Formal verification of quantum compilers

Hanru Jiang
Peng Cheng Laboratory
2019/09/12

Goal

Contract-based verification of a realistic quantum compiler

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MICRO 2019 poster

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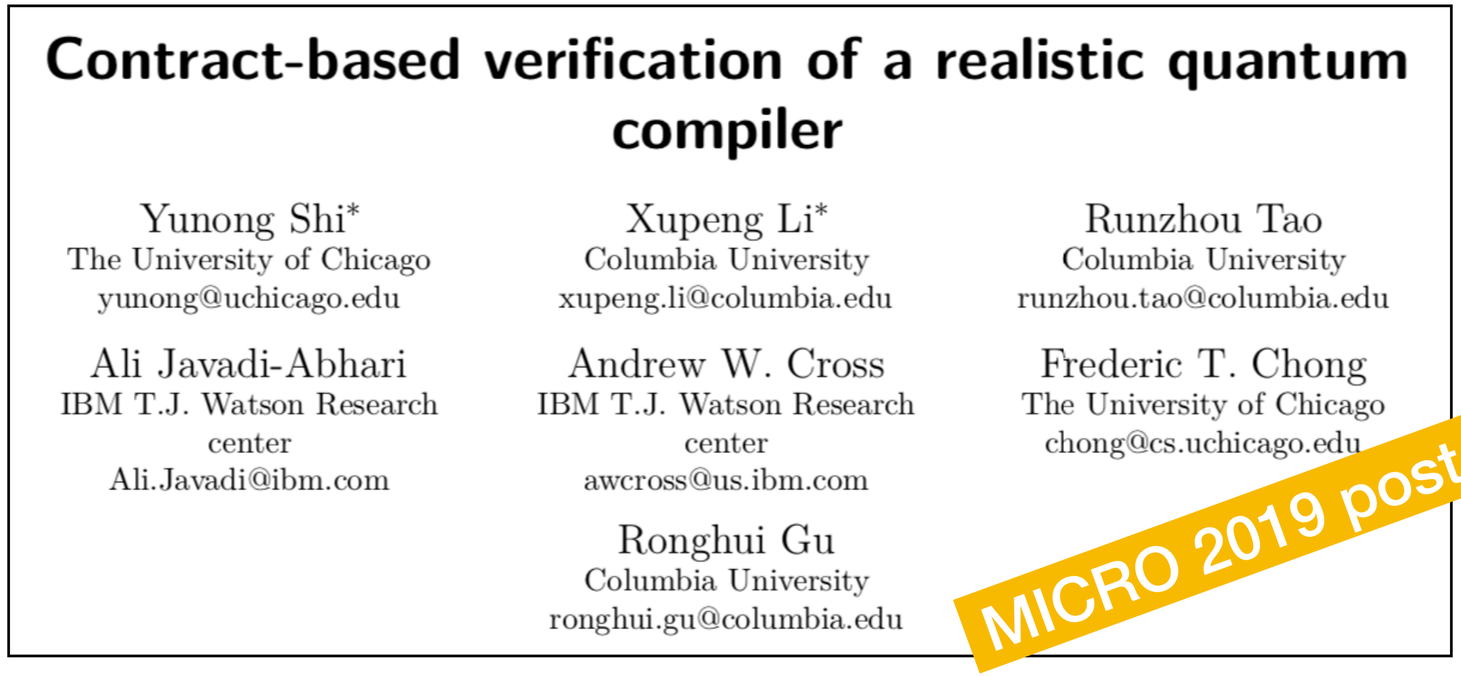

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MICRO 2019 poster

- Building **quantum compilers**…
- that are guaranteed to be **bug-free**…
- with **low verification burden**.

