Basic introduction to Bayesian inference

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Today's plan

Part 1:

- Three ways to write (and read) a model
- (Generalised) linear regression

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We will not explicitly cover:

- Bayes theorem
- Priors

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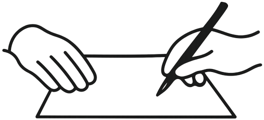
6. Map my parameters to my questions
7. Your models become **more purposeful**

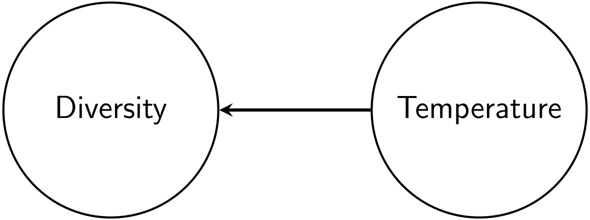
Three ways to write (and read) a model

1. Graphical
2. Code
3. Maths



The graphical way

The effect of temperature on diversity.



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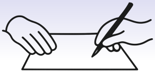

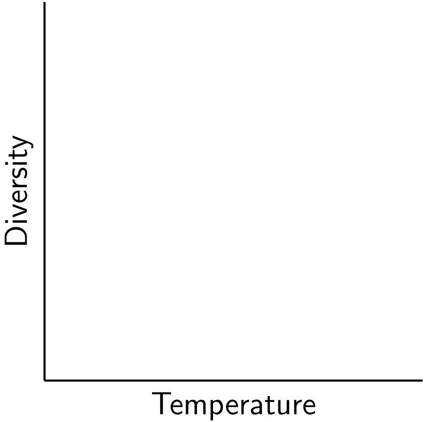

Label axis units & ticks, add data points, draw regression line

The code way

```
lm(Y ~ 1 + X)
```

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```

```
 Family: gaussian  ( identity )
Formula:          y ~ 1 + x

Dispersion estimate for gaussian family (sigma^2): 0.681

Conditional model:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -0.03087    0.08296  -0.372    0.710
x            0.03079    0.07936   0.388    0.698
```

The maths way

$$Y = a + bX + \epsilon$$

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$$\begin{aligned} Y &\sim \mathsf{Normal}(\mu, \sigma_\epsilon) \\ \mu &= a + bX \end{aligned}$$

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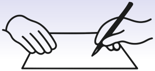

$$Y = \alpha + \beta X + \epsilon$$

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Label α and β on your graph (try μ, Y and σ if you want)

Back to the graphical way

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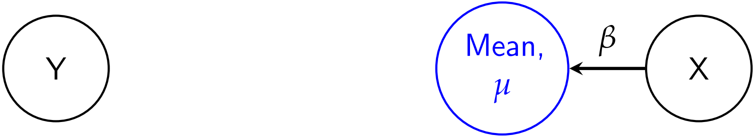

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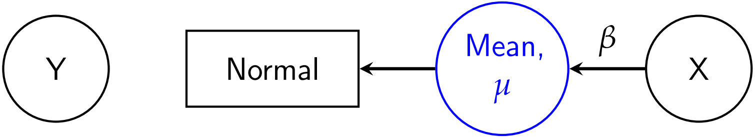

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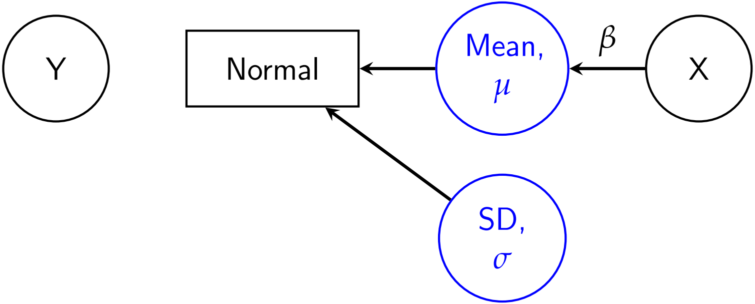

