# 6. zadatak - ANFIS

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18. prosinca 2020.

U sklopu ovog zadatka potrebno je izraditi neuro-fuzzy sustav ANFIS koji koristi Takagi-Sugeno-Kang (TSK) zaključivanje. Kod TSK zaključivanja, konsekvens je obična funkcija ulaznih varijabli. Sustav ima dva ulaza i jedan izlaz. Time se definira preslikavanje $\mathbb{R}^2 \mapsto \mathbb{R}$. Skup primjera za učenje je: $\{((x_1, y_1), z_1), ..., ((x_N, y_N), z_N))\}$, a veličina skupa za učenje je $N$.
Neuro-fuzzy sustav je definiran s $m$ pravila $R$:
$R_1$: Ako $x$ je $A_1$ i $y$ je $B_1$ onda $f = p_1x + q_1y + r_1$
$R_2$: Ako $x$ je $A_2$ i $y$ je $B_2$ onda $f = p_2x + q_2y + r_2$
...
$R_m$: Ako $x$ je $A_m$ i $y$ je $B_m$ onda $f = p_mx + q_my + r_m$
Pri tome su $A_i$ i $B_i$ neizraziti skupovi. Neka su $a_i$ i $b_i$ parametri neizrazitog skupa $A_i$, a $c_i$ i $d_i$ parametri skupa $B_i$. Funkcije pripadnosti neizrazitih skupva $A_i$ i $B_i$ su modelirane sigmoidalnim funkcijama:

$$\begin{aligned} A_i(x) \equiv \mu_{A_i}(x) &= \frac{1}{1 + e^{b_i(x-a_i)}} \\ B_i(x) \equiv \mu_{B_i}(x) &= \frac{1}{1 + e^{d_i(x-c_i)}} \end{aligned} \tag{0.1}$$

Operator $I$ (tj. t-norma) u antecedentu se modelira umnoškom. Za svako neizrazito pravilo $R_i$ potrebno je odrediti pravila učenja koja ažuriraju 7 parametara: $a_i$, $b_i$, $c_i$, $d_i$, $p_i$, $q_i$ i $r_i$.

## Zadatak 1

Potrebno je izvesti postupak učenja koji se temelji na gradijentnom spustu prikladan za ovakvu mrežu. Uvode se pomoćne oznake:

$$\begin{aligned} \alpha_i &= A_i(x) \\ \beta_i &= B_i(y) \end{aligned} \tag{1.1}$$

Tada je jakost antecedenta:

$$\pi_i = t - norma(\alpha_i, \beta_i) = \alpha_i \cdot \beta_i. \tag{1.2}$$

Izlaz sustava je:

$$o_k = \frac{\sum_{i=1}^{m} \pi_i f_i}{\sum_{i=1}^{m} \pi_i} \tag{1.3}$$

Definira se funkcija pogreške za taj uzorak:

$$E_k = \frac{1}{2}(z_k - o_k)^2 \tag{1.4}$$

Za ažuriranje proizvoljnog parametra $\psi$ minimizacijom funkcije pogreške uporabom gradijentnog spusta koristi se izraz:

$$\psi(t+1) = \psi(t) - \eta \cdot \frac{\partial E_k}{\partial \psi} \tag{1.5}$$

Pri tome je $\eta$ stopa učenja. Potrebno je utvrditi parcijalne derivacije funkcije $E_k$ po svim parametrima. Računa se redom:

$$\frac{\partial E_k}{\partial f_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial f_i} \tag{1.6}$$

$$\begin{aligned} \frac{\partial E_k}{\partial o_k} &= -(z_k - o_k) \\ \frac{\partial o_k}{\partial f_i} &= \frac{\pi_i}{\sum_{j=1}^{m} \pi_j} \\ \frac{\partial E_k}{\partial f_i} &= -(z_k - o_k) \cdot \frac{\pi_i}{\sum_{j=1}^{m} \pi_j} \end{aligned} \tag{1.7}$$

Gradijent u odnosu na parametre $p_i$, $q_i$ i $r_i$:

$$\frac{\partial E_k}{\partial p_i} = \frac{\partial E_k}{\partial f_i} \cdot \frac{\partial f_i}{\partial p_i} \tag{1.8}$$

$$\begin{aligned} \frac{\partial f_i}{\partial p_i} &= x \\ \frac{\partial E_k}{\partial p_i} &= -(z_k - o_k) \cdot \frac{\pi_i}{\sum_{j=1}^{m} \pi_j} \cdot x \end{aligned} \tag{1.9}$$

$$\frac{\partial E_k}{\partial q_i} = \frac{\partial E_k}{\partial f_i} \cdot \frac{\partial f_i}{\partial q_i} \tag{1.10}$$

$$\begin{aligned} \frac{\partial f_i}{\partial q_i} &= y \\ \frac{\partial E_k}{\partial q_i} &= -(z_k - o_k) \cdot \frac{\pi_i}{\sum_{j=1}^{m} \pi_j} \cdot y \end{aligned} \tag{1.11}$$

$$\frac{\partial E_k}{\partial r_i} = \frac{\partial E_k}{\partial f_i} \cdot \frac{\partial f_i}{\partial r_i} \tag{1.12}$$

$$\begin{aligned} \frac{\partial f_i}{\partial r_i} &= 1 \\ \frac{\partial E_k}{\partial r_i} &= -(z_k - o_k) \cdot \frac{\pi_i}{\sum_{j=1}^{m} \pi_j} \end{aligned} \tag{1.13}$$

