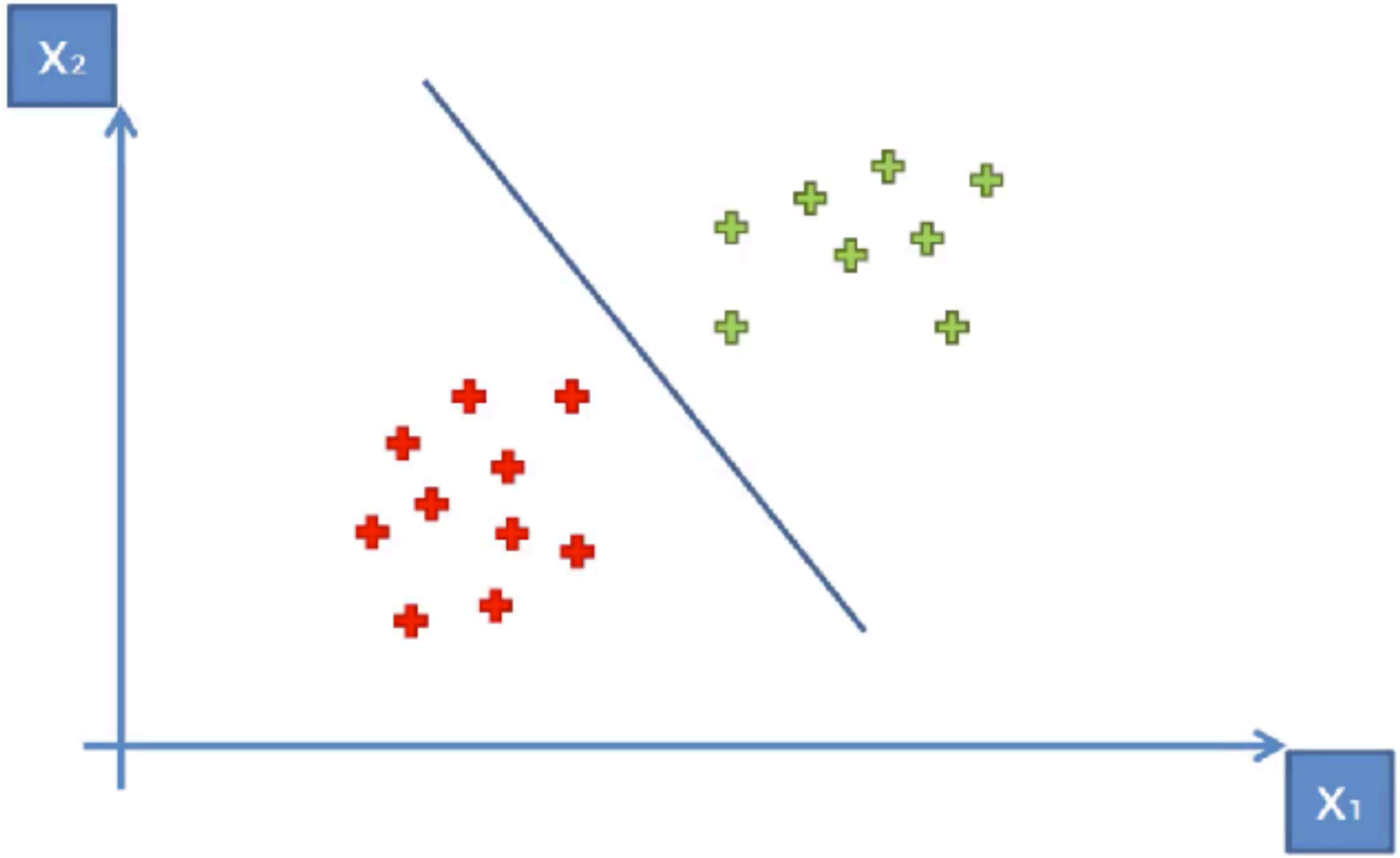
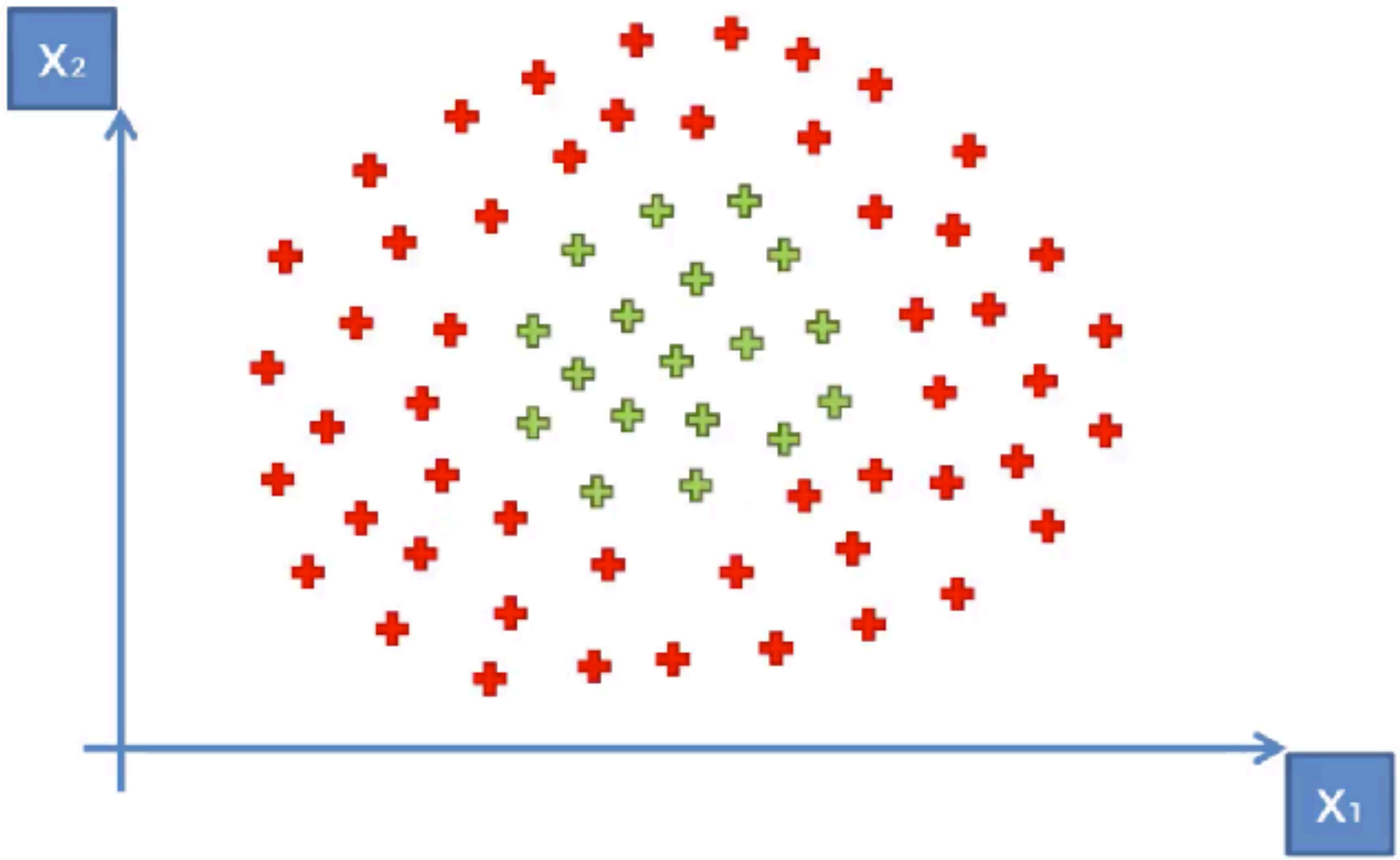
Kernel SVM Intuition

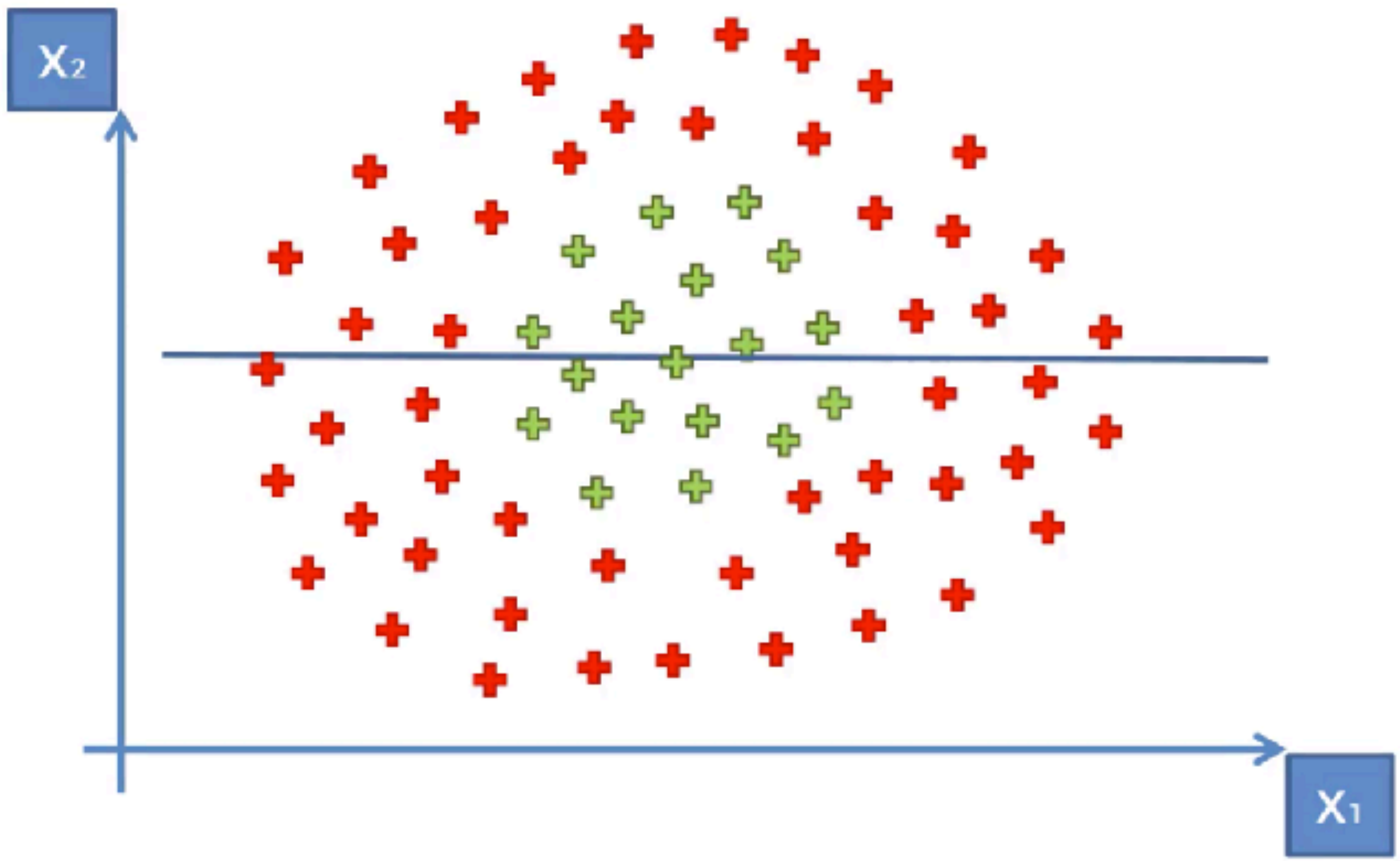
SVM separates well these points



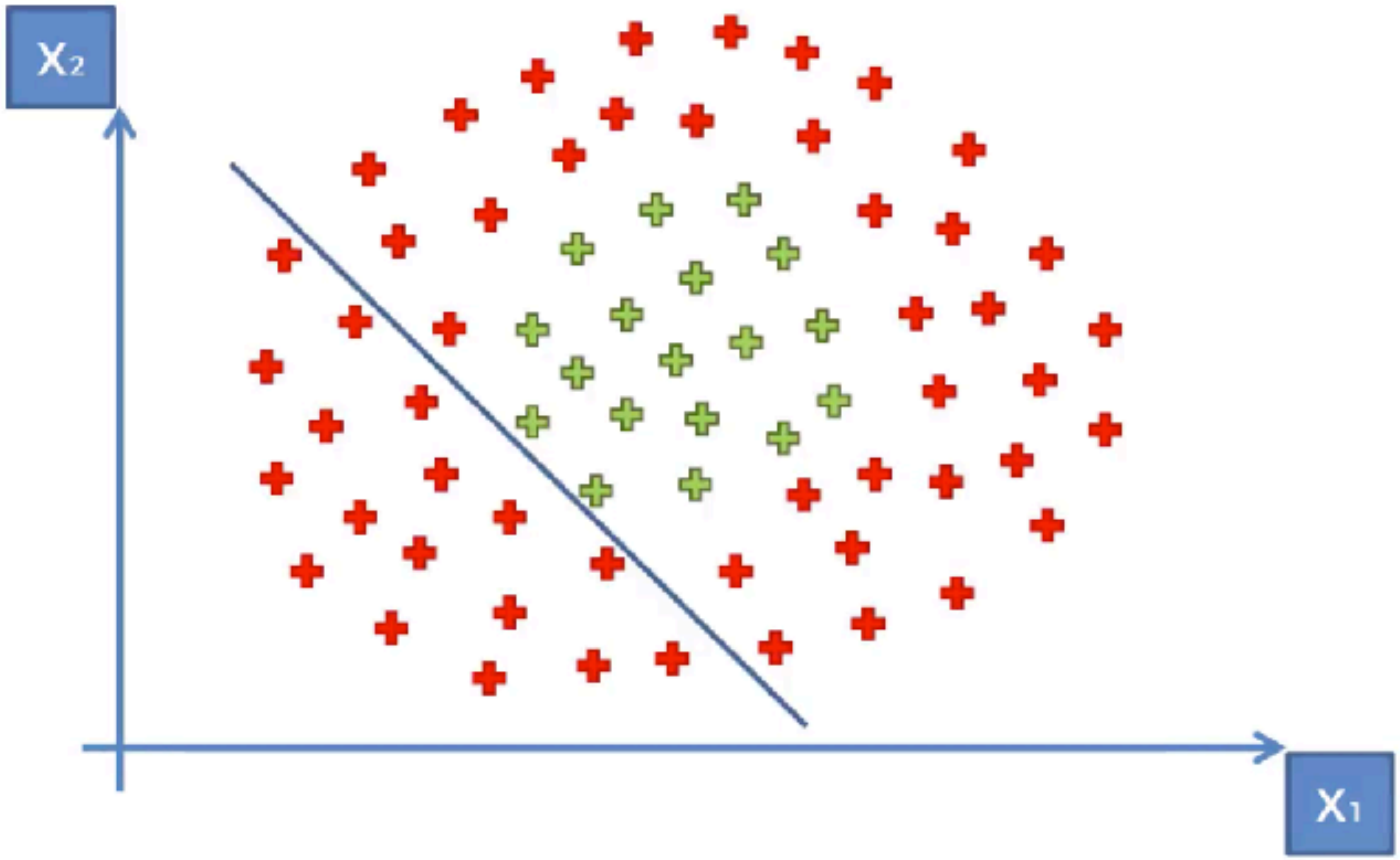
What about these points ?



