GR0877 SOLUTIONS

Detailed solutions

to the GRE Physics Test

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1. (**B**) Once the ball has been released, the only acting force on it is a gravitational force (the problem ignores friction). Since gravity has a zero horizontal component, the ball, as viewed from above, moves in a straight line. Thus, one should eliminate all but choices (B) and (D). The car is moving to the right, while the ball is thrown perpendicularly to this direction, so the initial velocity of the ball is directed to the south-east (and preserve this direction in the future). This is choice (B).

2. (**D**) Horizontal and vertical motions of the object are independent (if we ignore friction). Therefore, no one really needs the initial *horizontal* component of the ball's velocity to determine how long it will take to cover some distance in the *vertical* direction. The initial vertical component of the velocity is zero, so from $H = gt^2/2$ one gets $H = (9.8 \cdot 2^2)/2 = 9.8 \cdot 2 = 19.6$ (m).

3. (**E**) The dissipating power is $P = U^2/R$, where U is the voltage across the resistor R. So, if you double U, the power will *quadruple*. Comment: you cannot use $P = UI$, because if you change U the current I will also change.

4. (**E**) The loop exactly lies on a magnetic field line of the long wire. Thus, at every point of the loop the magnetic field due to the wire is parallel to the direction of the current I_2. $\mathbf{F}_{mag} = I_2 \int (d\mathbf{l} \times \mathbf{B})$.

5. (**A**) What else could be?

6. (**E**) The nth shell can accommodate $2n^2$ electrons. Thus, the total number of electrons is $2(1^2 + 2^2) = 2 \cdot 5 = 10$.

7. (**C**) From $m\overline{v^2}/2 = 3kT/2$ it's easy to obtain $\sqrt{3kT/m}$ which is choice (C).

8. (**D**) This is a straightforward application of the Stefan-Boltzmann law. If the temperature is increased by the factor of two, the radiated power and the mass of the ice that can be melt will increase by the factor of $2^4 = 16$.

9. (**E**) Statements II, I and III represent, respectively, the first, the second and the third Kepler's law. Thus, choice (E).

10. (**B**) From the conservation of energy one has $ks^2/2 = mv^2/2$ and $s = v\sqrt{m/k}$.

11. (**C**) The energy levels of a quantum harmonic oscillator are $E_n = \hbar\omega \left(n + \frac{1}{2}\right)$, for $n = 0, 1, 2, \dots$ The ground state is the lowest-energy state with $n = 0$ and $E = \frac{1}{2}\hbar\omega$.

12. (**C**) One of the Bohr's postulates is that the angular momentum of the electron is an integer multiple of $\hbar$: $mvr_n = n\hbar$. From this, one has $mv = n\hbar/r_n$ which is (C).

13. (**A**) On a log-log plot a function of the form $y = ax^m$ will appear as a straight line, where m is the slope of the line and a is the y-value corresponding to $x = 1$. Indeed, $\log_{10} y = \log_{10}(ax^m) = \log_{10} a + m \log_{10} x$. Thus, if $x = 1$, then $y = a$ and $m = \dfrac{\log_{10} y_2 - \log_{10} y_1}{\log_{10} x_2 - \log_{10} x_1} = \dfrac{\log_{10}(y_2/y_1)}{\log_{10}(x_2/x_1)}$). In our case, for $x = 1$, $y \approx 6$ and $m = \dfrac{\log_{10}(100/10)}{\log_{10}(300/3)} = \dfrac{1}{2}$. Therefore, $y = 6\sqrt{x}$.

14. (**B**) (See a wonderful book by John R. Taylor An introduction to error analysis, Chapter 7) If we have N separate measurements of a quantity x: $x_1 \pm \sigma_1$, $x_2 \pm \sigma_2$, $\dots$, $x_N \pm \sigma_N$, then the best estimate is the weighted average $x_{wav} = \dfrac{\sum_{i=1}^N w_i x_i}{\sum_{i=1}^N w_i}$, where $w_i = 1/\sigma_i^2$, and the uncertainty in x_{wav} is $\sigma_{wav} = \left(\sum_{i=1}^N w_i\right)^{-1/2}$. Thus, $\sigma_{wav} = \left(\frac{1}{1^2} + \frac{1}{2^2}\right)^{-1/2} = \sqrt{4/5} = 2/\sqrt{5}$.

15. (**E**) Remember a general formula for lenses $\frac{1}{F} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Here R_1 is the radius of curvature of the lens surface closest to the light source and R_2 is the radius of curvature of the lens surface farthest from the light source. Sign convention: the radius of curvature is positive if the center of spherical surface lies to the right from the lens and is negative if the center of the spherical surface lies to the left. Let the source of light be situated to the left from each of the lenses. Then, for (A) one has $R_1 = -R$, $R_2 = R$ and $F_A \sim -R/2$. For (B): $R_1 = \infty, R2 = R$, $F_B \sim -R$. (C): $R_1 = \infty, R2 = -R$, $F_C \sim R$. For (D) and (E): $R_1 = R$, $R_2 = -R$, $F \sim R/2$. But $R_E < R_D$, therefore $F_E < F_D$.

16. (**D**) When unpolarized light passes through the first polarizer it loses one-half of its intensity. Why half? Here is an explanation. A beam of unpolarized light is nothing but a uniform mixture of linear polarizations at all possible angles. When a perfect polarizer is placed in a polarized beam of light, the intensity of the light that passes through is $I = I_0 \cos^2\theta$ (Malus' law). The average value of $\cos^2\theta$ is 1/2, therefore the intensity is half of the initial: $I_{first} = I_0/2$. After the first polarizer the light is linearly polarized and the fraction of light that passes through the second polarizer is (again, according to Malus' law): $I = I_{first}\cos^2 45° = I_0/4$ or 25%.

