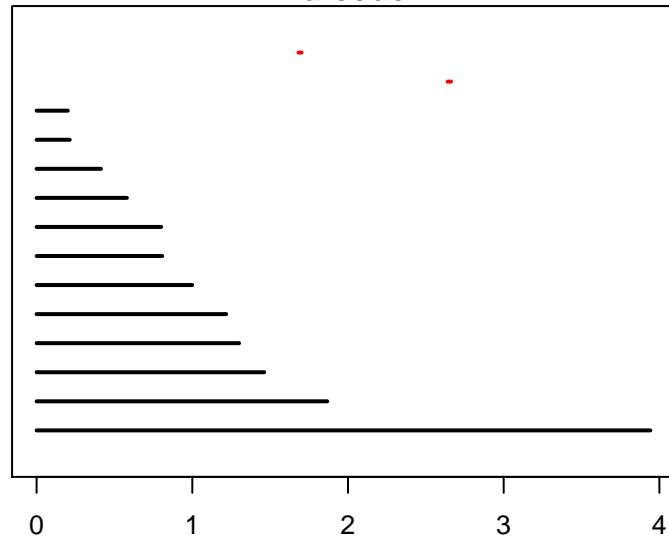
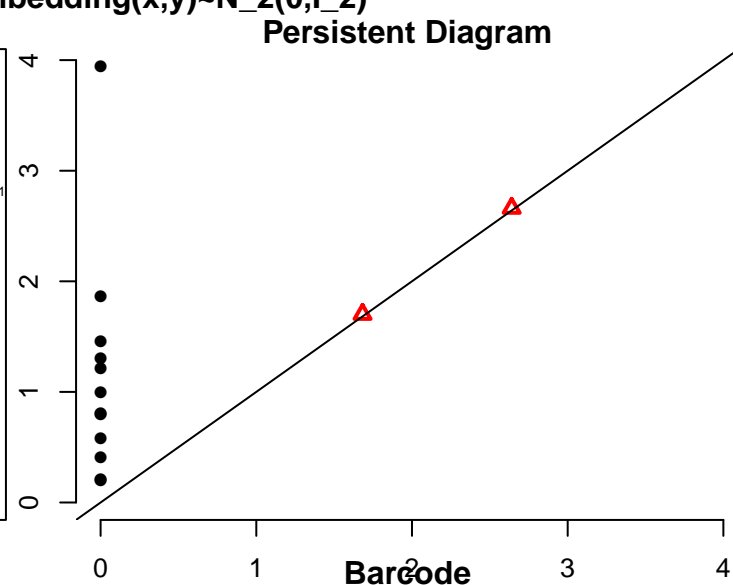
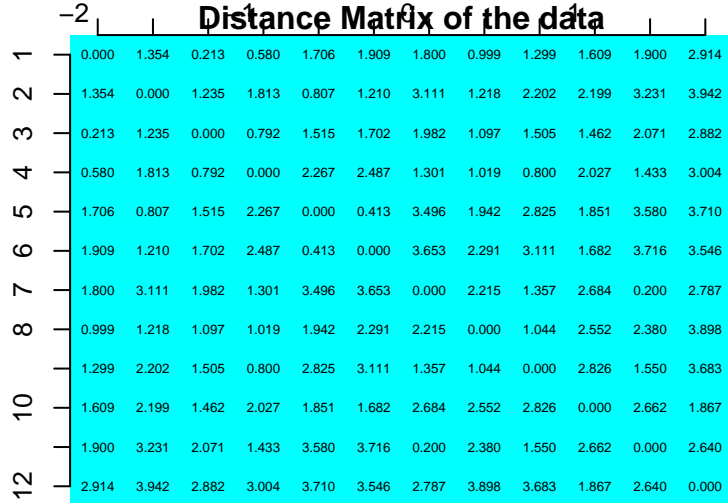
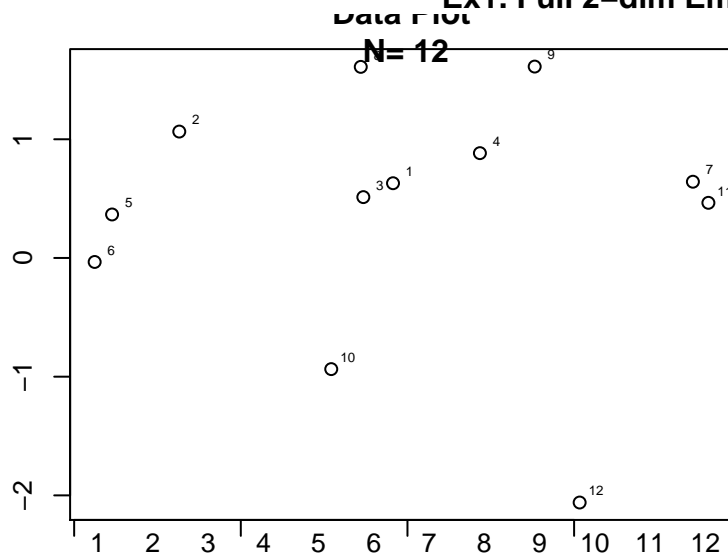
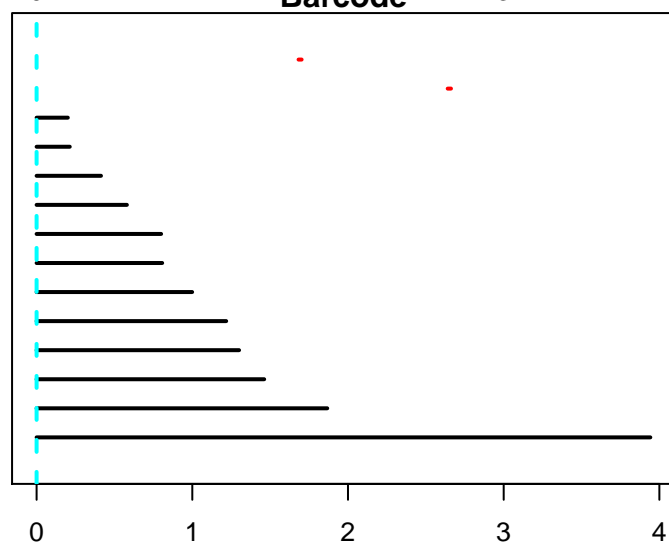
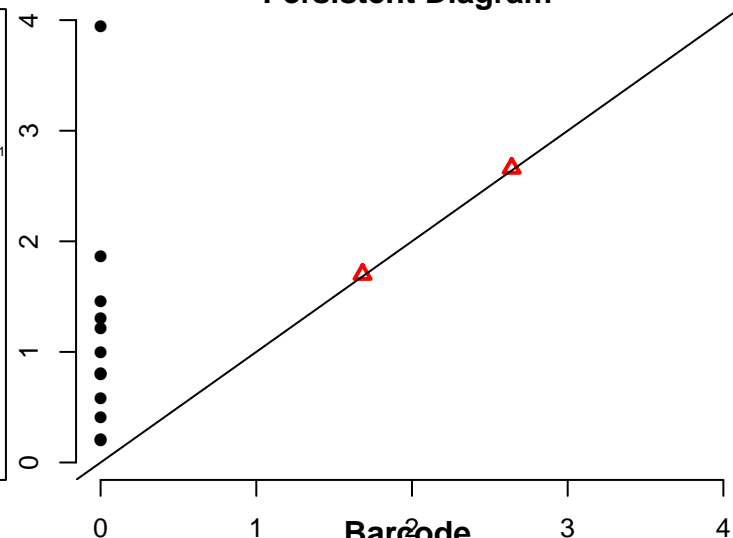
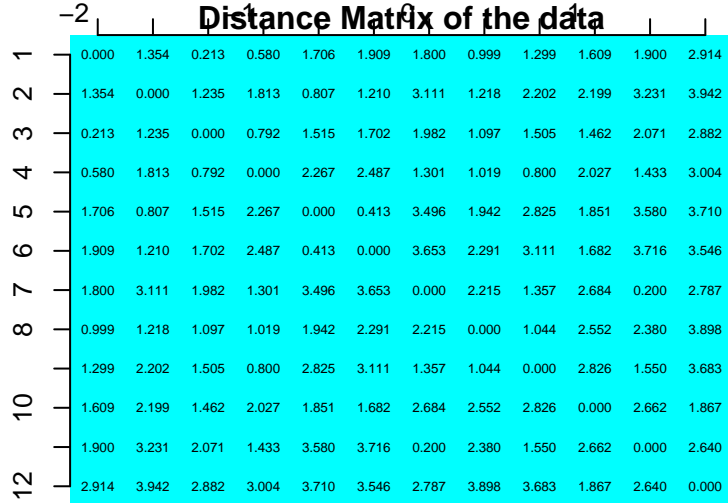
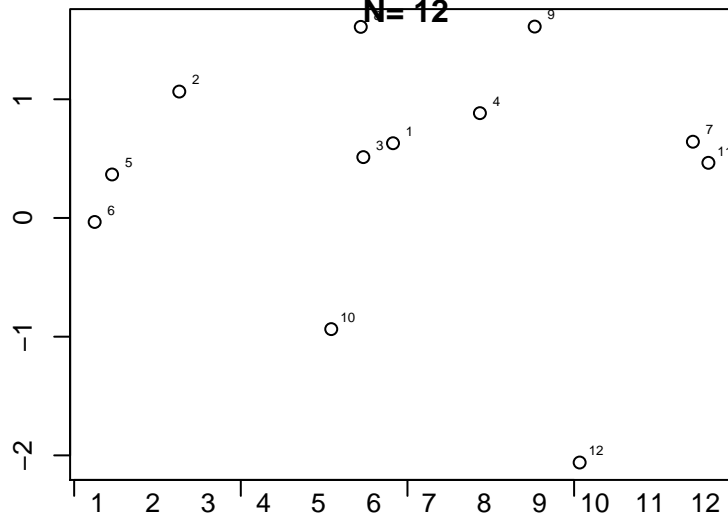


Result and Frame-by-frame plots for Example 1

# Ex1. Full 2-dim Embedding(x,y)~N\_2(0,I\_2)

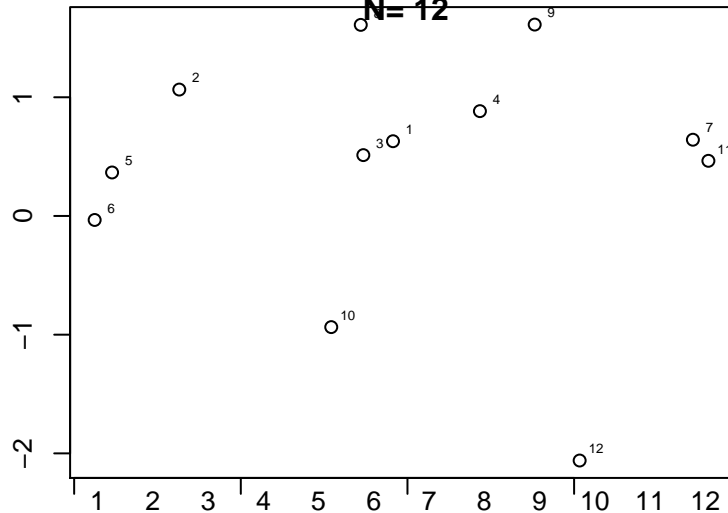


This is the 'Frame' at Euclidean distance = 0

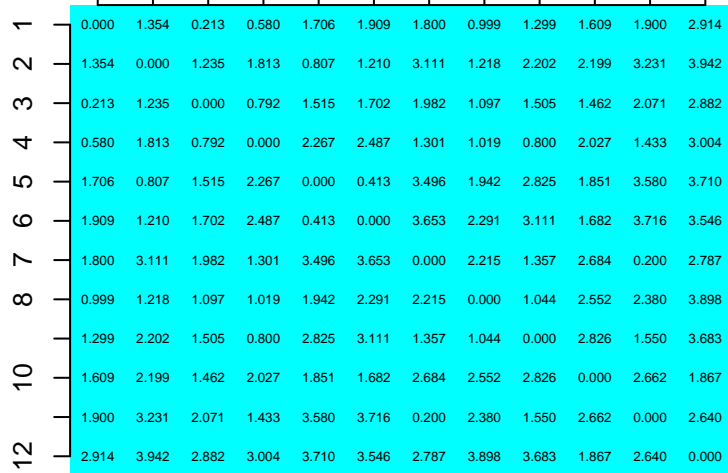


Data Plot

N=12

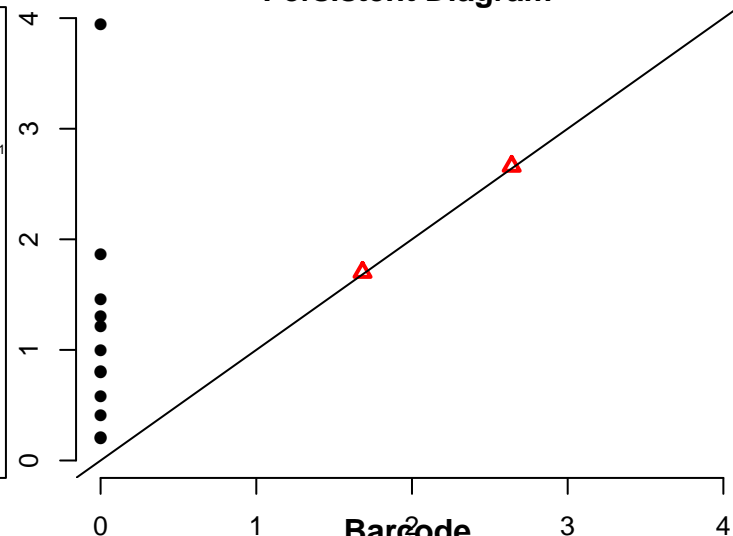


Distance Matrix of the data

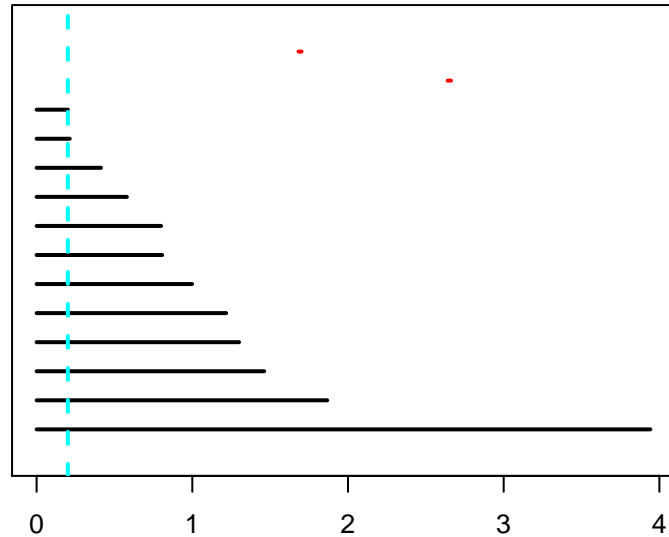


This is the 'Frame' at Euclidean distance = 0.2

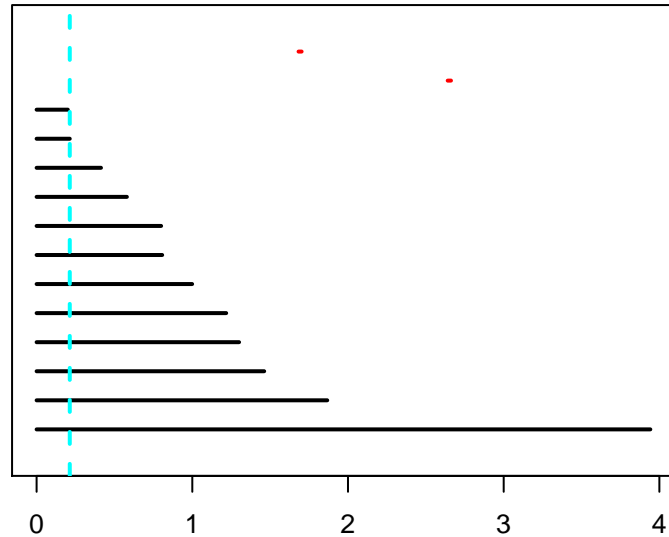
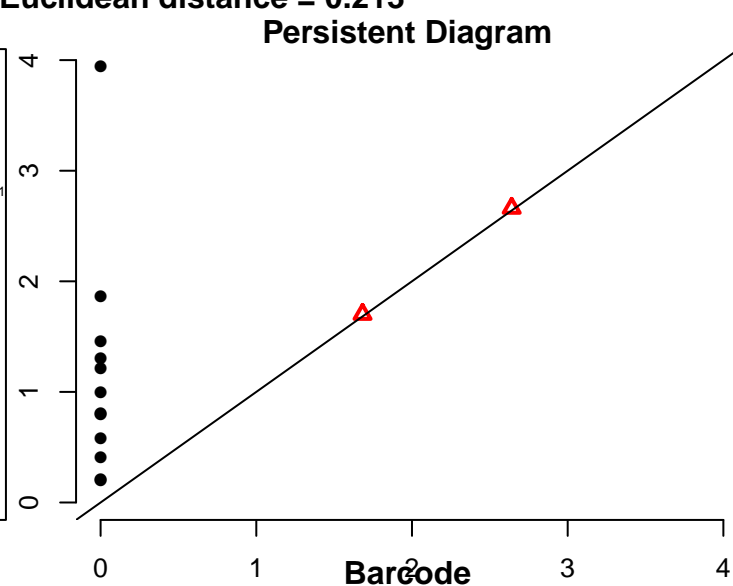
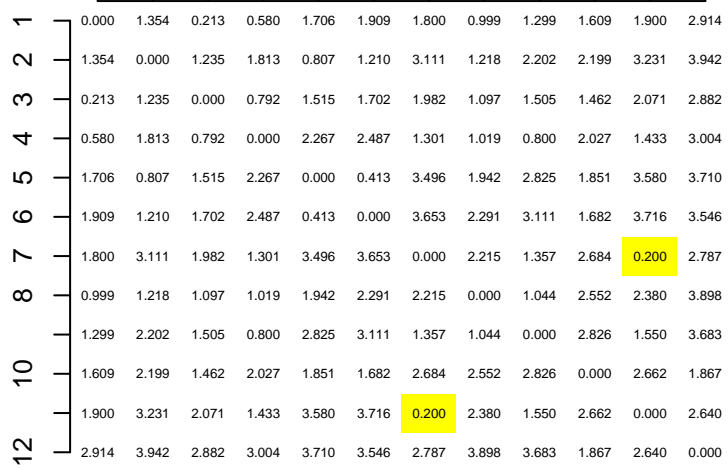
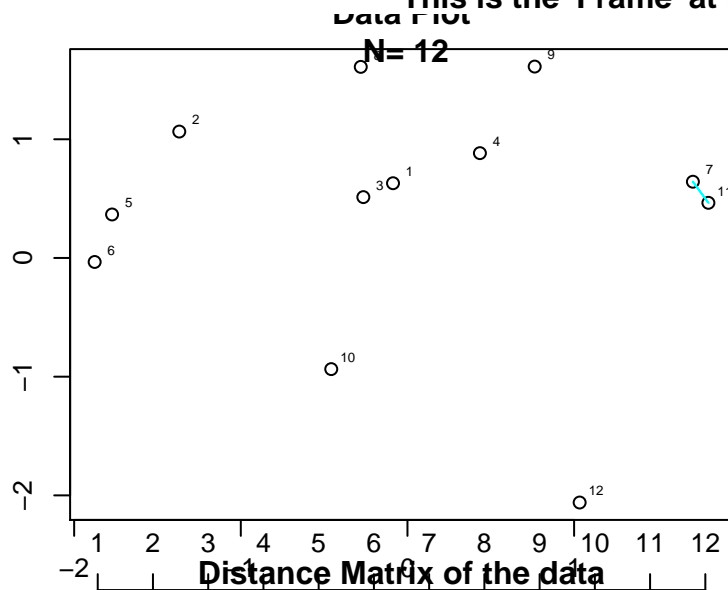
Persistent Diagram



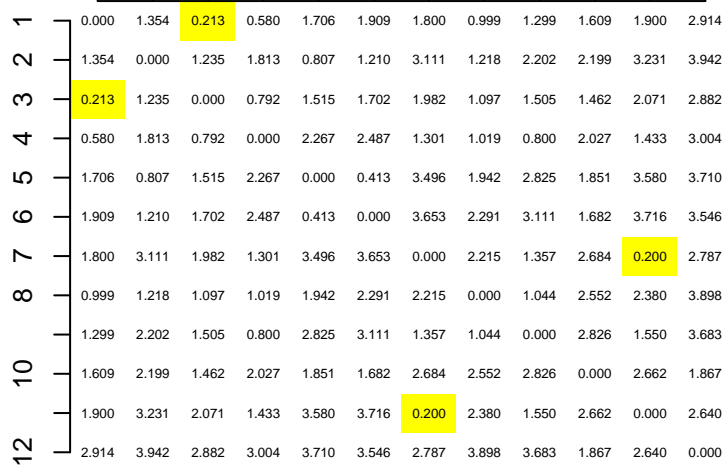
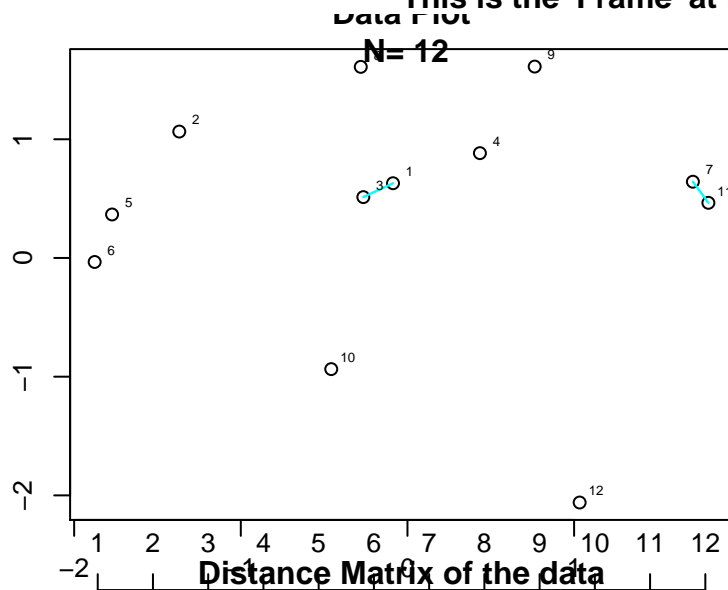
Barcode



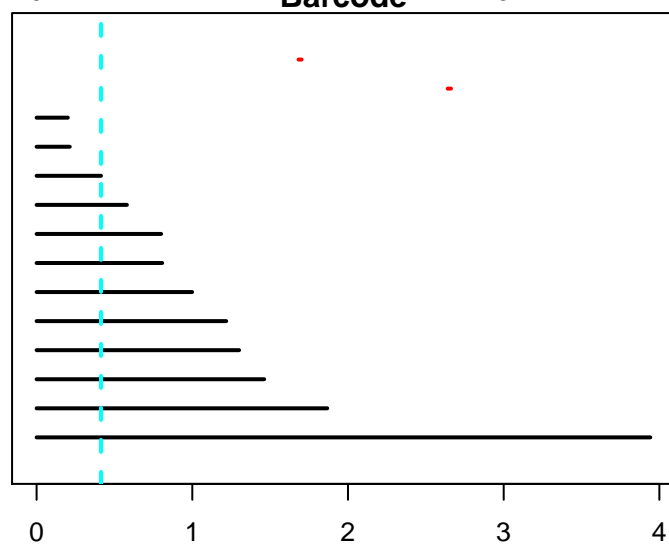
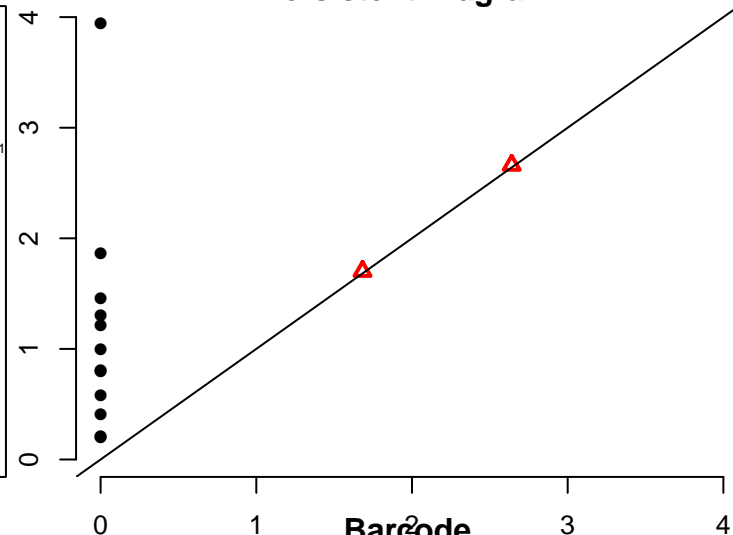
This is the 'Frame' at Euclidean distance = 0.213



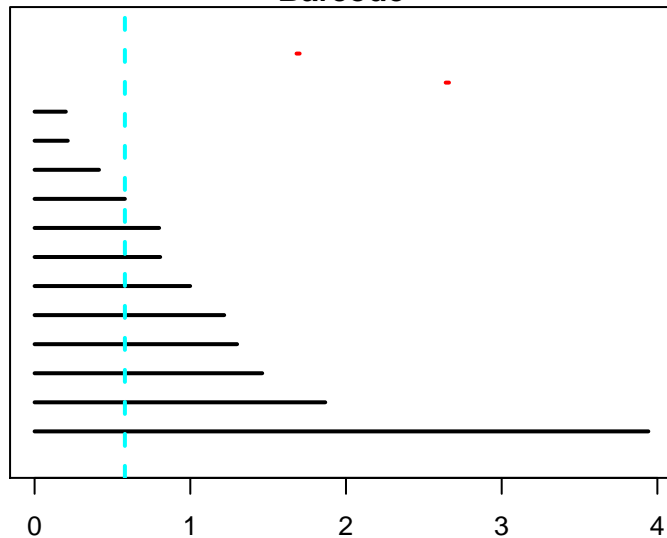
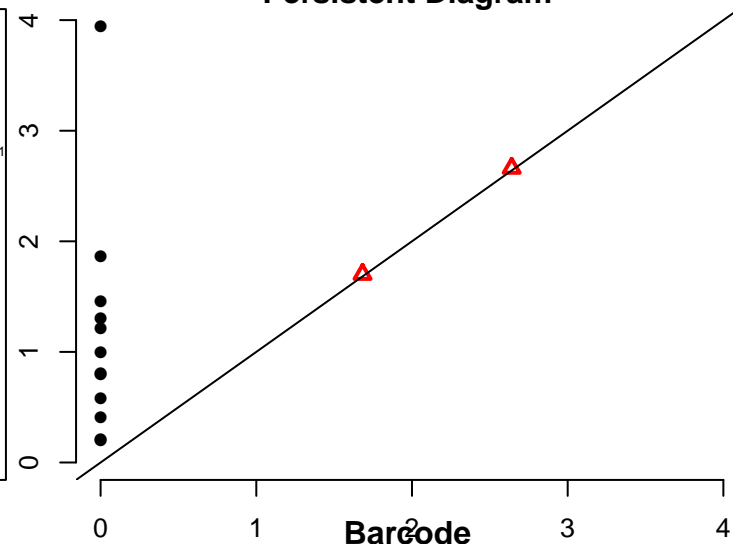
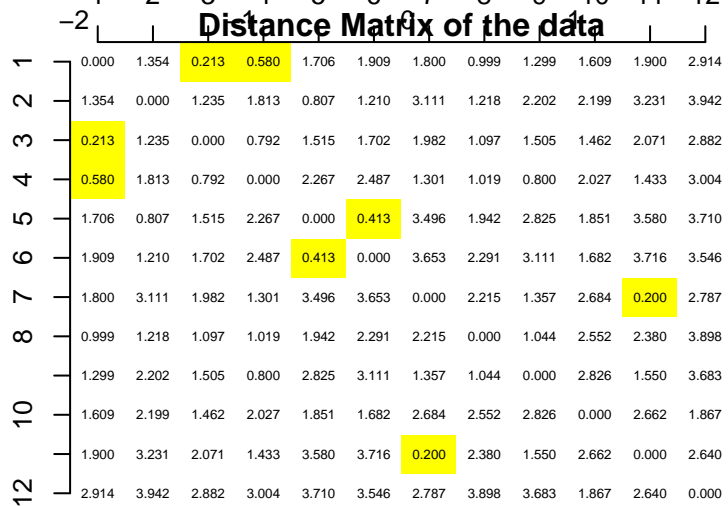
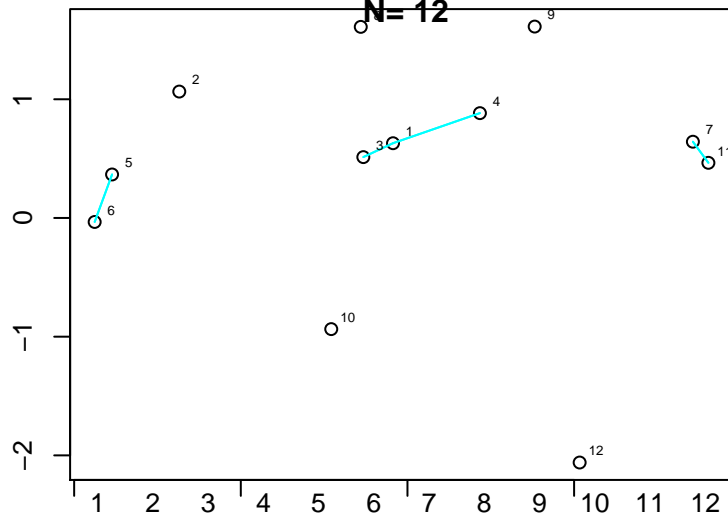
This is the 'Frame' at Euclidean distance = 0.413



