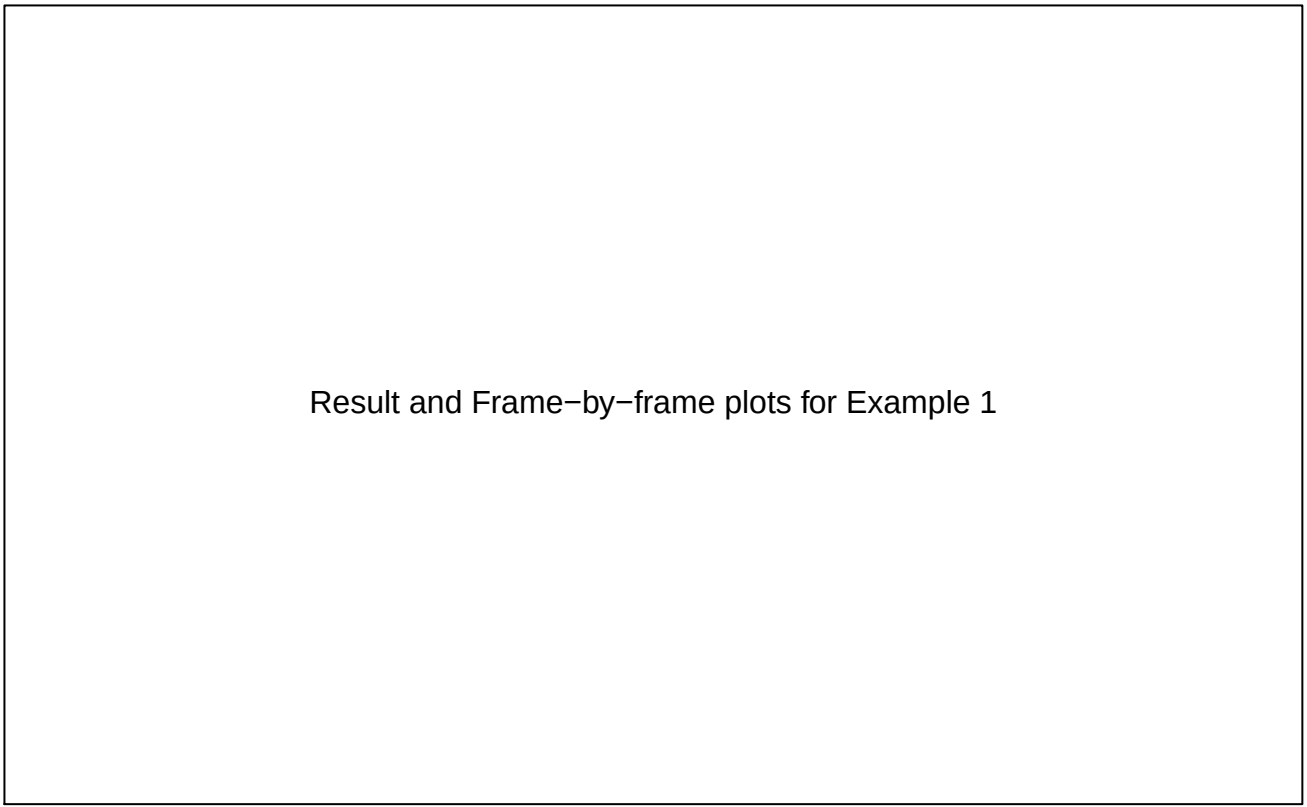
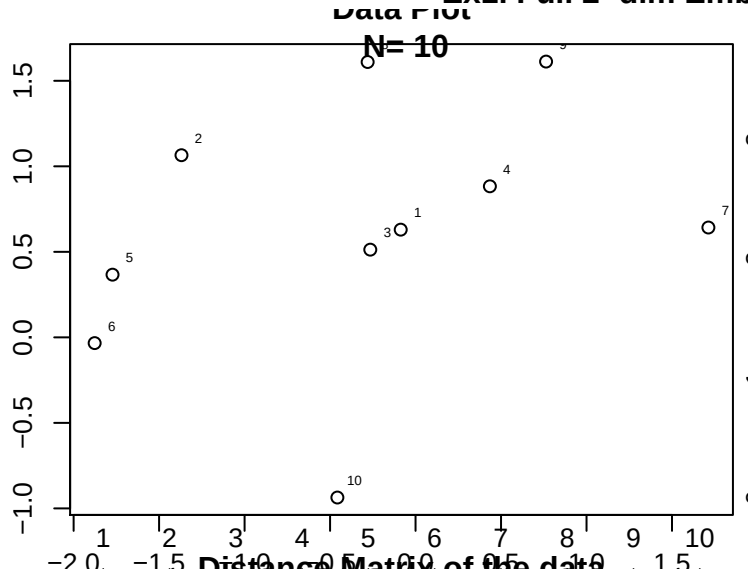
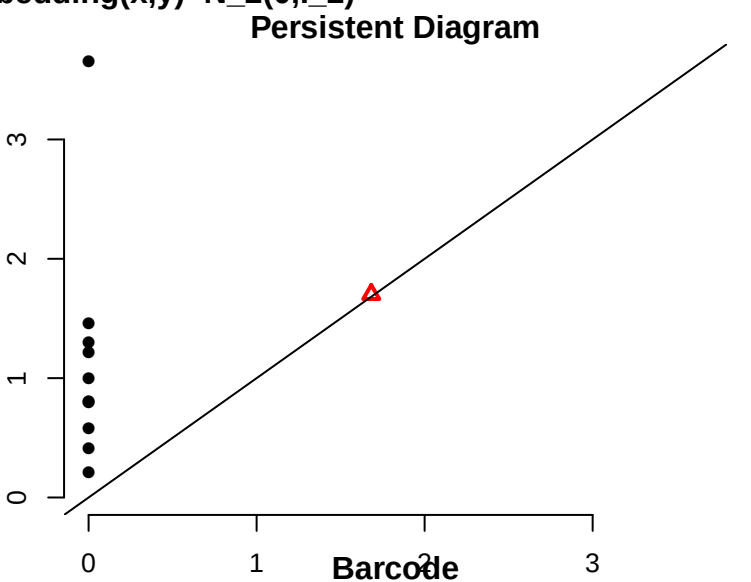