17. (**A**) Let the Gaussian surface be a cylinder of radius r and length L. The axis of the cylinder coincides with the wire. Gauss's law: $2\pi RLE = \lambda L/\epsilon_0$, from which $E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$. The answer can also be found by elimination of choices, since (A) is the only choice with correct dimensions.

18. (**C**) This is an application of Lenz's law. As the magnet *enters* the loop, the flux through the loop increases. The induced current generates a magnetic field that is opposing the bar magnet's field. This current is counter-clockwise (that is, form b to a). As the magnet *leaves* the loop, the flux decreases and the current flows clockwise. Choice (E).

19. (**A**) According to Wien's law $\lambda_1 T_1 = \lambda_2 T_2 = b$, from which $\lambda_2 = \lambda_1 T_1/T_2 = 500 \cdot 6000/300 = 10^4$ (nm) or 10 μm.

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20. (**A**) The temperature dependence of the radiation is $T_r \propto a^{-1}$, where a is a cosmic scale factor. Thus, when the temperature was higher by the factor of 4 $(12/3 = 4)$, the distances were four times less than today. See Misner, Thorne and Wheeler, Gravitation, Chapter 28.

21. (**C**) For an adiabatic process $PV^\gamma = const$. If one use $PV/T = const$ for an ideal gas, one has $P = const \cdot T/V$. Plugging this into $PV^\gamma = const$ yields $TV^{\gamma-1} = const$.

22. (**C**) $E = \gamma m_e c^2 \equiv 4m_e c^2$. From this, $\gamma = 4$ and $v = c\sqrt{1 - 1/\gamma^2} = \sqrt{15}c/4$. The momentum: $p = \gamma m_e v = \sqrt{15} m_e c$.

23. (**B**) Imagine the earth (S frame, x-axis is to the right and passes through spaceships) and two spaceships approach to the earth from the left and right with equal speeds v. Let's move to a reference frame of the *left* spaceship (system S', two systems are oriented the same way). Then, according to the velocity addition rule, velocity of the *right* spaceship in this frame of reference is: $u'_r = \dfrac{u_r - V}{1 - u_r V/c^2}$, where $u_r = -v$ is the velocity of the right spaceship in the S frame and V denotes the speed of S' relative to S, $V = v$. Thus, $u'_r = \dfrac{-2v}{1 + v^2/c^2}$. But this is exactly the speed with which two spaceships approach one another (as seeing from their reference frames). And we know from the length contraction formula that, if the relative speed between two frames is u, a stick at rest with respect to one reference frame is observed from the other reference frame to be contracted by the factor of $\gamma = 1/\sqrt{1 - u^2/c^2} = l_0/l$. Problem statement suggests $\gamma = l_0/l = 1/0.6 = 5/3$. Substituting u'_r into the equation for γ and working with units in which $c = 1$:

$$\frac{5}{3} = \frac{1}{\sqrt{1 - \frac{4v^2}{(1+v^2)^2}}} = \frac{1+v^2}{1-v^2},$$

$5 - 5v^2 = 3 + 3v^2$, $2 = 8v^2$, $v = 0.5$ or $v = 0.5c$ as in choice (B).

24. (**B**) To the observer, a meter stick l_0 that passes through with the speed $v = 0.8c$ is observed (sorry for tautology) to be contracted by the factor of $\gamma = 1/\sqrt{1-(0.8)^2} = \frac{5}{3}$. Thus, the time it takes the stick to pass the observer is $\Delta t = \frac{l_0}{\gamma v} = \frac{3}{5 \cdot 0.8 \cdot 3 \cdot 10^8} = 2.5$ (ns).

25. (**E**) The only choice which guarantees that the functions are both normalized and mutually orthogonal is (E).

26. (**D**) The probability that the electron would be found between r and $r + dr$ is $P = |\psi|^2 dV = |\psi|^2 4\pi r^2 dr = p(r)dr$. The most probable value is given by the maximum of the probability density $p(r)$ (take the second derivative if you want to convince yourself that it is indeed the maximum): $dP/dr = 4\pi r^2 d|\psi|^2/dr + 8\pi r|\psi|^2 = 0$. Plugging in: $-\frac{4\pi r^2}{\pi a_0^3}\frac{2}{a_0}e^{-2r/r_0} + \frac{8\pi r}{\pi a_0^3}e^{-2r/r_0} = 0$. From this, one has $r = a_0$ which is nothing else but the Bohr radius.

27. (**C**) This is an application of the energy-time uncertainty principle: $\Delta E \Delta t \gtrsim h$. Rewriting ΔE as $h\Delta\nu$ one has $\Delta\nu \sim \frac{1}{\Delta t} = \frac{1}{\tau} = \frac{1}{1.6 \cdot 10^{-9}} \approx 600$ (MHz), which is closest to (C).

28. (**D**) $2(kx^2/2) = K(x/2)^2/2$ gives $K = 8k$.

29. (**C**) In elastic collisions energy is conserved. So $Mv^2/2 = M(v/2)^2/2 + Mu^2/2$ from which one has $u = \frac{\sqrt{3}}{2}v$.

30. (**D**) I bet one half of the test takers who did this wrong (according to the official practice book only 51 per cent of all 14,395 examinees who took the Physics Test between July 1, 2007 and June 30, 2010 answered this question correctly) simply mixed up choices (C) and (D).

31. (**C**) Applying Archimedes' principle: $\rho g V = \rho_{water} \cdot g(3V)/4 + \rho_{oil} \cdot gV/4 \implies \rho = \frac{3}{4}\rho_{water} + \frac{1}{4}\rho_{oil} = 750 + 200 = 950$ (kg/m^3).

