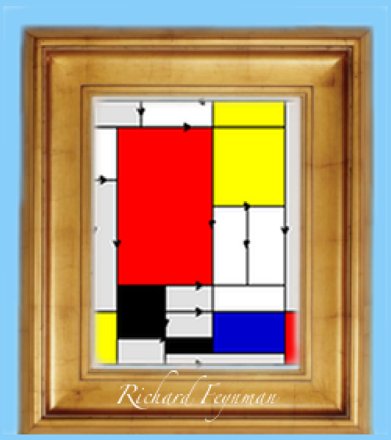
Complete Solutions to the Physics GRE



Exam #9277

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Cover Design: Taylor Faucett

Cover Image: "Composition with red, yellow blue and black" and Feynman

BiBTeX:

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@Book{Faucett9677,
     author   =  {Taylor Faucett},
     series   =  {Complete Solutions to the Physics GRE},
     title    =  {Exam #9277},
     pages    =  {36--38},
     year     =  {2010},
     edition  =  {first},
}
```

Contents

List of Figures

Chapter 1

Physics GRE Solutions

1.1 PGRE9277 #1

1. The wave function of a particle is $e^{i(kx-\omega t)}$, where x is distance, t is time, and k and ω are positive real numbers. The x-component of the momentum of the particle is

(A) 0

(B) $\hbar\omega$

(C) $\hbar k$

(D) $\frac{\hbar\omega}{c}$

(E) $\frac{\hbar k}{\omega}$

Recommended Solution

The momentum operator from quantum mechanics is

$$\hat{P} = \frac{\hbar}{i}\nabla\psi \tag{1.1}$$

If we substitute in the wave function $\psi = e^{i(kx-\omega t)}$,

$$\begin{aligned}
\hat{P} &= \frac{\hbar}{i}\frac{\partial}{\partial x}\left(e^{i(kx-\omega t)}\right) && (1.2)\\
&= \frac{\hbar k i}{i}e^{i(kx-\omega t)} && (1.3)\\
&= \hbar k\psi && (1.4)
\end{aligned}$$

Correct Answer
(C)

1.2 PGRE9277 #2

2. The longest wavelength x-ray that can undergo Bragg diffraction in a crystal for a given family of planes of spacing d is

(A) $\frac{d}{4}$

(B) $\frac{d}{2}$

(C) d

(D) $2d$

(E) $4d$

Recommended Solution

Bragg diffraction describes the phenomena by which specific angles of incident and wavelengths of x-rays will generate a peak in reflected radiation. From Bragg diffraction we get Bragg's law,

$$2d\sin(\theta) = n\lambda \tag{1.5}$$

From Bragg's law, it's clear that the wavelength for any given n will be maximized when $\theta = 90° = \pi/2$ making the LHS $2d$.

Correct Answer
(D)

1.3 PGRE9277 #3

3. The ratio of the energies of the K characteristic x-rays of carbon ($Z = 6$) to those of magnesium ($Z = 12$) is most nearly

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) 4

Recommended Solution

From the Bohr Model, we get the approximation of any Hydrogen like atoms as

$$E_n = -\frac{Z^2 R_e}{n^2} \tag{1.6}$$

for the ratio between carbon and magnesium, the only component of our approximation that will change is Z, so take the ratio of the 2 values,

$$\begin{aligned} \frac{E_C}{E_{Mg}} &= \frac{Z_C^2}{Z_{Mg}^2} && (1.7) \\ &= \frac{6^2}{12^2} && (1.8) \\ &= \frac{1}{4} && (1.9) \end{aligned}$$

Correct Answer
(A)

1.4 PGRE9277 #4

Questions 4-5

The magnitude of the Earth's gravitational force on a point mass is $F(r)$, where r is the distance from the Earth's center to the point mass. Assume the Earth is a homogeneous sphere of radius R.

4. What is $\frac{F(R)}{F(2R)}$?

(A) 32
(B) 8
(C) 4
(D) 2
(E) 1

5. Suppose there is a very small shaft in the Earth such that the point mass can be placed at a radius of $R/2$. What is $\frac{F(R)}{F\left(\frac{R}{2}\right)}$?

(A) 8

(B) 4

(C) 2

(D) $\frac{1}{2}$

(E) $\frac{1}{4}$

Recommended Solution

Recall that the force due to gravity between two objects of mass m_1 and m_2 is proportional to the inverse squared value of the radius,

$$F = G\frac{m_1 m_2}{R^2} \tag{1.10}$$

thus, if we double the radius (i.e. $R \to 2R$) then we get

$$\begin{aligned} \frac{F(R)}{F(2R)} &= \frac{1/R^2}{1/(2R)^2} && (1.11) \\ &= \frac{4R^2}{R^2} && (1.12) \\ &= 4 && (1.13) \end{aligned}$$

Correct Answer
(C)

1.5 PGRE9277 #5

Questions 4-5

The magnitude of the Earth's gravitational force on a point mass is $F(r)$, where r is the distance from the Earth's center to the point mass. Assume the Earth is a homogeneous sphere of radius R.

4. What is $\frac{F(R)}{F(2R)}$?

(A) 32
(B) 8
(C) 4
(D) 2
(E) 1

5. Suppose there is a very small shaft in the Earth such that the point mass can be placed at a radius of $R/2$. What is $\frac{F(R)}{F\left(\frac{R}{2}\right)}$?

(A) 8

(B) 4

(C) 2

(D) $\frac{1}{2}$

(E) $\frac{1}{4}$

Recommended Solution

In this problem, the point mass is located inside of the earth, specifically half way between the earth center and its surface. We can't just use the inverse square law in this form so utilize the part of the problem that tells us to assume the planet is homogenous. From this, we can calculate the mass of the earth, and the point mass, as being proportional to its density, ρ, by

$$M = \frac{4}{3}\pi R^3 \rho \tag{1.14}$$

so the gravitational force it wields is

$$F(R) = \frac{GM}{R^2} = \frac{G}{R^2}\left(\frac{4}{3}\pi R^3 \rho\right) = \frac{4}{3}\pi R \rho \tag{1.15}$$

compare this to the case of of $R/2$,

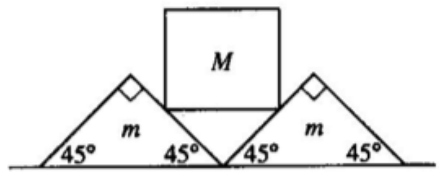
$$F(R/2) = \frac{G}{(R/2)^2}\left(\frac{4}{3}\pi(R/2)^3\rho\right) = \frac{4}{3}\pi R\frac{1}{2}\rho \tag{1.16}$$

finally, take the ratio of the two equations to get

$$\frac{F(R)}{F(R/2)} = \frac{\frac{4}{3}\pi R\rho}{\frac{4}{3}\pi R\frac{1}{2}\rho} = 2 \tag{1.17}$$

Correct Answer
(C)

1.6 PGRE9277 #6



6. Two wedges, each of mass m, are placed next to each other on a flat floor. A cube of mass M is balanced on the wedges as shown above. Assume no friction between the cube and the wedges, but a coefficient of static friction $\mu < 1$ between the wedges and the floor. What is the largest M that can be balanced as shown without motion of the wedges?

(A) $\frac{m}{\sqrt{2}}$

(B) $\frac{\mu m}{\sqrt{2}}$

(C) $\frac{\mu m}{1-\mu}$

(D) $\frac{2\mu m}{1-\mu}$

(E) All M will balance.

Recommended Solution

For starters, throw out option (E) as it can't be true that the system is in balance for all conceivable values of M of the block. second, get rid of (A) because it doesn't account for the coefficient of friction. If you aren't convinced we need it, consider that when $\mu = 0$, we should see our equation go to 0, which isn't true of (A). Next, eliminate (B) because when we maximize the coefficient of friction at $\mu = 1$, then no amount of mass should move the wedges and the equation should blow up. We can't make any more reasonable simplifications so if you struggle with the mechanics, at least you can guess. However, to solve between (C) and (D), let's consider the influence of the block on just one wedge. The block has a force downward which, because of the 45° angle between it and the block, generates a vertical force and horizontal force on the wedge (in fact it is the normal force of the block at its angle of incidence on the wedge). Since the angle is 45°, we can find the amount of force the block is putting out by

$$\tan(\theta) = \frac{F_G}{F_{G-x}} \tag{1.18}$$

$$F_{G-x} = F_G \tan(45^\circ) \tag{1.19}$$

$$F_{G-x} = F_G \tag{1.20}$$

which tells us that the horizontal force is equivalent to the vertical force of the block. Now, since half of the force will be used on each block, if we are only considering one block, the horizontal force generated by the block will be

$$F_{G-x} = \frac{1}{2}Mg \tag{1.21}$$

As the block applies the force, the frictional force of the wedge will try to resist it. From this, we know that the wedge will begin to move when the applied force over powers the frictional force,

$$F_{G-x} + f > 0 \tag{1.22}$$

and since the frictional force is $f = \mu F_N$, we find the normal force of the wedge by summing the vertical forces

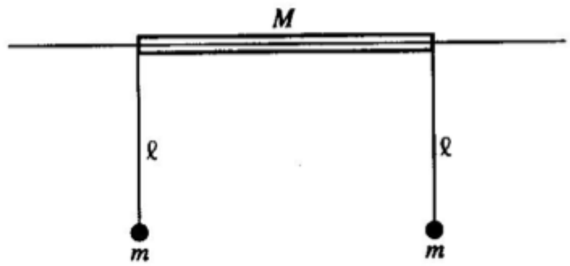
$$\begin{aligned} F_N &= -F_G && (1.23) \\ &= -\left(m + \frac{M}{2}\right) && (1.24) \end{aligned}$$

where the mass on the wedge is the wedges mass plus half of the blocks mass (i.e. $M/2$). combine our equations and solve to get

$$\begin{aligned} F_{G-x} + f &> 0 && (1.25) \\ \frac{1}{2}Mg - \mu\left(m + \frac{M}{2}\right) &> 0 && (1.26) \\ M - 2\mu m - \mu M &> 0 && (1.27) \\ M(1-\mu) &> 2\mu m && (1.28) \\ M &> \frac{2\mu m}{(1-\mu)} && (1.29) \end{aligned}$$

Correct Answer
(D)

1.7 PGRE9277 #7



7. A cylindrical tube of mass M can slide on a horizontal wire. Two identical pendulums, each of mass m and length ℓ, hang from the ends of the tube, as shown above. For small oscillations of the pendulums in the plane of the paper, the eigenfrequencies of the normal modes of oscillation of this system are 0, $\sqrt{\frac{g(M+2m)}{\ell M}}$, and

(A) $\sqrt{\frac{g}{\ell}}$

(B) $\sqrt{\frac{g}{\ell}\frac{M+m}{M}}$

(C) $\sqrt{\frac{g}{\ell}\frac{m}{M}}$

(D) $\sqrt{\frac{g}{\ell}\frac{m}{M+m}}$

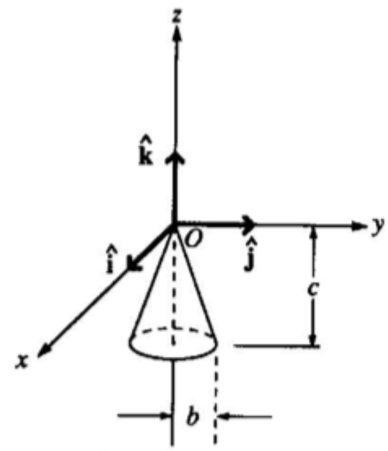
(E) $\sqrt{\frac{g}{\ell}\frac{m}{M+2m}}$

Recommended Solution

For the given apparatus, there are 3 possible modes. The first one they give to us is a normal mode of 0 (i.e. no frequency). The next mode given represents the 2 masses swaying in the same direction. Finally, we need to consider the last mode which occurs when masses sway in opposite directions, in which case it doesn't matter what the masses are and we can choose (A).

Correct Answer
(A)

1.8 PGRE9277 #8



8. A solid cone hangs from a frictionless pivot at the origin O, as shown above. If $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are unit vectors, and a, b, and c are positive constants, which of the following forces $\mathbf{F}$ applied to the rim of the cone at a point P results in a torque τ on the cone with a negative component τ_z?

(A) $\mathbf{F} = a\hat{\mathbf{k}}$, P is $(0, b, -c)$

(B) $\mathbf{F} = -a\hat{\mathbf{k}}$, P is $(0, -b, -c)$

(C) $\mathbf{F} = a\hat{\mathbf{j}}$, P is $(-b, 0, -c)$

(D) $\mathbf{F} = a\hat{\mathbf{j}}$, P is $(b, 0, -c)$

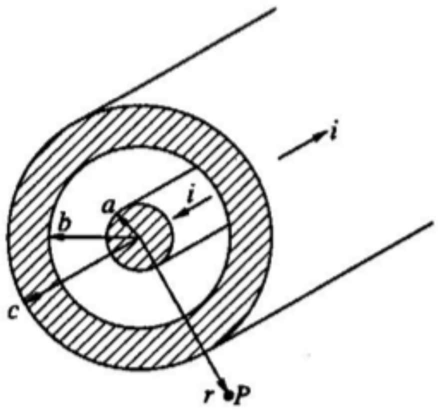
(E) $\mathbf{F} = -a\hat{\mathbf{k}}$, P is $(-b, 0, -c)$

Recommended Solution

The description in this problem is a little bit ridiculous but once you figure out what is going on, the problem is relatively easy. Torque is positive or negative based on the right hand rule and from this, we know that we want the cone to rotate in a clockwise direction about the z-direction when viewing the cone from above ($+\hat{k}$). Any force in the $\hat{k}$ isn't going to get our cone spinning so eliminate (A), (B) and (E). Next, looking at (C) and (D) it should be apparent that (C) will give us a negative torque (which is what we want) while (D) gives us a positive torque.

Correct Answer
(C)

1.9 PGRE9277 #9



9. A coaxial cable having radii a, b, and c carries equal and opposite currents of magnitude i on the inner and outer conductors. What is the magnitude of the magnetic induction at point P outside of the cable at a distance r from the axis?

(A) Zero (B) $\frac{\mu_0 i r}{2\pi a^2}$ (C) $\frac{\mu_0 i}{2\pi r}$

(D) $\frac{\mu_0 i}{2\pi r} \frac{c^2 - r^2}{c^2 - b^2}$ (E) $\frac{\mu_0 i}{2\pi r} \frac{r^2 - b^2}{c^2 - b^2}$

Recommended Solution

The intent behind coaxial cable shielding is to eliminate (at least in theory) the presence of an E&M field outside of the cable to reduce interference with other electronic equipment. This leads us to choice (A).

Correct Answer
(A)

Alternate Solution

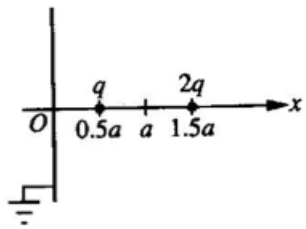
As our distance from the cable blows up to infinity ($r \to \infty$) we would expect the magnetic field to go to 0, which eliminates (B), (D) and (E). Next, recall that the magnetic field of a single, infinitely long cable can be found from Amperes law

$$\vec{B} = \frac{\mu_0 i}{2\pi R} \tag{1.30}$$

which is identical to (C). It is unreasonable to assume that adding a shielding element won't alter this equation with some dependence on a & b, so we are left with (A).

Correct Answer
(A)

1.10 PGRE9277 #10



10. Two positive charges of q and $2q$ coulombs are located on the x-axis at $x = 0.5a$ and $1.5a$, respectively, as shown above. There is an infinite, grounded conducting plane at $x = 0$. What is the magnitude of the net force on the charge q?

(A) $\frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}$

(B) $\frac{1}{4\pi\epsilon_0}\frac{3q^2}{2a^2}$

(C) $\frac{1}{4\pi\epsilon_0}\frac{2q^2}{a^2}$

(D) $\frac{1}{4\pi\epsilon_0}\frac{3q^2}{a^2}$

(E) $\frac{1}{4\pi\epsilon_0}\frac{7q^2}{2a^2}$

Recommended Solution

First, let's recall the inverse square law for the 2 charges q_1 and q_2

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \tag{1.31}$$

because of the infinite, grounded conducting plane at $x = 0$, we will get image charges of $-q$ and $-2q$ at $x = -0.5\ a$ and $x = -1.5\ a$ respectively.

This tells us that we will get three charges pushing on charge q at $x = 0.5\ a$. The first charge will be $2q$ at $x = 1.5\ a$ which will oppose q to the left. The other two charges, $-q$ and $-2q$ will attract q to the left as well. Sum all of the forces on q to get

$$\begin{aligned} F &= \frac{q}{4\pi\epsilon_0}\left[\frac{2q}{a^2}(-\hat{x}) + \frac{-q}{a^2}(\hat{x}) + \frac{-2q}{(2a)^2}(\hat{x})\right] && (1.32)\\ &= \frac{q}{4\pi\epsilon_0}\left[\frac{-4q}{2a^2} - \frac{2q}{2a^2} - \frac{q}{2a^2}\right] && (1.33)\\ &= \frac{1}{4\pi\epsilon_0}\frac{7}{2}\frac{q^2}{a^2} && (1.34) \end{aligned}$$

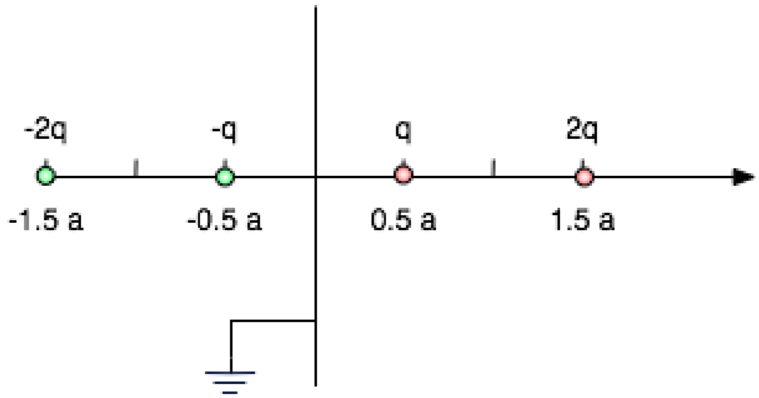
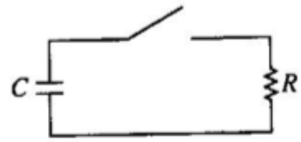


Figure 1.1: Mirror (image) charges induced as a result of an infinite grounding plate

Correct Answer
(E)

1.11 PGRE9277 #11



11. The capacitor in the circuit shown above is initially charged. After closing the switch, how much time elapses until one-half of the capacitor's initial stored energy is dissipated?

