mV_{b,f}^2 - mV_{B,f}^2 \tag{1.45}$$

and

$$mV_{b,0} = mV_{b,f} + 2mV_{B,f} \tag{1.46}$$

With two equations and three unknowns, we can get a relationship between any of the two variables. Combine 1.45 and 1.46 in order to get a relationship for $V_{b,0}$ with $V_{b,f}$ and $V_{b,0}$ with $V_{B,f}$.

$$\begin{aligned} V_{B,f} &= \frac{2}{3}V_{b,0} && (1.47) \\ V_{b,f} &= \frac{1}{3}V_{b,0} && (1.48) \end{aligned}$$

This tells us that the final velocity of the ball b is 1/3 of its initial velocity and the block B leaves the collision at 2/3 the initial velocity of the ball. Once the ball leaves at its velocity of

$V_{b,f}$, it will move up to its final height as the kinetic energy becomes potential energy. The kinetic energy is

$$U_f = \frac{1}{2}mV_{b,f}^2 = \frac{1}{2}m\left(\frac{1}{3}V_{b,0}\right)^2 \tag{1.49}$$

$$U_f = \frac{1}{2}m\left(\frac{1}{9}V_{b,0}^2\right) = \frac{1}{9}\left(\frac{1}{2}mV_{b,0}^2\right) \tag{1.50}$$

$$U_f = \frac{1}{9}mgh \tag{1.51}$$

Comparing the final energy to the intial, we get

$$mgh_f = \frac{1}{9}mgh_0 \tag{1.52}$$

$$h_f = \frac{1}{9}h_0 \tag{1.53}$$

Correct Answer
(A)

1.8 PGRE9677 #8

The quickest way to solve this problem is to recognize that the problem is describing a damped harmonic oscillator. The object, in this case a particle with mass m, oscillates with a damping force proportional to $F = -bv$ applied to the particle. We know that the force applied to the particle is fighting against the oscillation because it is always in the opposite direction of the velocity and this confirms that this is a damped harmonic oscillator. If the force was positive and adding to the force of the oscillation, then we would have a driven harmonic oscillator. Now, consider the period of damped harmonic oscillator in comparison to an unhindered SHO without the opposing force. Strictly speaking, a damped oscillator doesn't have a well-defined period and without knowing the specific values for mass, spring constant, etc we don't know whether we are talking about a system that is underdamped, over damped or critically damped. Nevertheless, we can see that all of the damped oscillations (Figure 1.7 will experience an increase in the period.

Correct Answer
(A)

8. A particle of mass m undergoes harmonic oscillation with period T_0. A force f proportional to the speed v of the particle, $f = -bv$, is introduced. If the particle continues to oscillate, the period with f acting is

(A) larger than T_0
(B) smaller than T_0
(C) independent of b
(D) dependent linearly on b
(E) constantly changing

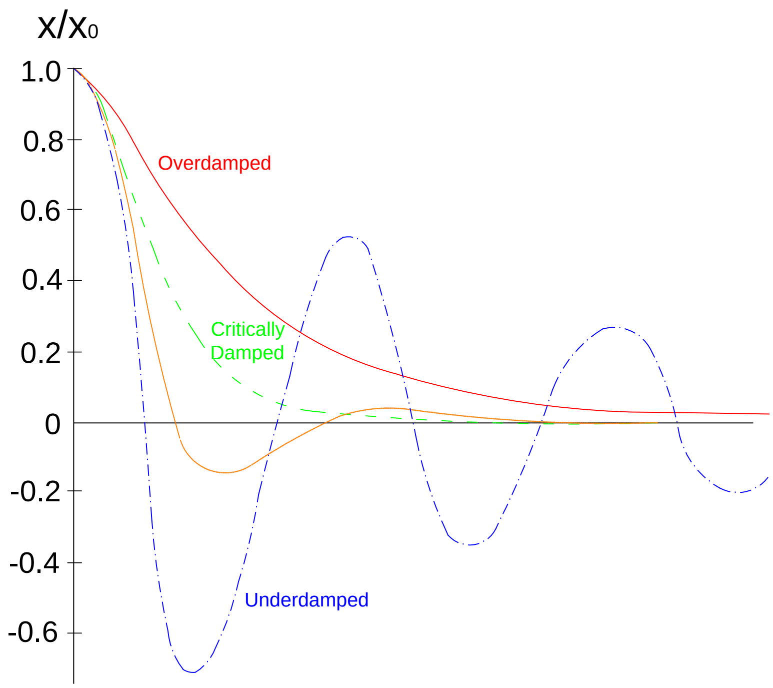
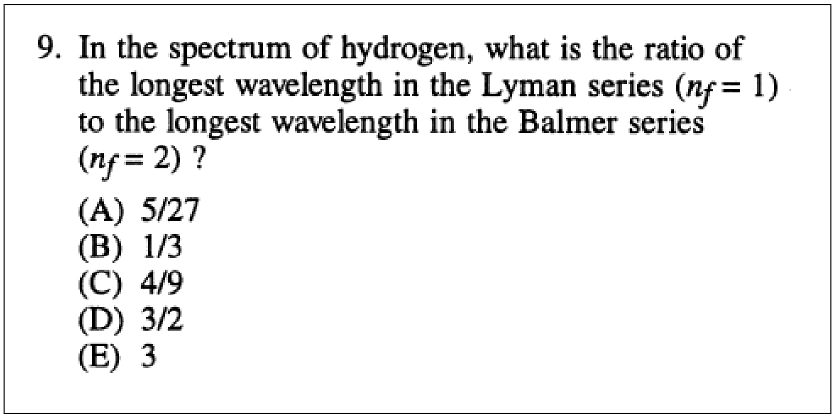


Figure 1.7: Damped harmonic oscillators

1.9 PGRE9677 #9



9. In the spectrum of hydrogen, what is the ratio of the longest wavelength in the Lyman series ($n_f = 1$) to the longest wavelength in the Balmer series ($n_f = 2$) ?

(A) 5/27
(B) 1/3
(C) 4/9
(D) 3/2
(E) 3

Recommended Solution

The Lyman and Balmer series both refer to different types of transitions of an electron in a hydrogen atom from one radial quantum level (n) to another. The Lyman series is a description of all such transitions from n=r to n=1, such that $r \geq 2$ and is an integer. The first Lyman transition (commonly called Lyman-alpha) is n=2 going to n=1, the second (Lyman-beta) involves a transition of n=3 to n=1, etc. The Balmer series, on the other hand, involves transitions from some n=s to n=2, such that $s \geq 3$ and is an integer. The longest wavelength for both series involves the smallest transition, i.e. n=2 going to n=1 for the Lyman Series and n=3 going to n=2 for the Balmer. The

Rydberg formula can then be used to find the wavelength for each of the two transitions

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{1.54}$$

For this problem we won't need to compute anything, just compare λ_L and λ_B. Doing this for the shortest Lyman transition gives

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \tag{1.55}$$

$$\frac{1}{\lambda_L} = \frac{3}{4}R \tag{1.56}$$

$$\lambda_L = 4/(3R) \tag{1.57}$$

and for the Balmer transition

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \tag{1.58}$$

$$\frac{1}{\lambda_B} = \frac{5}{36}R \tag{1.59}$$

$$\lambda_B = 36/(5R) \tag{1.60}$$

making the ratio

$$\lambda_L/\lambda_B = \frac{4/(3R)}{36/(5R)} = 5/27 \tag{1.61}$$

Correct Answer
(A)

1.10 PGRE9677 #10

10. Internal conversion is the process whereby an excited nucleus transfers its energy directly to one of the most tightly bound atomic electrons, causing the electron to be ejected from the atom and leaving the atom in an excited state. The most probable process after an internal conversion electron is ejected from an atom with a high atomic number is that the

(A) atom returns to its ground state through inelastic collisions with other atoms
(B) atom emits one or several x-rays
(C) nucleus emits a γ-ray
(D) nucleus emits an electron
(E) nucleus emits a positron

Recommended Solution

In internal conversion, one of the inner electrons of the molecule is ejected. Because of this, one of the outer electrons will drop down a level to fill the space and some form of electromagnetic radiation is released. From this, you can immediately remove (A), (D) and (E). Now you simply have to decide whether the electromagnetic radiation will correspond to the energy level of X-Rays or γ rays. As it turns out, this electronic transition will release X-rays. However, if you don't know this, you may be able to reason through to the answer. Consider that γ rays are the highest energy form of electromagnetic radiation and these will generally be a result of an energy transition for a nucleus. An electron transition, on the other hand, involves smaller exchanges of energy and correspond to the lower energy X-rays.

Correct Answer
(B)

1.11 PGRE9677 #11

11. A beam of neutral hydrogen atoms in their ground state is moving into the plane of this page and passes through a region of a strong inhomogeneous magnetic field that is directed upward in the plane of the page. After the beam passes through this field, a detector would find that it has been

(A) deflected upward
(B) deflected to the right
(C) undeviated
(D) split vertically into two beams
(E) split horizontally into three beams

Recommended Solution

In this problem, ETS is testing your knowledge of physics history. If you recall the Stern-Gerlach experiment (1922), this problem becomes quite easy. The Stern-Gerlach experiment involved firing neutral silver atoms through an inhomogenous magnetic field. The classical understanding (i.e. before Stern-Gerlach) would have suggested no deflection because the atoms are neutral in charge and have no orbital angular momentum and thus generate no magnetic dipole. However, this experiment showed that the beam split into two distinct beams, adding evidence to the ultimate conclusion that electrons have a spin property of 1/2. Keep in mind that, in general, the number of distinct beams generated after hitting the magnetic field will be equal to $2S+1$. Silver has a single unpaired electron and so $S = 1/2$, giving the original Stern-Gerlach experiment its two beams. In this problem, ETS went easy on us and gave us hydrogen to work with, ~~which also has $S = 1/2$ and thus will split into two beams~~.

Correct Answer
(D)

1.12 PGRE9677 #12

12. The ground-state energy of positronium is most nearly equal to

(A) −27.2 eV
(B) −13.6 eV
(C) −6.8 eV
(D) −3.4 eV
(E) 13.6 eV

Recommended Solution

When positronium questions pop up on the GRE, they generally can and should be solved by its relation to the hydrogen atom. In this case, you have to recall that the ground state energy of Hydrogen is equal to 1 Rydberg $= -13.6$ eV. Positronium involves an electron-positron pair while hydrogen involves a proton-electron pair. There is no difference between the two in terms of charge but there is a significant difference in mass. Since Rydberg's constant is mass dependent (Equation 1.62), we have to alter the original Rydberg constant

$$R_{hydrogen} = \frac{m_e m_p}{m_e + m_p} \frac{e^4}{8c\epsilon_0^2 h^3} \tag{1.62}$$

Which becomes,

$$\begin{aligned} R_{positronium} &= \frac{m_e m_e}{m_e + m_e} \frac{e^4}{8c\epsilon_0^2 h^3} &(1.63)\\ &= \frac{m_e}{2} \frac{e^4}{8c\epsilon_0^2 h^3} &(1.64) \end{aligned}$$

To convince yourself that this makes the Rydberg constant half as large, recall that the ratio of the proton mass to the electron mass is approximately 1836:1[1]. Calculating the effective mass with an electron and proton, we get

$$\frac{m_p m_e}{m_p + m_e} = \frac{1 * 1836}{1 + 1836} \approx 1 \tag{1.65}$$

Calculating the effective mass for the electron/positron pair, with their equivalent masses, gives us

$$\frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2} \tag{1.66}$$

Since the energy is proportional to the Rydberg constant, the ground state energy of positronium must be half of the hydrogen ground state energy

[1] It isn't necessary to know this quantity in order to arrive at this simplification. All that is really important is that the difference between the two masses is significant

$$E_{positronium} = \frac{E_{hydrogen}}{2} = -6.8eV \tag{1.67}$$

again, if you aren't convinced, consider the Rydberg equation for hydrogen

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{1.68}$$

and since $E = h\nu = \frac{hc}{\lambda}$

$$E = \frac{hc}{\lambda} = hc\frac{R}{2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{1.69}$$

Correct Answer
(C)

1.13 PGRE9677 #13

13. A 100-watt electric heating element is placed in a pan containing one liter of water. Although the heating element is on for a long time, the water, though close to boiling, does not boil. When the heating element is removed, approximately how long will it take the water to cool by 1° C ? (Assume that the specific heat for water is 4.2 kilojoules/kilogram °C.)

(A) 20 s
(B) 40 s
(C) 60 s
(D) 130 s
(E) 200 s

Recommended Solution

The problem gives us the specific heat of water as 4.2 kJ/kg which should be a strong hint that you will need to use the equations for heat absorption into a solid,

$$Q = cm\Delta T \tag{1.70}$$

We are also given the power of the heating element, meaning we know the amount of energy input. Additionally, because the problem states that the water never manages to boil, even if it comes close, the water must be outputting energy at an equal rate or at least very close to it. We then just need to know how long it will take for energy to dissipate from the system equivalent to a change in temperature of $1°C$. Using the definition of power,

$$P = W/\Delta t \tag{1.71}$$

Or equivalently

$$W = P\Delta t \tag{1.72}$$

We can then combine the two equations (Q=W) to get

$$cm\Delta T = P\Delta t \tag{1.73}$$

When making substitutions, keep in mind that 1 L of water in mass is 1 kg, giving

$$\left(4.2\,\frac{kJ}{kg}\right)(1\ kg)(1°C) = (100\ watts)(\Delta t) \tag{1.74}$$

We can get everything into the same units by converting watts to kJ. Specifically, 1 watt = 1 J/s = 0.001 so 100 watt = 100 J/s = 0.1 kJ/s.

$$4.2kJ = 0.1\frac{kJ}{s}\Delta t \tag{1.75}$$

Then solving for Δt gives

$$\Delta t \approx 40 \text{ sec} \tag{1.76}$$

Correct Answer
(C)

1.14 PGRE9677 #14

14. Two identical 1.0-kilogram blocks of copper metal, one initially at a temperature $T_1 = 0°$ C and the other initially at a temperature $T_2 = 100°$ C, are enclosed in a perfectly insulating container. The two blocks are initially separated. When the blocks are placed in contact, they come to equilibrium at a final temperature T_f. The amount of heat exchanged between the two blocks in this process is equal to which of the following? (The specific heat of copper metal is equal to 0.1 kilocalorie/kilogram °K.)

(A) 50 kcal
(B) 25 kcal
(C) 10 kcal
(D) 5 kcal
(E) 1 kcal

Recommended Solution

This is one of those rare "Plug-n-Chug" problems on the PGRE. Cherish it! In this problem we have 2 copper blocks in an insulated container. This tells us that the total energy of the system is conserved. Since both blocks are of the same mass, we know the final temperature of both blocks will reach equilibrium at 50 kcal each. Since heat travels from high temperatures to low temperatures, all energy transfer will take place from the block with $T_2 = 100°C$ to the block $T_1 = 0°C$ and so we only need to consider this one direction of energy transfer. Using our equation for heat absorption for a solid/liquid body, we can plug in all known values

$$Q = cm\Delta T \tag{1.77}$$
$$Q = (0.1kCal/kg\ K)(1kg)(50\ K) \tag{1.78}$$
$$Q = 5\ kCal \tag{1.79}$$

Correct Answer
(D)

Additional Note

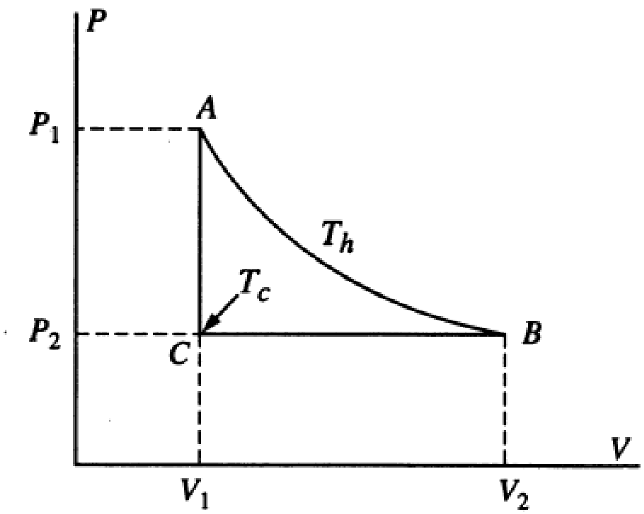
In this specific problem ETS has been very nice to us by making both blocks of the same mass. In the case that they gave us blocks of different masses, we wouldn't be able to easily conclude that the final temperature of each block would be at $50°C$. If, for example, all other values were the same but the block at $T_1 = 0°C$ had a mass of $m_1 = 1\ kg$ and the block at $T_2 = 100°C$ had a mass of $m_2 = 2\ kg$, then we would have to solve for the final temperature. To do so, consider that the system is insulated so no energy can leave. This means that the total energy transfer will be 0 and the sum of the energy transfers between the two blocks will sum to 0. This gives us

$$Q_1 + Q_2 = cm_1(T_f - T_{i,1}) + cm_2(T_f - T_{i,2}) = 0 \tag{1.80}$$

c is the same for both so cancel it out and solve for T_f,

$$T_f = 33.3^\circ C \tag{1.81}$$

1.15 PGRE9677 #15



15. Suppose one mole of an ideal gas undergoes the reversible cycle $ABCA$ shown in the P-V diagram above, where AB is an isotherm. The molar heat capacities are C_p at constant pressure and C_v at constant volume. The net heat added to the gas during the cycle is equal to

(A) RT_hV_2/V_1

(B) $-C_p(T_h-T_c)$

(C) $C_v(T_h-T_c)$

(D) $RT_h\ln V_2/V_1 - C_p(T_h-T_c)$

(E) $RT_h\ln V_2/V_1 - R(T_h-T_c)$

Recommended Solution

The thermodynamic process goes through its entire cycle, so energy is conserved, $\Delta U = 0$. From this and the first law of thermodynamics, $\Delta Q = \Delta U + \Delta W$, we know that the total heat will just be the sum of its work terms.

$$\Delta Q_{net} = \Delta W_{net} = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA} \tag{1.82}$$

The work equation for a thermodynamic system is $W = \int PdV$ and we have the ideal gas law, $PV = nRT$, so

$$W_{AB} = \int_{V_1}^{V_2} PdV \tag{1.83}$$

$$= \int_{V_1}^{V_2} \frac{nRT}{V} dV \tag{1.84}$$

$$= nRTln(\frac{V_2}{V_1}) \tag{1.85}$$

Next, moving from B to C, we get

$$W_{BC} = P\Delta V = nR\Delta T \tag{1.86}$$

Finally, taking C to A, there is no change in volume so we would expect to get a work of 0, i.e.

$$W_{CA} = P\Delta V = P(0) = 0 \tag{1.87}$$

Adding up all of the components (Equations 1.85, 1.86 and 1.87), we get

$$W_{net} = W_{AB} + W_{BC} + W_{CA} \tag{1.88}$$

$$= nRTln(V_2/V_1) + nR\Delta T + 0 \tag{1.89}$$

Since the problem specifies we have one mole of gas, Equation 1.89 becomes

$$W_{net} = RTln(V_2/V_1) + R\Delta T \tag{1.90}$$

Finally, consider the work equation for path BC to realize that $\Delta T = (T_c - T_h)$. Reversing the two heats, as we see in all of the possible solutions, will result in a negative sign coming out, giving a final result of

$$W_{net} = RTln(V_2/V_1) - R(T_h - T_c) \tag{1.91}$$

Correct Answer
(E)

1.16 PGRE9677 #16

16. The mean free path for the molecules of a gas is approximately given by $\frac{1}{\eta\sigma}$, where η is the number density and σ is the collision cross section. The mean free path for air molecules at room conditions is approximately

(A) 10^{-4} m
(B) 10^{-7} m
(C) 10^{-10} m
(D) 10^{-13} m
(E) 10^{-16} m

Recommended Solution

The problem gives us the equation for the mean free path as $\frac{1}{\eta\sigma}$. To get the density of air, use the ideal gas law and use the gas constant $R = 8.2 \times 10^{-5} \frac{\text{m}^3 \text{ Atm}}{\text{K Mol}}$

$$PV = nRT \tag{1.92}$$
$$\frac{n}{V} = \frac{P}{RT} \tag{1.93}$$

Which should give you

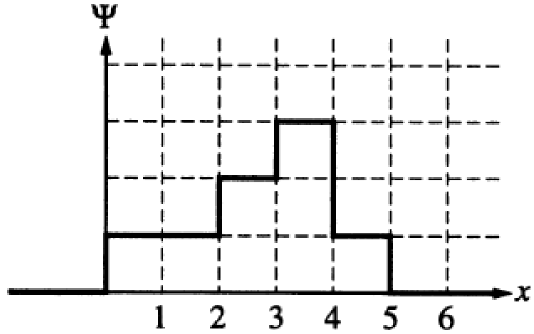
$$\frac{6 \times 10^{23}}{2.4 \times 10^{-2}} \frac{\text{Mol}}{\text{m}^3} = 3 \times 10^{25} \frac{\text{Mol}}{\text{m}^3} \tag{1.94}$$

Approximate the size of any given air molecule as being about 1 nm so the collision cross section is 1 nm^2 which is 1×10^{-18} m^2. Put this into the mean free path equation provided and you have

$$\frac{1}{\eta\sigma} = \frac{1}{(3 \times 10^{25} \text{ Mol/m}^3)(1 \times 10^{-18} \text{ m}^2)} \approx 1 \times 10^{-7} \tag{1.95}$$

Correct Answer
(B)

1.17 PGRE9677 #17



17. The wave function for a particle constrained to move in one dimension is shown in the graph above ($\Psi = 0$ for $x \leq 0$ and $x \geq 5$). What is the probability that the particle would be found between $x = 2$ and $x = 4$?