Gradijent u odnosu na parametre $a_i$, $b_i$, $c_i$ i $d_i$:

$$\frac{\partial E_k}{\partial a_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \pi_i} \cdot \frac{\partial \pi_i}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial a_i} \qquad [1.14]$$

$$\frac{\partial E_k}{\partial b_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \pi_i} \cdot \frac{\partial \pi_i}{\partial \alpha_i} \cdot \frac{\partial \alpha_i}{\partial b_i} \qquad [1.15]$$

$$\begin{gathered}\frac{\partial o_k}{\partial \pi_i} = \frac{f_i \cdot \sum_{j=1}^m \pi_j - \sum_{j=1}^m f_j \pi_j}{\left(\sum_{j=1}^m \pi_j\right)^2} = \frac{\sum_{j=1, j\neq i}^m (f_i - f_j) \cdot \pi_j}{\left(\sum_{j=1}^m \pi_j\right)^2} \\ \frac{\partial \pi_i}{\partial \alpha_i} = \beta_i \\ \frac{\partial \alpha_i}{\partial a_i} = b_i \cdot \alpha_i \cdot (1 - \alpha_i) \\ \frac{\partial \alpha_i}{\partial b_i} = -(x - a_i) \cdot \alpha_i \cdot (1 - \alpha_i)\end{gathered} \qquad [1.16]$$

$$\frac{\partial E_k}{\partial a_i} = -(z_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^m (f_i - f_j) \cdot \pi_j}{\left(\sum_{j=1}^m \pi_j\right)^2} \cdot \beta_i \cdot b_i \cdot \alpha_i \cdot (1 - \alpha_i) \qquad [1.17]$$

$$\frac{\partial E_k}{\partial a_i} = (z_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^m (f_i - f_j) \cdot \pi_j}{\left(\sum_{j=1}^m \pi_j\right)^2} \cdot \beta_i \cdot (x - a_i) \cdot \alpha_i \cdot (1 - \alpha_i) \qquad [1.18]$$

$$\frac{\partial E_k}{\partial c_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \pi_i} \cdot \frac{\partial \pi_i}{\partial \beta_i} \cdot \frac{\partial \beta_i}{\partial c_i} \qquad [1.19]$$

$$\frac{\partial E_k}{\partial d_i} = \frac{\partial E_k}{\partial o_k} \cdot \frac{\partial o_k}{\partial \pi_i} \cdot \frac{\partial \pi_i}{\partial \beta_i} \cdot \frac{\partial \beta_i}{\partial d_i} \qquad [1.20]$$

$$\begin{gathered}\frac{\partial \pi_i}{\partial \beta_i} = \alpha_i \\ \frac{\partial \beta_i}{\partial c_i} = d_i \cdot \beta_i \cdot (1 - \beta_i) \\ \frac{\partial \beta_i}{\partial d_i} = -(y - c_i) \cdot \beta_i \cdot (1 - \beta_i)\end{gathered} \qquad [1.21]$$

Nakon izvedenih svih parcijalnih derivacija, mogu se zapisati izrazi za ažuriranje svih parametara:

$$p_i(t+1) = p_i(t) + \eta(z_k - o_k) \cdot \frac{\pi_i}{\sum_{j=1}^m \pi_j} \cdot x \qquad [1.22]$$

$$q_i(t+1) = q_i(t) + \eta(z_k - o_k) \cdot \frac{\pi_i}{\sum_{j=1}^m \pi_j} \cdot y \qquad [1.23]$$

$$r_i(t+1) = r_i(t) + \eta(z_k - o_k) \cdot \frac{\pi_i}{\sum_{j=1}^m \pi_j} \qquad [1.24]$$

$$a_i(t+1) = a_i(t) + \eta(z_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^{m} (f_i - f_j) \cdot \pi_j}{\left(\sum_{j=1}^{m} \pi_j\right)^2} \cdot \beta_i \cdot b_i \cdot \alpha_i \cdot (1 - \alpha_i) \quad [1.25]$$

$$b_i(t+1) = b_i(t) - \eta(z_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^{m} (f_i - f_j) \cdot \pi_j}{\left(\sum_{j=1}^{m} \pi_j\right)^2} \cdot \beta_i \cdot (x - a_i) \cdot \alpha_i \cdot (1 - \alpha_i) \quad [1.26]$$

$$c_i(t+1) = c_i(t) + \eta(z_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^{m} (f_i - f_j) \cdot \pi_j}{\left(\sum_{j=1}^{m} \pi_j\right)^2} \cdot \alpha_i \cdot d_i \cdot \beta_i \cdot (1 - \beta_i) \quad [1.27]$$

