Field Synthesis Proof Illustration Live Script

supplementary material to:

Universal Light-Sheet Generation with Field Synthesis

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Field Synthesis Theorem for an Ideal Line Scan

We want to prove that the sum of the intensity of individual line scans produces a light-sheet illumination pattern equivalent to the average intensity created by a scanned light-sheet.

Let F(x,z) describe the electric field produced by illuminating the entire mask in the back pupil plane. $\hat{F}(k_x, k_z)$ represents the mask at the back pupil plane, the Fourier transform of the electric field.

An individual line scan is represented by the function $T_a(x, z)$ and has a Fourier transform $\hat{T}_a(k_x, k_z) = \hat{F}(k_x, k_z)\delta(k_x - a)$.

The intensity of the illumination is represented by the square modulus $|\cdot|^2$. The intensity created by illuminating the entire annular mask is $|F(x, z)|^2$. The intensity of an individual line scan is $|T_a(x, z)|^2$.

A sum of the intensity of individual line scans can be expressed as $\sum_a |T_a(x, z)|^2$.

An scanned light sheet has a z-profile equivalent to the average of then intensity of F(x,z) over the x-dimension: $\frac{1}{N}\sum_{x'} |F(x', z)|^2$.

Thus we want to prove that $\sum_a |T_a(x, z)| = \frac{1}{N}\sum_{x'} |F(x', z)|^2$

Setup

```
N = 512;
center = floor(N/2+1);

% Annulus mask at the back focal plane
F_hat = createAnnulus(N, (82+88)/2, 10);
F_hat = imgaussfilt(double(F_hat),0.5);

% Delta function representing the line scan
L_hat = zeros(N);
L_hat(:,center) = 1;

% Position of the line scan for demonstration
a = -57;

% x and z coordinates
x = ceil(-N/2):floor(N/2-1);
z = x;
```

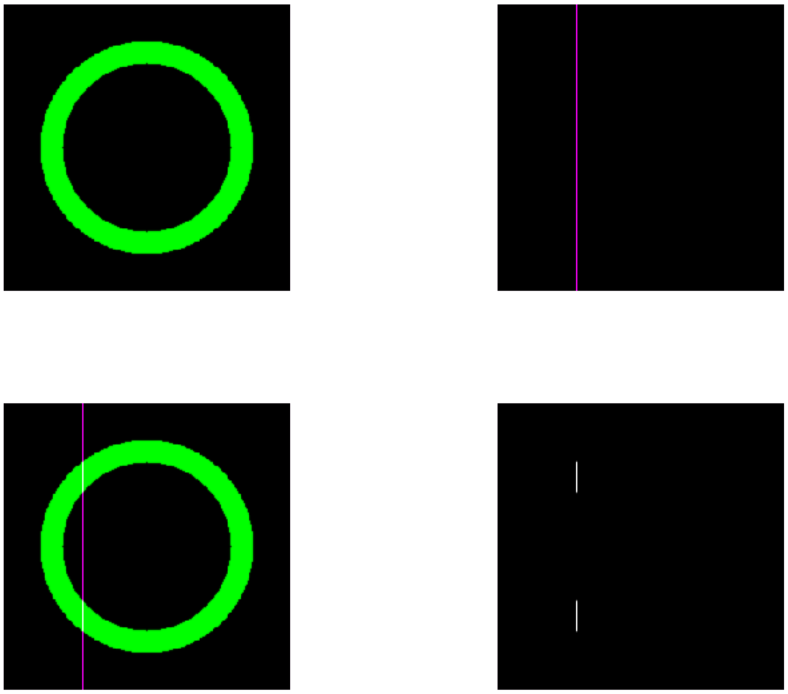
```
% Zoom levels
% 64x64 zoom level
xlims_highzoom = [-1 1]*64+center;
ylims_highzoom = [-1 1]*64+center;

% 128x128 zoom level
xlims_mediumzoom = [-1 1]*128+center;
ylims_mediumzoom = [-1 1]*128+center;

% Utility function to do shifts from back focal plane to object plane
% Note that the shifts are necessary to have the coefficients where fftw
% will expect them
% 0.1 Shift the frequency space representation so the zeroth frequency is at
%     matrix index (1,1)
% 0.2 Perform a 2-D inverse Fourier Transform
% 0.3 Shift the object space representation so that "center" is located
%     in the center of the image
doInverse2DFourierTransformWithShifts = @(X) fftshift( ifft2( ifftshift(X) ) );
```

Definition of $\hat{T}_a$, frequency space presentation of each line scan in the back focal plane

We first start at the back pupil plane where we multiply a ring by a line on a pixel-by-pixel basis.

$$\hat{T}_a(k_x, k_z) = \hat{F}(k_x, k_z)\delta(k_x - a)$$

```
% Calculate T_a_hat

L_hat_shifted = circshift(L_hat,[0 a]);
T_a_hat = F_hat.*L_hat_shifted;

figure;


% Green annulus only
subplot(2,2,1); % Upper left corner
him = imshowpair(F_hat,zeros(512));
xlim(xlims_mediumzoom); ylim(ylims_mediumzoom);
text(1,1,'F_hat','Color','green');


% Delta magenta only
subplot(2,2,2); % Upper right corner
him = imshowpair(zeros(512),L_hat_shifted);
xlim(xlims_mediumzoom); ylim(ylims_mediumzoom);

% Green annulus, magenta delta
subplot(2,2,3); % Lower left corner
him = imshowpair(F_hat,L_hat_shifted);
xlim(xlims_mediumzoom); ylim(ylims_mediumzoom);

% White overlap only
subplot(2,2,4); % Lower right corner
him = imshow(T_a_hat,[]);
xlim(xlims_mediumzoom); ylim(ylims_mediumzoom);
```