(A) 17/64

(B) 25/64

(C) 5/8

(D) $\sqrt{5/8}$

(E) 13/16

Recommended Solution

Recall from quantum mechanics that we can get the probability of finding a particle in any position by taking the integral of the squared wave function,

$$P_{ab} = \int_a^b |\Psi(x)|^2 dx \tag{1.96}$$

The integral of a curve is just the area underneath it and since a plot of the function is provided, we can quickly find the area. However, be aware that the curve represented is Ψ, and we want the area under the curve for Ψ^2. For this reason, we must square every piece of the plot and then take the area under the curve. Since we are concerned with the probability of the particle being located between $x = 2$ to $x = 4$, we need to compare that with the total area of the squared wave function. Doing so, from left to right, we have

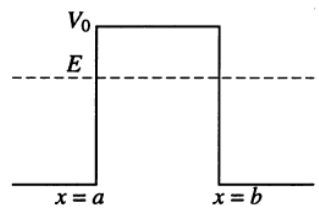
$$Area_{2\to4} = (2)^2 + (3)^2 = 13 \tag{1.97}$$

$$Area_{0\to6} = (1)^2 + (1)^2 + (2)^2 + (3)^2 + (1)^2 + (0)^2 = 16 \tag{1.98}$$

thus the probability is 13/16

Correct Answer
(E)

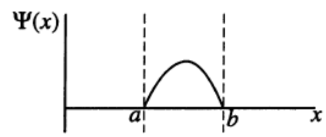
1.18 PGRE9677 #18

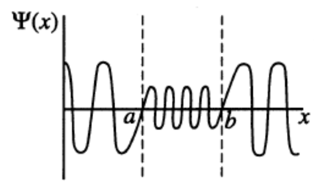


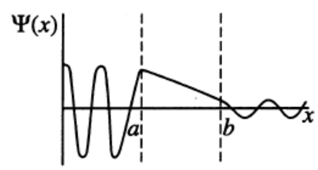
18. Consider a potential of the form

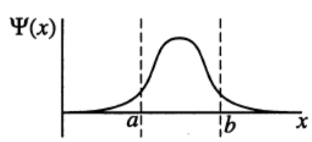
$$V(x) = 0, \ x \le a$$
$$V(x) = V_0, \ a < x < b$$
$$V(x) = 0, \ x \ge b$$

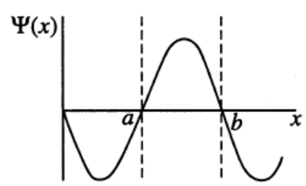
as shown in the figure above. Which of the following wave functions is possible for a particle incident from the left with energy $E < V_0$?

(A)


(B)


(C)


(D)


(E)


Recommended Solution

Recall the "infinite square well" that everybody does as their first, and frequently only, exactly solvable quantum system. You should recall that the wave function with infinite potential barriers on each side restricted the wave function to the area between the potential walls, with the exception of a small bit of tunneling on each side of the infinite potentials. This should tell you that no matter what the wave function looks like, it better have an amplitude that is lessened by interacting with a greater potential wall than it would in the open space. From this, eliminate (A), (D) and (E). Now, you just need to decide if the the wavefunction will be able to maintain its amplitude, frequency etc as its trying to tunnel through the wall. Again, you should recall from the infinite square well that this wasn't the case. Instead, the amplitude of your wave function continually dropped and approached $\Psi(x) = 0$. (C) shows this characteristic drop but (B) does not.

Correct Answer
(C)

1.19 PGRE9677 #19

19. When alpha particles are directed onto atoms in a thin metal foil, some make very close collisions with the nuclei of the atoms and are scattered at large angles. If an alpha particle with an initial kinetic energy of 5 MeV happens to be scattered through an angle of 180°, which of the following must have been its distance of closest approach to the scattering nucleus? (Assume that the metal foil is made of silver, with $Z = 50$.)

(A) $1.22 \times 50^{1/3}$ fm
(B) 2.9×10^{-14} m
(C) 1.0×10^{-12} m
(D) 3.0×10^{-8} m
(E) 1.7×10^{-7} m

Recommended Solution

This problem is a simpler case of Ruterford scattering. We are told that the scattering angle is 180° which is the maximum possible scattering angle. This means that the alpha particle is contacting the silver atom head on and all of the kinetic energy of the electron is becoming potential energy, resulting in Equation 1.99

$$\frac{1}{2}mv^2 = 5\ MeV = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r} \tag{1.99}$$

ϵ_0 is given in the list of constants in the beginning of the test. q for an alpha particle is $2(1.6 \times 10^{-19})$ C and q for silver is $50(1.6 \times 10^{-19})$ C. Converting the known kinetic energy into more convenient units gives, $5\ MeV \approx 8 \times 10^{-13}$ J and plugging everything in and solving for r, you get

$$r = \frac{2(1.6 \times 10^{-19}\text{ C})\ 50(1.6 \times 10^{-19}\text{ C})}{4\pi(8.85 \times 10^{-12})(8 \times 10^{-13}\text{ J})} \approx 2.9 \times 10^{-14}\text{ m} \tag{1.100}$$

Correct Answer
(B)

1.20 PGRE9677 #20

20. A helium atom, mass 4u, travels with nonrelativistic speed v normal to the surface of a certain material, makes an elastic collision with an (essentially free) surface atom, and leaves in the opposite direction with speed $0.6\,v$. The atom on the surface must be an atom of

(A) hydrogen, mass 1u
(B) helium, mass 4u
(C) carbon, mass 12u
(D) oxygen, mass 16u
(E) silicon, mass 28u

Recommended Solution

In this problem we are told that an elastic collision occurs, which tells us that energy and momentum will be conserved. From this, write the equation for each.

Momentum: $P_{total} = mV_0 = -mV_1 + \mu V_2$

Energy: $E_{total} = \frac{1}{2}mV_0^2 = \frac{1}{2}mV_1^2 + \frac{1}{2}\mu V_2^2$

Substitute in $0.6V_0 = V_1$ and simplify to get the momentum equation

$$mV_0 = -0.6\ mV_0 + \mu V_2 \tag{1.101}$$
$$1.6\ mV_0 = \mu V_2 \tag{1.102}$$
$$\left(\frac{1.6\ mV_0}{\mu}\right) = V_2 \tag{1.103}$$

and the energy equation

$$mV_0^2 = 0.6^2\ mV_0^2 + \mu V_2^2 \tag{1.104}$$
$$0.64\ mV_0^2 = \mu V_2^2 \tag{1.105}$$
$$\left(\frac{0.64\ mV_0^2}{\mu}\right) = V_2^2 \tag{1.106}$$

For your momentum equation, square both sides and set the resulting equation equal to your energy equation

$$\frac{2.56\ m^2V_0^2}{\mu^2} = \frac{0.64\ mV_0^2}{\mu} \tag{1.107}$$

Now simplify and you are left with only m and μ. Solving should give you

$$\mu = 4\ m \tag{1.108}$$

and since $m = 4\ u$,

$$\mu = 16\ u \tag{1.109}$$

Correct Answer
(D)

Additional Note

The method by which you solve this problem is identical to that used in problem #7 (See Section 1.7) on this same test (PGRE9677). It's all a matter of recognizing that energy and momentum are conservered in an elastic collision.

1.21 PGRE9677 #21

21. The period of a physical pendulum is $2\pi\sqrt{I/mgd}$, where I is the moment of inertia about the pivot point and d is the distance from the pivot to the center of mass. A circular hoop hangs from a nail on a barn wall. The mass of the hoop is 3 kilograms and its radius is 20 centimeters. If it is displaced slightly by a passing breeze, what is the period of the resulting oscillations?

(A) 0.63 s
(B) 1.0 s
(C) 1.3 s
(D) 1.8 s
(E) 2.1 s

Recommended Solution

From the Parallel-Axis theorem, we know that the moment of inertia of any object is equal to the sum of the objects inertia through its center of mass and Mh^2

$$I = I_{com} + Mh^2 = MR^2 + MR^2 = 2MR^2 \tag{1.110}$$

plug the provided values into Equation 1.110 to get

$$T = 2\pi\sqrt{\frac{2MR^2}{MgR}} = 2\pi\sqrt{\frac{2R}{g}} \tag{1.111}$$

$$T = 2\pi\sqrt{\frac{2(0.2\ cm)}{10\ m/s^2}} \approx 1.2 \tag{1.112}$$

Correct Answer
(C)

1.22 PGRE9677 #22

22. The curvature of Mars is such that its surface drops a vertical distance of 2.0 meters for every 3600 meters tangent to the surface. In addition, the gravitational acceleration near its surface is 0.4 times that near the surface of Earth. What is the speed a golf ball would need to orbit Mars near the surface, ignoring the effects of air resistance?

(A) 0.9 km/s
(B) 1.8 km/s
(C) 3.6 km/s
(D) 4.5 km/s
(E) 5.4 km/s

Recommended Solution

The problem tells us that the golf ball is orbiting mars, which tells us that the height of the golf ball relative to the surface of the planet is constant. The easiest way to deal with the provided information is to utilize the kinetmatic equation

$$y - y_0 = v_0 t - \frac{1}{2}at^2 \tag{1.113}$$

We can assume the initial velocity is 0 and the vertical change is 2 m, giving

$$2\ m = -\frac{1}{2}(-0.4g)t^2 \tag{1.114}$$

Solving for t to figure out how much time passes for the orbital motion gives

$$t = \sqrt{4\ m/0.4g} \approx 1 \text{ sec} \tag{1.115}$$

Since velocity is change in position over time, which is 1 second, we get a final velocity of

$$v = \frac{\Delta X}{1 \text{ sec}} = 3600 \text{ m/s} = 3.6 \text{ km/s} \tag{1.116}$$

Correct Answer
(C)

1.23 PGRE9677 #23

23. Suppose that the gravitational force law between two massive objects were $\mathbf{F}_{12} = \hat{\mathbf{r}}_{12} G m_1 m_2 / r_{12}^{2+\epsilon}$, where ϵ is a small positive number. Which of the following statements would be FALSE?

(A) The total mechanical energy of the planet-Sun system would be conserved.
(B) The angular momentum of a single planet moving about the Sun would be conserved.
(C) The periods of planets in circular orbits would be proportional to the $(3 + \epsilon)/2$ power of their respective orbital radii.
(D) A single planet could move in a stationary noncircular elliptical orbit about the Sun.
(E) A single planet could move in a stationary circular orbit about the Sun.

Recommended Solution

The problem specifies that ϵ, what ever value it might be, is small. Assume that the value of epsilon is so small as to be effectively irrelevant. Under this condition, look for statements which conform to known orbital phenomena.

(A) There's no reason to assume that energy isn't conserved in this scenario. Mechanical energy (Work) is $W = \int F \cdot ds$ and this won't interfere with the conservation of energy. Additionally, we should only expect conservation of energy to fail if there is some fricitional force applied to the object and the problem never mentions such a force.

(B) Angular momentum is conserved as long as there are no external torque on the system. ϵ wouldn't impose an external torque.

(C) This is consistent with Kepler's third law of planetary motion (i.e. $P^2 \propto a^3$)

(D) Noncircular orbits are rare and from process of elimination (see (E)) we can see that this is the only false proposition

(E) Circular type orbits are relatively common and so we should expect this to be true.

Correct Answer
(D)

1.24 PGRE9677 #24

24. Two identical conducting spheres, A and B, carry equal charge. They are initially separated by a distance much larger than their diameters, and the force between them is F. A third identical conducting sphere, C, is uncharged. Sphere C is first touched to A, then to B, and then removed. As a result, the force between A and B is equal to

(A) 0
(B) $F/16$
(C) $F/4$
(D) $3F/8$
(E) $F/2$

Recommended Solution

This problem can be solved with nothing more than Coulomb's law,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} \tag{1.117}$$

We start with two spheres, each with a charge of q. When the uncharged sphere touches sphere A, electrons are passed to the uncharged sphere until they each reach equilibrium. In this case, equilibrium involves half of the charge ending up on each sphere. Now, the initially uncharged sphere has a charge of $\frac{1}{2}q$ and sphere B has a charge of $1q$. When these two come in to contact, they equilibrate again. The average for these two charges is

$$\frac{(\frac{1}{2}+1)}{2}\,q = 3/4q \tag{1.118}$$

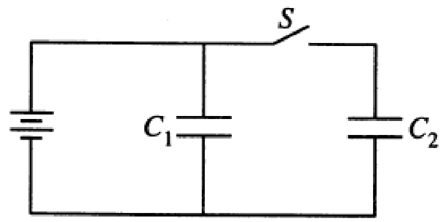
The initially uncharged sphere leaves itself and sphere B having a charge of $3/4\ q$. Knowing the charge for sphere A and sphere B, plug this into the equation for Coulomb's Law to get

$$\frac{1}{4\pi\epsilon_0}\frac{(1/2\ q)(3/4\ q)}{r^2} = \frac{3}{8}\left(\frac{1}{4\pi\epsilon_0}\frac{q_A q_B}{r^2}\right) \tag{1.119}$$

i.e. the force is now 3/8 of its original charge.

Correct Answer
(D)

1.25 PGRE9677 #25



25. Two real capacitors of equal capacitance ($C_1 = C_2$) are shown in the figure above. Initially, while the switch S is open, one of the capacitors is uncharged and the other carries charge Q_0. The energy stored in the charged capacitor is U_0. Sometime after the switch is closed, the capacitors C_1 and C_2 carry charges Q_1 and Q_2, respectively; the voltages across the capacitors are V_1 and V_2; and the energies stored in the capacitors are U_1 and U_2. Which of the following statements is INCORRECT?

(A) $Q_0 = \frac{1}{2}(Q_1 + Q_2)$

(B) $Q_1 = Q_2$

(C) $V_1 = V_2$

(D) $U_1 = U_2$

(E) $U_0 = U_1 + U_2$

Recommended Solution

Generally, I would recommend going through each possible solution to make sure you are finding the "best solution". However, in this problem one of the options stood out as being clearly false. Before the switch is thrown, current flows through the path of C_1 and charges it to energy U_0. After the switch has been thrown, C_1 will still charge to energy U_0 so we know $U_0 = U_1$. Without knowing, U_2, we can see that (E) violates this result (unless $U_2 = 0$ which is clearly not the case). I'd also like to point out that (D) and (E) disagree with one another (again, unless one of the energies is 0) so you can immediately determine that your answer must be one of these two.

Correct Answer
(E)

Alternate Solution

Before the switch is thrown, current is flowing through the path of C_1 and charges it to U_0 which means that the initial capacitance referred to in the problem is $C_0 = C_2$. Also, from the problem,

$C_1 = C_2$ so all three capacitors are equivalent. Additionally, all potentials are equal in a parallel circuit (i.e. $V_0 = V_1 = V_2$), giving $Q_0/V = Q_1/V = Q_2/V$

(A) $Q_0 = Q_1 = Q_2$ (which will be shown in part (B)) and half of the sum of two identical things will equal itself.

(B) Since $Q_0/V = Q_1/V = Q_2/V$, multiply the V out and get $Q_0 = Q_1 = Q_2$

(C) As was mentioned previously, the potential in a parallel circuit is always equivalent across all capacitors in the circuit. $V_0 = V_1 = V_2$

(D) Since all capacitors have the same capacitance and the same voltage, by $U = \frac{1}{2}CV^2$, we get $U_0 = U_1 = U_2$

(E) As was demonstrated in (D), $U_0 = U_1 = U_2$ so it has to be the case that $U_1 + U_2 = 2U_0$. (E) is false.

Correct Answer
(E)

1.26 PGRE9677 #26

26. A series RLC circuit is used in a radio to tune to an FM station broadcasting at 103.7 MHz. The resistance in the circuit is 10 ohms and the inductance is 2.0 microhenries. What is the best estimate of the capacitance that should be used?

(A) 200 pF
(B) 50 pF
(C) 1 pF
(D) 0.2 pF
(E) 0.02 pF

Recommended Solution

If you know the equation for frequency in an RLC circuit, then this problem is relatively straightforward. This question is primarily testing your ability to do unit conversions and multiply very large and/or very small numbers. The equation for the resonance frequency in an RLC circuit is

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{1.120}$$

$$f^2 = \frac{1}{4\pi^2 LC} \tag{1.121}$$

We know that the final frequency should be 103.7 MHz which we can simplify as 100 MHz. Squaring this value gives us 1.0×10^4 MHz. Converting this to hz, we get 1×10^{10} hz. The inductance is 2.0 microhenries which is equivalent to $2.0\ \Omega \cdot s$. Rearranging the previous equation to solve for capacitance, gives

$$C = \frac{1}{4\ \pi^2(2.0\ \Omega \cdot s)(1.0 \times 10^{10}\ hz)} \tag{1.122}$$

To make things easier, let's set $\pi^2 = 10$, which makes $4\pi^2 = 40$. Substituting everything into Equation 1.122,

$$C = \frac{1}{8 \times 10^{11}} \text{ Farads} \tag{1.123}$$

Finally, convert to pF by recalling that 1×10^{12} pF = 1 F, giving a final value of $C = 0.8$ pF which is closest to (C) 1 pF.

Correct Answer
(C)

1.27 PGRE9677 #27

27. In laboratory experiments, graphs are employed to determine how one measured variable depends on another. These graphs generally fall into three categories: linear, semilog (logarithmic *versus* linear), and log-log. Which type of graph listed in the third column below would NOT be the best for plotting data to test the relationship given in the first and second columns?

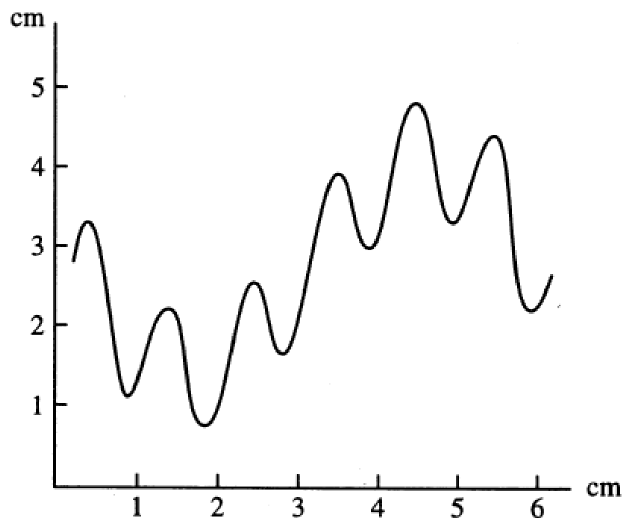
	Relation	Variables Plotted	Type of Graph
(A)	$dN/dt \propto e^{-2t}$	Activity *vs*. time for a radioactive isotope	Semilog
(B)	$eV_s = hf - W$	Stopping potential *vs*. frequency for the photoelectric effect	Linear
(C)	$s \propto t^2$	Distance *vs*. time for an object undergoing constant acceleration	Log-log
(D)	$V_{out}/V_{in} \propto 1/\omega$	Gain *vs*. frequency for a low-pass filter	Linear
(E)	$P \propto T^4$	Power radiated *vs*. temperature for blackbody radiation	Log-log

Recommended Solution

Logarithmic scaling is great for exponential curves and inverse curves (i.e. negative exponentials). From this, you know that (A), (C), (D) and (E) all should be either log-log or semilog. (A), (C), and (E) fulfill this requirement but (D) doesn't. Just to be sure that (D) is the "Best Solution" check (B) to see that we would expect it to utilize a linear graph just as the solution suggests.

Correct Answer
(D)

1.28 PGRE9677 #28



28. The figure above represents the trace on the screen of a cathode ray oscilloscope. The screen is graduated in centimeters. The spot on the screen moves horizontally with a constant speed of 0.5 centimeter/millisecond, and the vertical scale is 2 volts/centimeter. The signal is a superposition of two oscillations. Which of the following are most nearly the observed amplitude and frequency of these two oscillations?

	Oscillation 1	Oscillation 2
(A)	5V, 250Hz	2.5V, 1000Hz
(B)	1.5V, 250Hz	3V, 1500Hz
(C)	5V, 6Hz	2V, 2Hz
(D)	2.5V, 83Hz	1.25V, 500Hz
(E)	6.14V, 98Hz	1.35V, 257Hz

Recommended Solution

The wave on this oscilloscope clearly displays the sum of two separate waves with different frequencies. The wave with the larger frequency has a wavelength of about 6 cm. The speed of the wave is given as $v = 0.5$ cm/ms. Using the relationship of a linearly traveling wave through a homogeneous medium, we can calculate the frequency

$$\lambda = \frac{v}{f} \tag{1.124}$$

$$f = \frac{v}{\lambda} \tag{1.125}$$

Plug in the values from the problem into Equation 1.125

$$f = \frac{(0.5\text{ cm/ms})}{(6\text{ cm})} = \frac{1}{12\text{ ms}} \tag{1.126}$$

However, we want the frequency in Hz, so convert Equation 1.126 to get

$$f = \frac{1}{12\text{ ms}}\left(\frac{1000\text{ ms}}{1\text{ s}}\right) = 83\text{ Hz} \tag{1.127}$$

Which agrees with option (D).