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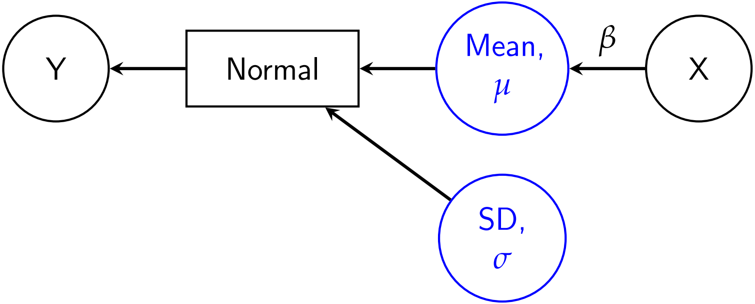

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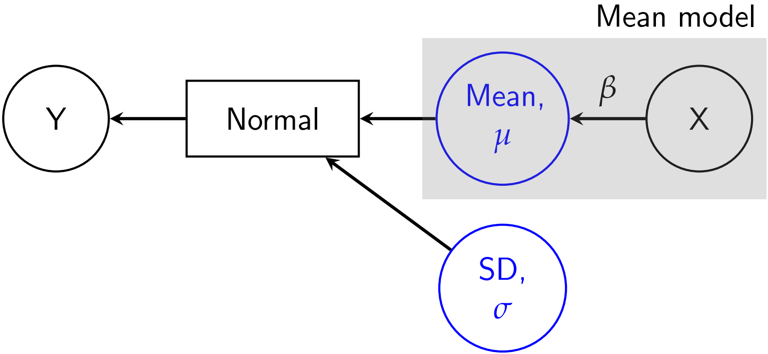

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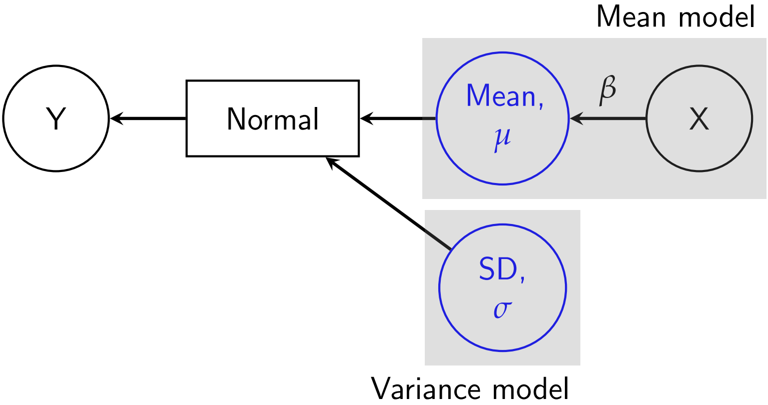

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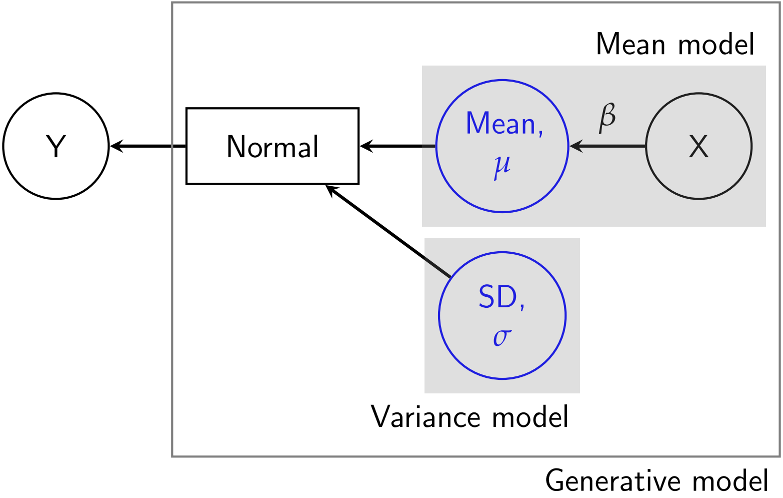

What happens if the response is not Normal?

Write the model in a general way — GLMs!