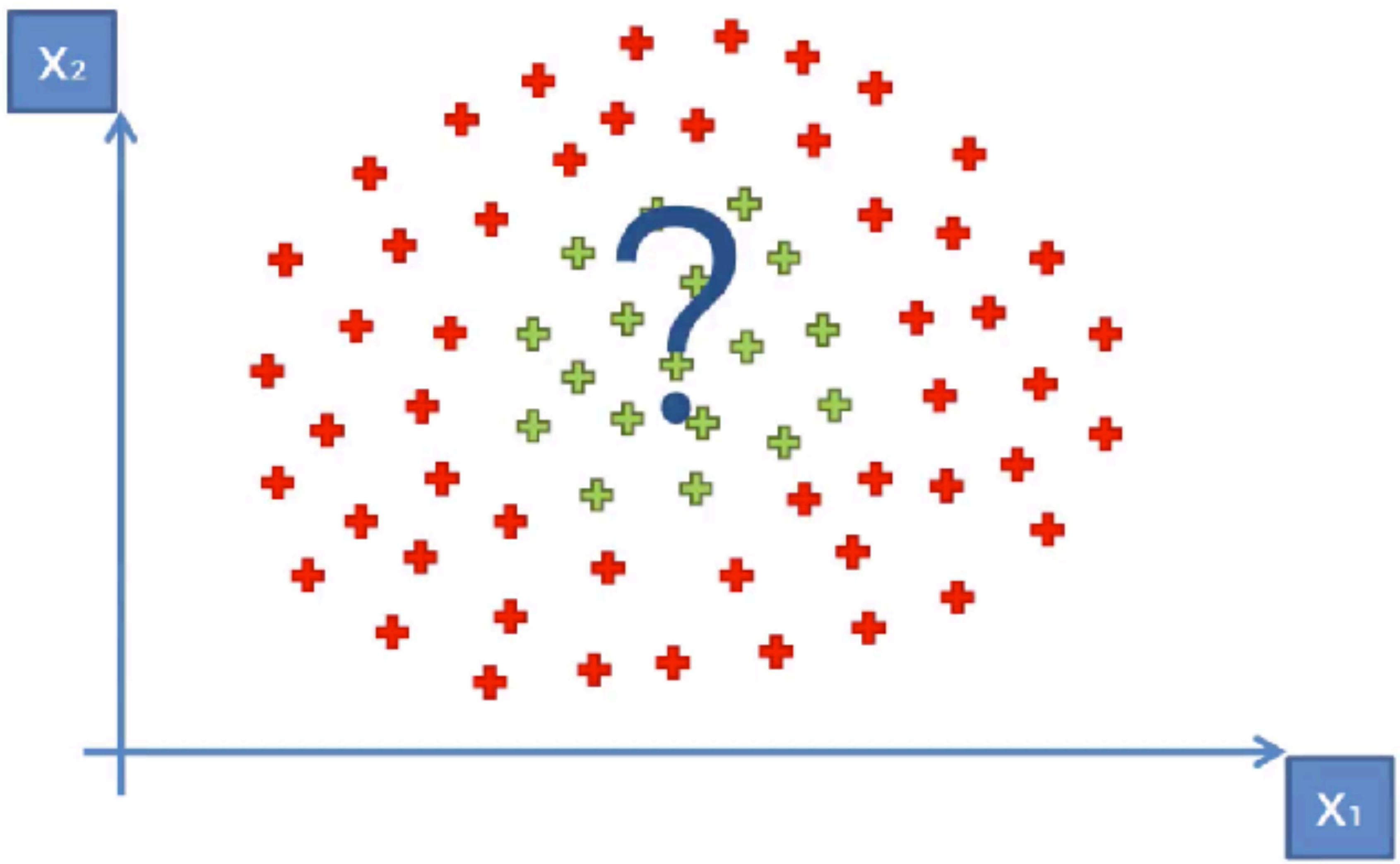
What about these points ?



What about these points ?



What about these points ?

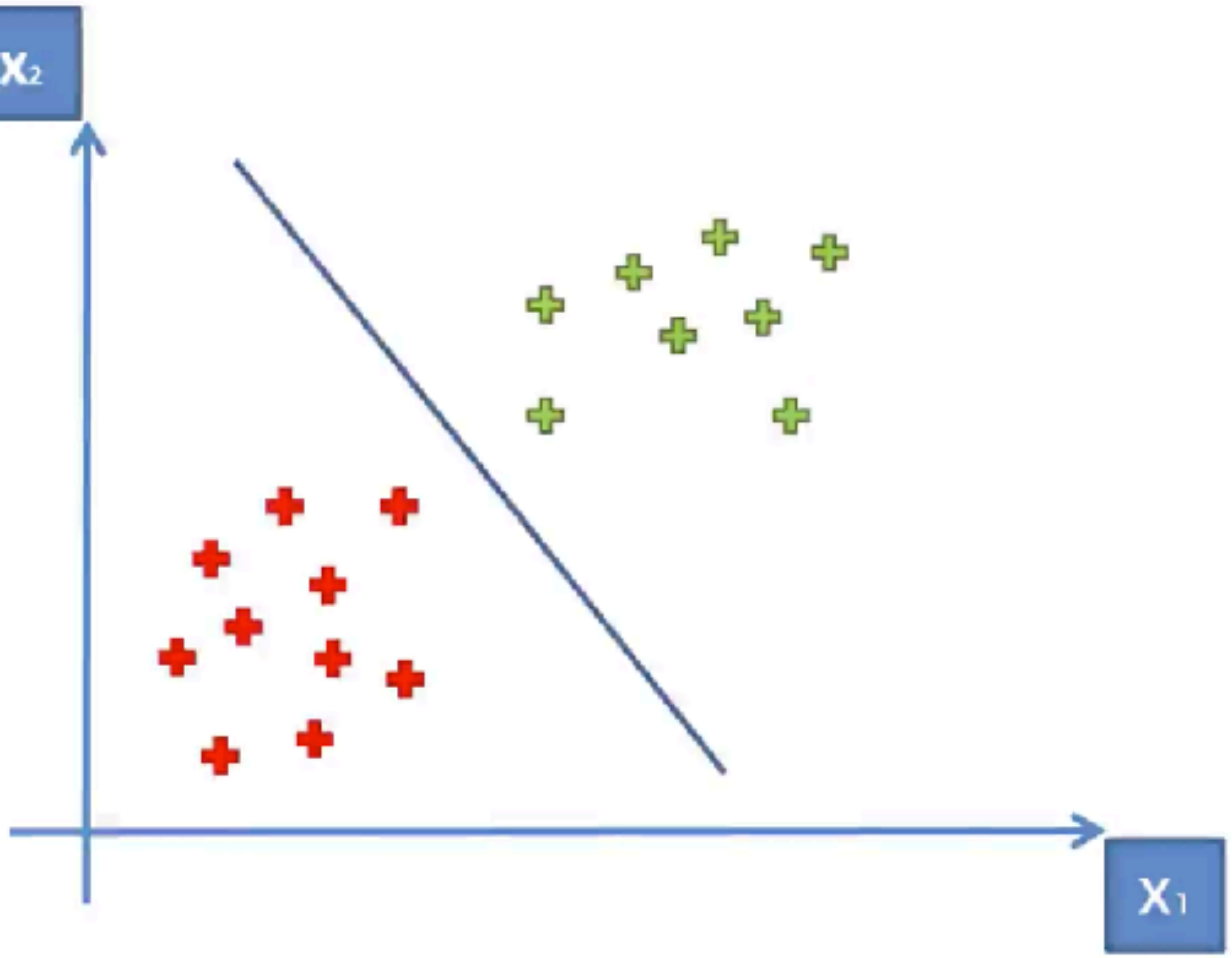


Why ?

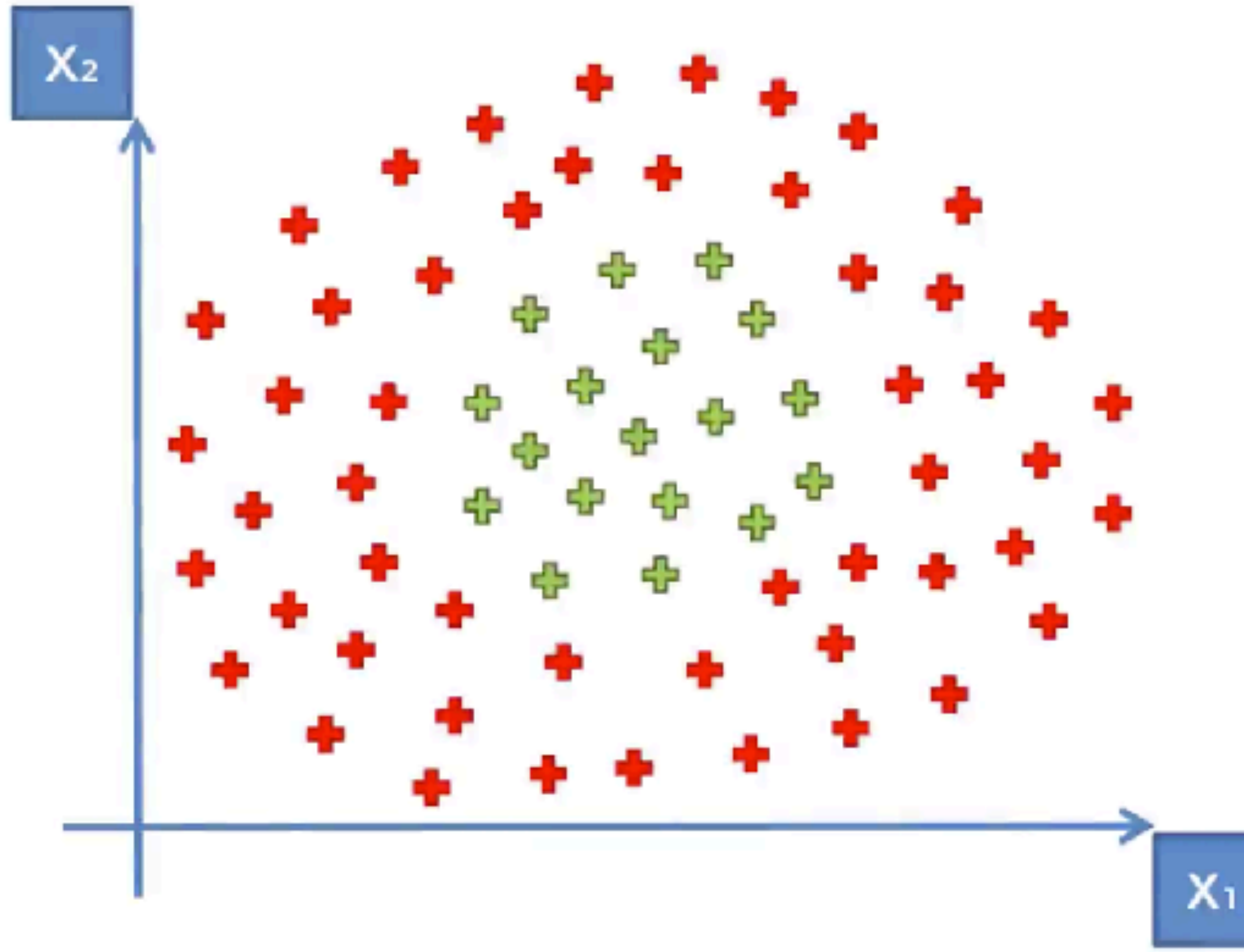
Because the data points are not LINEARLY SEPARABLE