Correct Answer
(D)

1.29 PGRE9677 #29

29. The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the physical constants G, $\hbar$, and c. Which of the following correctly gives the Planck length?

(A) $G\hbar c$

(B) $G\hbar^2 c^3$

(C) $G^2\hbar c$

(D) $G^{\frac{1}{2}}\hbar^2 c$

(E) $(G\hbar/c^3)^{\frac{1}{2}}$

Recommended Solution

The units for the Planck length should be a length (probably meters). The units for the three constants are

Gravitational Constant: $G \equiv \frac{m^3}{kg \cdot s^2}$

Reduced Planck's Constant: $\hbar \equiv \frac{m^2 \cdot kg}{s}$

Speed of Light: $c \equiv \frac{m}{s}$

We know that there can't be a unit of seconds in the final result and since all instances of seconds shows up as a denominator, we know at some point we will have to do some division. This only happens in option (E) .

Correct Answer
(E)

Alternate Solution

Using the units for the three constants, multiply out each one to figure out which of these gives us a result in meters

(A) $G\hbar c = \left(\frac{m^3}{kg \cdot s^2}\right)\left(\frac{m^2 \cdot kg}{s}\right)\left(\frac{m}{s}\right) = \frac{m^6}{s^4}$

(B) $G\hbar^2 c^3 = \left(\frac{m^3}{kg \cdot s^2}\right)\left(\frac{m^2 \cdot kg}{s}\right)^2\left(\frac{m}{s}\right)^3 = \frac{m^{10} \cdot kg}{s^7}$

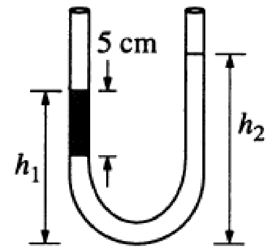
(C) $G^2\hbar c = \left(\frac{m^3}{kg \cdot s^2}\right)^2\left(\frac{m^2 \cdot kg}{s}\right)\left(\frac{m}{s}\right) = \frac{m^9}{kg \cdot s^6}$

(D) $G^{1/2}\hbar^2 c = \left(\frac{m^3}{kg\cdot s^2}\right)^{1/2}\left(\frac{m^2\cdot kg}{s}\right)^2\left(\frac{m}{s}\right) = \frac{m^{13/2}kg^{3/2}}{s^4}$

(E) $\left(G\hbar/c^3\right)^{1/2} = \sqrt{\left(\frac{m^3}{kg\cdot s^2}\right)\left(\frac{m^2\cdot kg}{s}\right)/\left(\frac{m}{s}\right)^3} = m$

Correct Answer
(E)

1.30 PGRE9677 #30



30. An open-ended U-tube of uniform cross-sectional area contains water (density 1.0 gram/centimeter^3) standing initially 20 centimeters from the bottom in each arm. An immiscible liquid of density 4.0 grams/centimeter^3 is added to one arm until a layer 5 centimeters high forms, as shown in the figure above. What is the ratio h_2/h_1 of the heights of the liquid in the two arms?

(A) 3/1
(B) 5/2
(C) 2/1
(D) 3/2
(E) 1/1

Recommended Solution

This problem can be solved with just a bit of clever reasoning. First, consider that the initial state of the system is water at equilibrium and at 20 cm from the bottom of the curve. We can get a good approximation of the total mass of the water by assuming that all of the water is accounted for by the 40 cm of vertical tube length (i.e. ignoring the curve). There is likely not much of a difference between the two values and as long as we are consistent with this assumption, deviations won't present themselves in the final ratio. So, we assume that with 40 cm of water at 1 g/cm^3, we have roughly 40 g of water (Technically we would have 40 g/cm^2 but because the tube is the same size throughout, the cross-sectional slices with units of cm^2 will be the same for all parts and we might as well treat it as 1). Now, we can add to that the more dense liquid which we know accounts for 5 cm of the tube and has a density 4 times greater than the water. The total mass of the more dense liquid is 20 g giving a grand total of 60 g of liquid. The system will be at equilibrium when pressure on both sides of the tube is equal. Keeping in mind that pressure is proportional to density (i.e. $P = \rho g h$) and density is proportional to mass (i.e. $\rho = m/V$) we can conclude, and it should seem reasonable that, the system will be at equilibrium when we have equal amounts of mass in each side of the tube. On the left side, we have 20 g of the denser liquid, leaving the 40 g of water. Leaving 10 g of water on the left side and the remaining 30 g of water on the right, we get 30 g of liquid on each side. Now that it is in equilibrium, figure the amount of height taken up by each liquid. On the left, 20 g of the dense liquid is taking up 5 cm of space (as marked on the diagram) and the 10 g of water is adding an extra 10 cm, giving 15 cm on the left. On the right, we just have 30 g of water which makes 30 cm of liquid. The ratio of the right side to the left side is 30 cm/15 cm or 2/1.

Correct Answer
(C)

Alternate Solution

A more rigorous method of tackling this problem is to use the equation for pressure as $P = \rho g h$ with the condition that the system is at equilibrium when the pressure on the left side is the same as on the right. Setting $P_1 = P_2$, we have to note that P_1 is actually composed of two different liquids with different heights and different densities, i.e. $P_1 = P_{dense} + P_{water,1} = P_{water,2}$. Substituting in values, we have

$$(\rho_{dense}\text{ g }(5\text{ cm})) + (\rho_{water}\text{ g }(h_1 - 5\text{ cm})) = \rho_{water}\text{ g }h_2 \tag{1.128}$$

$$(4\text{ g}/cm^3)(5\text{ cm}) + (1\text{ g}/cm^3)(h_1 - 5\text{ cm}) = (1\text{ g}/cm^3)h_2 \tag{1.129}$$

Then using the fact that $h_1 + h_2 = 45$ cm, you will have 2 equations and 2 unknowns. When you solve for h_1 and h_2, you should get $h_1 = 15$ cm and $h_2 = 30$ cm, making the ratio $h_2/h_1 = 30\text{ cm}/15\text{ cm} = 2/1$.

Correct Answer
(C)

1.31 PGRE9677 #31

31. A sphere of mass m is released from rest in a stationary viscous medium. In addition to the gravitational force of magnitude mg, the sphere experiences a retarding force of magnitude bv, where v is the speed of the sphere and b is a constant. Assume that the buoyant force is negligible. Which of the following statements about the sphere is correct?

(A) Its kinetic energy decreases due to the retarding force.
(B) Its kinetic energy increases to a maximum, then decreases to zero due to the retarding force.
(C) Its speed increases to a maximum, then decreases back to a final terminal speed.
(D) Its speed increases monotonically, approaching a terminal speed that depends on b but not on m.
(E) Its speed increases monotonically, approaching a terminal speed that depends on both b and m.

Recommended Solution

(A) Any object which falls through a medium which resists its motion will continually increase its velocity until it reaches some terminal velocity. If it helps, think of the mass falling through air resistance as the "viscous medium". In order for the retarding force to decrease kinetic energy, velocity would have to decrease at some point, which it doesn't.

(B) As in part (A), the mass will reach some terminal velocity, however after it has done that it will maintain that velocity, not slow down and stop.

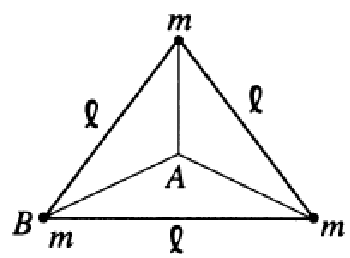
(C) The terminal velocity of an object, by definition, is the maximum speed which the mass can reach in a given medium. In other words, the maximum speed and the terminal speed are the same, so the object can't decrease its speed from a maximum speed to a terminal speed.

(D) For (D) and (E) we finally have an accurate description of terminal velocity, however we now need to decide whether the speed of the object is dependent on b and m or just b. You might be inclined to think that speed isn't dependent on mass because of the classical, well known results of Galileo which demonstrates that objects fall at the same speed regardless of their mass. Keep in mind that this is only true in a vacuum and drawing out a force diagram should make it quite clear that the speed is dependent on mass (i.e. $F_{net} = ma = mg - bv$). If you still aren't convinced, ask yourself why a feather falls more slowly than a brick in a real world scenario.

(E) This solution is the same as (D) except that it correctly identifies b and m as being variables of velocity

Correct Answer
(E)

1.32 PGRE9677 #32



32. Three equal masses m are rigidly connected to each other by massless rods of length ℓ forming an equilateral triangle, as shown above. The assembly is to be given an angular velocity ω about an axis perpendicular to the triangle. For fixed ω, the ratio of the kinetic energy of the assembly for an axis through B compared with that for an axis through A is equal to

(A) 3
(B) 2
(C) 1
(D) 1/2
(E) 1/3

Recommended Solution

Rotational kinetic energy is $U_k = \frac{1}{2}I\omega^2$. The moment of inertia for a point mass about a radius R is $I = mR^2$. For the rotation about point A, we need to determine the length between each mass and point A. Since the line between B and A bisects a 60° angle, we can make a right triangle and use trigonometry to find the length as

$$\cos(30°) = \frac{l/2}{R} \Longrightarrow R_A = \frac{l}{\sqrt{3}} \tag{1.130}$$

Which gives us the rotational kinetic energy equation as

$$U_{k-A} = \frac{1}{2}(3m)R^2\omega^2 = \frac{1}{2}(3m)\left(\frac{l^2}{3}\right)\omega^2 = \frac{1}{2}ml^2\omega^2 \tag{1.131}$$

The rotational kinetic energy around point B is easier to calculate because we have just two masses rotating at length l, giving us

$$U_{k-B} = \frac{1}{2}2ml^2\omega^2 = ml^2\omega^2 \tag{1.132}$$

Comparing Equations 1.131 and 1.132, we can see that U_{k-B} is twice as big as U_{k-A}

Correct Answer
(B)

1.33 PGRE9677 #33

33. A diatomic molecule is initially in the state $\Psi(\Theta, \Phi) = (5Y_1^{\,1} + 3Y_5^{\,1} + 2Y_5^{\,-1})/(38)^{1/2}$, where $Y_\ell^{\,m}$ is a spherical harmonic. If measurements are made of the total angular momentum quantum number ℓ and of the azimuthal angular momentum quantum number m, what is the probability of obtaining the result $\ell = 5$?

(A) 36/1444
(B) 9/38
(C) 13/38
(D) $5/(38)^{1/2}$
(E) 34/38

Recommended Solution

From quantum mechanics, recall that the probability of finding the state of any given operator can be found by Equation 1.133

$$P = \int \langle \psi | A | \psi \rangle \tag{1.133}$$

There are two terms with quantum number $l = 5$ which have coefficients of 2 and 3. This gives a total of $3^2 + 2^2 = 13$. Thus, the probability is 13 out of a total sum of 38 (i.e. $2^2 + 3^2 + 5^2 = 38$) so the probability is 13/38.

Correct Answer
(C)

1.34 PGRE9677 #34

34. When the beta-decay of ^{60}Co nuclei is observed at low temperatures in a magnetic field that aligns the spins of the nuclei, it is found that the electrons are emitted preferentially in a direction opposite to the ^{60}Co spin direction. Which of the following invariances is violated by this decay?

(A) Gauge invariance
(B) Time invariance
(C) Translation invariance
(D) Reflection invariance
(E) Rotation invariance

Recommended Solution

(A) Gauge invariance deals with an invariance of charge. This isn't violated simply due to a preferential direction.

(B) Time invariance doesn't deal with spin or Electromagnetic interactions. Besides, time invariance is rarely violated in any context.

(C) Translation invariance deals with the invariance of system equations under any translational frame. This isn't related to this problem.

(D) Reflection invariance is violated in this instance because the preferential direction in one frame changes if we were to create a reflection of that frame. Consider, for example, the way in which things will look reversed when viewed in a mirror. If particles are moving in a preferential direction, say +x, then there exists some reflected frame in which the particles move in the preferential direction of -x.

(E) Rotational invariance deals with maintaining system equations under any rotation of the system frame. This isn't violated in this example.

Correct Answer
(D)

1.35 PGRE9677 #35

35. The wave function for identical fermions is anti-symmetric under particle interchange. Which of the following is a consequence of this property?

(A) Pauli exclusion principle
(B) Bohr correspondence principle
(C) Heisenberg uncertainty principle
(D) Bose-Einstein condensation
(E) Fermi's golden rule

Recommended Solution

(A) Recall from doing atomic energy level diagrams that the Pauli exclusion principle tells us that electrons in an atom can't have the same set of 4 quantum numbers: n, l, m_l and m_s. In general, it also tells us that fermions may not simultaneously have the same quantum state as another fermion. More rigorously, it tells us that for two identical fermions, the total quantum state of the two is anti-symmetric.

(B) The Bohr correspondence principle says that quantum mechanical effects yield classical results under large quantum numbers. Definitely not the answer.

(C) The Heisenberg uncertainty principle tells us that the information of two related aspects of a quantum state can only ever be known with inverse amounts of certainty. Said plainly, the more you know about one component of the state of a system, the less you know about another[2]. This isn't related to the quantum state of fermions.

(D) A Bose-Einstein condensate is a gas of weakly interacting bosons which can be cooled sufficiently to force them to their ground state energies. This doesn't have anything to do with fermions.

(E) Fermi's Golden Rule involves the rate of transition from one energy eigenstate into a continuum of eigenstates. This is clearly not the right answer.

Correct Answer
(A)

[2] The typical example is momentum and position

1.36 PGRE9677 #36

36. A lump of clay whose rest mass is 4 kilograms is traveling at three-fifths the speed of light when it collides head-on with an identical lump going the opposite direction at the same speed. If the two lumps stick together and no energy is radiated away, what is the mass of the composite lump?

(A) 4 kg
(B) 6.4 kg
(C) 8 kg
(D) 10 kg
(E) 13.3 kg

Recommended Solution

When an object with mass moves at relativistic speeds, the mass of the object increases. You can always remember this because as an object approaches the speed of light, the mass approaches infinity, hence why massive objects can't travel at the speed of light. When the two lumps of clay hit one another the sum of the masses in non-relativistic terms would be 8 kg and thus at relativistic speeds, the mass must be higher. We can eliminate (A), (B) and (C) from this fact and, if you can't figure out the next step, then you at least have the problem down to two solutions. Relating the rest mass of the combined lumps (M) to the total energy of the system for the two masses separately ($2m$)gives

$$E_{net} = 2\gamma mc^2 = Mc^2 \tag{1.134}$$

$$2\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}m = M \tag{1.135}$$

Plug all of the values provided in the problem into Equation 1.135 to get

$$M = \frac{1}{\sqrt{1-(\frac{3}{5}c)^2/c^2}}(4\text{kg}) = 10\text{kg} \tag{1.136}$$

Correct Answer
(D)

1.37 PGRE9677 #37

37. An atom moving at speed $0.3c$ emits an electron along the same direction with speed $0.6c$ in the internal rest frame of the atom. The speed of the electron in the lab frame is equal to

(A) $0.25c$
(B) $0.51c$
(C) $0.66c$
(D) $0.76c$
(E) $0.90c$

Recommended Solution

Hooray for plug-n-chug physics problems. Relativistic addition of velocities is solved with

$$u' = \frac{u+v}{1+\frac{vu}{c^2}} \tag{1.137}$$

Plug in the values given and, shortly thereafter, chug to get

$$u' = \frac{0.3c + 0.6c}{1 + \frac{(0.3c)(0.6c)}{c^2}} = \frac{0.9c}{1.18} = 0.76c \tag{1.138}$$

Correct Answer
(D)

Alternate Solution

Velocity addition "sort of" still works in relativistic terms but it isn't quite as simple as finding the sum of velocities. We know that the if we could just add up the velocities, then the velocity of the particle would be $0.9c$. However, our ability to continually gain additional velocity drops off as we approach the speed of light, so the speed must be less than $0.9c$ and we eliminate (E). Since addition still "sort of works", we would expect the speed to at least be greater than the speed of just the particle at $0.6c$, so we can eliminate (A) and (B). Finally, you just have to decide whether (C) or (D) is a more reasonable speed. $0.66c$ is barely larger than the speed of the particle on its own so (D) is a more reasonable solution.

Correct Answer
(D)

1.38 PGRE9677 #38

38. What is the speed of a particle having a momentum of 5 MeV/c and a total relativistic energy of 10 MeV ?

(A) c

(B) $0.75\,c$

(C) $\frac{1}{\sqrt{3}}\,c$

(D) $\frac{1}{2}\,c$

(E) $\frac{1}{4}\,c$

Recommended Solution

Consider Einstein's equations for relativistic energy and relativistic momentum,

Relativistic energy: $E_{rel} = \gamma mc^2$

Relativistic momentum: $P_{rel} = \gamma mv$

From these 2 equations, it should be clear that the only way we will get velocity from them is to divide P_{rel} over E_{rel}, which results in

$$\frac{P_{rel}}{E_{rel}} = \frac{\gamma mv}{\gamma mc^2} = \frac{5\ MeV/c}{10\ MeV} = \frac{1}{2}\ c \tag{1.139}$$

Correct Answer
(D)

1.39 PGRE9677 #39

39. Which of the following atoms has the lowest ionization potential?

(A) ${}^{2}_{4}\mathrm{He}$

(B) ${}^{7}_{14}\mathrm{N}$

(C) ${}^{8}_{16}\mathrm{O}$

(D) ${}^{18}_{40}\mathrm{Ar}$

(E) ${}^{55}_{133}\mathrm{Cs}$

Recommended Solution

Ionization potential is lowest for atoms with full valence shells or nearly full valence shells because they generally don't want to lose electrons. However, atoms with 1 or 2 additional electrons will be very likely to lose an electron and will have high ionization potential.

(A) Full valence shell

(B) Nearly full valence shell

(C) Nearly full valence shell

(D) Full valence shell

(E) Cs has one additional electron, so it is most likely to lose that electron

Correct Answer
(E)

1.40 PGRE9677 #40

40. If a singly ionized helium atom in an $n = 4$ state emits a photon of wavelength 470 nanometers, which of the following gives the approximate final energy level, E_f, of the atom, and the n value, n_f, of this final state?

	E_f (eV)	n_f
(A)	−6.0	3
(B)	−6.0	2
(C)	−14	2
(D)	−14	1
(E)	−52	1

Recommended Solution

From the Rydberg Formula,we get

$$E = E_0\left(\frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2}\right) \tag{1.140}$$

Finding the values needed for the equation gives us,

$$\lambda_2 = 470 \text{ nm} \tag{1.141}$$

$$E_0 = 4 \times 13.6eV \approx 55eV \tag{1.142}$$

$$E = \frac{hc}{\lambda} \approx 2 \text{ eV} \tag{1.143}$$

Then, plug in our recently calculated values (Equations 1.141 1.142 and 1.143) to solve for λ_1

$$2eV = 55eV\left(\frac{1}{\lambda_1^2} - \frac{1}{16}\right) \Longrightarrow \lambda_1 \approx 3 \tag{1.144}$$

Which is only true of solution (A)

Correct Answer
(A)

1.41 PGRE9677 #41

41. A $3p$ electron is found in the $^3P_{3/2}$ energy level of a hydrogen atom. Which of the following is true about the electron in this state?

(A) It is allowed to make an electric dipole transition to the $^2S_{1/2}$ level.
(B) It is allowed to make an electric dipole transition to the $^2P_{1/2}$ level.
(C) It has quantum numbers $\ell = 3$, $j = 3/2$, $s = 1/2$.
(D) It has quantum numbers $n = 3$, $j = \ell$, $s = 3/2$.
(E) It has exactly the same energy as it would in the $^3D_{3/2}$ level.

Recommended Solution

You should be able to immediately eliminate (D) and (E). Option (D) clearly can't be correct because it suggests that the electron has a spin quantum number of $s = 3/2$ which is never true (it must always be $s = \pm 1/2$). Option (E) can't be true because you can't, by definition, move to a different energy level and maintain the same energy. Next, recall that the angular quantum number for P corresponds to $L = 1$ and, in general

$$(S, P, D, F, \ldots) \longrightarrow (0, 1, 2, 3, \ldots) \tag{1.145}$$

so (C) should have $L = 1$, not $L = 3$. Finally, between (A) and (B), you must recall your quantum number selection rules, specifically that transitions are allowed for $\Delta L = \pm 1$ so (A) is allowed because L moves from $P = 1$ to $S = 0$ while (B) violates this allowed transition.

Correct Answer
(A)

1.42 PGRE9677 #42

42. Light of wavelength 500 nanometers is incident on sodium, with work function 2.28 electron volts. What is the maximum kinetic energy of the ejected photoelectrons?