Result and Frame−by−frame plots for Example 1

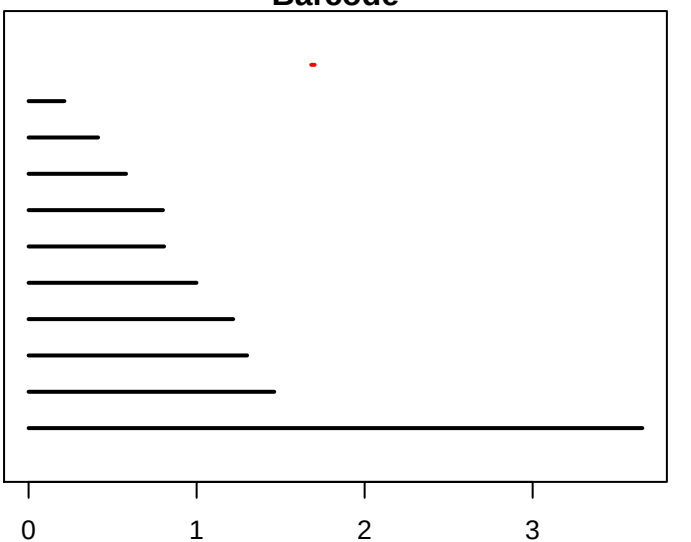
Ex1. Full 2−dim Embedding(x,y)~N_2(0,I_2)