Result

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- **CertiQ** — a mostly-automated verification framework…

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```
from qiskit import QuantumCircuit, Aer, execute

qc = QuantumCircuit(2, 2)

qc.h(0)
qc.cx(0, 1)
qc.measure([0, 1], [0, 1])

backend = Aer.get_backend('qasm_simulator')
job_sim = execute(qc, backend)
sim_result = job_sim.result()

print(sim_result.get_counts(qc))
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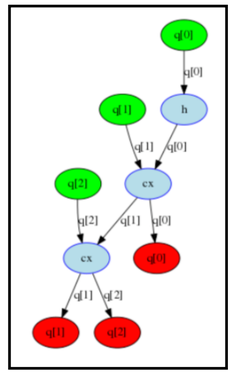
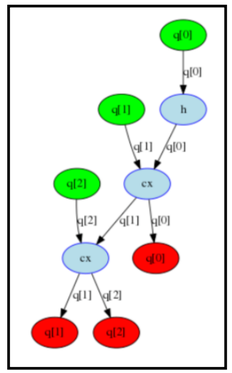
Converter



Transpilers



Assembler

qobj

...

Qiskit Circuit Description | DAG-representation | DAG-representation | Runnable qobj

Execution of Qiskit-Terra circuits: https://github.com/Qiskit/qiskit-terra/blob/master/qiskit/execute.py

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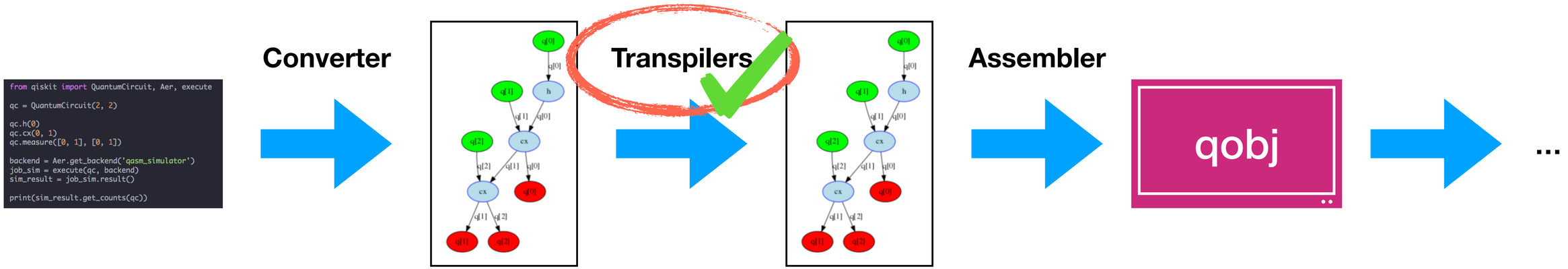


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class BasicSwap(TranformationPass):
  def __init__(self, layout=None, coupling_map):
     self.layout = layout
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- **Input**: transpiler pass + **contract** (specification, e.g. circuit equivalence)
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Challenges

- Checking circuit equivalence is **QMA** (quantum version of NP).
- Code base grows rapidly, need **modular** proof technique.
- Hard to **automate** the proof.

Contributions

For proving transpiler correctness **efficiently**

- Small step operational semantics for **equivalent circuit transformations**.
- Contract-based specification for **modular verification**.
- Combining SMT solver, symbolic execution, etc. for proof automation.
- Verified 7 transpilers, detected 3 bugs.

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What is transpiler correctness?

- Denote circuit equivalence as $\vDash [\![C]\!] = [\![C']\!]$
- Correct(transpiler) iff:
  - $\forall C, C' . transpiler(C) = C' \implies \vDash [\![C]\!] = [\![C']\!]$
- CertiQ contracts are actually more expressive
  - Transpiler must terminate
  - Resulting circuit conform to coupling-map of quits