(A) 0.03 eV
(B) 0.2 eV
(C) 0.6 eV
(D) 1.3 eV
(E) 2.0 eV

Recommended Solution

From the photoelectric effect, the equation for maximum kinetic energy is

$$U_K = h\nu - \phi = \frac{hc}{\lambda} - \phi \tag{1.146}$$

where ϕ is the Work Function. Plug all of your known values in and round everything to 1 significant figure to simplify things,

$$\begin{aligned} U_K &= \frac{(4 \times 10^{-15}\ eV \cdot s)(3 \times 10^{8}\ m/s)}{5 \times 10^{-7}\ m} - \phi & (1.147) \\ &= \frac{12 \times 10^{-7}\ eV \cdot m}{5 \times 10^{-7}\ m} - \phi & (1.148) \\ &= \frac{12}{5} - \phi & (1.149) \\ &= 2.4 - \phi & (1.150) \end{aligned}$$

Finally, Plug in your value for the work function into equation 1.150 and solve

$$U_K = 2.4 - 2.28 = 0.12 \tag{1.151}$$

which is closest to (B).

Correct Answer
(B)

1.43 PGRE9677 #43

43. The line integral of $\mathbf{u} = y\mathbf{i} - x\mathbf{j} + z\mathbf{k}$ around a circle of radius R in the xy-plane with center at the origin is equal to

(A) 0
(B) $2\pi R$
(C) $2\pi R^2$
(D) $\pi R^2/4$
(E) $3R^3$

Recommended Solution

The line integral in this problem moves about a circle in the xy-plane with its center at 0. Since we will be doing a line integral, we should probably know the general solution for a line integral about a curve, C

$$\oint_C f(s) = \int_a^b f(t)\,|f'(t)| \tag{1.152}$$

So in our case, we need to solve the integral

$$\int_a^b f(x,y,z)\,|f'(x,y,z)| \tag{1.153}$$

Since the circle is contained entirely in the xy-plane, $z = 0$. Additionally, since we are dealing with a circle, we will want to convert our values for x and y into their parametric equivalents. Specifically, $x = R\cos(\theta)$ and $y = R\sin(\theta)$. Replacing and x and y with our parametric equations gives us 1.155

$$\int_a^b \left(y\frac{dx}{d\theta} - x\frac{dy}{d\theta} \right)\, d\theta \tag{1.154}$$

$$\int_0^{2\pi} \left[(R\,sin(\theta))(-R\,sin(\theta)) - (R\,cos(\theta))(R\,cos(\theta))\right]\, d\theta \tag{1.155}$$

Multiplying out and simplifying, you should get

$$\int_0^{2\pi} -R^2\left[sin^2(\theta) + cos^2(\theta)\right]\, d\theta \tag{1.156}$$

Since $\sin^2(\theta) + \cos^2(\theta) = 1$, Equation 1.156 becomes

$$\int_0^{2\pi} -R^2 d\theta = -2\pi R^2 \tag{1.157}$$

The negative sign in 1.157 (and in fact the sign in general) is dependent on the direction in which you traverse the curve so we are primarily concerned with the magnitude of the solution.

Correct Answer
(C)

Alternate Solution

Kelvin-Stokes theorem tells us that the surface integral of the curl over a vector field is equivalent to the line integral around that vector field in a Euclidean 3-space. Mathematically, that is

$$\int (\nabla \times f) \cdot da = \oint f \cdot dl \tag{1.158}$$

Which means that if we can take the curl of our function, u, and the integral of it over our area, then we have the line integral of the same function. Recall that the curl of a function can be calculated by taking the determinant of the matrix

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

Applying this to our function, u and taking the determinant gives

$$\hat{i}\left(\frac{\partial z}{\partial y} + \frac{\partial x}{\partial y}\right) + \hat{j}\left(\frac{\partial z}{\partial x} - \frac{\partial y}{\partial z}\right) + \hat{k}\left(\frac{-\partial x}{\partial x} - \frac{\partial y}{\partial y}\right) \tag{1.159}$$

Since x, y and z are not functions of one another, everything in Equation 1.159 goes to 0 with the exception of the final term, which becomes -2. Substituting $\nabla \times u$ into Equation 1.158,

$$\int_0^{2\pi} -2 \cdot dA = -2A \tag{1.160}$$

where the area of the circle is πR^2, so

$$\oint u \cdot dl = -2A = -2\pi R^2 \tag{1.161}$$

Correct Answer
(C)

1.44 PGRE9677 #44

44. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to

$$v(x) = \beta x^{-n},$$

where β and n are constants and x is the position of the particle. What is the acceleration of the particle as a function of x ?

(A) $-n\beta^2 x^{-2n-1}$

(B) $-n\beta^2 x^{-n-1}$

(C) $-n\beta^2 x^{-n}$

(D) $-\beta x^{-n+1}$

(E) $-\beta x^{-2n+1}$

Recommended Solution

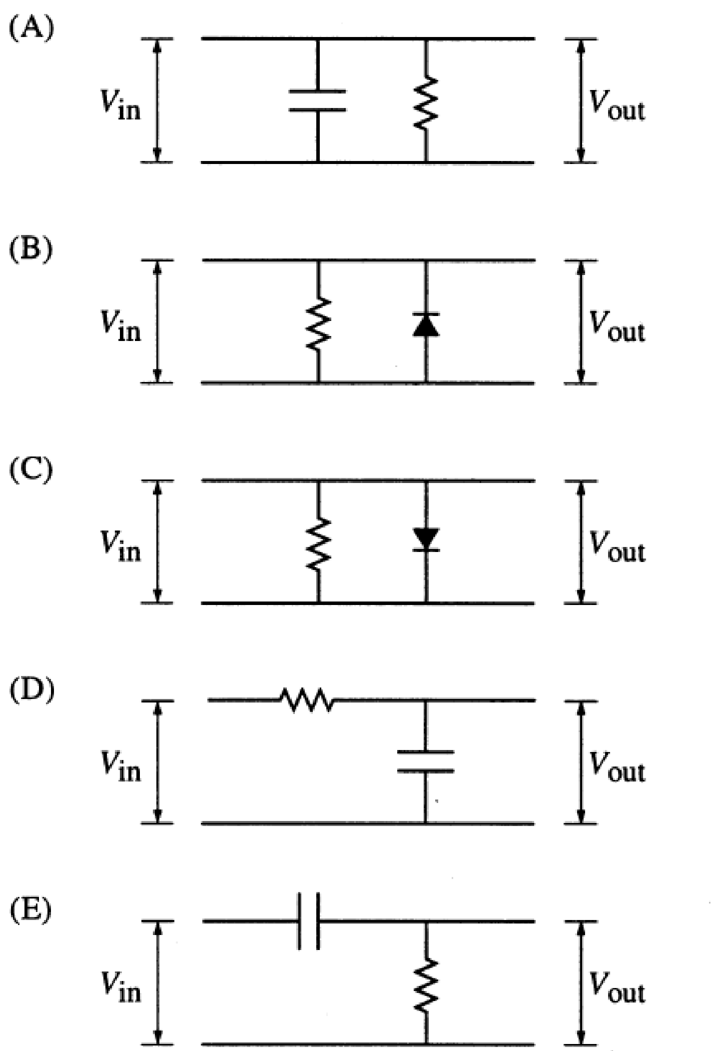
Velocity is given and since the derivative of velocity is acceleration, take the derviative. Take note, however, that v is a function of position (x) and position is a function of time (t) so use chain rule to get

$$\begin{aligned} a &= \frac{dv}{dx}\frac{dx}{dt} && (1.162) \\ &= \frac{dv}{dx}(v) && (1.163) \\ &= \left(-n\beta x^{-n-1}\right)\left(\beta x^{-n}\right) && (1.164) \\ &= -n\beta\,^2 x^{-2n-1} && (1.165) \end{aligned}$$

Correct Answer
(A)

1.45 PGRE9677 #45

45. The circuits below consist of two-element combinations of capacitors, diodes, and resistors. V_{in} represents an ac-voltage with variable frequency. It is desired to build a circuit for which $V_{out} \approx V_{in}$ at high frequencies and $V_{out} \approx 0$ at low frequencies. Which of the following circuits will perform this task?

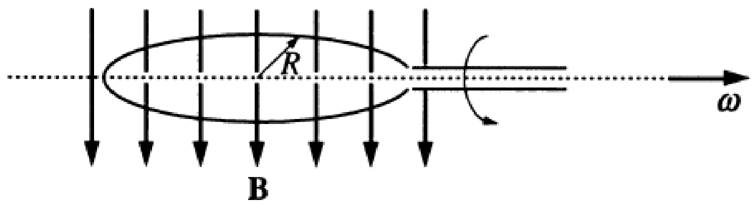


Recommended Solution

A simple method of generating a low-pass filter in a circuit involves placing a resistor in series with a load and a capacitor in parallel with that same load. low-frequency signals that attempt to pass through the circuit will be blocked by the capacitor and will be forced to pass through the load instead. Meanwhile, high-frequency signals will be able to bypass the capacitor with little to no effect. Of the possible solutions, only (D) provides us with a resistor in series with a load and a capacitor in parallel with that load.

Correct Answer
(D)

1.46 PGRE9677 #46



46. A circular wire loop of radius R rotates with an angular speed ω in a uniform magnetic field $\mathbf{B}$, as shown in the figure above. If the emf $\mathcal{E}$ induced in the loop is $\mathcal{E}_0 \sin \omega t$, then the angular speed of the loop is

(A) $\mathcal{E}_0 R/B$

(B) $2\pi \mathcal{E}_0/R$

(C) $\mathcal{E}_0/(B\pi R^2)$

(D) $\mathcal{E}_0^2/(BR^2)$

(E) $\tan^{-1}(\mathcal{E}_0/Bc)$

Recommended Solution

From Faraday's law of induction, we know that the potential (Electromotive Force) for a loop of wire is

$$|\varepsilon| = N\left|\frac{d\phi_B}{dt}\right| \tag{1.166}$$

Since there is only one loop, it simplifies to

$$|\varepsilon| = \left|\frac{d\phi_B}{dt}\right| \tag{1.167}$$

The problem tells us that the potential is, $\varepsilon = \varepsilon_0 \sin(\omega t)$, so we can set that equal to Equation 1.167 and substitute in $\phi_B = B \cdot dA$, to get

$$\varepsilon_0 \sin(\omega t) = \frac{d}{dt}(B \cdot dA) \tag{1.168}$$

move dt over and integrate both sides

$$\int \varepsilon_0 \sin(\omega t)\; dt = \int B \cdot dA \tag{1.169}$$

$$-\varepsilon_0 \cos(\omega t)/\omega = B \cdot A = B\pi R^2 \tag{1.170}$$

Finally, rearrange to solve for ω,

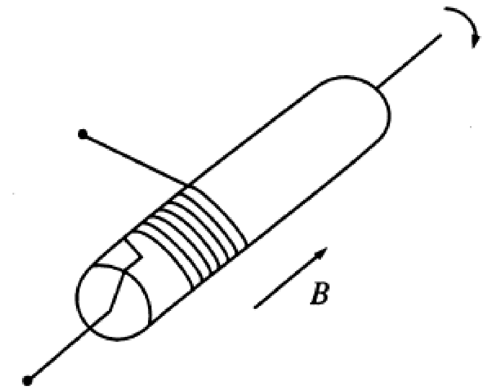
$$\omega = \frac{-\varepsilon_0 \cos(\omega t)}{B\pi R^2} \tag{1.171}$$

and consider when the angular velocity is maximized, $t = 0$, to get

$$\omega = \frac{-\varepsilon_0}{B\pi R^2} \tag{1.172}$$

Correct Answer
(C)

1.47 PGRE9677 #47



47. A wire is being wound around a rotating wooden cylinder of radius R. One end of the wire is connected to the axis of the cylinder, as shown in the figure above. The cylinder is placed in a uniform magnetic field of magnitude B parallel to its axis and rotates at N revolutions per second. What is the potential difference between the open ends of the wire?

(A) 0

(B) $2\pi NBR$

(C) πNBR^2

(D) BR^2/N

(E) πNBR^3

Recommended Solution

Faraday's law gives us potential for a changing magnetic field as

$$|\epsilon| = \left|\frac{d\phi_B}{dt}\right| \tag{1.173}$$

Where ϕ_B is the flux of the magnetic field through some area which is

$$\phi_B = \int B \cdot dA \tag{1.174}$$

The area through which the flux passes is just the area of a circle with radius R so $\phi_B = B\pi R^2$

$$|\epsilon| = \frac{d}{dt}\left(B\pi R^2\right) \tag{1.175}$$

and since the flux is changing from the the rotation and the loops rotate at $N\frac{\text{rev}}{s}$,

$$|\epsilon| = NB\pi R^2 \tag{1.176}$$

Correct Answer
(C)

Alternate Solution

Alternatively, you can go through a process of elimination by removing unlikely or impossible choices. (A) is clearly wrong because, by Faraday's law, a changing magnetic field will generate a potential. (D) is also clearly wrong because it suggests that potential decreases as the rate of variance in the magnetic field increases. For (B), (C) and (E), check the units.

(B) $2\pi NBR \equiv (rev)(kg/s^2A)(m) \equiv \frac{rev}{A}\frac{kg\ m}{s^2}$

(C) $\pi NBR^2 \equiv (rev)(kg/s^2A)(m^2) \equiv \frac{rev}{A}\frac{kg\ m^2}{s^2} = \frac{rev}{A}\ J$

(E) $NBR^3 \equiv (rev)(kg/s^2A)(m^3) \equiv \frac{rev}{A}\frac{kg\ m^3}{s^2}$

Of these, only (C) has the right units.

Correct Answer
(C)

1.48 PGRE9677 #48

48. The half-life of a π^+ meson at rest is 2.5×10^{-8} second. A beam of π^+ mesons is generated at a point 15 meters from a detector. Only $\frac{1}{2}$ of the π^+ mesons live to reach the detector. The speed of the π^+ mesons is

(A) $\frac{1}{2}c$

(B) $\sqrt{\frac{2}{5}}c$

(C) $\frac{2}{\sqrt{5}}c$

(D) c

(E) $2c$

Recommended Solution

Mesons are hadrons so they have mass, which means they can't exceed the speed of light and also can't match the speed of light (i.e. the mass-less photon) so we eliminate (D) and (E). Next, let's try to treat the motion as if it is non-relativistic. Only half of the π^+ mesons make it through the 15 meters, thus the amount of time to move the 15 meters is 1 half life or 2.5×10^{-8} seconds. From this, try

$$v = \frac{\Delta X}{\Delta t} = \frac{15 \text{ m}}{2.5 \times 10^{-8}} \approx 5 \times 10^8 \text{m/s} \tag{1.177}$$

The value gives us a speed faster than light, meaning that what ever the actual speed is, it needs to be analyzed using relativistic equations. This tells us that the velocity must be very near c but less than it.

(A) $\frac{1}{2}$ C: Not nearly fast enough for relativistic influence to take effect

(B) $\sqrt{\frac{2}{5}}$ $C \approx 0.6$ C: Very similar to (A) and likely not fast enough to have significant relativistic effect.

(C) $\frac{2}{\sqrt{5}}$ $C \approx 0.9$ C: This value is the closest to justifying relativistic influence so it is the most likely to be correct.

Correct Answer
(C)

Alternate Solution

To calculate the value exactly, start with your equation for "proper time",

$$\Delta\tau^2 = \Delta t^2 - \Delta X^2 \tag{1.178}$$

$$\Delta t^2 = \Delta\tau^2 + \Delta X^2 \tag{1.179}$$

We know, from the half-life given in the problem and the fact that only half of the sample makes it through the 15 meters, that it takes one half-life of time to move 15 meters. Substitute in 15 for the position and 15/2 for the time,

$$\Delta t^2 = (15/2)^2 + 15^2 = 15^2\left[\left(\frac{1}{2}\right)^2 + 1\right] = 225\left(\frac{1}{4} + 1\right) \tag{1.180}$$

Take the square root of both sides to get

$$\Delta t = \sqrt{225\left(\frac{1}{4} + 1\right)} = 15\left(\sqrt{5/4}\right) \tag{1.181}$$

Now using the basic definition for velocity (i.e. $v = \Delta X/\Delta t$), with distance $\Delta X = 15$, we get

$$\frac{\Delta X}{\Delta t} = \frac{15}{15\sqrt{5/4}} = \frac{1}{\sqrt{5/4}} = \frac{2}{\sqrt{5}} \tag{1.182}$$

Correct Answer
(C)

1.49 PGRE9677 #49

49. The infinite xy-plane is a nonconducting surface, with surface charge density σ, as measured by an observer at rest on the surface. A second observer moves with velocity $v\hat{\mathbf{x}}$ relative to the surface, at height h above it. Which of the following expressions gives the electric field measured by this second observer?

(A) $\frac{\sigma}{2\epsilon_0}\,\hat{\mathbf{z}}$

(B) $\frac{\sigma}{2\epsilon_0}\sqrt{1-v^2/c^2}\,\hat{\mathbf{z}}$

(C) $\frac{\sigma}{2\epsilon_0\sqrt{1-v^2/c^2}}\,\hat{\mathbf{z}}$

(D) $\frac{\sigma}{2\epsilon_0}\left(\sqrt{1-v^2/c^2}\,\hat{\mathbf{z}} + v/c\,\hat{\mathbf{x}}\right)$

(E) $\frac{\sigma}{2\epsilon_0}\left(\sqrt{1-v^2/c^2}\,\hat{\mathbf{z}} - v/c\,\hat{\mathbf{y}}\right)$

Recommended Solution

Relativistic influences on Electricity and Magnetism occurs proportionally to the lorentz factor, γ. Since $E_z = \frac{\sigma}{2\epsilon_0}$, we would expect the Electric field to be influenced by

$$E_z\gamma = \frac{\sigma}{2\epsilon_0}\frac{1}{\sqrt{1-v^2/C^2}} \tag{1.183}$$

Correct Answer
(C)

1.50 PGRE9677 #50

50. In inertial frame S, two events occur at the same instant in time and $3c \cdot$ minutes apart in space. In inertial frame S', the same events occur at $5c \cdot$ minutes apart. What is the time interval between the events in S'?

(A) 0 min
(B) 2 min
(C) 4 min
(D) 8 min
(E) 16 min

Recommended Solution

The space time interval equation is

$$\Delta S^2 = -(c\ \Delta t)^2 + \Delta X^2 \tag{1.184}$$

The space time interval is $S = 3C \cdot$ minutes and the position interval is $\Delta X = 5c \cdot$ mintues. Plug these into Equation 1.184

$$9C^2 = -(C\ \Delta t)^2 + 25C^2 \tag{1.185}$$

Then, solve for Δt to get

$$\Delta t = 4 \text{ minutes} \tag{1.186}$$

Correct Answer
(C)

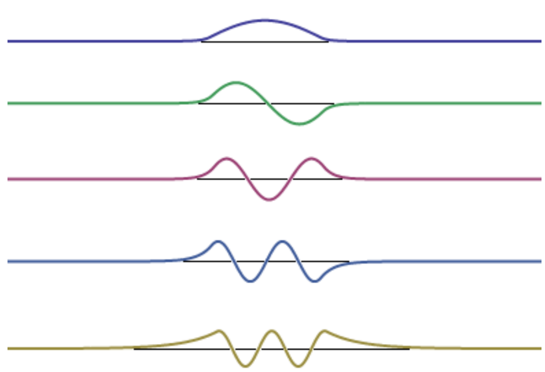
1.51 PGRE9677 #51

51. The solution to the Schrödinger equation for a particle bound in a one-dimensional, infinitely deep potential well, indexed by quantum number n, indicates that in the middle of the well the probability density vanishes for

(A) the ground state $(n = 1)$ only
(B) states of even n $(n = 2, 4, \dots)$
(C) states of odd n $(n = 1, 3, \dots)$
(D) all states $(n = 1, 2, 3 \dots)$
(E) all states except the ground state

Recommended Solution

The solution to the infinite square well (also known as the particle in a box) is a sine function in which the number of nodes on the wave is $n + 1$. In the ground state, you will have 2 nodes at the end of the infinite walls with the peak of the wave at exactly the middle. For $n = 2$, we get 3 nodes with two peaks (or a peak and a trough if you insist) and the middle of the wave falls on a node. If you keep checking all of the values for n, you will find that all states with even values for n result in a node in the middle of the well.



Correct Answer
(B)

Alternate Solution

The solution to the infinite square well is

$$\psi_n(x,t) = A \sin\left(\frac{n\pi x}{L}\right) \tag{1.187}$$

If we are only concerned with the middle position, set $x = L/2$, giving

$$\psi_n(x,t) = A \sin\left(\frac{n\pi}{2}\right) \tag{1.188}$$

and this equation will go to zero any time you are taking the sine of integer values of π (i.e. $\sin(0), \sin(\pi), \sin(2\pi), \ldots$).

Correct Answer
(B)

1.52 PGRE9677 #52

52. At a given instant of time, a rigid rotator is in the state $\psi(\theta, \phi) = \sqrt{3/4\pi}\ \sin\theta \sin\phi$, where θ is the polar angle relative to the z-axis and ϕ is the azimuthal angle. Measurement will find which of the following possible values of the z-component of the angular momentum, L_z ?