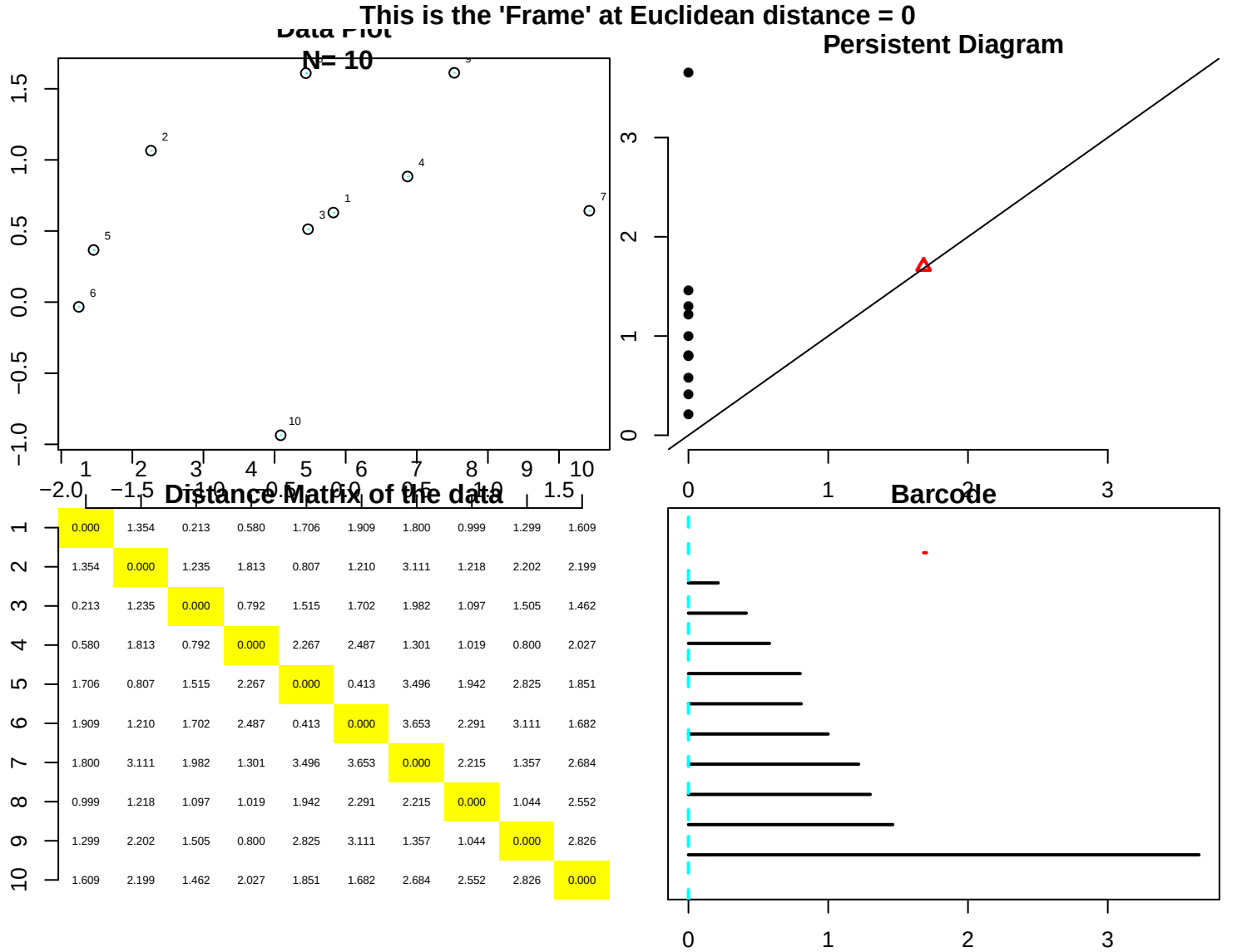


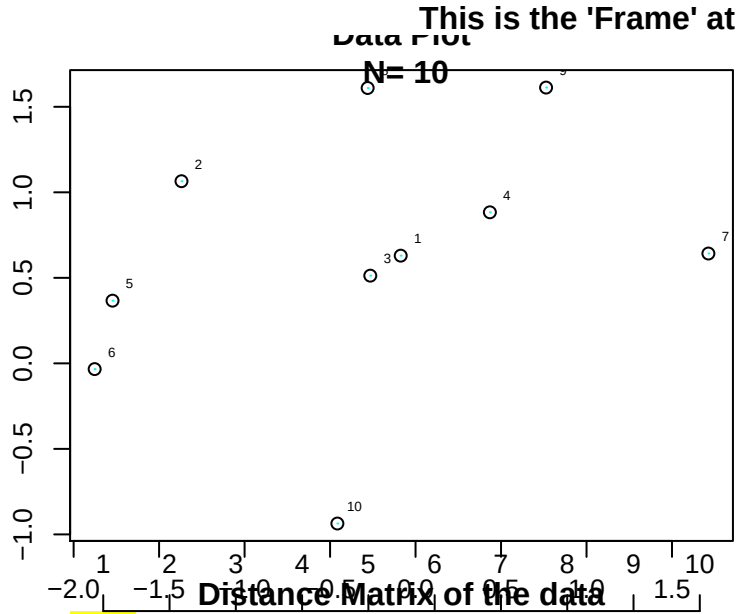


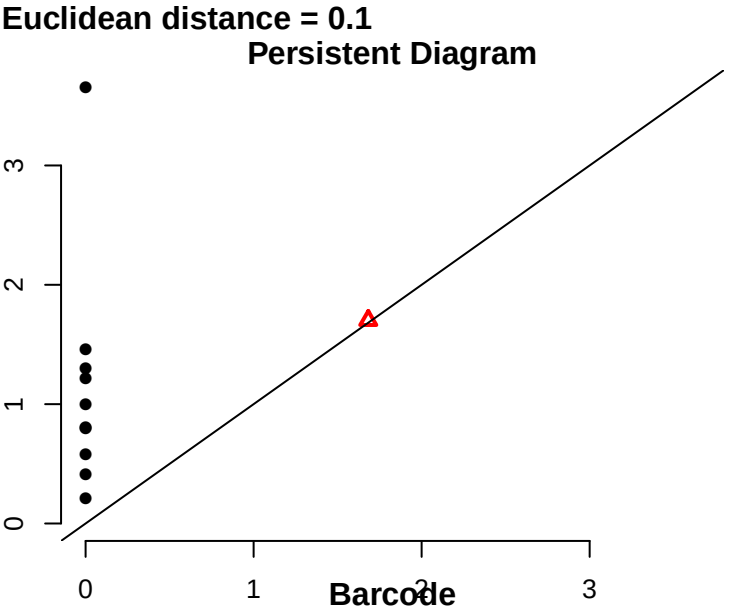
Distance Matrix of the data

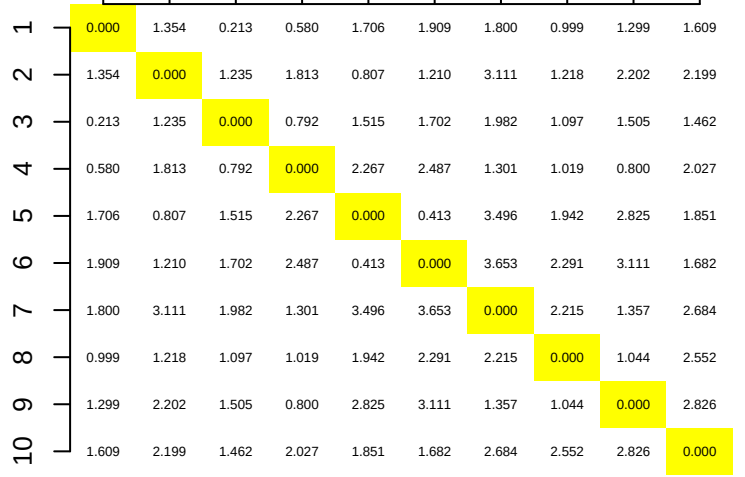
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000