Linear Separability

Linearly Separable

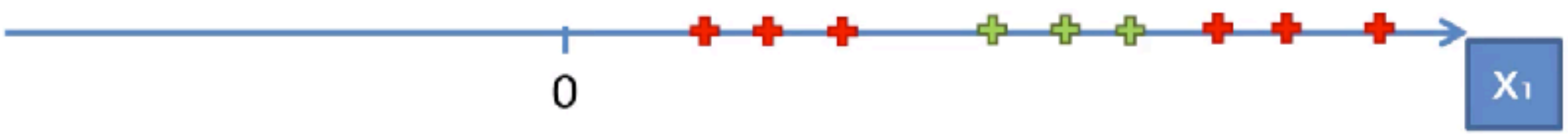


Not Linearly Separable

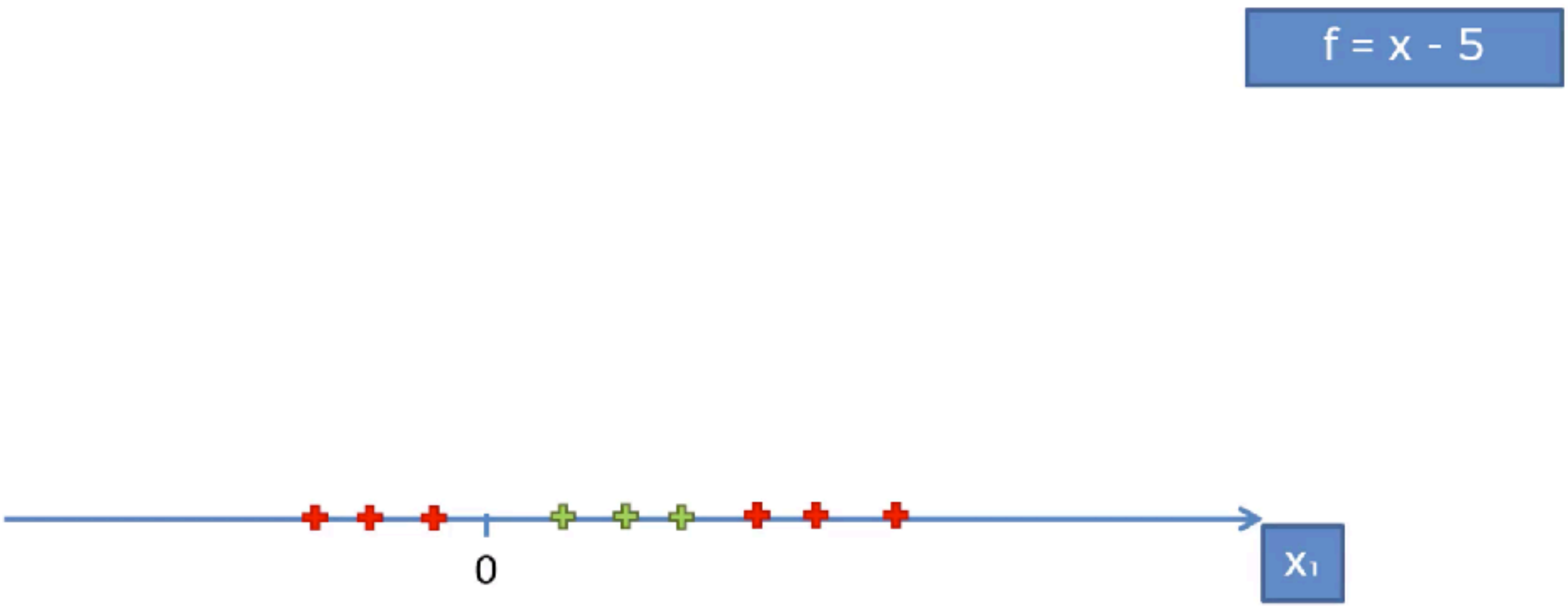


A Higher-Dimensional Space

Mapping to a Higher Dimension



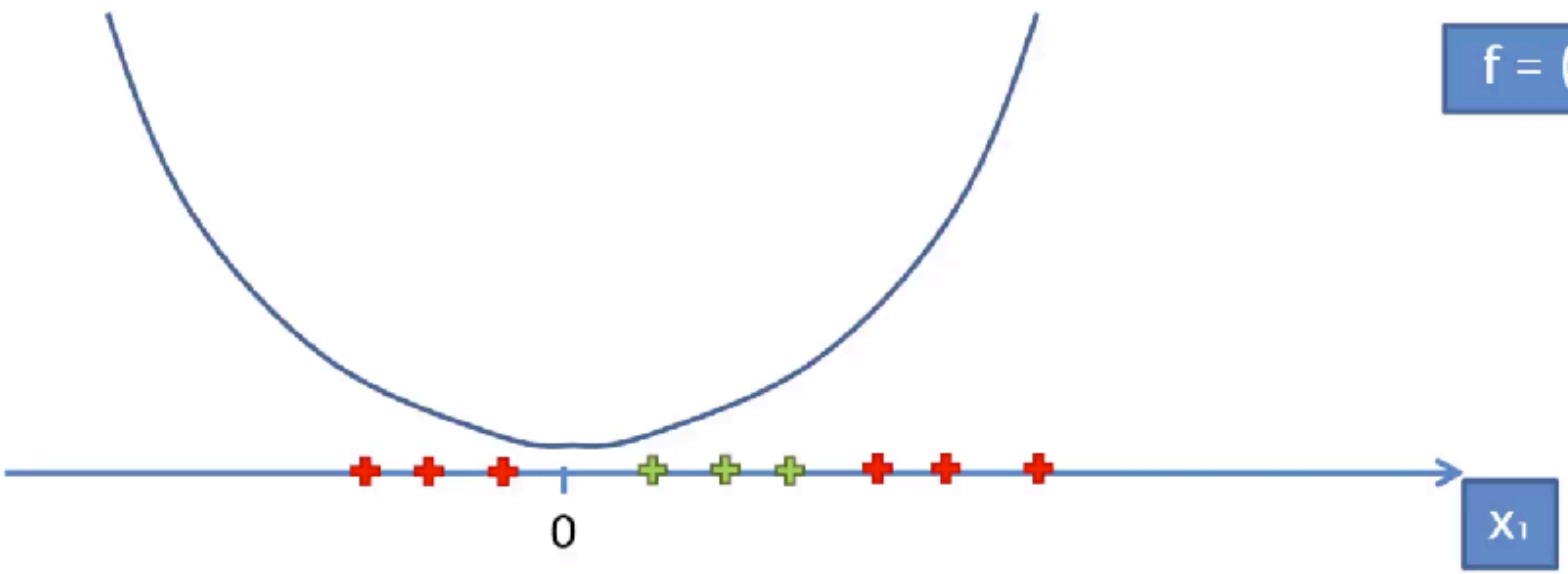
Mapping to a Higher Dimension



Mapping to a Higher Dimension

f = x - 5

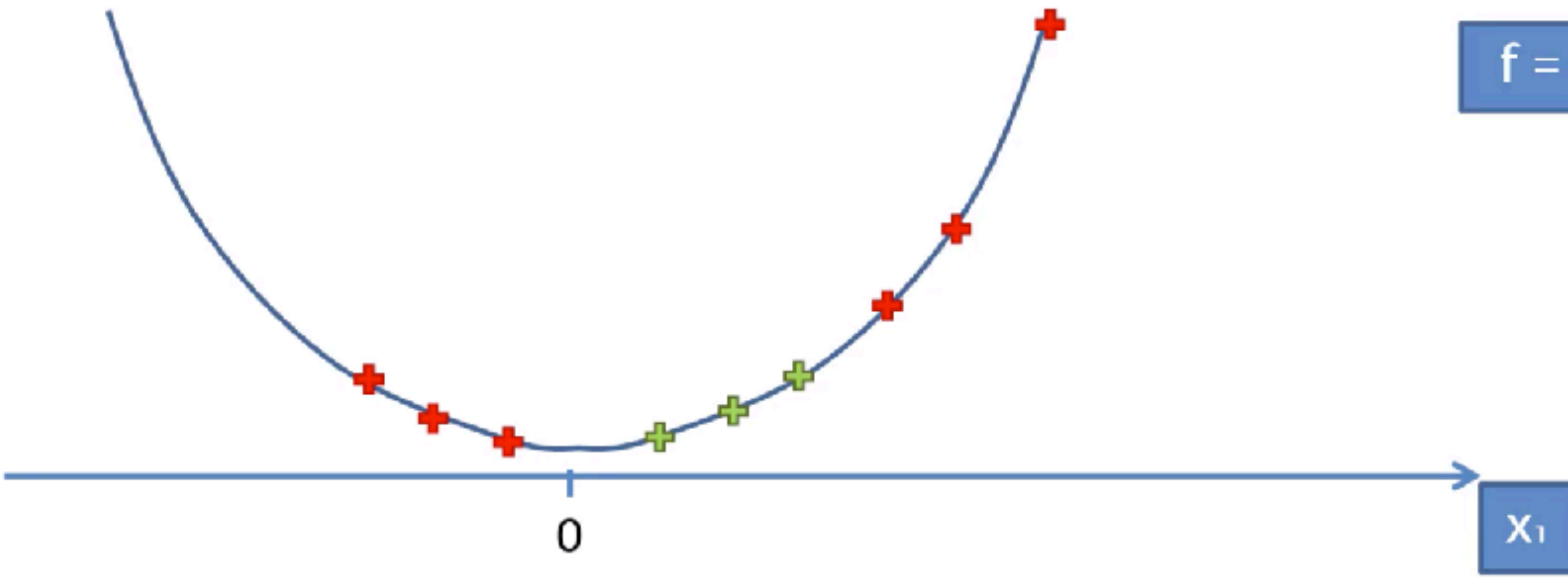
f = (x – 5)^2



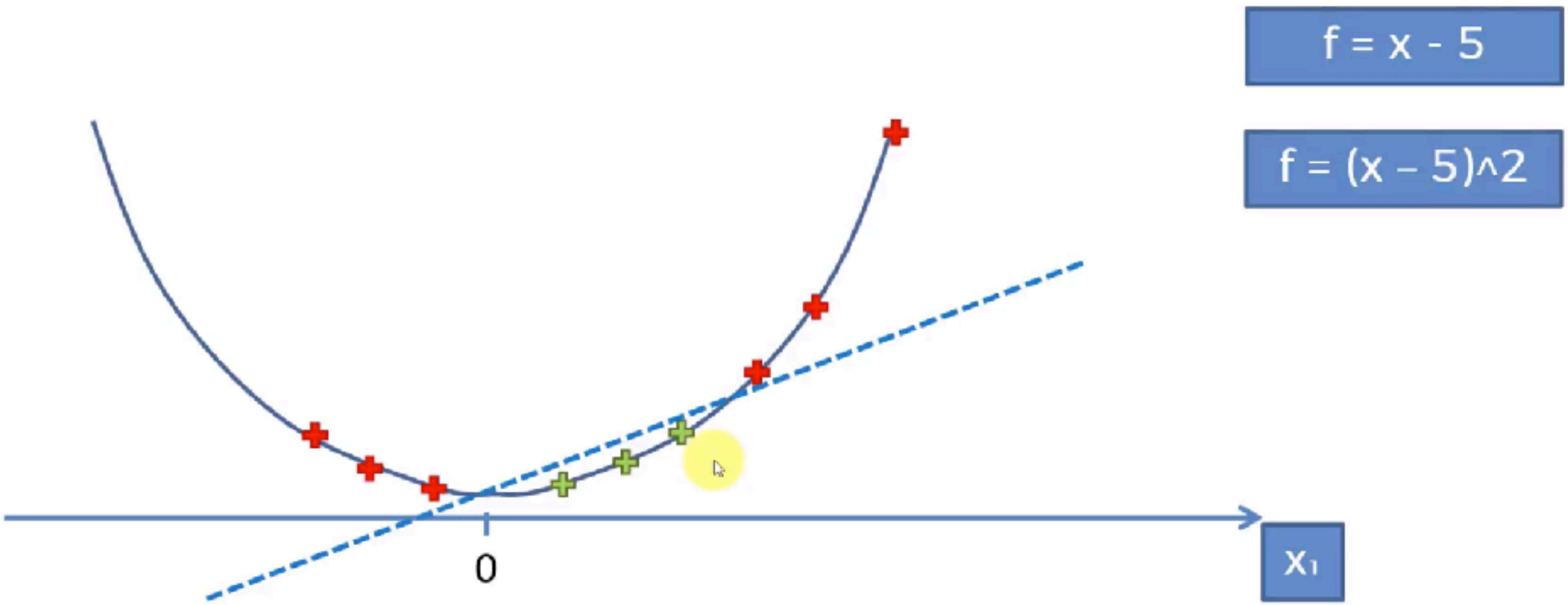
Mapping to a Higher Dimension

$f = x - 5$

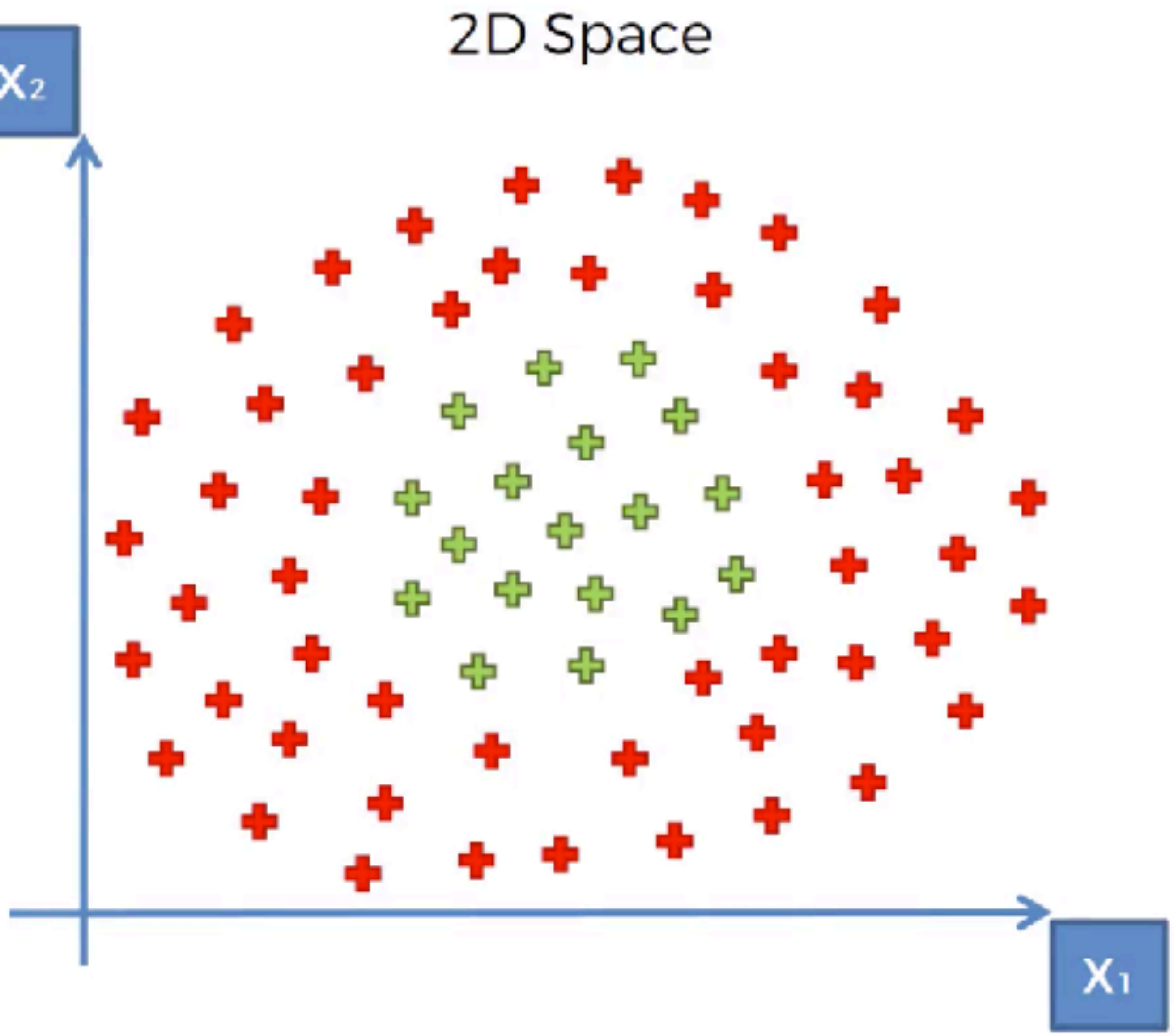
$f = (x - 5)^2$