(A) 0
(B) $\hbar/2,\ -\hbar/2$
(C) $\hbar, -\hbar$
(D) $2\hbar,\ -2\hbar$
(E) $\hbar,\ 0,\ -\hbar$

Recommended Solution

For the spherical harmonics, we get a sine term for the harmonic

$$Y_1^1 = -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin(\theta)e^{i\pi\phi} \tag{1.189}$$

and

$$Y_1^{-1} = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin(\theta)e^{i\pi\phi} \tag{1.190}$$

which means that $m = \pm 1$. The eigenvalues of a spherical harmonic can be found with

$$L_Z\psi = m\hbar\psi \tag{1.191}$$

Plugging in our value for m gives

$$L_Z\psi = \pm 1\hbar\psi \tag{1.192}$$

which is (C).

Correct Answer
(C)

1.53 PGRE9677 #53

53. Positronium is the bound state of an electron and a positron. Consider only the states of zero orbital angular momentum ($\ell = 0$). The most probable decay product of any such state of positronium with spin zero (singlet) is

(A) 0 photons
(B) 1 photon
(C) 2 photons
(D) 3 photons
(E) 4 photons

Recommended Solution

Positronium atoms can only decay into even numbered groupings of photons (to conserve spin) so we can eliminate choices (B) and (D).

Positronium atoms are very unstable and since energy must be conserved, there is no way that the atom will decay without releasing some photons so we can eliminate (A).

Finally, between 2 photons and 4 photons, consider how silly a feynman diagram will look with 4 photon lines emanating from the interaction. In case I wasn't being clear, it is wicked silly.

Correct Answer
(C)

1.54 PGRE9677 #54

Questions 54-55 concern a plane electromagnetic wave that is a superposition of two independent orthogonal plane waves and can be written as the real part of $\mathbf{E} = \hat{\mathbf{x}} E_1 \exp(i[kz - \omega t]) + \hat{\mathbf{y}} E_2 \exp(i[kz - \omega t + \pi])$, where k, ω, E_1, and E_2 are real.

54. If $E_2 = E_1$, the tip of the electric field vector will describe a trajectory that, as viewed along the z-axis from positive z and looking toward the origin, is a

(A) line at 45° to the $+x$-axis
(B) line at 135° to the $+x$-axis
(C) clockwise circle
(D) counterclockwise circle
(E) random path

55. If the plane wave is split and recombined on a screen after the two portions, which are polarized in the x- and y-directions, have traveled an optical path difference of $2\pi/k$, the observed average intensity will be proportional to

(A) $E_1^2 + E_2^2$
(B) $E_1^2 - E_2^2$
(C) $(E_1 + E_2)^2$
(D) $(E_1 - E_2)^2$
(E) 0

Recommended Solution

Electromagnetic waves are typically thought of as being composed of a magnetic and electric wave moving in the same direction and oscillating orthogonally (Figure 1.8).

The summation of any two or more of these waves will occur in one of two forms. One of the forms involves waves oscillating in the same plane as one another and the other involves oscillations in different plans. In general, any waves in the same plane will result in field vectors also in the same plane, resulting in a trajectory that moves in only 2 dimensions. Field vectors in an alternate plane will sum to a 3 dimensional rotational trajectory. For the specific problem here, the second wave is rotated π or 180° so the field vectors lie in the same plane and the final trajectory must be linear. Based on this, you can eliminate (C), (D), and (E). In order to decide between (A) and (B), realize that two waves perfectly in phase (i.e. with no rotation) will have field vectors in the same quadrant and so the angle will be 45°. On the other hand, a wave rotated π from the other will be in different quadrants and so the angle will be 135°.

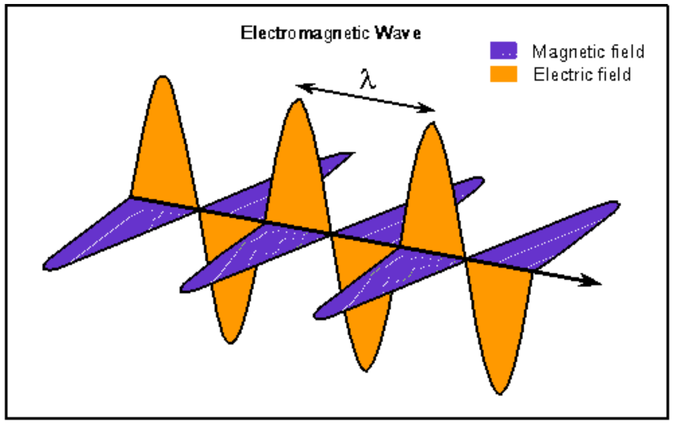


Figure 1.8: Classical view of elctromagnetic wave behavior

Correct Answer
(B)

1.55 PGRE9677 #55

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(A) $E_1^2 + E_2^2$
(B) $E_1^2 - E_2^2$
(C) $(E_1 + E_2)^2$
(D) $(E_1 - E_2)^2$
(E) 0

Recommended Solution

By Malus' Law, the intensity of a an electromagnetic wave after passing through a perfect polarizer is

$$I = \frac{1}{2}c\epsilon_0 E_0^2 \cos^2(\theta) \tag{1.193}$$

The optical path difference for the second wave is $z = 2\pi/k$. Plugging this into the original equation and letting $z \to 0$ and $t \to 0$.

$$E = \hat{x}E_1 e^{i(kz-\omega t)} + \hat{y}E_2 e^{i(kz-wt+\pi)} = \hat{x}E_1 e^0 + \hat{y}E_2 e^{i(2\pi+\pi)} \tag{1.194}$$

Recall Euler's identity, $e^{i\pi} = 1$, so the Equation 1.194 becomes

$$E = E_1 + E_2 \tag{1.195}$$

Since the two waves are decoupled, the magnitude of the entire wave will be the magnitude of each individual wave added separately

$$I = (E_1)^2 + (E_2)^2 \tag{1.196}$$

Correct Answer
(A)

1.56 PGRE9677 #56

56. A light source is at the bottom of a pool of water (the index of refraction of water is 1.33). At what minimum angle of incidence will a ray be totally reflected at the surface?

(A) 0°
(B) 25°
(C) 50°
(D) 75°
(E) 90°

Recommended Solution

Immediately eliminate choice (A) because there is no way you are going to get an angle of 0° between the water surface and the light source. Next, eliminate (E) because an angle of 90° will result in no bending of the light and so there will be no total internal reflection. As for the last 3, you can probably intuitively figure that it won't be at 25° simply because that isn't a very steep angle. However, if we want to be more rigorous, we must use Snell's law. Recall that total internal inflection occurs in any instance in which using Snell's law would require you to take the sine of an angle and get a value that isn't possible. In our scenario, if we apply an angle of 50°, we get

$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{n_2}{n_1} \tag{1.197}$$

$$\sin(\theta_2) = 1.33\ \sin(50°) \tag{1.198}$$

$$= 1.02 \tag{1.199}$$

However, there is no angle at which the sine function will give you a value larger than 1 and this means you've reached total internal reflection. Admittedly, it is a bit rude on the part of ETS to simultaneously give you relatively complicated decimals and lesser used angles[3] with no calculator, but at the very least you could quickly get rid of the angle 25° by using this method. Then, knowing that ETS is looking for the minimum angle for total internal reflection, do an approximation (e.g. 45° as an approximation for $\theta = 50°$) to determine which potential angle is closer to pushing our value over 1.

Correct Answer
(C)

[3] i.e. not one of the angles we've all memorized from the unit circle

1.57 PGRE9677 #57

57. Consider a single-slit diffraction pattern for a slit of width d. It is observed that for light of wavelength 400 nanometers, the angle between the first minimum and the central maximum is 4×10^{-3} radians. The value of d is

(A) 1×10^{-5} m

(B) 5×10^{-5} m

(C) 1×10^{-4} m

(D) 2×10^{-4} m

(E) 1×10^{-3} m

Recommended Solution

The equation relating slit width for a single slit diffraction to wavelength is

$$d \sin(\theta) = \lambda \tag{1.200}$$

$$d = \frac{\lambda}{\sin(\theta)} \tag{1.201}$$

Convert the wavelength into meters, giving

$$d = \frac{(4 \times 10^{-7} \text{ m})}{\sin(4 \times 10^{-3} \text{ rad})} \tag{1.202}$$

For small angles, we can make the approximation $\sin(\theta) = \theta$, which gives us

$$d = \frac{(4 \times 10^{-7} \text{ m})}{(4 \times 10^{-3})} \tag{1.203}$$

$$d = 1 \times 10^{-4} \text{ m} \tag{1.204}$$

Correct Answer
(C)

1.58 PGRE9677 #58

58. A collimated laser beam emerging from a commercial HeNe laser has a diameter of about 1 millimeter. In order to convert this beam into a well-collimated beam of diameter 10 millimeters, two convex lenses are to be used. The first lens is of focal length 1.5 centimeters and is to be mounted at the output of the laser. What is the focal length, f, of the second lens and how far from the first lens should it be placed?

	f	Distance
(A)	4.5 cm	6.0 cm
(B)	10 cm	10 cm
(C)	10 cm	11.5 cm
(D)	15 cm	15 cm
(E)	15 cm	16.5 cm

Recommended Solution

We are trying to convert a "well-collimated" laser with diameter of 1 mm to a "well-collimated" laser of 10 mm, which represents a magnification of 10×. Lens magnification must have the same proportionality for the focal length as it does for the separation distance, so we would expect a focal length that is ten times larger than the 1.5 cm lens, i.e. 15 cm. We can eliminate (A), (B) and (C) from this. Finally, we need to decide whether the distance for the new lens will be 15 cm or 16.5 cm. Assuming everything is done properly, the first lens will have its focus at 1.5 cm and the second lens will have its focus at a distance of 15 cm. For ideal collimation, we will want the foci to be at the same location, so

$$15 \text{ cm} + 1.5 \text{ cm} = 16.5 \text{ cm} \tag{1.205}$$

Correct Answer
(E)

1.59 PGRE9677 #59

59. The approximate number of photons in a femtosecond (10^{-15}s) pulse of 600 nanometers wavelength light from a 10-kilowatt peak-power dye laser is

(A) 10^3
(B) 10^7
(C) 10^{11}
(D) 10^{15}
(E) 10^{18}

Recommended Solution

The energy of a single photon with wavelength, $\lambda = 600$ nm can be found by

$$E = \frac{hc}{\lambda} \tag{1.206}$$

The total number of photons emitted per second will be equal to the total energy generated per second divided by the energy of a single photon, i.e.

$$\#\text{ of photons } = \frac{100\text{ watts}}{hc/\lambda} \tag{1.207}$$

Plug everything in and rounding all of our numbers to simplify the mental math, you should get

$$\#\text{ of photons} = \frac{(1.0 \times 10^4\text{ W})(6 \times 10^{-7}\text{ m})}{(6.63 \times 10^{-34}\text{ J} \cdot \text{s})(3 \times 10^8\text{ m/s})} \approx 3 \times 10^{23}\ \frac{1}{\text{s}} \tag{1.208}$$

However, the question asks for the number of photons per femtosecond, so convert the previous solution to the right units to get

$$\#\text{ of photons} \approx 3 \times 10^{23}\ \frac{1}{\text{s}} \approx 3 \times 10^8\ \frac{1}{\text{fs}} \tag{1.209}$$

Which is closest to (B)

Correct Answer
(B)

1.60 PGRE9677 #60

60. The Lyman alpha spectral line of hydrogen (λ = 122 nanometers) differs by 1.8×10^{-12} meter in spectra taken at opposite ends of the Sun's equator. What is the speed of a particle on the equator due to the Sun's rotation, in kilometers per second?

(A) 0.22
(B) 2.2
(C) 22
(D) 220
(E) 2200

Recommended Solution

Keep in mind while we work this problem that we don't necessarily want the most accurate answer, just the quickest method to the correct choice. The Lyman alpha line is the spectral line corresponding to a hydrogen atom transition from level n=2 to n=1. The question poses this problem in a way that would indicate that we should use the Doppler shift equation for light. If we wanted to be extremely accurate, and let me stress how much we DON'T want this, then we would need to use the relativistic equations for the Doppler shift. Note, however, that the largest possible speed for the particle given in the answers, is 2200 km/s. Convert that to m/s and compare it to the speed of light, for our purposes we will just call it $C = 3.0 \times 10^8$ m/s. The fastest this particle could possibly be moving, according to the potential solutions, would be roughly 0.73% the speed of light

$$\frac{2.2 \times 10^6 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}} = 0.0073 \tag{1.210}$$

Relativistic effects will be "relatively" negligible at these speeds so let's just use the non-relativistic equation,

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{C} \tag{1.211}$$

With,

$$\lambda = 122 \text{ nm} = 1.22 \times 10^{-7} \text{ m} \tag{1.212}$$
$$\Delta\lambda = 1.8 \times 10^{-12} \text{ m} \tag{1.213}$$
$$C \approx 3.0 \times 10^8 \text{ m} \tag{1.214}$$

Since we only care about an approximation, pretend that the two wavelength values of 1.22 and 1.8 are just 1. Plug this all in to get

$$\frac{(3.0 \times 10^8 \text{ m})(1.0 \times 10^{-12} \text{ m})}{(1.0 \times 10^{-7} \text{ m})} = 3.0 \times 10^3 \text{ m/s} \tag{1.215}$$

CAREFUL! The problem asked for the answer in km/s, not m/s like we've solved for. Convert 3.0×10^3 m/s to get 3.0 km/s which is closest to answer (B).

Correct Answer
(B)

1.61 PGRE9677 #61

61. A sphere of radius R carries charge density proportional to the square of the distance from the center: $\rho = Ar^2$, where A is a positive constant. At a distance of $R/2$ from the center, the magnitude of the electric field is

(A) $A/4\pi\epsilon_0$
(B) $AR^3/40\epsilon_0$
(C) $AR^3/24\epsilon_0$
(D) $AR^3/5\epsilon_0$
(E) $AR^3/3\epsilon_0$

Recommended Solution

Gauss's law gives us

$$\vec{E} \cdot d\vec{A} = q/\epsilon_0 \tag{1.216}$$

Taking the integral of both sides and substituting the equation for surface area of a sphere for A_s

$$\int \vec{E} \cdot d\vec{A_s} = EA = \int_0^{R/2} \frac{\rho A_s}{\epsilon_0} dr \tag{1.217}$$

$$E\left(4\pi r^2\right) = \int_0^{R/2} \frac{1}{\epsilon_0} \left(A(r)^2\right)\left(4\pi(r)^2\right) dr \tag{1.218}$$

$$E4\pi\left(\frac{R}{2}\right)^2 = \frac{4\pi A}{5\epsilon_0}\left(\frac{R}{2}\right)^5 \tag{1.219}$$

Start canceling things out and you get

$$E = \frac{A}{5\epsilon_0}\left(\frac{R}{2}\right)^3 = \frac{AR^3}{40\epsilon_0} \tag{1.220}$$

Correct Answer
(B)

1.62 PGRE9677 #62

62. Two capacitors of capacitances 1.0 microfarad and 2.0 microfarads are each charged by being connected across a 5.0-volt battery. They are disconnected from the battery and then connected to each other with resistive wires so that plates of opposite charge are connected together. What will be the magnitude of the final voltage across the 2.0-microfarad capacitor?

(A) 0 V
(B) 0.6 V
(C) 1.7 V
(D) 3.3 V
(E) 5.0 V

Recommended Solution

We can calculate Q_1 and Q_2 when the battery is in the system, using

$$Q_1 = C_1 V = (1.0 \text{ mF})(5.0 \text{ V}) = 5 \text{ mF V} \tag{1.221}$$

$$Q_2 = C_2 V = (2.0 \text{ mF})(5.0 \text{ V}) = 10 \text{ mF V} \tag{1.222}$$

Once the battery is removed the problem tells us that the capacitors are connected to one another such that the "opposite charges are connected together". Doing this, we get Figure 1.9

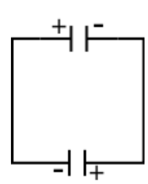


Figure 1.9: Circuit with opposite charges connected together

The potential for both capacitors will be the same so we can calculate V for both capacitors with

$$V = \frac{Q_{eq}}{C_{eq}} \tag{1.223}$$

For a parallel circuit, the orientation of this circuit has its capacitors flipped so $Q_{eq} = Q_2 - Q_1$

$$V = \frac{Q_2 - Q_1}{C_1 + C_2} = \frac{(10\ mF\ V) - (5\ mF\ V)}{(1\ mF) + (2\ mF)} = \frac{5}{3} V \approx 1.7\ V \tag{1.224}$$

Correct Answer
(C)

1.63 PGRE9677 #63

63. According to the Standard Model of elementary particles, which of the following is NOT a composite object?

(A) Muon
(B) Pi-meson
(C) Neutron
(D) Deuteron
(E) Alpha particle

Recommended Solution

(A) Muon: The Muon is one of the leptons, which are the fundamental particles. Muons are similar to electrons in that they have a negative charge and a spin of 1/2. The Muon IS NOT a composite object

(B) Pi-Meson: All Pi-Mesons (also known as a Pion) are composed of some combination of the first generation quarks (Up quark and Down Quark). The Pion IS a composite particle

(C) Neutron: Neutrons have a tri-quark arrangement, 1 Up quark and 2 Down quarks. The Neutron IS a composite particle

(D) Deuteron: A deuteron, the nucleus of a Deuterium atom, is composed of a proton and a neutron as opposed to hydrogen which has just a proton. The Deuteron IS a composite particle

(E) Alpha particle: Alpha particles are composed of 2 Neutrons and 2 Protons. The Alpha particle IS a composite particle.

Correct Answer
(A)

1.64 PGRE9677 #64

64. The binding energy of a heavy nucleus is about 7 million electron volts per nucleon, whereas the binding energy of a medium-weight nucleus is about 8 million electron volts per nucleon. Therefore, the total kinetic energy liberated when a heavy nucleus undergoes symmetric fission is most nearly

(A) 1876 MeV
(B) 938 MeV
(C) 200 MeV
(D) 8 MeV
(E) 7 MeV

Recommended Solution

In symmetric fission, the heavy nucleus is split into two equal halves that we are going to assume are both an example of a "medium-weight nucleus", per the problem description. The kinetic energy after fission will be the difference between the initial total energy and the energy remaining in the two "medium-weight" nuclei.

$$\Delta E = E_{\text{heavy N}} - E_{\text{2 medium N}} \tag{1.225}$$

The energy of the heavy nucleus is given in the description as 8 million eV/nucleon and the energy of the 2 medium nuclei is 7 million eV/nucleon.

$$\Delta E = (1 \text{ nucleus})\left(8\,\frac{\text{MeV}}{\text{nucleon}}\right)(N) - (2 \text{ nuclei})\left(7\,\frac{\text{MeV}}{\text{nucleon}}\right)\left(\frac{1}{2}N\right) \tag{1.226}$$

$$\Delta E = (8 \text{ MeV} - 7 \text{ MeV})N = N(1 \text{ MeV}) \tag{1.227}$$

Then taking some kind of heavy nucleus, say Uranium-238 (N=238), you get

$$\Delta E = 238 \text{ MeV} \tag{1.228}$$

Which is roughly (C).

Correct Answer
(C)

1.65 PGRE9677 #65

65. A man of mass m on an initially stationary boat gets off the boat by leaping to the left in an exactly horizontal direction. Immediately after the leap, the boat, of mass M, is observed to be moving to the right at speed v. How much work did the man do during the leap (both on his own body and on the boat) ?

(A) $\frac{1}{2}Mv^2$

(B) $\frac{1}{2}mv^2$

(C) $\frac{1}{2}(M+m)v^2$

(D) $\frac{1}{2}\left(M+\frac{M^2}{m}\right)v^2$

(E) $\frac{1}{2}\left(\frac{Mm}{M+m}\right)v^2$

Recommended Solution

The quickest method for solving this problem is to consider when the mans mass goes to infinity. When this occurs, the man won't move at all, do to his infinite mass, and so the only energy will involve the movement of the boat with kinetic energy $U_k = \frac{1}{2}Mv^2$. The only choice that gives this equation at the limit is (D).

Correct Answer
(D)

Alternate Solution

Energy and momentum are conserved, so we get the following equations

$$P_{tot} = mv - MV = 0 \tag{1.229}$$
$$mv = MV \tag{1.230}$$

$$U_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 \tag{1.231}$$

All of the possible solutions involve only M, v and m, so solve for V.

$$V = \frac{mv}{M} \tag{1.232}$$

Substitute this into Equation 1.231

$$U_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2 \tag{1.233}$$

$$\frac{1}{2}\left(m + \frac{m^2}{M}\right)v^2 \tag{1.234}$$

Correct Answer
(D)

1.66 PGRE9677 #66

66. When it is about the same distance from the Sun as is Jupiter, a spacecraft on a mission to the outer planets has a speed that is 1.5 times the speed of Jupiter in its orbit. Which of the following describes the orbit of the spacecraft about the Sun?