Equivalent Circuit Transformation

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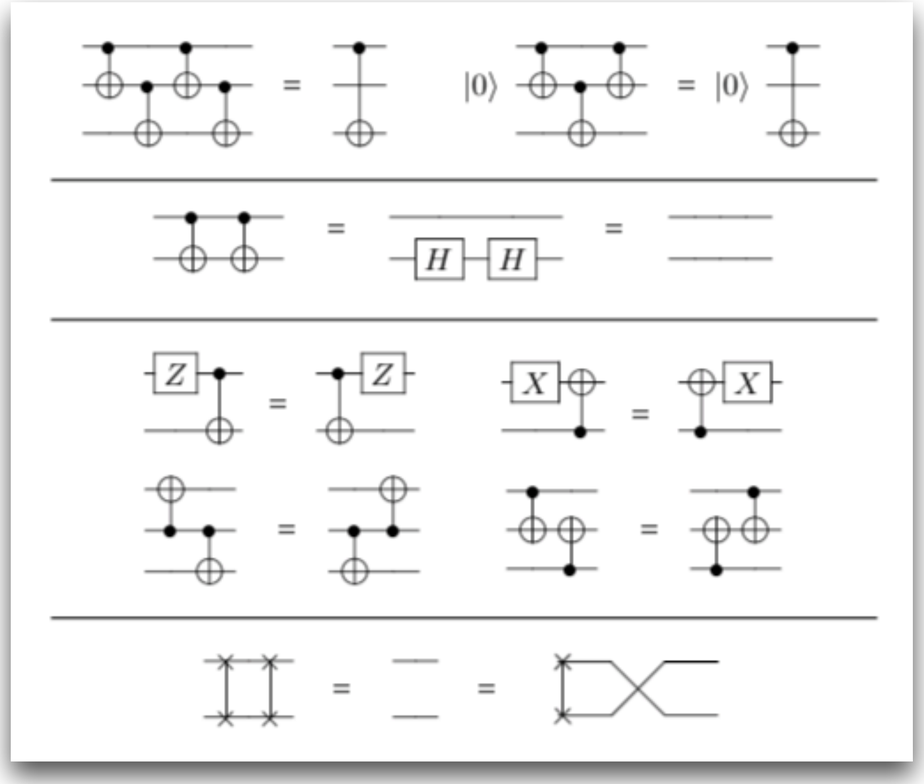

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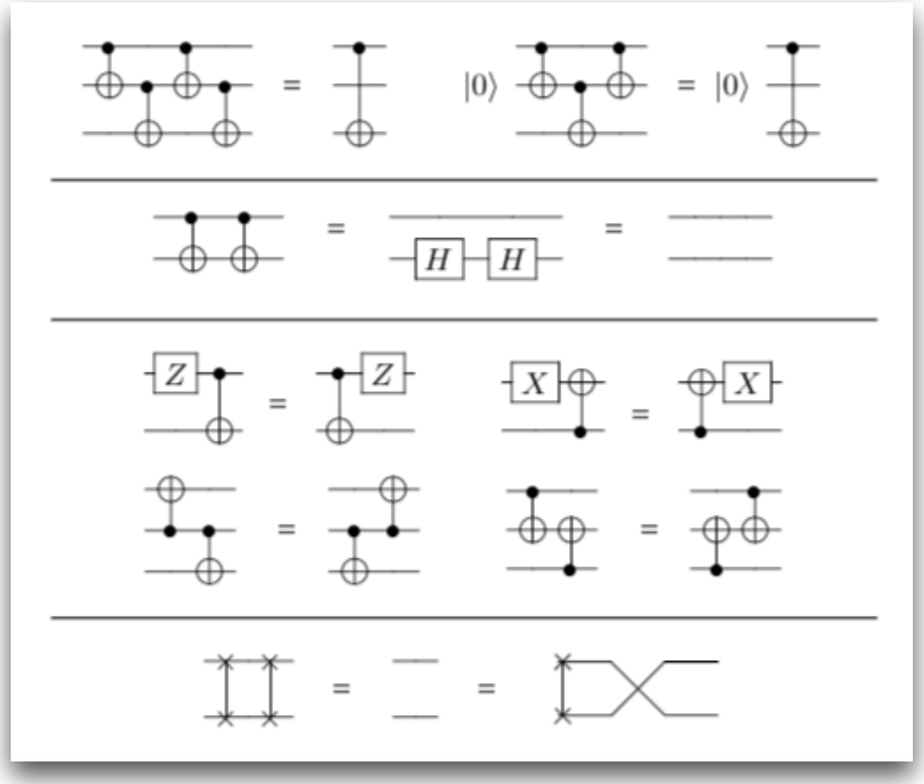