(A) RC

(B) $\frac{RC}{2}$

(C) $\frac{RC}{4}$

(D) $2RC\ln(2)$

(E) $\frac{RC\ln(2)}{2}$

Recommended Solution

Recall that the energy of the capacitor is

$$U = \frac{1}{2}CV^2 \tag{1.35}$$

Next, use Kirchhoff's second law which tells us that the sum of all voltages about a closed circuit is zero, to get

$$\begin{aligned} V_C + V_R &= 0 & (1.36)\\ \frac{Q}{C} + IR &= 0 & (1.37)\\ \frac{Q}{C} + \dot{Q}R & & (1.38) \end{aligned}$$

where $I = \dot{Q}$ because current is defined as a moving charge, Q. Rearrange the previous equation and integrate to get

$$\begin{aligned} \frac{Q}{C} &= -\frac{dQ}{dt}R & (1.39)\\ -\int \frac{dt}{RC} &= \int \frac{dQ}{Q} & (1.40)\\ -\frac{t}{RC} &= ln\left(\frac{Q}{Q_0}\right) & (1.41) \end{aligned}$$

$$Q = Q_0 e^{-t/RC} \tag{1.42}$$

From this, we also conclude that

$$I = I_0 e^{-t/RC} \tag{1.43}$$
$$V = V_0 e^{-t/RC} \tag{1.44}$$

From our initial energy equation, $U = \frac{1}{2}CV^2$, we get a voltage equation

$$V = \sqrt{\frac{2U}{C}} \tag{1.45}$$

and so our voltage equation is

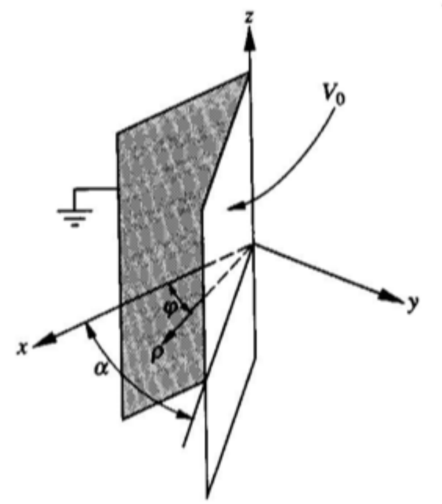
$$\sqrt{\frac{2U}{C}} = \sqrt{\frac{2U_0}{C}} e^{-t/RC} \tag{1.46}$$
$$U = U_0 e^{-2t/RC} \tag{1.47}$$

since we are concerned with the point at which half of the energy has dissipated, substitute in $U = U_0/2$

$$\frac{U_0}{2} = U_0 e^{-2t/RC} \tag{1.48}$$
$$1 = 2e^{-2t/RC} \tag{1.49}$$
$$e^{-2t/RC} = 2 \tag{1.50}$$
$$-\frac{2t}{RC} = ln(2) \tag{1.51}$$
$$t = \frac{RCln(2)}{2} \tag{1.52}$$

Correct Answer
(E)

1.12 PGRE9277 #12



12. Two large conducting plates form a wedge of angle α as shown in the diagram above. The plates are insulated from each other; one has a potential V_0 and the other is grounded. Assuming that the plates are large enough so that the potential difference between them is independent of the cylindrical coordinates z and ρ, the potential anywhere between the plates as a function of the angle φ is

(A) $\frac{V_0}{\alpha}$

(B) $\frac{V_0\varphi}{\alpha}$

(C) $\frac{V_0\alpha}{\varphi}$

(D) $\frac{V_0\varphi^2}{\alpha}$

(E) $\frac{V_0\alpha}{\varphi^2}$

Recommended Solution

Recall ~~LaPlace's~~ equation

$$\nabla^2 V = 0 \tag{1.53}$$

since the problem only tells us to concern ourselves with the ϕ component, we can integrate LaPlace's equation to get

$$\frac{d^2V(\phi)}{d\phi^2} = 0 \tag{1.54}$$

$$\frac{dV(\phi)}{d\phi} = A \tag{1.55}$$

$$V(\phi) = A\phi + B \tag{1.56}$$

since we have the initial condition $V(0) = 0$, we know that $B = 0$

$$V(\phi) = A\phi \tag{1.57}$$

since $V(a) = V_0$,

$$V_0 = Aa \tag{1.58}$$

$$A = \frac{V_0}{a} \tag{1.59}$$

compare this to the equations general form, $V = A\phi$, to get

$$\frac{V_0}{a} = \frac{V}{\phi} \tag{1.60}$$

$$V = \frac{V\phi}{a} \tag{1.61}$$

Correct Answer
(B)

1.13 PGRE9277 #13

13. Listed below are Maxwell's equations of electromagnetism. If magnetic monopoles exist, which of these equations would be INCORRECT?

I. Curl $\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$

II. Curl $\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$

III. div $\mathbf{D} = \rho$

IV. div $\mathbf{B} = 0$

(A) IV only
(B) I and II
(C) I and III
(D) II and IV
(E) III and IV

Recommended Solution

If you are like me, you probably learned and memorized Maxwell's equation, $\nabla \cdot \vec{B} = 0$ as the "There ain't no magnetic monopoles" law. For this reason, you know you can immediately get IV as one of the laws that becomes INCORRECT. Next, consider that if we don't require the magnetic field to curl back on itself in order to force the divergence of the magnetic field to zero, then it is possible to get the electric field to not curl which tells us II could also be INCORRECT and we choose (D).

Correct Answer
(D)

1.14 PGRE9277 #14

14. The total energy of a blackbody radiation source is collected for one minute and used to heat water. The temperature of the water increases from 20.0 °C to 20.5 °C. If the absolute temperature of the blackbody were doubled and the experiment repeated, which of the following statements would be most nearly correct?

(A) The temperature of the water would increase from 20 °C to a final temperature of 21 °C.
(B) The temperature of the water would increase from 20 °C to a final temperature of 24 °C.
(C) The temperature of the water would increase from 20 °C to a final temperature of 28 °C.
(D) The temperature of the water would increase from 20 °C to a final temperature of 36 °C.
(E) The water would boil within the one-minute time period.

Recommended Solution

From Stefan-Boltzmann's Law, we get the power radiation of a black body as

$$j^* = uT^4 \tag{1.62}$$

which tells us that doubling the temperature of the black body will alter the power proportional to the fourth power

$$\begin{aligned} j^* &= u(2T)^4 && (1.63) \\ &= 16(uT^4) && (1.64) \end{aligned}$$

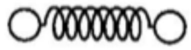
since power is energy over time and heat energy is

$$Q = mc\Delta T \tag{1.65}$$

we get that a unit increase in energy will increase temperature by $0.5°C$ and, therefore, 16 units of energy increase will get a change in temperature of $8°C$.

Correct Answer
(C)

1.15 PGRE9277 #15



15. **A classical model of a diatomic molecule is a springy dumbbell, as shown above, where the dumbbell is free to rotate about axes perpendicular to the spring. In the limit of high temperature, what is the specific heat per mole at constant volume?**

(A) $\frac{3}{2}R$

(B) $\frac{5}{2}R$

(C) $\frac{7}{2}R$

(D) $\frac{9}{2}R$

(E) $\frac{11}{2}R$

Recommended Solution

Heat capacity of a molecule is determined by the number of degrees of freedom of the molecule. For example, in a monatomic gas, the heat capacity is

$$C_V = \frac{3}{2}R \tag{1.66}$$

where the 3 comes from our 3 degrees of translational freedom $(\hat{x}, \hat{y}, \hat{z})$. For a springy, diatomic molecule, we have to then include additional degrees of freedom for its rotation and its vibration

$$C_V = \frac{3}{2}R + R_{rot} + R_{vib} = \frac{7}{2}R \tag{1.67}$$

Correct Answer
(C)

1.16 PGRE9277 #16

16. An engine absorbs heat at a temperature of 727° C and exhausts heat at a temperature of 527° C. If the engine operates at maximum possible efficiency, for 2000 joules of heat input the amount of work the engine performs is most nearly

(A) 400 J
(B) 1450 J
(C) 1600 J
(D) 2000 J
(E) 2760 J

Recommended Solution

The maximum efficiency of a Carnot engine (a theoretically, perfectly efficient heat engine) is

$$\eta = \frac{W}{Q_H} = 1 - \frac{T_C}{T_H} \tag{1.68}$$

where η is the maximum efficiency, W is the work done by the system, Q_H is the heat input, T_C is the absolute temperature of the cold reservoir and T_H is the absolute temperature of the hot reservoir. Convert temperatures to units of Kelvin to get

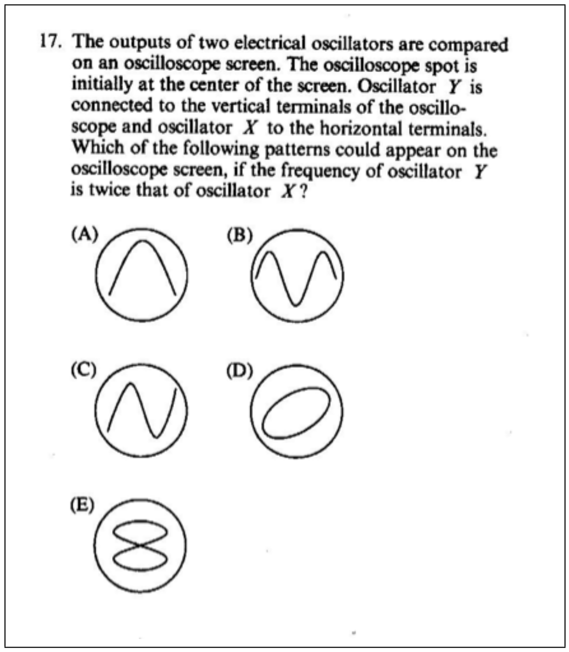
$$\eta = 1 - \frac{T_C}{T_H} \tag{1.69}$$
$$= 1 - \frac{800 \text{ K}}{1000 \text{ K}} \tag{1.70}$$
$$= 1 - 0.8 \tag{1.71}$$
$$= 0.2 \tag{1.72}$$

equate η to the work over heat equation with a heat of $Q_H = 2000$ J to get

$$\frac{W}{Q_H} = 0.2 \tag{1.73}$$
$$W = (0.2)(2000 \text{ J}) \tag{1.74}$$
$$= 400 \text{ J} \tag{1.75}$$

Correct Answer
(A)

1.17 PGRE9277 #17

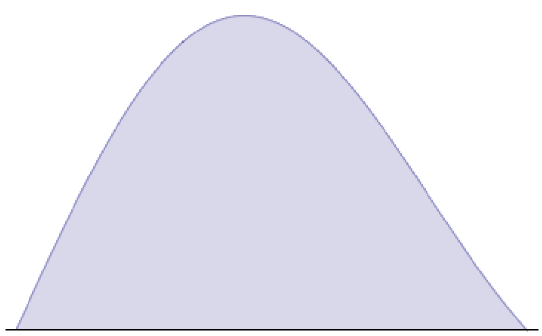

17. The outputs of two electrical oscillators are compared on an oscilloscope screen. The oscilloscope spot is initially at the center of the screen. Oscillator Y is connected to the vertical terminals of the oscilloscope and oscillator X to the horizontal terminals. Which of the following patterns could appear on the oscilloscope screen, if the frequency of oscillator Y is twice that of oscillator X?

Recommended Solution

The problem tells us that the frequency in the x-direction is twice that of the y-direction, so we know that the oscilloscope will be plotting

$$f(t) = \sin(\omega t) + \sin(2\omega t) \tag{1.76}$$

Using your copy of Mathematica provided in the front of your test booklet, plot the function to get the figure below



However, in the case that your test booklet doesn't have Mathematica, we can eliminate options (E) and (D) because the superposition of two sine waves shouldn't give us either of the two curves. Next, eliminate (B) and (C) because both represent just a function of $\sin(\omega t)$ or $\cos(\omega t)$, not a superposition of trig functions.

Correct Answer
(A)

1.18 PGRE9277 #18

18. In transmitting high frequency signals on a coaxial cable, it is important that the cable be terminated at an end with its characteristic impedance in order to avoid

(A) leakage of the signal out of the cable
(B) overheating of the cable
(C) reflection of signals from the terminated end of the cable
(D) attenuation of the signal propagating in the cable
(E) production of image currents in the outer conductor

Recommended Solution

With coaxial cables, impedance matching is necessary because differences in characteristic impedance can result in signal reflection, particularly in the case of a damaged/kinked line or an incorrectly/damaged termination to the cable.

Correct Answer
(C)

1.19 PGRE9277 #19

19. Which of the following is most nearly the mass of the Earth? (The radius of the Earth is about 6.4×10^6 meters.)

(A) 6×10^{24} kg
(B) 6×10^{27} kg
(C) 6×10^{30} kg
(D) 6×10^{33} kg
(E) 6×10^{36} kg

Recommended Solution

Use Newton's second law and our gravitational force law,

$$F = ma \tag{1.77}$$
$$F = G\frac{mM}{r^2} \tag{1.78}$$

combine the two and cancel m to get

$$ma = G\frac{mM}{r^2} \tag{1.79}$$
$$a = G\frac{M}{r^2} \tag{1.80}$$
$$M = \frac{gr^2}{G} \tag{1.81}$$

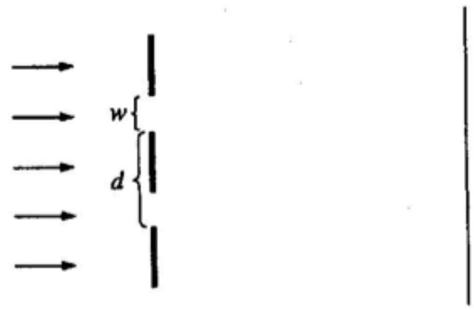
to simplify the mental math, assume that $G = 6\times10^{-11}$ m^3/kg$\cdot$s^2, $g = 10$ m/s^2 and $r = 6\times10^6$ m

$$M = \frac{gr^2}{G} \tag{1.82}$$
$$= \frac{\left(10 \text{ m/s}^2\right)\left(6 \times 10^6 \text{ m}\right)^2}{\left(6 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2\right)} \tag{1.83}$$
$$= \frac{6 \times 10^{13}}{6 \times 10^{-11}} \tag{1.84}$$
$$= 1 \times 10^{24} \text{ kg} \tag{1.85}$$

which is closest to (A).

Correct Answer
(A)

1.20 PGRE9277 #20



20. In a double-slit interference experiment, d is the distance between the centers of the slits and w is the width of each slit, as shown in the figure above. For incident plane waves, an interference maximum on a distant screen will be "missing" when

(A) $d = \sqrt{2}w$

(B) $d = \sqrt{3}w$

(C) $2d = w$

(D) $2d = 3w$

(E) $3d = 2w$

Recommended Solution

based on the given diagram, we know it can't be true that $d < \omega$ so (C) and (E) can be eliminated. Next, recall that the equation for constructive interference in double slit diffraction is

$$d \sin(\theta) = m_1 \lambda \tag{1.86}$$

additionally, we know that we will get a "missing" interference maximum when the constructive double slit equation coincides with the single slit diffraction, so we use

$$\omega sin(\theta) = m_2 \lambda \tag{1.87}$$

get both equations equal to $\sin(\theta)$ and set them equal to one another and solve for d,

$$\begin{aligned} \frac{\omega}{m_2} &= \frac{d}{m_1} && (1.88) \\ d &= \frac{m_1}{m_2}\omega && (1.89) \end{aligned}$$

comparing this to (A) and (B), we aren't going to get an irrational number (i.e. $\sqrt{2}$ or $\sqrt{3}$) with some fraction of integers so we can confidently choose (D).

Correct Answer
(D)

1.21 PGRE9277 #21

21. A soap film with index of refraction greater than air is formed on a circular wire frame that is held in a vertical plane. The film is viewed by reflected light from a white-light source. Bands of color are observed at the lower parts of the soap film, but the area near the top appears black. A correct explanation for this phenomenon would involve which of the following?

    I. The top of the soap film absorbs all of the light incident on it; none is transmitted.
    II. The thickness of the top part of the soap film has become much less than a wavelength of visible light.
    III. There is a phase change of 180° for all wavelengths of light reflected from the front surface of the soap film.
    IV. There is no phase change for any wavelength of light reflected from the back surface of the soap film.

    (A) I only
    (B) II and III only
    (C) III and IV only
    (D) I, II, and III
    (E) II, III, and IV

Recommended Solution

In thin film optics, and most optics in general, I is unequivocally silly so eliminate all choices which include it, i.e. (A) and (D). Next, consider III and IV and recognize that both are correct. More specifically, it is true that we will get a phase change as the light transitions from a lower index of refraction to a higher one (as it enters the bubble) and no phase change as it transitions from a higher index of refraction to a lower one (as it exits the bubble). Eliminate any options that don't include both of these choices, specifically (C). Finally, when considering option II, recall that equations for thin film optics

Constructive Interference $2t = \lambda/2$

Destructive Interference $2t = \lambda$

which tells us that in either case, the thickness of the bubble is generally less than the wavelength (i.e. half or a quarter)

Correct Answer
(B)

1.22 PGRE9277 #22

22 A simple telescope consists of two convex lenses, the objective and the eyepiece, which have a common focal point P, as shown in the figure above. If the focal length of the objective is 1.0 meter and the angular magnification of the telescope is 10, what is the optical path length between objective and eyepiece?

(A) 0.1 m
(B) 0.9 m
(C) 1.0 m
(D) 1.1 m
(E) 10 m

Recommended Solution

The quick and easy solution, dare I say the "Plug-n-chug" method, is to use the magnification equation for a convex lens

$$M = \frac{f_{Obj}}{f_{eye}} \tag{1.90}$$

We are given the objective focal length, $f_{obj} = 1.0$ meter and the magnification $M = 10$ so we solve for f_{eye},

$$\begin{aligned} f_{eye} &= \frac{f_{obj}}{M} && (1.91)\\ &= \frac{1.0 \text{ m}}{10} && (1.92)\\ &= 0.1 \text{ m} && (1.93) \end{aligned}$$

Lastly, we get the total distance from the sum of the two focal lengths,

$$\begin{aligned} d &= f_{obj} + f_{eye} && (1.94)\\ &= 1.0 \text{ m} + 0.1 \text{ m} && (1.95)\\ &= 1.1 \text{ m} && (1.96) \end{aligned}$$

Correct Answer
(D)

1.23 PGRE9277 #23

23. The Fermi temperature of Cu is about 80,000 K. Which of the following is most nearly equal to the average speed of a conduction electron in Cu ?

(A) 2×10^{-2} m/s
(B) 2 m/s
(C) 2×10^{2} m/s
(D) 2×10^{4} m/s
(E) 2×10^{6} m/s

Recommended Solution

The average speed of a conduction electron is described by the Fermi velocity, with equation

$$v_f = \sqrt{\frac{2E_f}{m}} \tag{1.97}$$

where the Fermi energy, E_f, is related to the Fermi temperature, T_f, by

$$E_f = kT_f \tag{1.98}$$

where k is Boltzmann's constant. Plug everything into our equation to get

$$\begin{aligned} v_f &= \sqrt{\frac{2E_f}{m}} && (1.99) \\ &= \sqrt{\frac{2kT_f}{m}} && (1.100) \end{aligned}$$

simplify the values for the electron mass, fermi temperature and Boltzmann's constant

$$\begin{aligned} v_f &= \sqrt{\frac{2kT_f}{m}} && (1.101) \\ &= \sqrt{\frac{2(1\times 10^{-23}\ \mathrm{J/K})(80,000\ \mathrm{K})}{10\times 10^{-31}\ \mathrm{kg}}} && (1.102) \\ &\approx \sqrt{\frac{10\times 10^{31}}{16\times 10^{19}\ \mathrm{m^2/s^2}}} && (1.103) \\ &\approx 1\times 10^{6}\ \mathrm{m/s} && (1.104) \end{aligned}$$

Correct Answer
(E)

1.24 PGRE9277 #24

24. Solid argon is held together by which of the following bonding mechanisms?

(A) Ionic bond only
(B) Covalent bond only
(C) Partly covalent and partly ionic bond
(D) Metallic bond
(E) van der Waals bond

Recommended Solution

Argon, like the other noble gases, has a full valence shell. Ionic bonding is bonding between a metal and a non-metal and also requires that one atom lack electrons from its valence shell and the other have an excess charge (think along the lines of the salt molecule with $Na^+ + Cl^-$) so we can eliminate (A). Covalent bonding, on the other hand, involves the sharing of electrons to fill out the valence shell when an atom is lacking electrons in its valence shell, but again Argon isn't missing any electrons so we eliminate (B) and then eliminate (C). Finally, since Argon isn't a metal, eliminate (D). As it turns out, argon bonds to other argon atoms by induced dipoles via the Van der Waals force.