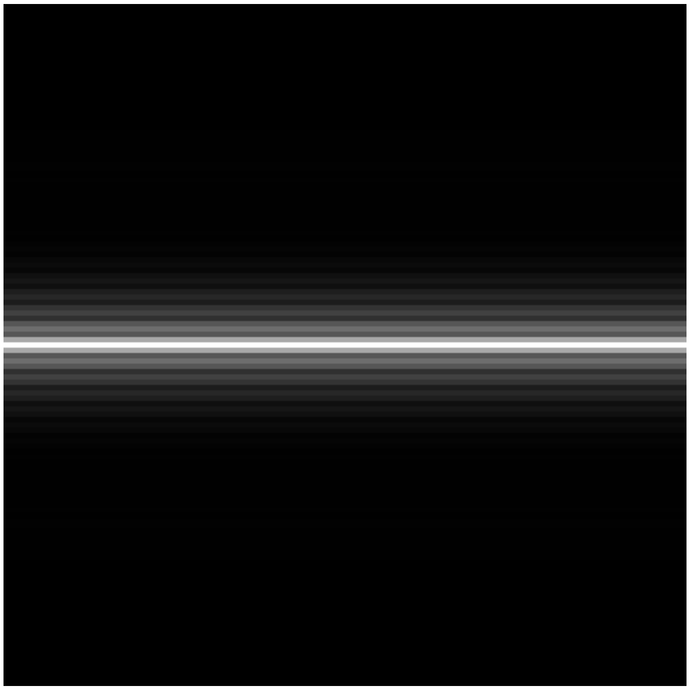
Equation 1

$$\sum_a |T_a(x,z)|^2 = \sum_a |\mathcal{F}^{-1}\{\hat{T}_a(k_x,k_z)\}(x,z)|^2$$

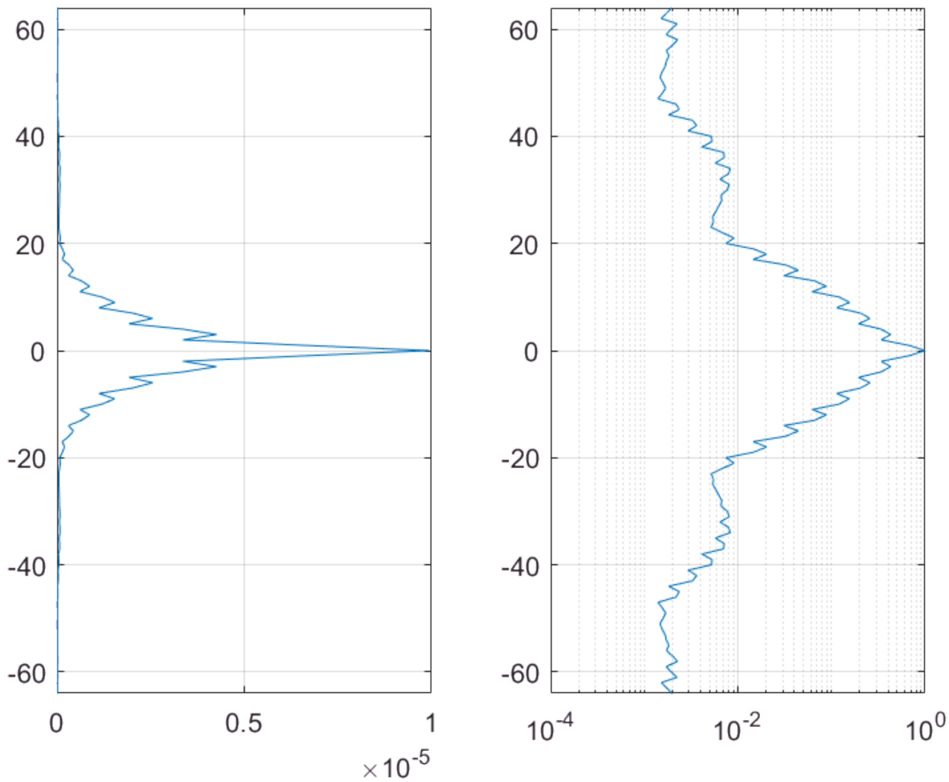
In equation 1, we state the inverse 2-D Fourier transform relationship between the electric field of an instaneous line scan, $T_a(x,z)$.

```
% Save our example "a" for later use
a_selected = a;
T_a_hat_selected = T_a_hat;
L_hat_shifted_selected = L_hat_shifted;

a_sequence = -256:255;
fsGeneratedField = zeros(N);
for a = a_sequence
    L_hat_shifted = circshift(L_hat,[0 a]);
    T_a_hat = F_hat.*L_hat_shifted;
    % 1.1 Shift the frequency space representation so the zeroth frequency is
    %     at matrix index (1,1)
    % 1.2 Perform a 2-D inverse Fourier Transform
    T_a = ifft2(ifftshift(T_a_hat));
    fsGeneratedField = fsGeneratedField + abs(T_a).^2;
end

% 1.3 Shift the object space representation so that "center" is located
%     in the center of the image
fsGeneratedField = fftshift(fsGeneratedField);

figure;
```

```
imshow(fsGeneratedField,[]);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```



```
fsGeneratedFieldSlice = fsGeneratedField(:,center);

figure;
subplot(1,2,1);
plot(fsGeneratedFieldSlice,z);
grid on;
ylim(xlims_highzoom-center);

subplot(1,2,2);
semilogx(mat2gray(fsGeneratedFieldSlice)+1e-5,z);
grid on;
ylim(xlims_highzoom-center);
```




```
% Restore our example "a"
a = a_selected;
T_a_hat = T_a_hat_selected;
L_hat_shifted = L_hat_shifted_selected;
```

Equation 2

In equation 2, we substitute in the frequency space representation at the back focal plane which we just defined above.

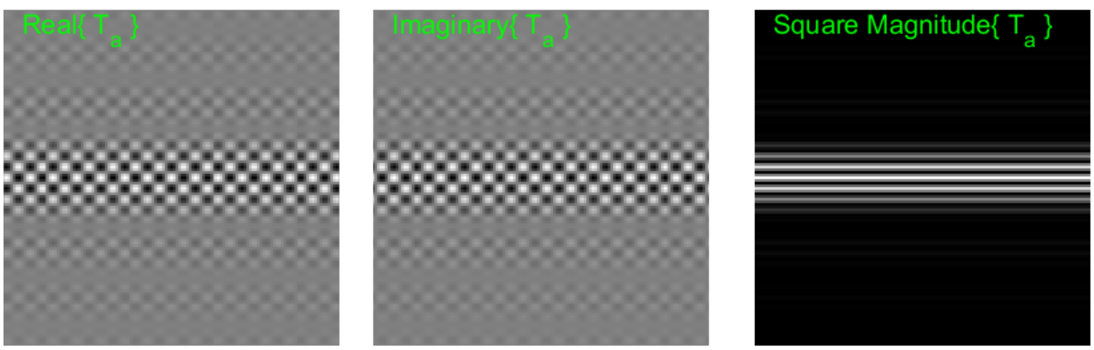
$$\sum_a |T_a(x,z)|^2 = \sum_a |\mathcal{F}^{-1}\{\hat{F}(k_x,k_z)\delta(k_x-a)\}(x,z)|^2$$

Note that because $\hat{T}_a$ is not conjugate symmetric, $\hat{T}_a(k_x,k_z) \neq \hat{T}_a^*(-k_x,-k_z)$, the range of T_a is complex valued.

To illustrate this we will focus on a single instaneous line scan at "a". Change the variable "a" above to view another slice.

```
% 2.1 Shift the frequency space representation so the zeroth frequency
%     is at matrix index (1,1)
% 2.2 Perform a 2-D inverse Fourier Transform
T_a = ifft2(ifftshift(T_a_hat));
% 2.3 Shift the object space representation so that "center" is located
%     in the center of the image
T_a = fftshift(T_a);
% Subsequent inverse Fourier Transforms will use
```

```
% doInverse2DFourierTransformWithShifts as defined in setup
%  T_a = doInverse2DFourierTransformWithShifts(T_a_hat);

figure;
subplot(1,3,1);
him = imshow(real(T_a),[]);
him.Parent.Position = [0 0 0.3 1];
text(256-56,256-56,'Real\{ T_a \}','Color','green');
xlim(xlims_highzoom); ylim(xlims_highzoom);

subplot(1,3,2);
him = imshow(imag(T_a),[]);
him.Parent.Position = [ 0.33 0 0.3 1];
text(256-56,256-56,'Imaginary\{ T_a \}','Color','green');
xlim(xlims_highzoom); ylim(xlims_highzoom);

subplot(1,3,3);
him = imshow(abs(T_a).^2,[]);
him.Parent.Position = [0.67 0 0.3 1];
text(256-56,256-56,'Square Magnitude\{ T_a \}','Color','green');
xlim(xlims_highzoom); ylim(xlims_highzoom);
```



Equation 3

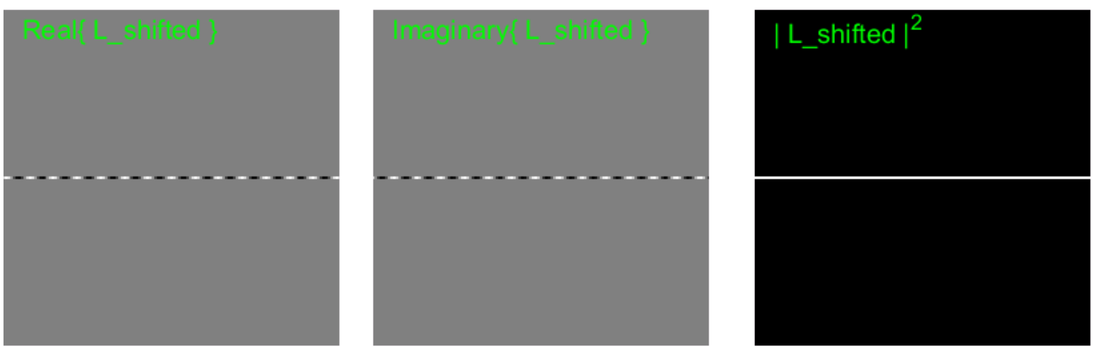
$$\sum_a |T_a(x,z)|^2 = \sum_a |\mathcal{F}^{-1}\left\{\hat{F}(k_x,k_z)\right\} **\mathcal{F}^{-1}\left\{\delta(k_x - a)\right\}(x,z)|^2$$

Next we apply the the 2-D Convolution Theorem to observe that T_a is the 2-D convolution of the inverse Fourier Transform of each term.

