Memo on Financial risk modelling and portfolio optimization with R

Chapter 6: Suitable distributions for returns

Fitting stock returns to the GHD

In this subsection the daily returns of Hewlett Packard (HWP) stock are fitted to the GHD and its special cases, the HYP and NIG. The sample runs from 31 December 1990 to 2 January 2001 and consists of 2529 observations.

Return calculation

```
library(ghyp)
library(timeSeries)
library(fBasics)
data(DowJones30)
y <- timeSeries(DowJones30[, "HWP"], charvec = as.character(DowJones30[, 1]))
yret <- na.omit(diff(log(y)) * 100)
str(yret)
```

```
## Time Series:
##  Name:               object
## Data Matrix:
##  Dimension:          2528 1
##  Column Names:       TS.1
##  Row Names:          1991-01-02  ...  2001-01-02
## Positions:
##  Start:              1991-01-02
##  End:                2001-01-02
## With:
##  Format:             %Y-%m-%d
##  FinCenter:          GMT
##  Units:              TS.1
##  Title:              Time Series Object
##  Documentation:      Sun Apr 22 12:40:16 2018
```

```
library(moments)
skewness(yret)
```

```
##      TS.1
## -0.258369
```

```
kurtosis(yret)
```

```
##     TS.1
## 7.817651
```

Fitting

For comparison of the fitted distributions, the empirical distribution (EDF) is first retrieved from the data with the function ef(). Then the returns are fitted to GHD, HYP, and NIG distributions. In each case, possible asymmetries in the data are allowed (i.e., non-zero skewness).

```
ef <- density(yret)
ghdfit <- fit.ghypuv(yret, symmetric = FALSE, control = list(maxit = 1000))
hypfit <- fit.hypuv(yret, symmetric = FALSE, control = list(maxit = 1000))
nigfit <- fit.NIGuv(yret, symmetric = FALSE, control = list(maxit = 1000))
```

```
ghdfit
```

```
## Asymmetric Generalized Hyperbolic Distribution:
##
## Parameters:
##      lambda   alpha.bar          mu       sigma       gamma
## -2.27034466  0.05096678  0.09308063  2.58440051 -0.01296031
##
## log-likelihood:
## -5837.037
##
##
## Call:
## fit.ghypuv(data = yret, symmetric = FALSE, control = list(maxit = 1000))
```

```
hypfit
```

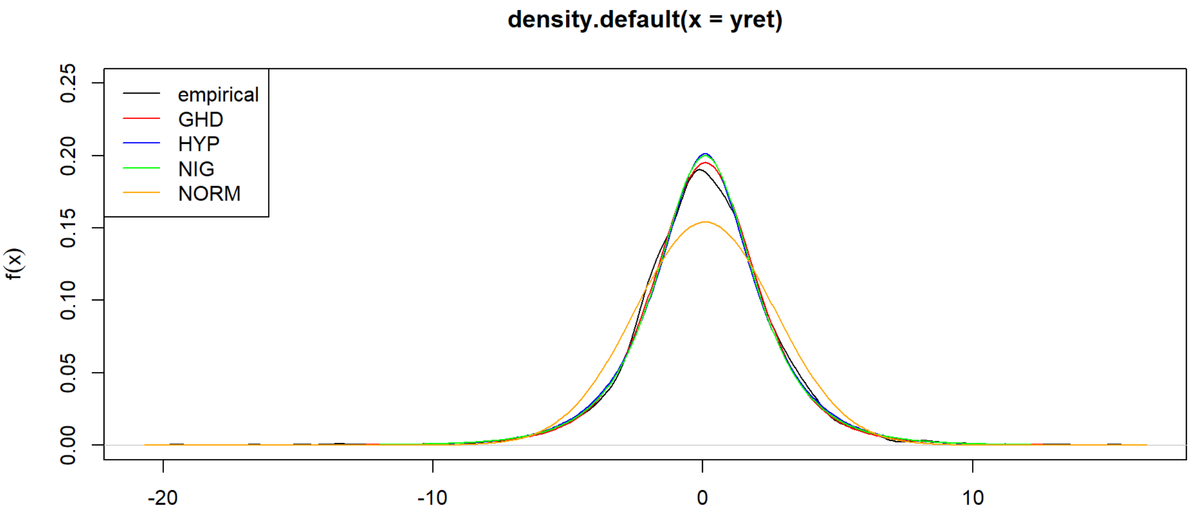
```
## Asymmetric Hyperbolic Distribution:
##
## Parameters:
##  alpha.bar         mu      sigma      gamma
## 0.90308162 0.06937404 2.52807083 0.01117653
##
## log-likelihood:
## -5848.207
##
##
## Call:
## fit.hypuv(data = yret, symmetric = FALSE, control = list(maxit = 1000))
```

```
nigfit
```

```
## Asymmetric Normal Inverse Gaussian Distribution:
##
## Parameters:
##    alpha.bar          mu       sigma       gamma
## 1.109829745 0.078023919 2.552470366 0.002564615
##
## log-likelihood:
## -5841.948
##
##
## Call:
## fit.NIGuv(data = yret, symmetric = FALSE, control = list(maxit = 1000))
```