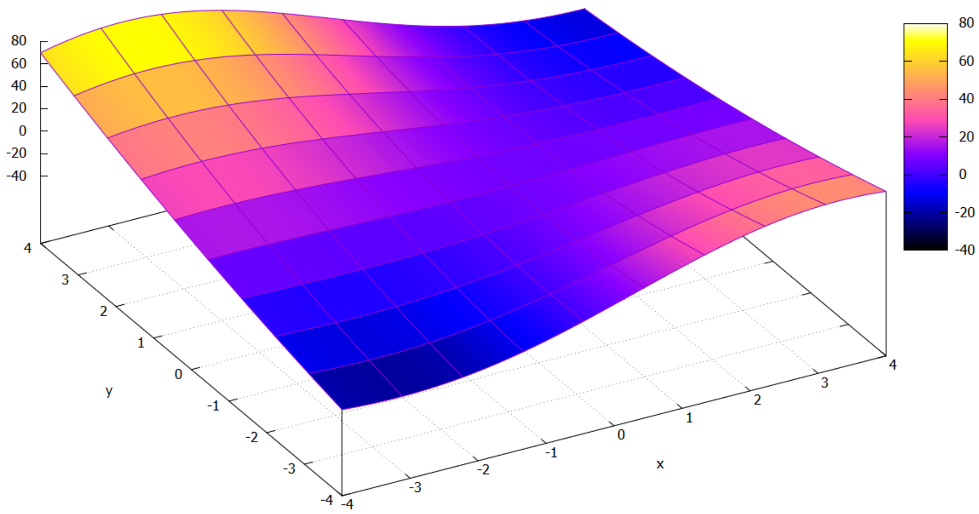
$$d_i(t+1) = d_i(t) - \eta(z_k - o_k) \cdot \frac{\sum_{j=1, j\neq i}^{m} (f_i - f_j) \cdot \pi_j}{\left(\sum_{j=1}^{m} \pi_j\right)^2} \cdot \alpha_i \cdot (y - c_i) \cdot \beta_i \cdot (1 - \beta_i) \quad [1.28]$$

# Zadatak 2

Potrebno je generirati skup podataka za učenje. Zadana je funkcija:

$$f(x, y) = \left((x-1)^2 + (y+2)^2 - 5xy + 3\right) \cdot cos^2\left(\frac{x}{5}\right) \quad [2.1]$$

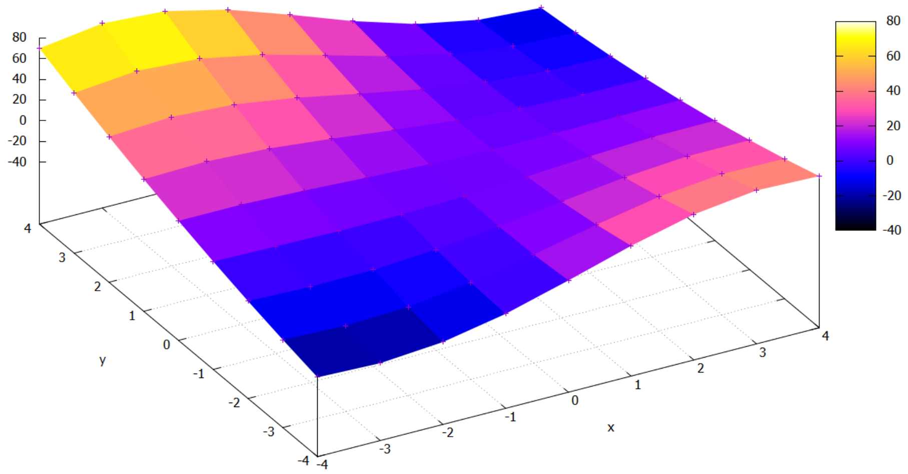
koja je definira nad domenom $x \in [-4, 4]$, $y \in [-4, 4]$. Skup podataka za učenje čine uzorci funkcije po svim cijelim brojevima iz domene.