```
F = doInverse2DFourierTransformWithShifts(F_hat);

figure;
subplot(1,3,1);
him = imshow(real(F),[]);
him.Parent.Position = [0 0 0.3 1];
text(256-56,256-56,'Real\{ F \}','Color','green');
% colormap(gca,hot);
xlim(xlims_highzoom); ylim(xlims_highzoom);


subplot(1,3,2);
him = imshow(imag(F),[0 1]);
him.Parent.Position = [ 0.33 0 0.3 1];
% colormap(gca,hot);
text(256-56,256-56,'Imaginary\{ F \}','Color','green');
xlim(xlims_highzoom); ylim(xlims_highzoom);


subplot(1,3,3);
him = imshow(abs(F).^2,[]);
him.Parent.Position = [0.67 0 0.3 1];
text(256-56,256-56,'| F |^2','Color','green');
% colormap(gca,hot);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```

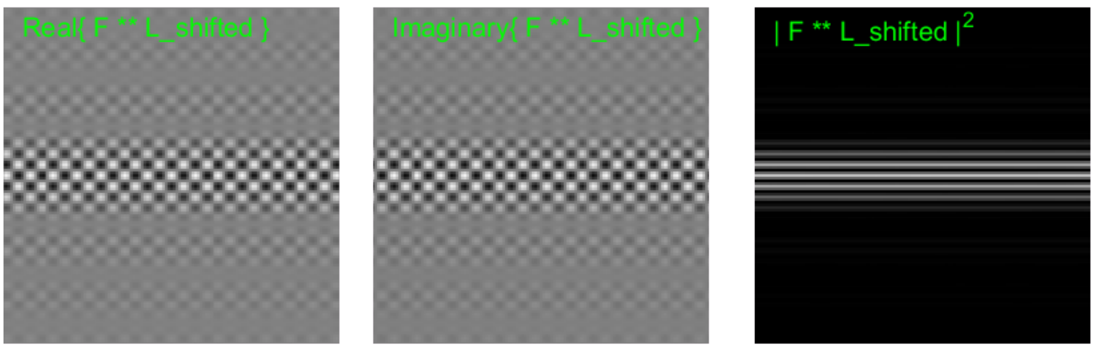

```
L_shifted = doInverse2DFourierTransformWithShifts(L_hat_shifted);

figure;
subplot(1,3,1);
him = imshow(real(L_shifted),[]);
him.Parent.Position = [0 0 0.3 1];
```

```
text(256-56,256-56,'Real\{ L\_shifted \}','Color','green','interpreter','tex');
% colormap(gca,hot);
xlim(xlims_highzoom); ylim(xlims_highzoom);


subplot(1,3,2);
him = imshow(imag(L_shifted),[]);
him.Parent.Position = [ 0.33 0 0.3 1];
% colormap(gca,hot);
text(256-56,256-56,'Imaginary\{ L\_shifted \}','Color','green');
xlim(xlims_highzoom); ylim(xlims_highzoom);


subplot(1,3,3);
him = imshow(abs(L_shifted).^2,[]);
him.Parent.Position = [0.67 0 0.3 1];
text(256-56,256-56,'| L\_shifted |^2','Color','green');
colormap(gca,hot);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```




```
T_a_by_conv = conv2(F,L_shifted,'same');

figure;
subplot(1,3,1);
him = imshow(real(T_a_by_conv),[]);
him.Parent.Position = [0 0 0.3 1];
text(256-56,256-56,'Real\{ F ** L\_shifted \}','Color','green','interpreter','tex');
% colormap(gca,hot);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```

```
subplot(1,3,2);
him = imshow(imag(T_a_by_conv),[]);
him.Parent.Position = [ 0.33 0 0.3 1];
% colormap(gca,hot);
text(256-56,256-56,'Imaginary\{ F ** L\_shifted \}','Color','green');
xlim(xlims_highzoom); ylim(xlims_highzoom);


subplot(1,3,3);
him = imshow(abs(T_a_by_conv).^2,[]);
him.Parent.Position = [0.67 0 0.3 1];
text(256-56,256-56,'| F ** L\_shifted |^2','Color','green');
% colormap(gca,hot);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```



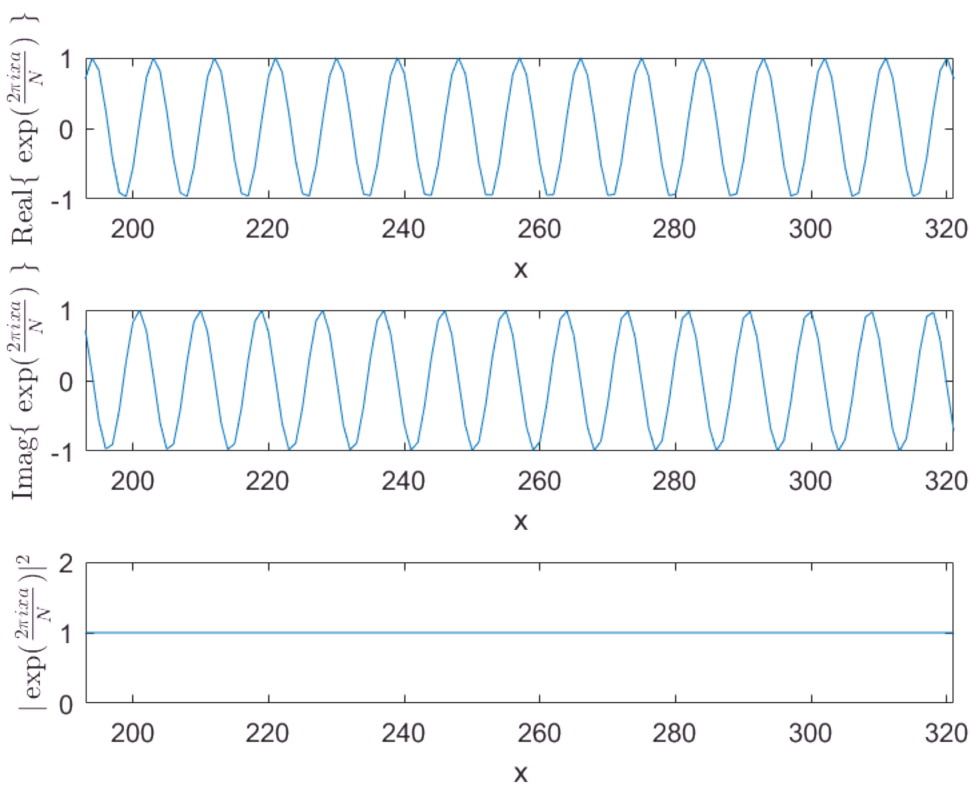
Equation 4

$$\sum_a |T_a(x,z)|^2 = \sum_a \left| F(x,z) ** \frac{1}{N}\,\delta(z)\,\exp\left(\frac{2\pi i x a}{N}\right)\right|^2$$

The inverse 2-D Fourier Transform of $\delta(k_x - a)$ is $\frac{1}{N}\,\delta(z)\,\exp\left(\frac{2\pi i x a}{N}\right)$. Below we illustrate that the product of the complex exponential and $\delta(z)$ is the same as $\mathcal{F}^{-1}\{\delta(k_x - a)\}$.

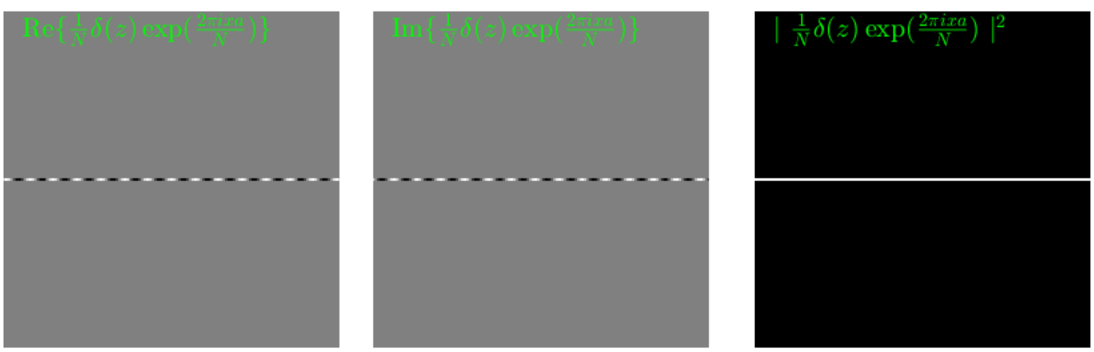
```
delta_z = zeros(512);
delta_z(center,:) = 1/N;