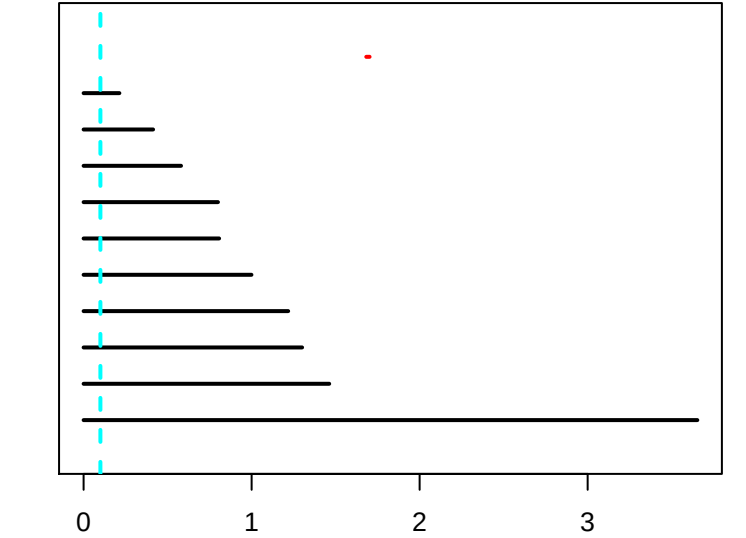



This is the 'Frame' at Euclidean distance = 0
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.354 0.213 0.580 1.706 1.909 1.800 0.999 1.299 1.609
1.354 0.000 1.235 1.813 0.807 1.210 3.111 1.218 2.202 2.199
0.213 1.235 0.000 0.792 1.515 1.702 1.982 1.097 1.505 1.462
0.580 1.813 0.792 0.000 2.267 2.487 1.301 1.019 0.800 2.027
1.706 0.807 1.515 2.267 0.000 0.413 3.496 1.942 2.825 1.851
1.909 1.210 1.702 2.487 0.413 0.000 3.653 2.291 3.111 1.682
1.800 3.111 1.982 1.301 3.496 3.653 0.000 2.215 1.357 2.684
0.999 1.218 1.097 1.019 1.942 2.291 2.215 0.000 1.044 2.552
1.299 2.202 1.505 0.800 2.825 3.111 1.357 1.044 0.000 2.826
1.609 2.199 1.462 2.027 1.851 1.682 2.684 2.552 2.826 0.000

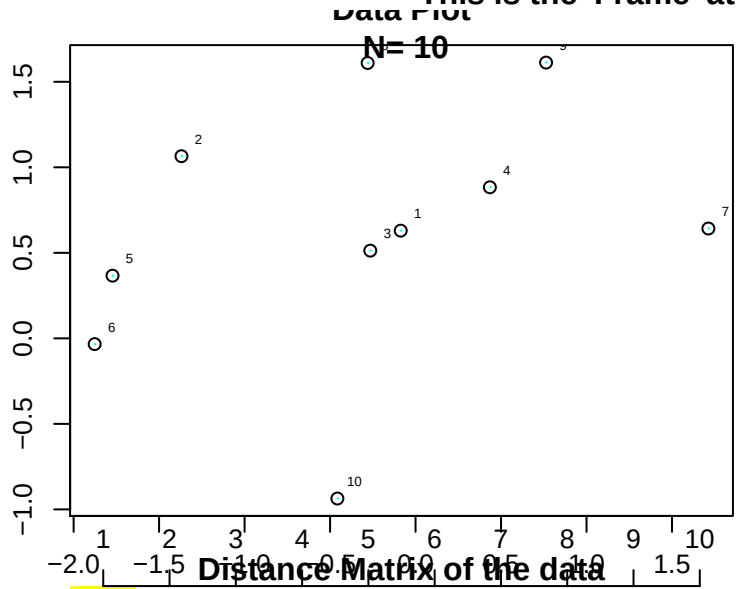

This is the 'Frame' at Euclidean distance = 0.1
Data Plot
N= 10
1
2
3
4
5
6
7
8
9
10
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
1
2
3
4
5
6
7
8
9
10

