

Origin		Order (of $\alpha$ )	Consequences	
Fine structure	<ol style="list-style-type: none"> <li>Replacing the electron kinematic energy term in Hamiltonian with the full relativistic form</li> <li>Spin-orbit couplings between the electron's orbital angular momentum and its spin, of the form <math>\mathbf{L}\cdot\mathbf{S}</math></li> </ol>	$\alpha^2$	<ul style="list-style-type: none"> <li>The hydrogen energy levels acquire a dependence on the <b>total</b> spin j</li> <li>Since <math>\mathbf{J}</math> commutes with <math>\mathbf{L}\cdot\mathbf{S}</math>, <math>m_j</math> is conserved.</li> <li>The energies are degenerate in <math>m_j</math></li> <li>The degeneracy in l is broken</li> </ul>	
Lamb shift	<ol style="list-style-type: none"> <li>Quantum electrodynamics correction</li> </ol>	$\alpha^3$	<ul style="list-style-type: none"> <li>Split the 2s and 2p levels with j=1/2 (which are degenerate even including fine structure since they both have the same j)</li> </ul>	
Hyperfine structure	<ol style="list-style-type: none"> <li>Magnetic dipole interaction between the spins of the electron and the proton, known as spin-spin coupling</li> </ol>	$\alpha^2$ (but suppressed by the ratio $m_e/m_p$ )	<ul style="list-style-type: none"> <li>The ground state of hydrogen is split depending on whether the two spins are in the singlet or triplet state.</li> <li>The triplet has the higher energy, roughly because the spins are aligned, and magnetic dipoles want to be anti-aligned</li> <li>The wavelength of the emitted photon in a transition between these two states is about 21cm, with energy about <math>5\times 10^{-6}</math> eV</li> </ul>	
Stark effect	<ol style="list-style-type: none"> <li>Placing an atom in external electric field, <math>\Delta H = e\mathbf{E}\cdot\mathbf{r}</math></li> </ol>	—	<ul style="list-style-type: none"> <li>There is not change in the ground state energy of hydrogen or any hydrogen atom, to first order in <math> \mathbf{E} </math></li> <li>The lowest-energy states to show a 1st-order shift are the n=2 state. The states with <math>m=\pm 1</math> are unperturbed, but the 2s state and the 2p state with m=0 are split.</li> <li>The magnitude of the splitting: from the dimensional analysis, <math>\Delta E = ke \mathbf{E} d</math>, where d is some length and k is a constant (which is not determined by this reasoning). The only length scale of the hydrogen atom is the Bohr radius, so <math>d=a_0</math></li> </ul>	
Zeeman effect	<ol style="list-style-type: none"> <li>Placing an atom in external magnetic field, <math>\Delta H = (e/2m)(\mathbf{L}+2\mathbf{S})\cdot\mathbf{B}</math> (e/2m: gyromagnetic ratio; the extra factor of 2 in front of the spin operator is because the quantum gyromagnetic ratio is twice the classical value)</li> </ol>	—	<ul style="list-style-type: none"> <li>In play are three operators, <math>\mathbf{L}</math>, <math>\mathbf{S}</math>, <math>\mathbf{J}=\mathbf{L}+\mathbf{S}</math>. Depending on the magnitude of <math>\mathbf{B}</math>, different combinations of these operators may be conserved, and hence different combinations of eigenvalues may label the energies.</li> </ul>	If $ \mathbf{B} $ is small: <ul style="list-style-type: none"> <li>The Zeeman Hamiltonian is a perturbation on top of fine structure, for which the energies are labeled by j, l, <math>m_j</math>.</li> <li>The weak-field Zeeman effect splits the j-states according to <math>m_j</math>, with the lower energy for the most negative <math>m_j</math>.               <ul style="list-style-type: none"> <li>The electron spin wants to be anti-aligned with the magnetic field, since it is energetically favorable</li> </ul> </li> <li>The splitting of energy levels according to spin can be seen in the <b>Stern-Gerlach experiment</b>, where an inhomogeneous magnetic field splits a beam of atoms into two, effectively performing a measurement of <math>m_j</math>.</li> </ul>
				If $ \mathbf{B} $ is large: <ul style="list-style-type: none"> <li>The fine structure as a perturbation on top of the Zeeman Hamiltonian</li> <li>In convention, we align <math>\mathbf{B}</math> to point in the <math>\hat{\mathbf{z}}</math>-direction <math>\Rightarrow L_z</math> and <math>S_z</math> both commute with the Zeeman Hamiltonian.               <ul style="list-style-type: none"> <li><math>l</math>, <math>m_l</math>, <math>m_s</math> are now conserved before fine structure comes into the picture</li> <li>Total spin j and <math>m_j</math> are <i>not</i> conserved, because the magnetic field provides an external torque</li> </ul> </li> <li>The energy of the Zeeman states depends on <math>m_l</math> and <math>m_s</math> in the same way as for the weak-field effect, and fine structure causes these states to develop a dependence on l.</li> </ul>