(A) Spiral
(B) Circle
(C) Ellipse
(D) Parabola
(E) Hyperbola

Recommended Solution

The problem clearly states that the spacecraft is "on a mission to the outer planets" meaning that it must have an orbit with an escape trajectory. Only Parabolic and Hyperbolic orbits are escape trajectories so eliminate (A), (B) and (C). Choosing between (D) and (E), go with (E) because hyperbolic orbits occur at high velocities, like 1.5 times the speed of Jupiter, while parabolic orbits happen at lower speeds and are generally more rare.

Correct Answer
(E)

1.67 PGRE9677 #67

67. A black hole is an object whose gravitational field is so strong that even light cannot escape. To what approximate radius would Earth (mass $= 5.98 \times 10^{24}$ kilograms) have to be compressed in order to become a black hole?

(A) 1 nm
(B) 1 μm
(C) 1 cm
(D) 100 m
(E) 10 km

Recommended Solution

Recall the equation for the event horizon (Schwarzschild radius) of an object is

$$R_e = \frac{2GM}{c^2} \tag{1.235}$$

The mass of the earth is given and G and C can be found in the list of constants in your GRE booklet. Plug these all in to get

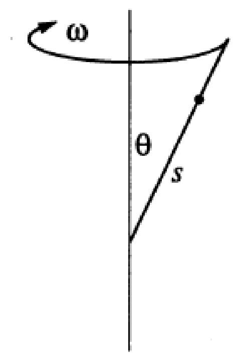
$$R_e = \frac{2(6.67 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2})(5.98 \times 10^{24}\ \mathrm{kg})}{(3.0 \times 10^8\ \mathrm{m/s})^2} \tag{1.236}$$

Rounding all of the numbers and multiplying out gives

$$R_e = \frac{84 \times 10^{13}\ \mathrm{m^3/s^2}}{9 \times 10^{16}\ \mathrm{m^2/s^2}} \approx 1 \times 10^{-3}\ \mathrm{m} = 1\ \mathrm{cm} \tag{1.237}$$

Correct Answer
(C)

1.68 PGRE9677 #68



68. A bead is constrained to slide on a frictionless rod that is fixed at an angle θ with a vertical axis and is rotating with angular frequency ω about the axis, as shown above. Taking the distance s along the rod as the variable, the Lagrangian for the bead is equal to

(A) $\frac{1}{2}m\dot{s}^2 - mgs\cos\theta$

(B) $\frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(\omega s)^2 - mgs$

(C) $\frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(\omega s\cos\theta)^2 + mgs\cos\theta$

(D) $\frac{1}{2}m(\dot{s}\sin\theta)^2 - mgs\cos\theta$

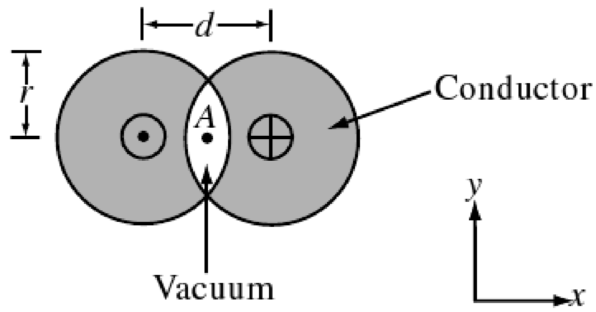
(E) $\frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(\omega s\sin\theta)^2 - mgs\cos\theta$

Recommended Solution

A quick trick to figuring out the solution to this is to consider which variables the Lagrangian must be dependent on. Looking over the diagram and considering that the Lagrangian is the difference between Kinetic and Potential energy, you should be able to convince yourself that it will have a dependence on (m, s, θ, ω). Look through each of the potential solutions to see that only (C) and (E) match this criteria. Now, consider when the angle between the rod and the vertical is 0 ($\theta = 0$). In this case, the potential energy term should go to 0 because the bead won't be able to move up the rod if the axis of rotation is parallel to the rod. Thus, we must have a term with a sine or tangent function in it so that the angle is forced to 0 in the potential energy term, which is only the case for (E).

Correct Answer
(E)

1.69 PGRE9677 #69



69. Two long conductors are arranged as shown above to form overlapping cylinders, each of radius r, whose centers are separated by a distance d. Current of density J flows into the plane of the page along the shaded part of one conductor and an equal current flows out of the plane of the page along the shaded portion of the other, as shown. What are the magnitude and direction of the magnetic field at point A ?

(A) $(\mu_0/2\pi)\pi dJ$, in the $+y$-direction
(B) $(\mu_0/2\pi)d^2J/r$, in the $+y$-direction
(C) $(\mu_0/2\pi)4d^2J/r$, in the $-y$-direction
(D) $(\mu_0/2\pi)Jr^2/d$, in the $-y$-direction
(E) There is no magnetic field at A.

Recommended Solution

Remember our good friend the right hand rule, which gives us, among other things, the direction of magnetic field vectors from a moving current (Note: this is the general principle behind how a solenoid works).

From the diagram given in the problem, we have a current going “into the page” on the right and coming out of the page on the left. If you've done everything right and aren't too embarrassed to be making hand gestures at your computer screen/test booklet, then you are giving the problem a thumbs up (I'm going to assume that this isn't a case of you approving of the GRE). From this, only (A) and (B) could be correct. The only difference between (A) and (B) is a dependence on r. If we were talking about current I, rather than current density, J, then we would care about the radius (i.e. the size) of the conductive cables. However, since the problem talks about everything in terms of a constant current density, J, we know there should be no dependence on r. In case you aren't convinced, compare Maxwell's equations for magnetic fields with dependence on I vs dependence on J.

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dl \times \hat{r}}{r^2} \tag{1.238}$$

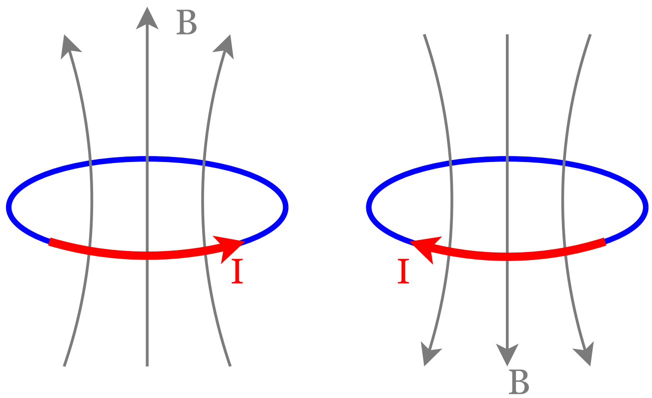


Figure 1.10: Magnetic fields generated as the result of a moving electric field

as compared to

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \tag{1.239}$$

Correct Answer
(A)

1.70 PGRE9677 #70

70. A charged particle, A, moving at a speed much less than c, decelerates uniformly. A second particle, B, has one-half the mass, twice the charge, three times the velocity, and four times the acceleration of particle A. According to classical electrodynamics, the ratio P_B/P_A of the powers radiated is

(A) 16
(B) 32
(C) 48
(D) 64
(E) 72

Recommended Solution

The Larmor Formula can be used to calculate the power radiated in non-relativistic motion of a charged particle

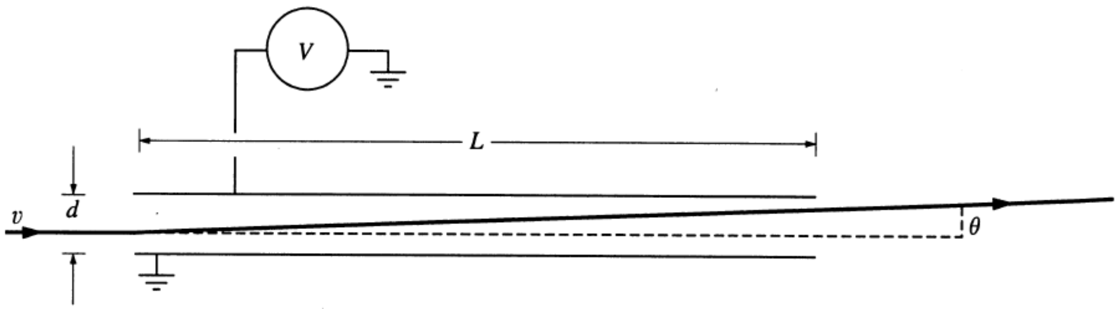
$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} \tag{1.240}$$

The problem states that particle B has half the mass ($\frac{1}{2}m$), twice the charge ($2q$), three times the velocity ($3v$) and four times the acceleration ($3a$). The Larmor Formula isn't dependent on mass or velocity so we are only concerned with charge and acceleration. Since the denominator of the Lamor Formula won't be altered for either particle, we only care about the numerator.

$$\frac{P_B}{P_A} = \frac{(q_B^2 a_B^2)}{(q_A^2 a_A^2)} = \frac{(4q_A^2 16a_A^2)}{(q_A^2 a_A^2)} = 64 \tag{1.241}$$

Correct Answer
(D)

1.71 PGRE9677 #71



71. The figure above shows the trajectory of a particle that is deflected as it moves through the uniform electric field between parallel plates. There is potential difference V and distance d between the plates, and they have length L. The particle (mass m, charge q) has nonrelativistic speed v before it enters the field, and its direction at this time is perpendicular to the field. For small deflections, which of the following expressions is the best approximation to the deflection angle θ ?

(A) arctan ($(L/d)(Vq/mv^2)$)
(B) arctan ($(L/d)(Vq/mv^2)^2$)
(C) arctan ($(L/d)^2(Vq/mv^2)$)
(D) arctan ($(L/d)(2Vq/mv^2)^{1/2}$)
(E) arctan ($(L/d)^{1/2}(2Vq/mv^2)$)

Recommended Solution

The angle of deflection for this particle can be calculated as

$$\tan(\theta) = \frac{\dot{y}}{\dot{x}} \tag{1.242}$$

We already know the velocity in the x direction as v, so

$$\tan(\theta) = \frac{\dot{y}}{v} \tag{1.243}$$

Force due to gravity will be minimal relative to the Lorentz force so we'll assume it is 0. This gives us a net force of

$$F_{net,y} = m\ddot{y} = q[\vec{E} + (\vec{v} \times \vec{B})] \tag{1.244}$$

The problem says nothing about a magnetic field existing (even though it should) so we'll assume $\vec{B} = 0$, giving

$$\begin{aligned} m\ddot{y} &= q\vec{E} = \frac{qV}{d} && (1.245) \\ \ddot{y} &= \frac{qV}{dm} && (1.246) \end{aligned}$$

We know that the velocity in the x-direction is $\dot{x} = L/t$ and the acceleration in the y-direction is $\ddot{y} = \dot{y}/t$. Solve for t in both equations and equate the two, to get

$$L/\dot{x} = \dot{y}/\ddot{y} \tag{1.247}$$

Solve for velocity in the y-direction to get

$$\dot{y} = L\ddot{y}/\dot{x} \tag{1.248}$$

We already solved for $\ddot{y}$ in Equation 1.246, so we can plug that in to get

$$\dot{y} = \frac{LqV}{dm\dot{x}} \tag{1.249}$$

then, plug 1.249 into 1.242 to get

$$\tan(\theta) = \frac{LqV}{dm\dot{x}v} \tag{1.250}$$

and since $\dot{x} = v$, we can solve for θ to get

$$\theta = \tan^{-1}\left(\frac{LVq}{dmv^2}\right) \tag{1.251}$$

Correct Answer
(A)

1.72 PGRE9677 #72

72. In a voltage amplifier, which of the following is NOT usually a result of introducing negative feedback?

(A) Increased amplification
(B) Increased bandwidth
(C) Increased stability
(D) Decreased distortion
(E) Decreased voltage gain

Recommended Solution

This solution is far from rigorous but it is the quickest way to solve the problem. Negative feedback and positive feedback function similarly in an electronic circuit as it does in acoustics. Positive feedback of an audio wave involves an increase in amplitude for the wave. Think along the lines of placing a microphone too close to a speaker and making that high pitched squeal. Negative feedback, on the other hand, should cancel out some of the amplitude. From this, you should immediately know that (A) can't be an aspect of negative feedback.

Correct Answer
(A)

1.73 PGRE9677 #73

73. The adiabatic expansion of an ideal gas is described by the equation $PV^{\gamma} = C$, where γ and C are constants. The work done by the gas in expanding adiabatically from the state $(V_i,\ P_i)$ to $(V_f,\ P_f)$ is equal to

(A) $P_f V_f$

(B) $\frac{(P_i + P_f)}{2}(V_f - V_i)$

(C) $\frac{P_f V_f - P_i V_i}{1 - \gamma}$

(D) $\frac{P_i\,({V_f}^{1+\gamma} - {V_i}^{1+\gamma})}{1 + \gamma}$

(E) $\frac{P_f\,({V_f}^{1-\gamma} - {V_i}^{1-\gamma})}{1 + \gamma}$

Recommended Solution

For a thermodynamic expansion, work done is

$$W = \int_{V_i}^{V_f} P\ dV \tag{1.252}$$

The problem gives us the ideal adiabatic expansion equation as

$$PV^{\gamma} = C \tag{1.253}$$

$$P = \frac{C}{V^{\gamma}} = CV^{-\gamma} \tag{1.254}$$

Making the substitution into the work equation and taking the integral should give us

$$W = C\int_{V_i}^{V_f} V^{-\gamma} dV = \left[\frac{C}{1-\gamma}V^{1-\gamma}\right]_{V_i}^{V_f} \tag{1.255}$$

At this point, you can see that the denominator must contain a $1-\gamma$ term and you can choose (C). However, if you want you can always substitute in C to get

$$\frac{P_f V_f - P_i V_i}{1-\gamma} \tag{1.256}$$

Correct Answer
(C)

1.74 PGRE9677 #74

74. A body of mass m with specific heat C at temperature 500 K is brought into contact with an identical body at temperature 100 K, and the two are isolated from their surroundings. The change in entropy of the system is equal to

(A) $(4/3)mC$
(B) $mC\ln(9/5)$
(C) $mC\ln(3)$
(D) $-mC\ln(5/3)$
(E) 0

Recommended Solution

Spontaneous events in a thermodynamic system always have positive value changes in entropy so get rid of (D) and (E). Additionally, since we know that the change in entropy is

$$\Delta S = \int_{T_1}^{T_2} \frac{dq}{T} \tag{1.257}$$

We are going to get a natural log component (unless one of our temperatures is 0 which is not the case) so eliminate (A). If you can't get any farther than this, at least you go it down to 2 choices and you can guess. The next step you should take is to use Equation 1.257 to calculate the net change in entropy as the sum change in entropy of the 2 masses

$$\Delta S_{net} = \Delta S_1 + \Delta S_2 \tag{1.258}$$

Because the masses are of the same size, they will both reach a temperature as an average of the two

$$\frac{500\text{ K} + 100\text{ K}}{2} = 300\text{ K} \tag{1.259}$$

then, using $dq = mCdT$ and equation 1.259, we get

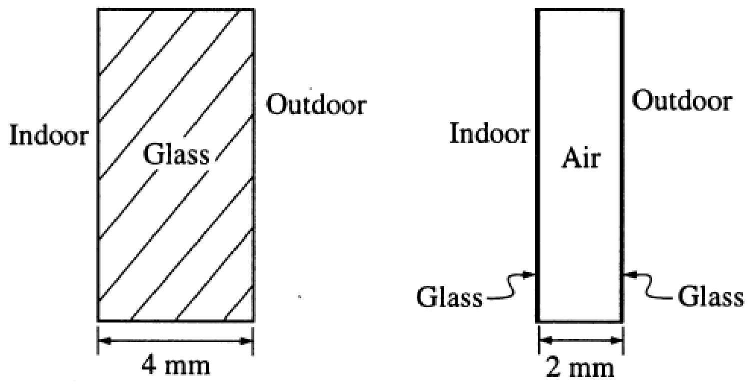
$$\Delta S_{net} = mC\left[\int_{100}^{300} \frac{dT}{T_1} + \int_{500}^{300} \frac{dT}{T_2}\right] = mC[ln(3) + ln(3/5)] = mCln(9/5) \tag{1.260}$$

Correct Answer
(B)

Additional Note

In the event that ETS didn't give us objects with the same mass, it should be relatively straightforward to calculate the final heat of the system by the conservation of energy (heat) as $Q_1 + Q_2 = 0$.

1.75 PGRE9677 #75



Window *A* Window *B*

75. Window *A* is a pane of glass 4 millimeters thick, as shown above. Window *B* is a sandwich consisting of two extremely thin layers of glass separated by an air gap 2 millimeters thick, as shown above. If the thermal conductivities of glass and air are 0.8 watt/meter °C and 0.025 watt/meter °C, respectively, then the ratio of the heat flow through window *A* to the heat flow through window *B* is

(A) 2
(B) 4
(C) 8
(D) 16
(E) 32

Recommended Solution

Fourier's law of heat conduction gives

$$\frac{\partial Q}{\partial t} = -k \oint_S \nabla T \cdot d\vec{A} \tag{1.261}$$

$$Q = -kA\frac{\Delta T}{\Delta X} \tag{1.262}$$

Which tells us that heat transfer is proportional to the thermal conductivity of the material, k and the cross-sectional area of the material but inversely related to the length the heat transfers through. In this problem, the cross sectional area is the same for both, so the ratio is

$$\frac{Q_A}{Q_B} = \frac{(0.8 \text{ watt/m}^{\circ}C)/(4 \text{ mm})}{(0.025 \text{ watt/m}^{\circ}C)/(2 \text{ mm})} = 16 \tag{1.263}$$

Correct Answer
(D)

1.76 PGRE9677 #76

76. A Gaussian wave packet travels through free space. Which of the following statements about the wave packet are correct for all such wave packets?

    I. The average momentum of the wave packet is zero.
    II. The width of the wave packet increases with time, as $t \to \infty$.
    III. The amplitude of the wave packet remains constant with time.
    IV. The narrower the wave packet is in momentum space, the wider it is in coordinate space.

    (A) I and III only
    (B) II and IV only
    (C) I, II, and IV only
    (D) II, III, and IV only
    (E) I, II, III, and IV

Recommended Solution

Analysis of Gaussian wave packets are fascinating because some of the more interesting and familiar quantum mechanical laws fall out of them. In particular, the Heisenberg Uncertainty principle is one of those results

$$\Delta x \; \Delta p \geq \frac{\hbar}{2} \tag{1.264}$$

This is relevant because the uncertainty principle tells us that the wavepackets momentum can never be 0, meaning I is not possible. Eliminate (A), (C), (E). Now, compare II and III. If it's true that the"width of the wave packet increases with time" then it isn't possible for the statement "Amplitude of the wave packet remains constant with time" to also be true, since the wave packet stretching in time would alter the amplitude. From this, (D) can't be true, leaving you with choice (B).

Correct Answer
(B)

1.77 PGRE9677 #77

77. Two ions, 1 and 2, at fixed separation, with spin angular momentum operators $\mathbf{S_1}$ and $\mathbf{S_2}$, have the interaction Hamiltonian $H = -J\, \mathbf{S_1} \cdot \mathbf{S_2}$, where $J > 0$. The values of $\mathbf{S_1}^2$ and $\mathbf{S_2}^2$ are fixed at $S_1(S_1+1)$ and $S_2(S_2+1)$, respectively. Which of the following is the energy of the ground state of the system?

(A) 0

(B) $-JS_1S_2$

(C) $-J[S_1(S_1+1) - S_2(S_2+1)]$

(D) $-(J/2)[(S_1+S_2)(S_1+S_2+1) - S_1(S_1+1) - S_2(S_2+1)]$

(E) $-\frac{J}{2}\left[\frac{S_1(S_1+1)+S_2(S_2+1)}{(S_1+S_2)(S_1+S_2+1)}\right]$

Recommended Solution

Recall that the expectation value for energy is given by the Hamiltonian operator as

$$\langle H(t)\rangle = \langle\psi|H(t)|\psi\rangle \tag{1.265}$$

We are given the Hamiltonian

$$H = -JS_1 \cdot S_2 \tag{1.266}$$

and spin operators

$$S_1^2\psi_1 = S_1(S_1+1)\psi_1 \tag{1.267}$$

$$S_2^2\psi_2 = S_2(S_2+1)\psi_2 \tag{1.268}$$

We can use the polynomial identity

$$a_1 \cdot a_2 = \frac{1}{2}\left[(a_1+a_2)^2 - a_1^2 - a_2^2\right] \tag{1.269}$$

to get

$$\langle \psi_1 | H(t) | \psi_2 \rangle = -\frac{J}{2}[(S_1 + S_2)^2 - S_1^2 - S_2^2] \tag{1.270}$$

Substitute in the spin operators (Equations 1.267 and 1.268) to get

$$\langle H(t) \rangle = -\frac{J}{2}[(S_1 + S_2)(S_1 + S_2 + 1) - S_1(S_1 + 1) - S_2(S_2 + 1)] \tag{1.271}$$

Correct Answer
(D)

1.78 PGRE9677 #78

78. In an n-type semiconductor, which of the following is true of impurity atoms?
    (A) They accept electrons from the filled valence band into empty energy levels just above the valence band.
    (B) They accept electrons from the filled valence band into empty energy levels just below the valence band.
    (C) They accept electrons from the conduction band into empty energy levels just below the conduction band.
    (D) They donate electrons to the filled valence band from donor levels just above the valence band.
    (E) They donate electrons to the conduction band from filled donor levels just below the conduction band.