complex_exp = exp(2*pi*1i*x*a/N);
```

```
figure;
subplot(3,1,1)
plot(real(complex_exp));
xlim(xlims_highzoom);
xlabel('x');
ylabel('Real\{ $ \exp(\frac{2 \pi i x a }{ N}) $ \}','interpreter','latex');

subplot(3,1,2);
plot(imag(complex_exp));
xlim(xlims_highzoom);
xlabel('x');
ylabel('Imag\{ $ \exp(\frac{2 \pi i x a }{ N}) $ \}','interpreter','latex');


subplot(3,1,3);
plot(abs(complex_exp).^2);
xlim(xlims_highzoom);
ylim([0 2]);
xlabel('x');
ylabel(' $ | \exp(\frac{2 \pi i x a }{ N}) |^2 $','interpreter','latex');
```




```
complex_exp = repmat(complex_exp,512,1);
L_shifted_by_product = delta_z.*complex_exp;

label = '$ \frac{1}{N}\delta(z)\exp(\frac{2 \pi i x a }{ N}) $';

figure;
subplot(1,3,1);
him = imshow(real(L_shifted_by_product),[]);
```

```
him.Parent.Position = [0 0 0.3 1];
text(256-56,256-56,['Re\{' label '\}'],'Color','green','interpreter','latex');
% colormap(gca,hot);
xlim(xlims_highzoom); ylim(xlims_highzoom);


subplot(1,3,2);
him = imshow(imag(L_shifted_by_product),[]);
him.Parent.Position = [ 0.33 0 0.3 1];
% colormap(gca,hot);
text(256-56,256-56,['Im\{' label '\}'],'Color','green','interpreter','latex');
xlim(xlims_highzoom); ylim(xlims_highzoom);


subplot(1,3,3);
him = imshow(abs(L_shifted_by_product).^2,[]);
him.Parent.Position = [0.67 0 0.3 1];
text(256-56,256-56,['$ | $ ' label ' $ |^2 $'],'Color','green','interpreter','latex');
colormap(gca,hot);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```

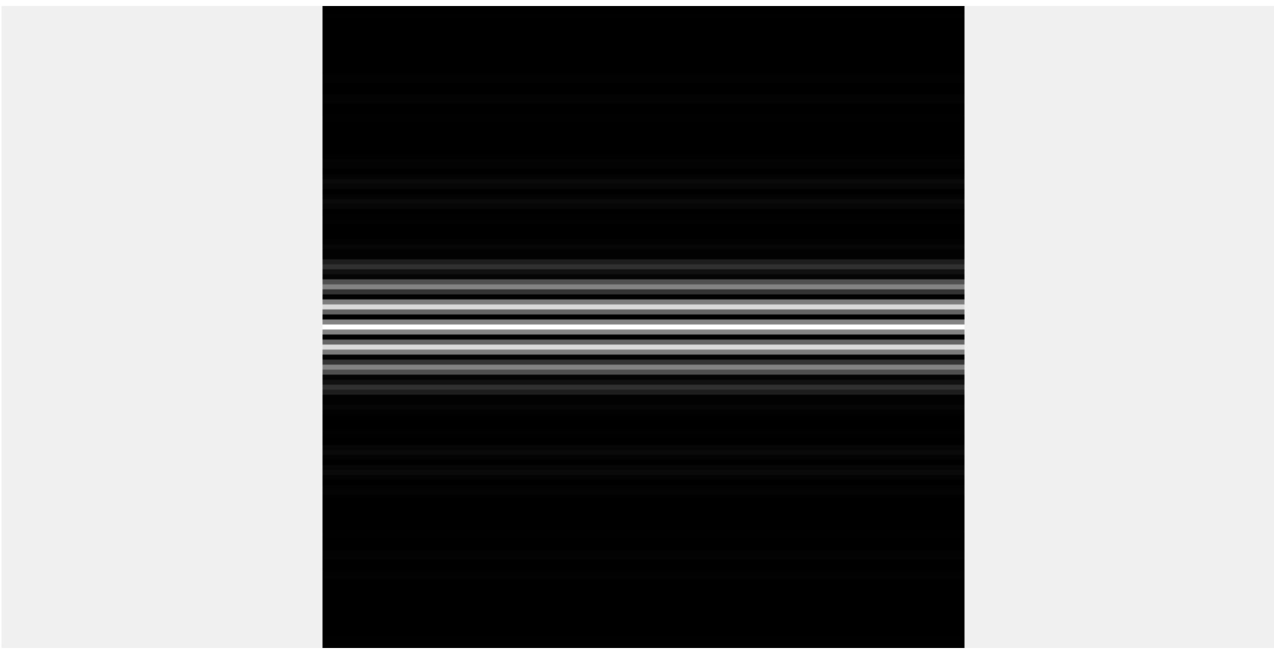


Equation 5

$$\sum_a |T_a(x,z)|^2 = \sum_a \left| \sum_{x'} \sum_{z'} \frac{1}{N} \left[F(x',z') \exp\left(\frac{2\pi i (x-x')a}{N}\right) \delta(z-z') \right] \right|^2$$

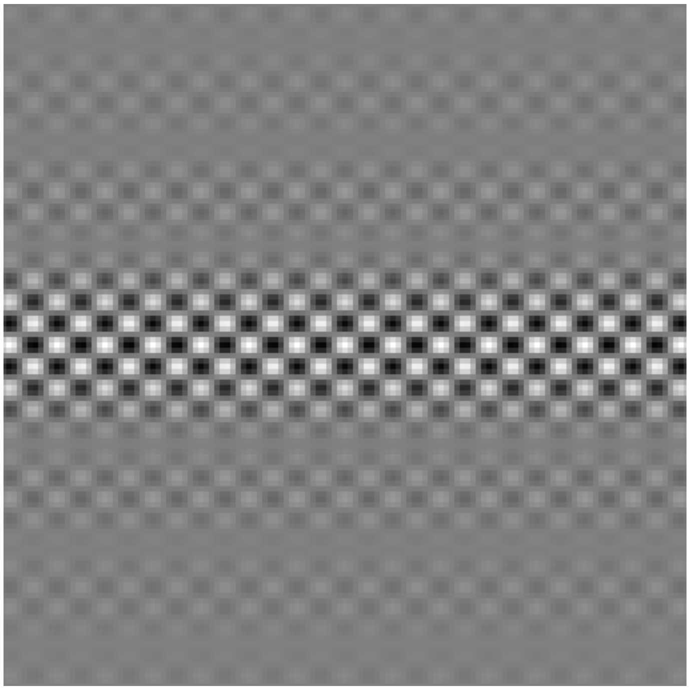
Next, we can use the definition of 2-D convolution to expand out the expression as two nested summations.

```
hfig = figure('visible','on');
him = imshow(zeros(N,N*2),[0 1]);
cumulativeSum = zeros(N);
% Technically, we are animated the commuted convolution here, which is equivalent
% Only iterate over 3 rows of z for brevity
for zp=((-1:1)*32)
    for xp=x
        F_shifted = circshift(F,[zp xp]).* complex_exp(center,xp+center);
        cumulativeSum = cumulativeSum + F_shifted.*(zp == 0);
        him.CData = [mat2gray(abs(F_shifted).^2) mat2gray(abs(cumulativeSum).^2)];
        drawnow;
    end
end
xlim(xlims_highzoom+N); ylim(xlims_highzoom);
```