32. (**A**) Who says Bernoulli's principle is unlikely on the PGRE? According to this principle, one has $P_0 + \frac{\rho v_0^2}{2} = P + \frac{\rho v^2}{2}$ (there is no $\rho g z$ term here because of horizontality of the pipe). Conservation of mass gives: $\rho v_0 S = \rho v S \implies v = 4v_0$, since $S = \pi r^2 = (\pi r_0^2)/4 = S_0/4$. Plugging $v = 4v_0$ into the first equation one finally obtain $P = P_0 + \frac{\rho v_0^2}{2} - 16\frac{\rho v_0^2}{2} = P_0 - \frac{15}{2}\rho v_0^2$.

33. (**E**) According to the first law of thermodynamics $dS = \frac{1}{T}dU + \frac{P}{T}dV = mc\frac{dT}{T} + \nu R\frac{dV}{V}$, where c is the specific heat (per one kilogram). Assuming water is incompressible fluid one has $dS = mc\frac{dT}{T}$. Integrating this from T_1 to T_2 one obtain $mc \ln\frac{T_2}{T_1}$.

34. (**C**) The first law of thermodynamics: $Q = \frac{3}{2}\nu R\Delta T$, $Q' = \frac{3}{2}\nu R\Delta T + p\Delta V = \frac{3}{2}\nu R\Delta T + \nu R\Delta T = \frac{5}{2}\nu R\Delta T$. Here we have used $PV = \nu RT$ to rewrite the second term. Thus, $Q' = 5Q/3$.

35. (**B**) Assuming a heat pump is an ideal Carnot engine for its efficiency one has $\eta = W/Q_H = 1 - T_C/T_H$, where W is the work done by the system, Q_H is the heat put into the system, T_C, T_H are the *absolute* temperatures of the cold and hot reservoirs. $W = Q_H(1 - T_C/T_H) = 15000 \cdot (1 - 280/300) = 15000/15 = 1000$ (J).

36. (**A**) The magnetic energy is $LI^2/2$. In LC-contour with initial conditions $q(t=0)=q_0$ and $I_0 \equiv (dq/dt)_0 = 0$ the charge on the capacitor is a cosine function of time and the current is, therefore, a sine function. Thus, magnetic energy is a square of the sine function of time with some proportionality factor. That is, it passes through the origin and has a form similar to (A).

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Dipoles: the electric fields goes as 1/r^3

37. (**E**) Clearly, the electric field is in the $-x$ direction, therefore, only (C) and (E) survive. The magnitude of the electric field at P is $E = 2E_q \cos\theta = 2E_{-q}\cos\theta$ where $Eq = E_{-q}$ are the magnitudes of the electric field at point P due to the charges q and $-q$, respectively; θ is the angle between x-axis and the line passing through the point P and charge $+q$. $\cos\theta = \dfrac{l/2}{\sqrt{r^2+l^2/4}} \implies E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{r^2+l^2/4}\dfrac{l/2}{\sqrt{r^2+l^2/4}}$. For $r \gg l$ we have $E \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{ql}{r^3}$.

38. (**E**) Magnetic field at distance a from a wire carrying a current I is $B = \dfrac{\mu_0 I}{2\pi a}$. From the picture (and right-hand rule) it is obvious that the magnetic field at point P due to the horizontal wire has the same magnitude, but opposite direction, than of the magnetic field at this point due to the vertical wire. Thus, the net magnetic field is zero.

39. (**C**) The lifetime of a muon in the laboratory is $\tau_l = \gamma\tau$, where $\gamma = 1/\sqrt{1-16/25} = 5/3$. The mean distance traveled is $l = v\tau_l = v\gamma\tau = \frac{4\cdot5}{5\cdot3}\cdot 3\cdot10^8\cdot2.2\cdot10^{-6} = 880$ (m).

40. (**B**) Conservation of momentum immediately suggests that after the decay particle m and massless particle have the same momentum p. From the relativistic energy-momentum equation (we use units in which $c=1$) $E^2-p^2=m^2$ one gets $E=p$ for massless particle. The conservation of energy: $M = p+\sqrt{m^2+p^2}$. Moving p to the left-hand side and squaring results in $(M-p)^2 = m^2+p^2$ or $M^2-2Mp = m^2$, from which one has $p = (M^2-m^2)/(2M)$. Clearly, the decay is possible if $M > m$.

41. (**B**) According to Einstein's photoelectric equation: $h\nu = \varphi + K_{max}$, where K_{max} is the maximum kinetic energy of the ejected electron. $K_{max} = eV$, where V is the stopping potential. From these two equations: $V = \frac{h}{e}\nu - \varphi/e$ and the slope is $\frac{h}{e}$.

42. (**E**) From the picture we see that the period of oscillations (this is 2π phase difference) is approximately 6 cm, while the difference between two points of equal voltage-level (say, zero-level) for two waveforms is 2 cm. Thus, the phase difference between waveforms is $2\cdot2\pi/6 = 2\pi/3$.

43. (**D**) If you took solid state physics course, even at introductory level, you should remember this.

tetrahedron: 四面體
octahedron: 八面體

44. (**D**) The attraction in cooper pair is due to the electron-phonon interaction.

See also 0877-solution-2.pdf
And review the general introduction of cooper pair in https://en.wikipedia.org/wiki/Cooper_pair

45. (**C**) According to the general Doppler formula $f_o = \dfrac{c+v_o}{c+v_s}f$, where f_o is observed frequency, c is the speed of sound waves in the medium, v_s and v_o are the speed of the source and the observer *relative to the medium*, respectively. Sign convention: the speed v_s is taken to be positive if the source is moving *away* from the observer at speed v_s, while v_o is taken to be positive if the observer is moving *toward* the source at speed v_o. In our case, the source and observer are moving in such a way that $v_o = v_s$ (and equal to the speed of the wind w). Thus, we have no Doppler effect *at all*: $f_o = f$.