Mapping to a Higher Dimension

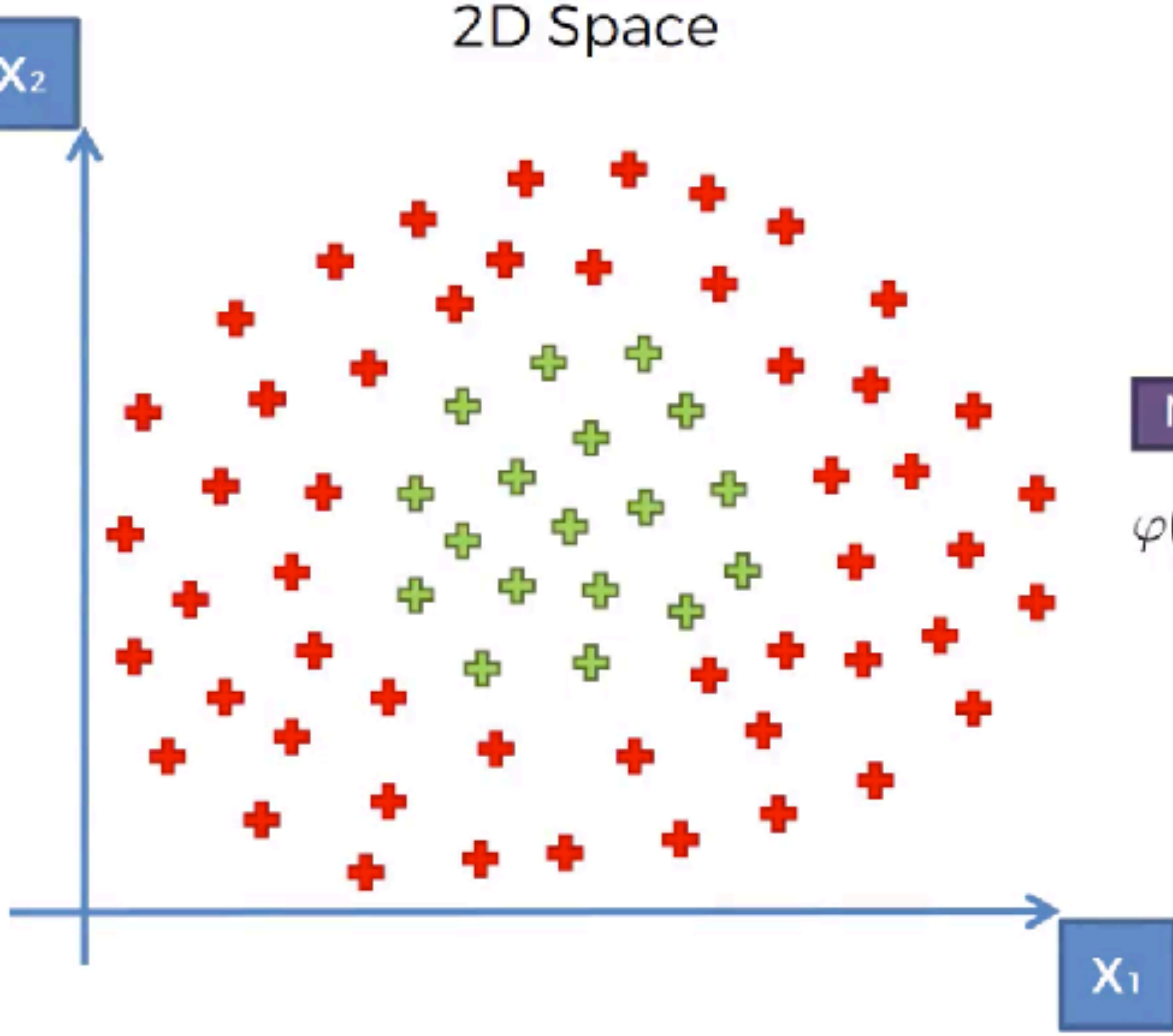


Mapping to a Higher Dimension



Mapping to a Higher Dimension

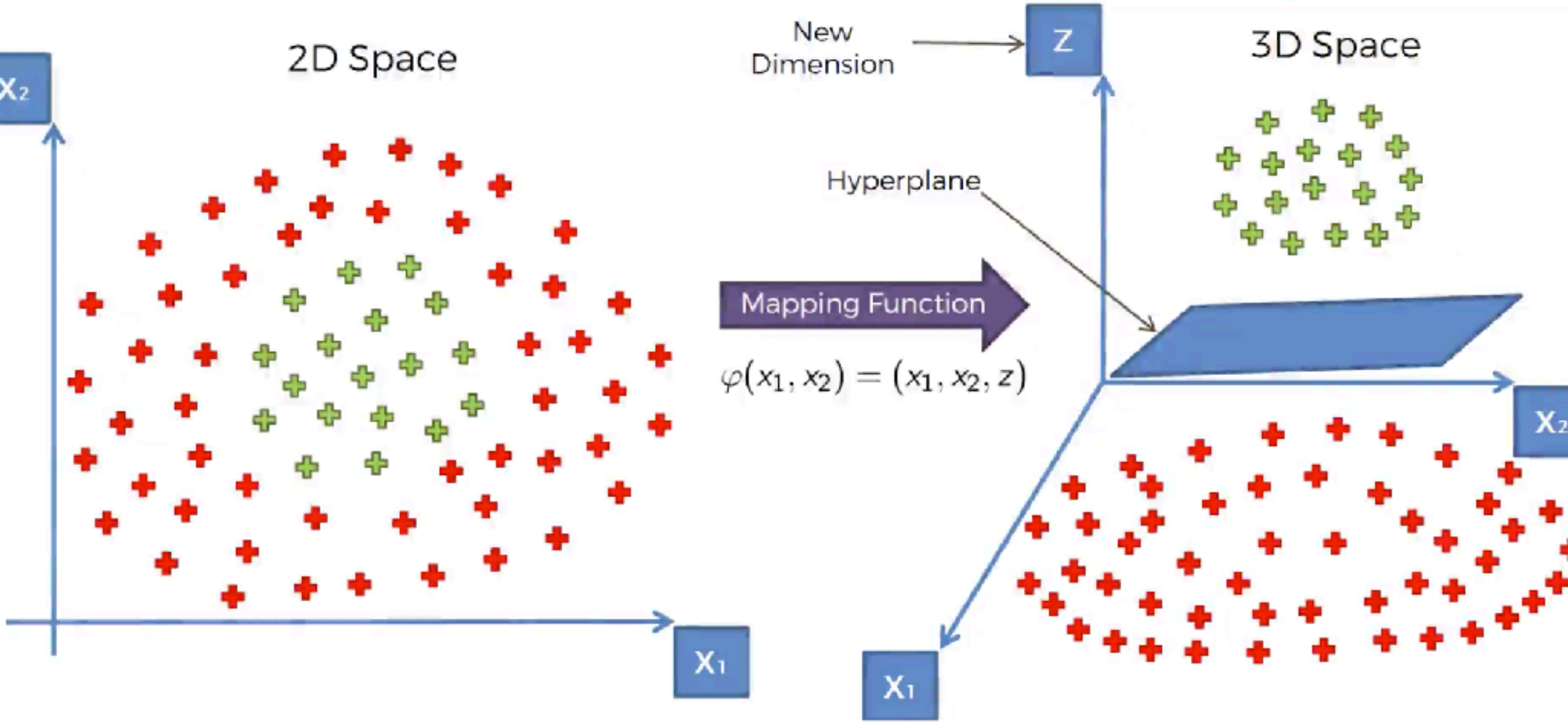
2D Space



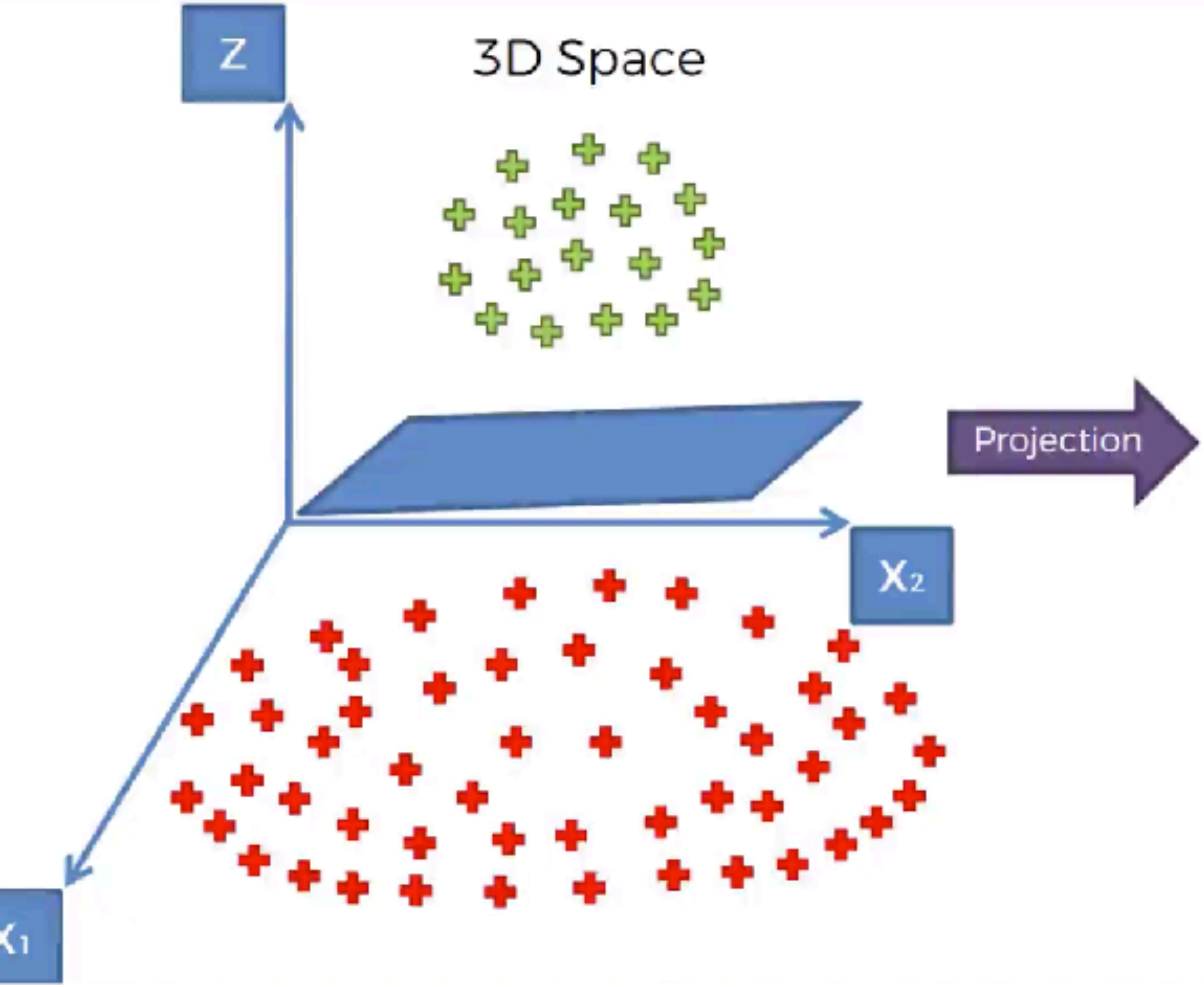
Mapping Function

$$\varphi(x_1, x_2) = (x_1, x_2, z)$$

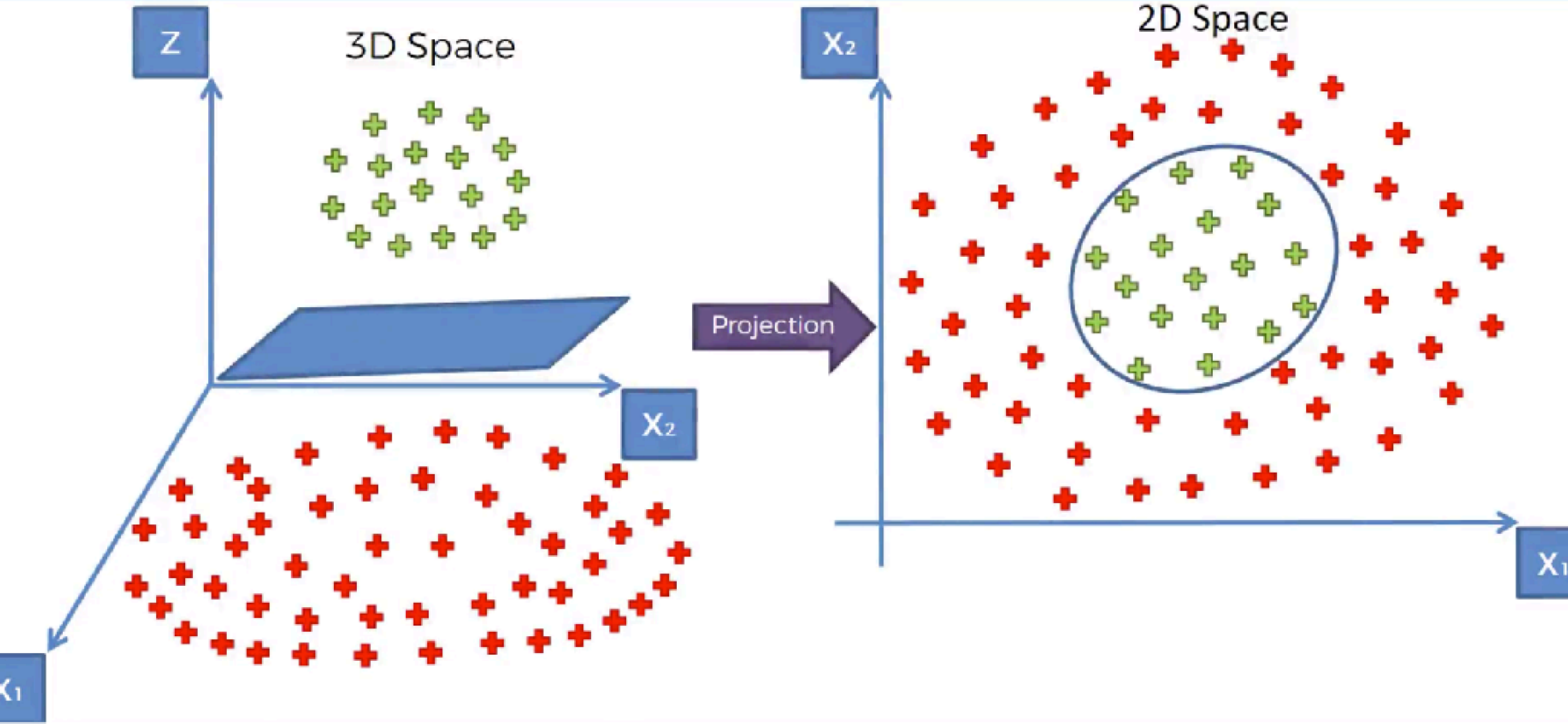
Mapping to a Higher Dimension



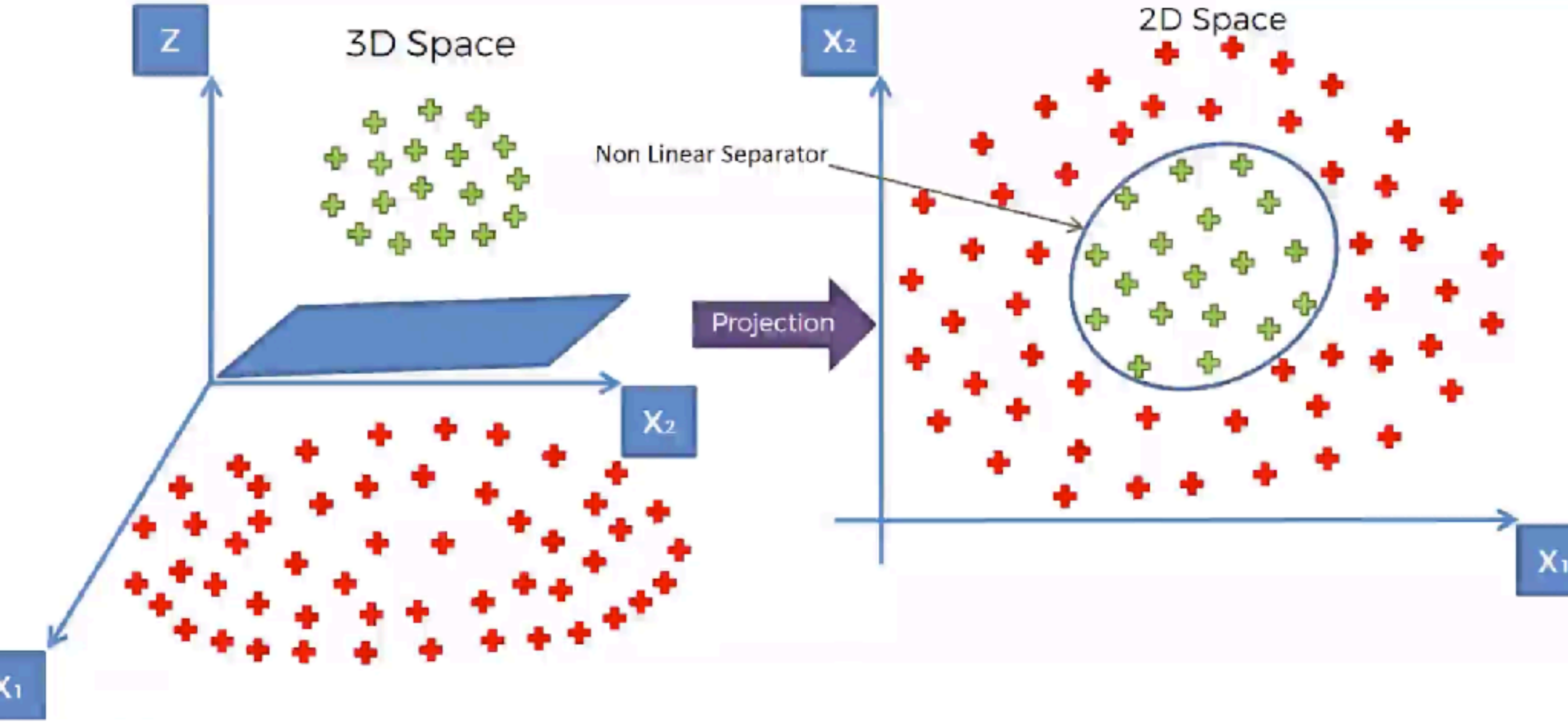
Projecting back to 2D Space



Projecting back to 2D Space



Projecting back to 2D Space



But there is a catch...