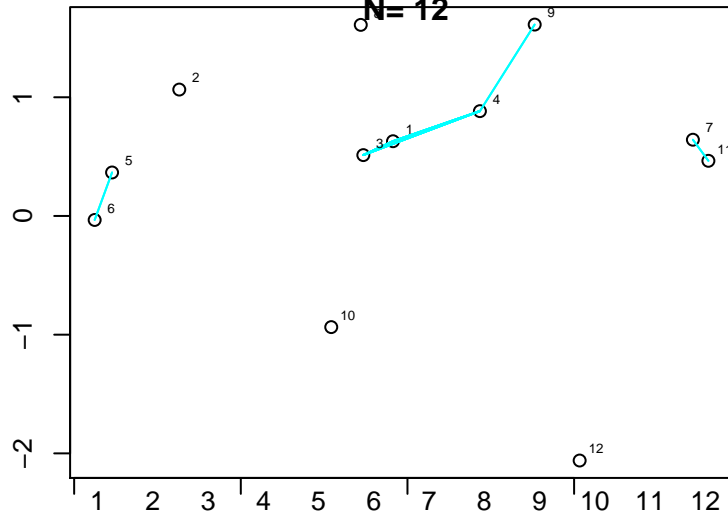
**Persistent Diagram**



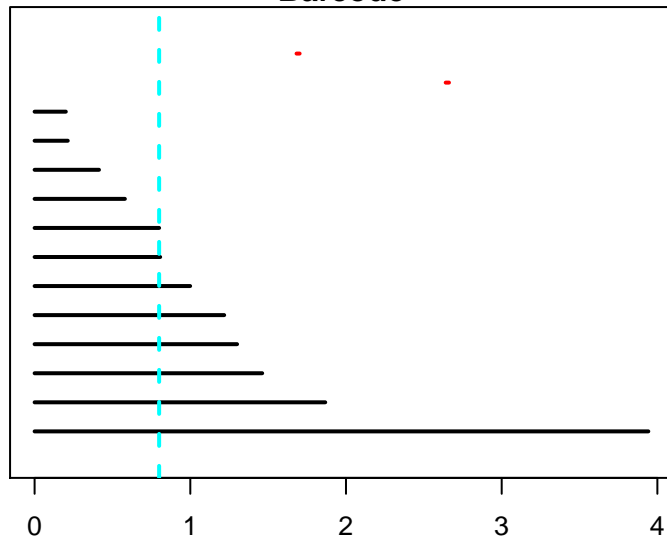
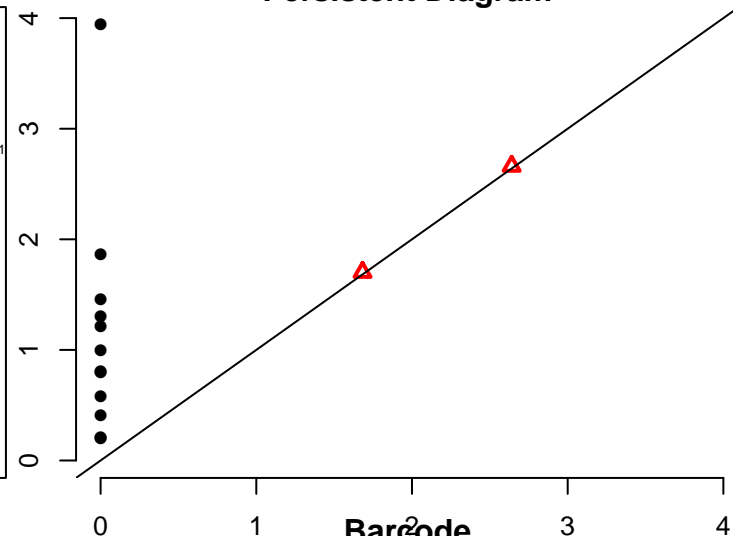
This is the 'Frame' at Euclidean distance = 0.58



This is the 'Frame' at Euclidean distance = 0.8

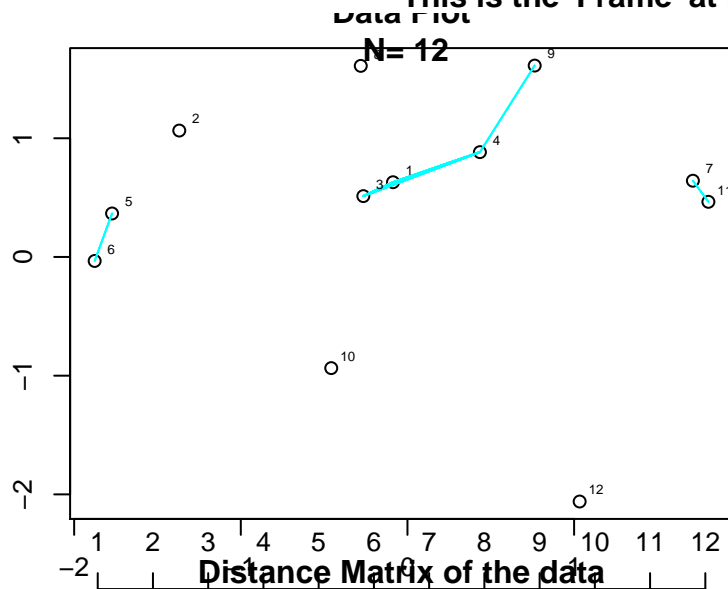


**Persistent Diagram**

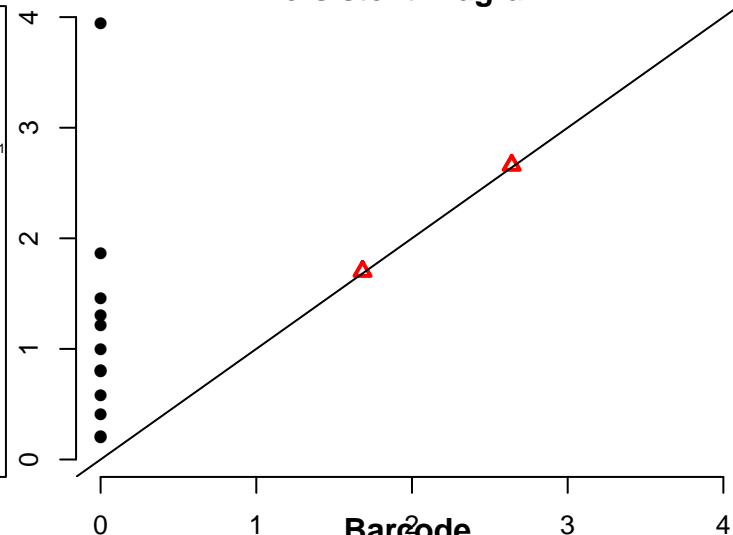




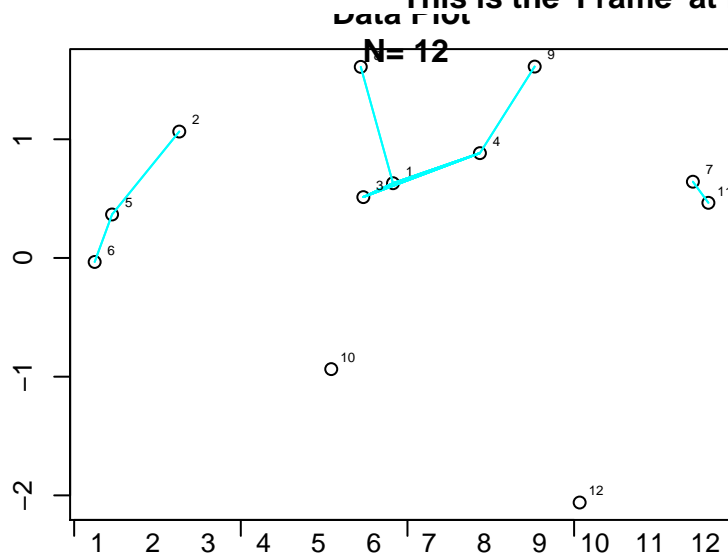
This is the 'Frame' at Euclidean distance = 0.807



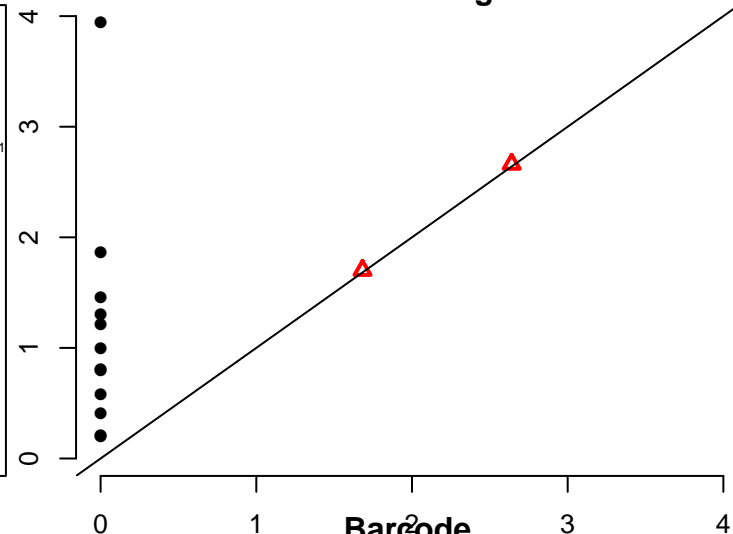
**Persistent Diagram**



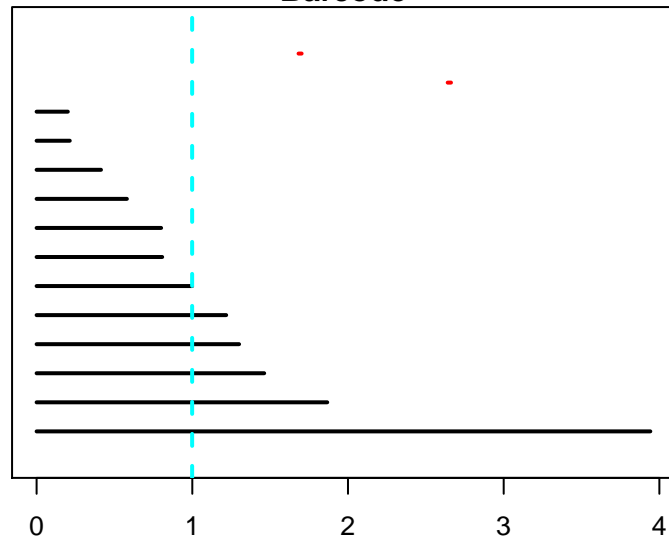
This is the 'Frame' at Euclidean distance = 0.999



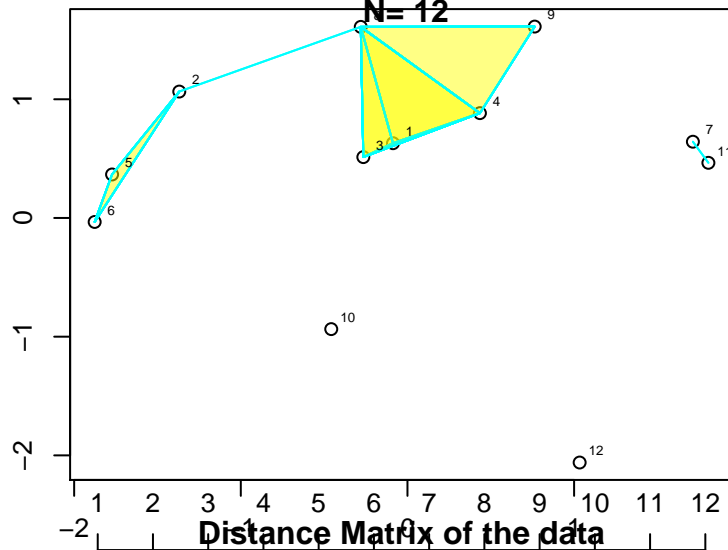
**Persistent Diagram**



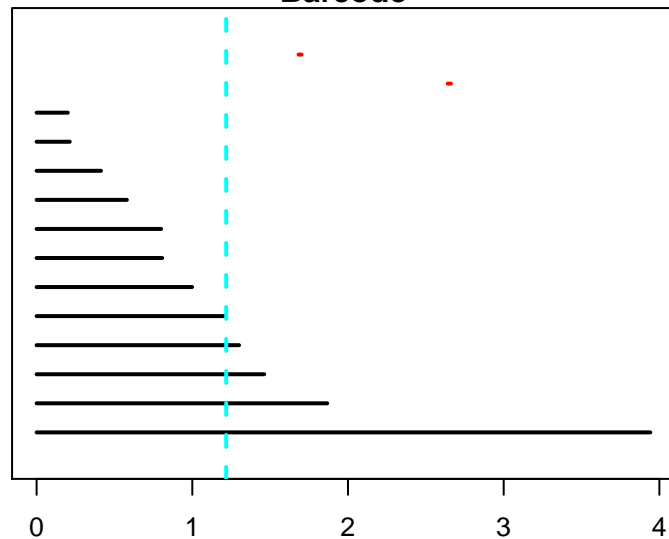
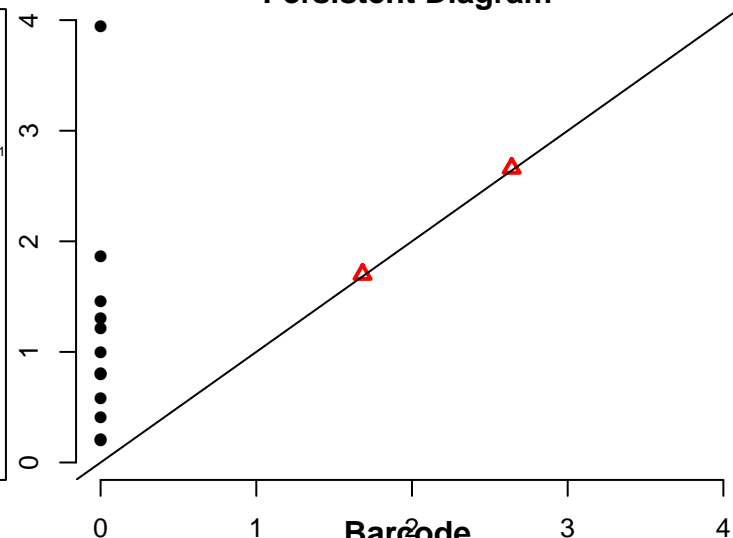
**Barcode**



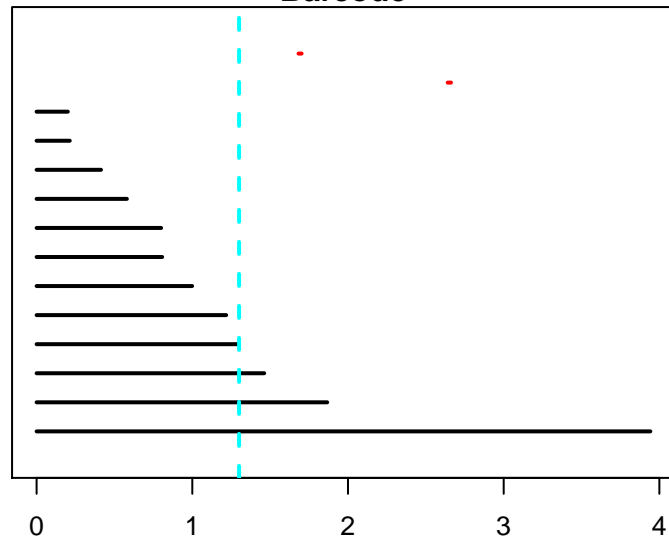
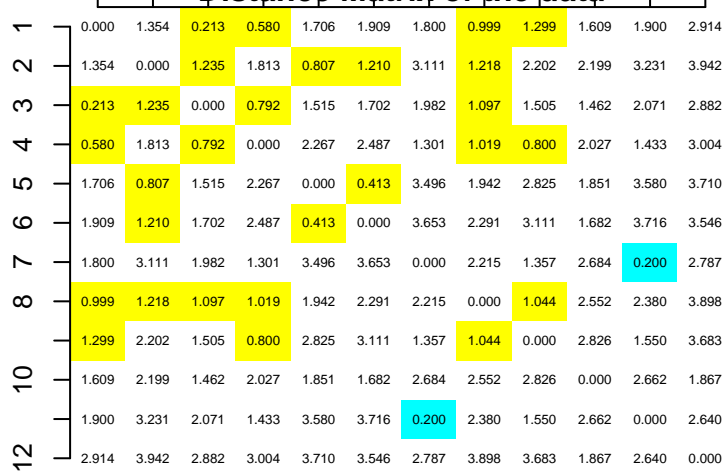
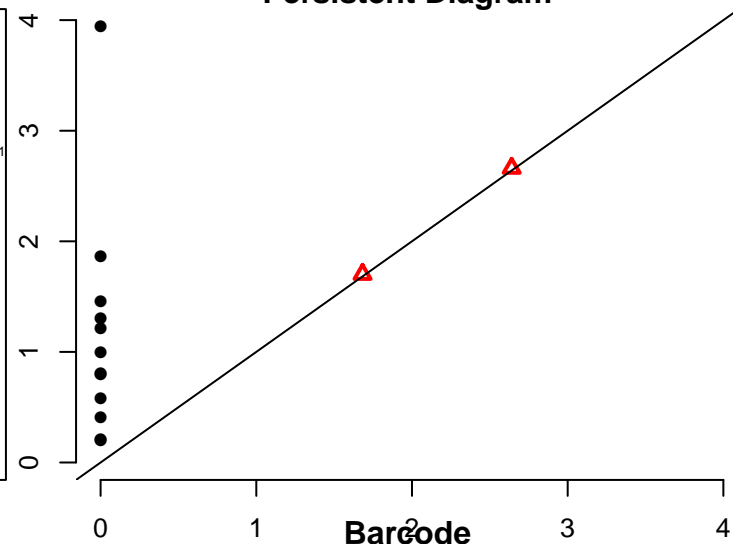
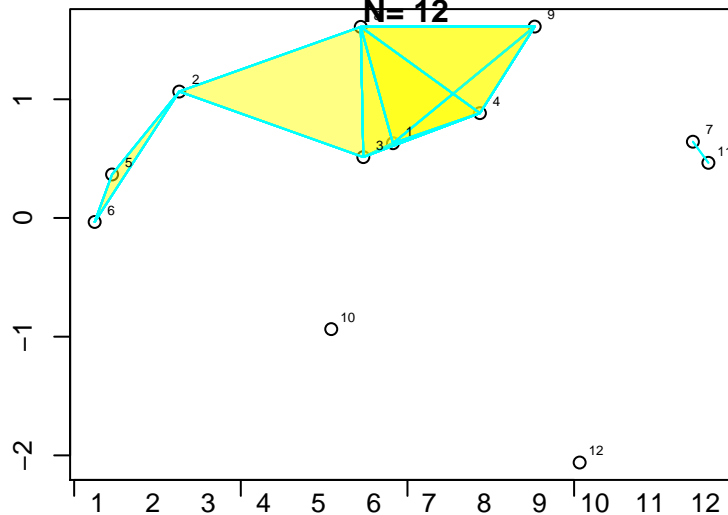
This is the 'Frame' at Euclidean distance = 1.22



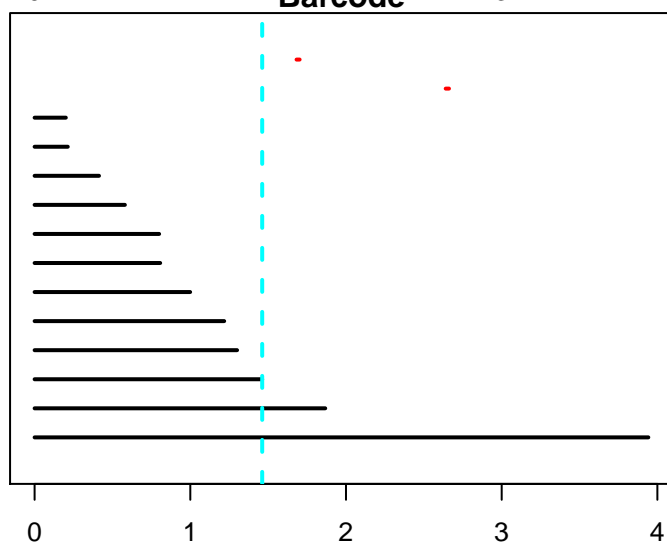
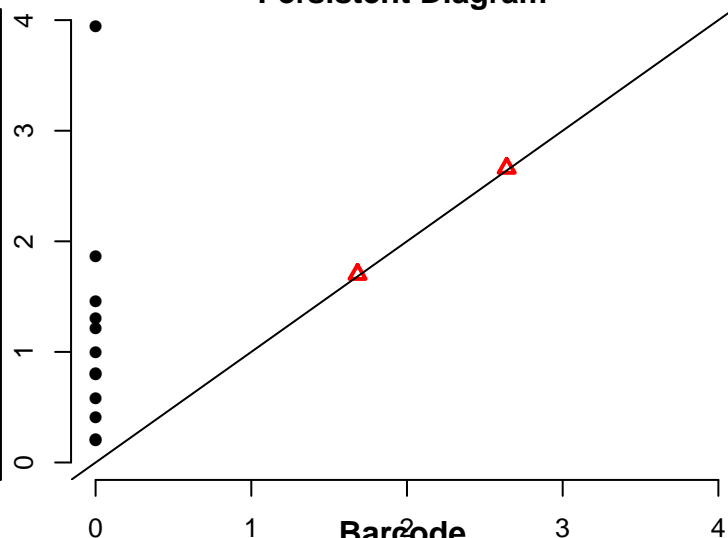
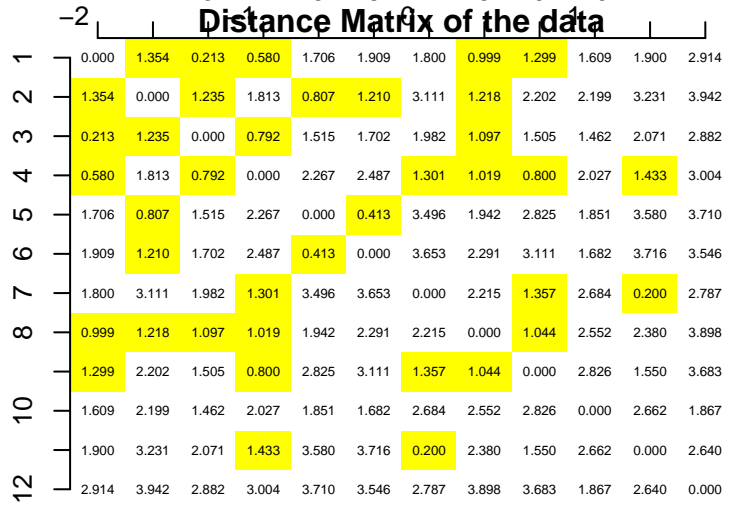
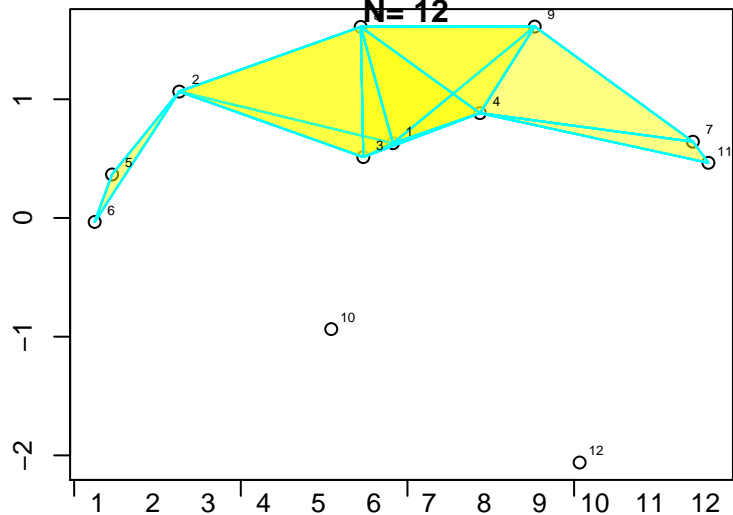
**Persistent Diagram**



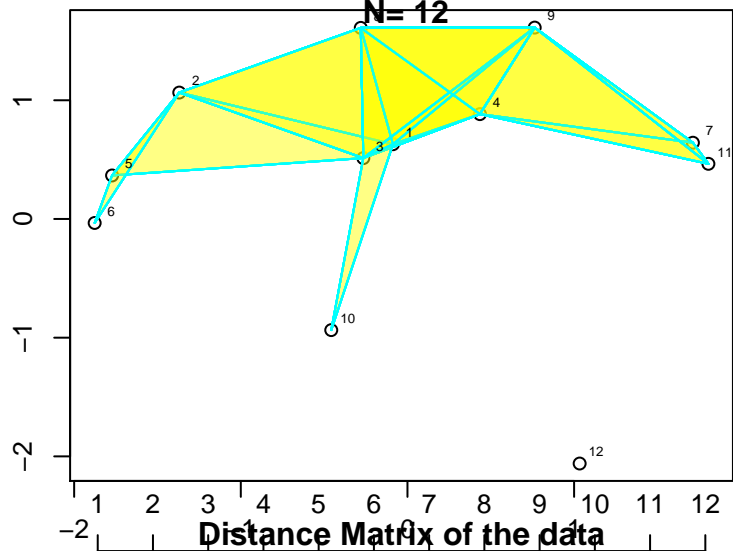
# This is the 'Frame' at Euclidean distance = 1.3