46. (**D**) The first minimum is determined by $d\sin\theta = \lambda$ or $d\sin\theta = c/\nu$. Thus, $\nu = \dfrac{c}{d\sin\theta} \approx \dfrac{350}{0.14\cdot0.7} = \dfrac{500}{0.14} \approx 7\cdot500 = 3500$ (Hz).

47. (D) For a pipe of length L, closed at one end and open at the other the resonant frequencies are given by $f = \frac{v}{\lambda} = \frac{nv}{4L}$, where $n = 1, 3, 5, \ldots$ and v is the speed of the sound. Thus, a fundamental frequency is $f(n=1) = \frac{v}{4L}$ and the next harmonic has a frequency $f(n=3) = \frac{3v}{4L}$. For this next harmonic $f(n=3) = 3f(n=1) = 3 \cdot 131 = 393$ (Hz).

48. (C) We have one NOR-gate with two inverted inputs, one AND-gate and one NAND-gate (you should remember this from your electrical engineering course to solve this problem). For NOR-gate with inverted inputs the output is $\overline{\overline{A} + \overline{B}}$, for NAND-gate $\overline{C \cdot D}$. Thus, $E = \overline{\overline{A} + \overline{B}} \cdot \overline{C \cdot D}$.

49. (D) The only choice in which free *atoms* involved is (D). See 0877-solution-2.pdf

50. (C) Dimension analysis works fine here. However, you can also derive needed expression using Newton's second law and Bohr's quantization rule. Indeed, the former gives $mv^2/r = Ze^2/(4\pi\epsilon_0 r^2)$, while the later $mvr = n\hbar$. From these two equations (excluding v): $r_n \propto \frac{n^2}{me^2 Z}$. Potential energy: $E_{pot} = -Ze^2/(4\pi\epsilon_0 r)$. The total energy: $E = E_{kin} + E_{pot} = -Ze^2/(8\pi\epsilon_0 r)$. Substituting r_n into the expression for the total energy yields $E \propto \frac{mZ^2 e^4}{n^2}$. To account for the motion of the nuclei we can treat m in the last equation as the reduced mass. Thus, choice (C).

51. (D) I: true; II: false, an atom can emit photons of light only with energy equal to the energy difference between two quantum states; III: true. 文字

52. (C) According to Bragg's law $2d \sin\theta = n\lambda$, where θ is the angle between incident ray and scattering planes. In our case $n = 1$ and $d = \frac{\lambda}{2\sin\theta} \approx 2\lambda = 0.500$ (nm).

53. (D) According to the problem statement, the angular momenta are the same: $m_1 v_1 R = m_2 v_2 R$, therefore $m_1/m_2 = v_2/v_1$. For the orbital periods one has $T_1 = 2\pi R/v_1$ and $T_2 = 2\pi R/v_2$ (orbits are circles!). Thus, $m_1/m_2 = v_2/v_1 = T_1/T_2 = 3$. Think about why cannot we apply Kepler's third law in its usual form.

54. (E) A solar-mass black hole would exert no more gravitational pull than our sun. Thus, the orbits would remain unchanged.

55. (A) Applying relativistic Doppler effect formula: $\frac{\lambda_0}{\lambda_{lab}} = \sqrt{\frac{1+\beta}{1-\beta}} \approx \frac{4}{3}$. Thus, $16 - 16\beta = 9 + 9\beta \implies \beta = 7/25 = 0.28$.

56. (D) To fly due north a pilot should point the plane in such a direction that $\mathbf{v} + \mathbf{u} \parallel SN$, where $\mathbf{v}$ is the velocity of the plain in still air, $\mathbf{u}$ is the velocity of the wind, and SN is the south-north line. According to the velocity addition formula, the velocity of the plane in the north direction is $\mathbf{V} = \mathbf{v} + \mathbf{u}$, $\mathbf{V} \perp \mathbf{u}$. From this we have $(\mathbf{V} - \mathbf{u})^2 = (\mathbf{v})^2$, $V^2 + u^2 = v^2$, $V = \sqrt{v^2 - u^2}$, $t = L/V = L/\sqrt{v^2 - u^2} = 500/\sqrt{200^2 - 30^2} = = 50/\sqrt{400 - 9} = 50/\sqrt{391}$ (h).

57. (B) For both figures accelerations of masses $2m$ and m are the same and can be calculated via Newton's second law for the whole system of two bodies: $3ma = F$, $a = F/(3m)$. For the first figure $ma = F_{12}$ and $F_{12} = F/3$. For the second figure $2ma = F_{12}$ and $F_{12} = 2F/3$. Thus, choice (B).

58. (**A**) The only force that provides acceleration a for the block B is the friction force due to the block A. Therefore, by Newton's second law, this force is equal to $m_B a = 10$ kg $\cdot$ 2 m/s$^2 = 20$ N.

59. (**C**) Elimination of choices. (A): wrong, the period must depends on a; (B): for a particular case $a = g$ the period blows up — wrong; (C): reasonable; (D): for $a = 0$, $T = 0$ — wrong; (E): for $a = 0$, T blows up — wrong. Thus, choice (C).

60. (**C**) Take a point $(x, 0, 0)$ on the x-axis. At this point the magnetic field due to the wire along $\hat{\mathbf{z}}$ is $\mu_0 I/(2\pi x)$; due to the wire crossing the first and third quadrants is $\dfrac{\mu_0 I}{2\pi x \sin 45°}$; due to the wire crossing the second and fourth quadrants is $\dfrac{\mu_0 I}{2\pi x \sin 45°}$. Thus, the net magnetic field at point $(x, 0, 0)$ is $\dfrac{\mu_0 I}{2\pi x}(1 + 2\sqrt{2})$.