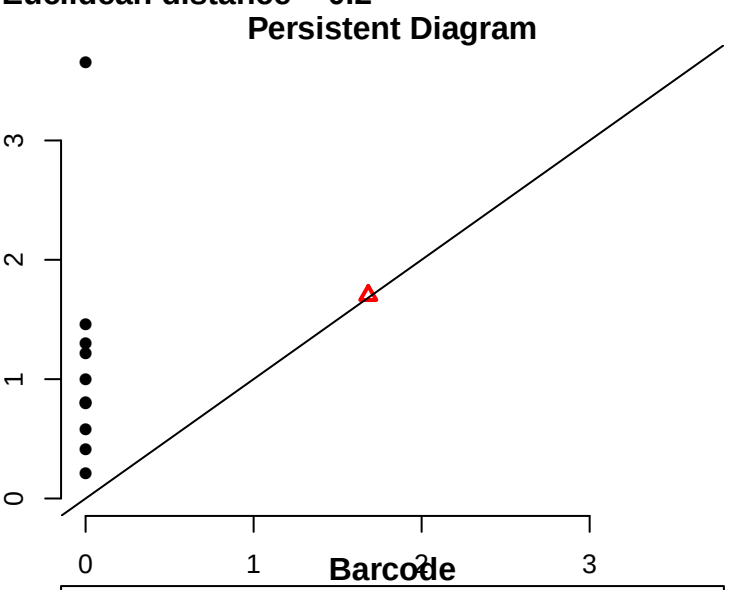

Persistent Diagram
0
1
2
3
0
1
2
3


Distance Matrix of the data
1 0.000 1.354 0.213 0.580 1.706 1.909 1.800 0.999 1.299 1.609
2 1.354 0.000 1.235 1.813 0.807 1.210 3.111 1.218 2.202 2.199
3 0.213 1.235 0.000 0.792 1.515 1.702 1.982 1.097 1.505 1.462
4 0.580 1.813 0.792 0.000 2.267 2.487 1.301 1.019 0.800 2.027
5 1.706 0.807 1.515 2.267 0.000 0.413 3.496 1.942 2.825 1.851
6 1.909 1.210 1.702 2.487 0.413 0.000 3.653 2.291 3.111 1.682
7 1.800 3.111 1.982 1.301 3.496 3.653 0.000 2.215 1.357 2.684
8 0.999 1.218 1.097 1.019 1.942 2.291 2.215 0.000 1.044 2.552
9 1.299 2.202 1.505 0.800 2.825 3.111 1.357 1.044 0.000 2.826
10 1.609 2.199 1.462 2.027 1.851 1.682 2.684 2.552 2.826 0.000


Barcode
0
1
2
3

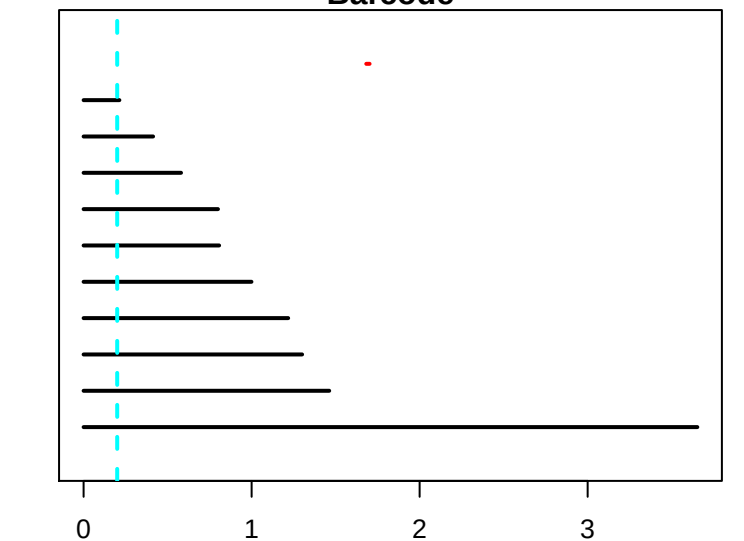
This is the 'Frame' at Euclidean distance = 0.2