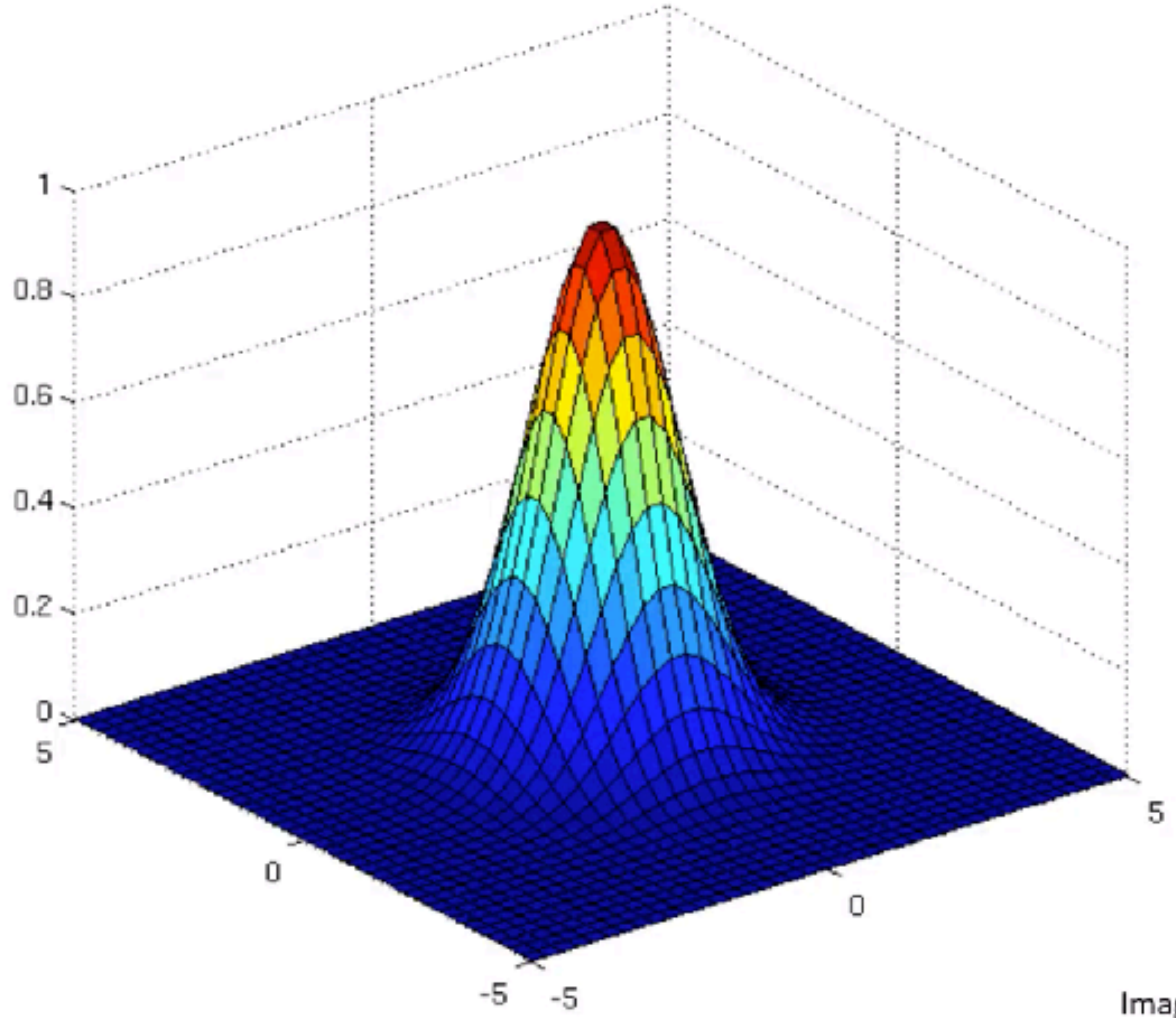
Mapping to a Higher Dimensional Space
can be highly compute-intensive

The Kernel Trick

The Gaussian RBF Kernel

$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\left\|\vec{x}-\vec{l}^i\right\|^2}{2\sigma^2}}$$

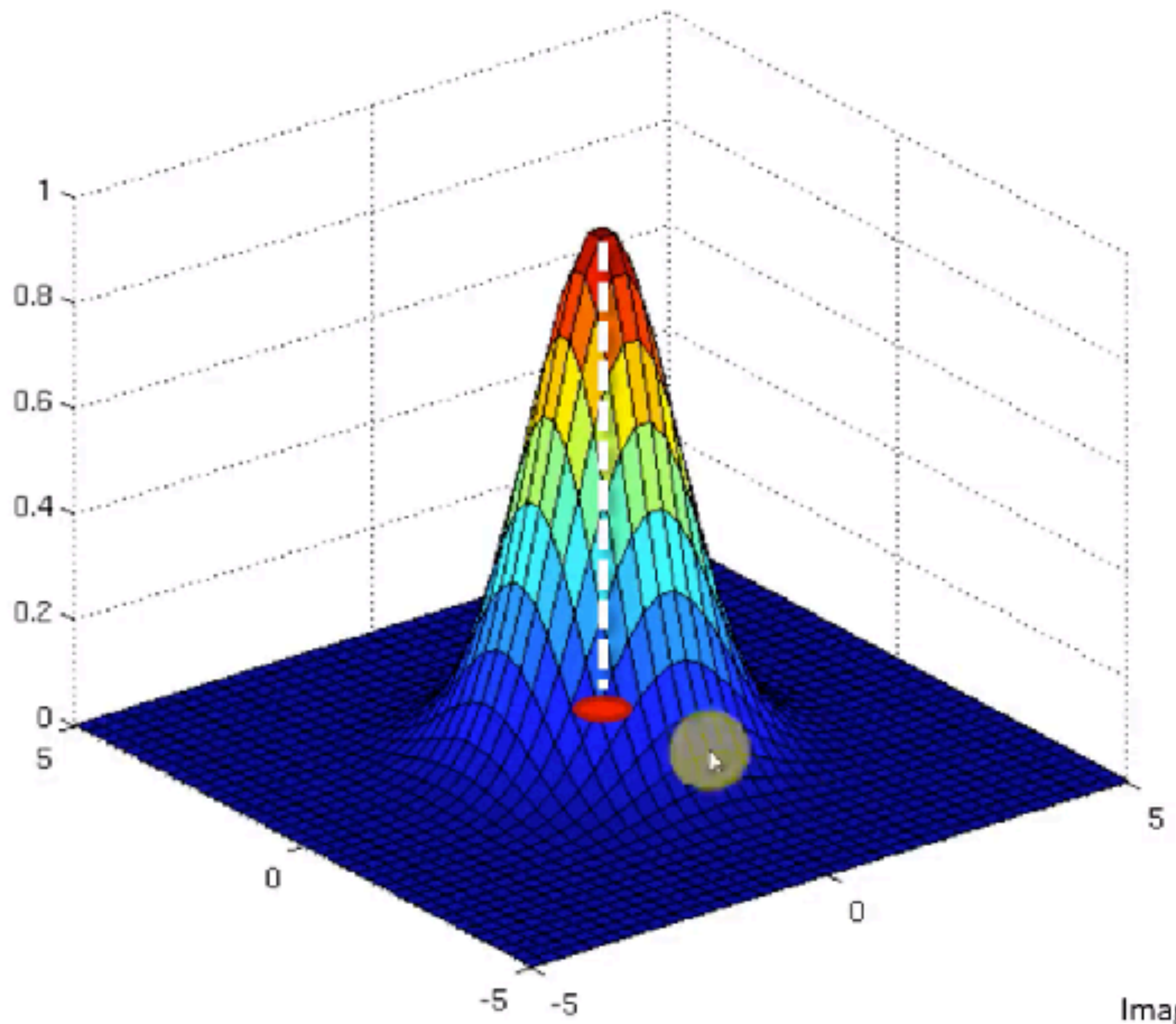
The Gaussian RBF Kernel



$$K\left(\vec{x}, \vec{l}^{i}\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^{i}\right\|^{2}}{2\sigma^{2}}}$$

Image source: http://www.cs.toronto.edu/~duvenaud/cookbook/index.htm

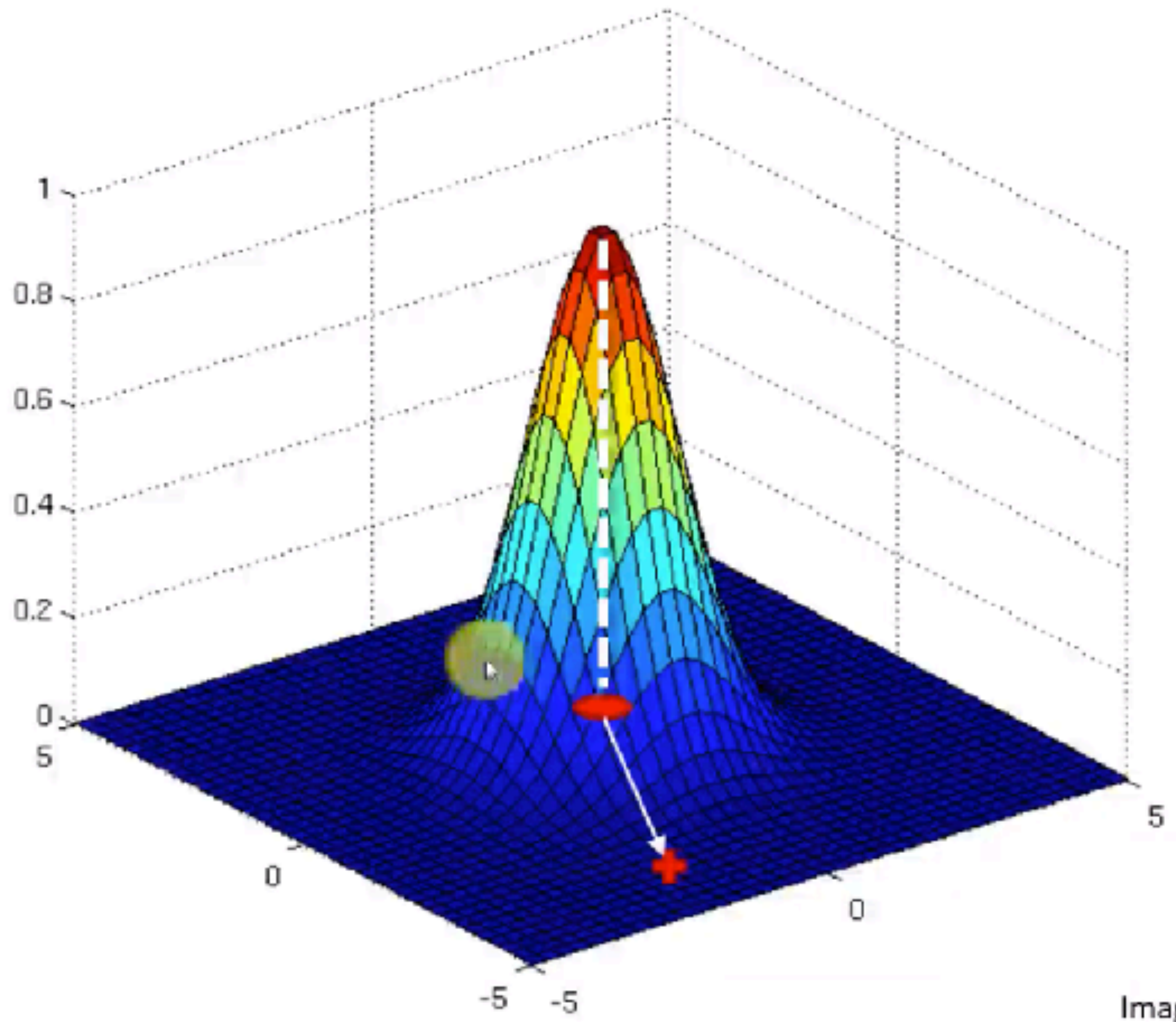
The Gaussian RBF Kernel



$$K\left(\vec{x}, \vec{l}^i\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^i\right\|^2}{2\sigma^2}}$$

Image source: http://www.cs.toronto.edu/~duvenaud/cookbook/index.htm

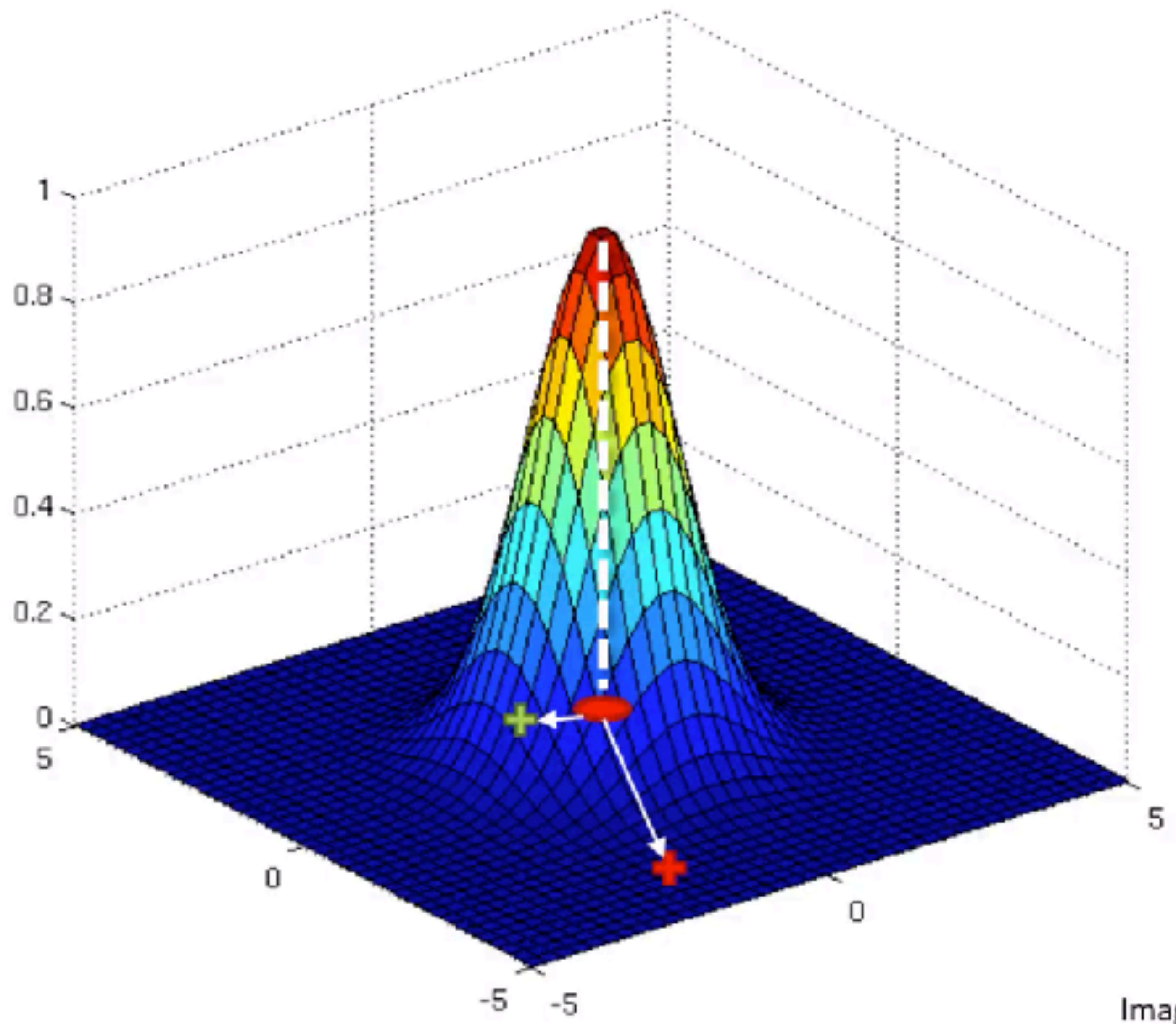
The Gaussian RBF Kernel



$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\|\vec{x}-\vec{l}^i\|^2}{2\sigma^2}}$$