61. (**E**) Newton's second law: $mv^2/R = qvB$ or $\dfrac{q}{m} \cdot d = const$ (v and B are constants). If we double $\dfrac{q}{m}$, the value of d should decrease by the factor of 2.

62. (**E**) Gauss's law: $\Phi_{total} = \Phi_A + \Phi = q/\epsilon_0$; $\Phi = q/\epsilon_0 - \Phi_A = = \dfrac{1 \cdot 10^{-9}}{8.85 \cdot 10^{-12}} + 100 \approx 100 + 100 = 200$ $(\mathrm{N} \cdot \mathrm{m}^2/C)$. Check the table of information in your test book for the numeric value of ϵ_0 (if you don't remember).

63. (**D**) Given example is a β^+ decay. Beta decay is mediated by the *weak* force.

64. (**D**) Eigenvalues of L^2 are $\hbar^2 l(l+1)$, eigenvalues of L_z are $m\hbar$. Thus, $(\sqrt{2}\hbar)^2 = \hbar^2 l(l+1)$, from which $l = 1$. For a given l, there are $2l + 1$ different values of m: $m = -l, -l+1, \ldots, l-1, l$. In our case, $m = -1, 0, 1$ and, therefore, the possible values of L_z are $-\hbar, 0, \hbar$.

65. (**C**) Step-by-step elimination. I: true: $E_n = \hbar\omega(n + 1/2)$, this is indeed evenly spaced spectrum; II: false, the potential function is a quadratic function, not linear; III: false, the expectation value of both potential and kinetic energy is *half* the total energy $\langle V \rangle = \langle T \rangle = \frac{1}{2}\hbar\omega(1 + 1/2)$. Thus, for $n = 0$, $\langle T \rangle \neq 0$. IV: true.

66. (**D**) To account for the fact that the nucleus has non-infinite mass and moves around atom's CM one must replace the electron mass (muon mass for the muonic atom) with the reduced mass μ. Noting that Rydberg R is proportional to μ we conclude that if we replace electron m_e with the muon m_μ the R is changed by the factor of $\dfrac{\mu_{muon}}{\mu_{electron}} = \dfrac{m_p m_\mu}{m_p + m_\mu} \dfrac{m_p + m_e}{m_p m_e}$.

67. (**D**) At any instant of time an electric field inside the parallel-plate capacitor is $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{Q}{\epsilon_0 S}$, where σ is a surface density of a charge Q on a positive plate. Differentiating this with respect to t gives $\dfrac{dE}{dt} = \dfrac{dQ/dt}{\epsilon_0 S} = \dfrac{I}{\epsilon_0 S} = \dfrac{9}{8.85 \cdot 10^{-12} \cdot (0.5)^2} \approx \dfrac{10^{12}}{0.25} = 4 \cdot 10^{12} \left(\dfrac{\mathrm{V}}{\mathrm{m} \cdot \mathrm{s}}\right)$.

68. (**D**) The total resistance of the circuit is equal to the resistance of three $R + R = 2R$ resistors connected in parallel: $\dfrac{1}{R_{total}} = \dfrac{1}{2R} + \dfrac{1}{2R} + \dfrac{1}{2R} = \dfrac{3}{2R}$, $R_{total} = \dfrac{2R}{3}$. The current flowing through the battery is $I = \dfrac{V}{R_{total}} = \dfrac{3V}{2R}$.

69. (**D**) The impedance of an ideal resistor is $Z_R = R$, and the impedance of an ideal capacitor is $Z_C = \dfrac{1}{i\omega C}$, where i is imaginary unit. The total impedance of the circuit is $Z = Z_R + Z_C =$

$R + \dfrac{1}{i\omega C}$. The total current is $I = V_i/Z = \dfrac{V_i}{R + \frac{1}{i\omega C}}$. The amplitude V_o of the output voltage is $V_o = IZ_C = \dfrac{V_i}{\left(R + \frac{1}{i\omega C}\right) i\omega C} = \dfrac{V_i}{iRC\omega + 1}$. From this one has $G = \dfrac{V_o}{V_i} = \dfrac{1}{iRC\omega + 1}$. Therefore, when $\omega \to \infty$, $G \to 0$ and when $\omega \to 0$, $G \to 1$. The only graph with such a behavior of G is (D).

70. (**A**) According to Faraday's law of induction $\dfrac{\Delta \Phi}{\Delta t} = -\mathcal{E}$. Substituting $\mathcal{E} = IR$, $I = \dfrac{\Delta q}{\Delta t}$ and $\Delta \Phi = B \Delta S$ one has $\Delta q = -\dfrac{B \Delta S}{R} = \dfrac{BS}{R} = \dfrac{0.5 \cdot 10 \cdot 10^{-4}}{5} = 10^{-4}$ (C).

71. (**B**) According to Newton's second law: $mv^2/R = qvB$ or $v \propto R$. Thus, $\dfrac{v_1}{v_2} = \dfrac{r_1}{r_2} = \dfrac{1}{3}$.

72. (**D**) A: bosons do not obey the Pauli exclusion principle; B: bosons do not have antisymmetric wave functions; C: fermions do not have symmetric wave functions; D: true; E: bosons do not obey the Pauli exclusion principle.

73. (**D**) See David Griffiths' Introduction to elementary particles, Chapter I (for both editions). It is strongly recommended to read and understand first two chapters from this book before your test to be in position to answer questions like this one.