Slika 1: Graf zadane funkcije

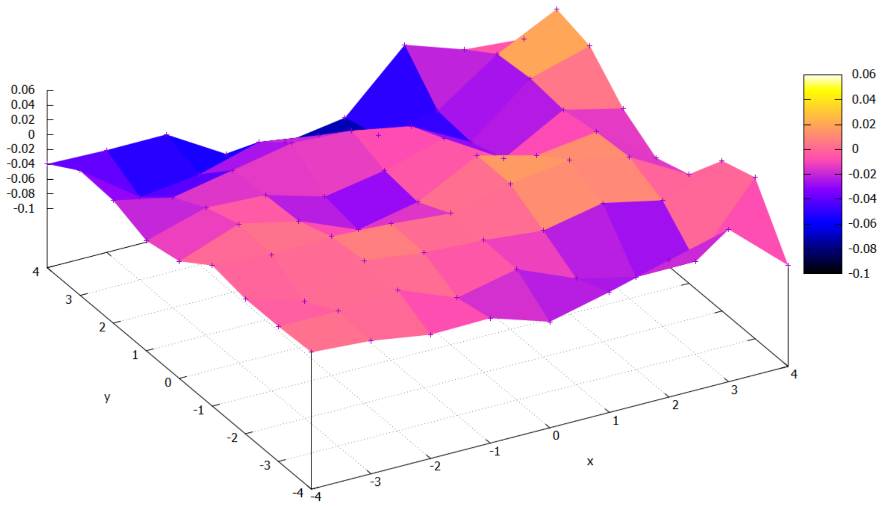
# Zadatak 3

Na temelju primjera za učenje potrebno je provesti učenje ANFIS sustava pomoću obje inačice algoritma učenja. Na slici 2 je prikazan graf funkcije koju je naučio ANFIS nad zadanom domenom. Naučeni sustav sadrži 10 pravila zaključivanja.



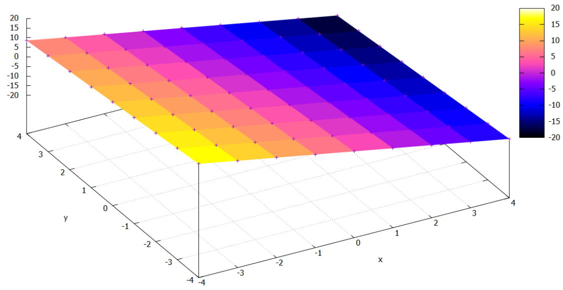

Slika 2: Graf funkcije koju je naučio ANFIS

Funkcija pogreške uzorka $\delta(x, y)$ je definirana kao razlika između vrijednosti koji sustav ANFIS daje na izlazu za točku $(x, y)$ i stvarne vrijednosti koju zadana funkcija poprima u točki $(x, y)$: $\delta(x, y) = o(x, y) - z(x, y)$. Kako se kreću odstupanja naučene funkcije i uzoraka za učenje prikazano je na slici 3.

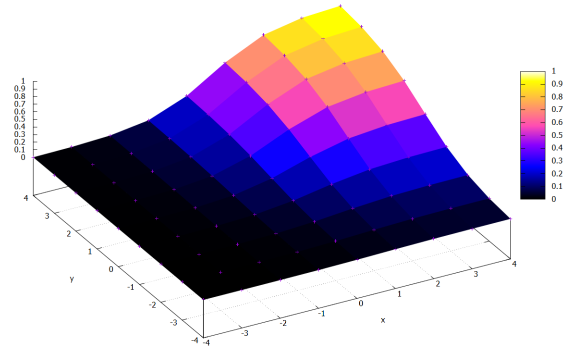



Slika 3: Odstupanja naučene funkcije i uzoraka za učenje

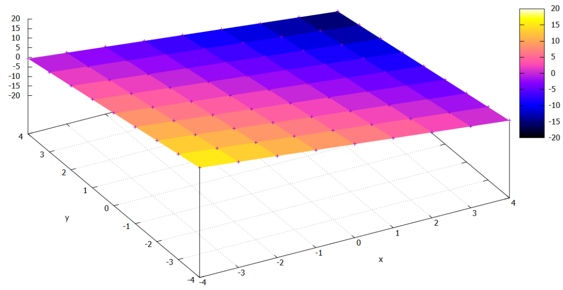
Na slikama 4, 5 i 6 su prikazani izgledi naučenih funkcija pripadnosti.