Correct Answer
(E)

1.25 PGRE9277 #25

25. In experiments located deep underground, the two types of cosmic rays that most commonly reach the experimental apparatus are

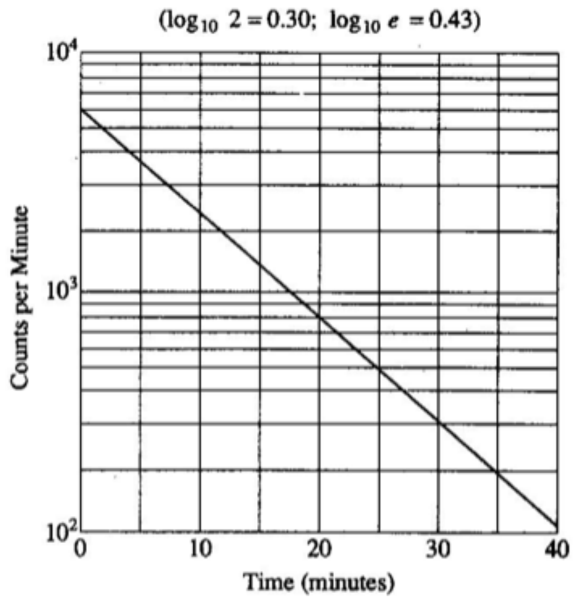
(A) alpha particles and neutrons
(B) protons and electrons
(C) iron nuclei and carbon nuclei
(D) muons and neutrinos
(E) positrons and electrons

Recommended Solution

The ability of a particle to pass through solid material is increased as the size of the particle decreases and as the speed of the particle increases. From this, we would expect larger particles like protons and neutrons to struggle to pass "deep underground" and we can eliminate (A), (B) and (C). Between (D) and (E), all of the listed particles are sufficiently fast and small but neutrinos have no charge, unlike positrons, electrons and muons so we should expect them to not be caught or repelled away from electrons in the matter they are trying to pass through and we should expect them to be in the final answer.

Correct Answer
(D)

1.26 PGRE9277 #26



26. A radioactive nucleus decays, with the activity shown in the graph above. The half-life of the nucleus is

(A) 2 min
(B) 7 min
(C) 11 min
(D) 18 min
(E) 23 min

Recommended Solution

At time $t = 0$, the number of counts is at $10^{3.5}$ counts. We can approximate this value by

$$10^{3.5} = 10^{7/2} = (10^7)^{1/2} = \sqrt{100 \cdot 100 \cdot 100 \cdot 10} = 10 \cdot 10 \cdot 10 \cdot \sqrt{10} \tag{1.105}$$

if we approximate $\sqrt{10} \approx 3$ then we get

$$10^{3.5} = 3000 \tag{1.106}$$

This means that half the counts will be 1500 and so at about 17 min, where the counts is $10^3 = 1000$ counts, we've passed our half way point and we can eliminate (D) and (E). For (A), (B) and (C), the counts are approximately

(A) $10^{3.4} \approx 2500$ counts

(B) $10^{3.2} \approx 1600$ counts

(C) $10^{3.1} \approx 1300$ counts

so we choose (B) which is the closest

Correct Answer
(B)

1.27 PGRE9277 #27

27. If a freely moving electron is localized in space to within Δx_0 of x_0, its wave function can be described by a wave packet $\psi(x,t) = \int_{-\infty}^{\infty} e^{i(kx-\omega t)} f(k)\,dk$, where $f(k)$ is peaked around a central value k_0. Which of the following is most nearly the width of the peak in k?

(A) $\Delta k = \dfrac{1}{x_0}$

(B) $\Delta k = \dfrac{1}{\Delta x_0}$

(C) $\Delta k = \dfrac{\Delta x_0}{x_0^2}$

(D) $\Delta k = \left(\dfrac{\Delta x_0}{x_0}\right) k_0$

(E) $\Delta k = \sqrt{k_0^2 + \left(\dfrac{1}{x_0}\right)^2}$

Recommended Solution

The width of the wave-function is determined by the relative size of Δp and Δx in the Heisenberg uncertainty principle

$$\Delta x\ \Delta p \geq \frac{\hbar}{2} \tag{1.107}$$

Recalling our momentum equation

$$p = \hbar k \tag{1.108}$$

where $\hbar$ is a constant, we get $\Delta p = \hbar\Delta k$. Combine this with the Heisenberg uncertainty principle and solve for Δk

$$\begin{aligned} \Delta x\ \Delta p &\geq \frac{\hbar}{2} && (1.109)\\ (\Delta x)\ (\hbar\Delta k) &\geq \frac{\hbar}{2} && (1.110)\\ \Delta k &\geq \frac{1}{2(\Delta x)} && (1.111) \end{aligned}$$

which gives us the inverse relationship between Δk and Δx like in (B).

Correct Answer
(B)

1.28 PGRE9277 #28

28. A system is known to be in the normalized state described by the wave function

$$\psi(\theta, \varphi) = \frac{1}{\sqrt{30}}(5\, Y_4^3 + Y_6^3 - 2\, Y_6^0),$$

where the $Y_\ell^m(\theta, \varphi)$ are the spherical harmonics. The probability of finding the system in a state with azimuthal orbital quantum number $m = 3$ is

(A) 0

(B) $\frac{1}{15}$

(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) $\frac{13}{15}$

Recommended Solution

In quantum mechanics, the probability of finding the system in a certain state is given by the integral over the squared wave function, or in our case $Y_l^m(\theta, \phi)$. The problem asks us about the state with azimuthal orbital quantum number, m=3, so we take the squared values of the form $Y_l^3(\theta, \phi)$. Since our wave function is already normalized, we just need to square the values of the first two terms of ψ

$$\psi_{m=3} = 5^2 + 1^2 = 26 \tag{1.112}$$

$$\psi_{total} = 5^2 + 1^2 + 2^2 = 30 \tag{1.113}$$

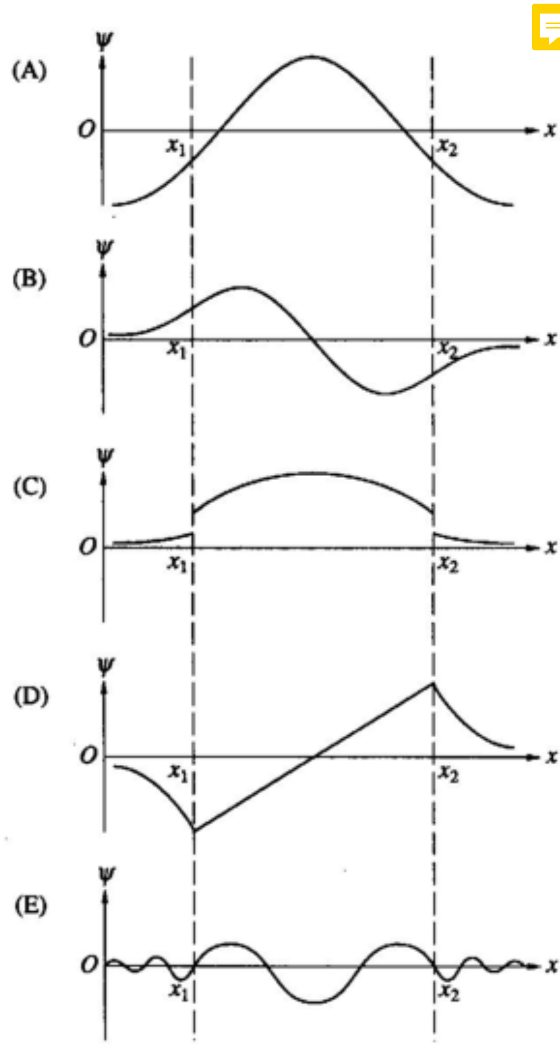
therefore, the probability of $\psi_{m=3}$ out of the total ψ_{total} is

$$\begin{aligned} \psi_{m=3}/\psi_{total} &= 26/30 \quad (1.114) \\ &= 13/15 \quad (1.115) \end{aligned}$$

Correct Answer
(E)

1.29 PGRE9277 #29

29. An attractive, one-dimensional square well has depth V_0 as shown above. Which of the following best shows a possible wave function for a bound state?



Recommended Solution

Questions regarding the infinite square well (particle in a box) and its related plot show up on nearly every test. For this reason, and simply because it is something you oughta know, you should memorize some of the fundamental aspects of the infinite square well graph. First, the solution to the infinite square well is sinusoidal, which allows us to eliminate (A), (C) and (D). Next, when the function impinges on the infinite barrier at x_1 and x_2, the amplitude continually decreases toward 0 and stops oscillating, which then allows us to eliminate (E).

Correct Answer
(B)

1.30 PGRE9277 #30

30. Given that the binding energy of the hydrogen atom ground state is $E_0 = 13.6$ eV, the binding energy of the $n = 2$ state of positronium (positron-electron system) is

(A) $8\ E_0$

(B) $4\ E_0$

(C) E_0

(D) $\frac{E_0}{4}$

(E) $\frac{E_0}{8}$

Recommended Solution

Considering how often the ground state energy of positronium comes up in the GRE, and because it's trivial to memorize, recall that its value is half that of hydrogen's ground state

$$E_{0,pos} = \frac{E_{0,hyd}}{2} = \frac{13.6 \text{ eV}}{2} = 6.8 \text{ eV} \tag{1.116}$$

Keep in mind that this is for the ground state but we need the binding energy in state $n = 2$. Using the Bohr equation, we see that the energy is inversely proportional to the squared value of the fundamental quantum number, n

$$E_n = \frac{Z^2 E_{0,pos}}{n^2} \tag{1.117}$$

since $Z = 1$, the final answer is

$$E_2 = \frac{E_0}{8} \tag{1.118}$$

Correct Answer
(E)

1.31 PGRE9277 #31

31. In a ^3S state of the helium atom, the possible values of the total electronic angular momentum quantum number are

(A) 0 only

(B) 1 only

(C) 0 and 1 only

(D) 0, $\frac{1}{2}$, and 1

(E) 0, 1, and 2

Recommended Solution

Recall our generic form for the Russel-Saunders term symbol

$$^{2s+1}L_J \tag{1.119}$$

the problem specifies that the helium atom has term symbol

$$^3S \tag{1.120}$$

so we know that $2s+1=3$, $J=1$ and $L=S$. Solve for s to get $s=1$ and then recalling the angular momentum quantum number can be found by

$$j = l + s \tag{1.121}$$

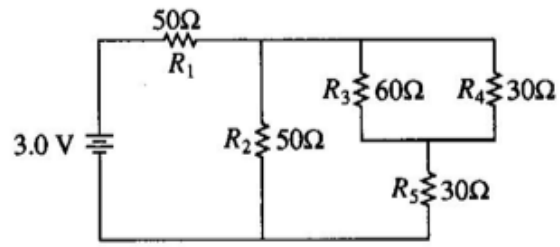
since $S = L$, and S corresponds to 0, (i.e. $(S, P, D, F, \ldots) \rightarrow (0, 1, 2, 3, \ldots)$), we finally get

$$j = 0 + 1 = 1 \tag{1.122}$$

Correct Answer
(B)

1.32 PGRE9277 #32

Questions 32-33



In the circuit shown above, the resistances are given in ohms and the battery is assumed ideal with emf equal to 3.0 volts.

32. The resistor that dissipates the most power is

(A) R_1 (B) R_2 (C) R_3

(D) R_4 (E) R_5

33. The voltage across resistor R_4 is

(A) 0.4 V
(B) 0.6 V
(C) 1.2 V
(D) 1.5 V
(E) 3.0 V

Recommended Solution

First, recall that the equivalent resistance of resistors in parallel can be found by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots \tag{1.123}$$

From Equation , we can see that the equivalent resistance of the R_3-R_4 system and R_2-R_5 system will have less resistance than R_1 on its own. Additionally, we know that power is related to voltage and current by

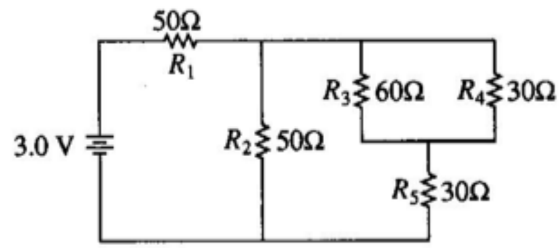
$$P = I^2 \cdot R = \frac{V^2}{R} \tag{1.124}$$

Which tells us that in addition to R_1 having the highest resistance, it will also be the case that the highest current will interact with this resistor and so we should expect R_1 to have the biggest amount of current to dissipate.

Correct Answer
(A)

1.33 PGRE9277 #33

Questions 32-33



In the circuit shown above, the resistances are given in ohms and the battery is assumed ideal with emf equal to 3.0 volts.

32. The resistor that dissipates the most power is
(A) R_1 (B) R_2 (C) R_3
(D) R_4 (E) R_5

33. The voltage across resistor R_4 is
(A) 0.4 V
(B) 0.6 V
(C) 1.2 V
(D) 1.5 V
(E) 3.0 V

Recommended Solution

Using the rules for resistors and in parallel, find the equivalent resistance of all resistors in the circuit as $R_{eq} = 75\ \Omega$. This tells us that the current of the entire circuit should be

$$I = \frac{V}{R} = \frac{3.0\text{ V}}{75\ \Omega} \tag{1.125}$$

Next, finding the equivalent resistance of resistors R_3, R_4 and R_5, you'll find that you have the same resistance in the RHS of the circuit as you do in the LHS (i.e .just resistor R_2). This tells us that half the current (3/150 A) will go through resistor R_2 and the other half will pass through $R_{3,4,5}$. Finally, find your equivalent resistance between R_3 and R_4 to get $2o\ \Omega$ and then solve for the voltage

$$V = IR = \left(\frac{3}{150}\text{ A}\right)(20\ \Omega) = 0.4\text{ V} \tag{1.126}$$

Correct Answer
(A)

1.34 PGRE9277 #34

34. A conducting cavity is driven as an electromagnetic resonator. If perfect conductivity is assumed, the transverse and normal field components must obey which of the following conditions at the inner cavity walls?

(A) $E_n = 0$, $B_n = 0$

(B) $E_n = 0$, $B_t = 0$

(C) $E_t = 0$, $B_t = 0$

(D) $E_t = 0$, $B_n = 0$

(E) None of the above

Recommended Solution

Because we are talking about electromagnetic waves in a waveguide, we can throw out all conditions which don't give the results as being orthogonal for the electric and magnetic fields separately, i.e. (A) and (C). Then, we can eliminate (B) and choose (C) because the electric field doesn't propagate in the same direction as the direction of current so the transverse electric field, E_t, should be 0.

Correct Answer
(D)

1.35 PGRE9277 #35

35. Light of wavelength 5200 angstroms is incident normally on a transmission diffraction grating with 2000 lines per centimeter. The first-order diffraction maximum is at an angle, with respect to the incident beam, that is most nearly

(A) 3°
(B) 6°
(C) 9°
(D) 12°
(E) 15°

Recommended Solution

An optical diffraction grating works in effectively the same way that a double slit, triple slit, etc interference except with more slits. Based on this similarity, we should try to use our generic equation for double slit diffraction

$$d\sin(\theta) = m\lambda \tag{1.127}$$

$$\theta = \arcsin\left(\frac{m\lambda}{d}\right) \tag{1.128}$$

where d is the distance between slits, θ is the angle of incidence, λ is the wavelength and $m = 0, 1, 2, \ldots$. The wavelength is given as 5200 angstroms and we can find the distance between slits by assuming that the 2000 slits are evenly spaced across each centimeter of the diffraction grating. Plug this all into Equation 1.128 to get

$$\theta = \arcsin\left(\frac{m\lambda}{d}\right) \tag{1.129}$$

$$= \arcsin\left(\frac{(1)(5200\text{angstroms})}{(0.0005\text{ cm})}\right) \tag{1.130}$$

$$= \arcsin(0.1) \tag{1.131}$$

$$\approx 6^\circ \tag{1.132}$$

Correct Answer
(B)

1.36 PGRE9277 #36

36. A plane-polarized electromagnetic wave is incident normally on a flat, perfectly conducting surface. Upon reflection at the surface, which of the following is true?

(A) Both the electric vector and magnetic vector are reversed.
(B) Neither the electric vector nor the magnetic vector is reversed.
(C) The electric vector is reversed; the magnetic vector is not.
(D) The magnetic vector is reversed; the electric vector is not.
(E) The directions of the electric and magnetic vectors are interchanged.

Recommended Solution

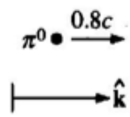
The problem tells us, quite explicitly might I add, that the surface which the E&M wave interacts with is a perfect conductor. This tells us that the net electric field must go to 0 as a result of the interaction. Since the electric field impinges on the surface with some value E_0, we know that the value afterwards must be equal and opposite,

$$E_1 = -E_0 \tag{1.133}$$

or, in words, its direction must reverse while maintaining the same magnitude. The magnetic field, however, won't change direction because of the conductor so (C) becomes the obvious choice.

Correct Answer
(C)

1.37 PGRE9277 #37



37. A π^0 meson (rest-mass energy 135 MeV) is moving with velocity $0.8c\,\hat{\mathbf{k}}$ in the laboratory rest frame when it decays into two photons, γ_1 and γ_2. In the π^0 rest frame, γ_1 is emitted forward and γ_2 is emitted backward relative to the π^0 direction of flight. The velocity of γ_2 in the laboratory rest frame is

(A) $-1.0c\,\hat{\mathbf{k}}$
(B) $-0.2c\,\hat{\mathbf{k}}$
(C) $+0.8c\,\hat{\mathbf{k}}$
(D) $+1.0c\,\hat{\mathbf{k}}$
(E) $+1.8c\,\hat{\mathbf{k}}$

Recommended Solution

If you get this problem wrong, you probably should give up any aspirations you've ever held which involve you being a physicist. The problem tells us that the π^0 meson decays into 2 photons which head off in opposite directions. Arguably, the most fundamental and important aspect of relativity is that photons travel at the speed of light, 1 C, in all reference frames. From this, we know the only solutions can be (A) or (D). Since the problem tells us that the photon, γ_2, proceeds in the "backwards" direction, the sign should be negative and we choose (A).

Correct Answer
(A)

1.38 PGRE9277 #38

38. Tau leptons are observed to have an average half-life of $\triangle t_1$ in the frame S_1 in which the leptons are at rest. In an inertial frame S_2, which is moving at a speed v_{12} relative to S_1, the leptons are observed to have an average half-life of $\triangle t_2$. In another inertial reference frame S_3, which is moving at a speed v_{13} relative to S_1 and v_{23} relative to S_2, the leptons have an observed half-life of $\triangle t_3$. Which of the following is a correct relationship among two of the half-lives, $\triangle t_1$, $\triangle t_2$, and $\triangle t_3$?

(A) $\triangle t_2 = \triangle t_1 \sqrt{1-(v_{12})^2/c^2}$

(B) $\triangle t_1 = \triangle t_3 \sqrt{1-(v_{13})^2/c^2}$

(C) $\triangle t_2 = \triangle t_3 \sqrt{1-(v_{23})^2/c^2}$

(D) $\triangle t_3 = \triangle t_2 \sqrt{1-(v_{23})^2/c^2}$

(E) $\triangle t_1 = \triangle t_2 \sqrt{1-(v_{23})^2/c^2}$

Recommended Solution

Before starting, let's take a look at the general time dilation equation

$$\Delta t' = \gamma \Delta t = \frac{\Delta t}{\sqrt{1-v^2/c^2}} \tag{1.134}$$

from this, we can see that ETS has, quite rudely, written things in terms of inverse Lorentz factors. At this point, I highly recommend that you quietly curse ETS under your breath and then re-write the equations in a more standard form,

(A) $\Delta t_1 = \gamma_{12}\Delta t_2$

(B) $\Delta t_3 = \gamma_{13}\Delta t_1$

(C) $\Delta t_3 = \gamma_{23}\Delta t_2$

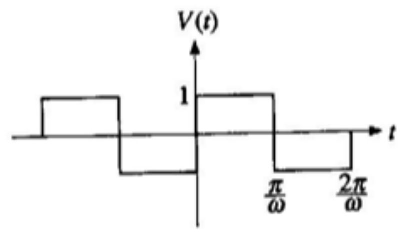
(D) $\Delta t_2 = \gamma_{23}\Delta t_3$

(E) $\Delta t_2 = \gamma_{23}\Delta t_1$

Right off the bat, eliminate (E) because it incorporates a Lorentz factor with frame 3 in it when the time for frame 3 isn't even represented. Next, we know from the time dilation effect that the time of a moving frame in relation to a stationary frame will appear to be longer in the stationary frame. If we let the stationary frame be S_1, then we see that (A) incorrectly concludes that time in the stationary frame would be longer in the stationary frame than the moving frame. (C) and (D) don't involve the stationary frame at all and we aren't given enough information to conclude anything about the relation between the two inertial frames, so those are both likely to be wrong. Only (B) correctly predicts the stationary vs inertial relationship

Correct Answer
(B)

1.39 PGRE9277 #39



39. If n is an integer ranging from 1 to infinity, ω is an angular frequency, and t is time, then the Fourier series for a square wave, as shown above, is given by which of the following?