74. (**E**) Using lensmakers equation one has $\dfrac{1}{R} + \dfrac{1}{l} = -\dfrac{2}{R}$ (note the minus sign). From this equation $l = -\frac{R}{3}$. The only choice that corresponds to this distance is (E), but if you don't want to invoke lensmakers equation just drop two rays onto the mirror, say, one from the tip of the object parallel to the lens' axis and the second from the tip of the object to the point where the lens' axis intersects the mirror, and you will see where the image is formed (and why it is upright).

75. (**B**) The shift for the wave reflecting off the top surface of the film is $\Delta_{top} = \lambda/2$ (since the light is reflecting from a higher-n medium). The shift for the wave reflecting off the bottom surface of the film is $2t$, where t is the thickness of the film. Thus, $\Delta = \Delta_{bottom} - \Delta_{top} = 2t - \lambda/2$. For constructive interference one has $2t - \lambda/2 = m\lambda$. But λ here is the wavelength *in the medium*, so we must replace it with λ/n: $2t - \dfrac{\lambda}{2n} = m\dfrac{\lambda}{n}$ or $4t = \frac{\lambda}{n}(2m+1)$. The film first appears bright for λ and $m = 0 \implies 4t = \dfrac{\lambda}{n}$. Then it appears bright for λ' and $m = 1 \implies 4t = \dfrac{3\lambda'}{n}$ and $\lambda' = \lambda/3 = 540/3 = 180$ (nm).

76. (**B**) According to Snell's law $\sin\theta = n \sin\beta$, where β is the angle of refraction. The critical angle α_c for the total internal reflection is determined by $\sin\alpha_c = 1/n$, where $\alpha_c + \beta = \pi/2$. From these three equations it is easy to obtain the critical angle of incidence: $\theta = \sin^{-1}\sqrt{n^2 - 1}$. Thus, we must choose between (A) and (B). Now, if we increase θ, β will increase and α will decrease, which prevents the total internal reflection. Therefore, choice (B) is correct.

77. (**C**) The average time between intermolecular collisions is $\tau = l/v = 1/(\sigma n v)$, where l is the mean free path, $v = \sqrt{\dfrac{3kT}{m}}$ is the root mean square speed, σ is the molecule's effective collision area. Thus, the time varies as the square root of m.

78. (**B**) According to the Gibb's distribution $P_i = \dfrac{1}{Z} \exp\left(-\dfrac{E_i}{kT}\right)$, where $Z = \sum_i \exp\left(-\dfrac{E_i}{kT}\right)$.

Thus, $P_2 = \dfrac{\exp\left(-\frac{E_2}{kT}\right)}{\exp\left(-\frac{E_1}{kT}\right) + \exp\left(-\frac{E_2}{kT}\right)}$.

79. (**D**) $P = \dfrac{RT}{V-b} - \dfrac{a}{V^2}$, $A = RT_0 \displaystyle\int_{V_1}^{V_2} \frac{dV}{V-b} - a\int_{V_1}^{V_2} \frac{dV}{V^2} = RT_0 \ln\left(\frac{V_2-b}{V_1-b}\right) + a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$.

80. (**C**) Clearly, $\nu_0 = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m_0}}$, $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{2k}{m}}$, where $\nu_0 = 1$ Hz, $m_0 = 1$ kg, $m = 8$ kg. Thus, $\nu = \nu_0\sqrt{\dfrac{2m_0}{m}} = 1 \cdot \sqrt{\dfrac{2\cdot 1}{8}} = 1/2$ (Hz).

81. (**B**) Kinetic energy of the moving disk is $T = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I_{c.m.}\omega^2 = \dfrac{1}{2}mv^2 + \dfrac{1}{2}\cdot\dfrac{1}{2}mR^2\left(\dfrac{v}{R}\right)^2 = \dfrac{1}{2}mv^2 + \dfrac{1}{4}mv^2 = \dfrac{3}{4}mv^2$. The conservation of energy gives $T = mgh$, from which one has $h = \dfrac{3v^2}{4g}$.

82. (**C**) Kinetic energy of the mass is $T = \frac{1}{2}mr^2\dot{\varphi}^2 + \frac{1}{2}m\dot{r}^2$. Potential energy of the spring is $U = \frac{1}{2}k(r-s)^2$. Lagrangian is $L = T - U$. Choice (D).

83. (**E**) From the Hamilton's equation $\dot{p}_\phi = -\frac{\partial H}{\partial \phi} = 0$ one has $p_\phi = const$.

84. (**E**) The x-coordinate of the rod's center of mass is $x_c = \dfrac{1}{M}\displaystyle\int_0^L xdm = \frac{1}{M}\int_0^L x\frac{2M}{L^2}xdx = \frac{2}{L^2}\int_0^L x^2dx = \frac{2}{L^2}\cdot\frac{L^3}{3} = \frac{2L}{3}$.

85. (**B**) As one can check for himself/herself the constant A is equal to $\sqrt{\dfrac{2}{L}}$ (this is from normalization condition $\displaystyle\int_0^L \psi^2(x)dx = 1$). The probability in question is: $\dfrac{2}{L}\displaystyle\int_{L/3}^{2L/3} \sin^2\left(\frac{3\pi x}{L}\right)dx = \frac{1}{L}\left[\int_{L/3}^{2L/3} dx - \int_{L/3}^{2L/3} \cos\left(\frac{6\pi x}{L}\right)dx\right] = \frac{1}{L}\left[\frac{L}{3} - 0\right] = \frac{1}{3}$.

86. (**B**) The eigenvalues of the matrix A are the solutions λ to the equation $\det(A - \lambda I) = 0$, where I is identity matrix. Thus, $A - \lambda I = \begin{pmatrix} 2-\lambda & i \\ -i & 2-\lambda \end{pmatrix}$ and $\det(A - \lambda I) = (2-\lambda)^2 + i^2 \equiv 0$. From this, one has $\lambda_1 = 1$, $\lambda_2 = 3$.