Recommended Solution

Semiconductors are useful devices because it is relatively straightforward to alter a semiconductors conductive properties by intentionally adding impurities to the semiconductor lattice. The process of adding these impurities is known as doping. Dopants in the lattice of an n-type semiconductor alter conductivity by donating their own weakly bound valence electrons to the material. This is precisely the description of (E).

In general, you should try to remember that dopants in an n-type semiconductor always contribute electrons to the lattice rather than take from the lattice (those would be p-type semiconductors). Once you've concluded that the only solutions could be (D) or (E) it should seem reasonable that electrons that are donated aren't going to get donated *to* a full valence shell.

Correct Answer
(E)

1.79 PGRE9677 #79

79. For an ideal diatomic gas in thermal equilibrium, the ratio of the molar heat capacity at constant volume at very high temperatures to that at very low temperatures is equal to

(A) 1
(B) 5/3
(C) 2
(D) 7/3
(E) 3

Recommended Solution

In its most general form, heat capacity C is given by

$$C = \left(\frac{\Delta Q}{\Delta T}\right) \tag{1.272}$$

Which at the very least gives us our temperature dependence for the heat capacity. Specific to this problem, recall the energy level diagrams from thermodynamics.

the heat capacity of an ideal gas is proportional to the sum of the degrees of freedom for each of the three energy levels. For a monotonic gas, particles will have three translational degrees of freedom corresponding to the three components of motion $(\vec{x}, \vec{y}, \vec{z})$

$$C_v = \frac{3}{2}R \tag{1.273}$$

For a diatomic molecule, we have to figure into the heat capacity the linear vibrational energy and the rotational energy

$$C_v = \frac{3}{2}R + R_{vib} + R_{rot} \tag{1.274}$$

From the energy level diagram (Figure 1.11), we can see that small changes in energy level correspond to translational motion. However, large quantities of energy are required for vibrational and rotational energy to play a part. From this, we get the low temperature (i.e. low energy) heat capacity as

$$C_{v-low} = \frac{3}{2}R \tag{1.275}$$

and high temperature (i.e. high energy) heat capacity as

$$C_{v-high} = \frac{3}{2}R + R_{vib} + R_{rot} = \frac{7}{2}R \tag{1.276}$$

So the ratio of high temperature heat capacity to low temperature heat capacity is

$$C_{v-high}/C_{v-low} = \frac{7/2\ R}{3/2\ R} = \frac{7}{3} \tag{1.277}$$

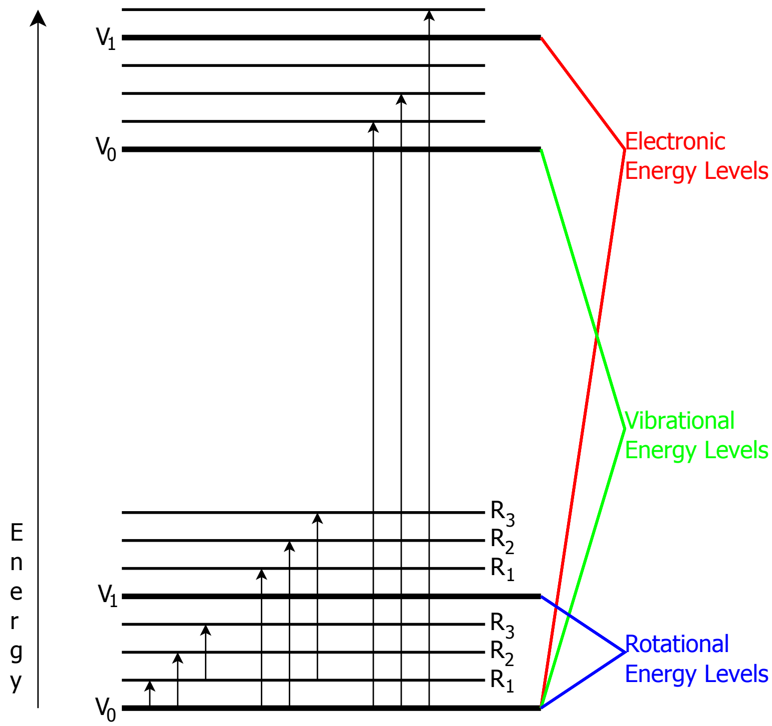
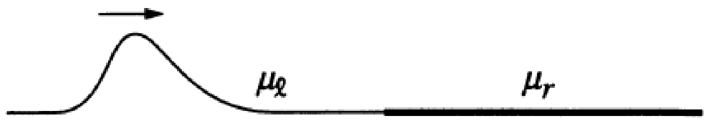


Figure 1.11: Energy level diagram and electron transitions

Correct Answer
(D)

1.80 PGRE9677 #80



80. A string consists of two parts attached at $x = 0$. The right part of the string ($x > 0$) has mass μ_r per unit length and the left part of the string ($x < 0$) has mass μ_ℓ per unit length. The string tension is T. If a wave of unit amplitude travels along the left part of the string, as shown in the figure above, what is the amplitude of the wave that is transmitted to the right part of the string?

(A) 1

(B) $\dfrac{2}{1 + \sqrt{\mu_\ell/\mu_r}}$

(C) $\dfrac{2\sqrt{\mu_\ell/\mu_r}}{1 + \sqrt{\mu_\ell/\mu_r}}$

(D) $\dfrac{\sqrt{\mu_\ell/\mu_r} - 1}{\sqrt{\mu_\ell/\mu_r} + 1}$

(E) 0

Recommended Solution

Immediately get rid of (A) and (E) because they suggest no dependence on the mass of either string. Next, consider the scenario in which the string on the right becomes infinitely massive (i.e. $\mu_r \to \infty$). When this occurs it won't be possible for any amount of energy on the left string to create an amplitude on an infinitely massive string on the right and so amplitude should go to 0. Under this condition (B) will become 2, (D) will go become -1 so these can't be correct. (C) is the only one which goes to 0 when $\mu_r \to \infty$.

Correct Answer
(C)

Alternate Solution

Consider the case when $\mu_l = \mu_r$. In this case, the two part string of different masses becomes a single string with one mass, call it μ, and the original amplitude of 1 will be maintained. From this (E) and (D) can be eliminated and (A) can be eliminated simply because it doesn't acknowledge the dependency on string mass. Finally, eliminate (D) because the amplitude doesn't go to 0 when the mass, μ_r goes to infinity.

Correct Answer
(C)

1.81 PGRE9677 #81

81. A piano tuner who wishes to tune the note D_2 corresponding to a frequency of 73.416 hertz has tuned A_4 to a frequency of 440.000 hertz. Which harmonic of D_2 (counting the fundamental as the first harmonic) will give the lowest number of beats per second, and approximately how many beats will this be when the two notes are tuned properly?

	Harmonic	Number of Beats
(A)	6	5
(B)	6	0.5
(C)	5	0.1
(D)	3	0.372
(E)	2	4.5

Recommended Solution

This is one of the few problems I would recommend doing the math in full gritty detail. The number of beats between two waves comes from the difference in frequency between them. Beats can be observed (heard), for example, when two musical instruments are out of tune. To minimize the beats with a frequency of 73.416 Hz for D_2, we will need the harmonic multiplied by that frequency to be very close to 440 Hz. Of the harmonics given, 6 is the most reasonable both from the perspective of quick mental math and from the perspective that ETS likes to keep you from getting the correct answer by knowing only one of the pieces of information (i.e. there are two solutions with a harmonic of 6). Multiplying everything out completely, you should get

$$(73.416 \text{ Hz})(6) = 440.49600 \text{ Hz} \approx 440.5 \text{ Hz} \tag{1.278}$$

Since the number of beats is just the difference between the two frequencies,

$$440.5 \text{ Hz} - 440.0 \text{ Hz} = 0.05 \text{ Beats} \tag{1.279}$$

Correct Answer
(B)

1.82 PGRE9677 #82

82. Consider two horizontal glass plates with a thin film of air between them. For what values of the thickness of the film of air will the film, as seen by reflected light, appear bright if it is illuminated normally from above by blue light of wavelength 488 nanometers?

(A) 0, 122 nm, 244 nm
(B) 0, 122 nm, 366 nm
(C) 0, 244 nm, 488 nm
(D) 122 nm, 244 nm, 366 nm
(E) 122 nm, 366 nm, 610 nm

Recommended Solution

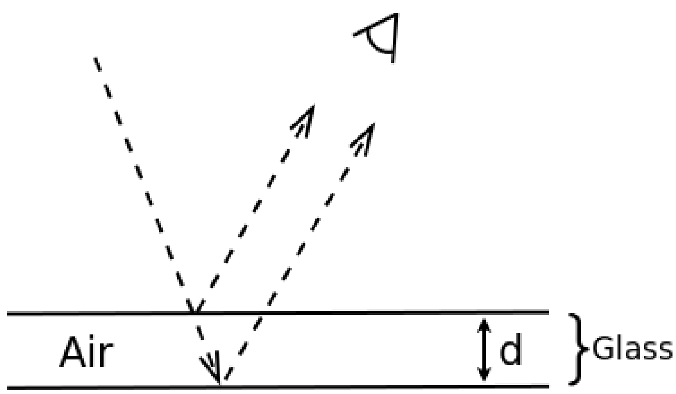


Figure 1.12: Reflection of light on a thin film

The equation for constructive interference of a thin film (Figure **??** is

$$2nd = \left(m + \frac{1}{2}\right)\lambda \tag{1.280}$$

Plug in $n = 1$ for air our value for the wavelength as 488 nm,

$$2d = m\lambda + \frac{1}{2}\lambda \tag{1.281}$$

$$d = m\left(\frac{448\ nm}{2}\right) + \left(\frac{448\ nm}{4}\right) = m(244\ nm) + 122\ nm \tag{1.282}$$

From this, we get

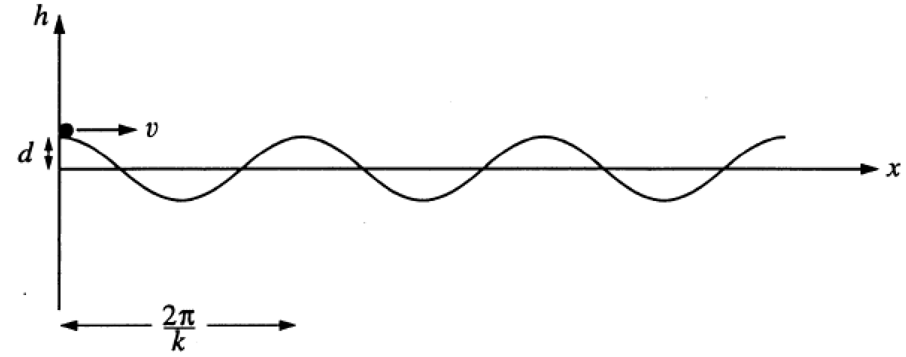
When $m = 0$**:** $d = 122\ nm$

When $m = 1$**:** $d = 366\ nm$

When $m = 2$**:** $d = 610\ nm$

Correct Answer
(E)

1.83 PGRE9677 #83



83. Consider a particle moving without friction on a rippled surface, as shown above. Gravity acts down in the negative h direction. The elevation $h(x)$ of the surface is given by $h(x) = d \cos(kx)$. If the particle starts at $x = 0$ with a speed v in the x direction, for what values of v will the particle stay on the surface at all times?

(A) $v \leq \sqrt{gd}$

(B) $v \leq \sqrt{\frac{g}{k}}$

(C) $v \leq \sqrt{gkd^2}$

(D) $v \leq \sqrt{\frac{g}{k^2 d}}$

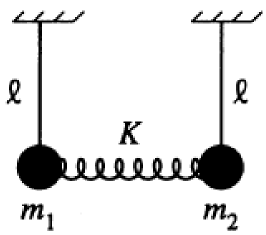
(E) $v > 0$

Recommended Solution

Note that when d goes to infinity, every peak and trough will be infinitely tall/short and you won't even need a velocity to get the ball to free fall (i.e. when $d \to \infty$ then $v \to 0$). The only equation which fits the bill is (D).

Correct Answer
(D)

1.84 PGRE9677 #84



84. Two pendulums are attached to a massless spring, as shown above. The arms of the pendulums are of identical lengths ℓ, but the pendulum balls have unequal masses m_1 and m_2. The initial distance between the masses is the equilibrium length of the spring, which has spring constant K. What is the highest normal mode frequency of this system?

(A) $\sqrt{g/\ell}$

(B) $\sqrt{\dfrac{K}{m_1+m_2}}$

(C) $\sqrt{\dfrac{K}{m_1}+\dfrac{K}{m_2}}$

(D) $\sqrt{\dfrac{g}{\ell}+\dfrac{K}{m_1}+\dfrac{K}{m_2}}$

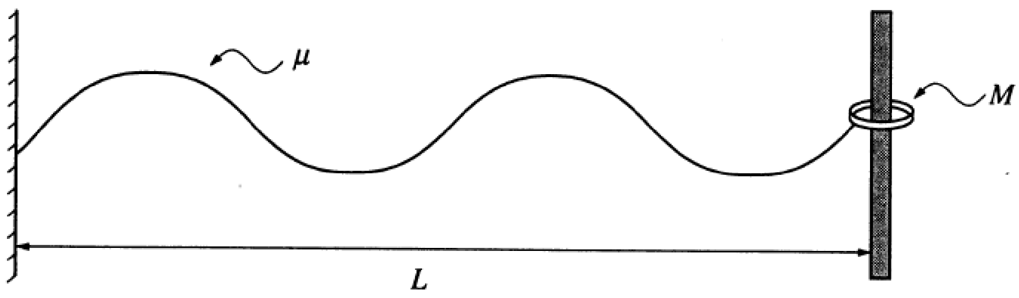
(E) $\sqrt{\dfrac{2g}{\ell}+\dfrac{K}{m_1+m_2}}$

Recommended Solution

Immediately get rid of (A) as it suggests that the normal mode has no dependence on either of the masses. Next, note that if we let the mass of one of our masses, say m_2 go to infinity, then the other mass will oscillate about that mass as if it were connected to a stationary object. When this happens, dependence on mass m_2 should disappear but dependence on m_1 should remain. For (B) and (E), allowing $m_2 \to \infty$ forces the entire term, including m_1 to disappear. Finally, comparing (C) and (D), get rid of (C) because it suggests that the normal mode has no dependence on the acceleration due to g or l, which is not true.

Correct Answer
(D)

1.85 PGRE9677 #85



85. Small-amplitude standing waves of wavelength λ occur on a string with tension T, mass per unit length μ, and length L. One end of the string is fixed and the other end is attached to a ring of mass M that slides on a frictionless rod, as shown in the figure above. When gravity is neglected, which of the following conditions correctly determines the wavelength? (You might want to consider the limiting cases $M \to 0$ and $M \to \infty$.)

(A) $\mu/M = \frac{2\pi}{\lambda} \cot \frac{2\pi L}{\lambda}$

(B) $\mu/M = \frac{2\pi}{\lambda} \tan \frac{2\pi L}{\lambda}$

(C) $\mu/M = \frac{2\pi}{\lambda} \sin \frac{2\pi L}{\lambda}$

(D) $\lambda = 2L/n$, $n = 1, 2, 3, \ldots$

(E) $\lambda = 2L/(n + \frac{1}{2})$, $n = 1, 2, 3, \ldots$

Recommended Solution

Taking the recommendation at the end of the problem, consider the limiting cases of $M \to 0$ and $M \to \infty$. Looking through the 5 options, you can immediately eliminate (D) and (E) because they both suggest that the mass of the string and the mass of the ring have no influence on the wavelength, which is not correct.

Considering the first limiting case, when $M \to 0$ then $\mu/M \to \infty$. In the case of (C), a sine function is going to limit the maximum and minimum values so we can eliminate (C). since $\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$ and $\tan(x) = \frac{\sin(x)}{\cos(x)}$, both can blow up to infinity if the bottom trig function goes to 0.

Considering the second limiting case, when $M \to \infty$ then $\mu/M \to 0$. It is also the case that when mass goes to infinity, the ring won't move from any amount of force placed on the string so what we have is a fixed end on the ring side. This means that the only wavelengths possible are lengths of $L = \frac{n\,\lambda}{2}$. This is the case because the fixed end on each side acts as a node and you have a bound standing wave. Checking this requirement with both (A) and (B), gives

(A) $\mu/M = 0 = cot(\frac{2\pi L}{\lambda}) \Longrightarrow cot^{-1}(0) = \pi/2 + n\ \pi = \frac{2\pi L}{\lambda} \Longrightarrow L = \frac{n}{2} + \frac{1}{4}$

(B) $\mu/M = 0 = \tan(\frac{2\pi L}{\lambda}) \Longrightarrow \tan^{-1}(0) = 0 + n\ \pi = \frac{2\pi L}{\lambda} \Longrightarrow L = \frac{n\lambda}{2}$

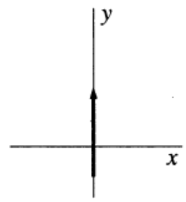
Of which, only (B) meets the necessary criteria.

Correct Answer
(B)

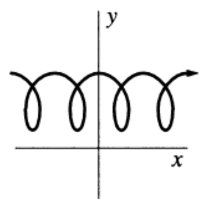
1.86 PGRE9677 #86

86. A positively charged particle is moving in the xy-plane in a region where there is a non-zero uniform magnetic field B in the $+z$-direction and a non-zero uniform electric field E in the $+y$-direction. Which of the following is a possible trajectory for the particle?

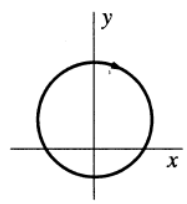
(A)



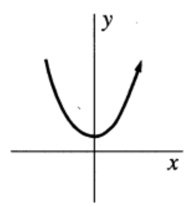
(B)



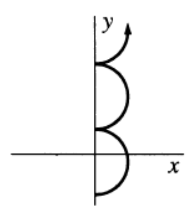
(C)



(D)



(E)

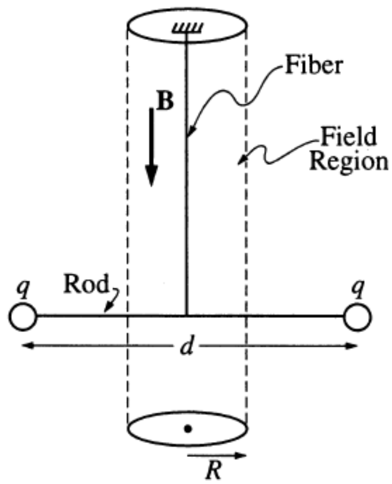


Recommended Solution

As a general rule, particles moving in an orthogonal direction to a magnetic field ($\vec{B}$) will exhibit cyclotron (helix shaped) motion with a direction of spin in agreement with the right hand rule. Of the choices, only (B) and (E) exhibit this phenomena and only (B) demonstrates actual helical motion.

Correct Answer
(B)

1.87 PGRE9677 #87



87. Two small pith balls, each carrying a charge q, are attached to the ends of a light rod of length d, which is suspended from the ceiling by a thin torsion-free fiber, as shown in the figure above. There is a uniform magnetic field $\mathbf{B}$, pointing straight down, in the cylindrical region of radius R around the fiber. The system is initially at rest. If the magnetic field is turned off, which of the following describes what happens to the system?

(A) It rotates with angular momentum qBR^2.

(B) It rotates with angular momentum $\frac{1}{4}qBd^2$.

(C) It rotates with angular momentum $\frac{1}{2}qBRd$.

(D) It does not rotate because to do so would violate conservation of angular momentum.

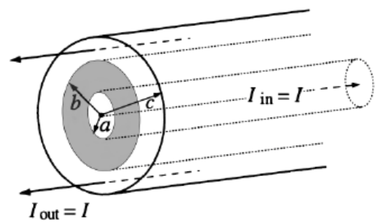
(E) It does not move because magnetic forces do no work.

Recommended Solution

Immediately eliminate (D) and (E) because they both suggest that charges in a magnetic field won't cause the pith balls to move which is incorrect. Eliminate (B) because it doesn't have a dependence on R and if R went to 0, the magnetic field would as well. Finally, consider that if we had a dependence on d as in (C) and let $d \to \infty$, then angular momentum would also need to be infinite which would violate conservation of the momentum built up from the magnetic field.

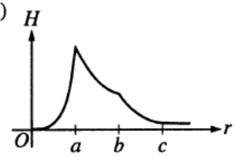
Correct Answer
(A)

1.88 PGRE9677 #88

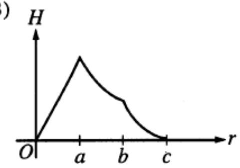


88. A coaxial cable has the cross section shown in the figure above. The shaded region is insulated. The regions in which $r < a$ and $b < r < c$ are conducting. A uniform dc current density of total current I flows along the inner part of the cable ($r < a$) and returns along the outer part of the cable ($b < r < c$) in the directions shown. The radial dependence of the magnitude of the magnetic field, H, is shown by which of the following?