The above is an animation. Run this section to see it. In a static document like a PDF, only the last frame will be shown. The cumulative sum also retains its complex character as is shown in the illustration of its real component.

```
figure; imshow(real(cumulativeSum),[]);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```

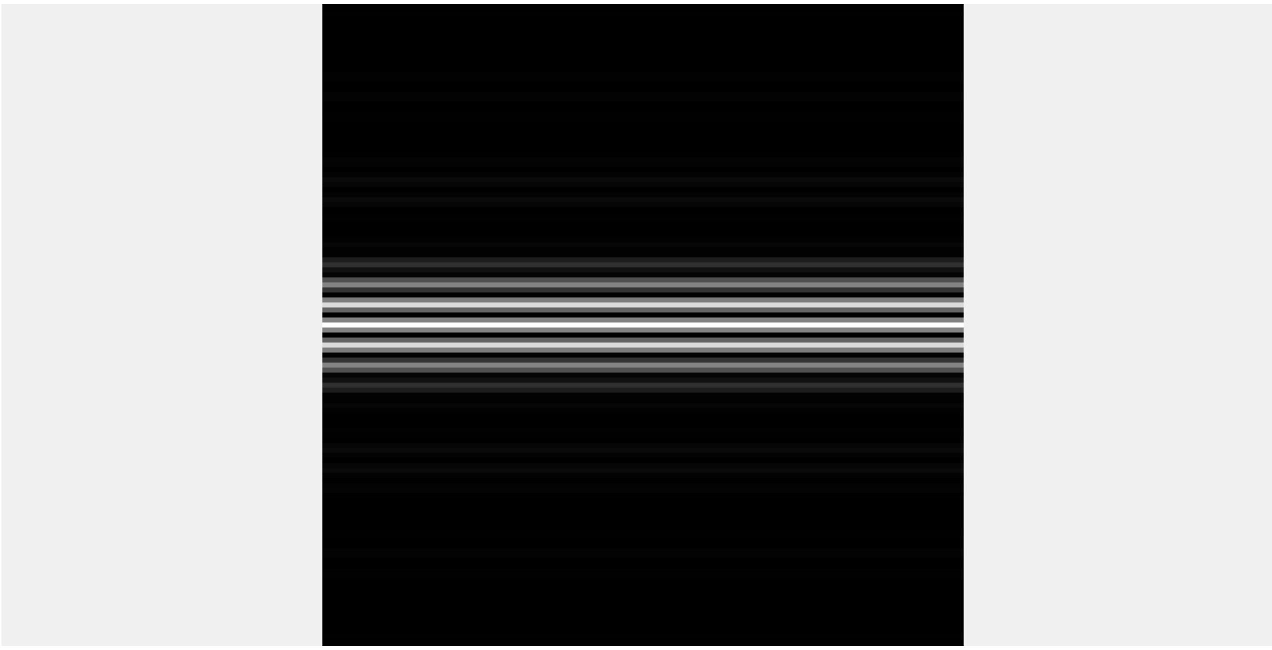


Equation 6

$$\sum_a |T_a(x,z)|^2 = \frac{1}{N^2}\sum_a \left| \exp\left(\frac{2\pi i x a}{N}\right) \sum_{x'} \left[F(x',z') \exp\left(-\frac{2\pi i x' a}{N}\right)\right]\right|^2$$

Because of the 1-D delta function $\delta(z - z')$ in the summation, we only need to perform the summation over x, the middle row. Only the term when $z' = z$ survives.

```
hfig = figure('visible','on');
him = imshow(zeros(N,N*2),[0 1]);
cumulativeSum = zeros(N);
% Technically, we are animated the commuted convolution here, which is equivalent
% We only iterate over on row
for xp=x
    F_shifted = circshift(F.*conj(complex_exp),[0 xp]); %exp(2*pi*1i*xp*a/N);
    cumulativeSum = cumulativeSum + F_shifted;
    him.CData = [mat2gray(abs(F_shifted).^2) mat2gray(abs(complex_exp.*cumulativeSum).^2)];
    drawnow;
end
xlim(xlims_highzoom+N); ylim(xlims_highzoom);
```

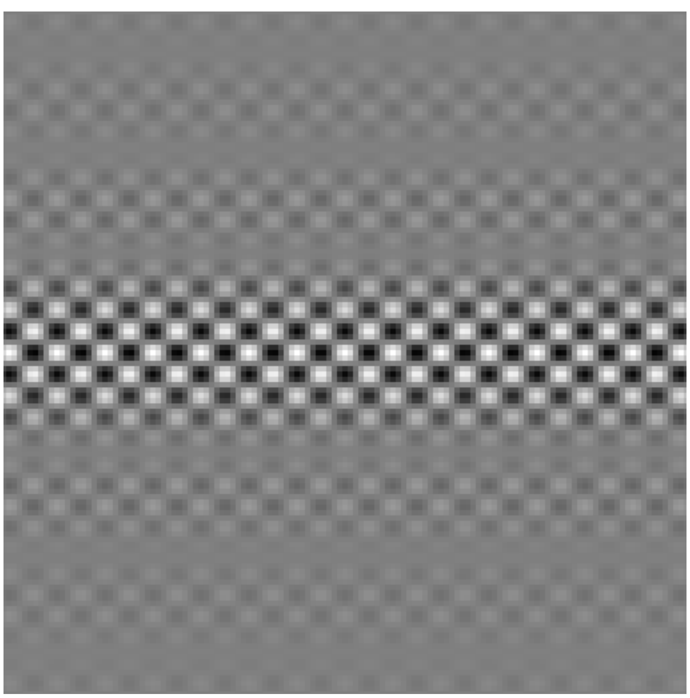


Note that the variable cumulativeSum now differs from T_a. Let's call that field Q_a and observe it is real valued.

$$Q_a(x,z) = \sum_{x'} \left[F(x',z') \exp\left(-\frac{2\pi i x' a}{N} \right) \right]$$

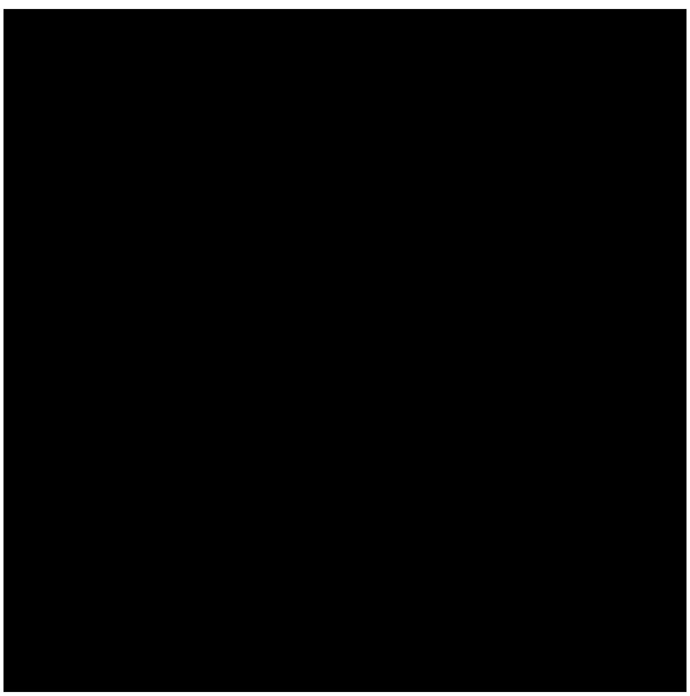
$$T_a(x,z) = \exp\left(\frac{2\pi i x a}{N} \right) Q_a(x,z)$$

```
Q_a = cumulativeSum;

figure; imshow(real(complex_exp.*Q_a),[]);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```



```
figure; imshow(real(Q_a),[]);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```

```
% Note that Q_a is real valued!
figure; imshow(imag(Q_a),[0 1]);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```