87. (**D**) $\sigma_x\sigma_y - \sigma_y\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} =$ $= 2i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = 2i\sigma_z$.

88. (**D**) Let us first determine the normalization constant A: $\chi^\dagger\chi = |A|^2((1-i)(1+i)+4) = 6|A|^2 = 1 \implies A = 1/\sqrt{6}$. Now, $\chi = a\chi_+ + \chi_- = \dfrac{1+i}{\sqrt{6}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \dfrac{2}{\sqrt{6}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, where χ_+ represents spin up and χ_- is spin down. When you measure S_z on a particle in the state χ, you could get $+\hbar/2$ with probability $|a|^2$ or $-\hbar/2$ with probability $|b|^2$. Thus, the probability of finding the particle with spin projection $S_z = -\hbar/2$ is $|\dfrac{2}{\sqrt{6}}|^2 = \dfrac{2}{3}$. For more on this, see section 4.4 of Griffiths' Introduction to quantum mechanics (2nd ed.) and, in particular, identical Example 4.2.

89. (**D**) The reflection coefficient R is defined in terms of the incident and reflected probability current density j: $R = \dfrac{|j_{refl}|}{|j_{inc}|}$, where $j_{refl} = -\dfrac{\hbar k}{m}|B|^2$ and $j_{inc} = \dfrac{\hbar k}{m}|A|^2$. Thus, $R = \dfrac{|B|^2}{|A|^2}$.

According to the standard boundary conditions, ψ is always continues and $d\psi/dx$ is continues (except at points where the potential becomes infinite). Therefore, for the point $x = 0$ one has $A + B = C$ for continuity of ψ and $Ak_1 - Bk_2 = Ck_2$ for continuity of $d\psi/dx$. From these two equations $\dfrac{B}{A} = \dfrac{k_1 - k_2}{k_1 + k_2}$ and $R = \dfrac{|B|^2}{|A|^2} = \left(\dfrac{k_1 - k_2}{k_1 + k_2}\right)^2$.

90. (**D**) For $a < r < b$ one has $\varphi(r) = -\int_a^r E dr = -\int_a^r \frac{Q}{4\pi\epsilon_0}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{a}\right)$; for $r > b$ $\varphi(r) = -\int_a^b E dr - \int_b^r E dr = -\int_a^b \frac{Q}{4\pi\epsilon_0}\frac{dr}{r^2} - 0 = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$.

91. (**C**) When $\nabla \times \mathbf{E} = 0$, the line integral of $\mathbf{E}$ around any closed loop, according to Stokes' theorem, is zero. Because of this, we can unambiguously talk about a scalar function $\varphi(r) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$ such that $\mathbf{E} = -\nabla\varphi$.

92. (**E**) According to Ampere's law $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$, where I_{enc} is the total current enclosed by the integration path: $I_{enc} = \int \mathbf{J} \cdot d\mathbf{a}$, $\mathbf{J}$ is the current density. For $r < R$ the enclosed current is zero, so $B = 0$. For $R < r < 2R$ the enclosed current is $I_{enc} = J\int_R^r 2\pi r dr = \frac{I}{3\pi R^2} \cdot 2\pi \cdot \frac{r^2 - R^2}{2} = \frac{I(r^2 - R^2)}{3R^2}$ and from Ampere's law $B \cdot 2\pi r = \mu_o I_{enc}$, so $B(r) = \frac{\mu_0 I(r^2 - R^2)}{6\pi r R^2} = \frac{\mu_0 I}{6\pi R^2}\left(r - \frac{R^2}{r}\right)$ which shows a linear character with respect to r on $R < r < 2R$. For $r > 2R$ the enclosed current is $I_{enc} = I$ and $B = \frac{\mu_0 I}{2\pi r}$ or $B \sim \frac{1}{r}$. The only graph that reflects such a behavior of B is (E).

93. (**B**) The capacitance of the parallel-plate capacitor filled with dielectric exceeds the vacuum value by a factor of the dielectric constant: $C = kC_{vac}$. Thus, the electromagnetic energy $U = \frac{1}{2}CV_0^2$ (if V_0 is unchanged) is increased by the same factor k: $U = \frac{1}{2}kC_{vac}V_0^2 = kU_0$. We are left with (B) and (E), but do not rush to choose the second one. You might think that, when you insert a dielectric between the plates, the electric field between them (that is, inside the dielectric) is reduced by the factor of k in comparison with the vacuum value $\frac{V_0}{d}$. But you forgot about the battery! In fact, when the dielectric is inserted, the surface charge density on the lates of the capacitor is changed due to the battery. Let's apply $\int \mathbf{D} \cdot d\mathbf{a} = Q_{fenc}$ to the gaussian pillbox one surface of which is inside the positive plate of the capacitor and the other one is inside the dielectric material (Q_{fenc} here is a *free* charge enclosed by the gaussian surface; pillbox's surfaces are parallel to the plate's surface). Noting that $\mathbf{D} = 0$ inside the metal plate, one has $DA = \sigma A \implies D = \sigma$. The electric field inside the dielectric is $E = \frac{D}{\epsilon} = \frac{D}{\epsilon_0 k} \implies E = \frac{\sigma}{\epsilon_0 k}$. We can find σ using the fact that V_0 is unchanged during the insertion process. $C_{vac} = \frac{Q_{vac}}{V_0} = \frac{\epsilon_0 S}{d}$, $C = \frac{Q}{V_0} = \frac{\epsilon_0 k S}{d}$, so $\sigma = \frac{Q}{S} = k\frac{Q_{vac}}{S} = k\sigma_{vac}$. Finally, $E = \frac{\sigma}{\epsilon_0 k} = \frac{\sigma_{vac}}{\epsilon_0} \equiv \frac{V_0}{d}$.