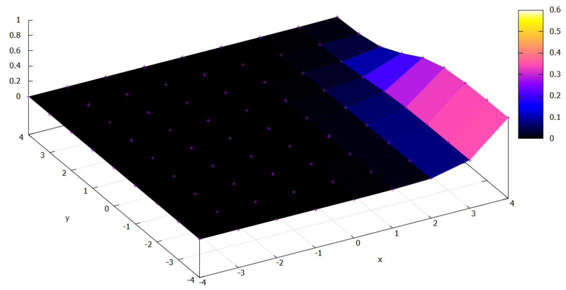
(a) Pravilo 1 - linearna funkcija



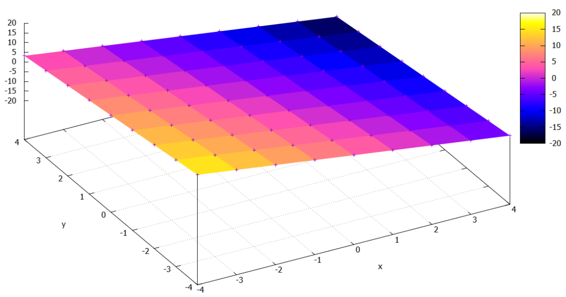
(b) Pravilo 1 - funkcija pripadnosti



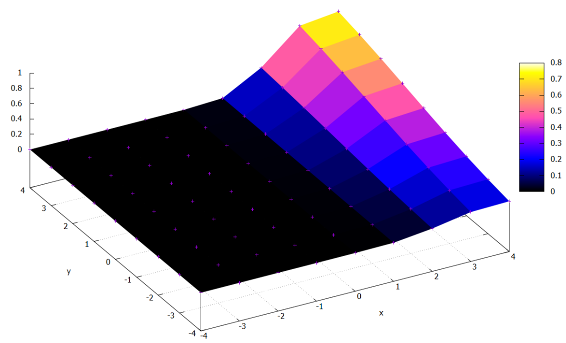
(c) Pravilo 2 - linearna funkcija



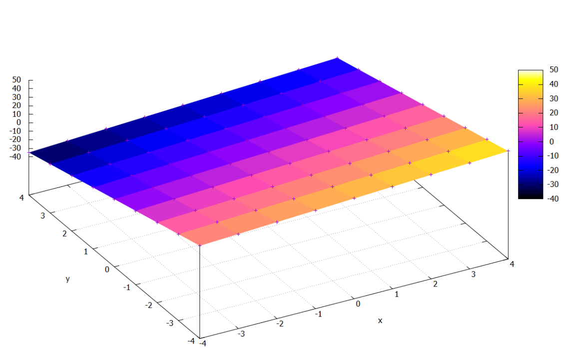
(d) Pravilo 2 - funkcija pripadnosti



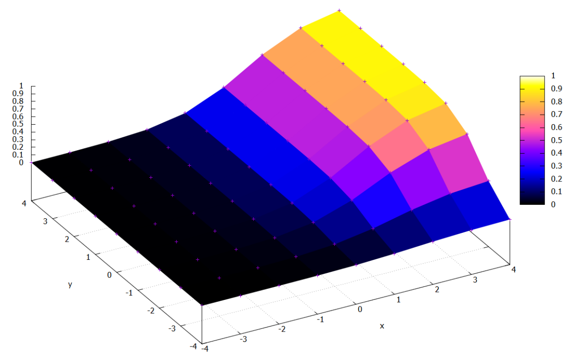
(e) Pravilo 3 - linearna funkcija



(f) Pravilo 3 - funkcija pripadnosti

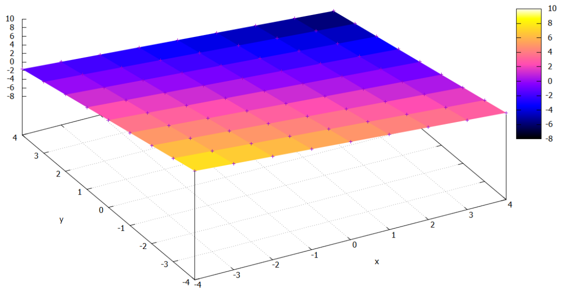


(g) Pravilo 4 - linearna funkcija

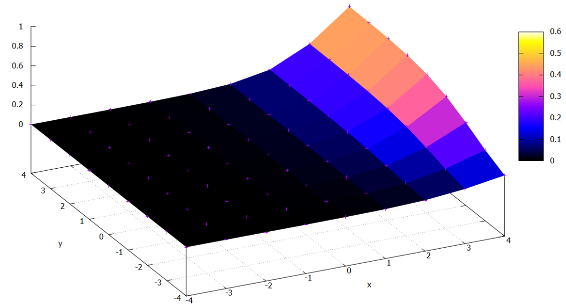


(h) Pravilo 4 - funkcija pripadnosti

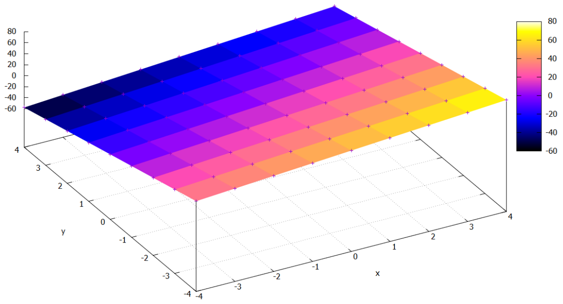