(A) $V(t) = \frac{4}{\pi}\sum_1^\infty \frac{1}{n}\sin(n\omega t)$

(B) $V(t) = \frac{4}{\pi}\sum_0^\infty \frac{1}{(2n+1)}\sin((2n+1)\omega t)$

(C) $V(t) = \frac{4}{\pi}\sum_1^\infty \frac{1}{n}\cos(n\omega t)$

(D) $V(t) = \frac{4}{\pi}\sum_0^\infty \frac{1}{(2n+1)}\cos((2n+1)\omega t)$

(E) $V(t) = -\frac{4}{\pi} + \frac{4}{\pi}\sum_1^\infty \frac{1}{n^2}\cos(n\omega t)$

Recommended Solution

Recall that the sine function is an odd function, just like the plot given in the problem, while the cosine function is an even function. Based on this, we should eliminate all solutions which utilize cosine functions, i.e. (C), (D), and (E). I should point out that there is always a possibility that the cosine function could be shifted to produce and sine-esque plot, however in our case none of the solutions feature the necessary shift. Next, to choose between (A) and (B), plug in $t = \pi/\omega$ which should give us an amplitude of $V(t) = -1$. in (A), we get

$$V(t) = \frac{4}{\pi}\sum_1^\infty \frac{1}{n}\sin\left(\frac{n\pi\omega}{\omega}\right) \tag{1.135}$$

$$= \frac{4}{\pi}\sum_1^\infty \frac{1}{n}\sin(n\pi) \tag{1.136}$$

From Equation 1.136, it should be clear that $V(\pi/\omega)$ will be 0 for all values of n, so our solution must be (B).

Correct Answer
(B)

1.40 PGRE9277 #40

40. A rigid cylinder rolls at constant speed without slipping on top of a horizontal plane surface. The acceleration of a point on the circumference of the cylinder at the moment when the point touches the plane is

(A) directed forward
(B) directed backward
(C) directed up
(D) directed down
(E) zero

Recommended Solution

The acceleration at any point on the cylinder will be equal to the sum of all its accelerations. Since the problem explicitly specifies that the cylinder doesn't slide, we know that there are no lateral forces to contribute. The only acceleration we have is the centripetal acceleration from its rotation which will be pointing toward the center of the cylinder. When the point under consideration is touching the surface of the plane, the acceleration must point up to point towards the cylinder center.

Correct Answer
(C)

1.41 PGRE9277 #41

Questions 41-42

A cylinder with moment of inertia 4 kg · m² about a fixed axis initially rotates at 80 radians per second about this axis. A constant torque is applied to slow it down to 40 radians per second.

41. The kinetic energy lost by the cylinder is

(A) 80 J
(B) 800 J
(C) 4000 J
(D) 9600 J
(E) 19,200 J

42. If the cylinder takes 10 seconds to reach 40 radians per second, the magnitude of the applied torque is

(A) 80 N · m
(B) 40 N · m
(C) 32 N · m
(D) 16 N · m
(E) 8 N · m

Recommended Solution

The kinetic energy of a rotating object is related to angular frequency, ω, and moment of inertia, I, by

$$E_K = \frac{1}{2} I \omega^2 \tag{1.137}$$

The moment of inertia is given as $I = 4\ \mathrm{kg \cdot m^2}$ and we are told that the initial angular frequency of 80 rad changes down to 40 rad. Using Equation 1.137 and accounting for the change in ω, we calculate the kinetic energy as

$$\begin{aligned}
E_K &= \frac{1}{2} I \left(\omega_f^2 - \omega_i^2\right) && (1.138)\\
&= \frac{1}{2}\left(4\ \mathrm{kg \cdot m^2}\right)\left((80\ \mathrm{rad/s})^2 - (40\ \mathrm{rad/s})^2\right) && (1.139)\\
&= \frac{1}{2}\left(4\ \mathrm{kg \cdot m^2}\right)(4800\ \mathrm{rad/s}) && (1.140)\\
&= 9600\ \mathrm{J} && (1.141)
\end{aligned}$$

Correct Answer
(D)

1.42 PGRE9277 #42

Questions 41-42

A cylinder with moment of inertia $4\ \text{kg}\cdot\text{m}^2$ about a fixed axis initially rotates at 80 radians per second about this axis. A constant torque is applied to slow it down to 40 radians per second.

41. The kinetic energy lost by the cylinder is

(A) 80 J
(B) 800 J
(C) 4000 J
(D) 9600 J
(E) 19,200 J

42. If the cylinder takes 10 seconds to reach 40 radians per second, the magnitude of the applied torque is

(A) $80\ \text{N}\cdot\text{m}$
(B) $40\ \text{N}\cdot\text{m}$
(C) $32\ \text{N}\cdot\text{m}$
(D) $16\ \text{N}\cdot\text{m}$
(E) $8\ \text{N}\cdot\text{m}$

Recommended Solution

Recall our equation for torque

$$\tau = I\alpha \tag{1.142}$$

where α is the angular acceleration. We are given the change in angular velocity as $\omega = 40$ rad/sec. Since the rate of change of angular velocity is given, we can find the average angular acceleration as

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{40\ \text{rad/s}}{10\ \text{s}} = 4\ \text{rad/s}^2 \tag{1.143}$$

Plug our angular acceleration value from Equation 1.143 into Equation 1.142 to get

$$\begin{aligned} \tau &= I\alpha && (1.144)\\ &= \left(4\ \text{kg}\cdot\text{m}^2\right)\left(16\ \text{rad/s}^2\right) && (1.145)\\ &= 16\ \text{N}\cdot\text{m} && (1.146) \end{aligned}$$

Correct Answer
(D)

1.43 PGRE9277 #43

43. If $\frac{\partial L}{\partial q_n} = 0$, where L is the Lagrangian for a conservative system without constraints and q_n is a generalized coordinate, then the generalized momentum p_n is

(A) an ignorable coordinate

(B) constant

(C) undefined

(D) equal to $\frac{d}{dt}\left(\frac{\partial L}{\partial q_n}\right)$

(E) equal to the Hamiltonian for the system

Recommended Solution

This problem is one of those infuriating exam questions that you either know, or you don't. In this instance, you can only be sure you've got the right answer if you recall that Noether's theorem tells us that p_n is a constant under the condition

$$p_n = \frac{\partial L}{\partial q_n} \tag{1.147}$$

Even if you don't know this, we can try to eliminate some of the options based on some common sense

(A) An ignorable coordinate is a coordinate that doesn't show up in the Lagrangian which is not the case

(B) I can't think of a compelling reason to eliminate this one

(C) There is no reason to assume that differentiating the Lagrangian with respect to q_n will be undefined except when $\partial q_n = 0$ which would be a poor assumption

(D) There is no mention made of a time dependence so it is unlikely that $\frac{\partial L}{\partial q_n} = \frac{d}{dt}\left(\frac{\partial L}{\partial q_n}\right)$.

(E) Keep in mind that the Lagrangian and Hamiltonian are both measures of Energy and it is not likely that you can differentiate only one but have them each keep the same units.

Correct Answer
(B)

1.44 PGRE9277 #44

44. A particle of mass m on the Earth's surface is confined to move on the parabolic curve $y = ax^2$, where y is up. Which of the following is a Lagrangian for the particle?

(A) $L = \frac{1}{2}m\dot{y}^2\left(1 + \frac{1}{4ay}\right) - mgy$

(B) $L = \frac{1}{2}m\dot{y}^2\left(1 - \frac{1}{4ay}\right) - mgy$

(C) $L = \frac{1}{2}m\dot{x}^2\left(1 + \frac{1}{4ax}\right) - mgx$

(D) $L = \frac{1}{2}m\dot{x}^2(1 + 4a^2x^2) + mgx$

(E) $L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + mgy$

Recommended Solution

Recall that the Langrangian is the difference between the kinetic and gravitational potential energy,

$$L = T - V \tag{1.148}$$

We can first eliminate (E) because the potential term most oppose the kinetic term and we need at least one minus sign. Next, we know the solution must have some potential energy term mgy so that all of the energy is kinetic at bottom of the parabola, so we eliminate (C) and (D). Finally, we know that the kinetic energy pieces should be adding together, not fighting one another, so we choose (A).

Correct Answer
(A)

1.45 PGRE9277 #45

45. A ball is dropped from a height h. As it bounces off the floor, its speed is 80 percent of what it was just before it hit the floor. The ball will then rise to a height of most nearly

(A) 0.94 h
(B) 0.80 h
(C) 0.75 h
(D) 0.64 h
(E) 0.50 h

Recommended Solution

Before the ball is dropped, the net energy of the system is all potential equal to mgh. Once the ball is released, the potential is converted to kinetic until the ball hits the ground and all mgh of the energy is now kinetic equal to $1/2mv_0^2$. The problem tells us that the velocity after collision is only 4/5 of its initial velocity, v_0, so the kinetic energy on its way back up is

$$T_f = \frac{1}{2}m\left(\frac{4}{5}v_0^2\right) \tag{1.149}$$

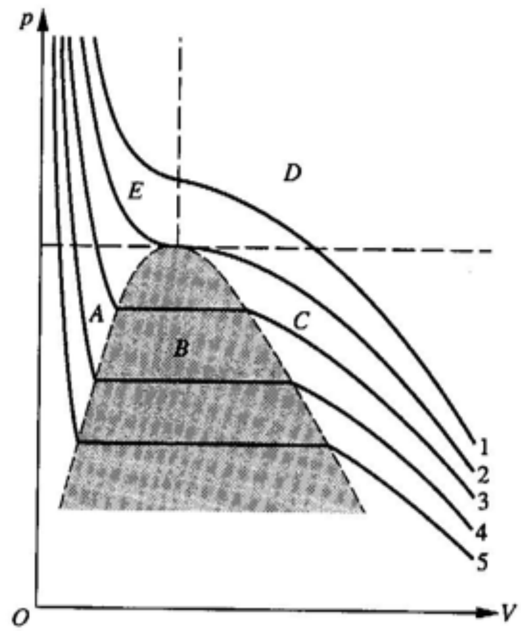
$$= \frac{1}{2}m\left(\frac{16}{25}v_0^2\right) \tag{1.150}$$

If we compare this to the original kinetic energy, it is clear that the final kinetic energy and, therefore, potential energy is $16/25 = 0.64$ times as big as its initial energy.

Correct Answer
(D)

1.46 PGRE9277 #46

Questions 46-47



Isotherms and coexistence curves are shown in the pV diagram above for a liquid-gas system. The dashed lines are the boundaries of the labeled regions.

46. Which numbered curve is the critical isotherm?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

47. In which region are the liquid and the vapor in equilibrium with each other?

(A) *A*
(B) *B*
(C) *C*
(D) *D*
(E) *E*

Recommended Solution

The critical isotherm refers to a curve that has the property that the derivative of the pressure with respect to the volume is 0

$$\frac{\partial P}{\partial V} = 0 \tag{1.151}$$

Of the curves shown, only curve 2 has a point where taking the tangent to the curve results in a horizontal line (i.e. the derivative of the curve is 0). This occurs at precisely the point where the vertical and horizontal dashed lines cross.

Correct Answer
(B)

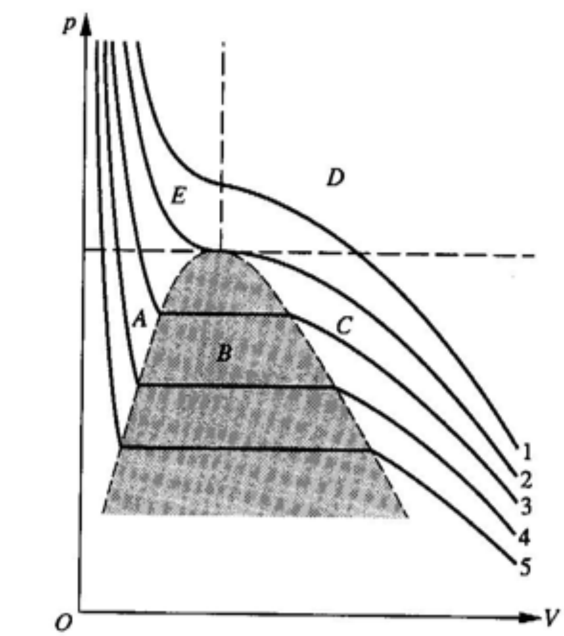
Alternate Solution

Without knowing anything about isotherms, we can eliminate some options through a bit of reasoning. First, eliminate curves 3, 4 and 5 because they are all effectively the same, especially when discussing everything in qualitative terms. Next, you can eliminate curve 1 because, unlike curve 2, it is the same as all other similar curves above the horizontal dashed line and we would expect the solution to be unique.

Correct Answer
(B)

1.47 PGRE9277 #47

Questions 46-47



Isotherms and coexistence curves are shown in the pV diagram above for a liquid-gas system. The dashed lines are the boundaries of the labeled regions.

46. Which numbered curve is the critical isotherm?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

47. In which region are the liquid and the vapor in equilibrium with each other?

(A) *A*
(B) *B*
(C) *C*
(D) *D*
(E) *E*

Recommended Solution

This problem asks us which region will have vapor and liquid in equilibrium which tells us that we expect to have both states present in the region. The quickest way to determine the answer is to consider each region in its limit

(A) In region A, the volume extends to extremely small sizes and this would cause vapor to get compressed to liquid. This likely won't support vapor and liquid phases at the same time.

(B) Region B represents a middle ground of pressure and volume so there are no glaring limit issues.

(C) In region C, the volume can blow up to infinity which would likely force everything to a vapor phase.

(D) Region D allows the volume and the pressure to blow up to infinity, meaning there will absolutely be states which either force everything to vapor or everything to liquid.

(E) Region E will allow the pressure to blow up to infinity which will force any vapor present into liquid.

Only (B) lacks a limiting value that could potentially ruin our equilibrium.

Correct Answer
(B)

1.48 PGRE9277 #48

48. The magnitude of the force F on an object can be determined by measuring both the mass m of an object and the magnitude of its acceleration a, where $F = ma$. Assume that these measurements are uncorrelated and normally distributed. If the standard deviations of the measurements of the mass and the acceleration are σ_m and σ_a, respectively, then σ_F/F is

(A) $\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_a}{a}\right)^2$

(B) $\left(\frac{\sigma_m}{m} + \frac{\sigma_a}{a}\right)^{\frac{1}{2}}$

(C) $\left[\left(\frac{\sigma_m}{m}\right)^2 + \left(\frac{\sigma_a}{a}\right)^2\right]^{\frac{1}{2}}$

(D) $\frac{\sigma_m \sigma_a}{ma}$

(E) $\frac{\sigma_m}{m} + \frac{\sigma_a}{a}$

Recommended Solution

At some point in your undergraduate career, you were probably forced to calculate some standard deviations by hand. If you did, then you likely used this equation

$$\sigma = \sqrt{\sum_{i=1}^{N} p_i \left(x_i - \bar{x}\right)^2} \tag{1.152}$$

From Equation 1.152, we know to look for a square root and we can then eliminate (A), (D) and (E) based on this condition. Next, we should expect to see σ^2 values rather than simply σ so we eliminate (B) and choose (C).

Correct Answer
(C)

1.49 PGRE9277 #49

49. Two horizontal scintillation counters are located near the Earth's surface. One is 3.0 meters directly above the other. Of the following, which is the largest scintillator resolving time that can be used to distinguish downward-going relativistic muons from upward-going relativistic muons using the relative time of the scintillator signals?

(A) 1 picosecond
(B) 1 nanosecond
(C) 1 microsecond
(D) 1 millisecond
(E) 1 second

Recommended Solution

Considering the amount of variation between the possible solutions, let's do an approximation. Muons move very quickly and the problem tells us that the motion is relativistic, let's approximate the speed of a muon going nearly the speed of light as just the speed of light

$$v_\mu = 3.0 \times 10^8 \text{ m/s} \tag{1.153}$$

Since the scintillators are 3.0 meters apart, we can find the time scale as

$$\begin{aligned}
\Delta t &= \frac{\Delta x}{v_\mu} && (1.154)\\
&= \frac{3.0 \text{ m}}{3.0 \times 10^8 \text{ m/s}} && (1.155)\\
&= 1 \times 10^{-8} \text{ s} && (1.156)\\
&= 10 \text{ nanoseconds} && (1.157)
\end{aligned}$$

so we will want to choose the nanosecond range, i.e. solution (B).

Correct Answer
(B)

1.50 PGRE9277 #50

50. The state of a quantum mechanical system is described by a wave function ψ. Consider two physical observables that have discrete eigenvalues: observable A with eigenvalues $\{\alpha\}$, and observable B with eigenvalues $\{\beta\}$. Under what circumstances can all wave functions be expanded in a set of basis states, each of which is a simultaneous eigenfunction of both A and B?

(A) Only if the values $\{\alpha\}$ and $\{\beta\}$ are nondegenerate
(B) Only if A and B commute
(C) Only if A commutes with the Hamiltonian of the system
(D) Only if B commutes with the Hamiltonian of the system
(E) Under all circumstances

Recommended Solution

First of all, eliminate (E) because we would never be so lucky that we could expand a wave function into basis states under any and all circumstances. Next, eliminate (C) and (D) because A and B as well as α and β are qualitatively identical to one another so if (C) was true, (D) should also be true and we can't choose both. Finally, eliminate (A) because whether or not the two eigenvalues are non-degenerate should have nothing to do with the basis functions of the wave function.

Correct Answer
(B)

1.51 PGRE9277 #51

Questions 51-53

A particle of mass m is confined to an infinitely deep square-well potential:

$$V(x) = \infty, x \le 0, x \ge a$$
$$V(x) = 0, 0 < x < a.$$

The normalized eigenfunctions, labeled by the quantum number n, are $\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$.

51. For any state n, the expectation value of the momentum of the particle is

(A) 0

(B) $\frac{\hbar n\pi}{a}$

(C) $\frac{2\hbar n\pi}{a}$

(D) $\frac{\hbar n\pi}{a}(\cos n\pi - 1)$

(E) $\frac{-i\hbar n\pi}{a}(\cos n\pi - 1)$

52. The eigenfunctions satisfy the condition $\int_0^a \psi_n{}^*(x)\,\psi_\ell(x)\,dx = \delta_{n\ell}$, $\delta_{n\ell} = 1$ if $n = \ell$, otherwise $\delta_{n\ell} = 0$. This is a statement that the eigenfunctions are

(A) solutions to the Schrödinger equation
(B) orthonormal
(C) bounded
(D) linearly dependent
(E) symmetric

53. A measurement of energy E will <u>always</u> satisfy which of the following relationships?

(A) $E \le \frac{\pi^2\hbar^2}{8ma^2}$

(B) $E \ge \frac{\pi^2\hbar^2}{2ma^2}$

(C) $E = \frac{\pi^2\hbar^2}{8ma^2}$

(D) $E = \frac{n^2\pi^2\hbar^2}{8ma^2}$

(E) $E = \frac{\pi^2\hbar^2}{2ma^2}$

Recommended Solution

Starting with classical momentum, $P = m\dot{x}$, recall that the analog for the expectation value of momentum is

$$\langle p \rangle = m\frac{d\langle x \rangle}{dt} \tag{1.158}$$

in the infinite square well, the expectation value for position is

$$\langle x \rangle = \frac{a}{2} \tag{1.159}$$

since $\langle x \rangle$ is nothing but constants, if we take the derivative of it then it goes to 0 and so does $\langle p \rangle$.

Correct Answer
(B)

Alternate Solution

More rigorously, we can use the general equation for expectation value

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^* \frac{\hbar}{i} \psi \; dx \tag{1.160}$$

we are given the wave function so if we plug everything in, we get

$$\begin{aligned} \langle p \rangle &= \int_{-\infty}^{\infty} \psi^* \frac{\hbar}{i} \psi dx & (1.161) \\ &= \frac{2n\pi\hbar}{a^2 i} \int_{-\infty}^{\infty} \sin\left(\frac{n\pi x}{a}\right) \cos\left(\frac{n\pi x}{a}\right) & (1.162) \end{aligned}$$

and since cosine and sine are orthogonal with respect to each other, integrating over all of x will result in each one canceling out the other and the total area is 0.