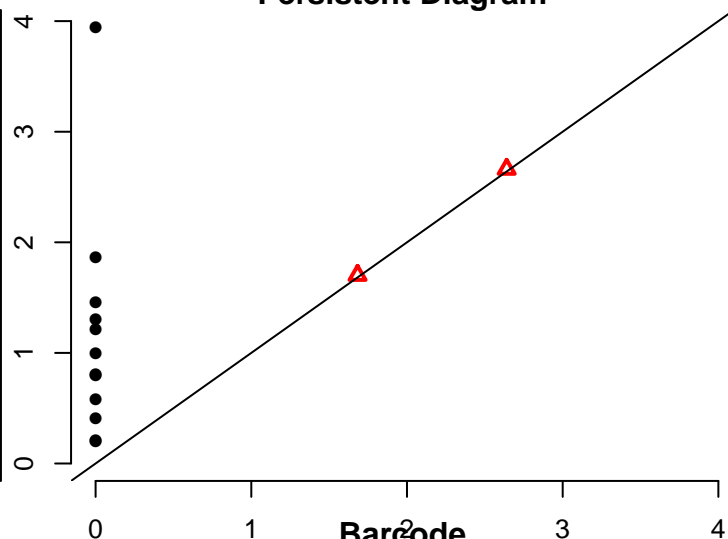
This is the 'Frame' at Euclidean distance = 1.46



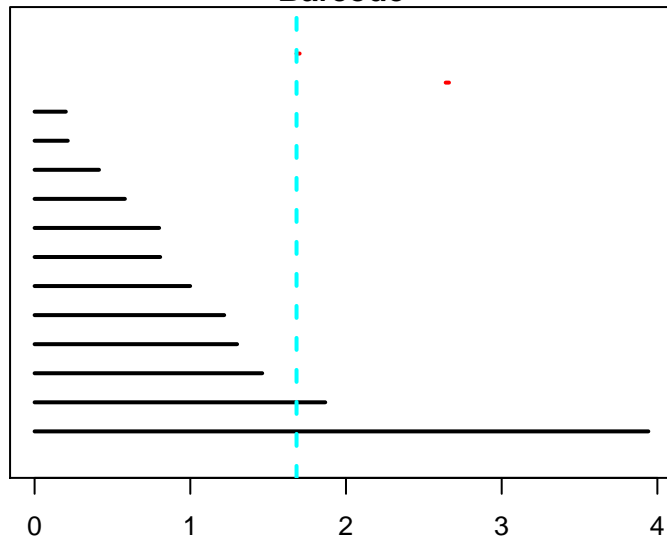
This is the 'Frame' at Euclidean distance = 1.68



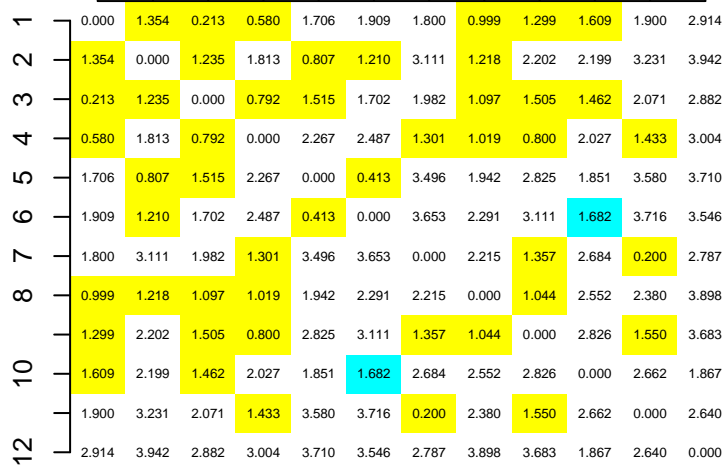
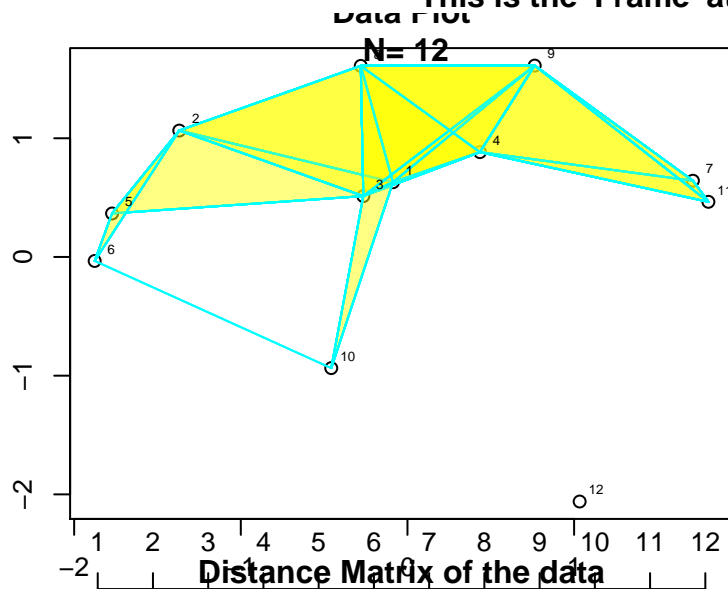
**Persistent Diagram**



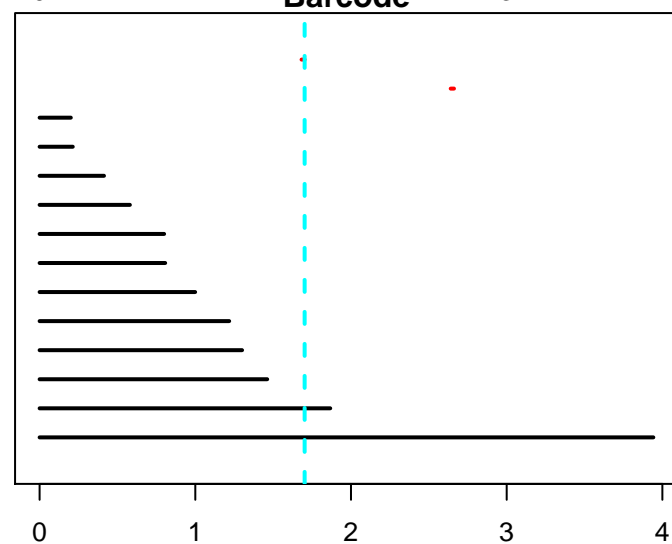
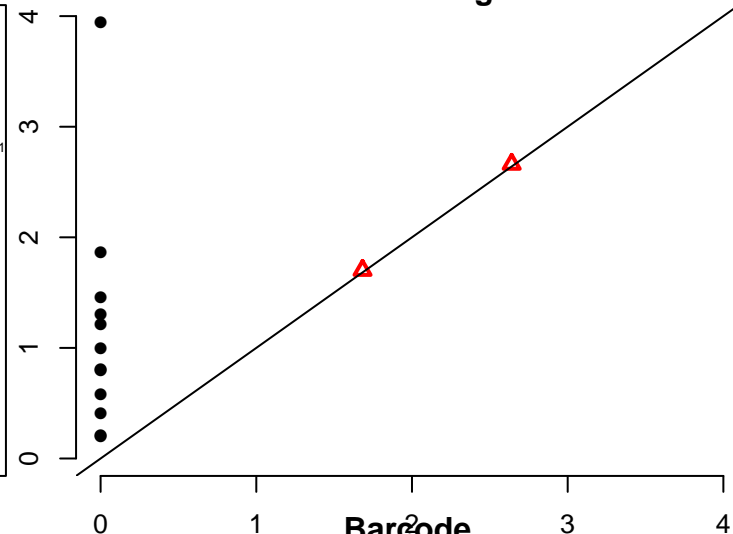
**Barcode**



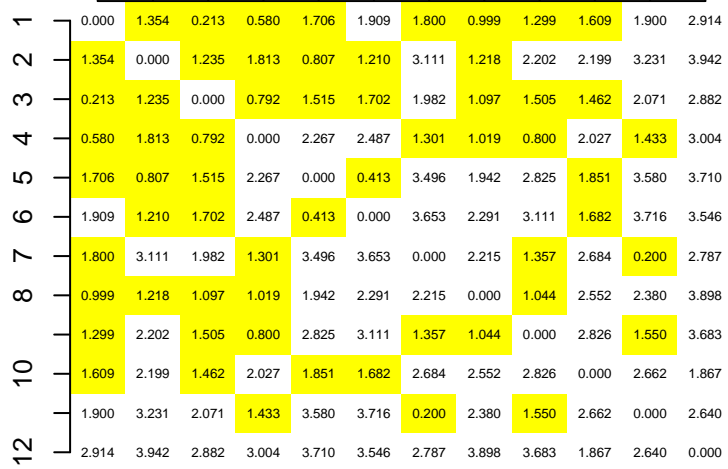
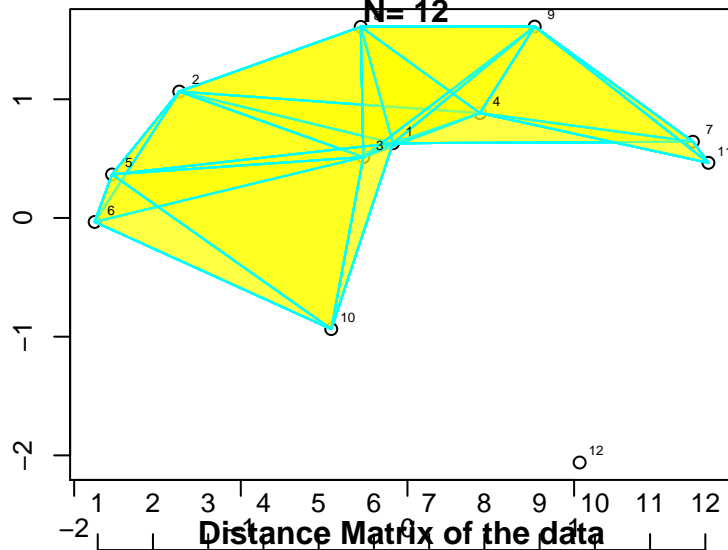
This is the 'Frame' at Euclidean distance = 1.7



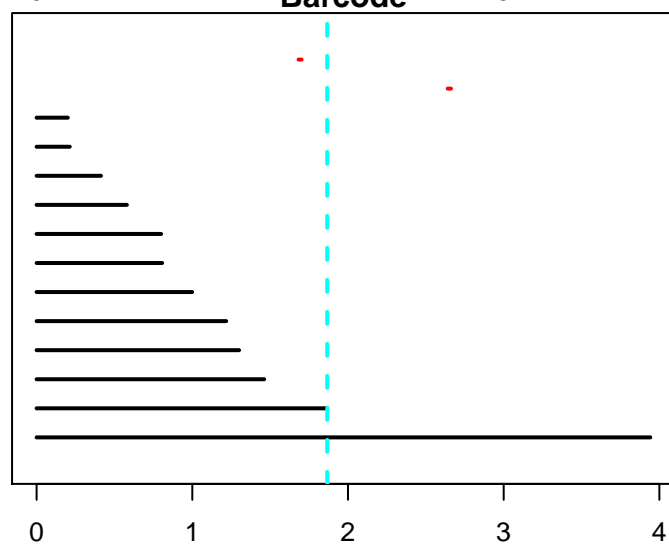
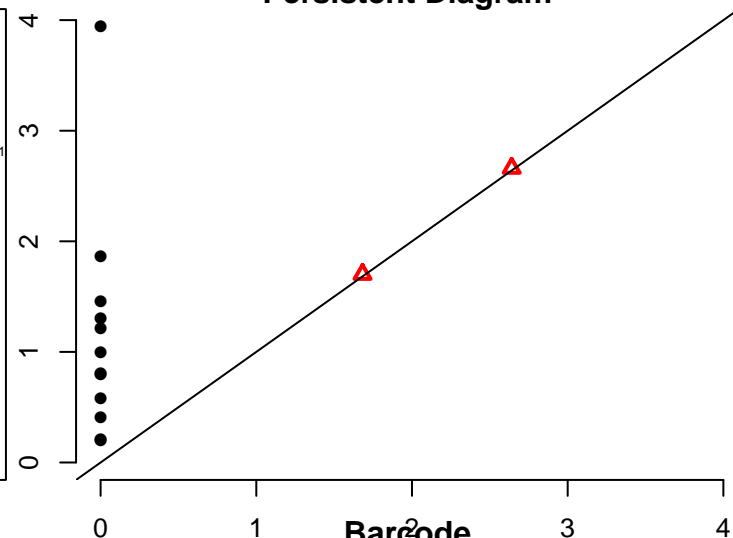
**Persistent Diagram**



This is the 'Frame' at Euclidean distance = 1.87

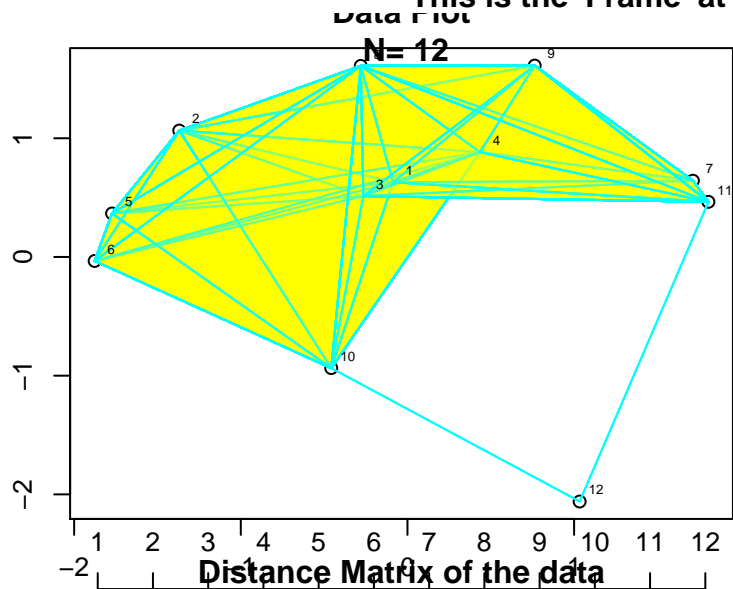


**Persistent Diagram**

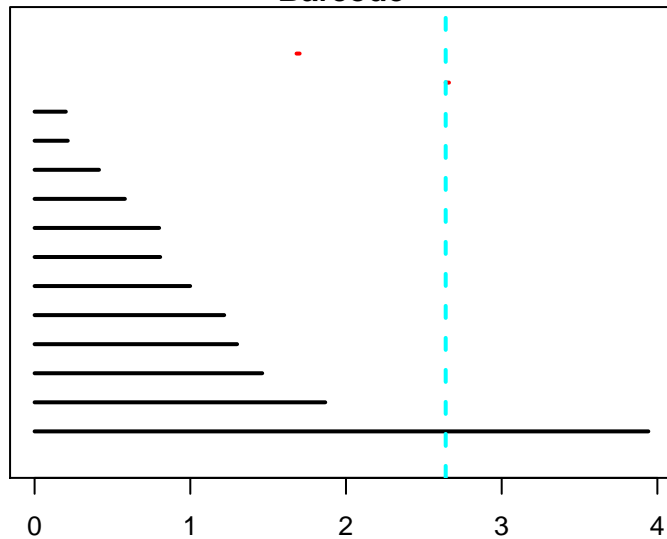
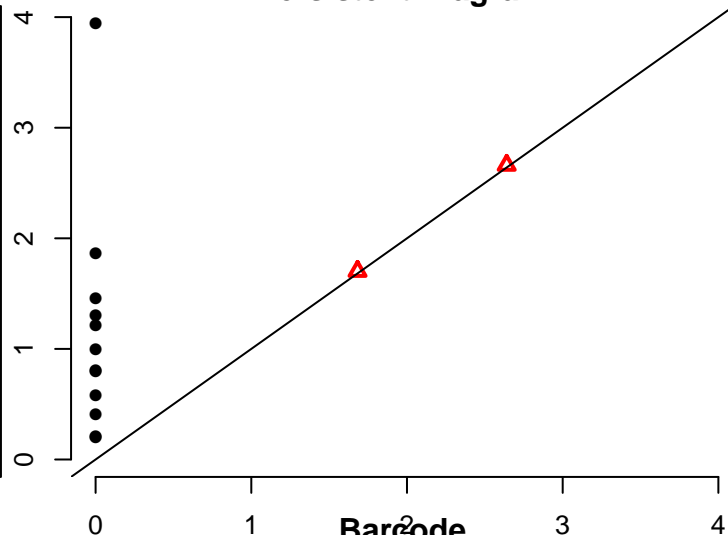




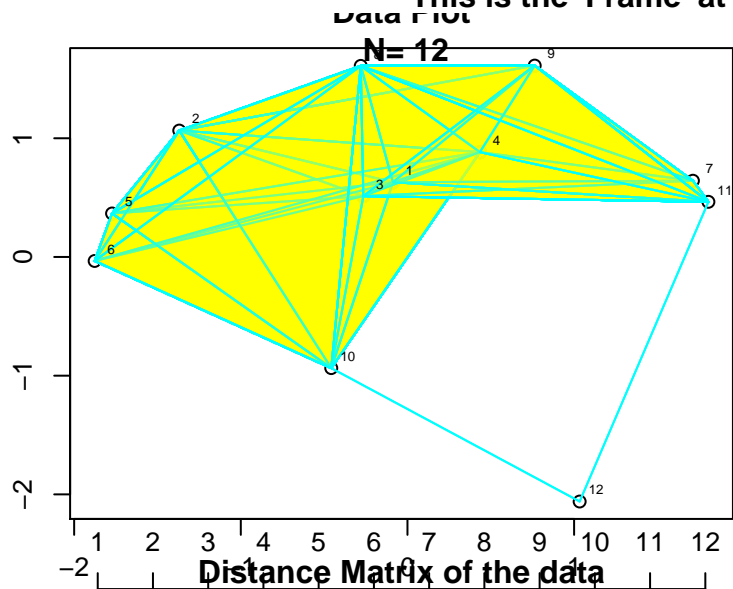
This is the 'Frame' at Euclidean distance = 2.64



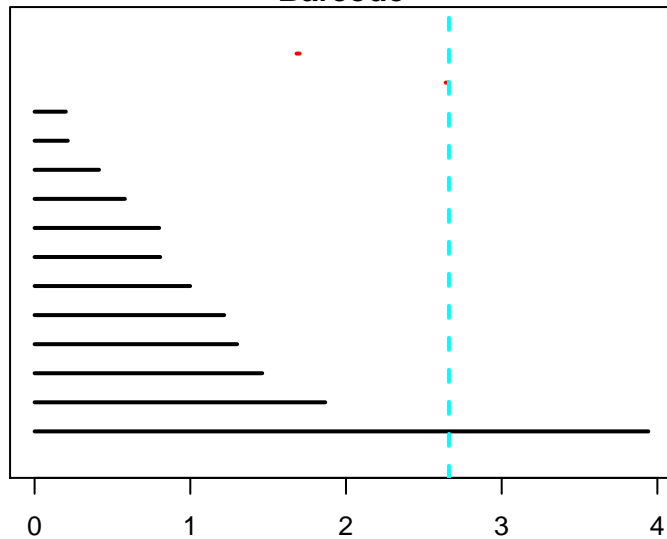
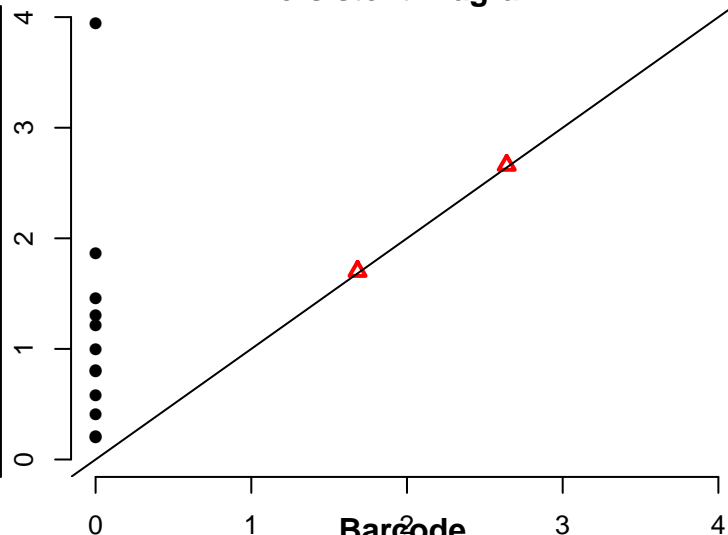
**Persistent Diagram**



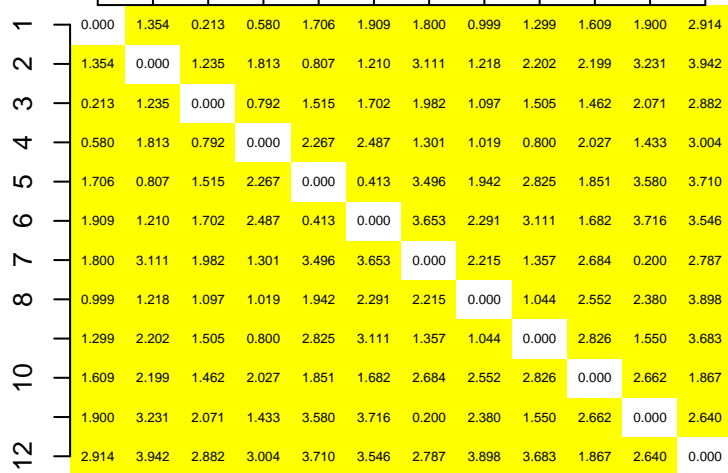
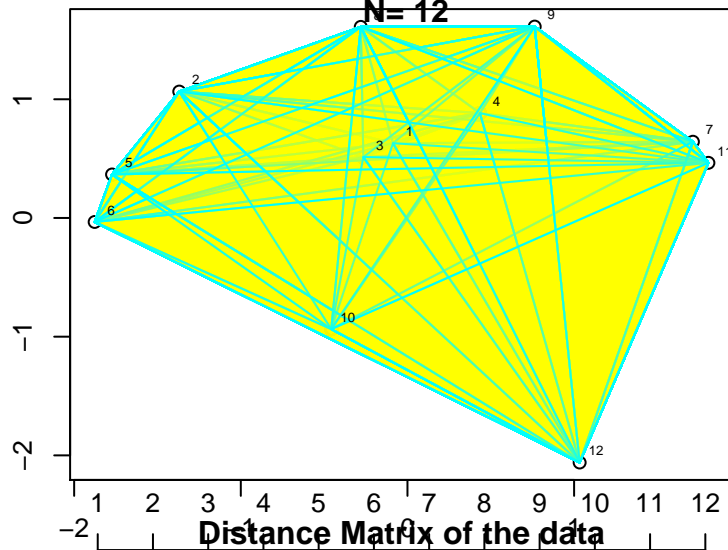
This is the 'Frame' at Euclidean distance = 2.66



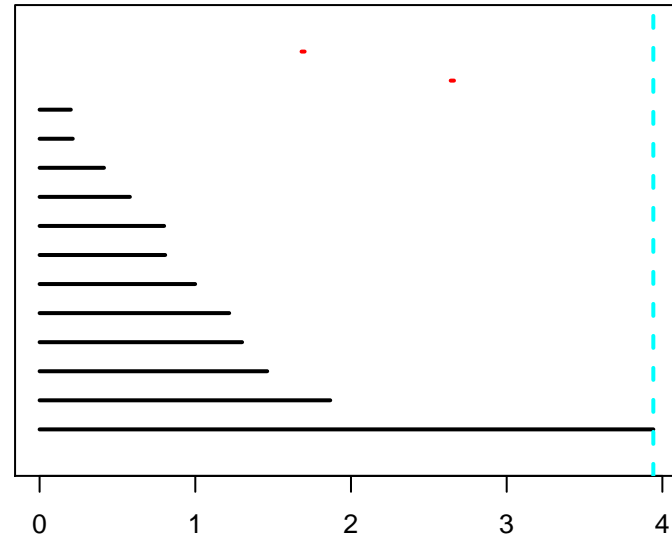
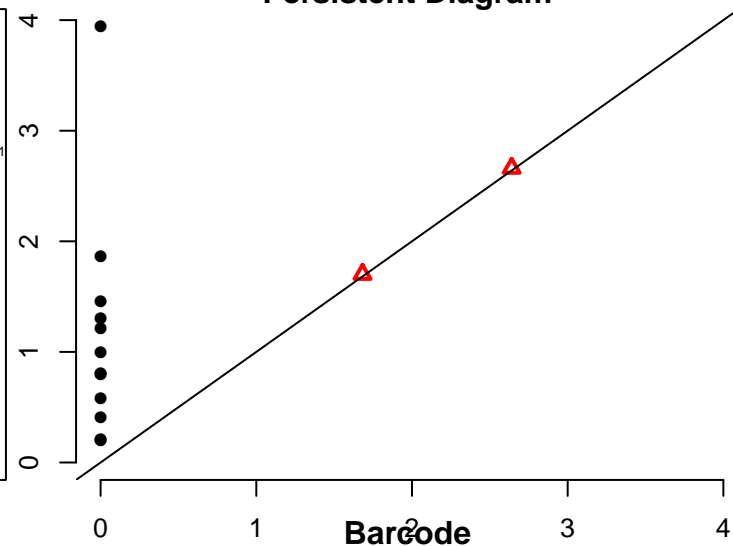
**Persistent Diagram**