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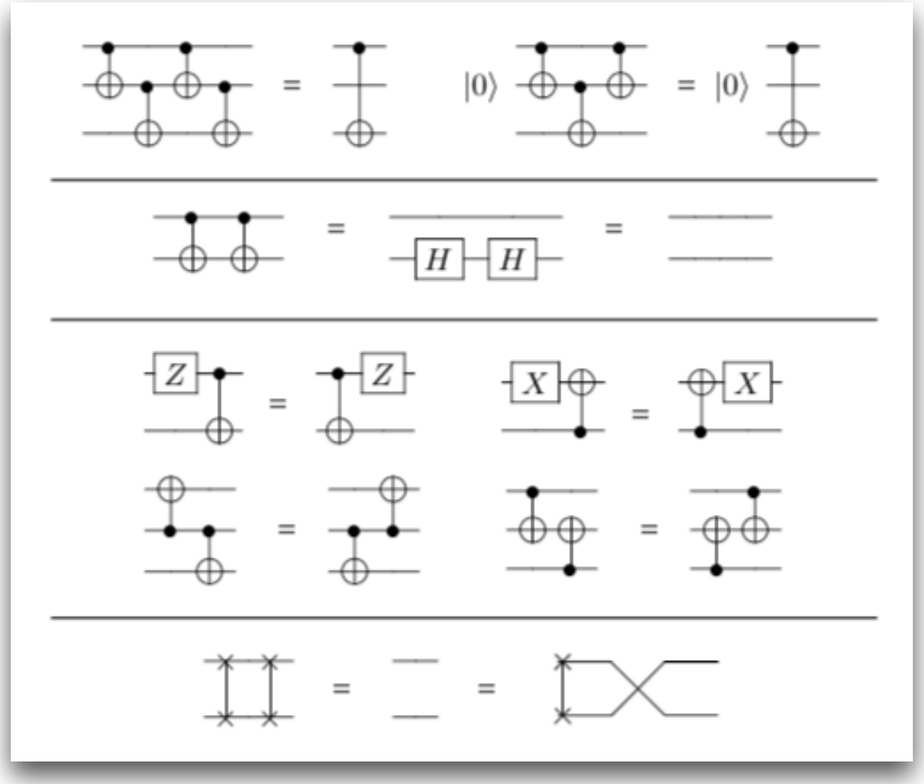

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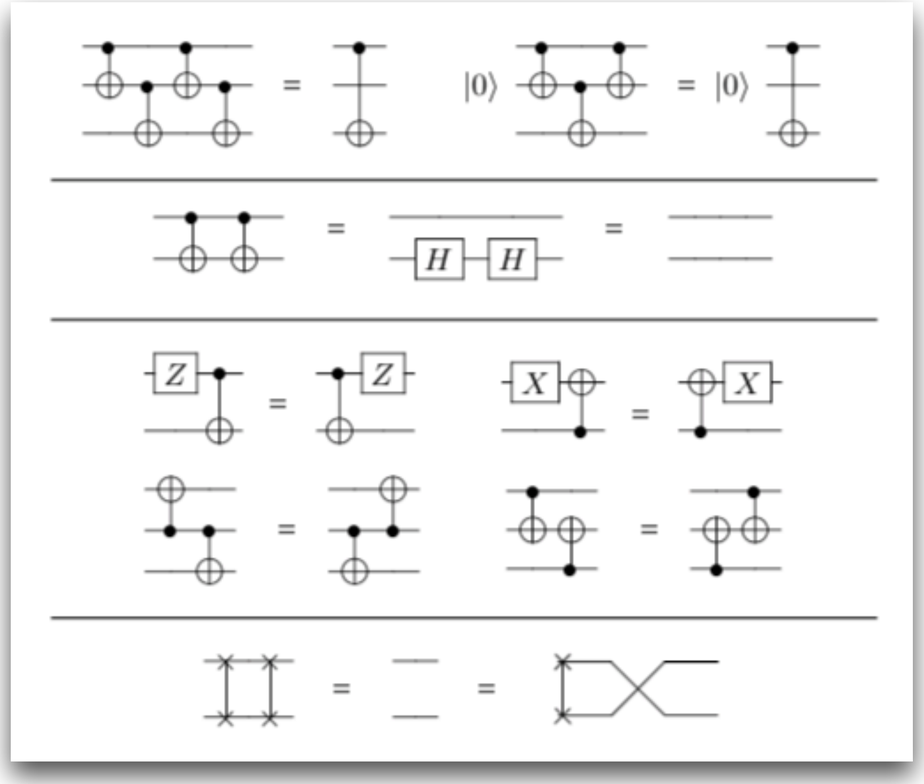