Slika 4: Naučena pravila 1 - 4



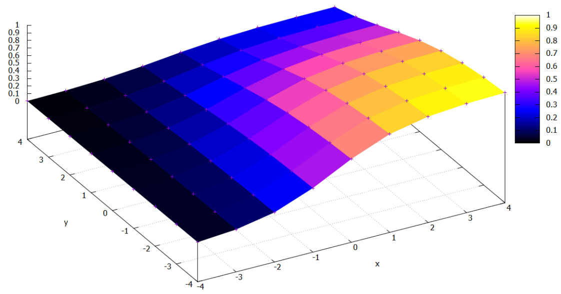
(a) Pravilo 5 - linearna funkcija



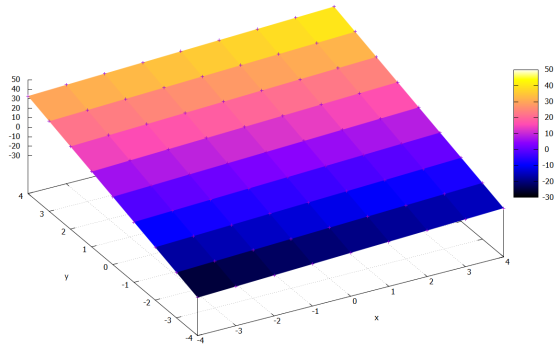
(b) Pravilo 5 - funkcija pripadnosti



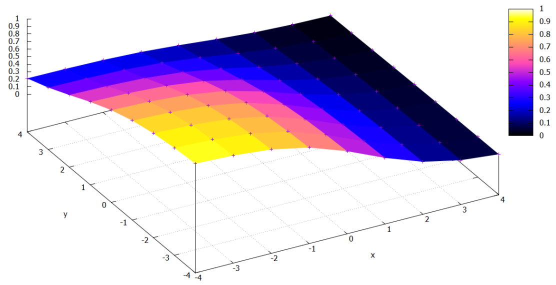
(c) Pravilo 6 - linearna funkcija



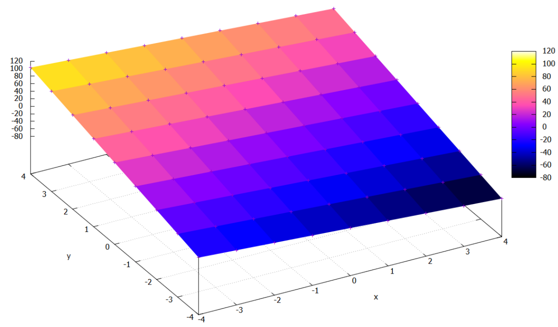
(d) Pravilo 6 - funkcija pripadnosti



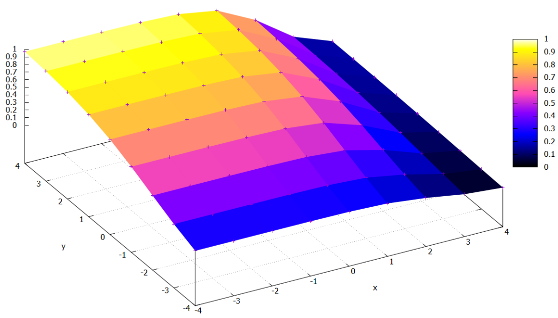
(e) Pravilo 7 - linearna funkcija



(f) Pravilo 7 - funkcija pripadnosti

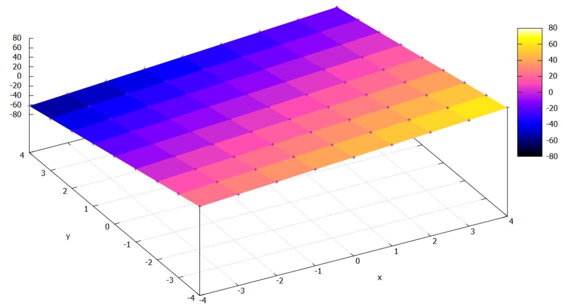


(g) Pravilo 8 - linearna funkcija

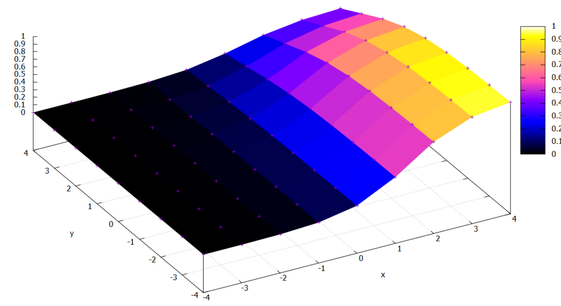


(h) Pravilo 8 - funkcija pripadnosti