This is the 'Frame' at Euclidean distance = 3.94

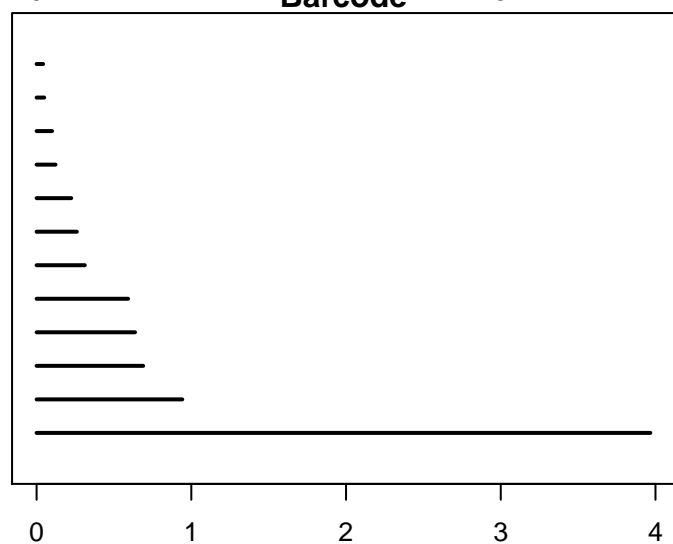
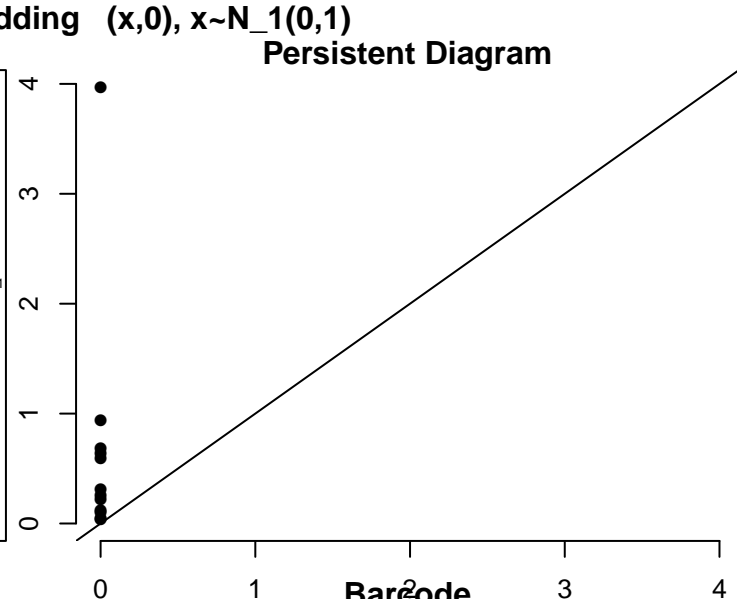
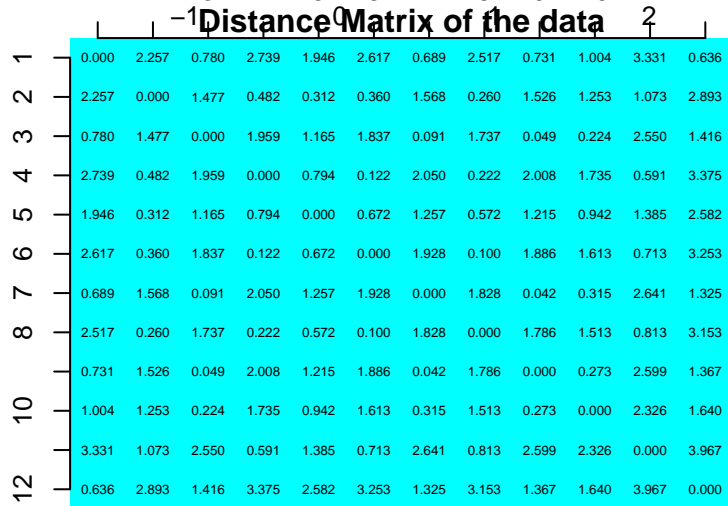
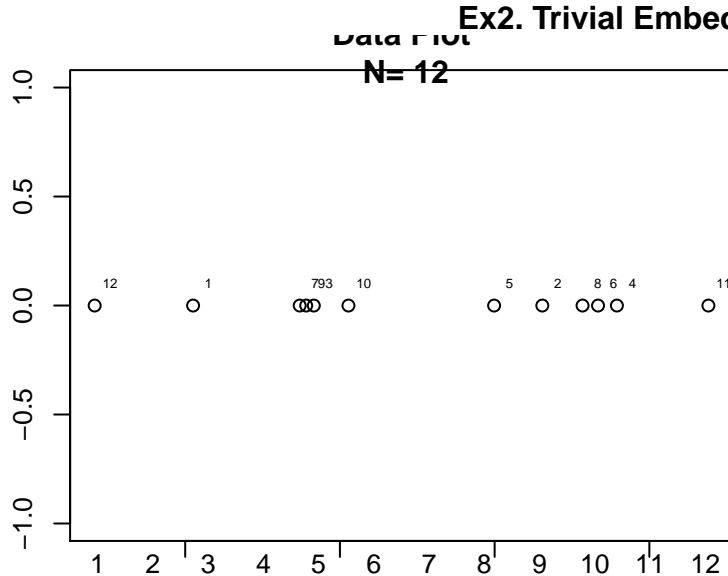


**Persistent Diagram**

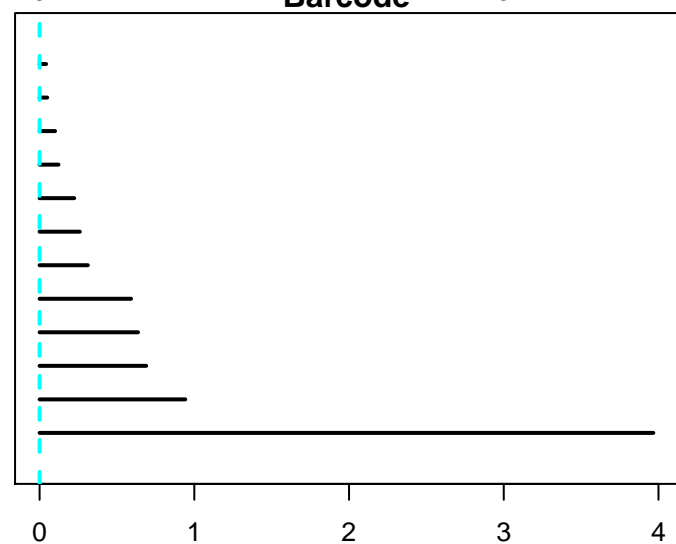
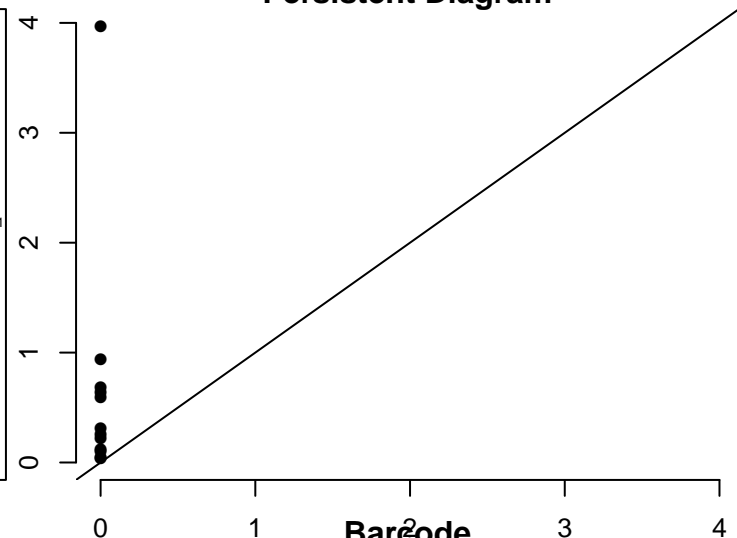
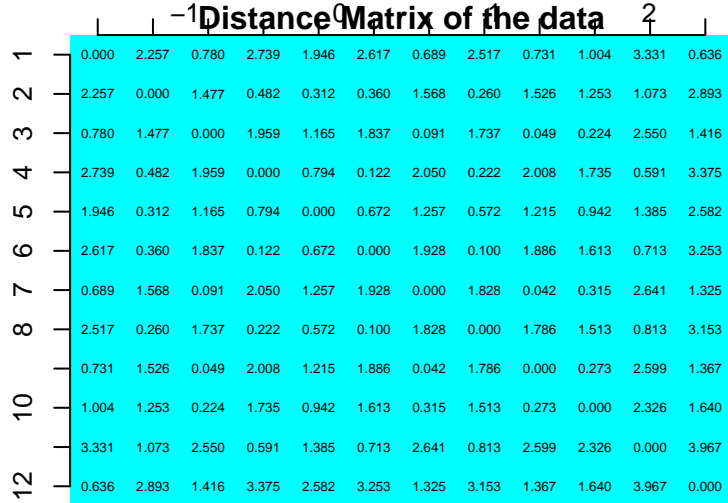
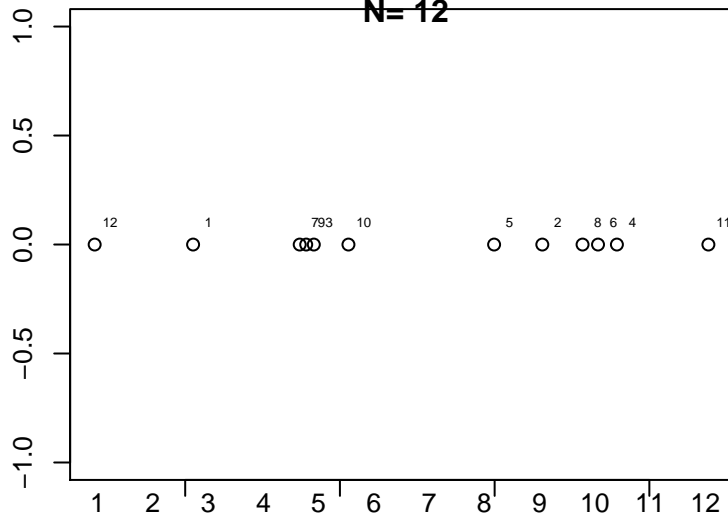


Result and Frame-by-frame plots for Example 2

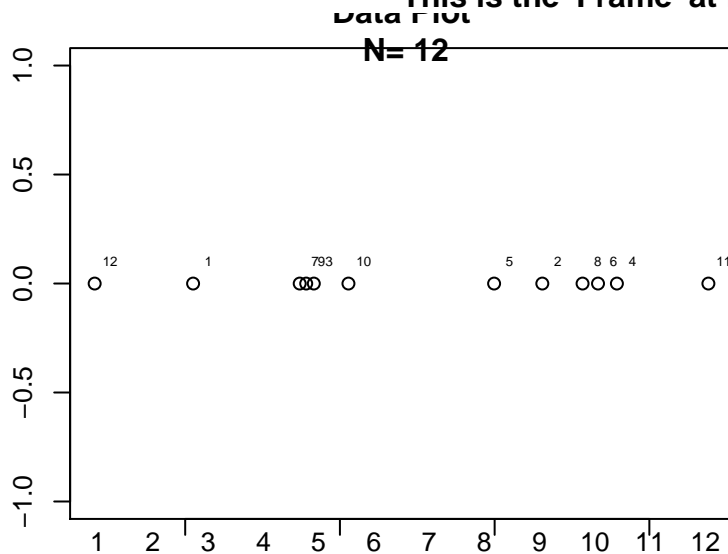
# Ex2. Trivial Embedding (x,0), x~N\_1(0,1)



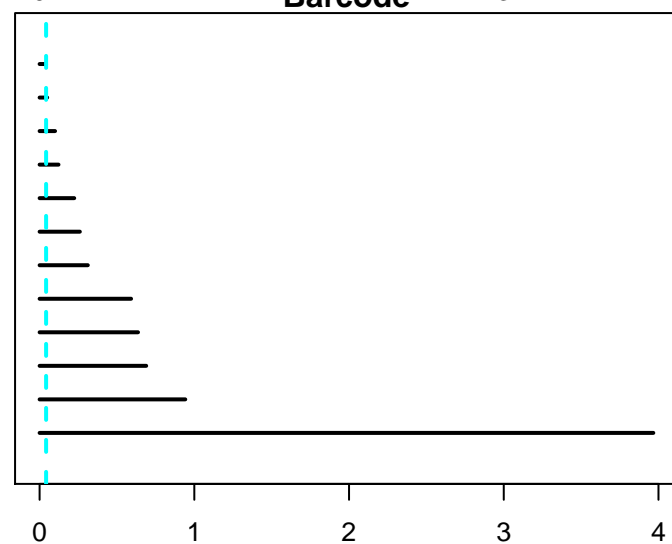
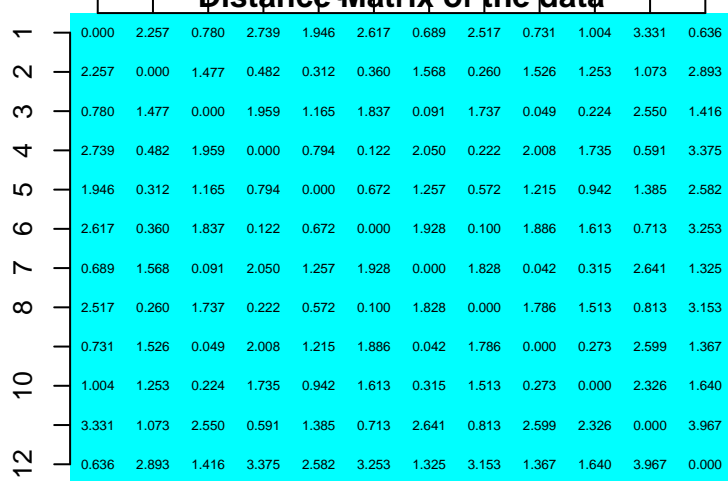
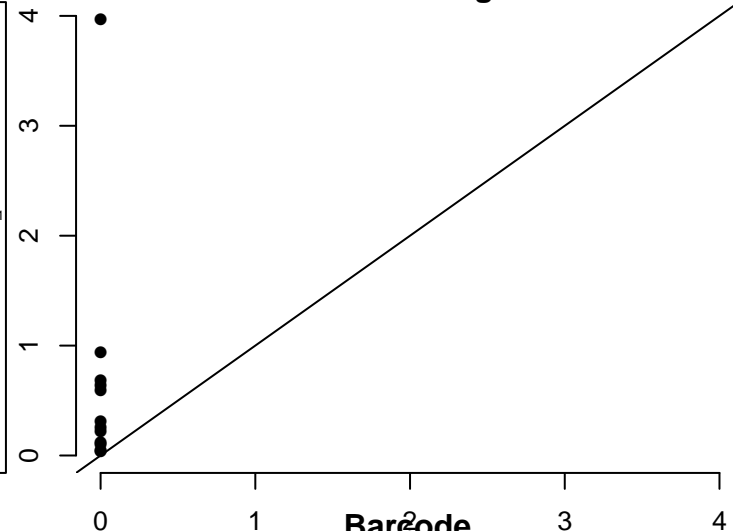
# This is the 'Frame' at Euclidean distance = 0



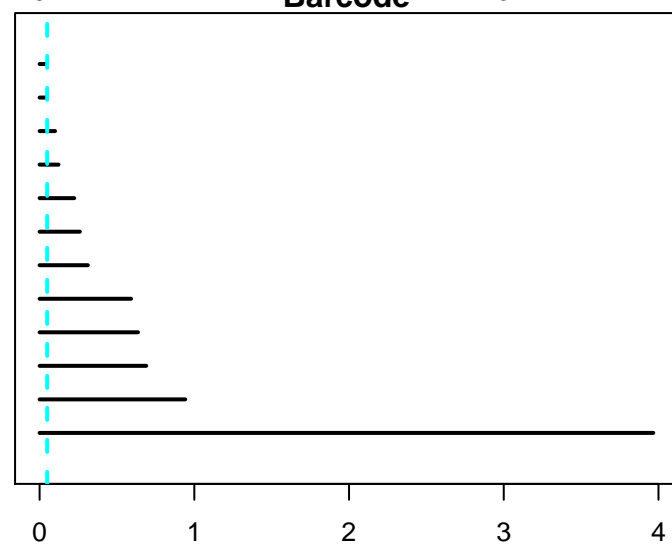
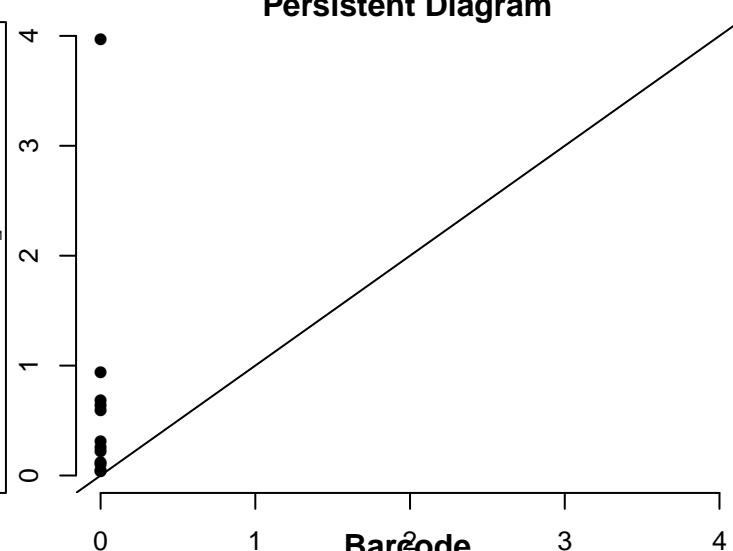
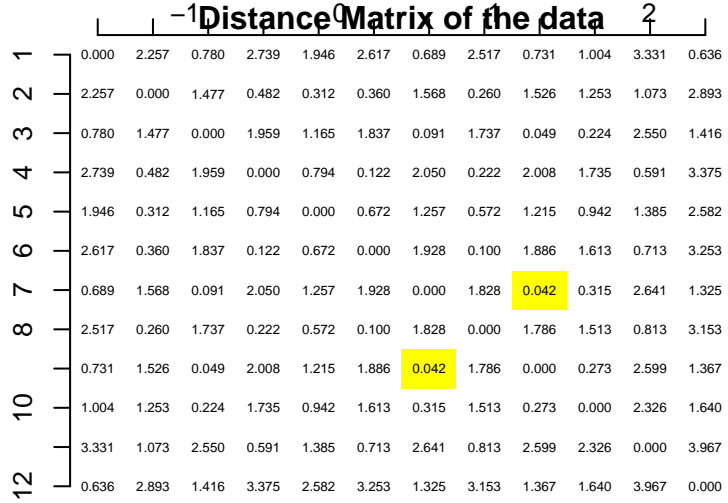
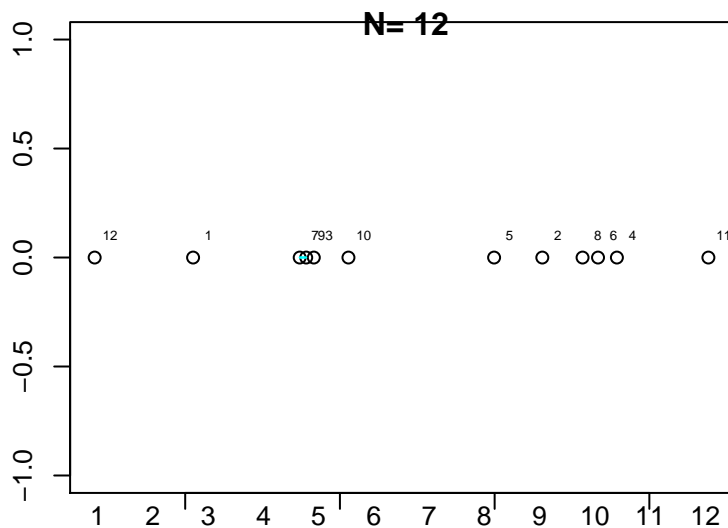
This is the 'Frame' at Euclidean distance = 0.042



**Persistent Diagram**

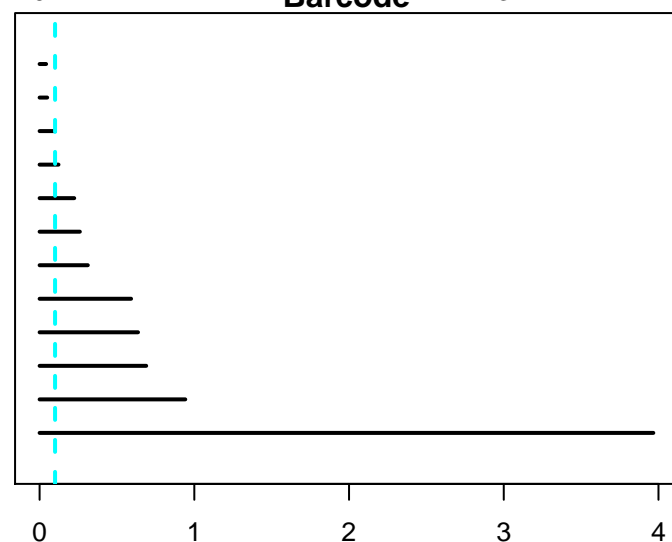
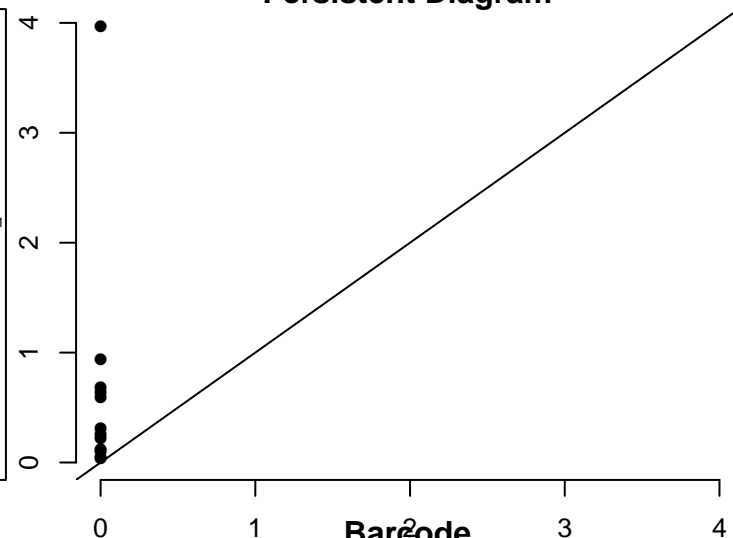
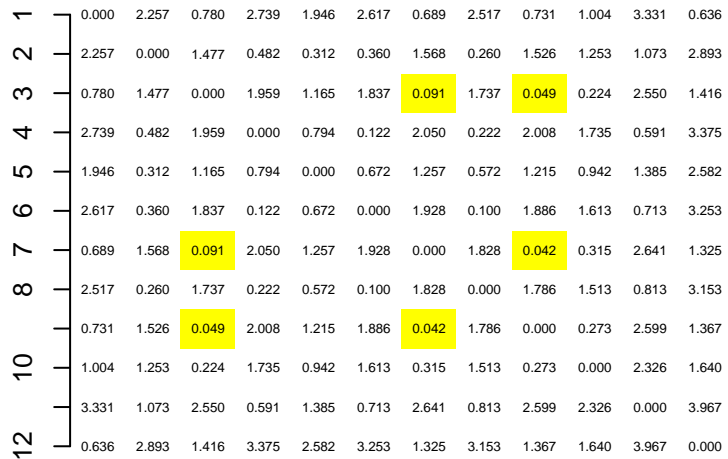
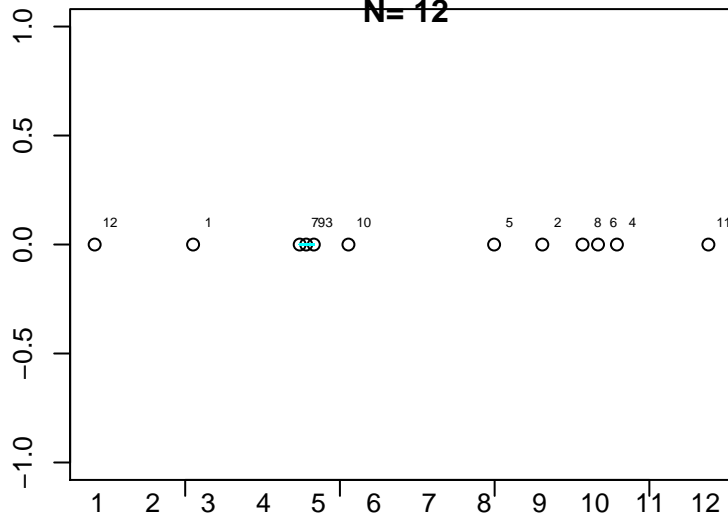


This is the 'Frame' at Euclidean distance = 0.0491

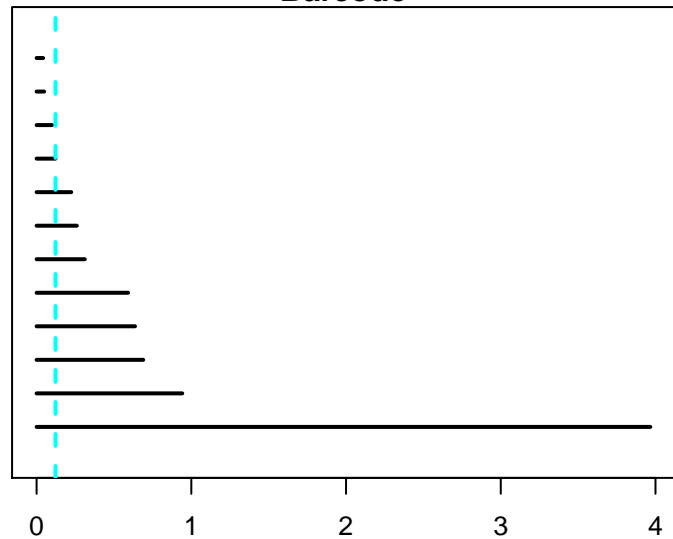
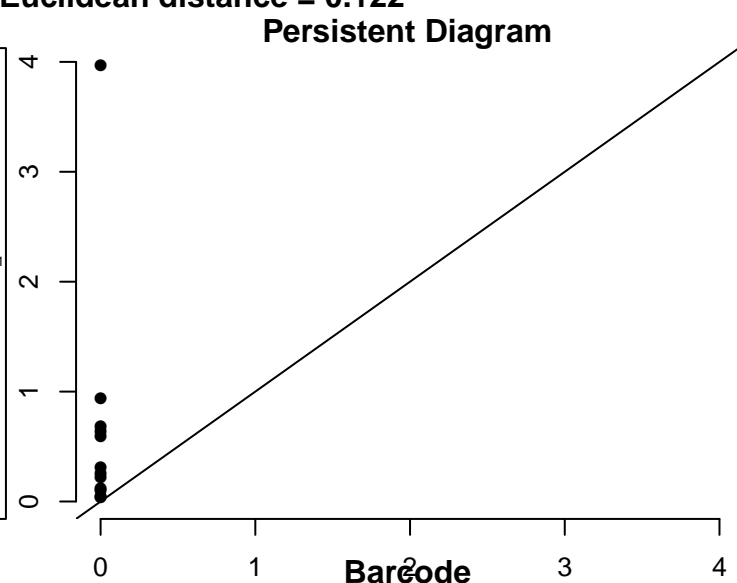
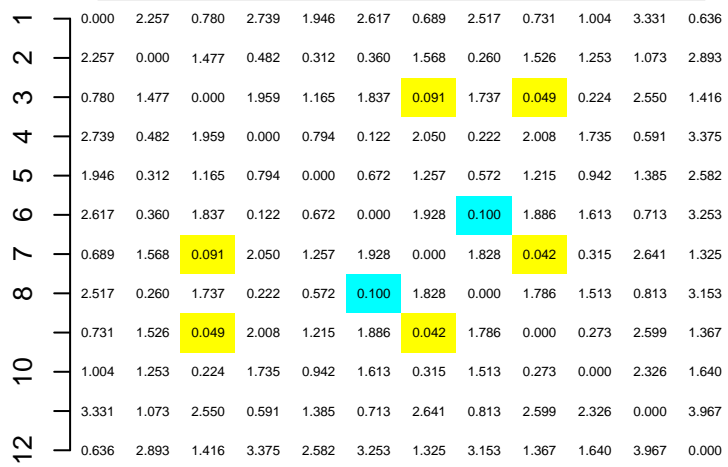
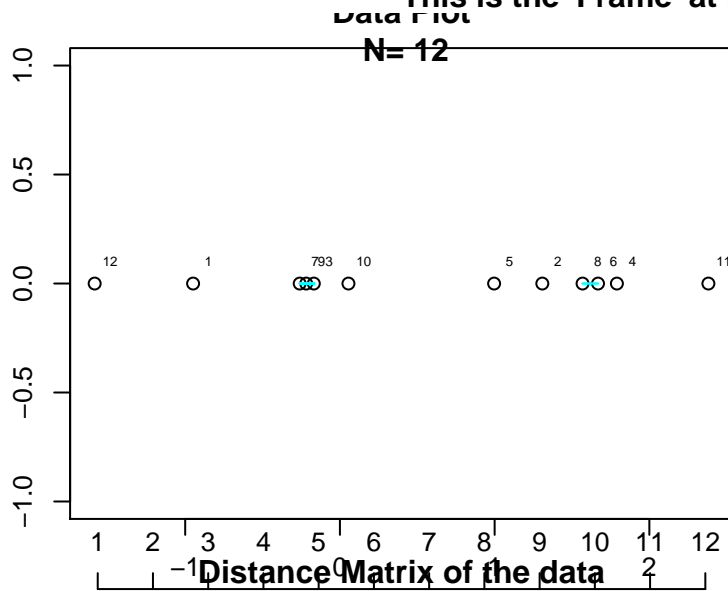