Image source: http://www.cs.toronto.edu/~duvenaud/cookbook/index.htm

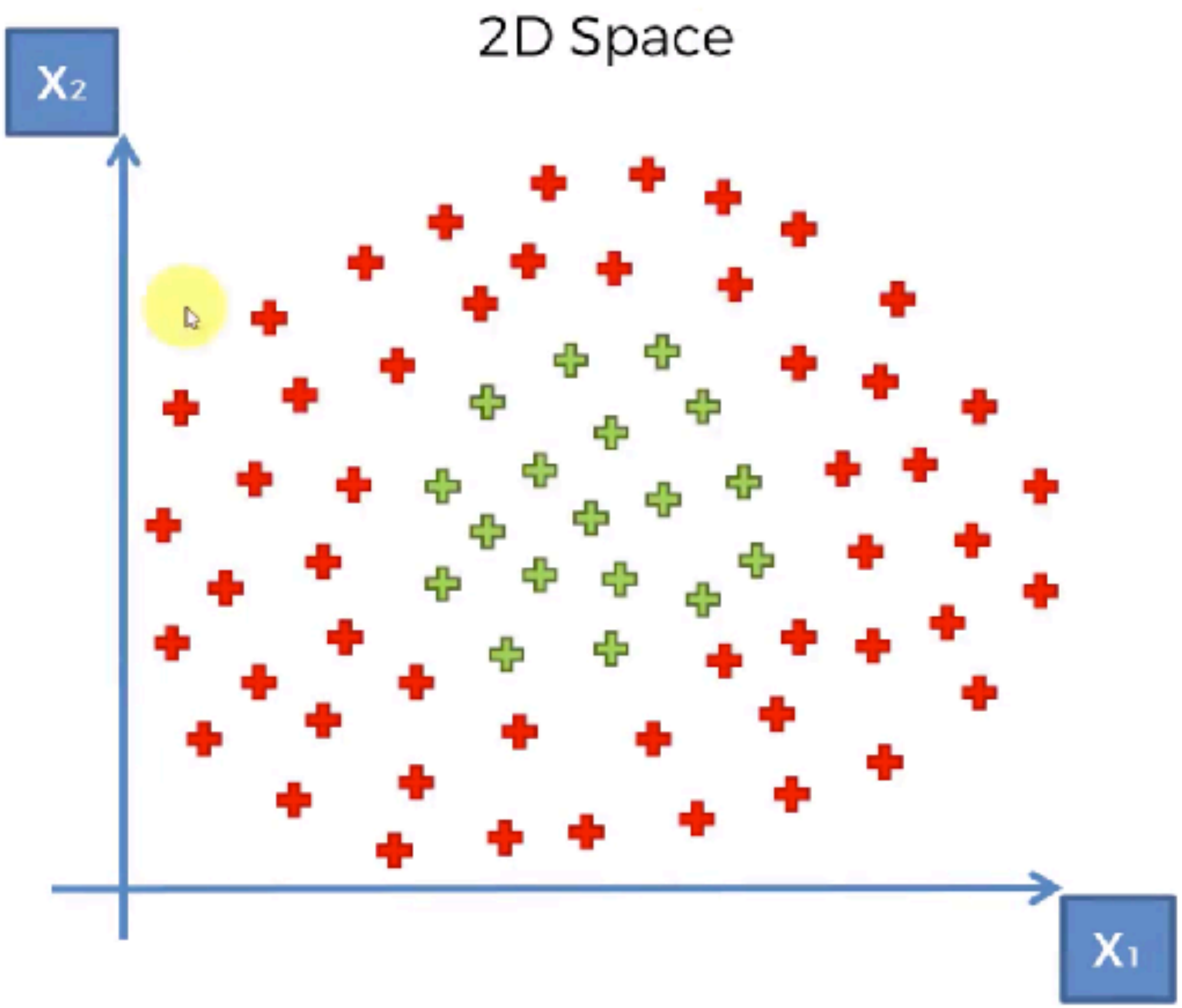
The Gaussian RBF Kernel

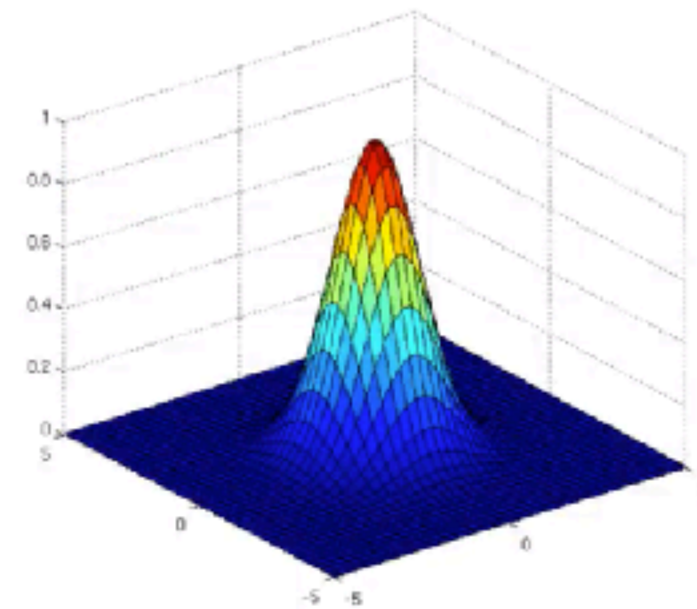


$$K(\vec{x},\vec{l}^{i}) = e^{-\frac{\left\|\vec{x}-\vec{l}^{i}\right\|^{2}}{2\sigma^{2}}}$$