Correct Answer
(B)

1.52 PGRE9277 #52

Questions 51-53

A particle of mass m is confined to an infinitely deep square-well potential:

$$V(x) = \infty,\ x \le 0,\ x \ge a$$
$$V(x) = 0,\ 0 < x < a.$$

The normalized eigenfunctions, labeled by the quantum number n, are $\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$.

51. For any state n, the expectation value of the momentum of the particle is

(A) 0

(B) $\frac{\hbar n\pi}{a}$

(C) $\frac{2\hbar n\pi}{a}$

(D) $\frac{\hbar n\pi}{a}(\cos n\pi - 1)$

(E) $\frac{-i\hbar n\pi}{a}(\cos n\pi - 1)$

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(B) $E \ge \frac{\pi^2\hbar^2}{2ma^2}$

(C) $E = \frac{\pi^2\hbar^2}{8ma^2}$

(D) $E = \frac{n^2\pi^2\hbar^2}{8ma^2}$

(E) $E = \frac{\pi^2\hbar^2}{2ma^2}$

Recommended Solution

The condition for orthonormality,

$$\langle n|m\rangle = \delta_{nm} \tag{1.163}$$

is a function of the Kronecker delta type, i.e.

$$\delta_{nn} = 1 \tag{1.164}$$
$$\delta_{nm} = 0 \tag{1.165}$$

This is precisely the description in the problem so we choose the orthonormality condition.

Correct Answer
(B)

1.53 PGRE9277 #53

Questions 51-53

A particle of mass m is confined to an infinitely deep square-well potential:

$$V(x) = \infty,\ x \leq 0,\ x \geq a$$
$$V(x) = 0,\ 0 < x < a.$$

The normalized eigenfunctions, labeled by the quantum number n, are $\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$.

51. For any state n, the expectation value of the momentum of the particle is

(A) 0

(B) $\frac{\hbar n\pi}{a}$

(C) $\frac{2\hbar n\pi}{a}$

(D) $\frac{\hbar n\pi}{a}(\cos n\pi - 1)$

(E) $\frac{-i\hbar n\pi}{a}(\cos n\pi - 1)$

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(A) solutions to the Schrödinger equation
(B) orthonormal
(C) bounded
(D) linearly dependent
(E) symmetric

53. A measurement of energy E will always satisfy which of the following relationships?

(A) $E \leq \frac{\pi^2\hbar^2}{8ma^2}$

(B) $E \geq \frac{\pi^2\hbar^2}{2ma^2}$

(C) $E = \frac{\pi^2\hbar^2}{8ma^2}$

(D) $E = \frac{n^2\pi^2\hbar^2}{8ma^2}$

(E) $E = \frac{\pi^2\hbar^2}{2ma^2}$

Recommended Solution

The energy of the infinite square well isn't constant so we can immediately eliminate (C) and (E). Next, we can eliminate (A) because, in theory, there shouldn't be an upper bound to the energy and (A) suggests that there is such a maximum energy. Lastly, you can quickly check the coefficients for (B) and (D) by recalling the energy equation derived form the Schrodinger equation,

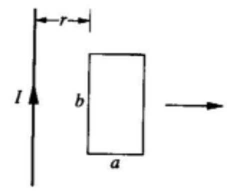
$$E = \frac{p^2}{2m} + V \tag{1.166}$$

since V = 0 inside the potential, we get $E = p^2/2m$ which tells us the coefficient should be 1/2 rather than 1/8 and so we can choose (B).

Correct Answer
(B)

1.54 PGRE9277 #54

Questions 54-55



A rectangular loop of wire with dimensions shown above is coplanar with a long wire carrying current I. The distance between the wire and the left side of the loop is r. The loop is pulled to the right as indicated.

54. What are the directions of the induced current in the loop and the magnetic forces on the left and the right sides of the loop as the loop is pulled?

	Induced Current	Force on Left Side	Force on Right Side
(A)	Counterclockwise	To the left	To the right
(B)	Counterclockwise	To the left	To the left
(C)	Counterclockwise	To the right	To the left
(D)	Clockwise	To the right	To the left
(E)	Clockwise	To the left	To the right

55. What is the magnitude of the net force on the loop when the induced current is i?

(A) $\frac{\mu_0 iI}{2\pi}\ln\left(\frac{r+a}{r}\right)$

(B) $\frac{\mu_0 iI}{2\pi}\ln\left(\frac{r}{r+a}\right)$

(C) $\frac{\mu_0 iI}{2\pi}\frac{b}{a}$

(D) $\frac{\mu_0 iI}{2\pi}\frac{ab}{r(r+a)}$

(E) $\frac{\mu_0 iI}{2\pi}\frac{r(r+a)}{ab}$

Recommended Solution

This problem is a left/right hand rule paradise (or nightmare depending on your familiarity with the rules). First of all, start by applying the right hand rule to the vertical wire with your thumb in the up direction. This tells us that the magnetic field is pointing into the plane on the side of the loop of wire. Now, apply the right hand rule of a magnetic field into the plane and into the loop such that your thumb is pointing into the plane and your hands are looping clockwise (Figure 1.2), allowing us to eliminate (A), (B) and (C).

Lastly, use the left hand rule (Figure 1.3) with your thumb in the direction of the current, pointer finger into the plane and middle finger in the direction of the resulting force to find that the left side of the loop goes to the left and the right side of the loop goes right, leaving us with (E).

Correct Answer
(B)

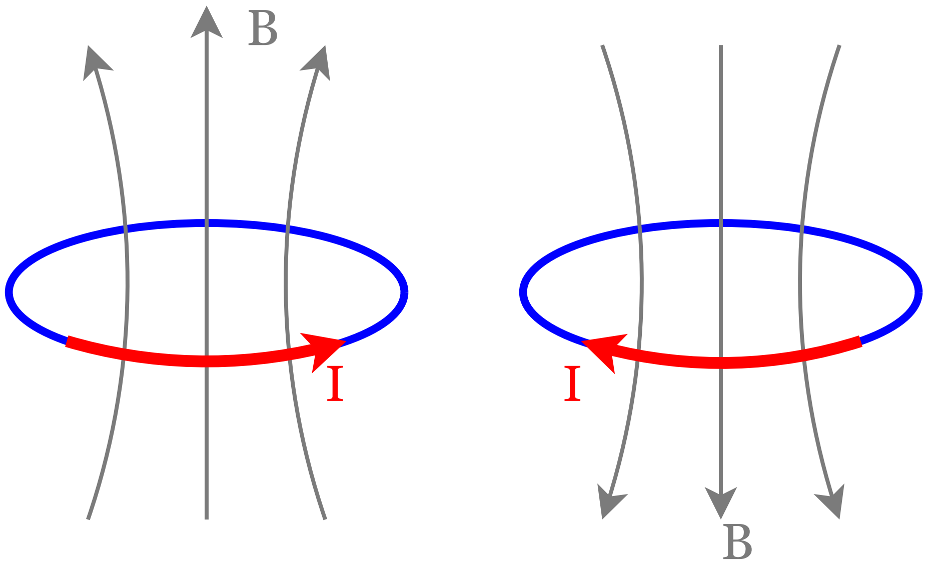


Figure 1.2: Right hand rule for a magnetic field passing through a loop of wire

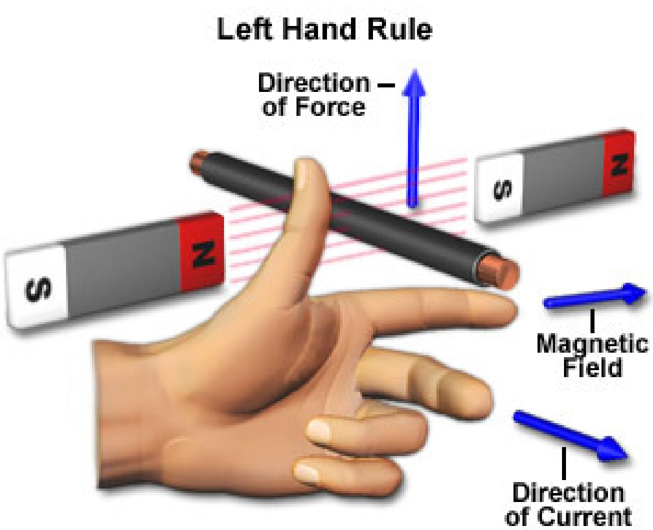
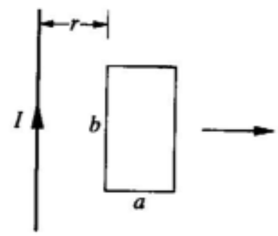


Figure 1.3: Left hand rule for a current through a magnetic field

1.55 PGRE9277 #55

Questions 54-55



A rectangular loop of wire with dimensions shown above is coplanar with a long wire carrying current I. The distance between the wire and the left side of the loop is r. The loop is pulled to the right as indicated.

54. What are the directions of the induced current in the loop and the magnetic forces on the left and the right sides of the loop as the loop is pulled?

	Induced Current	Force on Left Side	Force on Right Side
(A)	Counterclockwise	To the left	To the right
(B)	Counterclockwise	To the left	To the left
(C)	Counterclockwise	To the right	To the left
(D)	Clockwise	To the right	To the left
(E)	Clockwise	To the left	To the right

55. What is the magnitude of the net force on the loop when the induced current is i?

(A) $\frac{\mu_0 iI}{2\pi}\ln\left(\frac{r+a}{r}\right)$

(B) $\frac{\mu_0 iI}{2\pi}\ln\left(\frac{r}{r+a}\right)$

(C) $\frac{\mu_0 iI}{2\pi}\frac{b}{a}$

(D) $\frac{\mu_0 iI}{2\pi}\frac{ab}{r(r+a)}$

(E) $\frac{\mu_0 iI}{2\pi}\frac{r(r+a)}{ab}$

Recommended Solution

The quickest way to solve this problem is to consider the limits of the lengths a and b. If either of these go to 0 then the flux goes to 0 and so too does the force. For this reason, we can eliminate any solution that doesn't have some dependence on both a and b, i.e. (A) and (B). Next, note that (C) and (E) blows up to infinity when $a \to 0$ so eliminate both of these.

Correct Answer
(D)

1.56 PGRE9277 #56

56. If ν is frequency and h is Planck's constant, the ground state energy of a one-dimensional quantum mechanical harmonic oscillator is

(A) 0

(B) $\frac{1}{3}h\nu$

(C) $\frac{1}{2}h\nu$

(D) $h\nu$

(E) $\frac{3}{2}h\nu$

Recommended Solution

The general equation for energy of a quantum harmonic oscillator in state n is

$$E = \left(\frac{1}{2} + n\right) h\nu \tag{1.167}$$

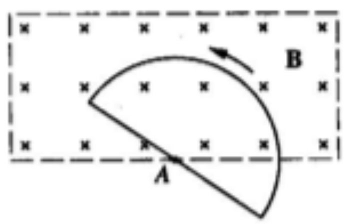
In its ground state, $n = 0$ so the solution should be

$$E = \left(\frac{1}{2} + 0\right) h\nu \tag{1.168}$$

$$= \frac{1}{2}h\nu \tag{1.169}$$

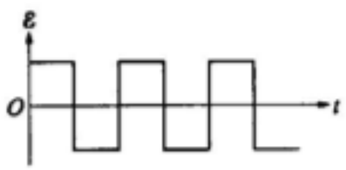
Correct Answer
(D)

1.57 PGRE9277 #57

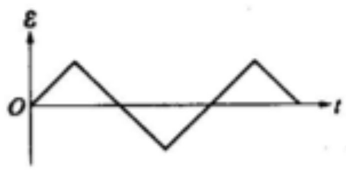


57. A uniform and constant magnetic field **B** is directed perpendicularly into the plane of the page everywhere within a rectangular region as shown above. A wire circuit in the shape of a semicircle is uniformly rotated counterclockwise in the plane of the page about an axis A. The axis A is perpendicular to the page at the edge of the field and directed through the center of the straight-line portion of the circuit. Which of the following graphs best approximates the emf $\mathcal{E}$ induced in the circuit as a function of time t?

(A)



(B)



(C)

(D)

(E)

Recommended Solution

Recall Faraday's law which states that a current will be induced in a conductor due to a change in magnetic flux

$$|\varepsilon| = \left| \frac{d\phi_B}{dt} \right| \tag{1.170}$$

from the description, we know that the half circle is rotating "uniformly" so the induced current should be constant and we can eliminate (C), (D) and (E). As the half circle begins to enter the rectangle, it will have a constantly increasing induced current and once it begins to exit the rectangle, it should have a constantly decreasing induced current. Option (A) gives us this feature but (B) suggest a constantly increasing increase in induced current, which is not what we want.

Correct Answer
(A)

1.58 PGRE9277 #58

58. The ground state configuration of a neutral sodium atom ($Z = 11$) is

(A) $1s^2 2s^2 2p^5 3s^2$
(B) $1s^2 2s^3 2p^6$
(C) $1s^2 2s^2 2p^6 3s$
(D) $1s^2 2s^2 2p^6 3p$
(E) $1s^2 2s^2 2p^5$

Recommended Solution

In ground state, the number of electrons on the atom should be the same as $Z = 11$. The quickest way to figure out the number of electrons proposed in each of the 5 options is to sum up all of the superscripts in each configuration. Doing so will eliminate option (E). Next, we can eliminate (A) because we should completely fill $2p$ to $2p^6$ before moving to the next energy level. Next, eliminate (B) because the s level can't have 3 electrons in it. Finally, recall your energy level diagrams (Figure 1.4) to see that we should progress to $3s$ after $2p$ as opposed to going from $2p$ to $3p$

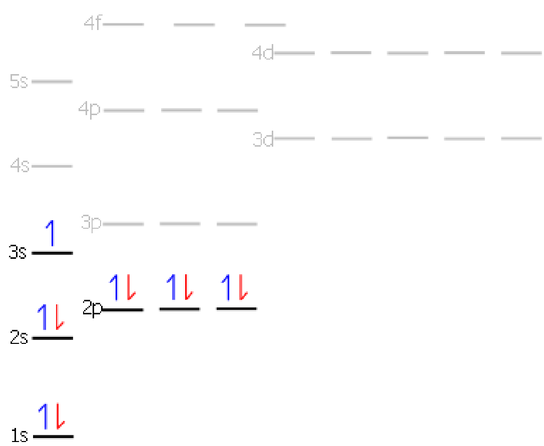


Figure 1.4: Energy level diagram of Sodium

Correct Answer
(C)

1.59 PGRE9277 #59

59. The ground state of the helium atom is a spin

(A) singlet
(B) doublet
(C) triplet
(D) quartet
(E) quintuplet

Recommended Solution

In its ground state, the helium atom has 2 electrons in the first shell which, by the Pauli Exclusion Principle, has one spin up and one spin down. The spin multiplicity, which is $2S+1$, is what determines whether an atom is a singlet, doublet, triplet, etc.

Singlet $2(0)+1=1$

Double $2(1/2)+1=2$

Triplet $2(1)+1=3$

In the case of helium, since we have two electrons with opposite directions of spin, they cancel to give us $S=0$ which is a singlet

Correct Answer
(A)

1.60 PGRE9277 #60

60. An electron in a metal has an effective mass $m^* = 0.1m_e$. If this metal is placed in a magnetic field of magnitude 1 tesla, the cyclotron resonance frequency, ω_c, is most nearly

(A) 930 rad/s

(B) 8.5×10^6 rad/s

(C) 2.8×10^{11} rad/s

(D) 1.8×10^{12} rad/s

(E) 7.7×10^{20} rad/s

Recommended Solution

If you recall the equation for cyclotron resonance frequency, this problem is a quick plug-n-chug problem

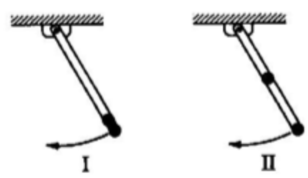
$$\omega_c = \frac{eB}{m} \tag{1.171}$$

from the problem description, we are given values for B and m and we can get the elementary charge value e from the front of the test booklet. Plug these values into Equation 1.171 and solve

$$\begin{aligned} \omega_c &= \frac{(1.6 \times 10^{-19} \text{ coulombs})\,(1 \text{ tesla})}{(0.1)\,(9 \times 10^{-31} \text{ kg})} && (1.172) \\ &\approx 2 \times 10^{12} \text{rad/s} && (1.173) \end{aligned}$$

Correct Answer
(D)

1.61 PGRE9277 #61



61. A long, straight, and massless rod pivots about one end in a vertical plane. In configuration I, shown above, two small identical masses are attached to the free end; in configuration II, one mass is moved to the center of the rod. What is the ratio of the frequency of small oscillations of configuration II to that of configuration I ?

(A) $(6/5)^{\frac{1}{2}}$

(B) $(3/2)^{\frac{1}{2}}$

(C) 6/5

(D) 3/2

(E) 5/3

Recommended Solution

Start by recalling the frequency equation for a pendulum,

$$\omega = \sqrt{\frac{mgr_{com}}{I}} \tag{1.174}$$

where the moment of inertia will always be that of a point mass, $I = mr^2$. for the first pendulum, all of the mass is located at the bottom of the pendulum which makes our center of mass at a distance r, making our frequency equation

$$\omega_I = \sqrt{\frac{2mgr}{2mr^2}} = \sqrt{\frac{g}{r}} \tag{1.175}$$

for the second pendulum, however, the masses are separated at a distance of $r/2$ and r which forces the center of mass to be $3/4\ r$, so the frequency equation becomes

$$\begin{aligned} \omega_{II} &= \sqrt{\frac{2mg(3/4)r}{(m(1/2\ r)^2) + (mr^2)}} && (1.176) \\ &= \sqrt{\frac{(6/4)g}{(5/4)r}} && (1.177) \\ &= \sqrt{\frac{6}{5}\frac{g}{r}} && (1.178) \end{aligned}$$

at which point it should be clear that pendulum II has a frequency of $\sqrt{6/5}$ that of pendulum I

Correct Answer
(A)

1.62 PGRE9277 #62

62. A mole of ideal gas initially at temperature T_0 and volume V_0 undergoes a reversible isothermal expansion to volume V_1. If the ratio of specific heats is $c_p/c_v = \gamma$ and if R is the gas constant, the work done by the gas is

(A) zero

(B) $RT_0(V_1/V_0)^\gamma$

(C) $RT_0(V_1/V_0 - 1)$

(D) $c_v T_0\left[1 - (V_0/V_1)^{\gamma-1}\right]$

(E) $RT_0 \ln(V_1/V_0)$

Recommended Solution

Anytime you get an increase in volume, you will be doing work so we should first eliminate (A). By the same type of reasoning, if $V_1 = V_0$ then the work should be 0 so we can eliminate (B). Next, because the problem tells us that we can treat the gas as an ideal gas, which should mean that the type of gas is irrelevant, we should be able to ignore specific heats because they are dependent on the type of gas. From this, eliminate (D). Lastly, we need to use the thermodynamic work equation and ideal gas law to see that the solution should have a natural log,

$$W = -\int_{V_0}^{V_1} PdV \tag{1.179}$$

$$P = \frac{nRT}{V} \tag{1.180}$$

Combine Equation 1.179 and Equation 1.180, and integrate to get

$$\begin{aligned} W &= -\int_{V_0}^{V_1} \left(\frac{nRT}{V}\right) dV & (1.181)\\ &= nRT\ [\ln(V_1) - \ln(V_2)] & (1.182)\\ &= nRT\ \ln\left(\frac{V_1}{V_0}\right) & (1.183) \end{aligned}$$

so we should choose (E)

Correct Answer
(E)

1.63 PGRE9277 #63

63. Which of the following is true if the arrangement of an isolated thermodynamic system is of maximal probability?

(A) Spontaneous change to a lower probability occurs.
(B) The entropy is a minimum.
(C) Boltzmann's constant approaches zero.
(D) No spontaneous change occurs.
(E) The entropy is zero.