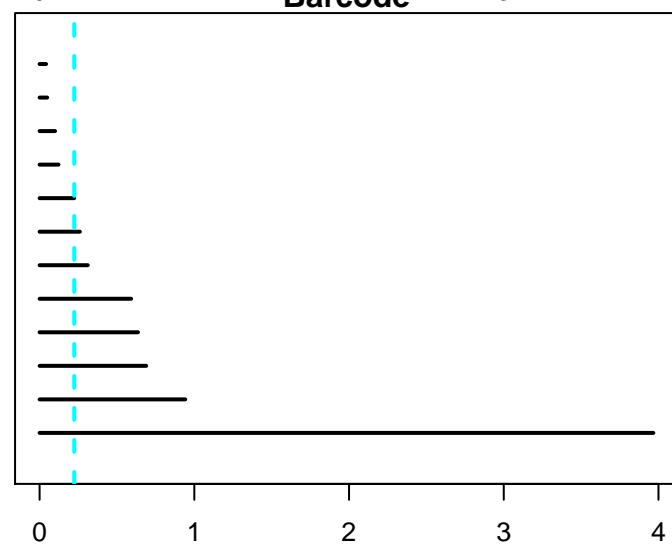
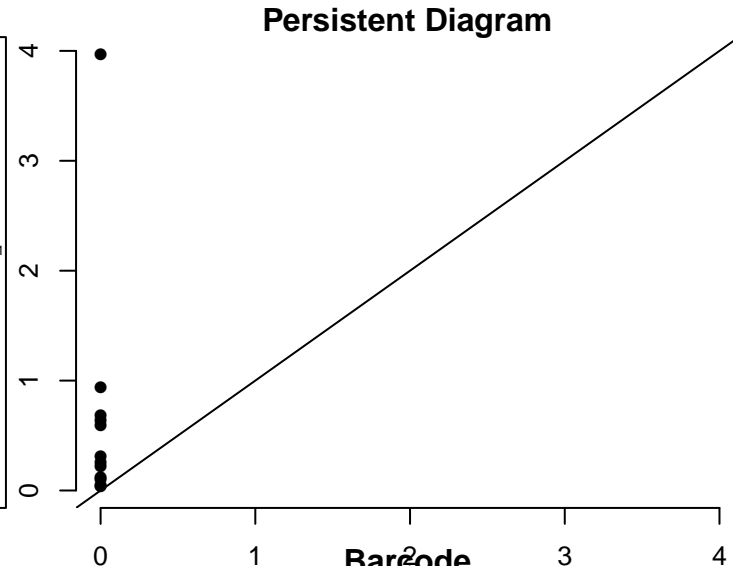
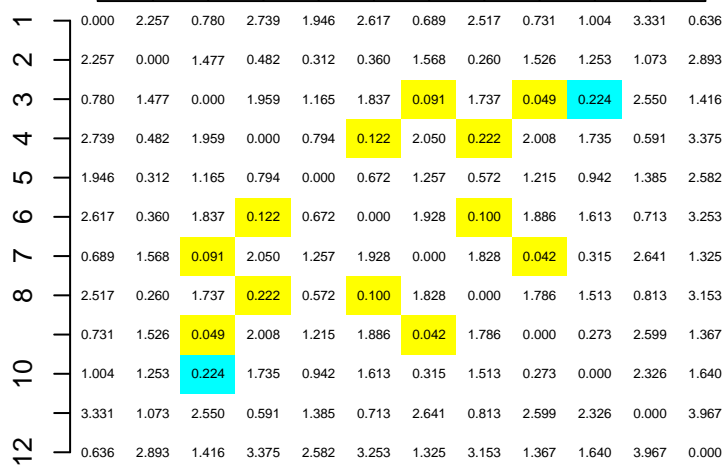
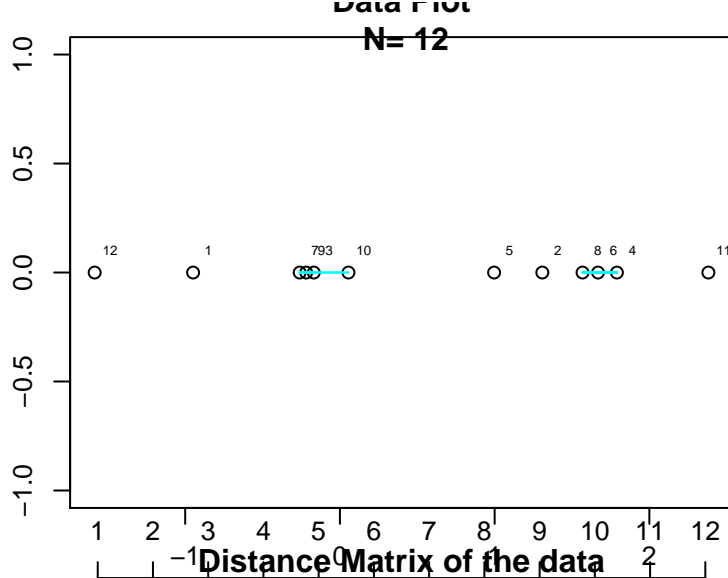
This is the 'Frame' at Euclidean distance = 0.0998



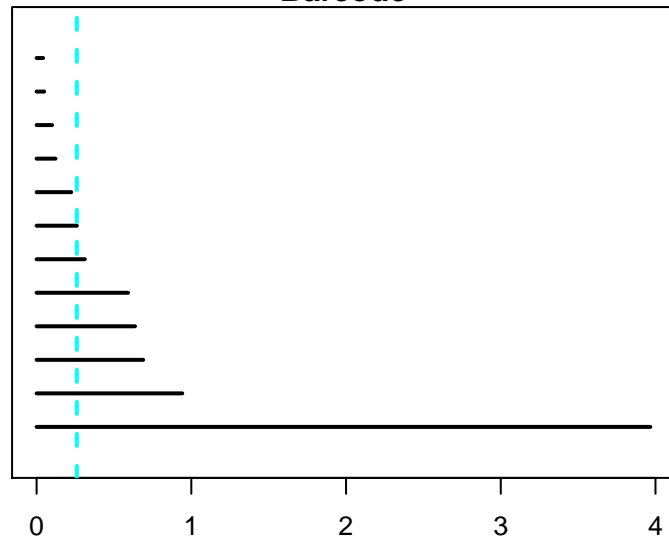
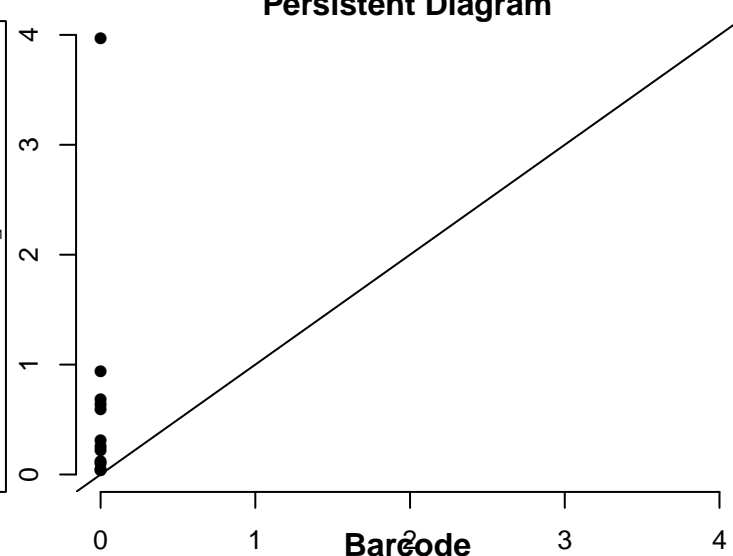
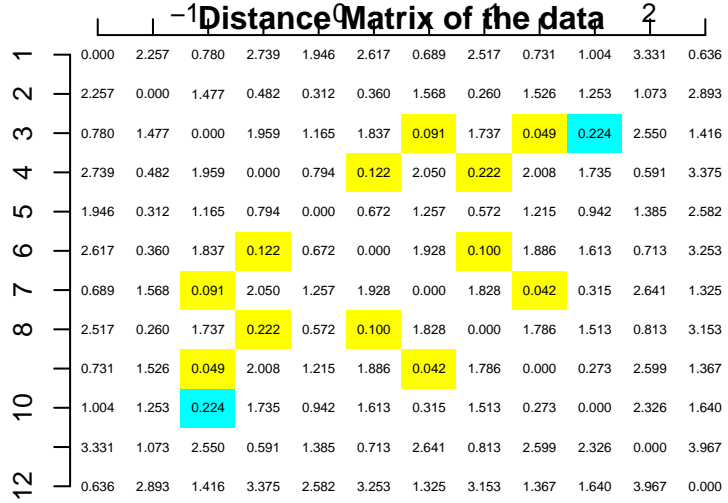
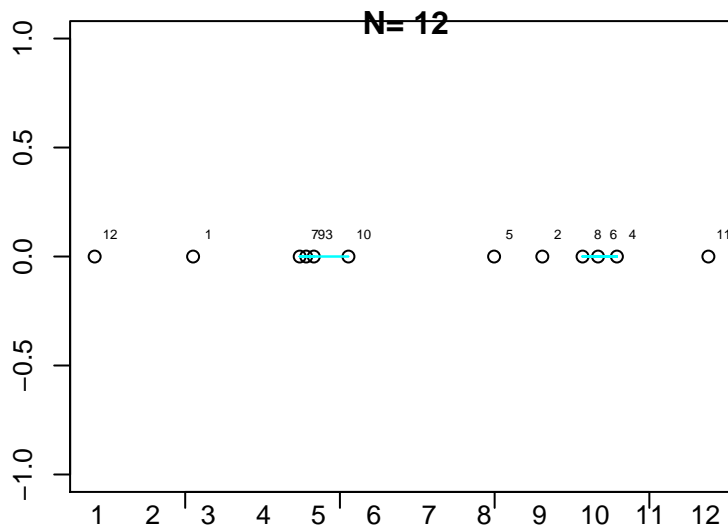
This is the 'Frame' at Euclidean distance = 0.122



This is the 'Frame' at Euclidean distance = 0.224

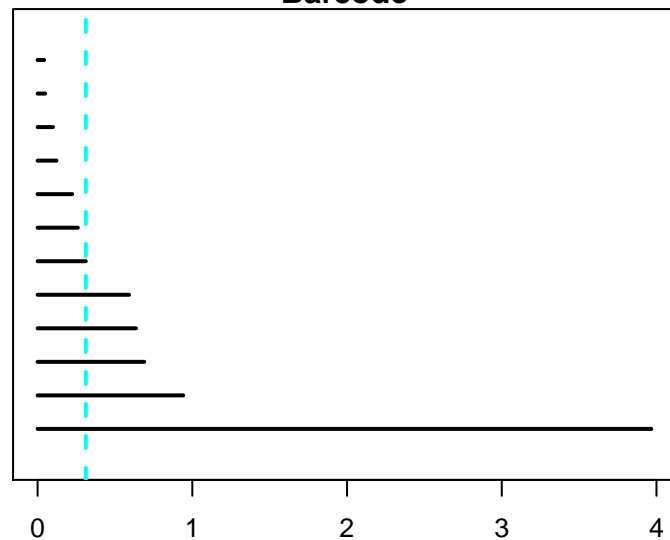
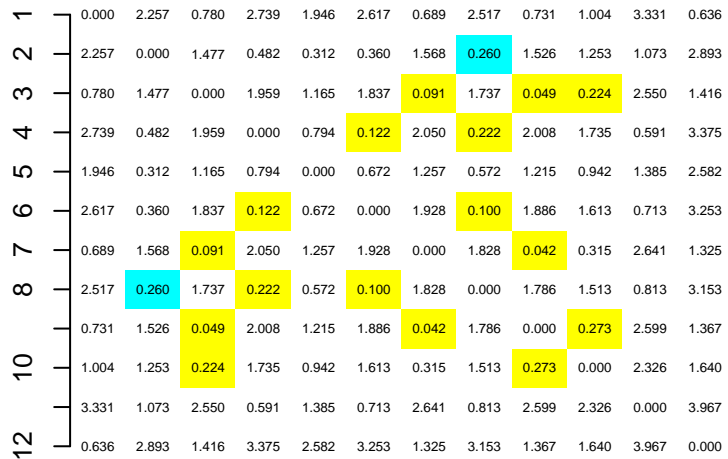
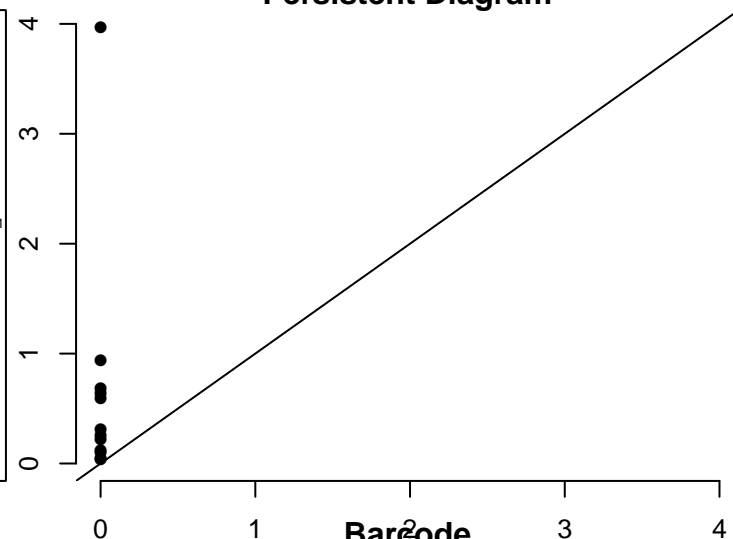
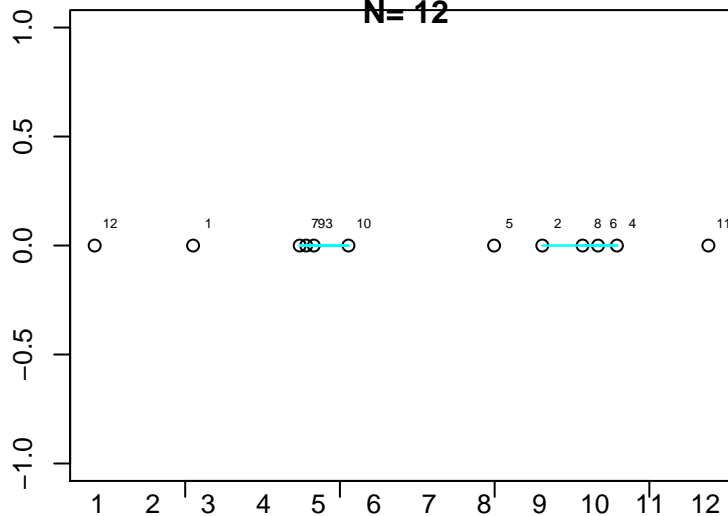


This is the 'Frame' at Euclidean distance = 0.26



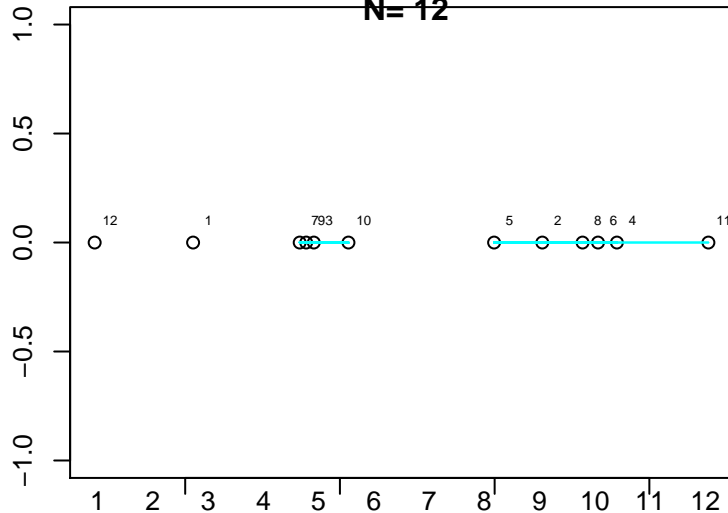
This is the 'Frame' at Euclidean distance = 0.312

Persistent Diagram

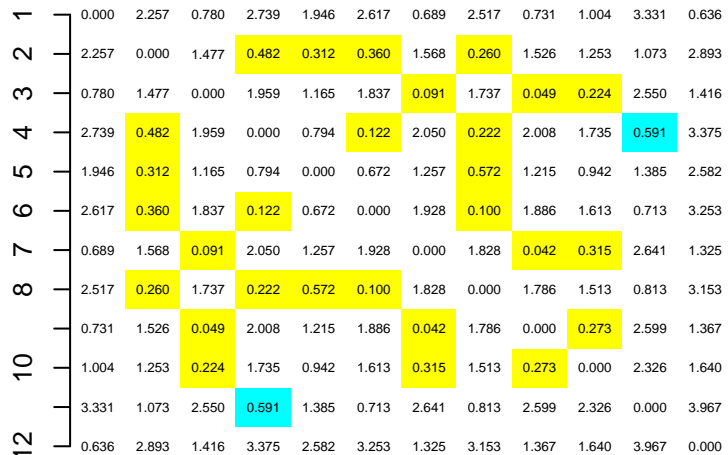


Data Plot

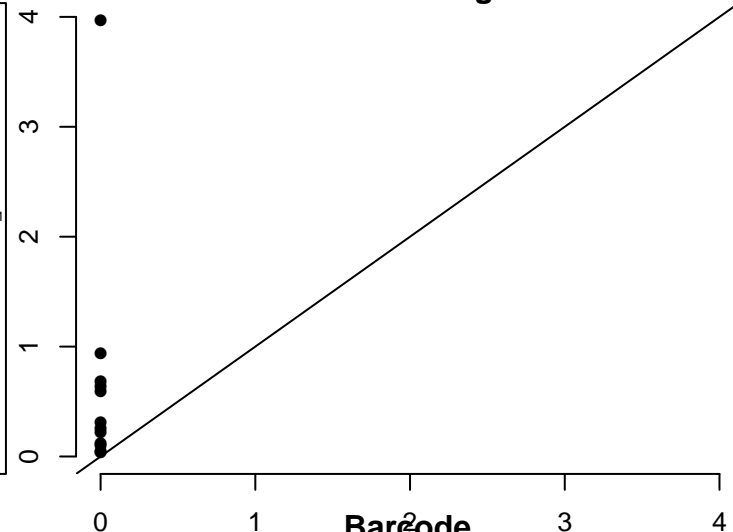
N=12



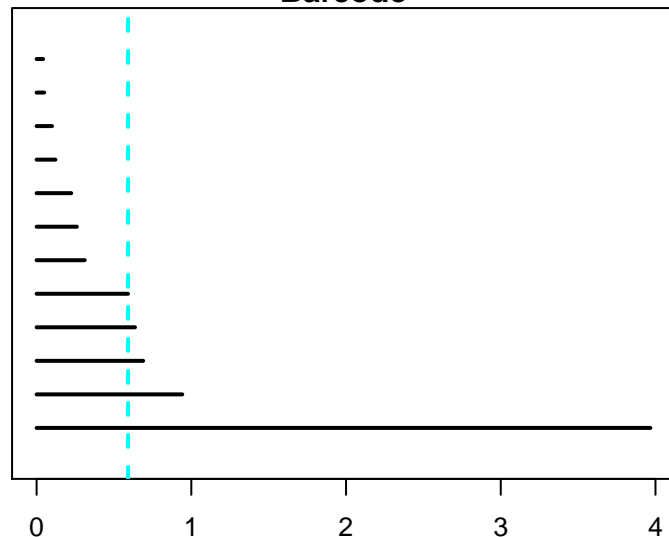
Distance Matrix of the data



Persistent Diagram

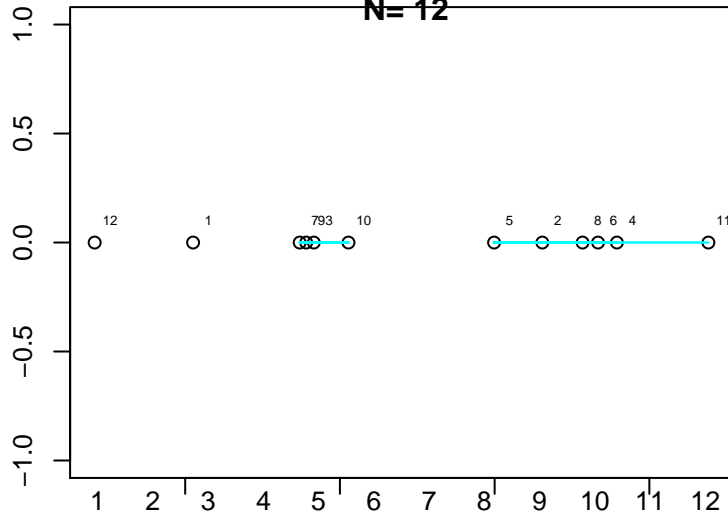


Barcode



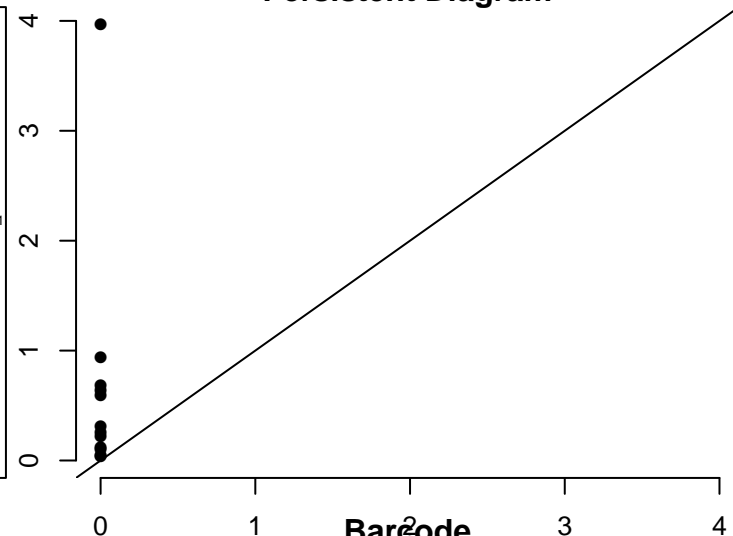
Data Plot

N=12

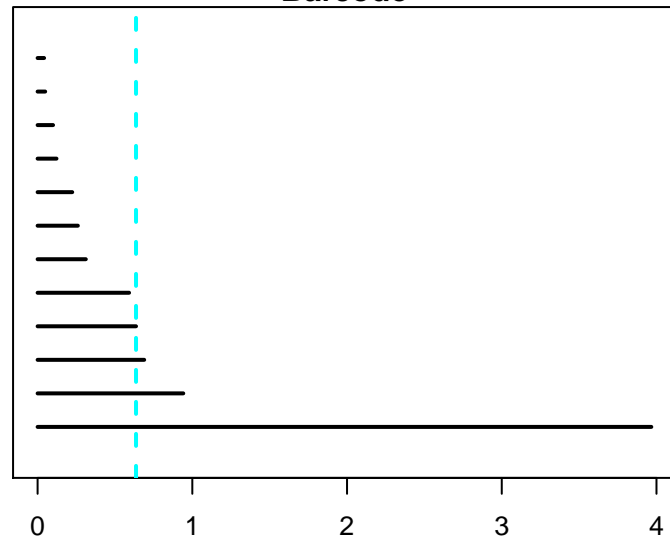


This is the 'Frame' at Euclidean distance = 0.636

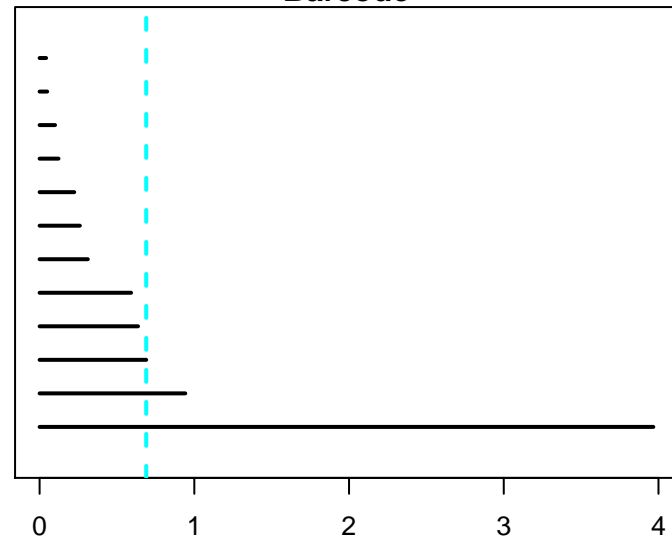
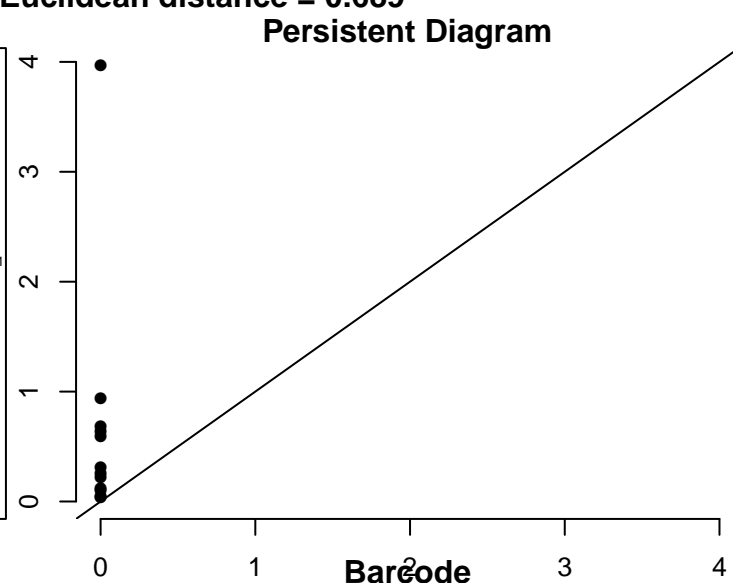
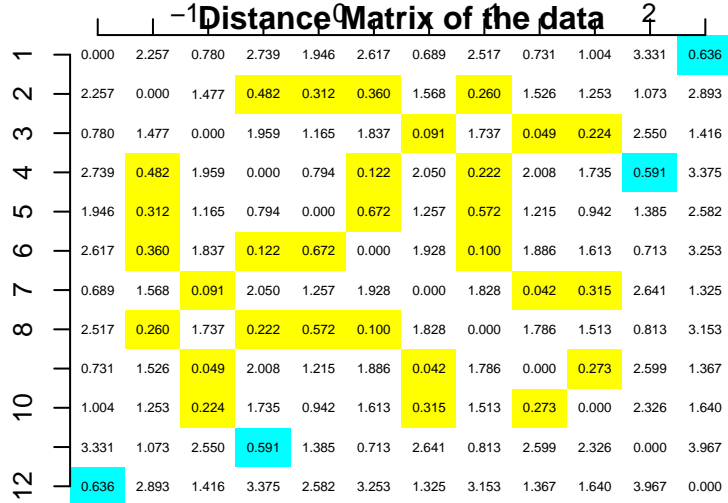
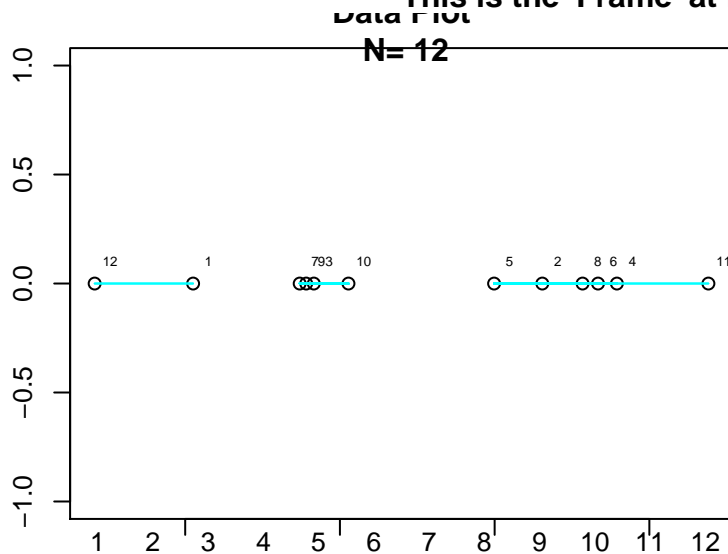
Persistent Diagram