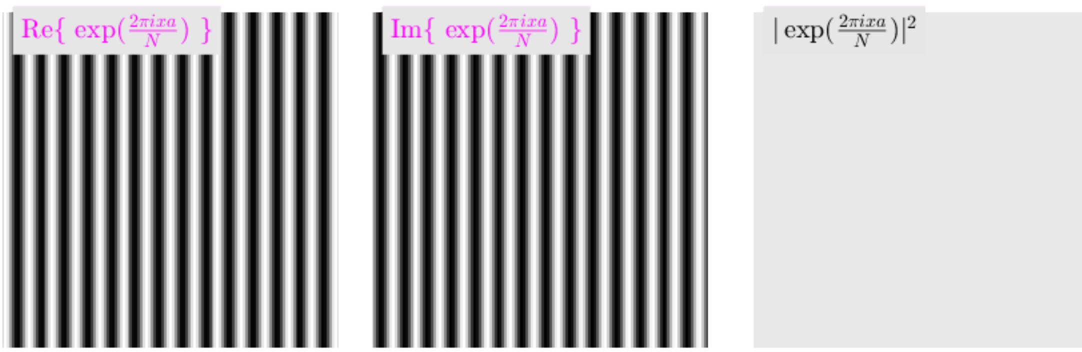


Equation 7

$$\sum_a |T_a(x,z)|^2 = \frac{1}{N^2}\sum_a \left|\exp\left(\frac{2\pi i\,(x)a}{N}\right)\right|^2 \left|\sum_{x'}\left[F(x',z')\exp\left(-\frac{2\pi i x' a}{N}\right)\right]\right|^2$$

The multiplicative property of the complex modulus allows us to factor out the square modulus of the first complex exponential. The square modulus of that complex exponential is unity, 1, everywhere.

```
label = '\exp(\frac{2 \pi i x a }{ N})';

figure;
subplot(1,3,1);
him = imshow(real(complex_exp),[]);
him.Parent.Position = [0 0 0.3 1];
text(256-56,256-56,['Re\{ $' label '$ \}'], ...
    'BackgroundColor',[0.9 0.9 0.9],'Color','magenta','interpreter','latex');
% colormap(gca,hot);
xlim(xlims_highzoom); ylim(xlims_highzoom);


subplot(1,3,2);
him = imshow(imag(complex_exp),[]);
him.Parent.Position = [ 0.33 0 0.3 1];
% colormap(gca,hot);
text(256-56,256-56,['Im\{ $' label '$ \}'], ...
    'BackgroundColor',[0.9 0.9 0.9],'Color','magenta','interpreter','latex');
xlim(xlims_highzoom); ylim(xlims_highzoom);


subplot(1,3,3);
% Adjust upper color limit so you can see the white square
him = imshow(abs(complex_exp).^2,[0 1.1]);
him.Parent.Position = [0.67 0 0.3 1];
text(256-56,256-56,['$ | ' label ' |^2 $'], ...
    'BackgroundColor',[0.9 0.9 0.9],'Color','black','interpreter','latex');
% colormap(gca,hot);
% colorbar;
xlim(xlims_highzoom); ylim(xlims_highzoom);
```



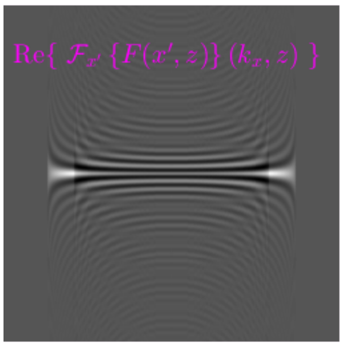
Equation 8

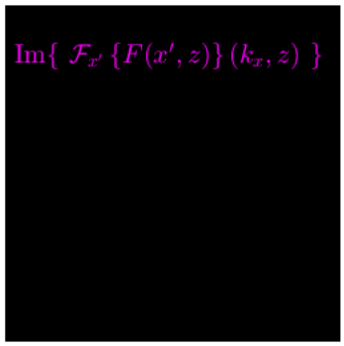
$$\sum_{a} |T_a(x,z)|^2 = \frac{1}{N^2} \sum_{a} \left| \mathcal{F}_{x'} \left\{ F(x',z) \right\} (a,z) \right|^2$$

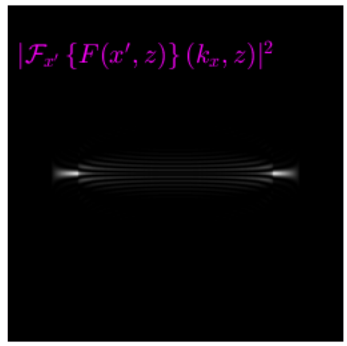
We can simplify the expression further using the definition of the 1-D Fourier Transform with respect to x'. Note that this creates a field where the x-dimension is frequency space and the z-dimension is in object space.

```
one_d_ft = fftshift(fft(ifftshift(F,2),[],2),2);
label = '\mathcal{F}_{x''} \left\{ F(x'',z) \right\}(k_x,z)';

figure;
subplot(1,3,1);
him = imshow(real(one_d_ft),[]);
him.Parent.Position = [0 0 0.3 1];
text(256-56-64,256-56-32,['Re\{ $ ' label ' $ \}'],'Color','magenta','interpreter','latex');
% colormap(gca,hot);
xlim(xlims_mediumzoom); ylim(xlims_mediumzoom);


subplot(1,3,2);
him = imshow(imag(one_d_ft),[0 1]);
him.Parent.Position = [ 0.33 0 0.3 1];
% colormap(gca,hot);
text(256-56-64,256-56-32,['Im\{ $ ' label ' $ \}'],'Color','magenta','interpreter','latex');
xlim(xlims_mediumzoom); ylim(xlims_mediumzoom);


subplot(1,3,3);
```

```
% Adjust upper color limit so you can see the white square
him = imshow(abs(one_d_ft).^2,[]);
him.Parent.Position = [0.67 0 0.3 1];
text(256-56-64,256-56-32,['$ | ' label ' |^2 $'],'Color','magenta','interpreter','latex');
% colormap(gca,hot);
% colorbar;
xlim(xlims_mediumzoom); ylim(xlims_mediumzoom);
```