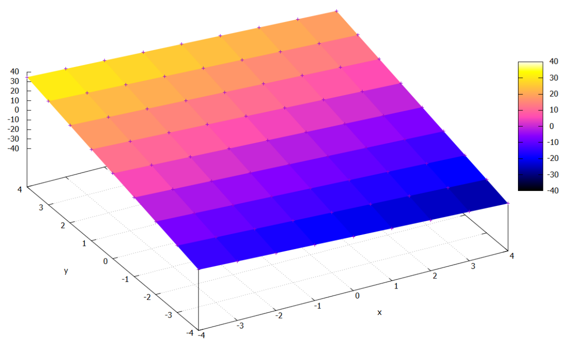
Slika 5: Naučena pravila 5 - 8



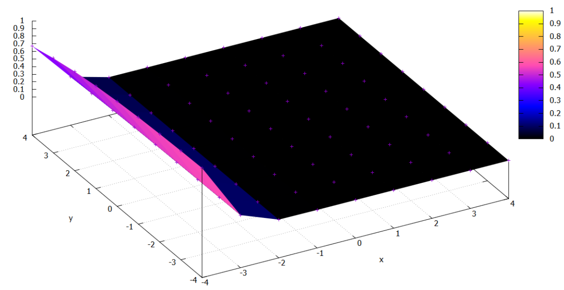
(a) Pravilo 9 - linearna funkcija



(b) Pravilo 9 - funkcija pripadnosti



(c) Pravilo 10 - linearna funkcija

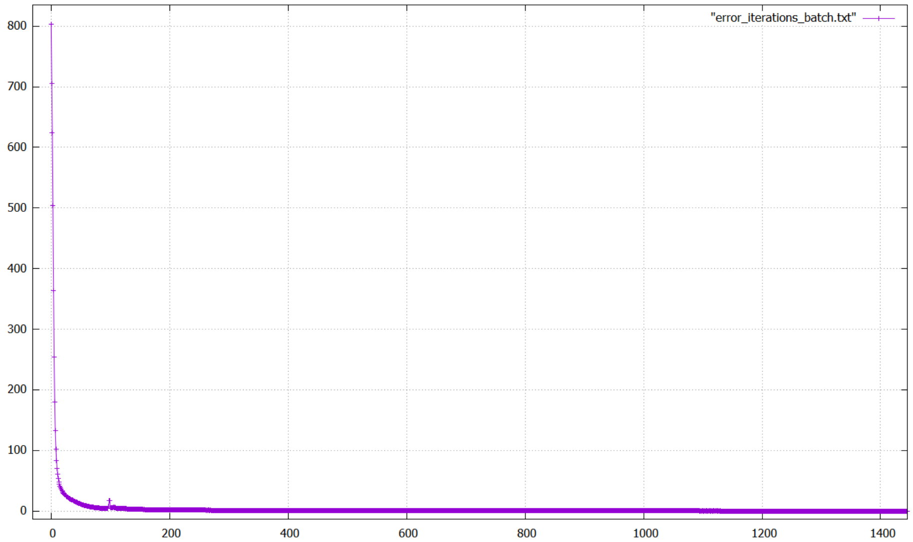


(d) Pravilo 10 - funkcija pripadnosti

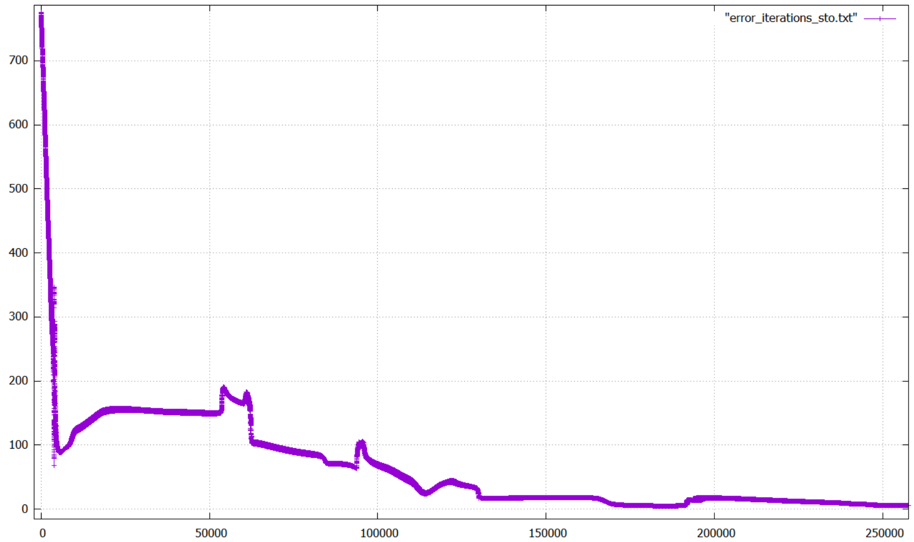
Slika 6: Naučena pravila 9 i 10

# Zadatak 4

Za neuro-fuzzy sustav s 10 pravila ponavlja se postupak učenja s obje inačice algoritma i nakon učenja se u datoteku pohrane pogreške koje je sustav radio nakon svake epohe učenja. Na slici 7 je prikazan graf kretanja pogreške ovisno o broju epohe.