	0.000	2.257	0.780	2.739	1.946	2.617	0.689	2.517	0.731	1.004	3.331	0.636
1	0.000	2.257	0.780	2.739	1.946	2.617	0.689	2.517	0.731	1.004	3.331	0.636
2	2.257	0.000	1.477	0.482	0.312	0.360	1.568	0.260	1.526	1.253	1.073	2.893
3	0.780	1.477	0.000	1.959	1.165	1.837	0.091	1.737	0.049	0.224	2.550	1.416
4	2.739	0.482	1.959	0.000	0.794	0.122	2.050	0.222	2.008	1.735	0.591	3.375
5	1.946	0.312	1.165	0.794	0.000	0.672	1.257	0.572	1.215	0.942	1.385	2.582
6	2.617	0.360	1.837	0.122	0.672	0.000	1.928	0.100	1.886	1.613	0.713	3.253
7	0.689	1.568	0.091	2.050	1.257	1.928	0.000	1.828	0.042	0.315	2.641	1.325
8	2.517	0.260	1.737	0.222	0.572	0.100	1.828	0.000	1.786	1.513	0.813	3.153
9	0.731	1.526	0.049	2.008	1.215	1.886	0.042	1.786	0.000	0.273	2.599	1.367
10	1.004	1.253	0.224	1.735	0.942	1.613	0.315	1.513	0.273	0.000	2.326	1.640
11	3.331	1.073	2.550	0.591	1.385	0.713	2.641	0.813	2.599	2.326	0.000	3.967
12	0.636	2.893	1.416	3.375	2.582	3.253	1.325	3.153	1.367	1.640	3.967	0.000



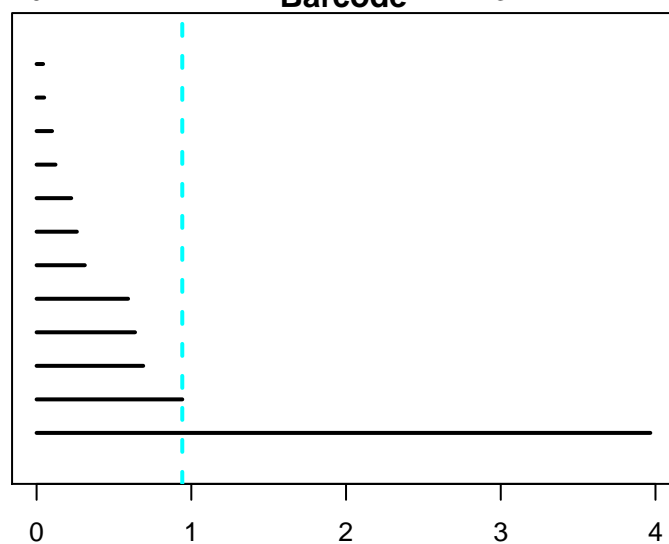
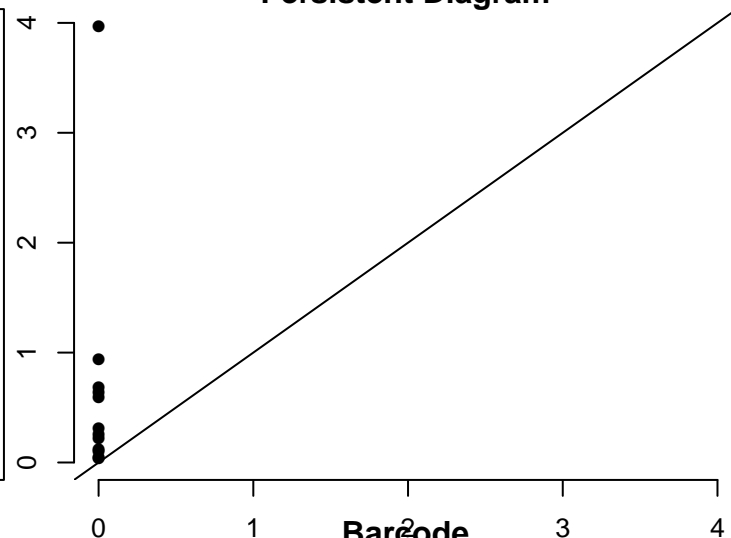
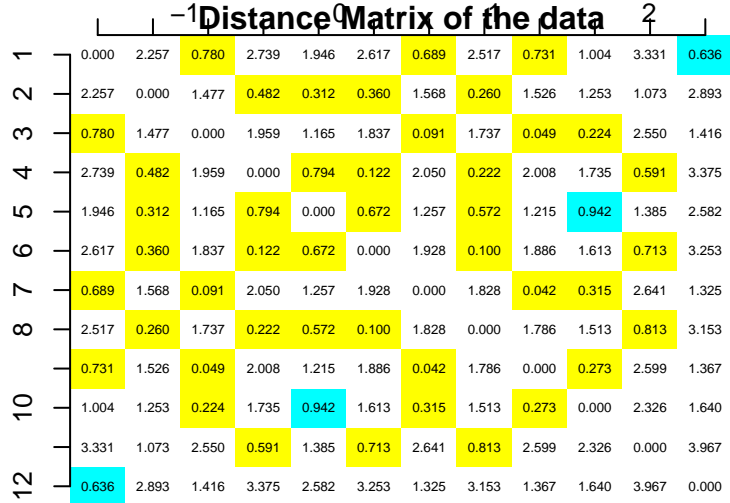
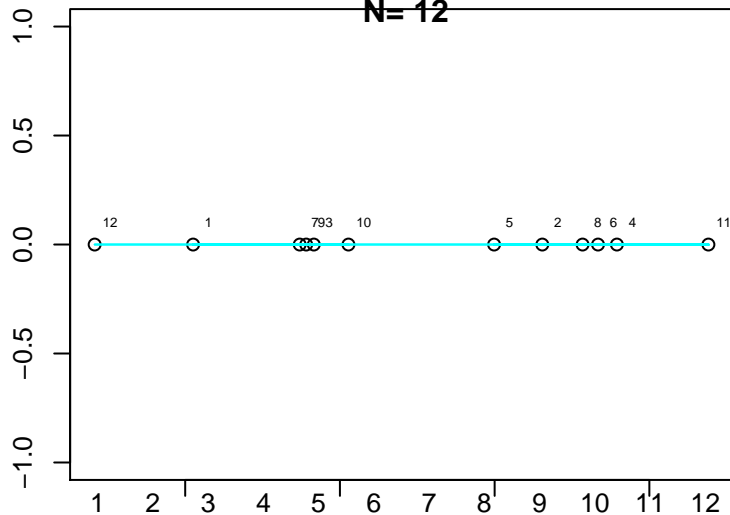
This is the 'Frame' at Euclidean distance = 0.689



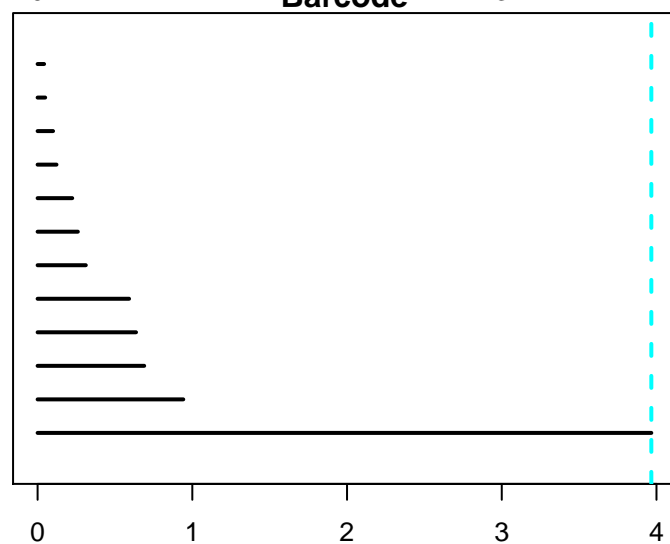
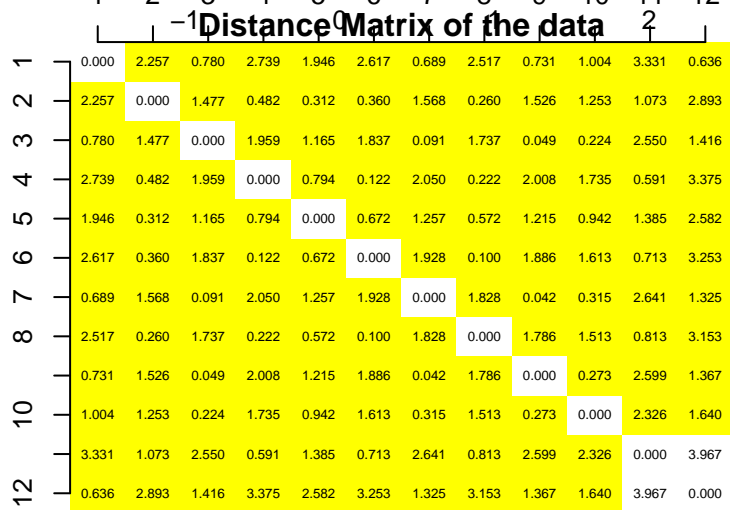
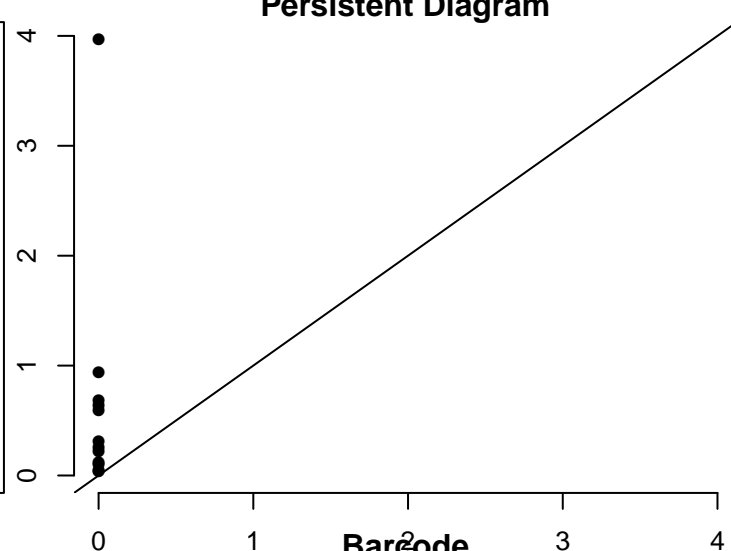
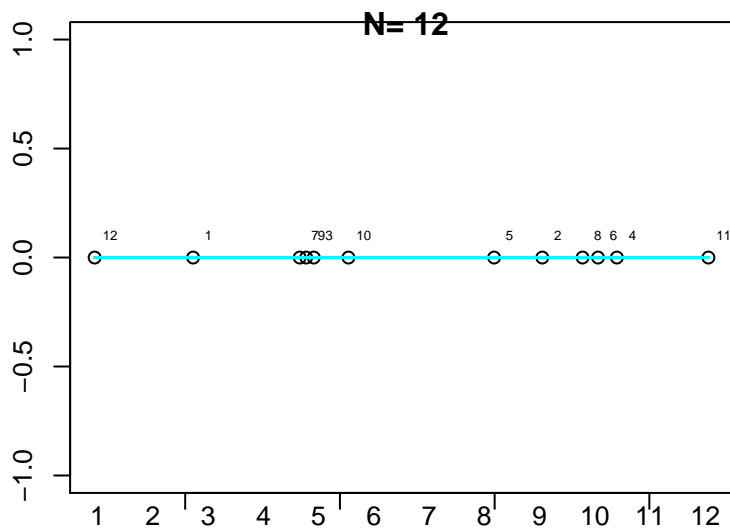


This is the 'Frame' at Euclidean distance = 0.942

Persistent Diagram



# This is the 'Frame' at Euclidean distance = 3.97

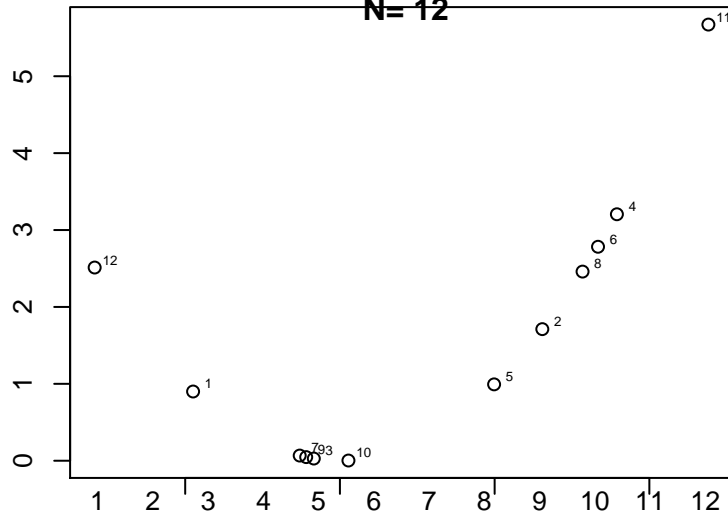


Result and Frame-by-frame plots for Example 3

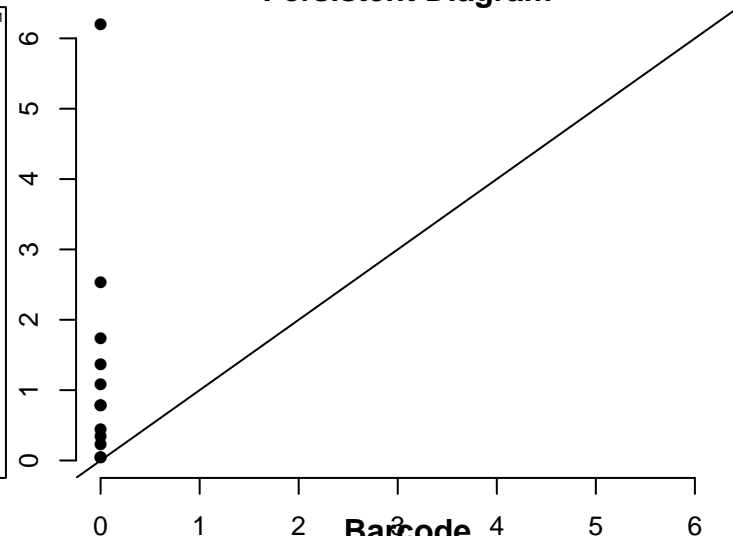
# Ex3. Quadratic Embedding ( $x, x^2$ ), $x \sim N_1(0,1)$

Data Plot

N=12



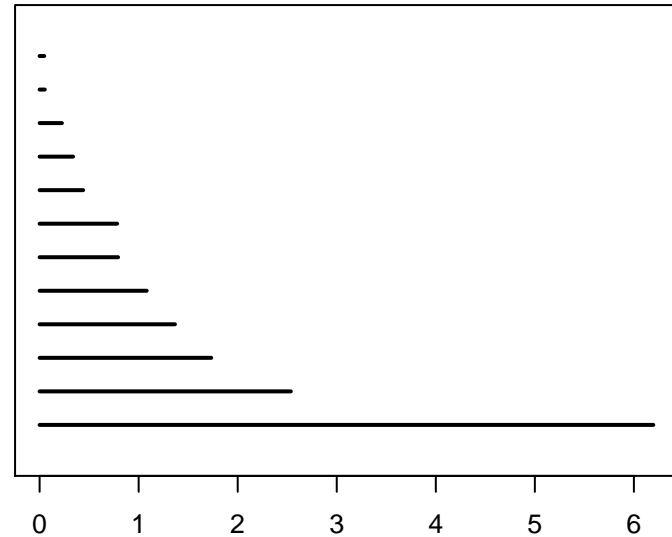
Persistent Diagram



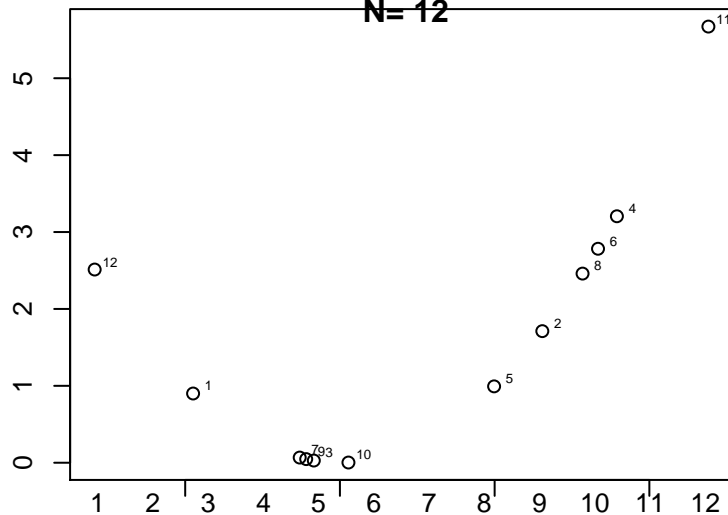
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	2.398	1.170	3.580	1.948	3.224	1.081	2.961	1.123	1.347	5.819	1.733
2	2.398	0.000	2.239	1.570	0.783	1.130	2.272	0.792	2.258	2.119	4.103	3.002
3	1.170	2.239	0.000	3.732	1.513	3.311	0.099	2.988	0.053	0.225	6.193	2.859
4	3.580	1.570	3.732	0.000	2.350	0.440	3.748	0.778	3.742	3.642	2.537	3.446
5	1.948	0.783	1.513	2.350	0.000	1.911	1.561	1.574	1.539	1.366	4.879	2.996
6	3.224	1.130	3.311	0.440	1.911	0.000	3.330	0.338	3.322	3.214	2.976	3.264
7	1.081	2.272	0.099	3.748	1.561	3.330	0.000	3.011	0.047	0.321	6.196	2.781
8	2.961	0.792	2.988	0.778	1.574	0.338	3.011	0.000	3.002	2.885	3.314	3.154
9	1.123	2.258	0.053	3.742	1.539	3.322	0.047	3.002	0.000	0.277	6.196	2.819
10	1.347	2.119	0.225	3.642	1.366	3.214	0.321	2.885	0.277	0.000	6.128	2.998
11	5.819	4.103	6.193	2.537	4.879	2.976	6.196	3.314	6.196	6.128	0.000	5.071
12	1.733	3.002	2.859	3.446	2.996	3.264	2.781	3.154	2.819	2.998	5.071	0.000

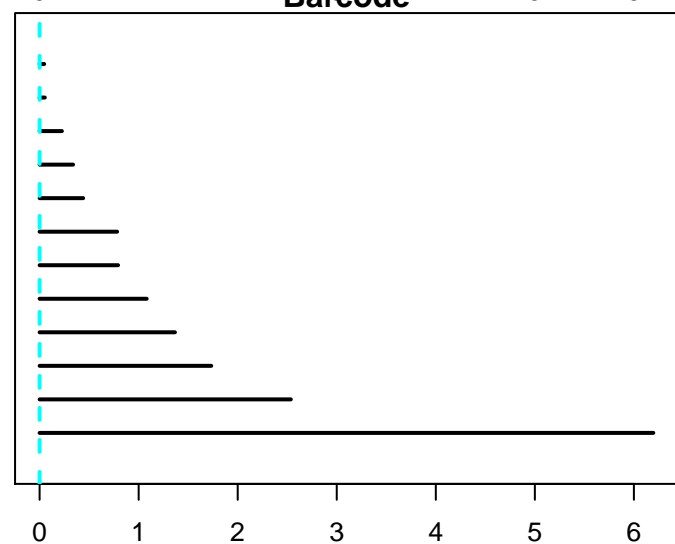
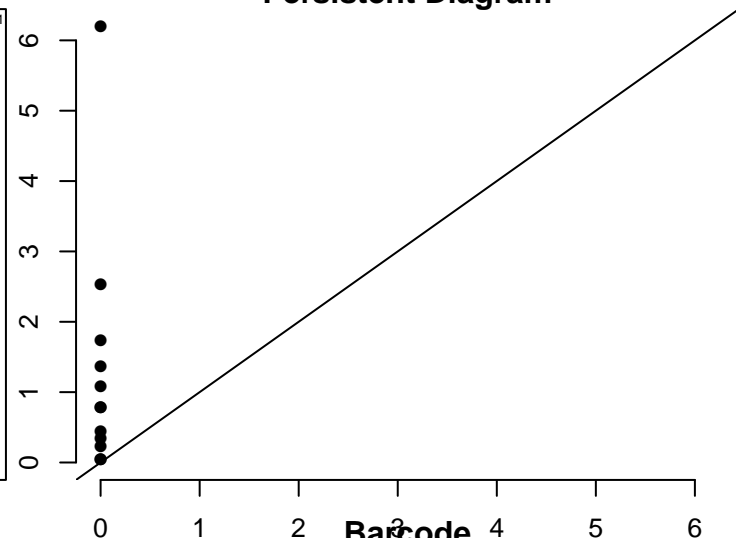
Barcode



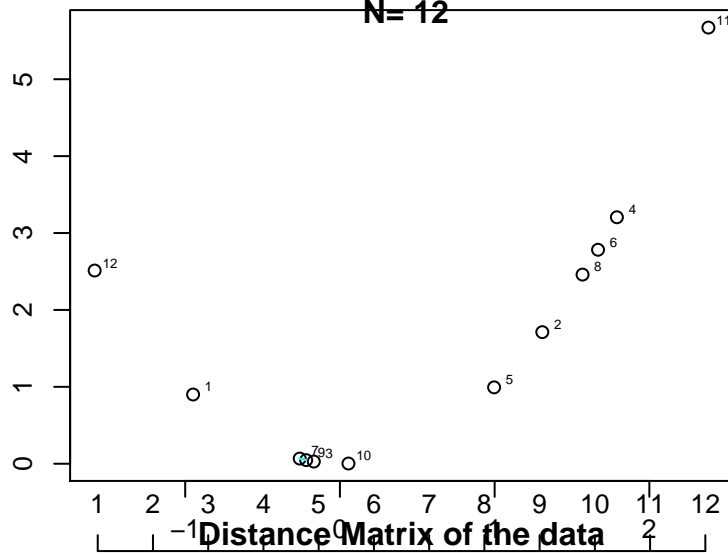
# This is the 'Frame' at Euclidean distance = 0



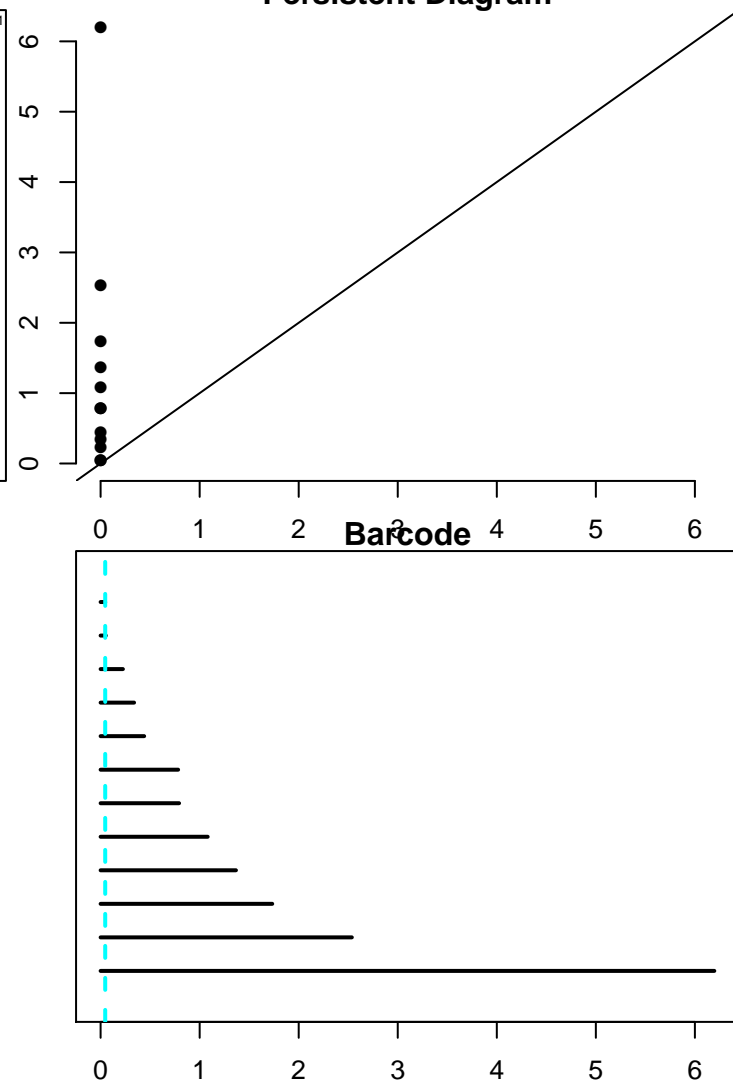
	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	2.398	1.170	3.580	1.948	3.224	1.081	2.961	1.123	1.347	5.819	1.733
2	2.398	0.000	2.239	1.570	0.783	1.130	2.272	0.792	2.258	2.119	4.103	3.002
3	1.170	2.239	0.000	3.732	1.513	3.311	0.099	2.988	0.053	0.225	6.193	2.859
4	3.580	1.570	3.732	0.000	2.350	0.440	3.748	0.778	3.742	3.642	2.537	3.446
5	1.948	0.783	1.513	2.350	0.000	1.911	1.561	1.574	1.539	1.366	4.879	2.996
6	3.224	1.130	3.311	0.440	1.911	0.000	3.330	0.338	3.322	3.214	2.976	3.264
7	1.081	2.272	0.099	3.748	1.561	3.330	0.000	3.011	0.047	0.321	6.196	2.781
8	2.961	0.792	2.988	0.778	1.574	0.338	3.011	0.000	3.002	2.885	3.314	3.154
9	1.123	2.258	0.053	3.742	1.539	3.322	0.047	3.002	0.000	0.277	6.196	2.819
10	1.347	2.119	0.225	3.642	1.366	3.214	0.321	2.885	0.277	0.000	6.128	2.998
11	5.819	4.103	6.193	2.537	4.879	2.976	6.196	3.314	6.196	6.128	0.000	5.071
12	1.733	3.002	2.859	3.446	2.996	3.264	2.781	3.154	2.819	2.998	5.071	0.000