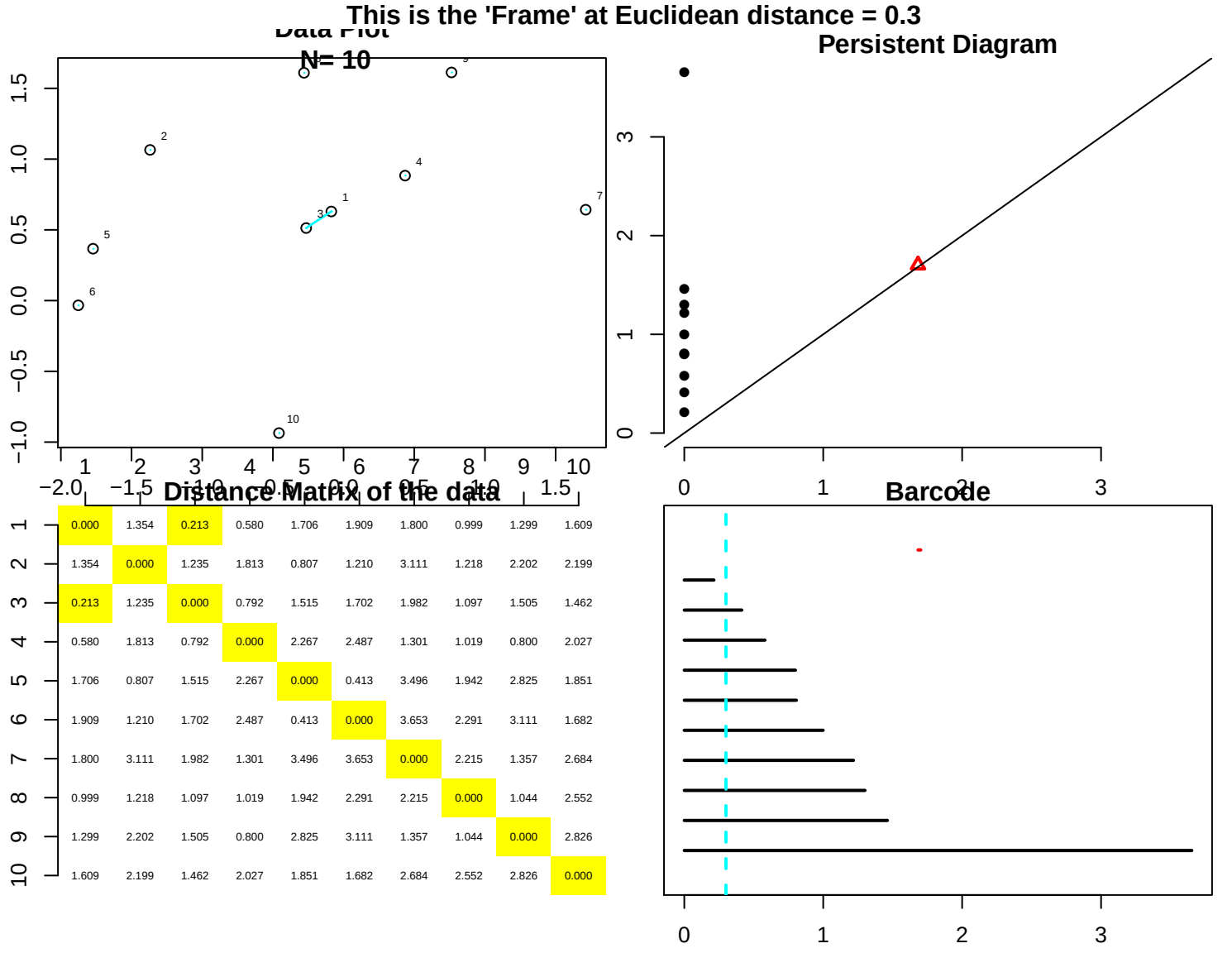




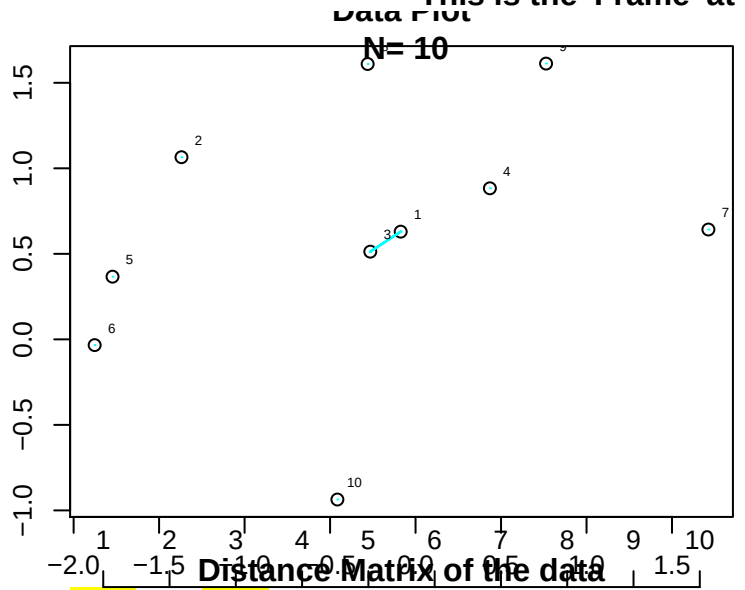
Distance Matrix of the data

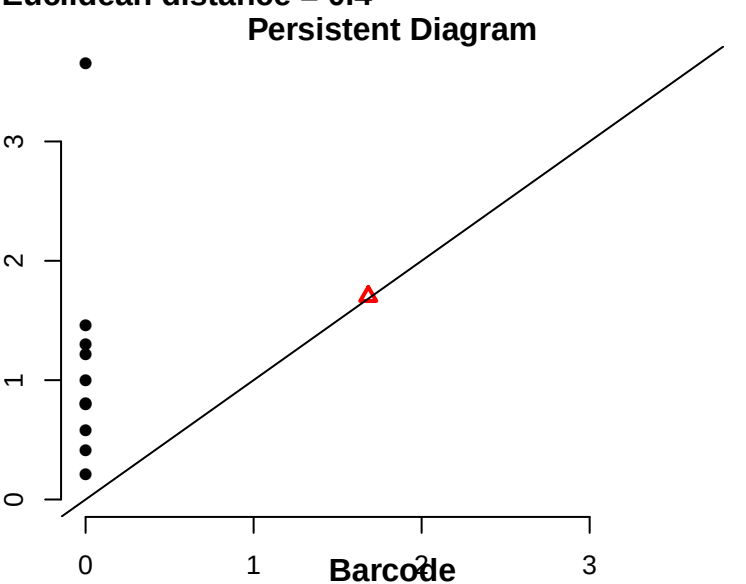
	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000