Recommended Solution

If you're clever, you'll notice that (A) and ~~(B)~~ are exactly opposite so they can't both be wrong and we know it must be one or the other. In order to choose between the two, recall that a system of maximal probability is in its most stable state so we would expect no spontaneous changes and we choose (D).

Correct Answer
(D)

1.64 PGRE9277 #64

64. If an electric field is given in a certain region by $E_x = 0$, $E_y = 0$, $E_z = kz$, where k is a nonzero constant, which of the following is true?

(A) There is a time-varying magnetic field.
(B) There is charge density in the region.
(C) The electric field cannot be constant in time.
(D) The electric field is impossible under any circumstances.
(E) None of the above

Recommended Solution

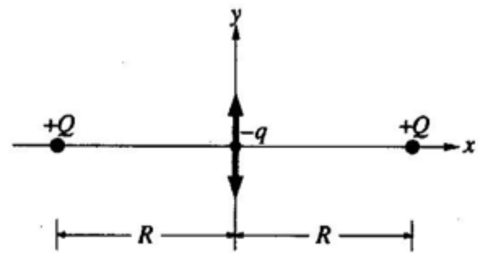
Immediately eliminate (D) because the presence of $E_z = kz$ guarantees we will have some sort of electric field. Next, eliminate (C) because there is nothing about (E_x, E_y, E_z) that forces the electric field to vary, especially considering that the only non-zero component is scaled by a constant k and z can be constant as well. Next, eliminate (A) because nothing about what is given demonstrates any time dependence and, for that matter, it says essentially the same thing as (C) which we already decided wasn't correct. If (B) is also untrue, then we choose (E). However, recall Gauss' law

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \tag{1.184}$$

which clearly states that we should get a charge density in the region of the electric field.

Correct Answer
(B)

1.65 PGRE9277 #65



65. Two point charges with the same charge $+Q$ are fixed along the x-axis and are a distance $2R$ apart as shown. A small particle with mass m and charge $-q$ is placed at the midpoint between them. What is the angular frequency ω of small oscillations of this particle along the y-direction?

(A) $\frac{Qq}{2\pi\epsilon_0 mR^2}$

(B) $\frac{Qq}{4\pi\epsilon_0 mR^2}$

(C) $\frac{Qq}{2\pi\epsilon_0 mR^3}$

(D) $\left(\frac{Qq}{4\pi\epsilon_0 mR^2}\right)^{\frac{1}{2}}$

(E) $\left(\frac{Qq}{2\pi\epsilon_0 mR^3}\right)^{\frac{1}{2}}$

Recommended Solution

We are looking for angular frequency so we should expect to get units of inverse time, recall the SI units for each variable/constant used in the solutions

$$\mathrm{Q/q} = \mathrm{C} \tag{1.185}$$
$$\epsilon_0 = \mathrm{C^2/N \cdot m} \tag{1.186}$$
$$m = \mathrm{kg} \tag{1.187}$$
$$R = \mathrm{m} \tag{1.188}$$

check each of the potential solutions

(A) $\frac{Qq}{2\pi\epsilon_0 mR^2} = \frac{\mathrm{m}}{\mathrm{sec}^2}$

(B) $\frac{Qq}{4\pi\epsilon_0 mR^2} = \frac{\mathrm{m}}{\mathrm{sec}^2}$

(C) $\frac{Qq}{2\pi\epsilon_0 mR^3} = \frac{1}{\mathrm{sec}^2}$

(D) $\sqrt{\frac{Qq}{4\pi\epsilon_0 mR^2}} = \frac{\sqrt{\text{m}}}{\text{sec}}$

(E) $\sqrt{\frac{Qq}{2\pi\epsilon_0 mR^3}} = \frac{1}{\text{sec}} = \text{sec}^{-1}$

Correct Answer
(E)

Alternate Solution

If you insist on doing this problem in the rigorous fashion, start with Coulomb's law

$$\frac{1}{4\pi\epsilon_0}\frac{qQ}{R^2} \tag{1.189}$$

since there are two charges pushing on the central charge q, we change this to account for both with

$$F = \frac{1}{4\pi\epsilon_0}\frac{2qQ}{R^2} \tag{1.190}$$

$$= \frac{1}{2\pi\epsilon_0}\frac{qQ}{R^2} \tag{1.191}$$

Next, recall that the force for an oscillating spring is

$$m\ddot{x} = -kx \tag{1.192}$$

and has angular frequency

$$\omega^2 = \frac{k}{m} \tag{1.193}$$

re-arrange Equation 1.192 to get everything in terms of k/m to get

$$\omega^2 = \frac{k}{m} = -\frac{\ddot{x}}{x} \tag{1.194}$$

or equivalently,

$$\omega^2 x = \ddot{x} \tag{1.195}$$

Finally, since $F = m\ddot{x}$, use Equation 1.195 with Equation 1.191 (and let $x = R$) to solve for angular frequency

$$m\ddot{x} = \frac{qQ}{2\pi\epsilon_0 R^2} \tag{1.196}$$

$$m\omega^2 x = \frac{qQ}{2\pi\epsilon_0 R^2} \tag{1.197}$$

$$\omega^2 = \frac{qQ}{2\pi\epsilon_0 mR^3} \tag{1.198}$$

$$\omega = \sqrt{\frac{qQ}{2\pi\epsilon_0 mR^3}} \tag{1.199}$$

Correct Answer
(E)

1.66 PGRE9277 #66

66. A thin uniform steel chain is 10 meters long with a mass density of 2 kilograms per meter. One end of the chain is attached to a horizontal axle having a radius that is small compared to the length of the chain. If the chain initially hangs vertically, the work required to slowly wind it up on to the axle is closest to

(A) 100 J
(B) 200 J
(C) 1,000 J
(D) 2,000 J
(E) 10,000 J

Recommended Solution

Recall from the Work-Energy theorem

$$W = \Delta E_K \tag{1.200}$$

Since energy is conserved, the kinetic energy used to move the chain up will be equal to the total potential energy at the top of the axle. The potential energy can be found by

$$W = E_G = mgh = (10 \text{ kg})(10 \text{ m/s}^2)(10 \text{ m}) = 1000 \text{ J} \tag{1.201}$$

Correct Answer
(C)

Alternate Solution

You could, if you aren't as industrious as I, set up a differential to account for the changing mass of the chain as it is lifted up. However, to do this quickly let's first consider our simple work equation

$$W = F\Delta x \tag{1.202}$$

To make a quick approximation, assume that $g = 10$ m/s^2 and make measurements for every 1 m of change in the chain which will account for a decrease in 20 N. The initial change of of 1 m with a 10 meter long chain with 2 kg per meter is 200 N $\cdot$ m. The next bit of work will be 180 N $\cdot$ m and then 140 N $\cdot$ m and so on to get

$$W_{net} = 200 + 180 + 160 + 140 + 120 + 100 + 80 + 60 + 40 + 20 = 1060 \text{ N} \cdot \text{m} \tag{1.203}$$

which is closest to (C).

Correct Answer
(C)

Alternate Solution

If you insist on doing things the hard way, start out with the integral form of work

$$W = -\int F \cdot dx \tag{1.204}$$

In our problem the force changes as the length changes and it changes proportional to

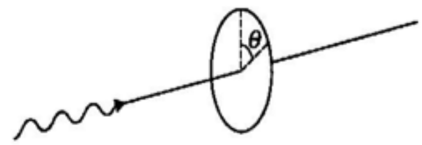
$$F = mg = (20\text{ kg} - 2l)g \tag{1.205}$$

that is to say that the mass is initially 20 kg and then decreases by 2 times the length of the chain. Plug Equation 1.204 into Equation 1.205 to get

$$\begin{aligned} W &= -\int_{10}^{0} (20\text{ kg} - 2l)g dl & (1.206)\\ &= \int_{0}^{10} 20g - 2gl\ dl & (1.207)\\ &= \Big[20gl - gl^2\Big]_0^{10} & (1.208)\\ &= (2000\text{ N}\cdot\text{m}) - (1000\text{ N}\cdot\text{m}) & (1.209)\\ &= 1000\text{ N}\cdot\text{m} & (1.210) \end{aligned}$$

Correct Answer
(C)

1.67 PGRE9277 #67



67. A steady beam of light is normally incident on a piece of polaroid. As the polaroid is rotated around the beam axis, the transmitted intensity varies as $A + B\cos 2\theta$, where θ is the angle of rotation, and A and B are constants with $A > B > 0$. Which of the following may be correctly concluded about the incident light?

(A) The light is completely unpolarized.
(B) The light is completely plane polarized.
(C) The light is partly plane polarized and partly unpolarized.
(D) The light is partly circularly polarized and partly unpolarized.
(E) The light is completely circularly polarized.

Recommended Solution

The intensity of the light that gets transmitted through the polaroid is given as

$$I = A + B\cos(2\theta) \tag{1.211}$$

Which tells us that one term can go to 0 when $\cos(2\theta)$ goes to 0 while the other term, the A term, can't. This tells us that the light is composed of two different types of polarizations and we eliminate (A), (B) and (E). Lastly, to distinguish between (C) and (D), recall Malus' law which states that plane polarized light has intensity proportional to

$$I = I_0\cos^2(\theta) \tag{1.212}$$

which we can re-write in a similar form as Equation 1.211 by the double angle identity

$$2\cos^2(a) = 1 + \cos(2a) \tag{1.213}$$

which gives us (C).

Correct Answer
(C)

1.68 PGRE9277 #68

68. The angular separation of the two components of a double star is 8 microradians, and the light from the double star has a wavelength of 5500 angstroms. The smallest diameter of a telescope mirror that will resolve the double star is most nearly

(A) 1 mm
(B) 1 cm
(C) 10 cm
(D) 1 m
(E) 100 m

Recommended Solution

To calculate optical resolution, we need to use the Rayleigh Criterion,

$$\sin(\theta) = 1.22\frac{\lambda}{d} \tag{1.214}$$

the angle and wavelength are given so we can re-arrange Equation 1.214 to solve for d,

$$d = \frac{1.22\lambda}{\sin(\theta)} \tag{1.215}$$

By a small angle approximation, which we can make because the angle is in microradians, let $\sin(\theta) = \theta$ and then convert all of the values into the same units to get

$$\begin{aligned} d &= \frac{1.22\lambda}{\theta} && (1.216)\\ &= \frac{1.22(5.5 \times 10^{-7}\text{ m})}{8.0 \times 10^{-6}\text{ m}} && (1.217)\\ &\approx 1 \times 10^{-1}\text{ m} && (1.218)\\ &\approx 10\text{ cm} && (1.219) \end{aligned}$$

which is closest to (C).

Correct Answer
(C)

Alternate Solution

Even without remembering the necessary equation, you eliminate some choices by a bit of common sense. Because we are talking about a telescope reflecting mirror, we can probably eliminate (A) and (B) as being ridiculously small to be a reflecting mirror on a telescope. On the other end of the spectrum, a 100 m reflecting mirror would be ridiculously too big. In fact, the largest telescopes on earth peak at or just sightly above 10 m so 100 m is very unlikely and we can eliminate (E). At this point, you can now guess between (C) and (D).

Correct Answer
(C)

1.69 PGRE9277 #69

69. A fast charged particle passes perpendicularly through a thin glass sheet of index of refraction 1.5. The particle emits light in the glass. The minimum speed of the particle is

(A) $\frac{1}{3}c$

(B) $\frac{4}{9}c$

(C) $\frac{5}{9}c$

(D) $\frac{2}{3}c$

(E) c

Recommended Solution

A photon travels through a medium with an index of refraction, n, according to the equation

$$v = \frac{c}{n} \tag{1.220}$$

The index of refraction of the glass is, $n = 1.5$, so we plug that in and solve

$$v = \frac{c}{3/2} \tag{1.221}$$

$$= \frac{2}{3}c \tag{1.222}$$

Correct Answer
(D)

1.70 PGRE9277 #70

70. A monoenergetic beam consists of unstable particles with total energies 100 times their rest energy. If the particles have rest mass m, their momentum is most nearly

(A) mc
(B) $10\ mc$
(C) $70\ mc$
(D) $100\ mc$
(E) $10^4\ mc$

Recommended Solution

Start off with the relativistic equation

$$E^2 = p^2c^2 + m^2c^4 \tag{1.223}$$

The problem tells us that the energy is $E = 100mc^2$ so we plug that into Equation 1.223,

$$(100mc^2)^2 = p^2c^2 + m^2c^4 \tag{1.224}$$
$$10000m^2c^4 = p^2c^2 + m^2c^4 \tag{1.225}$$

We could then combine terms with m^2c^4 but doing so will make almost no change to the $10000m^2c^4$ so let's just ignore it. Finally, solve for the p in Equation 1.225 to get

$$p^2c^2 = 10000m^2c^4 \tag{1.226}$$
$$p^2 = 10000m^2c^2 \tag{1.227}$$
$$p = 100mc \tag{1.228}$$

which is (D).

Correct Answer
(D)

1.71 PGRE9277 #71

Questions 71-73

A system in thermal equilibrium at temperature T consists of a large number N_0 of subsystems, each of which can exist only in two states of energy E_1 and E_2, where $E_2 - E_1 = \epsilon > 0$. In the expressions that follow, k is the Boltzmann constant.

71. For a system at temperature T, the average number of subsystems in the state of energy E_1 is given by

(A) $\frac{N_0}{2}$

(B) $\frac{N_0}{1 + e^{-\epsilon/kT}}$

(C) $N_0 e^{-\epsilon/kT}$

(D) $\frac{N_0}{1 + e^{\epsilon/kT}}$

(E) $\frac{N_0 e^{\epsilon/kT}}{2}$

72. The internal energy of this system at any temperature T is given by $E_1 N_0 + \frac{N_0 \epsilon}{1 + e^{\epsilon/kT}}$. The heat capacity of the system is given by which of the following expressions?

(A) $N_0 k \left(\frac{\epsilon}{kT}\right)^2 \frac{e^{\epsilon/kT}}{(1 + e^{\epsilon/kT})^2}$

(B) $N_0 k \left(\frac{\epsilon}{kT}\right)^2 \frac{1}{(1 + e^{\epsilon/kT})^2}$

(C) $N_0 k \left(\frac{\epsilon}{kT}\right)^2 e^{-\epsilon/kT}$

(D) $\frac{N_0 k}{2} \left(\frac{\epsilon}{kT}\right)^2$

(E) $\frac{3}{2} N_0 k$

73. Which of the following is true of the entropy of the system?

(A) It increases without limit with T from zero at $T = 0$.
(B) It decreases with increasing T.
(C) It increases from zero at $T = 0$ to $N_0 k \ln 2$ at arbitrarily high temperatures.
(D) It is given by $N_0 k \left[\frac{5}{2} \ln T - \ln p + \text{constant}\right]$.
(E) It cannot be calculated from the information given.

Recommended Solution

Start off by recalling that as temperature and, therefore, net energy of a system blows up to infinity, energy levels will start to become equally populated. Based on this, get rid of any solution that doesn't account for a temperature dependence, specifically (A). Also from this fact, we can eliminate any solution that doesn't give the average number as $N_0/2$ when $T \to \infty$, which would be (C). Finally, when the temperature is minimized (i.e. let $T = 0$), we would expect all N_0 of the particles to be at energy level E_1 so plug this into the remaining options to find

(B) $\frac{N_0}{1+e^{-k/\epsilon(0)}} = N_0$

(D) $\frac{N_0}{1+e^{k/\epsilon(0)}} = \infty$

(E) $\frac{N_0 e^{\epsilon/kT}}{2} = \infty$

So we choose (B).

Correct Answer
(B)

1.72 PGRE9277 #72

Questions 71-73

A system in thermal equilibrium at temperature T consists of a large number N_0 of subsystems, each of which can exist only in two states of energy E_1 and E_2, where $E_2 - E_1 = \epsilon > 0$. In the expressions that follow, k is the Boltzmann constant.

71. For a system at temperature T, the average number of subsystems in the state of energy E_1 is given by

(A) $\frac{N_0}{2}$

(B) $\frac{N_0}{1 + e^{-\epsilon/kT}}$

(C) $N_0 e^{-\epsilon/kT}$

(D) $\frac{N_0}{1 + e^{\epsilon/kT}}$

(E) $\frac{N_0 e^{\epsilon/kT}}{2}$

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(B) $N_0 k \left(\frac{\epsilon}{kT}\right)^2 \frac{1}{(1 + e^{\epsilon/kT})^2}$

(C) $N_0 k \left(\frac{\epsilon}{kT}\right)^2 e^{-\epsilon/kT}$

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(D) It is given by $N_0 k \left[\frac{5}{2} \ln T - \ln p + \text{constant}\right]$.
(E) It cannot be calculated from the information given.

Recommended Solution

Recall that heat capacity is the derivative of energy with respect to temperature,

$$C_V = \frac{\partial U}{\partial T} \tag{1.229}$$

the problem gives us the energy so we take the derivative of it

$$C_V = \frac{dU}{dT} \tag{1.230}$$

$$= \frac{d}{dT}(E_1 N_0) + \frac{d}{dT}\left(\frac{N_0 \epsilon}{1 + e^{\epsilon/kT}}\right) \tag{1.231}$$

$$= N_0 k \left(\frac{\epsilon}{kT}\right)^2 \frac{e^{\epsilon/kT}}{(1 + e^{\epsilon/kT})^2} \tag{1.232}$$

which is (A). The worst part of this problem is doing the quotient rule under pressure but you can recognize certain pieces that should be there and only do part of the derivative to get the right answer.

Correct Answer
(A)

1.73 PGRE9277 #73

Questions 71-73

A system in thermal equilibrium at temperature T consists of a large number N_0 of subsystems, each of which can exist only in two states of energy E_1 and E_2, where $E_2 - E_1 = \epsilon > 0$. In the expressions that follow, k is the Boltzmann constant.

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(B) $\frac{N_0}{1 + e^{-\epsilon/kT}}$

(C) $N_0 e^{-\epsilon/kT}$

(D) $\frac{N_0}{1 + e^{\epsilon/kT}}$

(E) $\frac{N_0 e^{\epsilon/kT}}{2}$

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(B) $N_0 k \left(\frac{\epsilon}{kT}\right)^2 \frac{1}{(1 + e^{\epsilon/kT})^2}$

(C) $N_0 k \left(\frac{\epsilon}{kT}\right)^2 e^{-\epsilon/kT}$

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(D) It is given by $N_0 k \left[\frac{5}{2} \ln T - \ln p + \text{constant}\right]$.
(E) It cannot be calculated from the information given.

Recommended Solution

We can immediately eliminate (B) because it is generally true that entropy increases as temperature increases. We can also eliminate (D) because entropy should approach 0 as temperature approaches 0, not approach some non-zero value. Eliminate (E) because we have more than enough information to pick a choice. Finally, we need to decide whether or not entropy has an upper limit (i.e. option

(C)) or goes off to infinity (i.e. option (A)). Because there is some temperature at which all energy levels get equally populated, we also have a temperature at which any further increases in temp won't result in a wider dispersion of the particles so we should choose (C).

Correct Answer
(C)

1.74 PGRE9277 #74

74. Two circular hoops, X and Y, are hanging on nails in a wall. The mass of X is four times that of Y, and the diameter of X is also four times that of Y. If the period of small oscillations of X is T, the period of small oscillations of Y is

(A) T (B) $T/2$ (C) $T/4$
(D) $T/8$ (E) $T/16$

Recommended Solution

We start off with the angular frequency equation,

$$\omega = \sqrt{\frac{mgR}{I}} \tag{1.233}$$

The moment of inertia can be found by using the parallel axis theorem

$$I = I_{COM} + mR^2 \tag{1.234}$$

where I_{COM} is the moment of inertia of the object about an axis passing through its center of mass. In the case of a loop, that moment of inertia about the center of mass is the same as a point particle at distance R so we get

$$I = I_{COM} + mR^2 \tag{1.235}$$
$$= mR^2 + mR^2 \tag{1.236}$$
$$= 2mR^2 \tag{1.237}$$

so the moment of inertia for rings X and Y is

$$I_X = 2(4m)(16R^2) \tag{1.238}$$
$$I_Y = 2mR^2 \tag{1.239}$$

plugging these into the angular frequency, ω, gives

$$\omega_X = \sqrt{\frac{(4m)g(4R)}{2(4m)(16R^2)}} \tag{1.240}$$
$$= \sqrt{\frac{4g}{32R}} \tag{1.241}$$

and

$$\omega_Y = \sqrt{\frac{mgR}{2mR^2}} \quad (1.242)$$

$$= \sqrt{\frac{g}{2R}} \quad (1.243)$$

comparing the two, we get

$$\frac{\omega_X}{\omega_Y} = \frac{\sqrt{g/8R}}{\sqrt{g/2R}} \quad (1.244)$$

$$= \sqrt{\frac{1}{4}} \quad (1.245)$$

$$= \frac{1}{2} \quad (1.246)$$

which is (B).