Densities

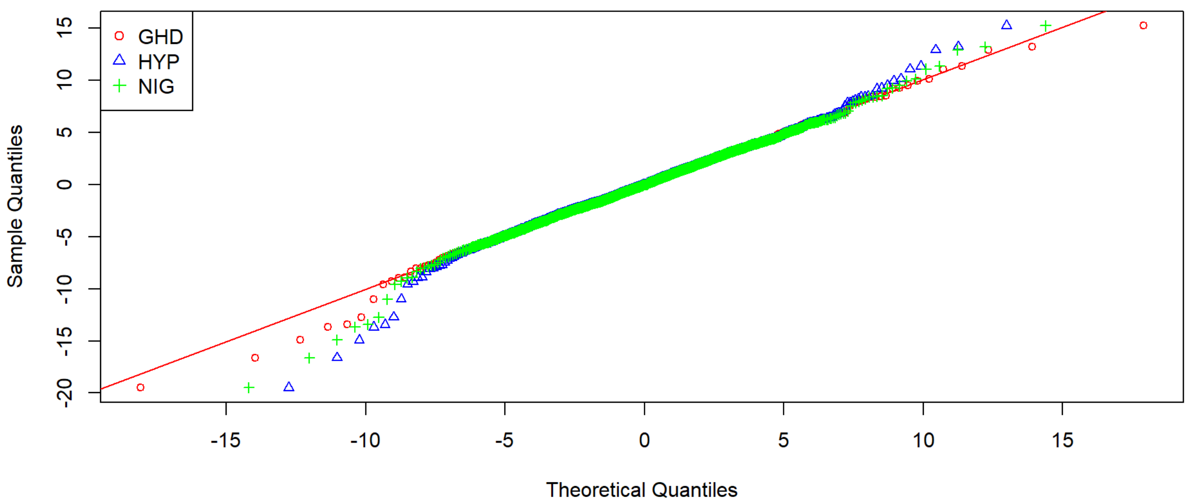
```
ghddens <- dghyp(ef$x, ghdfit)
hypdens <- dghyp(ef$x, hypfit)
nigdens <- dghyp(ef$x, nigfit)
nordens <- dnorm(ef$x, mean = mean(yret), sd = sd(c(yret[, 1])))
col.def <- c("black", "red", "blue", "green", "orange")

plot(ef, xlab = "", ylab = expression(f(x)), ylim = c(0, 0.25))
lines(ef$x, ghddens, col = "red")
lines(ef$x, hypdens, col = "blue")
lines(ef$x, nigdens, col = "green")
lines(ef$x, nordens, col = "orange")
legend("topleft",
       legend = c("empirical", "GHD", "HYP", "NIG", "NORM"), col = col.def, lty = 1
)
```



The rather poor description of the empirical return distribution for the Gaussian case is immediately evident from this plot. The normal distribution falls short of capturing the excess kurtosis of 4.811. Matters are different for the class of generalized hyperbolic distributions. In these instances the empirical distribution function is tracked rather well. The fitted HYP and NIG models almost coincide, and from this plot these two distributions cannot be discerned. The fitted GHD seems to mirror the returns slightly better. In particular, the values of the density are closer to their empirical counterparts around the median of the EDF. Ceteris paribus, this implies higher probability masses in the tails of the distribution compared to the λ-restricted HYP and NIG distributions.