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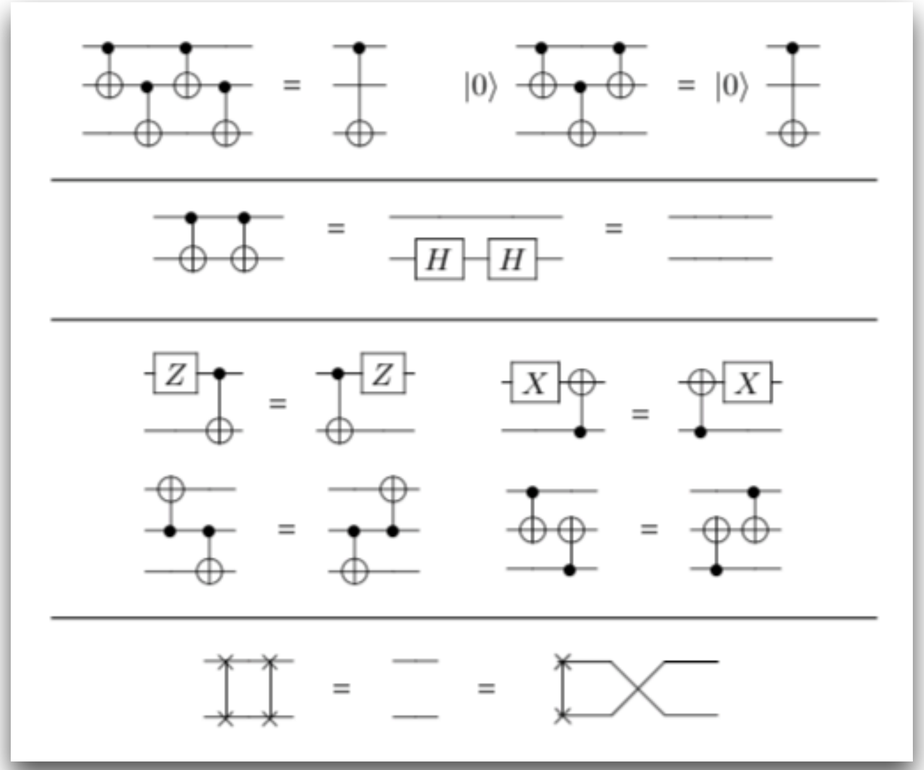

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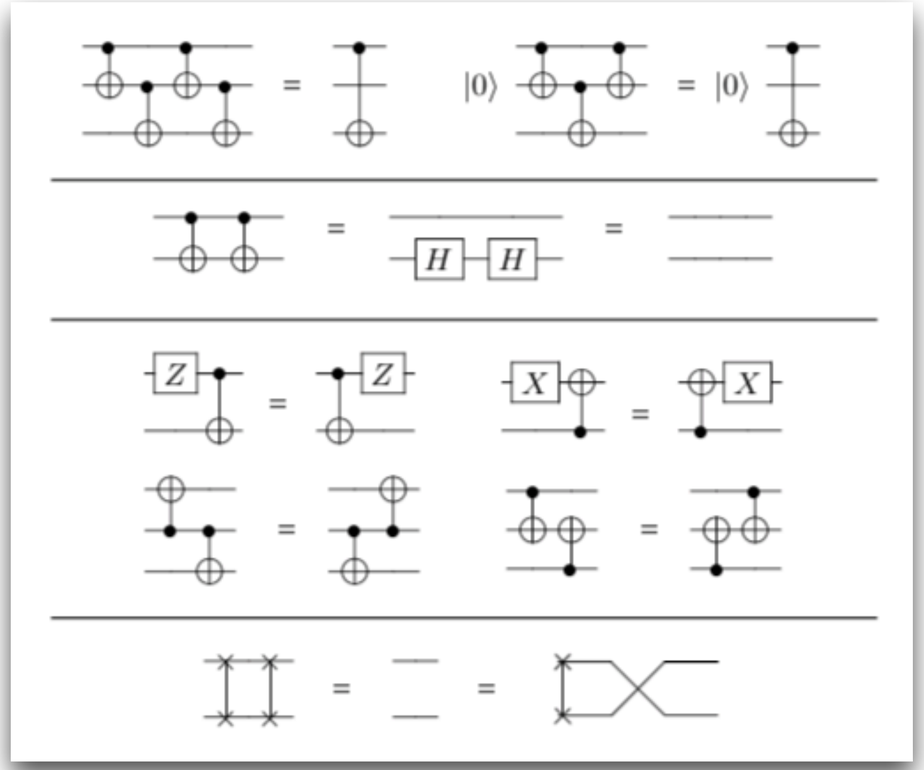