Correct Answer
(B)

1.75 PGRE9277 #75

$${}^{235}_{92}\mathrm{U} \rightarrow {}^{231}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

75. A uranium nucleus decays at rest into a thorium nucleus and a helium nucleus, as shown above. Which of the following is true?

(A) Each decay product has the same kinetic energy.
(B) Each decay product has the same speed.
(C) The decay products tend to go in the same direction.
(D) The thorium nucleus has more momentum than the helium nucleus.
(E) The helium nucleus has more kinetic energy than the thorium nucleus.

Recommended Solution

By conservation of momentum, we should have the original momentum equal to the sum of the individual momentums of the two atoms,

$$P_{net} = P_T + P_H \tag{1.247}$$
$$0 = m_T v_T + m_H v_H \tag{1.248}$$
$$v_H = -\frac{m_T}{m_H} v_T \tag{1.249}$$

If we combine Equation 1.249 with the kinetic energy equation for helium, we get

$$K_H = \frac{1}{2} m_H v_H^2 \tag{1.250}$$
$$= \frac{1}{2} m_H \frac{m_T^2}{m_H^2} v_T^2 \tag{1.251}$$
$$= \frac{1}{2} \frac{m_T^2}{m_H} v_T^2 \tag{1.252}$$
$$= \frac{m_T}{m_H} \left(\frac{1}{2} m_T v_T^2 \right) \tag{1.253}$$

and since $m_T > m_H$, the kinetic energy of the Helium atom must be larger than the kinetic energy of the Thorium atom.

Correct Answer
(E)

1.76 PGRE9277 #76

76. The configuration of three electrons ls2p3p has which of the following as the value of its maximum possible total angular momentum quantum number?

(A) $\frac{7}{2}$

(B) 3

(C) $\frac{5}{2}$

(D) 2

(E) $\frac{3}{2}$

Recommended Solution

The total angular momentum quantum number, j, is the sum of the spin angular momentum, s, and the orbital angular momentum number, l,

$$j = l + s \tag{1.254}$$

Since we have three electrons and all electrons have a spin of 1/2, the total spin angular momentum must be

$$s = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \tag{1.255}$$

$$s = \frac{3}{2} \tag{1.256}$$

Then, recalling the orbital angular momentum rules $(S, P, D, F, \ldots) \rightarrow (0, 1, 2, 3, \ldots)$, we get

$$l = 0 + 1 + 1 \tag{1.257}$$

$$l = 2 \tag{1.258}$$

Sum the values from Equation 1.256 and 1.258 to get

$$s = 2 + \frac{3}{2} \tag{1.259}$$

$$s = \frac{7}{2} \tag{1.260}$$

Correct Answer
(A)

1.77 PGRE9277 #77

77. Consider a heavy nucleus with spin $\frac{1}{2}$. The magnitude of the ratio of the intrinsic magnetic moment of this nucleus to that of an electron is

(A) zero, because the nucleus has no intrinsic magnetic moment
(B) greater than 1, because the nucleus contains many protons
(C) greater than 1, because the nucleus is so much larger in diameter than the electron
(D) less than 1, because of the strong interactions among the nucleons in a nucleus
(E) less than 1, because the nucleus has a mass much larger than that of the electron

Recommended Solution

If we keep in mind that the magnetic moment is a measure of the tendency of an object to align itself with a magnetic field. Although the magnetic moment of the nucleus and electrons are both non-zero (which let's us eliminate (A)), we can determine just from common sense that a very small and light weight particle will more easily change alignment to conform to the magnetic field than will a "heavy nucleus". This means that the ratio of magnetic moment between nucleus and electron should be less than 1, which eliminates all but (D) and (E). Between the two, we can comfortably choose (E) because, as I said previously, the mass of the nucleus is what resists the change in alignment.

Correct Answer
(E)

1.78 PGRE9277 #78

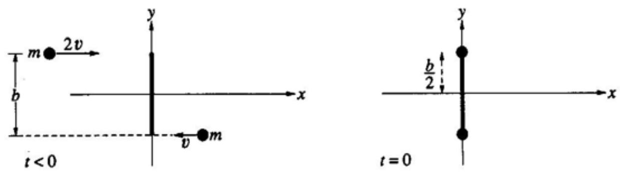


Figure I Figure II

78. One ice skater of mass m moves with speed $2v$ to the right, while another of the same mass m moves with speed v toward the left, as shown in Figure I. Their paths are separated by a distance b. At $t = 0$, when they are both at $x = 0$, they grasp a pole of length b and negligible mass. For $t > 0$, consider the system as a rigid body of two masses m separated by distance b, as shown in Figure II. Which of the following is the correct formula for the motion after $t = 0$ of the skater initially at $y = b/2$?

(A) $x = 2vt, y = b/2$

(B) $x = vt + 0.5b\sin(3vt/b)$, $y = 0.5b\cos(3vt/b)$

(C) $x = 0.5vt + 0.5b\sin(3vt/b)$, $y = 0.5b\cos(3vt/b)$

(D) $x = vt + 0.5b\sin(6vt/b)$, $y = 0.5b\cos(6vt/b)$

(E) $x = 0.5vt + 0.5b\sin(6vt/b)$, $y = 0.5b\cos(6vt/b)$

Recommended Solution

Note that at time $t = 0$, the velocity should be equal to the initial velocity $2v_0$ that it had right before grabbing the pole. To check this limit, take the derivative of all of the possible solutions at time $t = 0$ to see which of these correctly predicts the condition.

(A) $x' = v = 2v_0$

(B) $x' = v + \left(\frac{0.5b(3v_0)}{b}\right)\cos(0) = 2.5v_0$

(C) $x' = 0.5v_0 + \left(\frac{0.5b(3v_0)}{b}\right)\cos(0) = 2v_0$

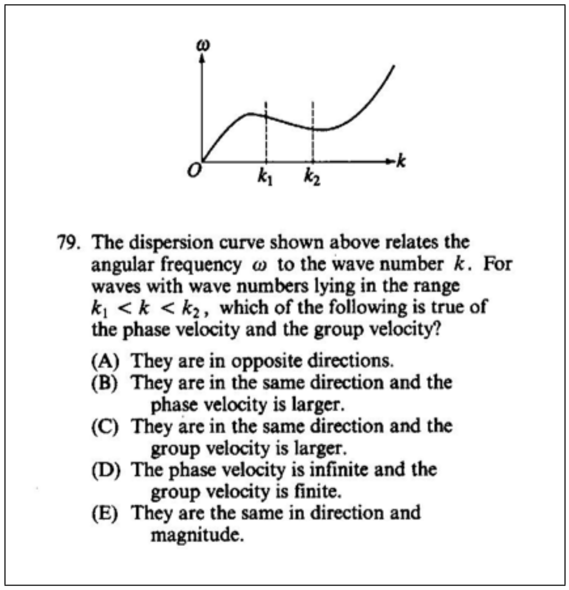
(D) $x' = v_0 + \left(\frac{0.5b(6v_0)}{b}\right)\cos(0) = 4v_0$

(E) $x' = 0.5v_0 + \left(\frac{0.5b(6v_0)}{b}\right)\cos(0) = 3.5v_0$

Only (A) and (C) meet this criteria and since (A) doesn't properly account for the rotation with a sine function, choose (C).

Correct Answer
(C)

1.79 PGRE9277 #79



79. The dispersion curve shown above relates the angular frequency ω to the wave number k. For waves with wave numbers lying in the range $k_1 < k < k_2$, which of the following is true of the phase velocity and the group velocity?

(A) They are in opposite directions.
(B) They are in the same direction and the phase velocity is larger.
(C) They are in the same direction and the group velocity is larger.
(D) The phase velocity is infinite and the group velocity is finite.
(E) They are the same in direction and magnitude.

Recommended Solution

The group velocity is the velocity with which the overall wave travels while the phase velocity is the rate at which the phase propagates. The relevant equations for group velocity and phase velocity are

$$v_g = \frac{\partial \omega}{\partial k} \tag{1.261}$$

$$v_p = \frac{\omega}{k} \tag{1.262}$$

take the derivative (tangent on the curve) between k_1 and k_2 to get a roughly constant negative value for the group velocity. The phase velocity, however, has a positive value because it's a negative slope with an inverse relationship. Since one is positive and the other is negative, they should be in opposite directions and we choose (A).

Correct Answer
(A)

1.80 PGRE9277 #80

80. A beam of electrons is accelerated through a potential difference of 25 kilovolts in an x-ray tube. The continuous x-ray spectrum emitted by the target of the tube will have a short wavelength limit of most nearly

(A) 0.1 Å

(B) 0.5 Å

(C) 2 Å

(D) 25 Å

(E) 50 Å

Recommended Solution

Start with the equation for the energy of an electromagnetic wave,

$$E = \frac{hc}{\lambda} \tag{1.263}$$

and now adjust it to solve for the wavelenth

$$\lambda = \frac{hc}{\lambda} \tag{1.264}$$

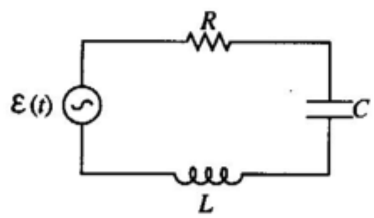
we know the energy is 25 kilovolts and can utilize Planck's constant and the speed of light constant from the front of our test booklet. Plug everything in to get

$$\begin{aligned}
\lambda &= \frac{(4 \times 10^{-15}\ \mathrm{eV \cdot s})(3.0 \times 10^{8}\ \mathrm{m/s})}{2.5 \times 10^{4}\ \mathrm{eV}} && (1.265)\\
&= \frac{12 \times 10^{-7}\ \mathrm{eV \cdot m}}{2.5\ \times 10^{4}\ \mathrm{eV}} && (1.266)\\
&= 4 \times 10^{-11}\ \mathrm{m} && (1.267)\\
&= 0.4\ \text{Angstroms} && (1.268)
\end{aligned}$$

which is nearly (B).

Correct Answer
(B)

1.81 PGRE9277 #81



81. In the RLC circuit shown above, the applied voltage is

$$\mathcal{E}(t) = \mathcal{E}_m \cos \omega t.$$

For a constant $\mathcal{E}_m$, at what angular frequency ω does the current have its maximum steady-state amplitude after the transients have died out?

(A) $\frac{1}{RC}$

(B) $\frac{2L}{R}$

(C) $\frac{1}{\sqrt{LC}}$

(D) $\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

(E) $\sqrt{\left(\frac{1}{RC}\right)^2 - \left(\frac{L}{R}\right)^2}$

Recommended Solution

In electronics, we will reach the max steady-state amplitude at the point when impedence is matched between

$$Z_L = Z_C \tag{1.269}$$

The inductor impedance is

$$Z_L = j\omega L \tag{1.270}$$

and the capacitor impedance is

$$Z_C = \frac{1}{j\omega C} \tag{1.271}$$

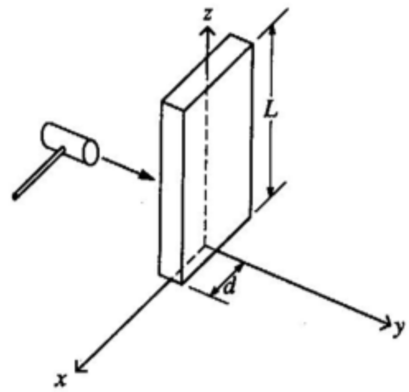
so applying Equations 1.270 and 1.271 to Equation 1.269 gives us

$$
\begin{aligned}
Z_L &= Z_C && (1.272)\\
j\omega L &= \frac{1}{j\omega C} && (1.273)\\
&= \omega^2 j^2 && (1.274)\\
&= \frac{1}{LC} && (1.275)\\
\omega j &= \sqrt{\frac{1}{LC}} && (1.276)
\end{aligned}
$$

which gives the same inverse LC dependence that (C) suggests.

Correct Answer
(C)

1.82 PGRE9277 #82



82. A thin plate of mass M, length L, and width $2d$ is mounted vertically on a frictionless axle along the z-axis, as shown above. Initially the object is at rest. It is then tapped with a hammer to provide a torque τ, which produces an angular impulse $\mathbf{H}$ about the z-axis of magnitude $H = \int \tau\, dt$. What is the angular speed ω of the plate about the z-axis after the tap?

(A) $\frac{H}{2Md^2}$

(B) $\frac{H}{Md^2}$

(C) $\frac{2H}{Md^2}$

(D) $\frac{3H}{Md^2}$

(E) $\frac{4H}{Md^2}$

Recommended Solution

First, recall the angular impulse **H** is proportional to the moment of inertia by

$$H = I\omega \tag{1.277}$$

the moment of inertia of a plate through its center is $(1/12)mL^2$, so with our length of $2d$, we get

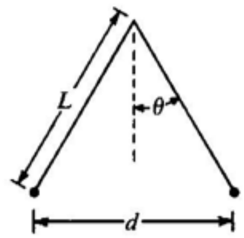
$$\begin{aligned} I &= \frac{1}{12}mL^2 && (1.278) \\ &= \frac{1}{12}m(2d)^2 && (1.279) \\ &= \frac{1}{3}md^2 && (1.280) \end{aligned}$$

now solve for ω in Equation 1.277 with Equation 1.277 plugged into it to get our final answer

$$\omega = \frac{H}{I} \tag{1.281}$$
$$= \frac{H}{(1/3)md^2} \tag{1.282}$$
$$= \frac{3H}{md^2} \tag{1.283}$$

Correct Answer
(D)

1.83 PGRE9277 #83



83. Two pith balls of equal mass M and equal charge q are suspended from the same point on long massless threads of length L as shown in the figure above. If k is the Coulomb's law constant, then for small values of θ, the distance d between the charged pith balls at equilibrium is

(A) $\left(\frac{2kq^2L}{Mg}\right)^{\frac{1}{3}}$

(B) $\left(\frac{kq^2L}{Mg}\right)^{\frac{1}{3}}$

(C) $\left(\frac{2kq^2L}{Mg}\right)^{\frac{1}{2}}$

(D) $\left(\frac{kq^2L}{Mg}\right)^{\frac{1}{2}}$

(E) $\frac{L}{4}$

Recommended Solution

We can get the right proportionality to figure out the solution by making a few small angle approximations. Start by summing the forces in both dimensions

$$F_A = -F_{T-x} \tag{1.284}$$

$$F_{T-y} = -F_G \tag{1.285}$$

by geometry, we can figure the length of d as

$$\sin(\theta) = \frac{d/2}{L} \tag{1.286}$$

$$d = 2L\sin(\theta) \tag{1.287}$$

we can also use some trigonometry to a relationship between tensions

$$\sin(\theta) = \frac{F_{T-x}}{F_T} \tag{1.288}$$

plug Equation 1.288 into Equation 1.287 to get

$$d = 2L\frac{F_{T-x}}{F_T} \tag{1.289}$$

Finally, get a relationship between F_{T-y} and F_T and apply the small angle approximation $\cos(\theta) \approx 1$ to get

$$\begin{aligned} \cos(\theta) &= \frac{F_{T-y}}{F_T} &(1.290)\\ 1 &= \frac{F_{T-y}}{F_T} &(1.291)\\ F_T &= F_{T-y} &(1.292) \end{aligned}$$

but since $F_{T-y} = -F_G = -mg$ and F_{T-x} is the force from Coulomb's law, substitute these values into Equation 1.289,

$$\begin{aligned} d &= 2L\frac{F_{T-x}}{F_T} &(1.293)\\ &= \frac{2Lkq^2}{d^2mg} &(1.294) \end{aligned}$$

which matches (A).

Correct Answer
(A)

1.84 PGRE9277 #84

84. An electron oscillates back and forth along the $+$ and $-$ x-axes, consequently emitting electromagnetic radiation. Which of the following statements concerning the radiation is NOT true?

(A) The total rate of radiation of energy into all directions is proportional to the square of the electron's acceleration.
(B) The total rate of radiation of energy into all directions is proportional to the square of the electron's charge.
(C) Far from the electron, the rate at which radiated energy crosses a perpendicular unit area decreases as the inverse square of the distance from the electron.
(D) Far from the electron, the rate at which radiated energy crosses a perpendicular unit area is a maximum when the unit area is located on the $+$ or $-$ x-axes.
(E) Far from the electron, the radiated energy is carried equally by the transverse electric and the transverse magnetic fields.

Recommended Solution

For choices (A) and (B), we can see that these are both true by the Larmor formula

$$P = \frac{e^2a^2}{6\pi\epsilon_0 c^3} \tag{1.295}$$

Next, we know that (C) must be true by the LinardWiechert potential which states, in a big hot mess,

$$\vec{E}(\vec{x},t) = q\left(\frac{\vec{n}-\vec{\beta}}{\gamma^2(1-\vec{\beta}\cdot\vec{n})^3R^2}\right) + \frac{q}{c}\left(\frac{\vec{n}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]}{(1-\vec{\beta}\cdot\vec{n})^3R}\right) \tag{1.296}$$

of which, we only car that $\vec{E}\propto 1/R^2$. Finally, we know that (E) is true because as we go off to infinity, both fields tend to 0. We are left with (D) so that must be our correct answer.

Correct Answer
(D)

1.85 PGRE9277 #85

85. A free electron (rest mass $m_e = 0.5\ \text{MeV/c}^2$) has a total energy of 1.5 MeV. Its momentum p in units of MeV/c is about

(A) 0.86
(B) 1.0
(C) 1.4
(D) 1.5
(E) 2.0

Recommended Solution

Start of with the relativistic energy equation

$$E = \gamma m_0 c^2 \tag{1.297}$$

the energy is given to us as $E = 1.5$ MeV and the mass is $m_e = 0.5\ \text{MeV/c}^2$,

$$\begin{aligned} 1.5\ \text{MeV} &= \gamma(0.5\ \text{MeV/c}^2)c^2 && (1.298)\\ \gamma &= 3 && (1.299)\end{aligned}$$

now, we want to use the relativistic momentum equation but to do so, we need the velocity of the electron. Using our result from Equation 1.299, solve for v,

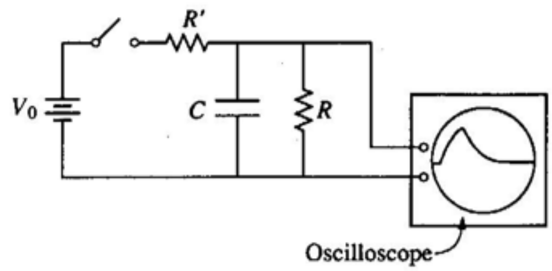
$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} && (1.300)\\ 3 &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} && (1.301)\\ 9\left(1-\frac{v^2}{c^2}\right) &= 1 && (1.302)\\ \frac{v^2}{c^2} &= \frac{8}{9} && (1.303)\\ v &= \frac{\sqrt{8}}{3}c && (1.304)\end{aligned}$$

Plug results from Equation 1.299 and 1.304 into the relativistic momentum equation to get the final answer

$$\begin{aligned} P &= \gamma m_0 v && (1.305)\\ &= 3(0.5\ \text{MeV/c}^2)(\sqrt{8}/3\ \text{c} && (1.306)\\ &= (0.5)\sqrt{8}\ \text{MeV/c} && (1.307)\\ &= 1.4\ \text{MeV/c} && (1.308)\end{aligned}$$

Correct Answer
(C)

1.86 PGRE9277 #86



86. The circuit shown above is used to measure the size of the capacitance C. The y-coordinate of the spot on the oscilloscope screen is proportional to the potential difference across R, and the x-coordinate of the spot is swept at a constant speed s. The switch is closed and then opened. One can then calculate C from the shape and the size of the curve on the screen plus a knowledge of which of the following?