This is the 'Frame' at Euclidean distance = 0.0465



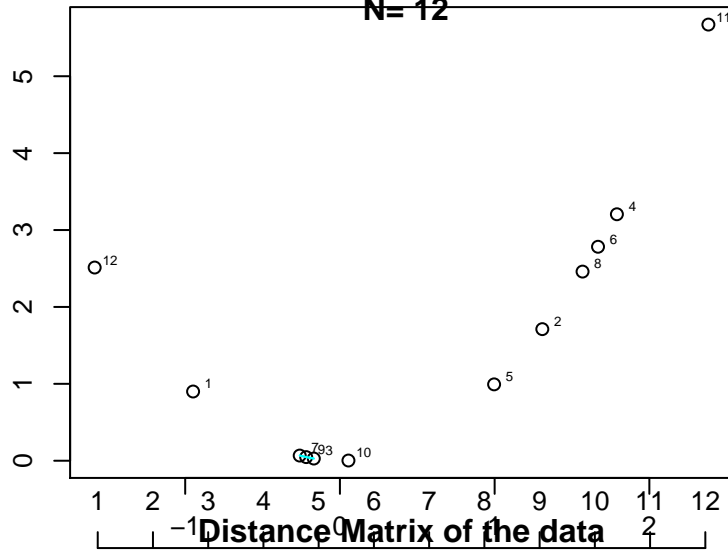
	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0.000	2.398	1.170	3.580	1.948	3.224	1.081	2.961	1.123	1.347	5.819	1.733	
1	2.398	0.000	2.239	1.570	0.783	1.130	2.272	0.792	2.258	2.119	4.103	3.002	
2	1.170	2.239	0.000	3.732	1.513	3.311	0.099	2.988	0.053	0.225	6.193	2.859	
3	3.580	1.570	3.732	0.000	2.350	0.440	3.748	0.778	3.742	3.642	2.537	3.446	
4	1.948	0.783	1.513	2.350	0.000	1.911	1.561	1.574	1.539	1.366	4.879	2.996	
5	3.224	1.130	3.311	0.440	1.911	0.000	3.330	0.338	3.322	3.214	2.976	3.264	
6	1.081	2.272	0.099	3.748	1.561	3.330	0.000	3.011	0.047	0.321	6.196	2.781	
7	2.961	0.792	2.988	0.778	1.574	0.338	3.011	0.000	3.002	2.885	3.314	3.154	
8	1.123	2.258	0.053	3.742	1.539	3.322	0.047	3.002	0.000	0.277	6.196	2.819	
9	1.347	2.119	0.225	3.642	1.366	3.214	0.321	2.885	0.277	0.000	6.128	2.998	
10	5.819	4.103	6.193	2.537	4.879	2.976	6.196	3.314	6.196	6.128	0.000	5.071	
11	1.733	3.002	2.859	3.446	2.996	3.264	2.781	3.154	2.819	2.998	5.071	0.000	
12													



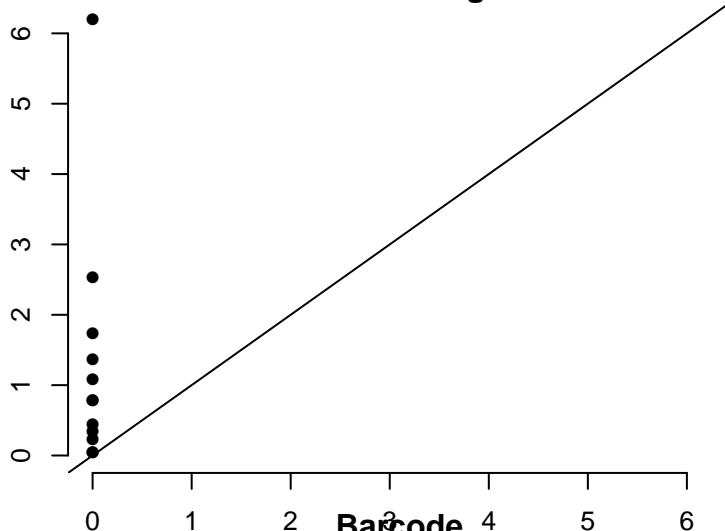
This is the 'Frame' at Euclidean distance = 0.0527

Data Plot

N=12

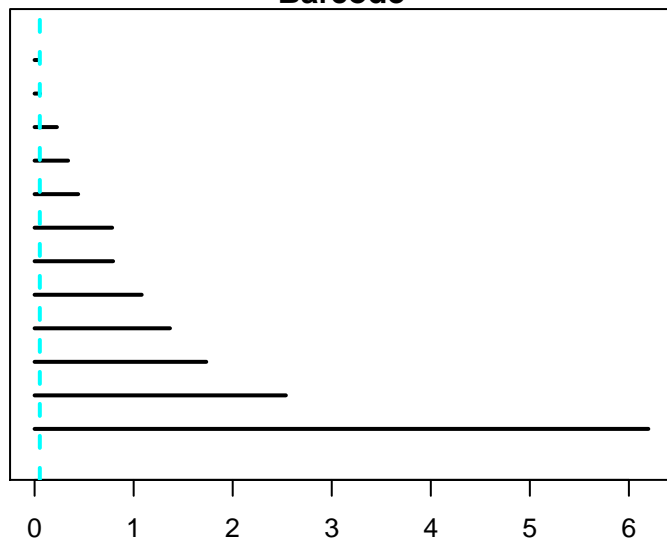


Persistent Diagram



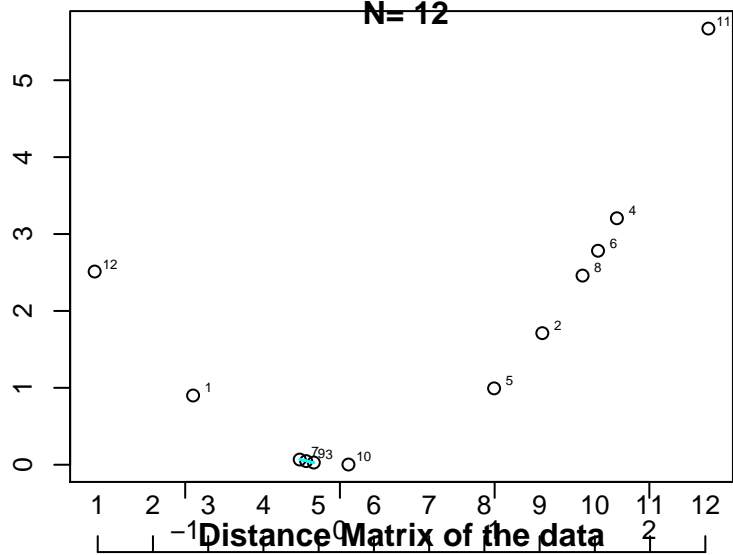
Distance Matrix of the data

0.000	2.398	1.170	3.580	1.948	3.224	1.081	2.961	1.123	1.347	5.819	1.733
2.398	0.000	2.239	1.570	0.783	1.130	2.272	0.792	2.258	2.119	4.103	3.002
1.170	2.239	0.000	3.732	1.513	3.311	0.099	2.988	0.053	0.225	6.193	2.859
3.580	1.570	3.732	0.000	2.350	0.440	3.748	0.778	3.742	3.642	2.537	3.446
1.948	0.783	1.513	2.350	0.000	1.911	1.561	1.574	1.539	1.366	4.879	2.996
3.224	1.130	3.311	0.440	1.911	0.000	3.330	0.338	3.322	3.214	2.976	3.264
1.081	2.272	0.099	3.748	1.561	3.330	0.000	3.011	0.047	0.321	6.196	2.781
2.961	0.792	2.988	0.778	1.574	0.338	3.011	0.000	3.002	2.885	3.314	3.154
1.123	2.258	0.053	3.742	1.539	3.322	0.047	3.002	0.000	0.277	6.196	2.819
1.347	2.119	0.225	3.642	1.366	3.214	0.321	2.885	0.277	0.000	6.128	2.998
5.819	4.103	6.193	2.537	4.879	2.976	6.196	3.314	6.196	6.128	0.000	5.071
1.733	3.002	2.859	3.446	2.996	3.264	2.781	3.154	2.819	2.998	5.071	0.000

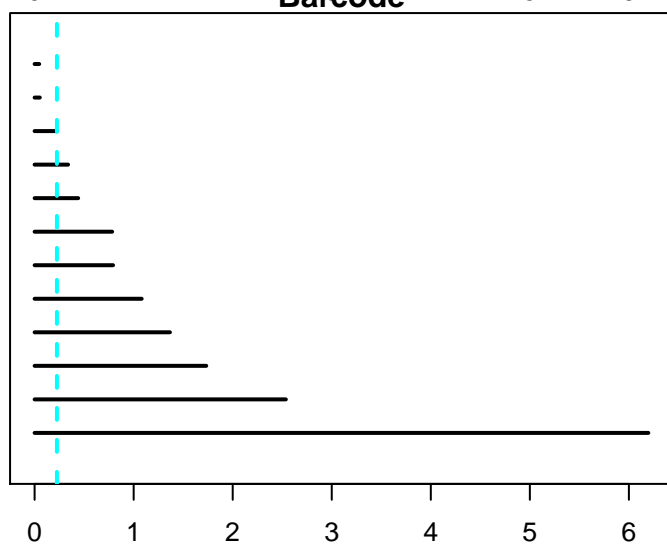
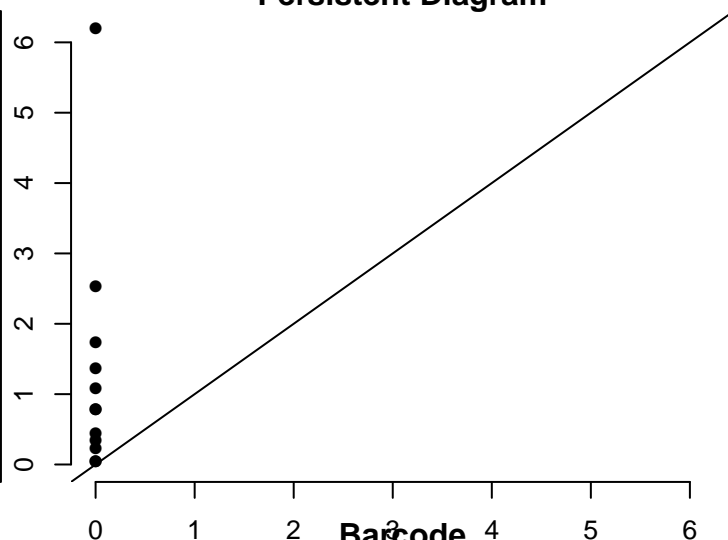


This is the 'Frame' at Euclidean distance = 0.225

Persistent Diagram

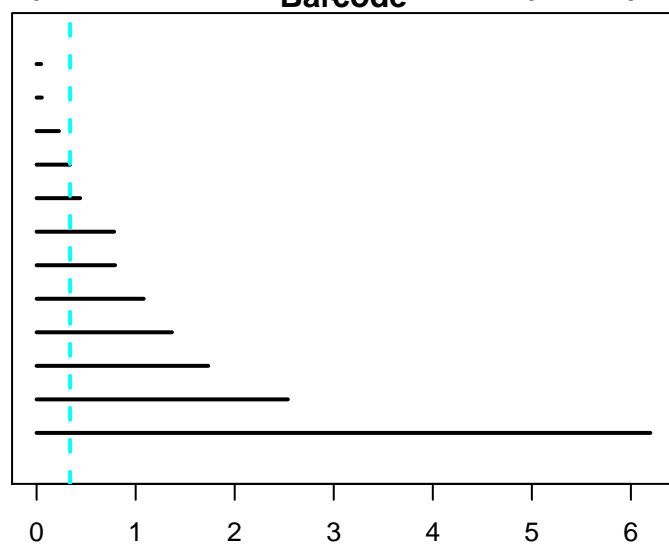
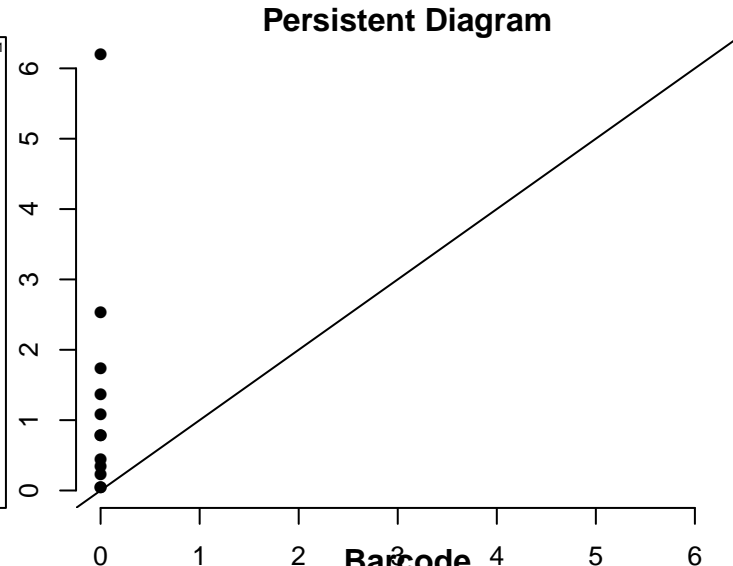
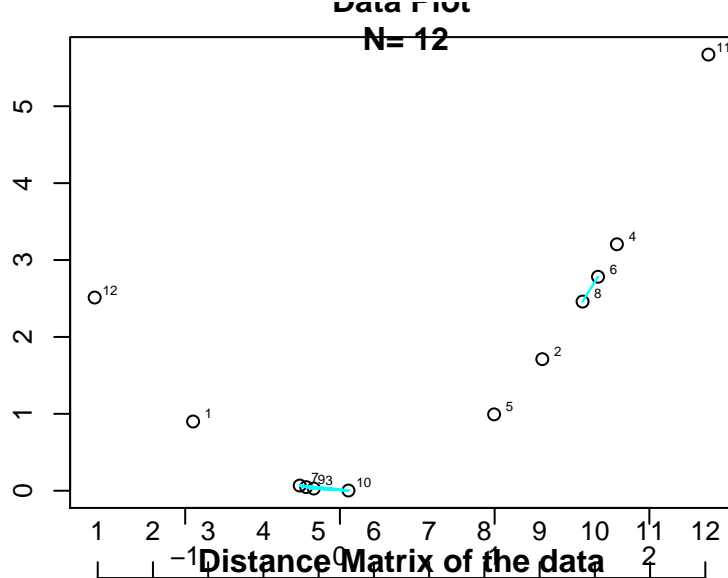


	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0.000	2.398	1.170	3.580	1.948	3.224	1.081	2.961	1.123	1.347	5.819	1.733	
1	2.398	0.000	2.239	1.570	0.783	1.130	2.272	0.792	2.258	2.119	4.103	3.002	
2	1.170	2.239	0.000	3.732	1.513	3.311	0.099	2.988	0.053	0.225	6.193	2.859	
3	3.580	1.570	3.732	0.000	2.350	0.440	3.748	0.778	3.742	3.642	2.537	3.446	
4	1.948	0.783	1.513	2.350	0.000	1.911	1.561	1.574	1.539	1.366	4.879	2.996	
5	3.224	1.130	3.311	0.440	1.911	0.000	3.330	0.338	3.322	3.214	2.976	3.264	
6	1.081	2.272	0.099	3.748	1.561	3.330	0.000	3.011	0.047	0.321	6.196	2.781	
7	2.961	0.792	2.988	0.778	1.574	0.338	3.011	0.000	3.002	2.885	3.314	3.154	
8	1.123	2.258	0.053	3.742	1.539	3.322	0.047	3.002	0.000	0.277	6.196	2.819	
9	1.347	2.119	0.225	3.642	1.366	3.214	0.321	2.885	0.277	0.000	6.128	2.998	
10	5.819	4.103	6.193	2.537	4.879	2.976	6.196	3.314	6.196	6.128	0.000	5.071	
11	1.733	3.002	2.859	3.446	2.996	3.264	2.781	3.154	2.819	2.998	5.071	0.000	
12													

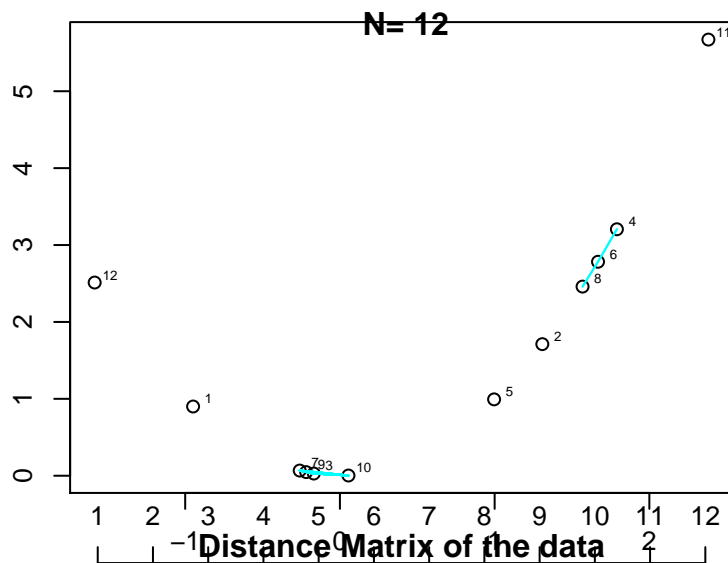




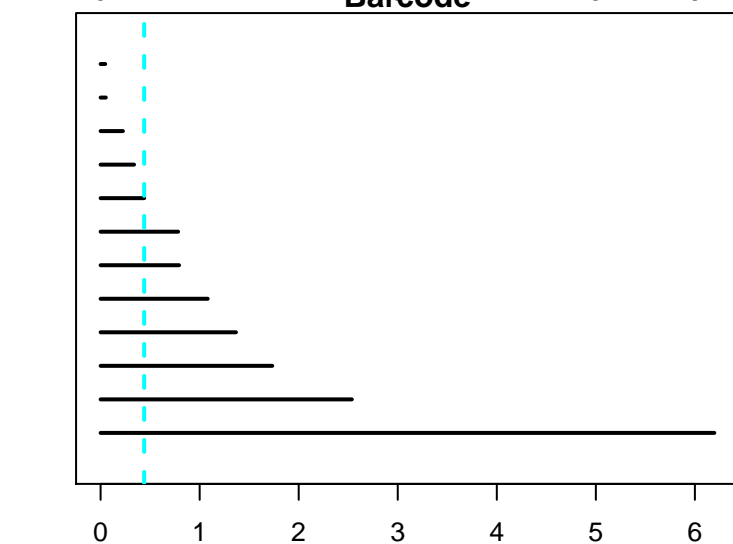
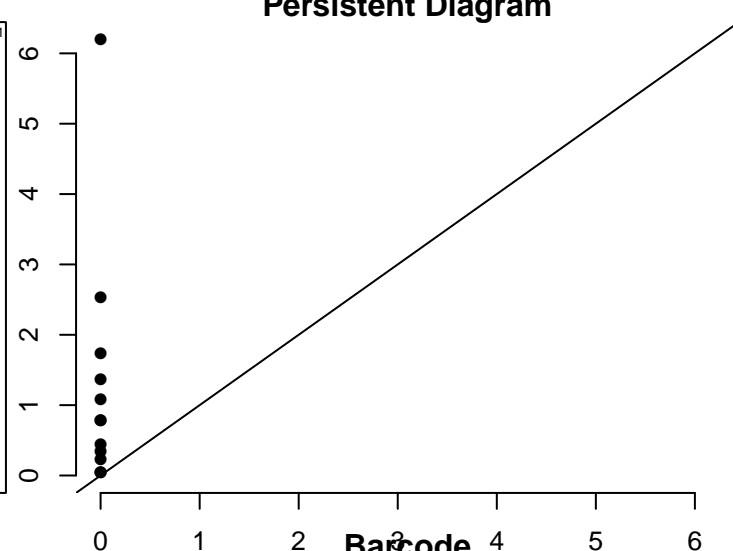
This is the 'Frame' at Euclidean distance = 0.338



This is the 'Frame' at Euclidean distance = 0.44

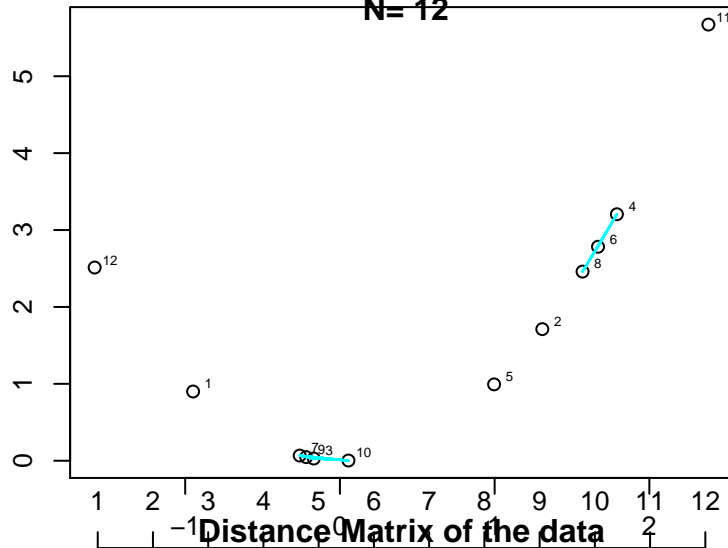


1	0.000	2.398	1.170	3.580	1.948	3.224	1.081	2.961	1.123	1.347	5.819	1.733
2	2.398	0.000	2.239	1.570	0.783	1.130	2.272	0.792	2.258	2.119	4.103	3.002
3	1.170	2.239	0.000	3.732	1.513	3.311	0.099	2.988	0.053	0.225	6.193	2.859
4	3.580	1.570	3.732	0.000	2.350	0.440	3.748	0.778	3.742	3.642	2.537	3.446
5	1.948	0.783	1.513	2.350	0.000	1.911	1.561	1.574	1.539	1.366	4.879	2.996
6	3.224	1.130	3.311	0.440	1.911	0.000	3.330	0.338	3.322	3.214	2.976	3.264
7	1.081	2.272	0.099	3.748	1.561	3.330	0.000	3.011	0.047	0.321	6.196	2.781
8	2.961	0.792	2.988	0.778	1.574	0.338	3.011	0.000	3.002	2.885	3.314	3.154
9	1.123	2.258	0.053	3.742	1.539	3.322	0.047	3.002	0.000	0.277	6.196	2.819
10	1.347	2.119	0.225	3.642	1.366	3.214	0.321	2.885	0.277	0.000	6.128	2.998
11	5.819	4.103	6.193	2.537	4.879	2.976	6.196	3.314	6.196	6.128	0.000	5.071
12	1.733	3.002	2.859	3.446	2.996	3.264	2.781	3.154	2.819	2.998	5.071	0.000

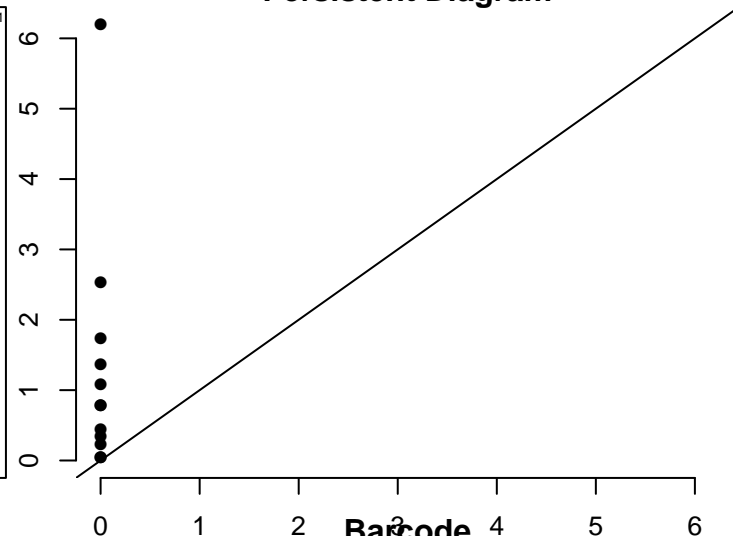


Data Plot

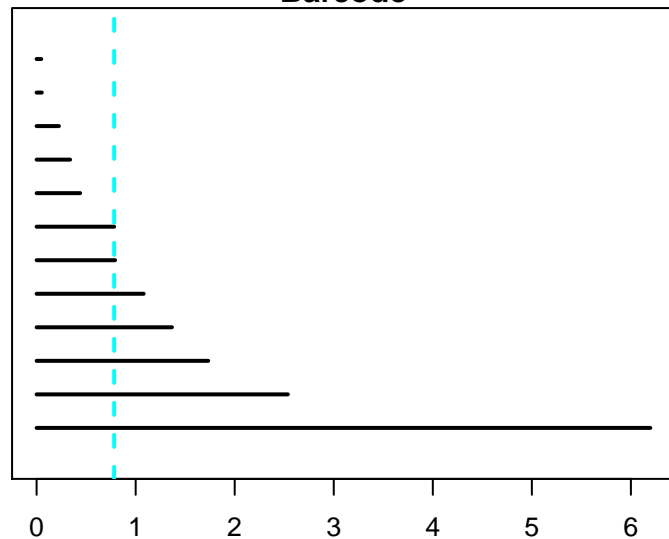
N=12



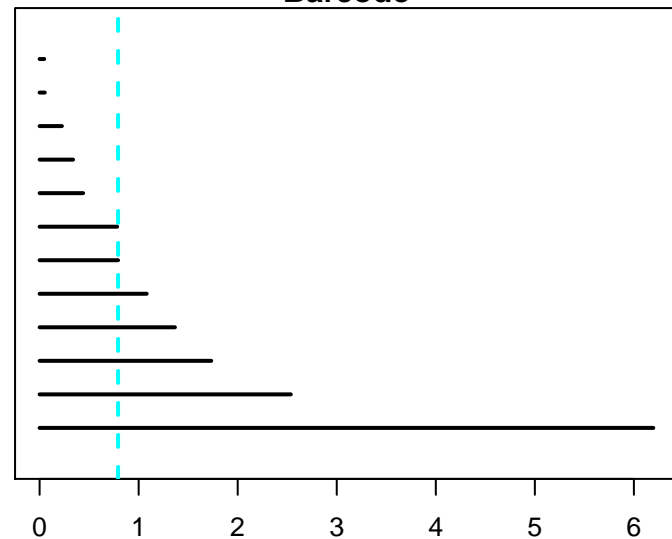
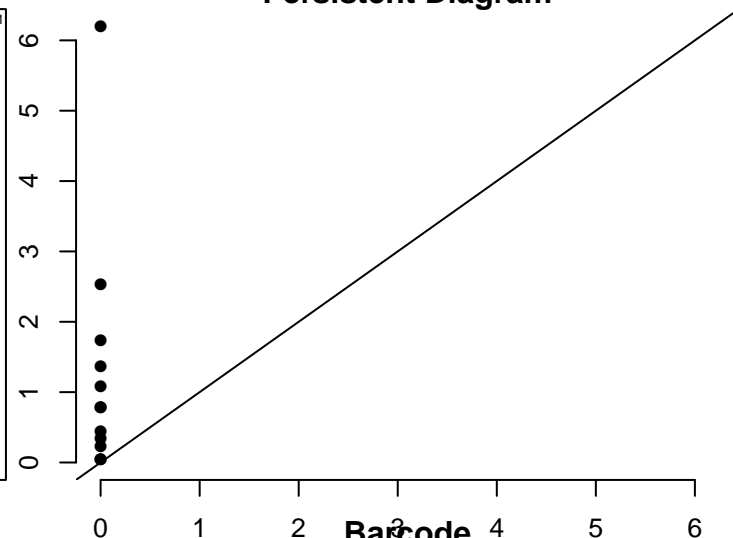
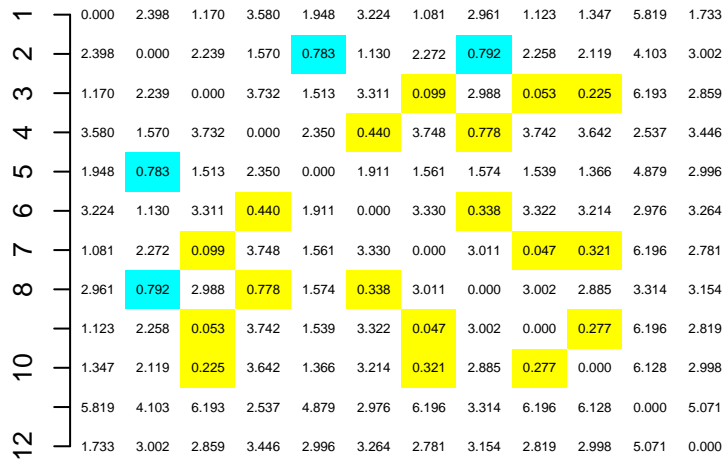
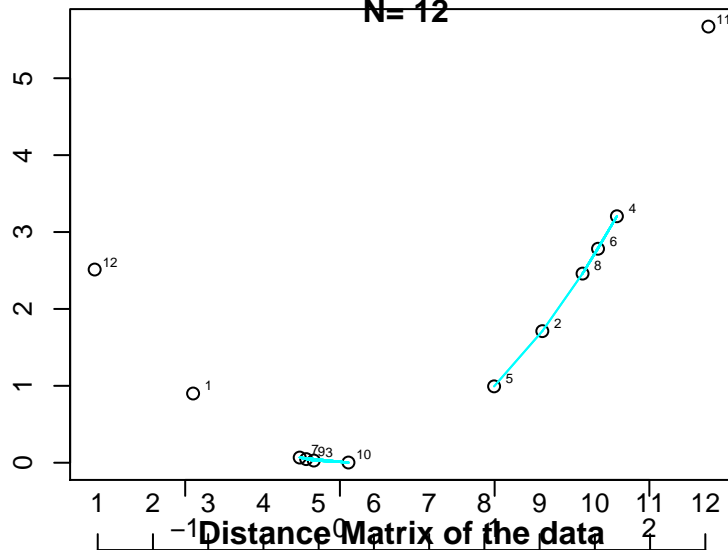
Persistent Diagram