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Y &\sim \text{Distribution}(\mu,\ \phi) \\
\text{Link-function}(\mu) &= a + bX
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Write the model in a general way — GLMs!

$$Y \sim \text{Distribution}(\mu,\ \phi)$$
$$\text{Link-function}(\mu) = a + bX$$

Distribution	Link function	Typically used for
Normal	Identity	Normal stuff...
Binomial	logit / probit	Binary, count with trials...
Poisson	log	Counts
Negative binomial	log	Counts but clustered
Lognormal	log	Skewed continuous
Beta	logit	Proportions

GLMs are "generators"

Binomial generator

$$\begin{aligned} Y &\sim \text{Binomial}(N,\ p) \\ \text{logit}(p) &= \alpha + \beta X \end{aligned}$$

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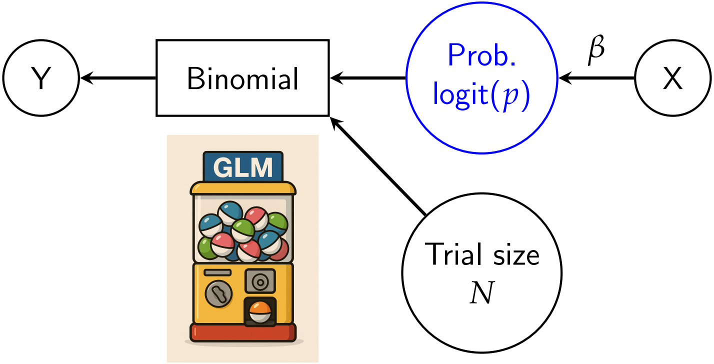

Poisson generator

$$
\begin{aligned}
Y &\sim \textsf{Poisson}(\lambda) \\
\log(\lambda) &= \alpha + \beta X
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$$\begin{tikzcd}
\text{Y} & \boxed{\text{Poisson}} \arrow[l] & \text{Rate } \log(\lambda) \arrow[l] & \text{X} \arrow[l, "\beta"']
\end{tikzcd}$$

Negative binomial generator

$$Y \sim \mathsf{Negbinom}(\mu,\ \phi)$$
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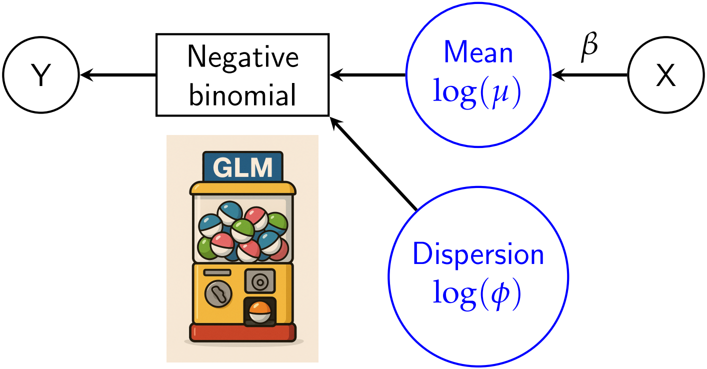

Back to the code way

```
glm(Y ~ 1 + X, family = binomial(link = logit))

glm(Y ~ 1 + X, family = poisson(link = log))

glm(Y ~ 1 + X, family = nbinom(link = log))
```

$$\vdots$$

Combining the ways

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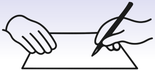

1. Express `Richness ~ 1 + Temperature` mathematically
2. Graph it out; including points, line, and as many labels as you can

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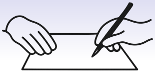

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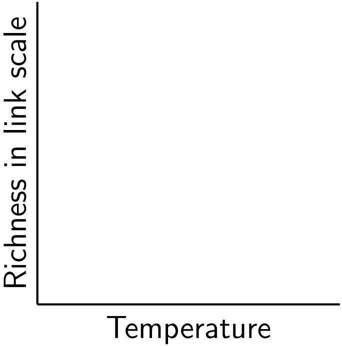

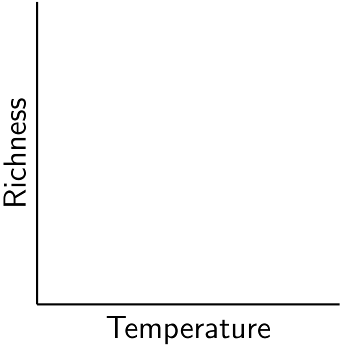

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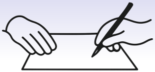
Without it, Bayesian inference is no different from frequentist approach — just another algorithm.

Open session



Express your analysis to a colleague in a way that is comfortable to *both of you*.