(A) V_0 and R

(B) s and R

(C) s and V_0

(D) R and R'

(E) The sensitivity of the oscilloscope

Recommended Solution

First, eliminate any option that includes V_0 (i.e. (A) and (C)) as the oscilloscope provides this data and therefore wouldn't be dependent on any of the other pieces of data being known. Next, recall that a capacitor discharges according to

$$V = V_0 e^{-t/RC} \tag{1.309}$$

which tells us that we will need R and we can eliminate (C) and (E). Additionally, we will need the time which we can derive from the sweep rate, s, so we get our final solution as (B).

Correct Answer
(B)

1.87 PGRE9277 #87

87. A particle of mass M moves in a circular orbit of radius r around a fixed point under the influence of an attractive force $F = \frac{K}{r^3}$, where K is a constant. If the potential energy of the particle is zero at an infinite distance from the force center, the total energy of the particle in the circular orbit is

(A) $-\frac{K}{r^2}$

(B) $-\frac{K}{2r^2}$

(C) 0

(D) $\frac{K}{2r^2}$

(E) $\frac{K}{r^2}$

Recommended Solution

To find the net energy, we need to sum the kinetic energy and potential energy of the particle

$$H = T + V \tag{1.310}$$

since the orbit is circular, we know that the centripetal force must be equivalent to the attractive force

$$\begin{aligned} F_c &= F_K \qquad (1.311) \\ \frac{mv^2}{r} &= \frac{K}{r^3} \qquad (1.312) \end{aligned}$$

In Equation 1.312, if we just multiply a 1/2 to the LHS and cancel out the r, this becomes our kinetic energy equaiton

$$\frac{1}{2}mv^2 = \frac{K}{2r^2} \tag{1.313}$$

Now for the potential energy, we use

$$V = -\int F \cdot dr \tag{1.314}$$

$$= -\int \frac{K}{r^3}\,dr \tag{1.315}$$
$$= \frac{K}{2r^2} \tag{1.316}$$

but because the potential is attractive, it becomes negative. We then sum the 2 potentials from Equation 1.313 and 1.316 to get

$$H = T + V \tag{1.317}$$
$$= \frac{K}{2r^2} - \frac{K}{2r^2} \tag{1.318}$$
$$= 0 \tag{1.319}$$

which is (C).

Correct Answer
(C)

1.88 PGRE9277 #88

88. A parallel-plate capacitor is connected to a battery. V_0 is the potential difference between the plates, Q_0 the charge on the positive plate, E_0 the magnitude of the electric field, and D_0 the magnitude of the displacement vector. The original vacuum between the plates is filled with a dielectric and then the battery is disconnected. If the corresponding electrical parameters for the final state of the capacitor are denoted by a subscript f, which of the following is true?

(A) $V_f > V_0$

(B) $V_f < V_0$

(C) $Q_f = Q_0$

(D) $E_f > E_0$

(E) $D_f > D_0$

Recommended Solution

According to the problem, this parallel plate capacitor is connected to a batter. As long as it is not removed, the voltage and electric field should not be altered, even if a dielectric is put in place. This tells us that (A), (B) and (D) must all be wrong. Next, we can find the before and after charge as

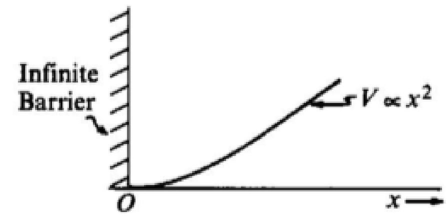
$$Q_0 = C_0 V_0 \tag{1.320}$$

$$Q_f = \kappa C_0 V_0 \tag{1.321}$$

so (C) must be wrong. This only leaves (E).

Correct Answer
(E)

1.89 PGRE9277 #89



89. The energy levels for the one-dimensional harmonic oscillator are $h\nu\left(n + \frac{1}{2}\right)$, $n = 0, 1, 2 \ldots$. How will the energy levels for the potential shown in the graph above differ from those for the harmonic oscillator?

(A) The term $\frac{1}{2}$ will be changed to $\frac{3}{2}$.
(B) The energy of each level will be doubled.
(C) The energy of each level will be halved.
(D) Only those for even values of n will be present.
(E) Only those for odd values of n will be present.

Recommended Solution

Recall the solution to the Infinite square well, which gives us a set of sinusoidal waves (Figure 1.5)

The first plot given represents $n = 0$, the second is $n = 1$, the third is $n = 2$ and so on. It should become clear that all even values for n peak at $x = 0$ and so these will be disrupted by the infinite potential at this point. The odd valued quantum numbers won't so they will remain. This description only matches (E).

Correct Answer
(E)

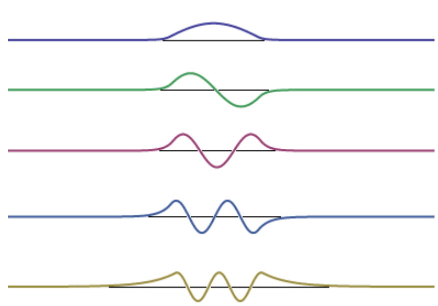


Figure 1.5: Plots of the solution to the infinite square well

1.90 PGRE9277 #90

90. The spacing of the rotational energy levels for the hydrogen molecule H_2 is most nearly

(A) 10^{-9} eV
(B) 10^{-3} eV
(C) 10 eV
(D) 10 MeV
(E) 100 MeV

Recommended Solution

It's worth memorizing the scale of energy spacing for the different energy levels

$$E = E_{trans} + E_{rot} + E_{vib} + E_{elec} \tag{1.322}$$

those being

$$E_{rot} \approx 0.001 \text{ eV} \tag{1.323}$$
$$E_{vib} \approx 0.1 \text{ eV} \tag{1.324}$$
$$E_{elec} \approx 1 \text{ eV} \tag{1.325}$$

from which we see that the rotational energy level should be around 10^{-3} which is (B).

Correct Answer
(B)

1.91 PGRE9277 #91

91. The particle decay $\Lambda \rightarrow p + \pi^-$ must be a weak interaction because

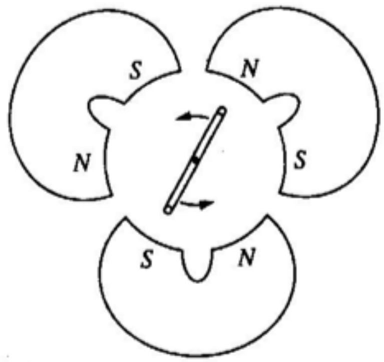
(A) the π^- is a lepton
(B) the Λ has spin zero
(C) no neutrino is produced in the decay
(D) it does not conserve angular momentum
(E) it does not conserve strangeness

Recommended Solution

Eliminate (A) because the pion isn't a lepton. Next, eliminate (B) because the Λ particle is a baryon so it must have spin 1/2. We can also quickly eliminate (D) because angular momentum is conserved. Lastly, eliminate (C) because it isn't true that all interactions that don't produce a neutrino are weak.

Correct Answer
(E)

1.92 PGRE9277 #92



92. A flat coil of wire is rotated at a frequency of 10 hertz in the magnetic field produced by three pairs of magnets as shown above. The axis of rotation of the coil lies in the plane of the coil and is perpendicular to the field lines. What is the frequency of the alternating voltage in the coil?

(A) $\frac{10}{6}$ Hz

(B) $\frac{10}{3}$ Hz

(C) 10 Hz

(D) 30 Hz

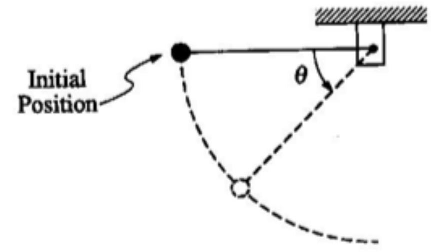
(E) 60 Hz

Recommended Solution

For a single coil of wire, it is relatively clear that a rotating magnet with frequency of 10 Hz will give us an alternating voltage of 10 Hz. However, for three coils, for every third of a rotation 10 Hz will have been generated for a single coil and a full rotation will have done 3 of these, making the net frequency ~~40~~ Hz

Correct Answer
(D)

1.93 PGRE9277 #93



93. The figure above shows a small mass connected to a string, which is attached to a vertical post. If the mass is released when the string is horizontal as shown, the magnitude of the total acceleration of the mass as a function of the angle θ is

(A) $g \sin \theta$

(B) $2g \cos \theta$

(C) $2g \sin \theta$

(D) $g\sqrt{3 \cos^2 \theta + 1}$

(E) $g\sqrt{3 \sin^2 \theta + 1}$

Recommended Solution

When the weight is released at time $t = 0$ and angle $\theta = 0$, the weight is essentially in free fall so it should have an acceleration of $a = g$. Plug in the $\theta = 0$ condition to check the limit

(A) $g \sin(0) = 0$

(B) $2g \cos(0) = 2g$

(C) $2g \sin(0) = 0$

(D) $g\sqrt{3\cos^2(0) + 1} = 2g$

(E) $g\sqrt{3\sin^2(0) + 1} = g$

and only (E) meets our criteria.

Correct Answer
(E)

1.94 PGRE9277 #94

94. Which of the following is a Lorentz transformation? (Assume a system of units such that the velocity of light is 1.)

(A) $x' = 4x$
$y' = y$
$z' = z$
$t' = .25t$

(B) $x' = x - .75t$
$y' = y$
$z' = z$
$t' = t$

(C) $x' = 1.25x - .75t$
$y' = y$
$z' = z$
$t' = 1.25t - .75x$

(D) $x' = 1.25x - .75t$
$y' = y$
$z' = z$
$t' = .75t - 1.25x$

(E) None of the above

Recommended Solution

The Lorentz transformation always takes the form of

$$t' = \gamma(t - vx) \tag{1.326}$$
$$x' = \gamma(x - vt) \tag{1.327}$$
$$y' = y \tag{1.328}$$
$$z' = z \tag{1.329}$$

which tells us that whatever the coefficients we have on x' and t', they should be the same with the variables swapped. This is only true of (C) so this must be our solution.

Correct Answer
(C)

1.95 PGRE9277 #95

95. A beam of 10^{12} protons per second is incident on a target containing 10^{20} nuclei per square centimeter. At an angle of 10 degrees, there are 10^{2} protons per second elastically scattered into a detector that subtends a solid angle of 10^{-4} steradians. What is the differential elastic scattering cross section, in units of square centimeters per steradian?

(A) 10^{-24}
(B) 10^{-25}
(C) 10^{-26}
(D) 10^{-27}
(E) 10^{-28}

Recommended Solution

Assuming that ETS hasn't given us a bunch of useless information in this problem, which is probably a pretty good assumption, we can arrive at the answer quickly with a bit of dimensional analysis. We are given, and should try to use,

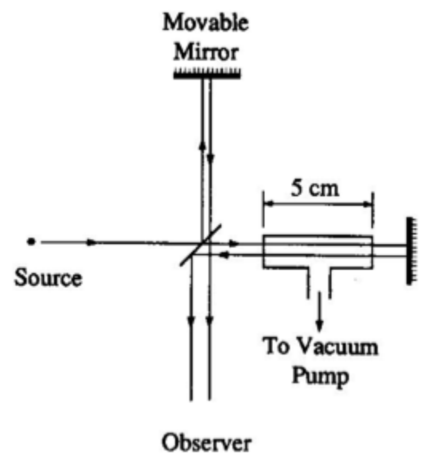
$$10^{12} \text{ proton/sec} \tag{1.330}$$
$$10^{20} \text{ nuclei/cm}^2 \tag{1.331}$$
$$10^{2} \text{ proton/sec} \tag{1.332}$$
$$10^{-4} \text{ steradians} \tag{1.333}$$

The only way to arrange these 4 values to get a final unit of cm^2/steradian is by

$$\frac{(10^2 \text{ protons/sec})}{(10^{20} \text{ nuclei/cm}^2)(10^{12} \text{ proton/sec})(10^{-4} \text{ steradians})} = 10^{-26} \text{ cm}^2\text{/steradian} \tag{1.334}$$

Correct Answer
(C)

1.96 PGRE9277 #96



96. A gas-filled cell of length 5 centimeters is inserted in one arm of a Michelson interferometer, as shown in the figure above. The interferometer is illuminated by light of wavelength 500 nanometers. As the gas is evacuated from the cell, 40 fringes cross a point in the field of view. The refractive index of this gas is most nearly

(A) 1.02
(B) 1.002
(C) 1.0002
(D) 1.00002
(E) 0.98

Recommended Solution

If you recognize that this setup is precisely the setup used to measure the index of refraction of air and you recall that the index of refraction of air is 1.000293, then you can quickly see that (C) is the correct answer.

Correct Answer
(C)

Alternate Solution

In a gas interferometer, a beam of light is passed to a partially silvered mirror which splits the beam into two parts. One part continues through the mirror and the other is reflected at a right angle. Ultimately, both beams arrive at the observer and create an interference pattern. We know that the optical path length is related to the index of refraction by nL, but since the light travels the distance L twice, we re-write it as $2nL$. Now, if we remove the gas from the system, our index of refraction must change (Δn) and the interference pattern will its fringes shift according to

$$\Delta n = \frac{m\lambda}{2L} \tag{1.335}$$

Since the index of refraction of most gases is nearly 1, we typically define the index of refraction of any gas as the index of refraction of a vacuum (i.e. $n = 1$) plus some additional factor kp, where k is some constant and p is the air pressure.

$$n_{gas} = 1 + kp \tag{1.336}$$

changes in index of refraction are proportional to changes in air pressure by

$$\Delta p = \Delta n k \tag{1.337}$$

so we combine Equations 1.335, 1.336 and 1.337 to get

$$n = 1 + \frac{m\lambda p}{2L\delta p} \tag{1.338}$$

to make a quick approximation, and because this information is given, lose the dependence on p (i.e. $p = \Delta p$) and solve to get

$$\begin{aligned} n &= 1 + \frac{m\lambda}{2L} && (1.339) \\ &= 1 + \frac{(40 \text{ fringes})(500 \text{ nm})}{2(5 \text{ cm})} && (1.340) \\ &= 1 + 0.0002 && (1.341) \\ &= 1.0002 && (1.342) \end{aligned}$$

which is (C).

Correct Answer
(C)

1.97 PGRE9277 #97

97. Lattice forces affect the motion of electrons in a metallic crystal, so that the relationship between the energy E and wave number k is not the classical equation $E = \hbar^2 k^2/2m$, where m is the electron mass. Instead, it is possible to use an effective mass m^* given by which of the following?

(A) $m^* = \frac{1}{2}\hbar^2 k \left(\frac{dk}{dE}\right)$

(B) $m^* = \frac{\hbar^2 k}{\left(\frac{dk}{dE}\right)}$

(C) $m^* = \hbar^2 k \left(\frac{d^2k}{dE^2}\right)^{\frac{1}{3}}$

(D) $m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)}$

(E) $m^* = \frac{1}{2}\hbar^2 m^2 \left(\frac{d^2E}{dk^2}\right)$

Recommended Solution

To start off, check the units of each potential solution to see that only (A) and (D) give some sort of mass

(A) $m^* = \frac{1}{2}\hbar^2 k \left(\frac{dk}{dE}\right) = \text{kg}$

(B) $m^* = \frac{\hbar^2 k}{\left(\frac{dk}{dE}\right)} = \frac{\text{m}^6\ \text{kg}^4}{\text{sec}^4}$

(C) $m^* = \hbar^2 k \left(\frac{d^2k}{dE^2}\right)^{1/3}$ = A big mess that clearly has extra units

(D) $m^* = \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} = \text{kg}$

(E) $m^* = \frac{1}{2}\hbar^2 m^2 \left(\frac{d^2E}{dk^2}\right) = \frac{\text{kg}^5\ \text{m}^8}{\text{sec}^4}$

At which point you can either make an educated guess or recall that you should be differentiating the energy with respect to wave number, as in (D), rather than differentiating wave number with respect to energy, like in (A).

Correct Answer
(D)

1.98 PGRE9277 #98

98. The matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

has three eigenvalues λ_i defined by $Av_i = \lambda_i v_i$. Which of the following statements is NOT true?

(A) $\lambda_1 + \lambda_2 + \lambda_3 = 0$

(B) λ_1, λ_2, and λ_3 are all real numbers.

(C) $\lambda_2\lambda_3 = +1$ for some pair of roots.

(D) $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 0$

(E) $\lambda_i^3 = +1, i = 1, 2, 3$

Recommended Solution

Recall that we find the eigenvalues from a matrix by finding the determinant of the characteristic equation

$$\det \begin{pmatrix} 0-\lambda & 1 & 0 \\ 0 & 0-\lambda & 1 \\ 1 & 0 & 0-\lambda \end{pmatrix}$$

You can, and for speed you should, use the quick method of finding the determinant of a 3 dimensional matrix (which I once heard called the "shoe string method")

$$\begin{pmatrix} -\lambda & 1 & 0 & -\lambda & 1 \\ 0 & -\lambda & 1 & 0 & -\lambda \\ 1 & 0 & -\lambda & 1 & 0 \end{pmatrix}$$

multiplying the diagonals and summing them (via "shoe string method"), you get

$$(-\lambda^3 + 1 + 0) - (0 + 0 + 0) = 0 \tag{1.343}$$
$$-\lambda^3 + 1 = 0 \tag{1.344}$$
$$\lambda^3 = 1 \tag{1.345}$$

Then from complex analysis, since we clearly only have 1 real solution (i.e. $\lambda = 1$) the rest of the solutions must be complex and, therefore, (B) must be a false statement.

Correct Answer
(B)

1.99 PGRE9277 #99

99. In perturbation theory, what is the first order correction to the energy of a hydrogen atom (Bohr radius a_0) in its ground state due to the presence of a static electric field E?

(A) Zero

(B) eEa_0

(C) $3eEa_0$

(D) $\frac{8e^2Ea_0^3}{3}$

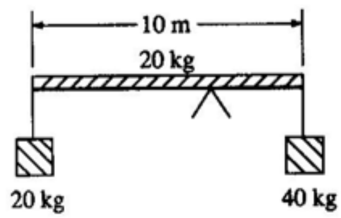
(E) $\frac{8e^2E^2a_0^3}{3}$

Recommended Solution

For this problem, you should be able to immediately recognize that the correct answer is (A). This is because the Hydrogen atom, unlike nearly every other problem in all of quantum mechanics, is an ideal and exactly solvable system. For this reason, there is no correction factor for the hydrogen atom in its ground state.

Correct Answer
(A)

1.100 PGRE9277 #100



100. **A uniform rod of length 10 meters and mass 20 kilograms is balanced on a fulcrum with a 40-kilogram mass on one end of the rod and a 20-kilogram mass on the other end, as shown above. How far is the fulcrum located from the center of the rod?**

(A) 0 m
(B) 1 m
(C) 1.25 m
(D) 1.5 m
(E) 2 m

Recommended Solution

In order to figure out the balancing point of the system, we need to find its net center of mass.

$$\frac{m_1x_1 + m_2x_2}{m_{total}} \tag{1.346}$$

We know the masses of each block are $m_1 = 20$ kg and $m_2 = 40$ kg and the positions are $x_1 = -5$ m and $x_2 = 5$ m, with respect to the center. We also know the total mass is the sum of the 2 blocks and the mass of the rod (i.e. 20 kg + 40 kg + 20 kg = 80 kg) so we can plug everything in and solve to get

$$\text{COM} = \frac{(20 \text{ kg})(-5 \text{ m}) + (40 \text{ kg})(4 \text{ m})}{80 \text{ kg}} \tag{1.347}$$

$$= \frac{100}{80} \text{ m} \tag{1.348}$$

$$= 1.25 \text{ m} \tag{1.349}$$

which is (C).

Correct Answer
(C)