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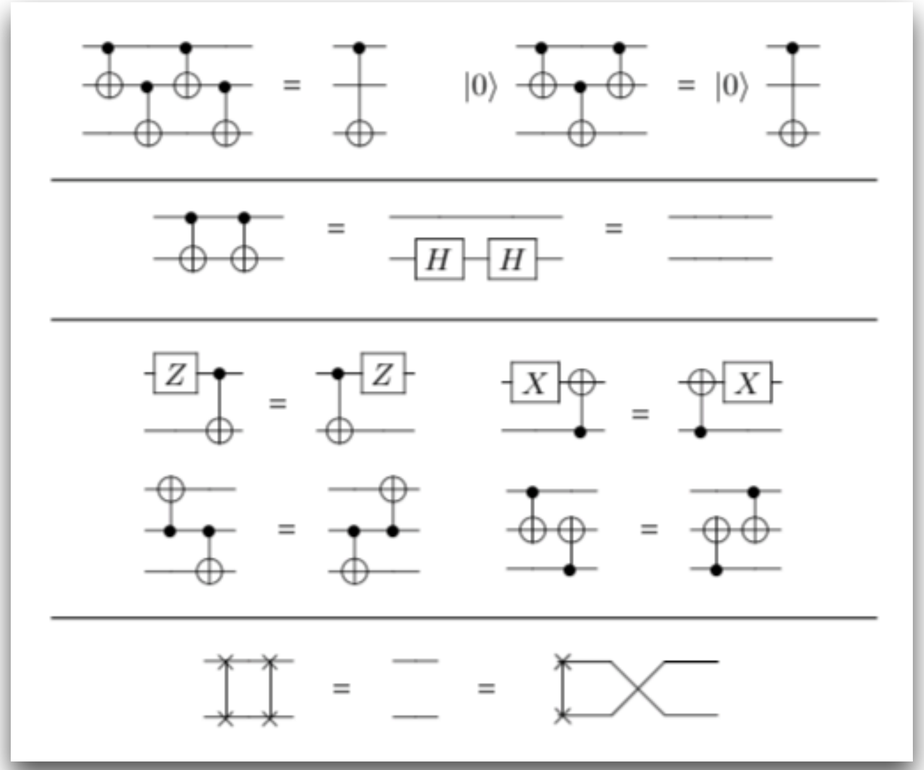

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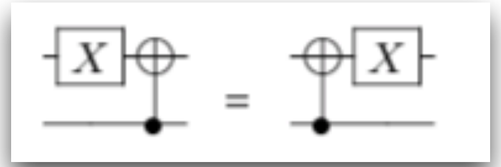

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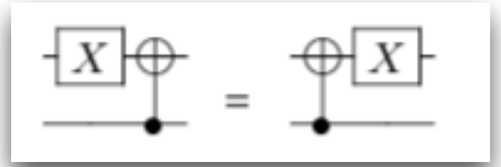

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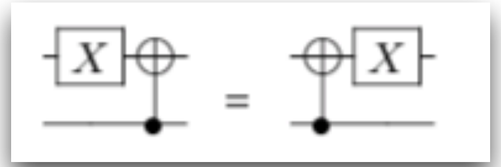