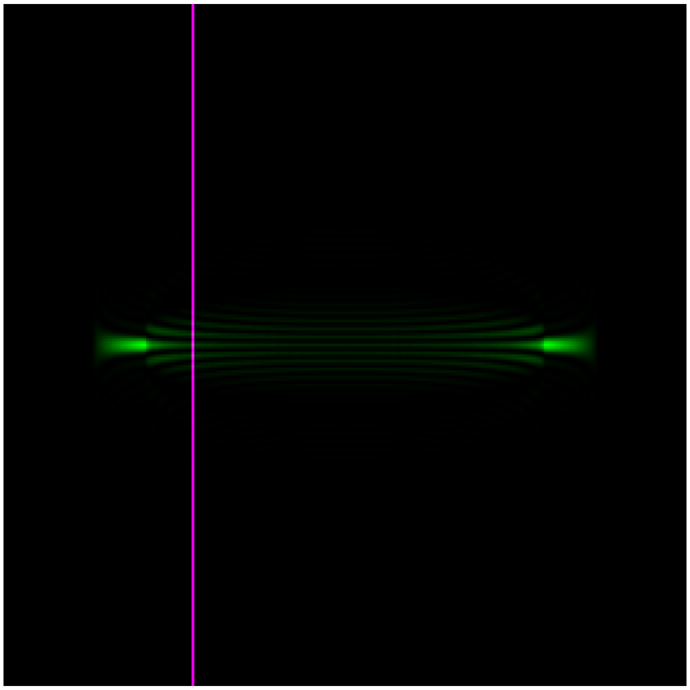






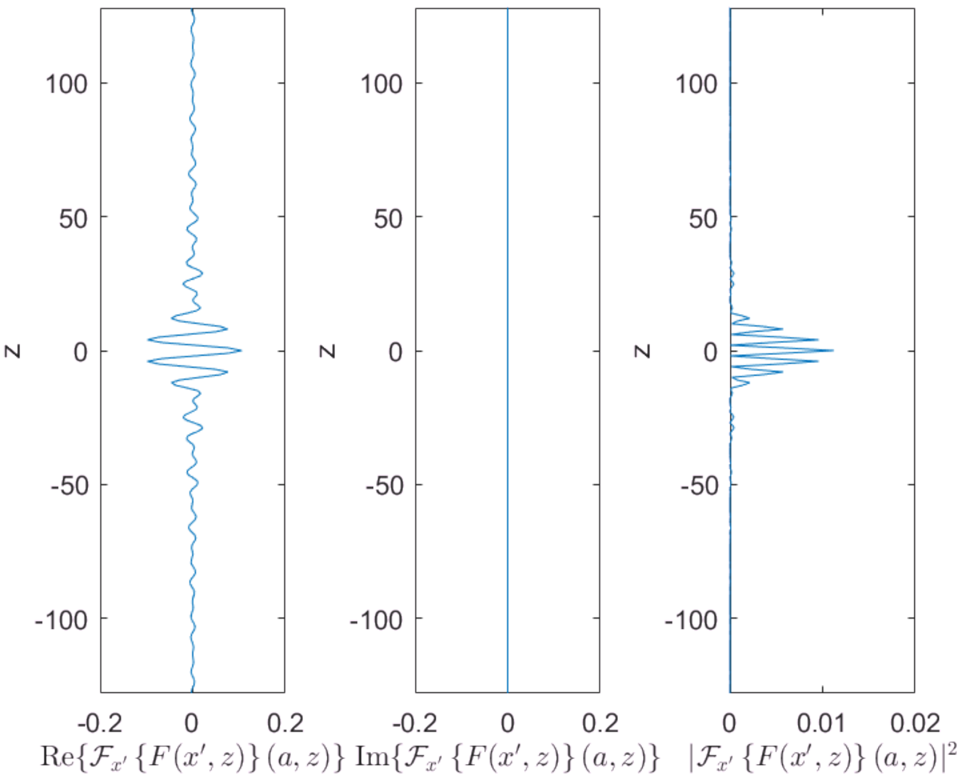
The z-profile of $|T_a(x,z)|^2$ and $|Q_a(x,z)|^2$ is a slice of this one-dimensional Fourier Transform of $F(x,z)$, $|\mathcal{F}_{x'}\{F(x',z)\}(a,z)|^2$

```
figure; imshowpair(abs(one_d_ft).^2,L_hat_shifted);
xlim(xlims_mediumzoom); ylim(xlims_mediumzoom);
```



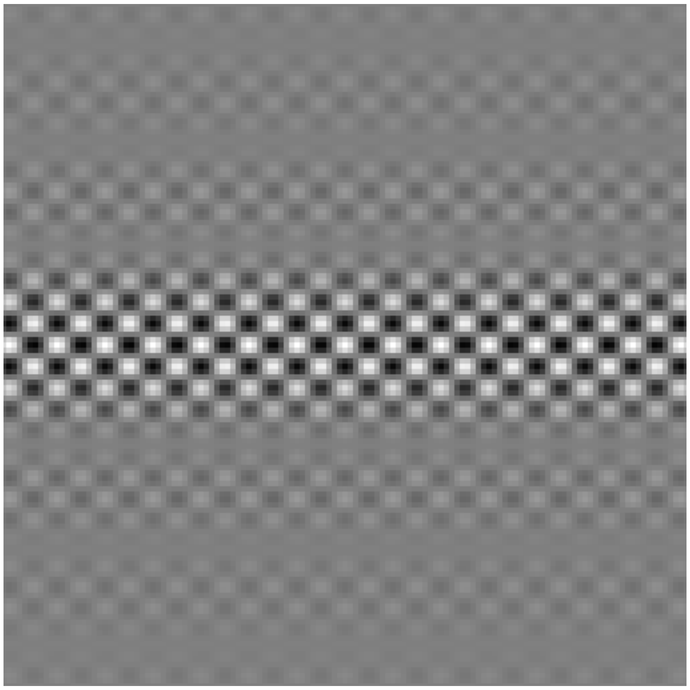
To see this, let the 1-D function $g_a(z) = \mathcal{F}_{x'}\{F(x', z)\}(a, z)$

```
z = x;
g_a = one_d_ft(:,a+center);
label = '\mathcal{F}_{x''} \left\{ F(x'',z) \right\}(a,z)';
figure;
subplot(1,3,1);
plot(real(g_a),z);
xl = xlim;
ylim(xlims_mediumzoom-center);
ylabel('z');
xlabel(['Re\{$ ' label ' $\}'],'interpreter','latex');

subplot(1,3,2);
plot(imag(g_a),z);
xlim(xl);
ylim(xlims_mediumzoom-center);
ylabel('z');
xlabel(['Im\{$ ' label ' $\}'],'interpreter','latex');


subplot(1,3,3);
plot(abs(g_a).^2,z);
ylim(xlims_mediumzoom-center);
ylabel('z');
xlabel(['$ |' label '|^2 $'],'interpreter','latex');
```

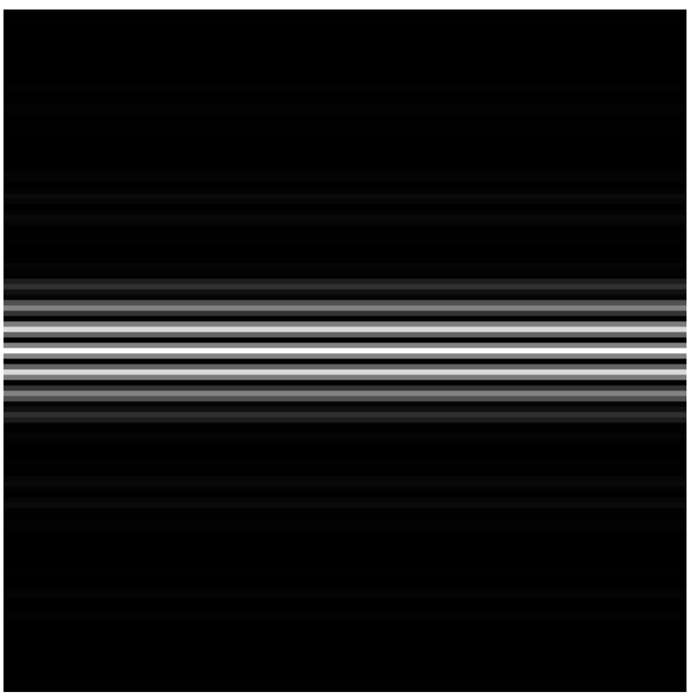



```
Q_a_from_FT = repmat(g_a,1,512);

figure; imshow(real(Q_a_from_FT),[]);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```

```
T_a_from_FT = complex_exp.*Q_a_from_FT;
figure; imshow(real(T_a_from_FT),[]);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```