Image source: http://www.cs.toronto.edu/~duvenaud/cookbook/index.htm

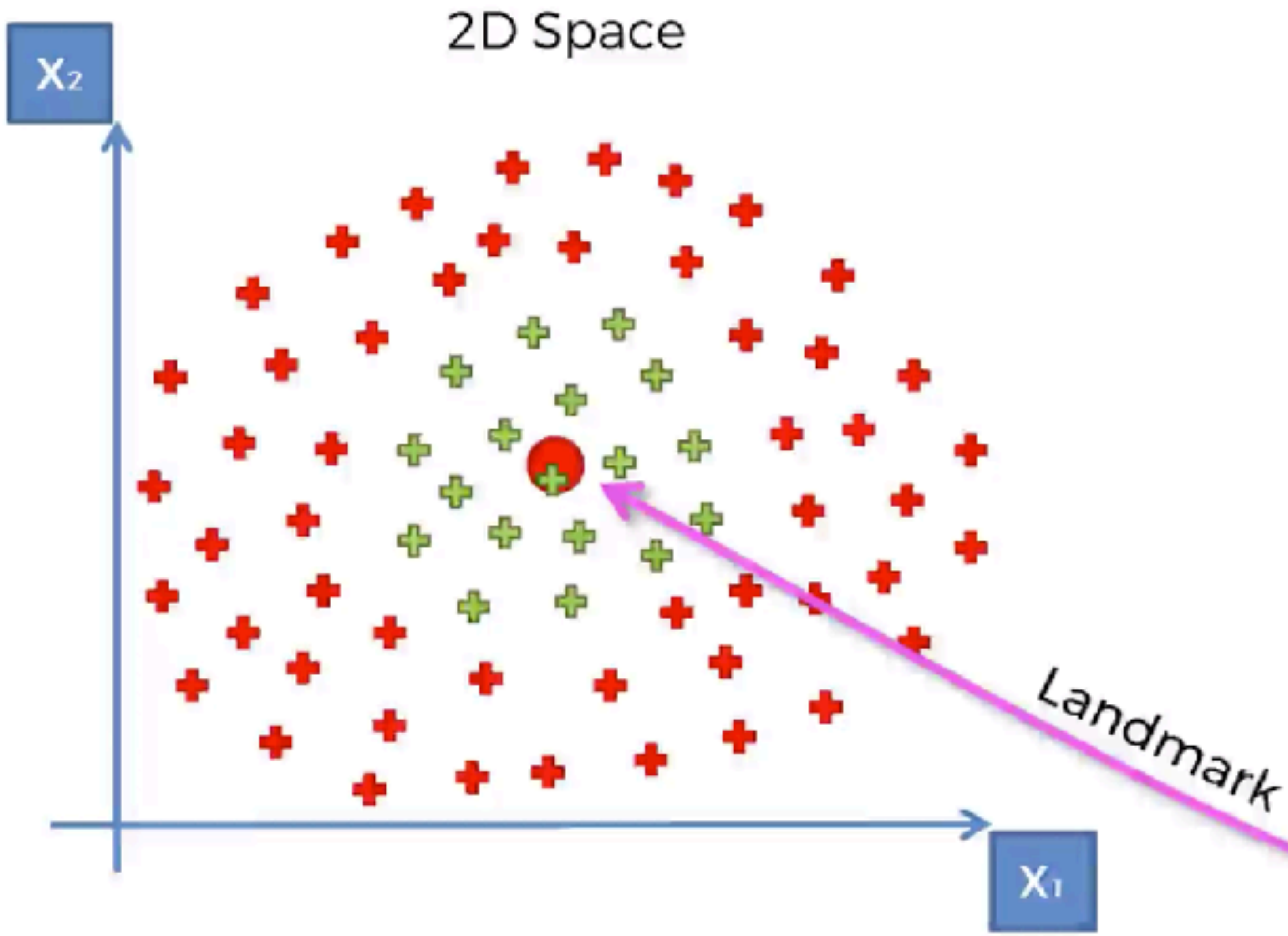
The Gaussian RBF Kernel

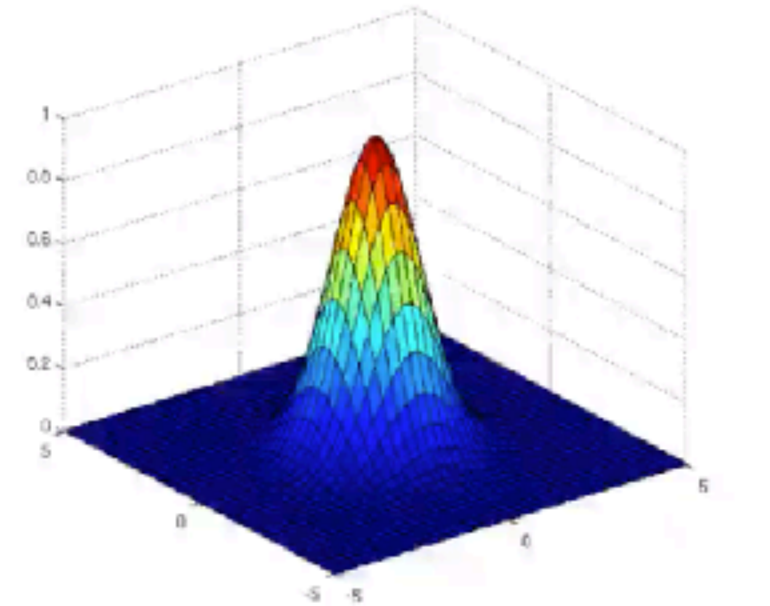




$$K(\vec{x}, \vec{l}^i) = e^{-\frac{\|\vec{x}-\vec{l}^i\|^2}{2\sigma^2}}$$

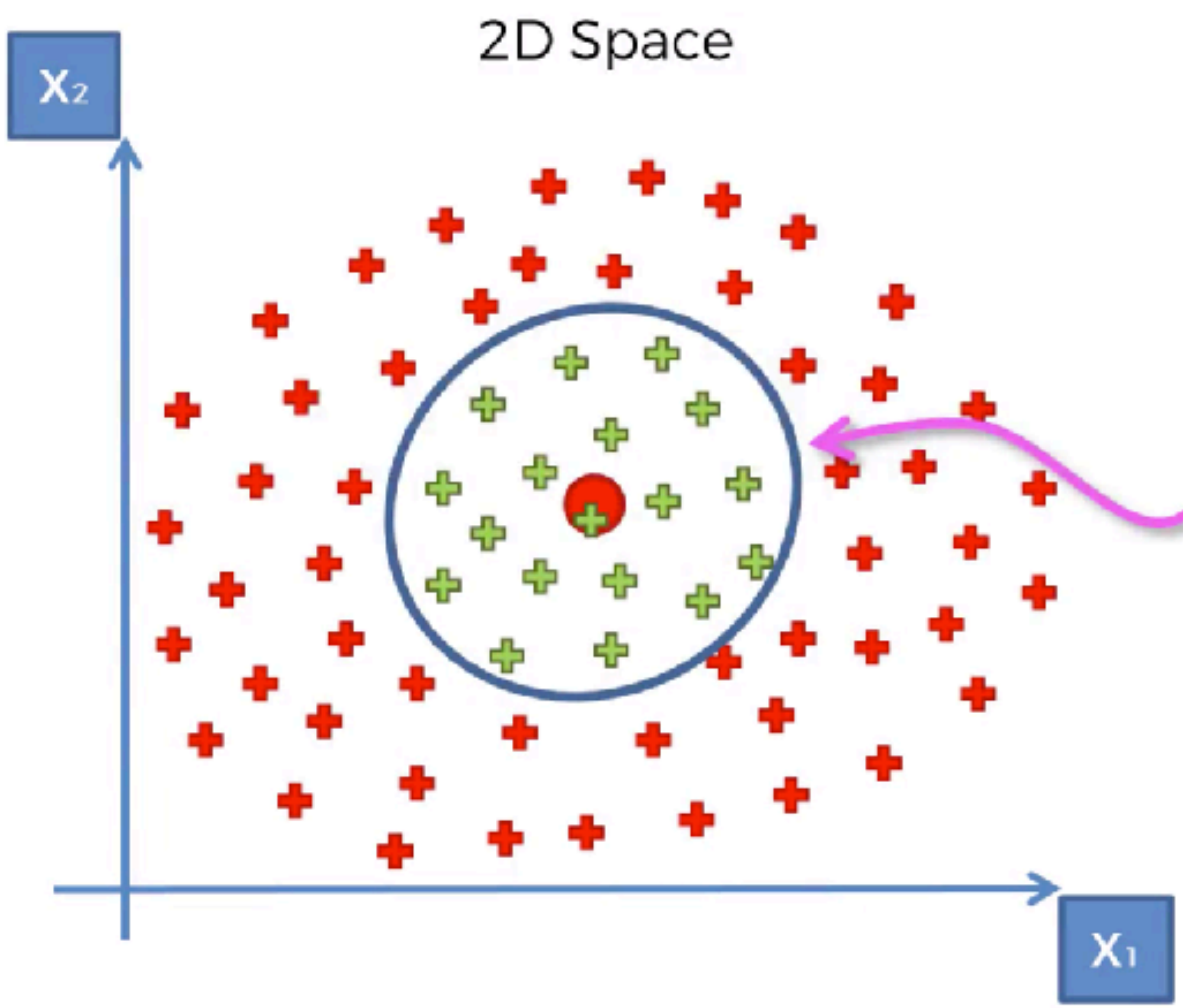
The Gaussian RBF Kernel

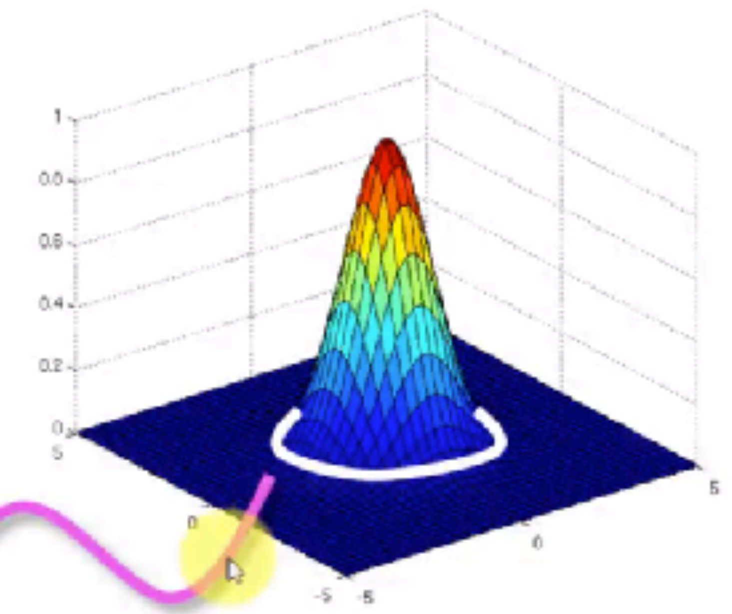




$$K\left(\vec{x}, \vec{l}^{i}\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^{i}\right\|^{2}}{2\sigma^{2}}}$$

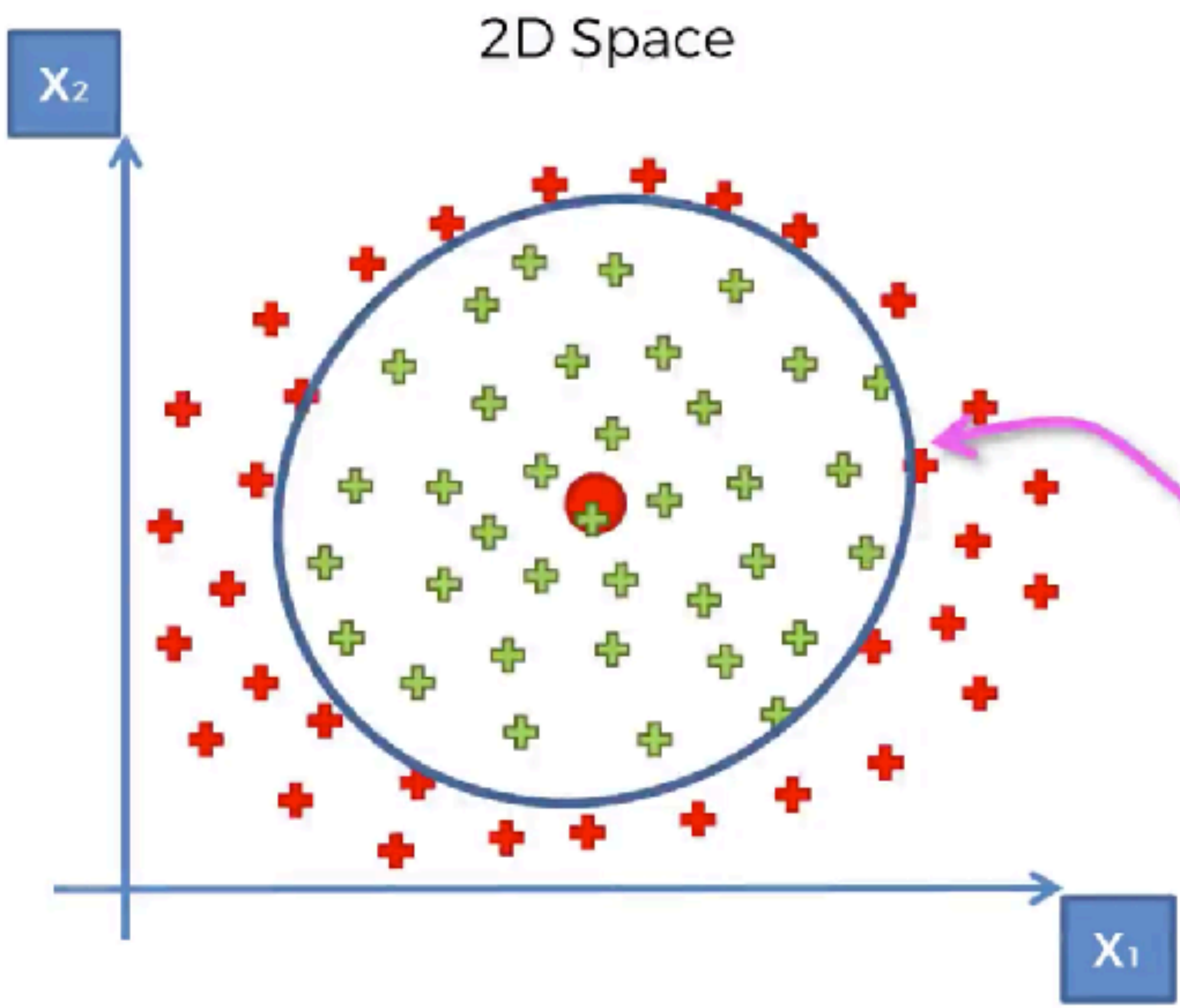
The Gaussian RBF Kernel



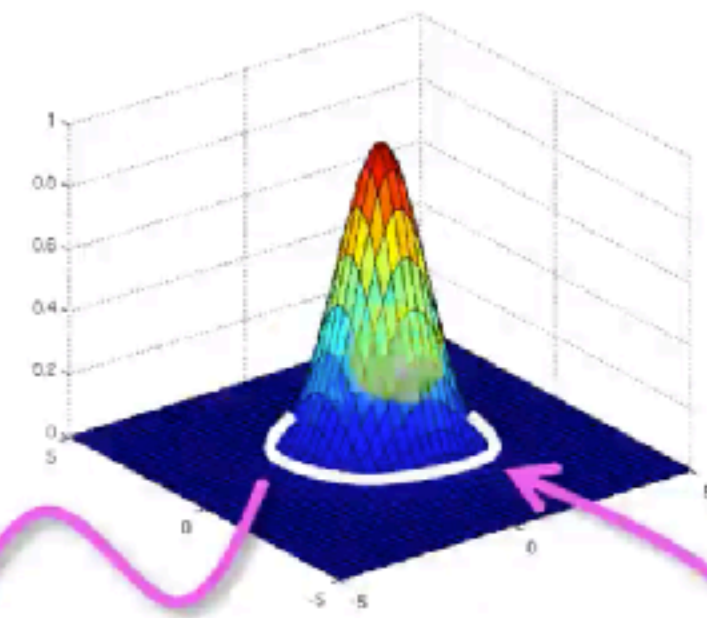


$$K\left(\vec{x}, \vec{l}^{i}\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^{i}\right\|^{2}}{2\sigma^{2}}}$$