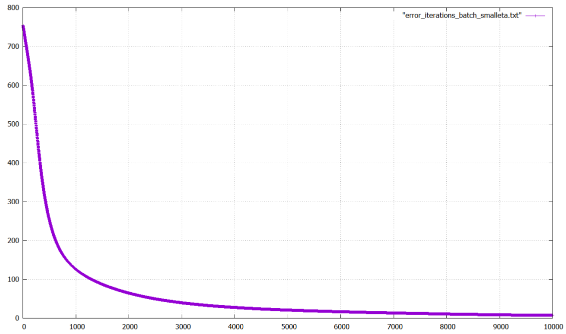



(a) Varijanta koja se temelji na izračunu pravog gradijenta

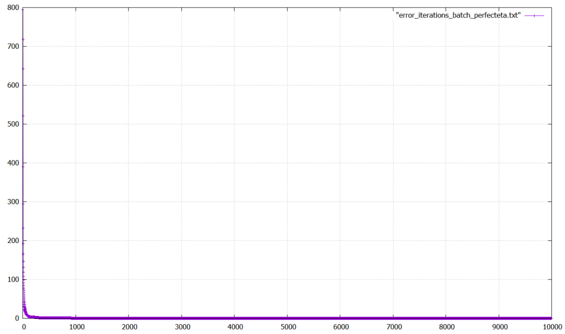



(b) Stohastička varijanta

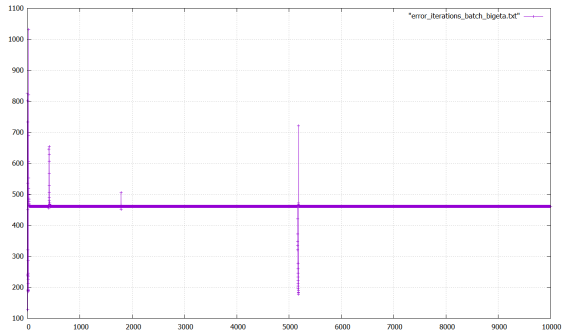
Slika 7: Graf kretanja pogreške ovisno o epohi




(a) Vrlo mala stopa učenja




(b) Prikladna stopa učenja

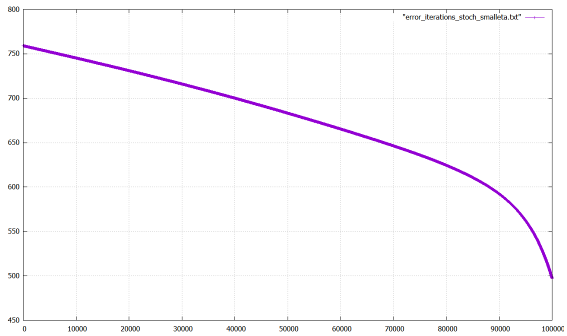



(c) Vrlo velika stopa učenja

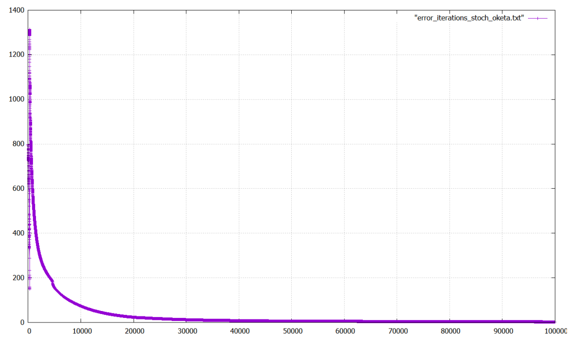
Slika 8: Varijanta koja se temelji na izračunu pravog gradijenta

# Zadatak 5

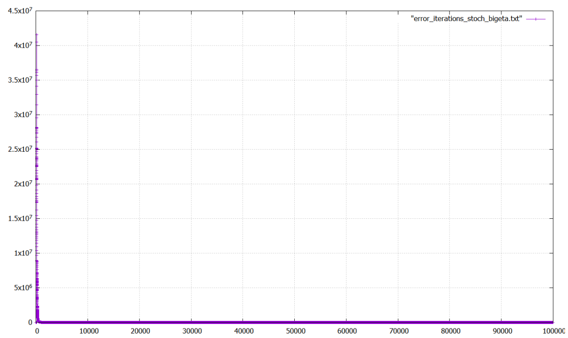
Za ANFIS sustav s 10 pravila za obje inačice algoritma ponavlja se postupak učenja za tri rubne vrijednosti stope učenja (vrlo mala, prikladna i vrlo velika). Na slici 8 je prikazan graf kretanja srednje kvadratrane pogreške ovisno o broju epohe za različite vrijednosti stope učenja za varijantu koja se temelji na izračunu pravog gradijenta, a na slici 9 za stohastičku varijantu.




(a) Vrlo mala stopa učenja




(b) Prikladna stopa učenja




(c) Vrlo velika stopa učenja

Slika 9: Stohastička varijanta