QQ-Plots

```
qqghyp(ghdfit, line = TRUE, ghyp.col = "red", plot.legend = FALSE,
       gaussian = FALSE, main = "", cex = 0.8)
qqghyp(hypfit, add = TRUE, ghyp.pch = 2, ghyp.col = "blue",
       gaussian = FALSE, line = FALSE, cex = 0.8)
qqghyp(nigfit, add = TRUE, ghyp.pch = 3, ghyp.col = "green",
       gaussian = FALSE, line = FALSE, cex = 0.8)
legend("topleft", legend = c("GHD", "HYP", "NIG"), col = col.def[-c(1,5)], pch = 1:
3)
```



As a second means of graphically comparing the fitted distributions, QQ plots are produced in the ensuing code lines of Listing 6.1. For clarity the marks of the fitted normal distribution have been omitted from the plot. The reader is encouraged to adopt the plot accordingly. What has already been concluded from the density becomes even more evident when the QQ plot is examined. The daily returns can be tracked better with the GHD than with the HYP and NIG distributions, especially in the tails. Furthermore-this conclusionwas less clear from the density plot-the returns can be slightly better explained by the NIG than by the HYP distribution.

Diagnostics

```
AIC <- stepAIC.ghyp(yret, dist = c("ghyp", "hyp", "NIG"), symmetric = FALSE,
                    control = list(maxit = 1000))
LRghdnig <- lik.ratio.test(ghdfit, nigfit)
LRghdhyp <- lik.ratio.test(ghdfit, hypfit)
```

```
AIC
```

```
## $best.model
## Asymmetric Generalized Hyperbolic Distribution:
##
## Parameters:
##      lambda   alpha.bar          mu       sigma       gamma
## -2.27034466  0.05096678  0.09308063  2.58440051 -0.01296031
##
## log-likelihood:
## -5837.037
##
##
## Call:
## stepAIC.ghyp(data = yret, dist = c("ghyp", "hyp", "NIG"), symmetric = FALSE,
control = list(maxit = 1000))
##
##
## $all.models
## $all.models[[1]]
## Asymmetric Generalized Hyperbolic Distribution:
##
## Parameters:
##      lambda   alpha.bar          mu       sigma       gamma
## -2.27034466  0.05096678  0.09308063  2.58440051 -0.01296031
##
## log-likelihood:
## -5837.037
##
##
## Call:
## stepAIC.ghyp(data = yret, dist = c("ghyp", "hyp", "NIG"), symmetric = FALSE,
control = list(maxit = 1000))
##
##
## $all.models[[2]]
## Asymmetric Hyperbolic Distribution:
##
## Parameters:
##  alpha.bar         mu      sigma      gamma
## 0.90308162 0.06937404 2.52807083 0.01117653
##
## log-likelihood:
## -5848.207
##
##
## Call:
## stepAIC.ghyp(data = yret, dist = c("ghyp", "hyp", "NIG"), symmetric = FALSE,
control = list(maxit = 1000))
##
##
## $all.models[[3]]
## Asymmetric Normal Inverse Gaussian Distribution:
##
## Parameters:
##   alpha.bar          mu       sigma       gamma
## 1.109829745 0.078023919 2.552470366 0.002564615
##
## log-likelihood:
```

```
## -5841.948
##
##
## Call:
## stepAIC.ghyp(data = yret, dist = c("ghyp", "hyp", "NIG"), symmetric = FALSE,
control = list(maxit = 1000))
##
##
##
## $fit.table
##   model symmetric    lambda  alpha.bar         mu    sigma        gamma
## 1  ghyp     FALSE -2.270345 0.05096678 0.09308063 2.584401 -0.012960312
## 3   NIG     FALSE -0.500000 1.10982974 0.07802392 2.552470  0.002564615
## 2   hyp     FALSE  1.000000 0.90308162 0.06937404 2.528071  0.011176535
##        aic       llh converged n.iter
## 1 11684.07 -5837.037      TRUE    646
## 3 11691.90 -5841.948      TRUE    147
## 2 11704.41 -5848.207      TRUE    225
```

```
LRghdnig
```

```
## $statistic
##           L
## 0.007370886
##
## $p.value
## [1] 0.001725835
##
## $df
## [1] 1
##
## $H0
## [1] FALSE
```

```
LRghdhyp
```

```
## $statistic
##            L
## 1.409629e-05
##
## $p.value
## [1] 2.284946e-06
##
## $df
## [1] 1
##
## $H0
## [1] FALSE
```

The latter is of most interest because it not only provides information about the AICs and the values of the log-likelihood (LLH), but also returns the estimates of the distribution parameters, whether a symmetric distribution has been fitted or not, whether the optimizer achieved convergence, and the number of iterations required. The conclusions drawn from the graphical inspection of the results are mirrored by their quantitative counterparts. Clearly, a GHD-based model is favored over the NIG and HYP distributions according to the AIC.