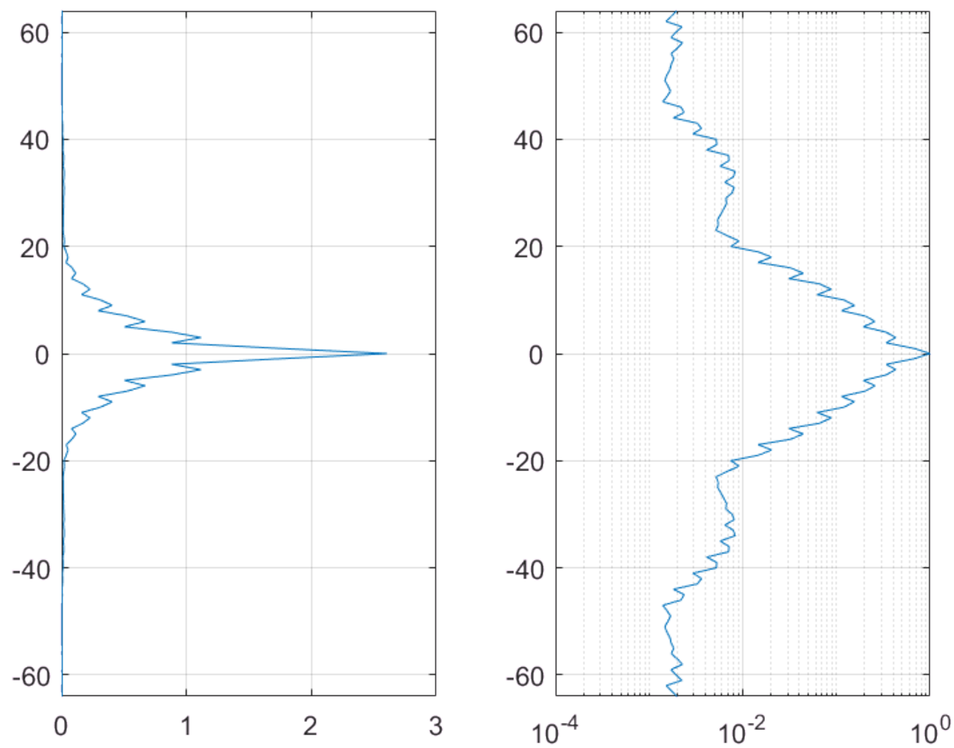
```
figure; imshow(abs(T_a_from_FT).^2,[]);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```



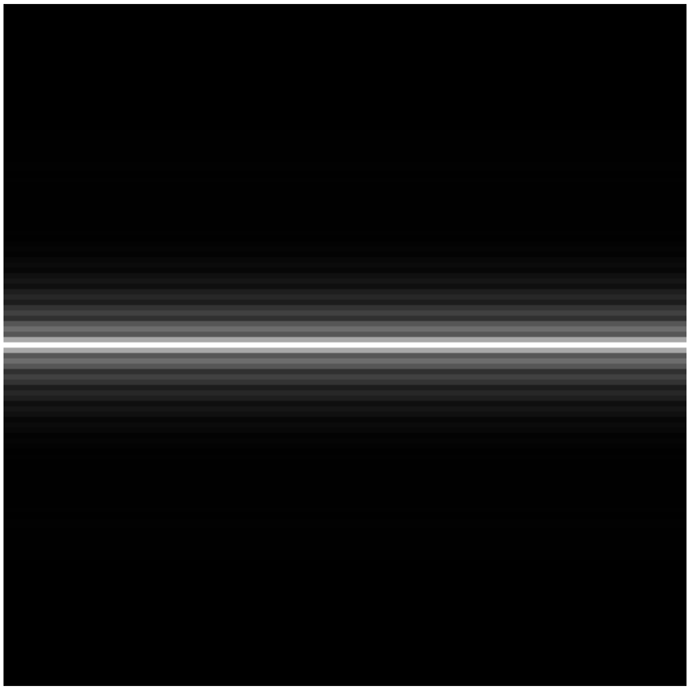
Now that we have followed each instaneous $T_a(x,z)$ and then $Q_a(x,z)$ through the manipulations, we now consider the summation over "a".

```
one_d_ft_projection = sum(abs(one_d_ft).^2,2);

figure;
subplot(1,2,1);
plot(one_d_ft_projection,z);
grid on;
ylim(xlims_highzoom-center);
```

```
subplot(1,2,2);
semilogx(mat2gray(one_d_ft_projection)+1e-5,z);
grid on;
ylim(xlims_highzoom-center);
```




```
one_d_ft_projection = repmat(one_d_ft_projection,1,512)/N;

figure;
imshow(one_d_ft_projection,[]);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```

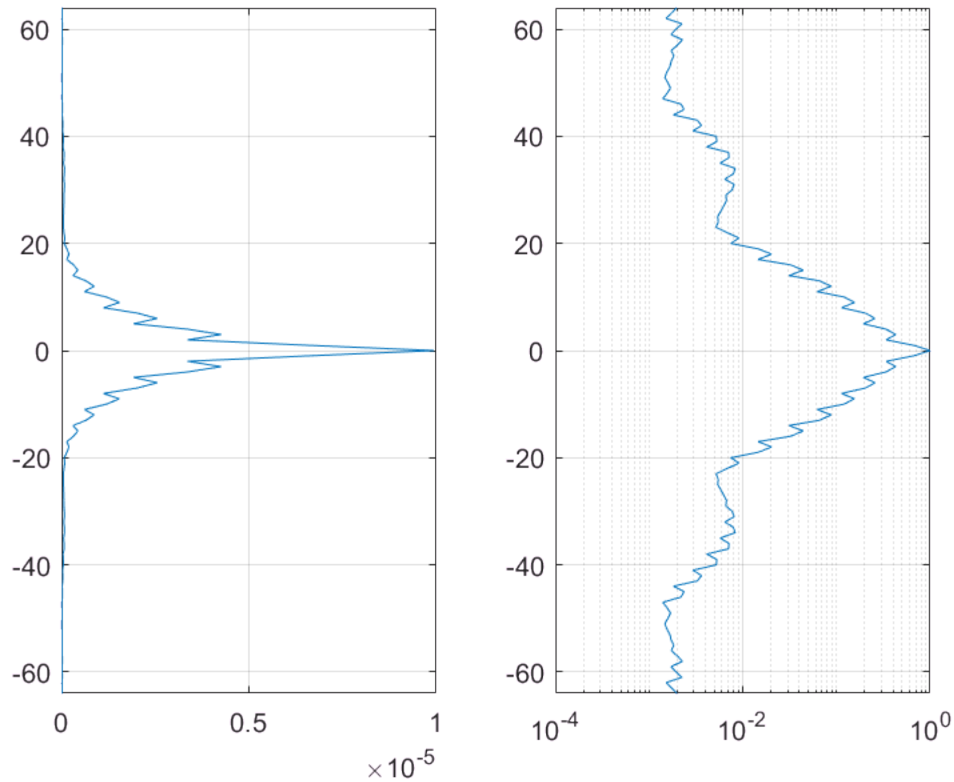


Equation 9

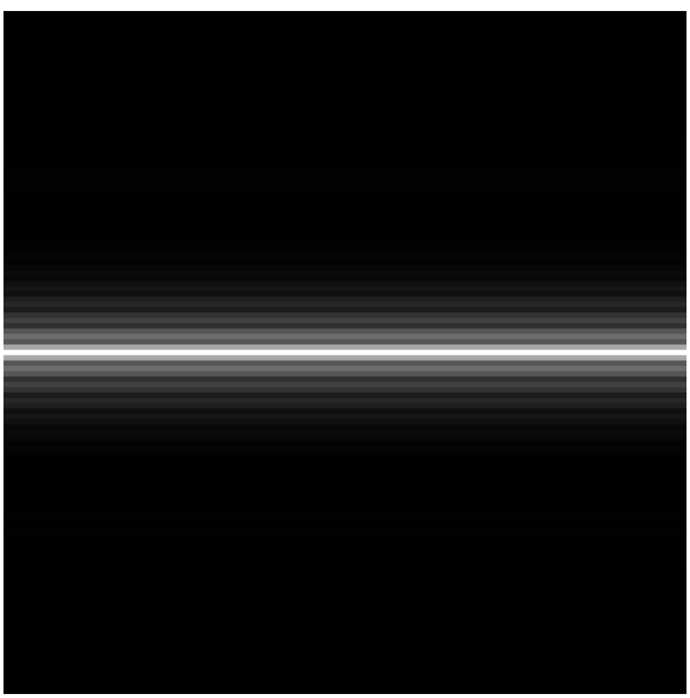
$$\sum_{a} |T_a(x,z)|^2 = \frac{1}{N} \sum_{x'} |F(x',z)|^2$$

Finally we use Parserval's theorem to equate the sum of the square modulus of the 1-D Fourier transform of the sequence with the sum of the square modulus of the original electric field produced by the mask. The sum of the square modulus of the electric field is a projection in the x' direction. This projection is created by the conventional manner of scanning a beam across a field to create a lightsheet.

```
F_projection = sum(abs(F).^2,2)/N;

figure;
subplot(1,2,1);
plot(F_projection,z);
grid on;
ylim(xlims_highzoom-center);

subplot(1,2,2);
semilogx(mat2gray(F_projection)+1e-5,z);
grid on;
ylim(xlims_highzoom-center);
```




```
F_projection = repmat(F_projection,1,512);
figure; imshow(F_projection,[]);
xlim(xlims_highzoom); ylim(xlims_highzoom);
```



```
% Is F_projection == fsGeneratedField ?
rmsDiff = sqrt(mean(abs(F_projection(:)-fsGeneratedField(:)).^2))
```

```
rmsDiff = 6.8544e-22
```

We have thus proved the Field Synthesis theorem.