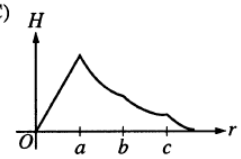
(A)



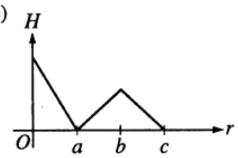
(B)



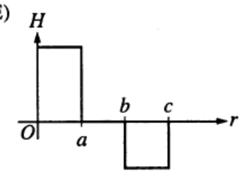
(C)



(D)



(E)



Recommended Solution

Immediately eliminate all solutions that suggest that the magnetic field is anything other than 0 at the origin, specifically (E) and (D). There are a number of ways you can convince yourself this criteria must be true. For example, consider integrating the magnetic field generated over an infinitely small surface area which would give you no magnetic field at all. Alternatively, recall that a larger magnetic field vector is indicative of a larger magnetic field and that the magnetic field vectors get smaller as R gets smaller, until it approaches $R = 0$ when $\vec{B} = 0$ (Figure 1.13).

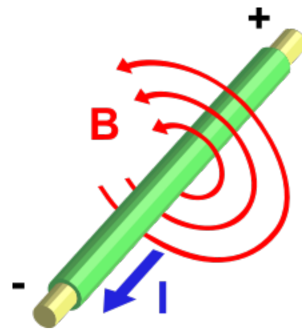
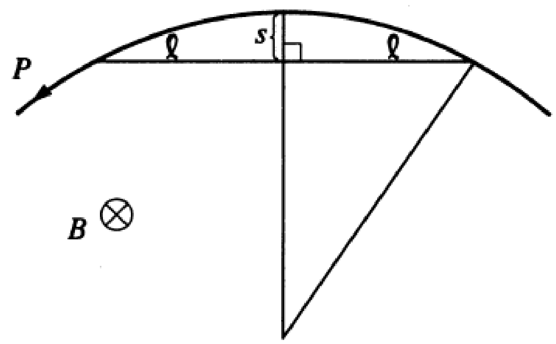


Figure 1.13: Magnetic field vectors decrease in magnitude as $R \to 0$

Next, eliminate all solutions which don't suggest that the magnetic field at point c and all radii larger than c is 0., i.e. (C) and (A). You can convince yourself that this must be the cases because the two cables have equivalent magnetic fields moving in opposite directions which will cancel each other out at and past point c.

Correct Answer
(B)

1.89 PGRE9677 #89



89. A particle with charge q and momentum p is moving in the horizontal plane under the action of a uniform vertical magnetic field of magnitude B. Measurements are made of the particle's trajectory to determine the "sagitta" s and half-chord length ℓ, as shown in the figure above. Which of the following expressions gives the particle's momentum in terms of q, B, s, and ℓ? (Assume $s << \ell$.)

(A) $qBs^2/2\ell$

(B) qBs^2/ℓ

(C) $qB\ell/s$

(D) $qB\ell^2/2s$

(E) $qB\ell^2/8s$

Recommended Solution

As soon as you see a charged particle in a magnetic field, think Lorentz force, $F = q(\vec{v} \times \vec{B})$. In our particular problem the velocity vector and magnetic field vectors are orthogonal, so the cross product of $\vec{v} \times \vec{B} = vB$. Since the object is rotating the net force on the particle should also be equal to the centripetal force, $F = \frac{m\ v^2}{R}$. Set these two equations equal to one another and solve for momentum, $p = mv$.

$$qvB = \frac{m\ v^2}{R} \tag{1.283}$$

$$m\ v = qBR \tag{1.284}$$

Using Pythagorean theorem, solve for the hypotenuse of the triangle drawn in the diagram, which also happens to be our radius R. You should get

$$R^2 = l^2 + (R-s)^2 \tag{1.285}$$

expand the $(R-s)^2$ term to get

$$R^2 = l^2 + R^2 - 2sR + s^2 \tag{1.286}$$

$$l^2 - 2sR + s^2 = 0 \tag{1.287}$$

Now, recall that $s << l$ so we can let $s = 0$. Solve for R, and you get

$$R = l^2/2s \tag{1.288}$$

Substitute that back into our previous equation and you have

$$p = qBl^2/2s \tag{1.289}$$

Correct Answer
(D)

Alternate Solution

First, we would expect the momentum of the particle to increase as the arc of the particles motion becomes more linear (i.e. as $s \to 0$). From this, you can immediately get rid of (A) and (B) because both predict that momentum goes to 0 as the sagitta goes to 0. Next, we know that the final result should have units of $N \cdot s$ or $\frac{kg\ m}{s}$. Recall that the unit for the magnetic field, the Tesla, is $\frac{kg}{A \cdot s^2}$ and units for electrical charge, the Coulomb, is $A \cdot s$. From this, you can see only (D) and (E) can have correct units, so get rid of (C). Finally, you will need to find some way to decide whether the denominator of the solution is $2s$ or $8s$. You can do this by recognizing, as we did in the "Recommended Solution", that the force on the particle will be a centripetal force, $\frac{m\ v^2}{R}$. Since m and v aren't lengths, we only care about the relationship between s and R. Using Pythagorean theorem, $R^2 = (R-s)^2 + l^2$. Expand everything and you will see that you get a $2s$ term. Eliminate choice (E) based on this

Correct Answer
(D)

1.90 PGRE9677 #90

90. THIS ITEM WAS NOT SCORED.

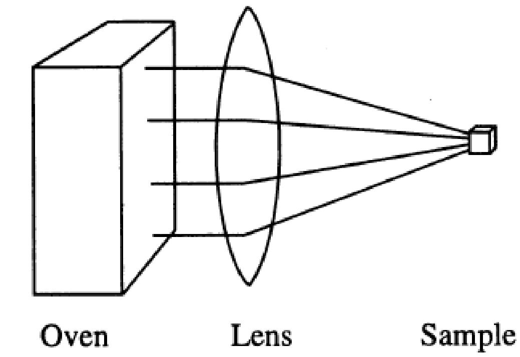
Recommended Solution

Pick your favorite letter between A, B, C, and D.

Alternate Solution

In the middle of the quiet exam room, with many other test takers concentrating on what could very well change their futures forever, stand up and walk to the nearest wall. Using a piece of tape, gum, other adhesive compound, attach your test booklet to the wall so that it is open and the possible solutions are facing you. Locate a long sharp object, perhaps a small flag pole or a wooden pointer and from 10 paces away, charge the test booklet while making horse noises (your motivation is a medieval knight in a jousting competition). Stab through your test booklet and check to see which of the 4 choices was punctured. If you managed to spear a bunch of white space or some non-related part of the problem, return to your starting point and charge the test again. Repeat this process until you've managed to skewer one of the 4 possible solutions. While this method is preferable to the Recommended Solution in both style and form, it can be more time consuming and so it should be your second method of attack. Note that the word "attac" in the previous sentence is used both figuratively and literally.

1.91 PGRE9677 #91



91. An experimenter needs to heat a small sample to 900 K, but the only available oven has a maximum temperature of 600 K. Could the experimenter heat the sample to 900 K by using a large lens to concentrate the radiation from the oven onto the sample, as shown above?

(A) Yes, if the volume of the oven is at least 3/2 the volume of the sample.
(B) Yes, if the area of the front of the oven is at least 3/2 the area of the front of the sample.
(C) Yes, if the sample is placed at the focal point of the lens.
(D) No, because it would violate conservation of energy.
(E) No, because it would violate the second law of thermodynamics.

Recommended Solution

There are a number of different descriptions of the second law of thermodynamics. One of those, the one relevant to this problem, is known as the Clausius statement, which says

> It is impossible to move heat from a high temperature source to a low temperature source unless external work is done

This tells us that the oven will transfer heat to the sample as long as the temperature of the oven is higher than the sample. However, if the temperature of the sample were able to get higher than the oven, then temperature flow would switch to moving from the sample back to the oven, which has become the lower temperature source. In other words, the temperature of the sample will never be able to exceed that of the oven because temperature flow will simply reverse any time it does. The sample can never achieve 900 K.

Correct Answer
(E)

1.92 PGRE9677 #92

92. A particle of mass m moves in a one-dimensional potential $V(x) = -ax^2 + bx^4$, where a and b are positive constants. The angular frequency of small oscillations about the minima of the potential is equal to

(A) $\pi(a/2b)^{1/2}$
(B) $\pi(a/m)^{1/2}$
(C) $(a/mb)^{1/2}$
(D) $2(a/m)^{1/2}$
(E) $(a/2m)^{1/2}$

Recommended Solution

First of all, we know that there will be a dependence on the mass of the particle so eliminate (A). Based on the statement about small oscillations, you can conclude that bx^4 term is not particularly influential so it likely doesn't depend on b and we can eliminate (C). For (B), ETS is trying to play off of your possibly memorized equation for angular frequency as

$$\omega = \frac{2\pi}{T} \tag{1.290}$$

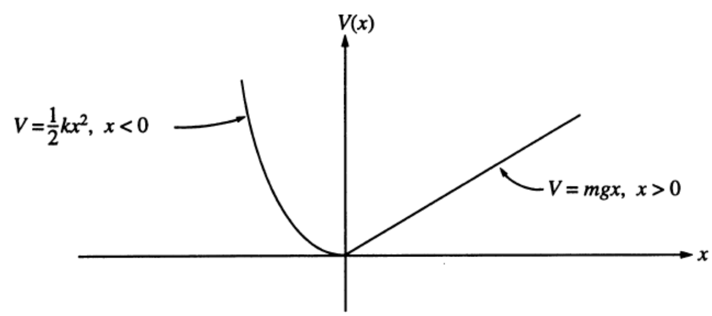
But don't be fooled because we shouldn't expect to get a π out of this oscillator. Finally, between (D) and (E), recall that the angular frequency for a SHO is

$$\omega = \sqrt{\frac{k}{m}} \tag{1.291}$$

Since angular frequency requires us to take the derivative of the potential and we are treating the potential as a 2nd order polynomial, you can bet we are going to get a 2 down from differentiation.

Correct Answer
(D)

1.93 PGRE9677 #93



93. A particle of mass m moves in the potential shown above. The period of the motion when the particle has energy E is

(A) $\sqrt{k/m}$

(B) $2\pi\sqrt{m/k}$

(C) $2\sqrt{2E/mg^2}$

(D) $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$

(E) $2\pi\sqrt{m/k} + 4\sqrt{2E/mg^2}$

Recommended Solution

If we let $k \to 0$ then $V = 0$. This tells us, at the very least, that the period of motion is dependent on k. (C) doesn't account for this, get rid of it. If $g \to 0$, again the period should be influenced. (A) and (B) don't appropriately account for this so eliminate them. Finally, recall that the period for a simple harmonic oscillator is

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{1.292}$$

But this only applies to one side of the equation so we want half of the term. One of the components should be $(\frac{1}{2})2\pi\sqrt{m/k} = \pi\sqrt{m/k}$. This requirement matches the first term of (D).

Correct Answer
(D)

1.94 PGRE9677 #94

94. A system consists of N weakly interacting subsystems, each with two internal quantum states with energies 0 and ϵ. The internal energy for this system at absolute temperature T is equal to

(A) $N\epsilon$

(B) $\frac{3}{2}NkT$

(C) $N\epsilon e^{-\epsilon/kT}$

(D) $\dfrac{N\epsilon}{(e^{\epsilon/kT}+1)}$

(E) $\dfrac{N\epsilon}{(1+e^{-\epsilon/kT})}$

Recommended Solution

Recall from Statistical Mechanics the fact that with infinite energy available to a system, the total possible energy states will be populated equally in order to minimize the total number of "alternative" microstates the system could occupy. From this, we know that as $T \to \infty$ we would expect a system with only two energy levels to each contain half of the total particles. Let $T \to \infty$ to see which solution fulfills this requirement.

(A) $N\epsilon$: No Temperature dependence so this isn't a possible solution.

(B) $\frac{3}{2}NK(\infty) = \infty$

(C) $N\epsilon e^{-\epsilon/k(\infty)} = \infty$

(D) $\frac{N\epsilon}{(e^{\epsilon/kT}+1)} = \frac{N\epsilon}{2}$

(E) $\frac{N\epsilon}{(1+e^{-\epsilon/kT})} = \frac{N\epsilon}{2}$

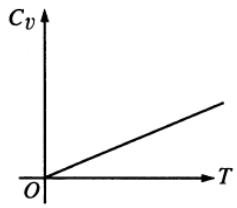
Both (D) and (E) fulfill the requirement, so now you can figure that (D) is correct because we would expect energy to approach 0 as $T \to 0$ which isn't true of (E).

Correct Answer
(D)

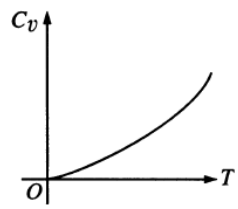
1.95 PGRE9677 #95

95. Which of the following curves is characteristic of the specific heat C_V of a metal such as lead, tin, or aluminum in the temperature region where it becomes superconducting?

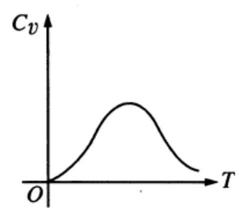
(A)



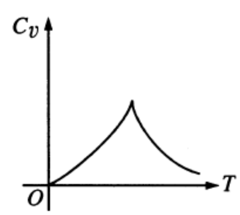
(B)



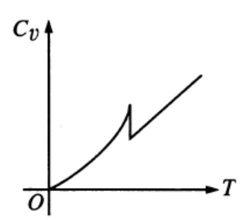
(C)



(D)



(E)



Recommended Solution

The key to solving this problem comes in the description, which tells us that we are concerned with the, "region where it BECOMES superconducting". When an object hits that temperature that enables it to exhibit superconductivity (a jump in resistance), the specific heat also changes instantly, meaning we would expect an instantaneous jump at some temperature. Of the 5 possible solutions, only (E) exhibits this jump.

Correct Answer
(E)

1.96 PGRE9677 #96

96. Which of the following reasons explains why a photon cannot decay to an electron and a positron ($\gamma \rightarrow e^+ + e^-$) in free space?

(A) Linear momentum and energy are not both conserved.
(B) Linear momentum and angular momentum are not both conserved.
(C) Angular momentum and parity are not both conserved.
(D) Parity and strangeness are not both conserved.
(E) Charge and lepton number are not both conserved.

Recommended Solution

One of the unique differences between the photon and the electron (well the photon and other particles) is its ability to maintain the same speed in all conceivable reference frames. From this, it should be clear that kinetic energy of a photon can never be 0 in any frame. However, it is also true of a particle with mass that a frame can be chosen in which kinetic energy and momentum can be 0.

Correct Answer
(A)

1.97 PGRE9677 #97

97. A particle of mass m has the wave function $\psi(x, t) = e^{i\omega t}[\alpha \cos(kx) + \beta \sin(kx)]$, where α and β are complex constants and ω and k are real constants. The probability current density is equal to which of the following? (Note: α^* denotes the complex conjugate of α, and $|\alpha|^2 = \alpha^*\alpha$.)

(A) 0

(B) $\hbar k/m$

(C) $\frac{\hbar k}{2m}(|\alpha|^2 + |\beta|^2)$

(D) $\frac{\hbar k}{m}(|\alpha|^2 - |\beta|^2)$

(E) $\frac{\hbar k}{2mi}(\alpha^*\beta - \beta^*\alpha)$

Recommended Solution

The probability current equation is

$$\vec{J}(x,t) = \frac{\hbar}{2mi}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right) \tag{1.293}$$

Take the derivative of both wave functions

$$\psi' = ke^{i\omega t}\left[-\alpha sin(kx) + \beta cos(kx)\right] \tag{1.294}$$

$$\psi^{*\,\prime} = -ke^{i\omega t}\left[-\alpha sin(kx) + \beta cos(kx)\right] \tag{1.295}$$

Plug everything in to get

$$\frac{\hbar}{2mi}\gamma \tag{1.296}$$

where γ is

$$\begin{aligned}\gamma \quad = \quad & \left[\left(e^{-i\omega t}\left[\alpha \text{ cos } kx + \beta \text{ sin } kx\right]\right)\left(ke^{i\omega t}\left[-\alpha \text{ sin } kx + \beta \text{ cos } kx\right]\right)\right] \\ - \quad & \left[\left(-ke^{i\omega t}\left[-\alpha \text{ sin } kx + \beta \text{ cos } kx\right]\right)\left(e^{i\omega t}\left[\alpha \text{ cos } kx + \beta \text{ sin } kx\right]\right)\right]\end{aligned}$$

At this point, you could simplify the equation and get an exact solution. However, it is much quicker to recognize that all terms with α^2 and β^2 cancel out and so you only get left with $\alpha\beta$ and $\beta\alpha$ terms. From this, you can get that the solution must be (E).

Correct Answer
(B)

1.98 PGRE9677 #98

98. A particle of mass m is acted on by a harmonic force with potential energy function $V(x) = m\omega^2x^2/2$ (a one-dimensional simple harmonic oscillator). If there is a wall at $x = 0$ so that $V = \infty$ for $x < 0$, then the energy levels are equal to

(A) $0,\ \hbar\omega,\ 2\hbar\omega,\ \ldots$

(B) $0,\ \frac{\hbar\omega}{2},\ \hbar\omega,\ \ldots$

(C) $\frac{\hbar\omega}{2},\ \frac{3\hbar\omega}{2},\ \frac{5\hbar\omega}{2},\ \ldots$

(D) $\frac{3\hbar\omega}{2},\ \frac{7\hbar\omega}{2},\ \frac{11\hbar\omega}{2},\ \ldots$

(E) $0,\ \frac{3\hbar\omega}{2},\ \frac{5\hbar\omega}{2},\ \ldots$

Recommended Solution

First off, eliminate solution that suggests the first energy level is ever 0. Having an energy of 0 is a problem because this would imply that we have a completely stationary particle which also then means that we have precisely defined the particles position and momentum (oops!). Next, recall the equation for the energy levels of the one-dimensional harmonic oscillator from QM is

$$V_n = \hbar\omega\left(n + \frac{1}{2}\right) = \frac{\hbar\omega}{2}, \frac{3\hbar\omega}{2} \frac{5\hbar\omega}{2} \frac{7\hbar\omega}{2}, \cdots \tag{1.297}$$

However, note that the problem tells us that there is an infinite potential wall at the center. This won't effect all of our odd termed energies because they always have a node at the center but even valued energies have a peak at this point and so they will be disrupted.

Taking only the odd valued energies,

$$V_n = \frac{3\hbar\omega}{2}, \frac{7\hbar\omega}{2}, \frac{11\hbar\omega}{2}, \frac{15\hbar\omega}{2}, \cdots \tag{1.298}$$

Correct Answer
(D)

1.99 PGRE9677 #99

99. The electronic energy levels of atoms of a certain gas are given by $E_n = E_1 n^2$, where $n = 1, 2, 3, \ldots$. Assume that transitions are allowed between all levels. If one wanted to construct a laser from this gas by pumping the $n = 1 \rightarrow n = 3$ transition, which energy level or levels would have to be metastable?

(A) $n = 1$ only

(B) $n = 2$ only

(C) $n = 1$ and $n = 3$ only

(D) $n = 1$, $n = 2$, and $n = 3$

(E) None

Recommended Solution

A metastable state in an n-level laser is any state that acts as an "energy trap" or an intermediate energy level. You can think of a metastable energy level as helping to support transitions from lower to higher levels by providing a stepping stone and temporary energy location for the transition. Metastable states are, by definition, only intermediate states so our solution shouldn't include $n = 1$ or $n = 3$. The only energy level between $n = 1$ and $n = 3$ is $n = 2$.

Correct Answer
(B)

1.100 PGRE9677 #100

100. The operator $\hat{a} = \sqrt{\frac{m\omega_0}{2\hbar}}(\hat{x} + i\frac{\hat{p}}{m\omega_0})$, when operating on a harmonic energy eigenstate Ψ_n with energy E_n, produces another energy eigenstate whose energy is $E_n - \hbar\omega_0$. Which of the following is true?

I. $\hat{a}$ commutes with the Hamiltonian.
II. $\hat{a}$ is a Hermitian operator and therefore an observable.
III. The adjoint operator $\hat{a}^\dagger \neq \hat{a}$.

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

Recommended Solution

By definition, a Hermitian operator is an observable operator and an observable operator commutes so If I is true, II is also true. Without looking at anything else, we know that the only possible combination of choices will be (I and II), (I, II and III) or (III only). We could go through the explanation for why I and II being true implies III is false and vice versa, but since all 3 being true isn't even an option, we won't bother. The only two options are

(C) III only

(D) I and II only

Consider that for an operator to be adjoint, it must satisfy the condition

$$\langle \hat{a}x, y\rangle = \langle x, \hat{a}^\dagger y\rangle \tag{1.299}$$

$\hat{a}^\dagger$ isn't technically a complex conjugate but it is analogous to one (at least it is in a Hilbert space in which you treat operators as complex numbers, which is true of $\hat{a}$). If we were to apply a complex conjugate to $\hat{a}$ then we would get a sign change at which point it would be clear that $\hat{a}^\dagger \neq \hat{a}$. Since III is true, I and II can't be true.

Correct Answer
(C)