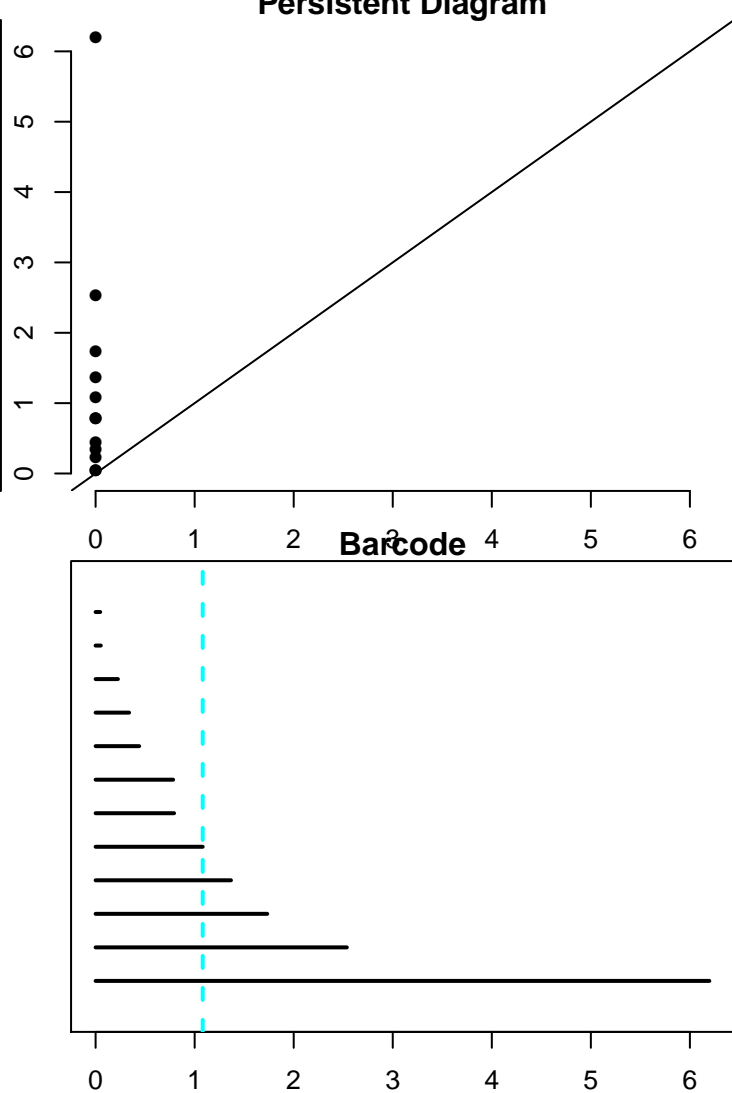
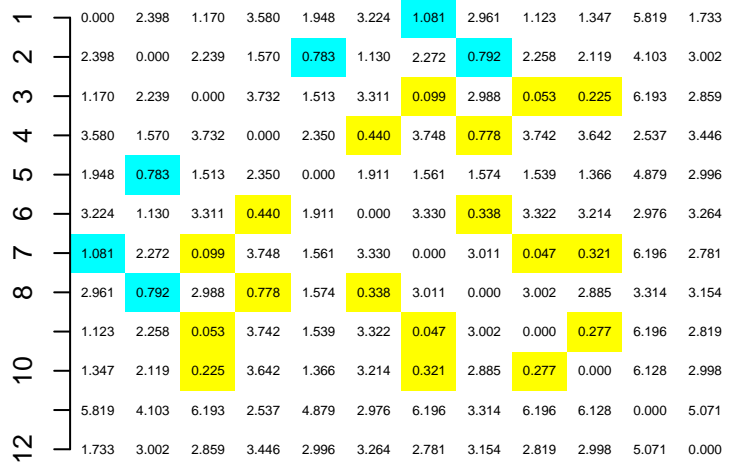
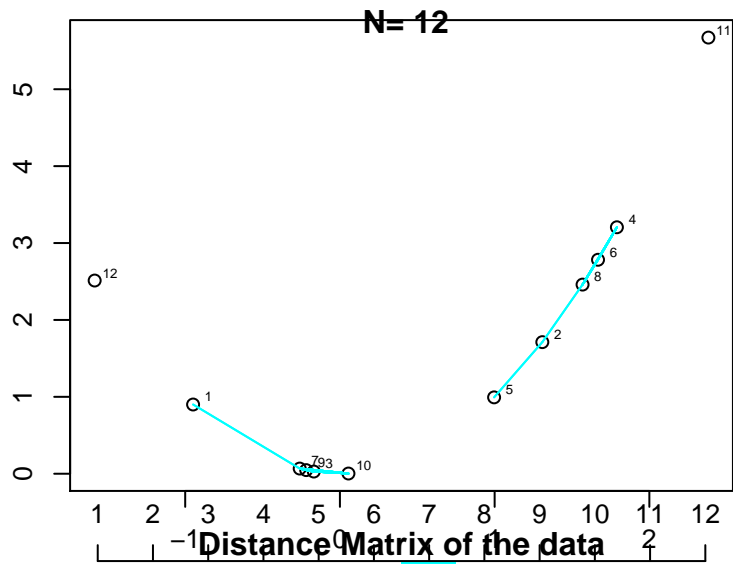
1	0.000	2.398	1.170	3.580	1.948	3.224	1.081	2.961	1.123	1.347	5.819	1.733
2	2.398	0.000	2.239	1.570	0.783	1.130	2.272	0.792	2.258	2.119	4.103	3.002
3	1.170	2.239	0.000	3.732	1.513	3.311	0.099	2.988	0.053	0.225	6.193	2.859
4	3.580	1.570	3.732	0.000	2.350	0.440	3.748	0.778	3.742	3.642	2.537	3.446
5	1.948	0.783	1.513	2.350	0.000	1.911	1.561	1.574	1.539	1.366	4.879	2.996
6	3.224	1.130	3.311	0.440	1.911	0.000	3.330	0.338	3.322	3.214	2.976	3.264
7	1.081	2.272	0.099	3.748	1.561	3.330	0.000	3.011	0.047	0.321	6.196	2.781
8	2.961	0.792	2.988	0.778	1.574	0.338	3.011	0.000	3.002	2.885	3.314	3.154
9	1.123	2.258	0.053	3.742	1.539	3.322	0.047	3.002	0.000	0.277	6.196	2.819
10	1.347	2.119	0.225	3.642	1.366	3.214	0.321	2.885	0.277	0.000	6.128	2.998
11	5.819	4.103	6.193	2.537	4.879	2.976	6.196	3.314	6.196	6.128	0.000	5.071
12	1.733	3.002	2.859	3.446	2.996	3.264	2.781	3.154	2.819	2.998	5.071	0.000



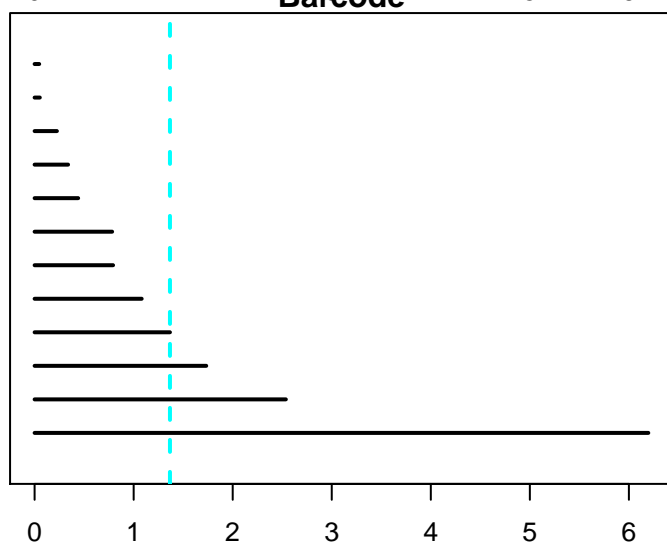
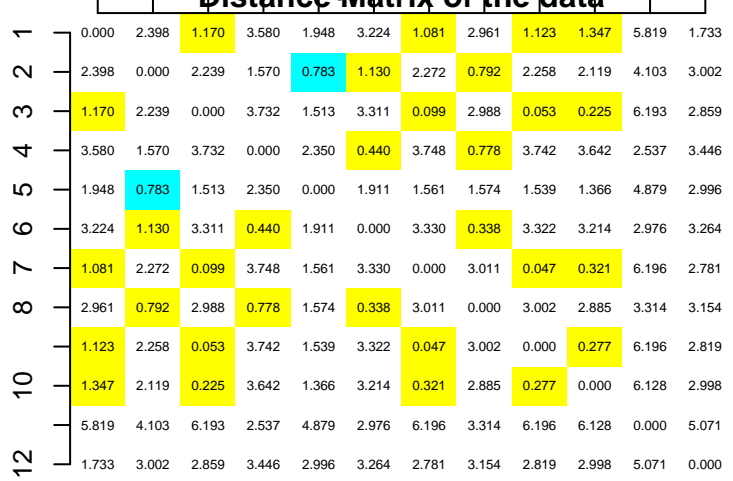
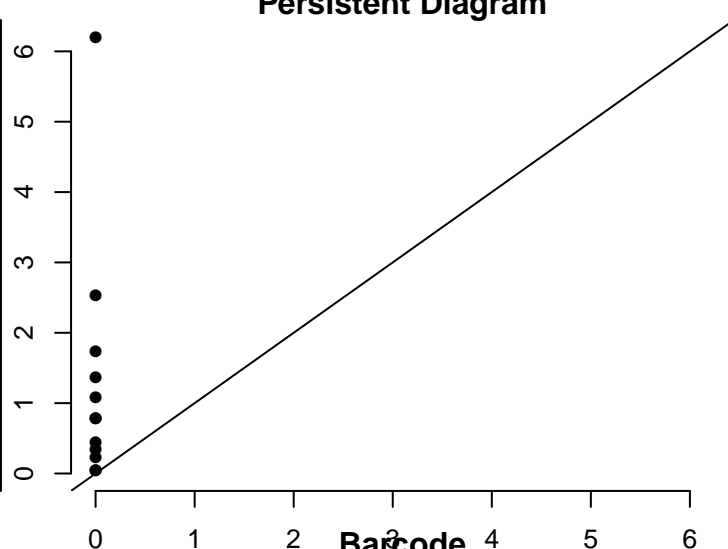
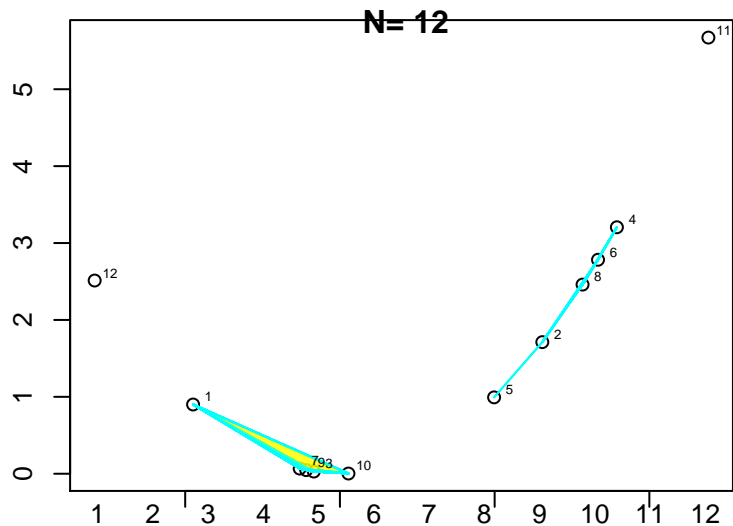
# This is the 'Frame' at Euclidean distance = 0.792



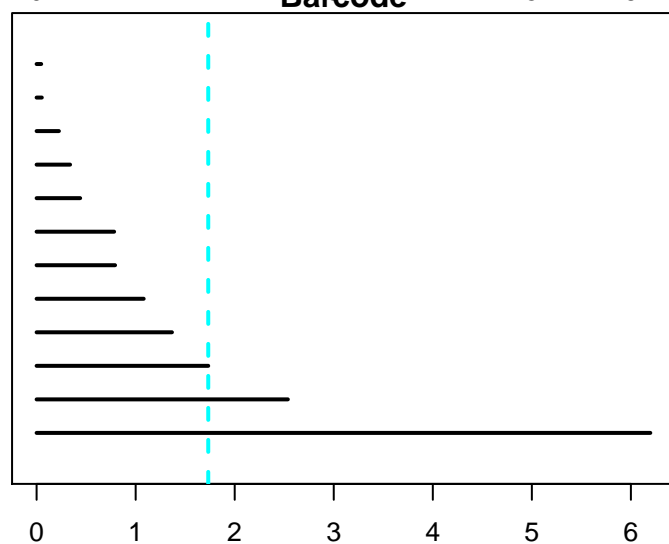
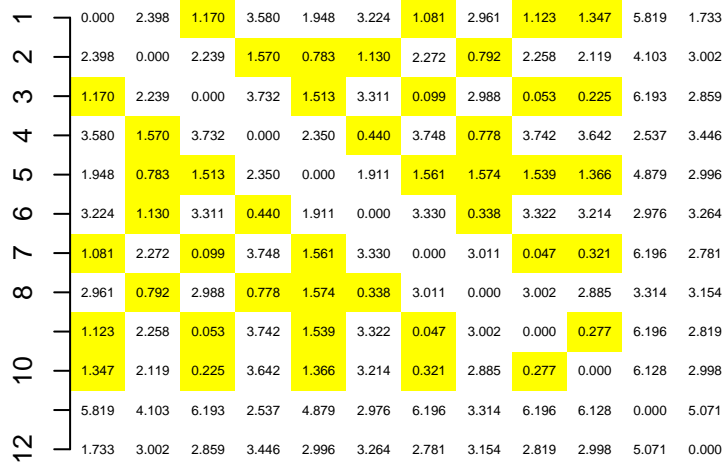
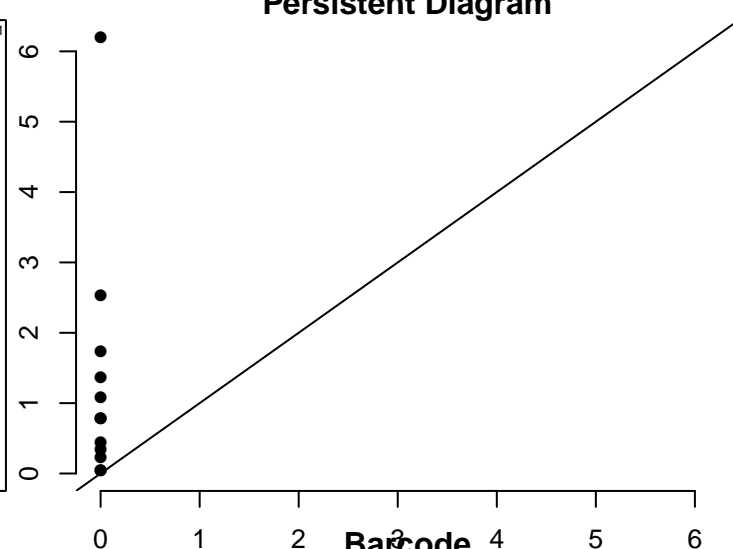
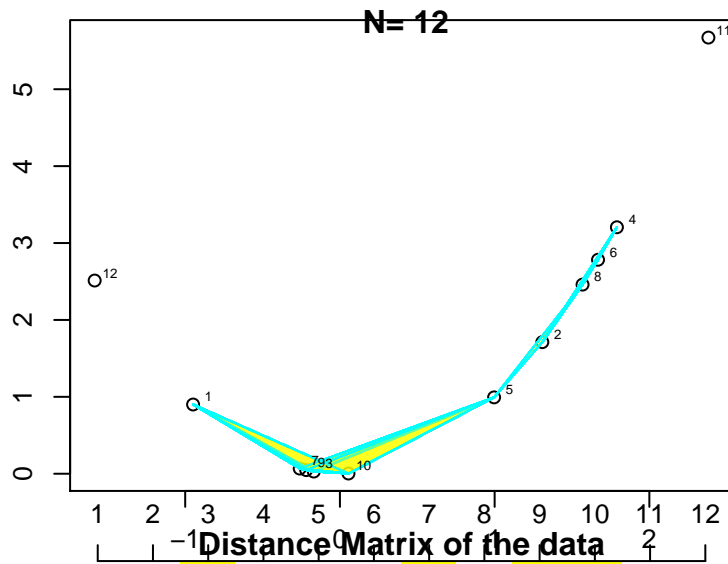
This is the 'Frame' at Euclidean distance = 1.08



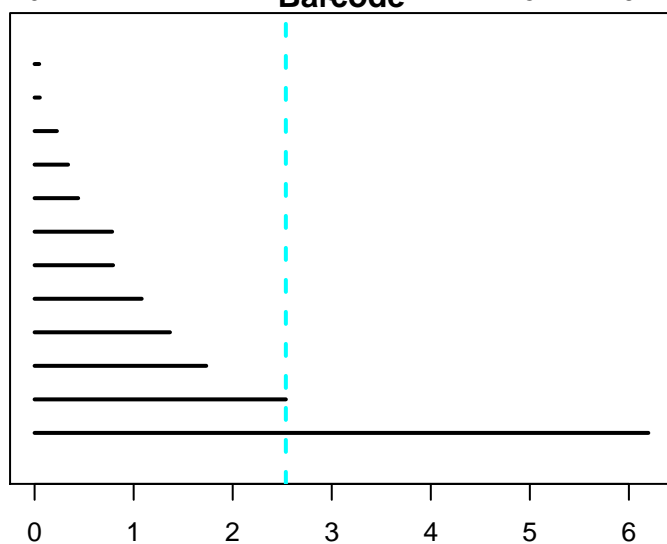
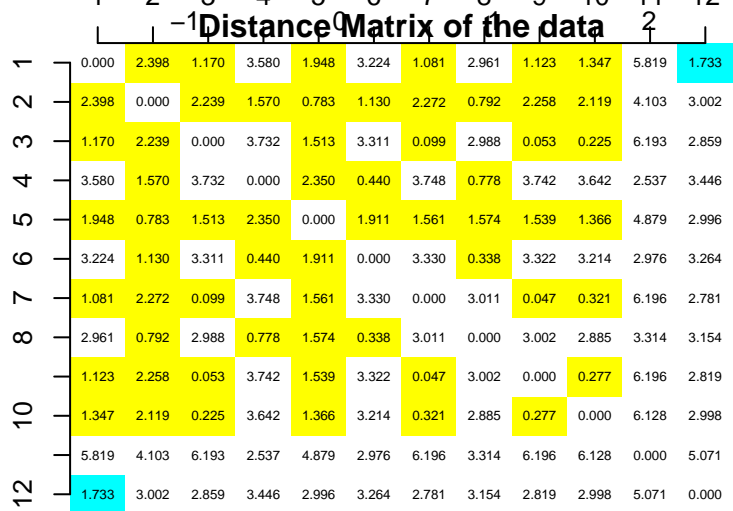
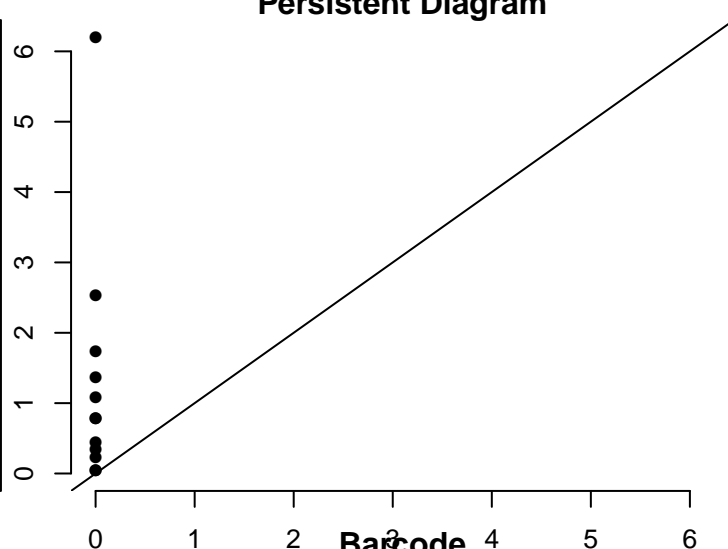
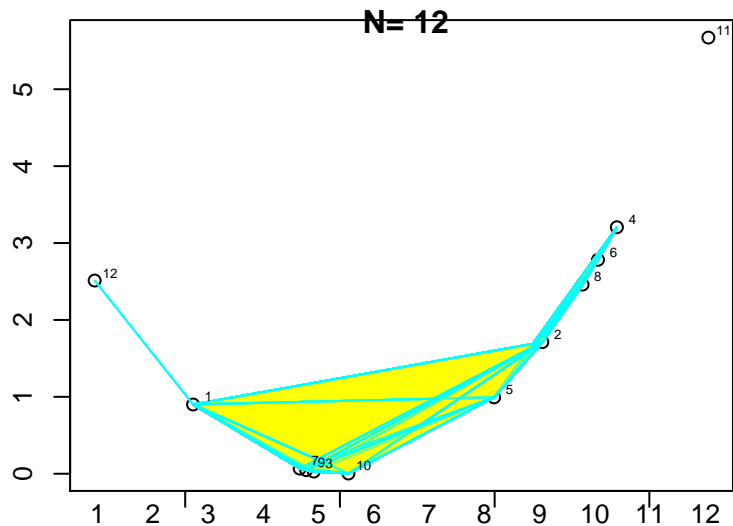
This is the 'Frame' at Euclidean distance = 1.37



This is the 'Frame' at Euclidean distance = 1.73

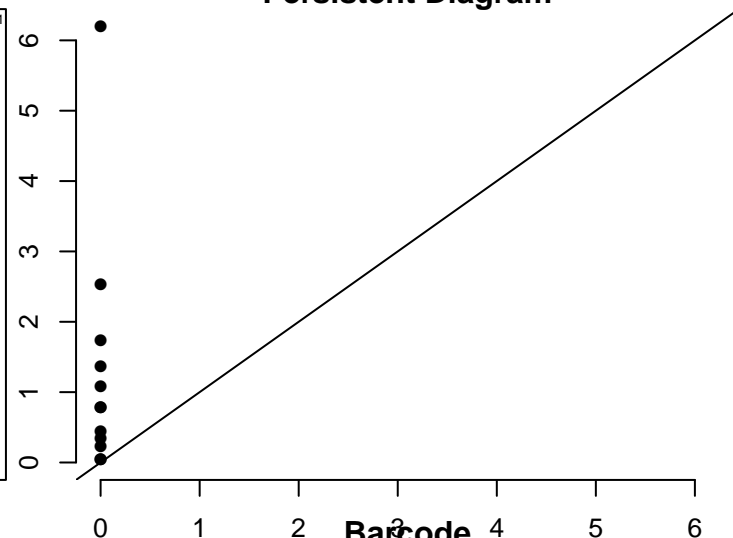
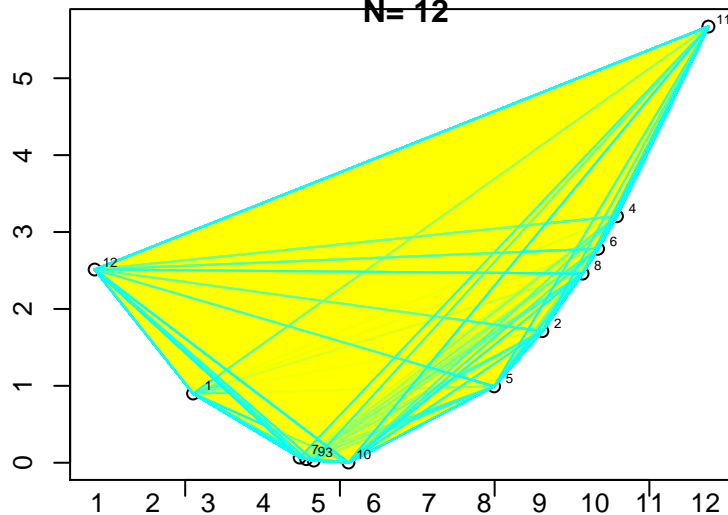


This is the 'Frame' at Euclidean distance = 2.54





This is the 'Frame' at Euclidean distance = 6.2



**Distance Matrix of the data**

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	2.398	1.170	3.580	1.948	3.224	1.081	2.961	1.123	1.347	5.819	1.733
2	2.398	0.000	2.239	1.570	0.783	1.130	2.272	0.792	2.258	2.119	4.103	3.002
3	1.170	2.239	0.000	3.732	1.513	3.311	0.099	2.988	0.053	0.225	6.193	2.859
4	3.580	1.570	3.732	0.000	2.350	0.440	3.748	0.778	3.742	3.642	2.537	3.446
5	1.948	0.783	1.513	2.350	0.000	1.911	1.561	1.574	1.539	1.366	4.879	2.996
6	3.224	1.130	3.311	0.440	1.911	0.000	3.330	0.338	3.322	3.214	2.976	3.264
7	1.081	2.272	0.099	3.748	1.561	3.330	0.000	3.011	0.047	0.321	6.196	2.781
8	2.961	0.792	2.988	0.778	1.574	0.338	3.011	0.000	3.002	2.885	3.314	3.154
9	1.123	2.258	0.053	3.742	1.539	3.322	0.047	3.002	0.000	0.277	6.196	2.819
10	1.347	2.119	0.225	3.642	1.366	3.214	0.321	2.885	0.277	0.000	6.128	2.998
11	5.819	4.103	6.193	2.537	4.879	2.976	6.196	3.314	6.196	6.128	0.000	5.071
12	1.733	3.002	2.859	3.446	2.996	3.264	2.781	3.154	2.819	2.998	5.071	0.000