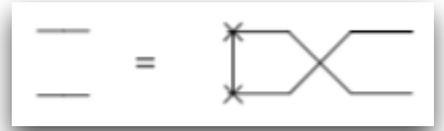

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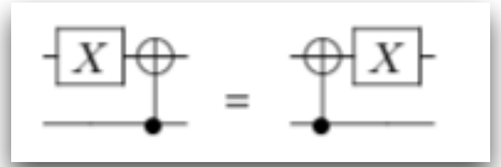
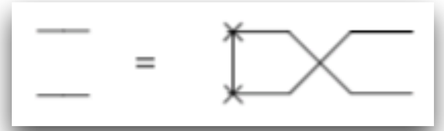
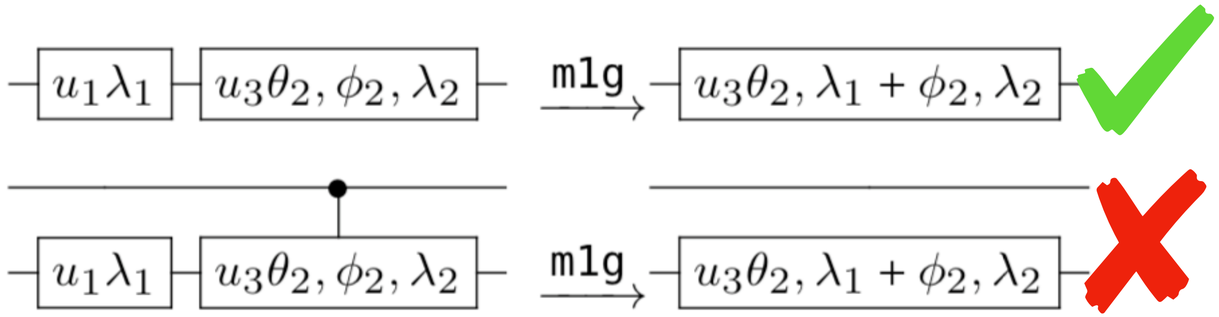




Have to re-implement ALL the passes to enable automated verification

Bugs Detected

- **`lookahead_swap`**: may not terminate
- **`optimize_1q_gate`**



- **`commutation`**
  - $U_1U_2 = U_2U_1 \wedge U_2U_3 = U_3U_2 \implies U_1U_3 = U_3U_1$
  - Does not hold in general, hold for gate set $\{\mathrm{CNOT}, \mathrm{X}, \mathrm{Z}, \mathrm{H}, \mathrm{T}, u_1, u_2, u_3\}$

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- **Underlying technique**: **SMT solver** & **validator**

```
class Layout():
  ... # omitted code
  # Contract of the Layout object
  def precondition(self): return True
  def invariant(self): return True
  def postcondition(self):
    i = fresh_int()
    return ForAll([i], self.p2v(self.v2p(i))==i)
```

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SMT solver

- **SMT** — Satisfiability Modulo Theories
    - decision problem for logical formulas…
    - w.r.t. background theories (e.g. integer, array, linear arithmetic, …)
    - expressed in first-order logic with equality.
- **SMT-solver**:
    - **Input**: (quantifier-free) formula, e.g. $(a = b) \wedge \neg(b - a = 0)$
    - **Output**: sat + model / unsat / out of time
- **Z3**: SMT-solver, available in C/C++, .NET, OCaml, Python, Java, Haskell

SMT solver applications

- Symbolic-execution based analysis and testing:
    - E.g., KLEE (llvm), SAGE (x86-binary),…
- Computer-aided verification
    - Encode pre-conditions/post-conditions/invariants into SMT formulas
    - E.g., VCC (verifier for concurrent C), Why3 (deductive program verification platform),..
- Program synthesis
    - Input: property P
    - Output: program satisfying P

Validator

- Type of validator:

$$\text{QuantumCircuit} \to \text{QuantumCircuit} \to \mathbb{B}$$

- Soundness:

$$\text{validator}(C_{in}, C_{out}) = \text{True} \implies \vdash [\![C_{in}]\!] = [\![C_{out}]\!]$$

$$\text{i.e. } C_{out} \to_e^* C_{in}$$

- Run after compilation.

- Support only swap and CNOT cancellation passes.

Strength

- **Relevant**
    - Verified 7 real transpilers, detected 3 bugs
- **Light-weighted** verification by introducing…
    - Equivalent circuit transformation
    - Z3, static analysis, contract continuation,…
- **Expressive** contract
    - circuit equivalence, conform with coupling-map, termination

Weakness

- Large **trusted computing base**:
    - Qiskit libs, Z3, equivalent circuits, static analyzer…
- Only 7/20? passes verified
- Passes need **re-implement** to enable automated verification
- Validators are **limited** to swap and CNOT cancellation passes
- **Not very clear** written, no contract example for transpiler
- The code is **not publicly available**

THE END