The Gaussian RBF Kernel

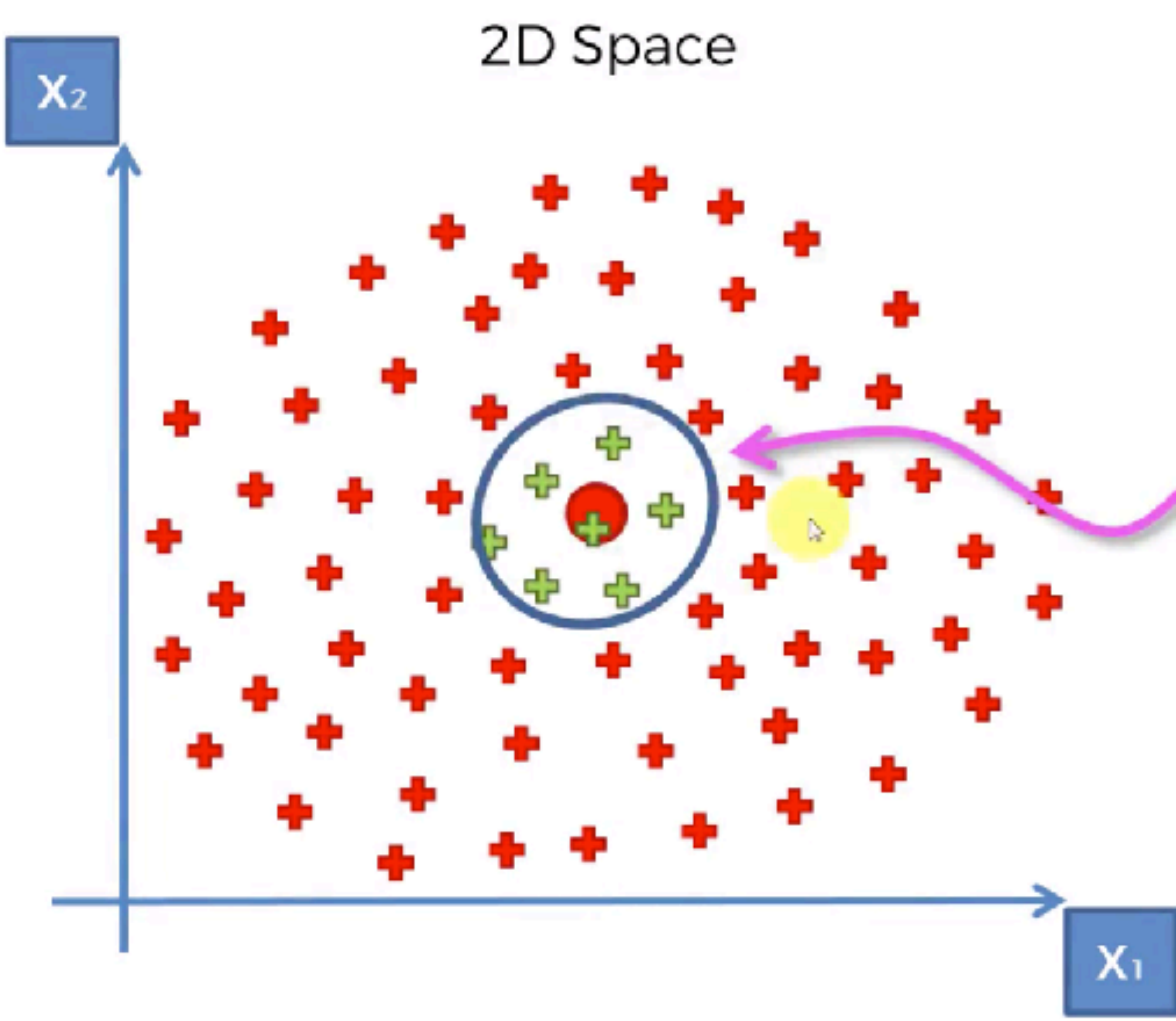


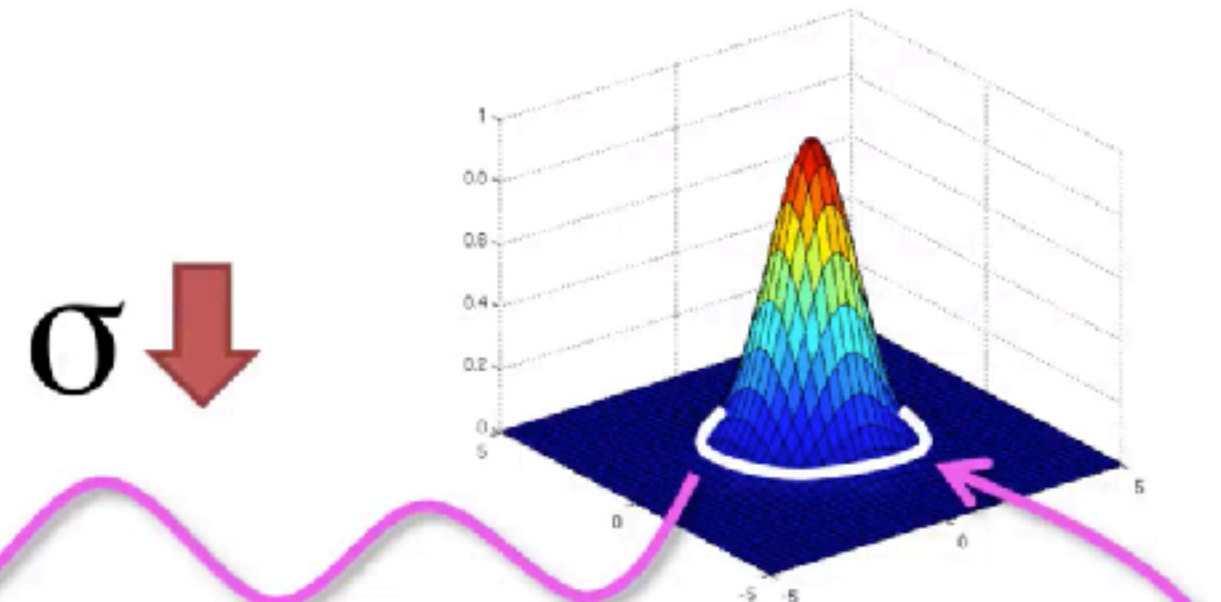
σ ⬆



$$K(\vec{x},\vec{l}^i) = e^{-\frac{\|\vec{x}-\vec{l}^i\|^2}{2\sigma^2}}$$

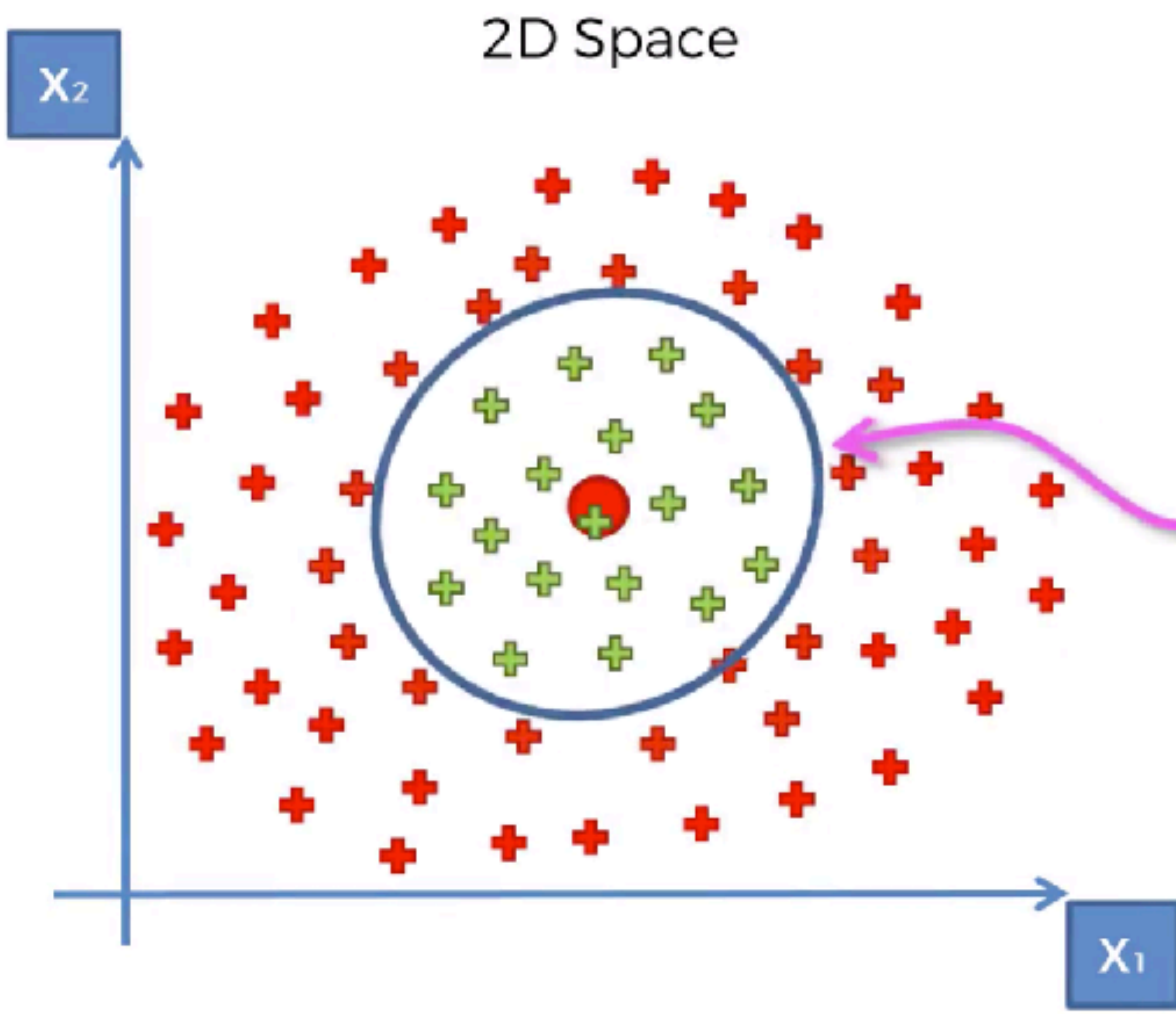
The Gaussian RBF Kernel

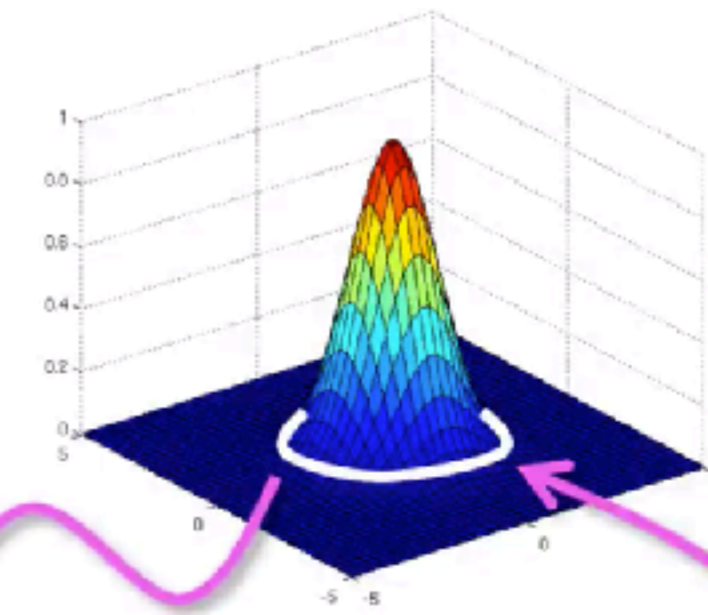




$$K(\vec{x},\vec{l}^i) = e^{-\frac{\|\vec{x}-\vec{l}^i\|^2}{2\sigma^2}}$$

The Gaussian RBF Kernel



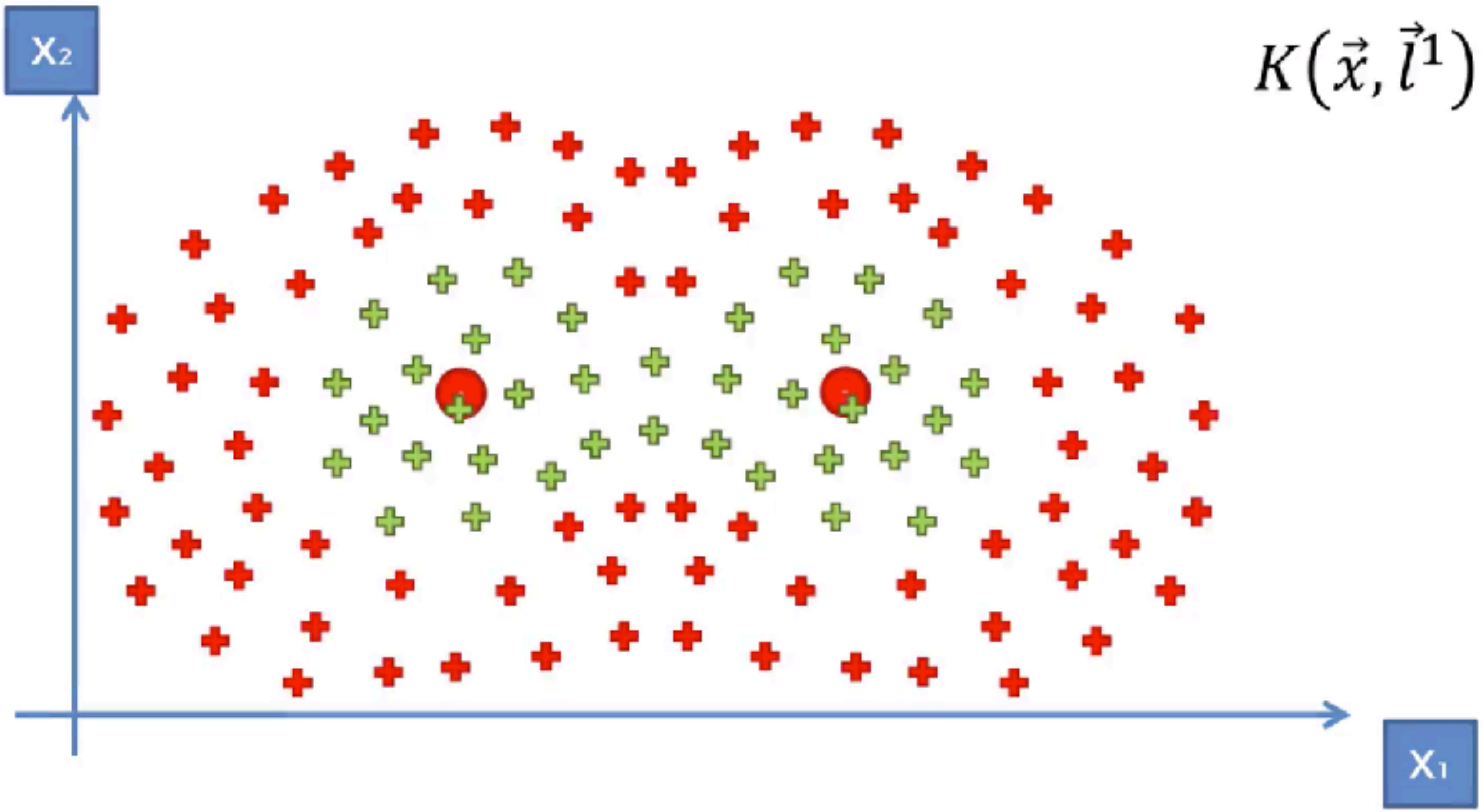


$$K\left(\vec{x}, \vec{l}^i\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^i\right\|^2}{2\sigma^2}}$$

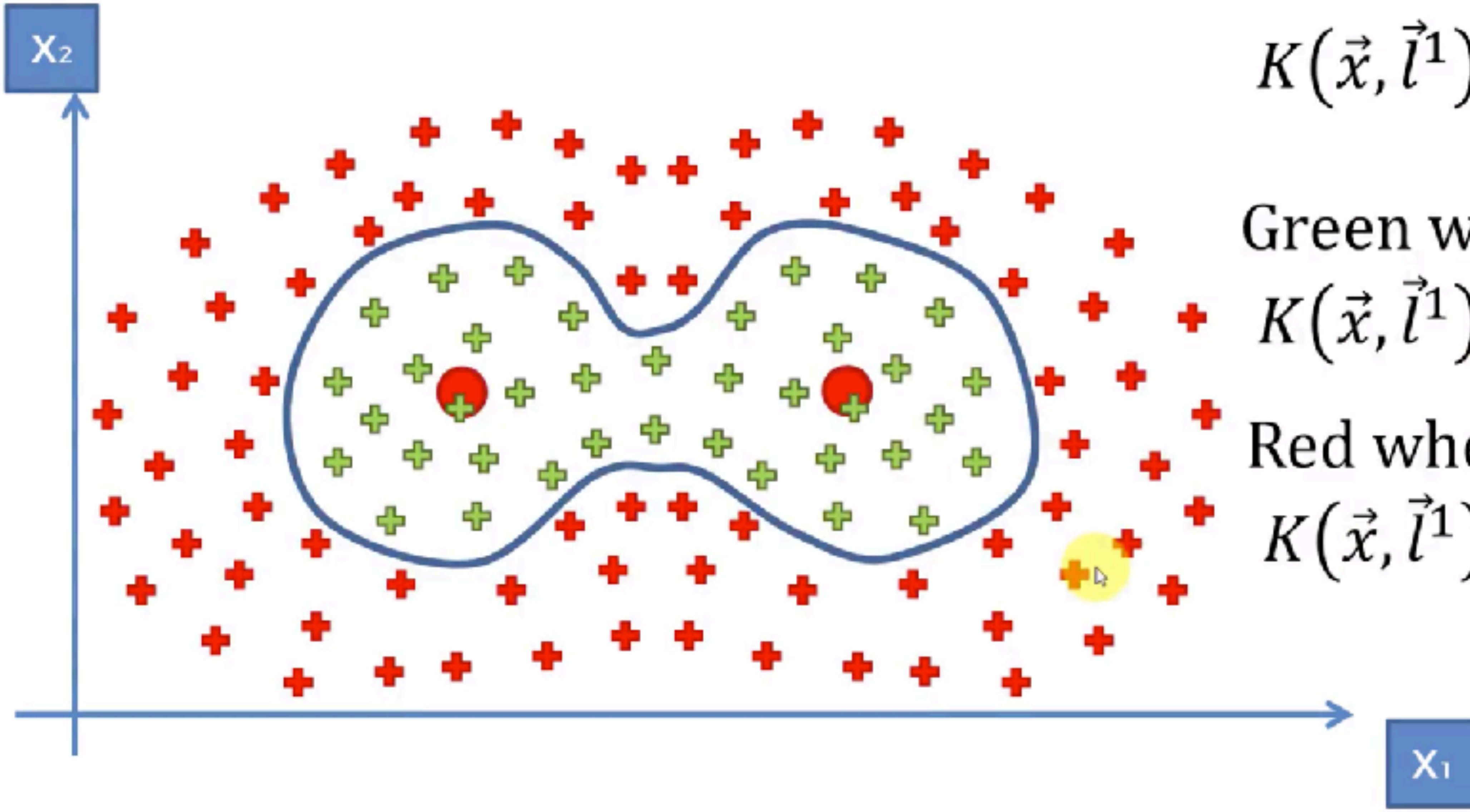
The Gaussian RBF Kernel

$$K\left(\vec{x},\vec{l}^{1}\right)+K\left(\vec{x},\vec{l}^{2}\right)$$

(Simplified Formula)



The Gaussian RBF Kernel



$$K(\vec{x}, \vec{l}^1) + K(\vec{x}, \vec{l}^2)$$

(Simplified Formula)

Green when:

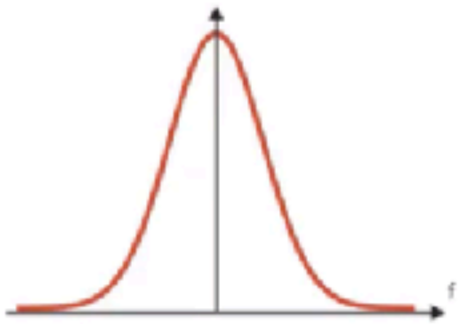
$$K(\vec{x}, \vec{l}^1) + K(\vec{x}, \vec{l}^2) > 0$$

Red when:

$$K(\vec{x}, \vec{l}^1) + K(\vec{x}, \vec{l}^2) = 0$$

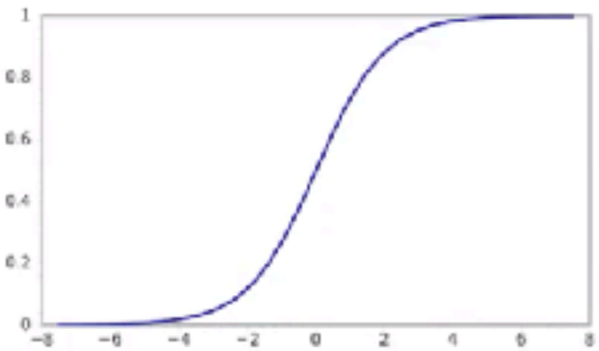
Types of Kernel Functions

Types of Kernel Functions



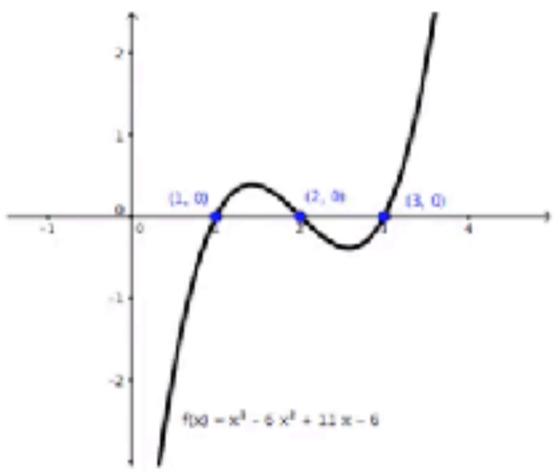
Gaussian RBF Kernel

$$K\left(\vec{x},\vec{l}^{i}\right) = e^{-\frac{\left\|\vec{x}-\vec{l}^{i}\right\|^{2}}{2\sigma^{2}}}$$



Sigmoid Kernel

$$K(X,Y) = \tanh(\gamma \cdot X^{T}Y + r)$$



Polynomial Kernel

$$K(X,Y) = (\gamma \cdot X^{T}Y + r)^{d}, \gamma >$$