94. (**C**) Lorentz transformation $t' = \gamma(t - \frac{v}{c^2}x)$ suggests that for the observer O' moving at constant speed v parallel to x-axis the event (flash of light) that occurs at $(t_0, x_0) = (0, 10)$ for the observer O is at time $t' = \gamma(0 - \frac{v}{c^2}x_0)$ (we assume O' is moving in the positive direction of x and at a zero time according to both observers the origins of their reference frames coincide, therefore t' is negative; this means that $(0, 10)$ event (as observed by O') occurs

$|t'|$ seconds *before* the other). Let $\alpha = \frac{ct'}{x_0} = \frac{3\cdot 10^8 \cdot 13\cdot 10^{-9}}{10} = 0.39$, then, using units in which $c = 1$, one has $\alpha = \gamma v = \frac{v}{\sqrt{1-v^2}}$ or $\alpha^2 = \frac{v^2}{1-v^2} \implies v = \frac{\alpha}{\sqrt{\alpha^2+1}} \approx \alpha = 0.39$ (for such an approximation the calculation error will be $\sim 7\%$). The closest choice is (C). If you calculate the square root in the above equation fairly, you will get $v \approx 0.363c$.

95. (**D**) As you can check for yourself $[J_x, J_y] = i\hbar J_z$ (the other two commutation relations can be obtain be cyclic permutation of the indexes $x \to y$, $y \to z$, $z \to x$; note also that I do not use the sign ˆ for the operators). Using a sweet relation one must remember (or derive if needed) on the PGRE $[AB, C] = A[B, C] + [A, C]B$ one has $[J_xJ_y, J_x] = J_x[J_y, J_x] + [J_x, J_x]J_y = -i\hbar J_xJ_z$ (here I've used the property that any operator commutes with itself).

96. (**D**) If you impure into tetravalent germanium atom Ge a pentavalent element such as As, P, Sb or N, then four of the five valence electrons introduced by impurity atoms come into contact with four neighboring atoms of Ge and form a stable shell of eight electrons, while the fifth electron will be weakly bound to the nucleus of the impurity atom. This creates an excess of negative (n-type) electron charge carriers. The only element that cannot be used to create an excess of negative charge is B, since it has only *three* valence electrons.

97. (**E**) According to Compton scattering formula, the difference in wavelength of the scattered λ' and incident λ photon is $\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$, where θ is the scattering angle. In our case, $\theta = 90°$ and $\lambda' - \lambda = \frac{h}{mc}$. But $E = \frac{hc}{\lambda}$, so $E' = \frac{hc}{\lambda'}$, where E' is the energy of the scattered photon. Thus, $\frac{hc}{E'} - \frac{hc}{E} = \frac{h}{mc}$, from which $E' = \frac{E\cdot mc^2}{E+mc^2}$.

98. (**D**) Lepton number L_e is conserved only for (D). Remember, $L_e = +1$ for the electron, the muon, and the neutrino, and $L_e = -1$ for the positron, the positive muon and the antineutrino (all other particles are given a lepton number of zero).

99. (**E**) In inertial reference frame associated with the platform, Newton's second law reads $m\mathbf{a} = \mathbf{f}_s + m\mathbf{g} + \mathbf{N}$, where $\mathbf{N}$ is a normal force perpendicular to the surface of contact, $m\mathbf{g}$ is the downward force of gravity. Acceleration $\mathbf{a}$ has a tangential component $\mathbf{a}_\tau$ and a radial component $\mathbf{a}_n$: $\mathbf{a} = a_\tau\hat{\boldsymbol{\tau}} + a_n\hat{\mathbf{n}}$, where $\hat{\boldsymbol{\tau}}$ is a unit vector tangent to the path pointing in the direction of motion at the chosen moment in time and $\hat{\mathbf{n}}$ is the unit (inward) normal vector to the particle's trajectory, $a_\tau = \frac{dv}{dt} = d(\omega r)/dt = r d\omega/dt = r\alpha$; $a_n = v^2/r = \omega^2 r^2/r = \omega^2 r$. Projecting Newton's second law of motion onto $\hat{\boldsymbol{\tau}}$ and $\hat{\mathbf{n}}$ gives $m\alpha r = f_s \sin\theta$ and $m\omega^2 r = f_s\cos\theta$. From these two equations $\alpha/\omega^2 = \tan\theta$, so $\theta = \tan^{-1}\left(\frac{\alpha}{\omega^2}\right)$.

100. (**E**) The partition function for a single QM harmonic oscillator is:

$$Z = \sum_{n=0}^{\infty} \exp\left(-\frac{E_n}{kT}\right) = \exp\left(-\frac{\hbar\omega}{2kT}\right)\sum_{n=0}^{\infty}\exp\left(-\frac{n\hbar\omega}{kT}\right).$$

$\sum_{n=0}^{\infty}\exp\left(-\frac{n\hbar\omega}{kT}\right) = 1 + \exp\left(-\frac{\hbar\omega}{kT}\right) + \exp\left(-\frac{2\hbar\omega}{kT}\right) + \ldots$ which is an infinite geometric series: $\sum_{n=0}^{\infty}\exp\left(-\frac{n\hbar\omega}{kT}\right) = \frac{1}{1-\exp\left(-\frac{\hbar\omega}{kT}\right)}$. Thus, $Z = \frac{\exp\left(-\frac{\hbar\omega}{2kT}\right)}{1-\exp\left(-\frac{\hbar\omega}{kT}\right)} = \frac{\exp\left(\frac{\hbar\omega}{2kT}\right)}{\exp\left(\frac{\hbar\omega}{kT}\right)-1}$.