This is the 'Frame' at Euclidean distance = 0.3
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode

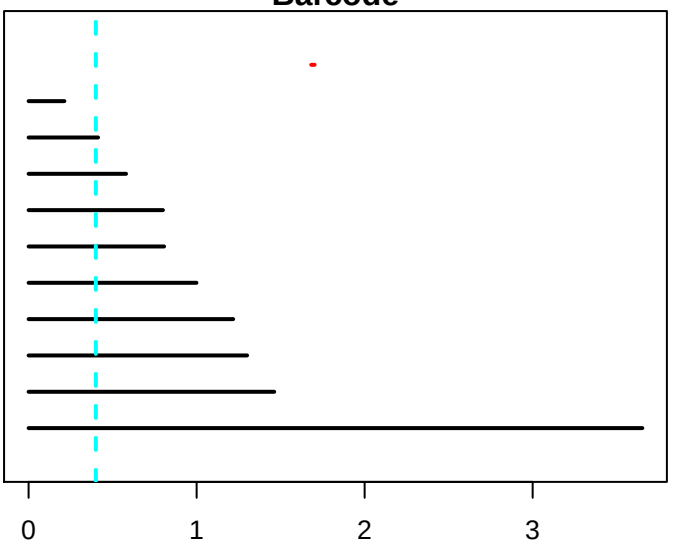
This is the 'Frame' at Euclidean distance = 0.4

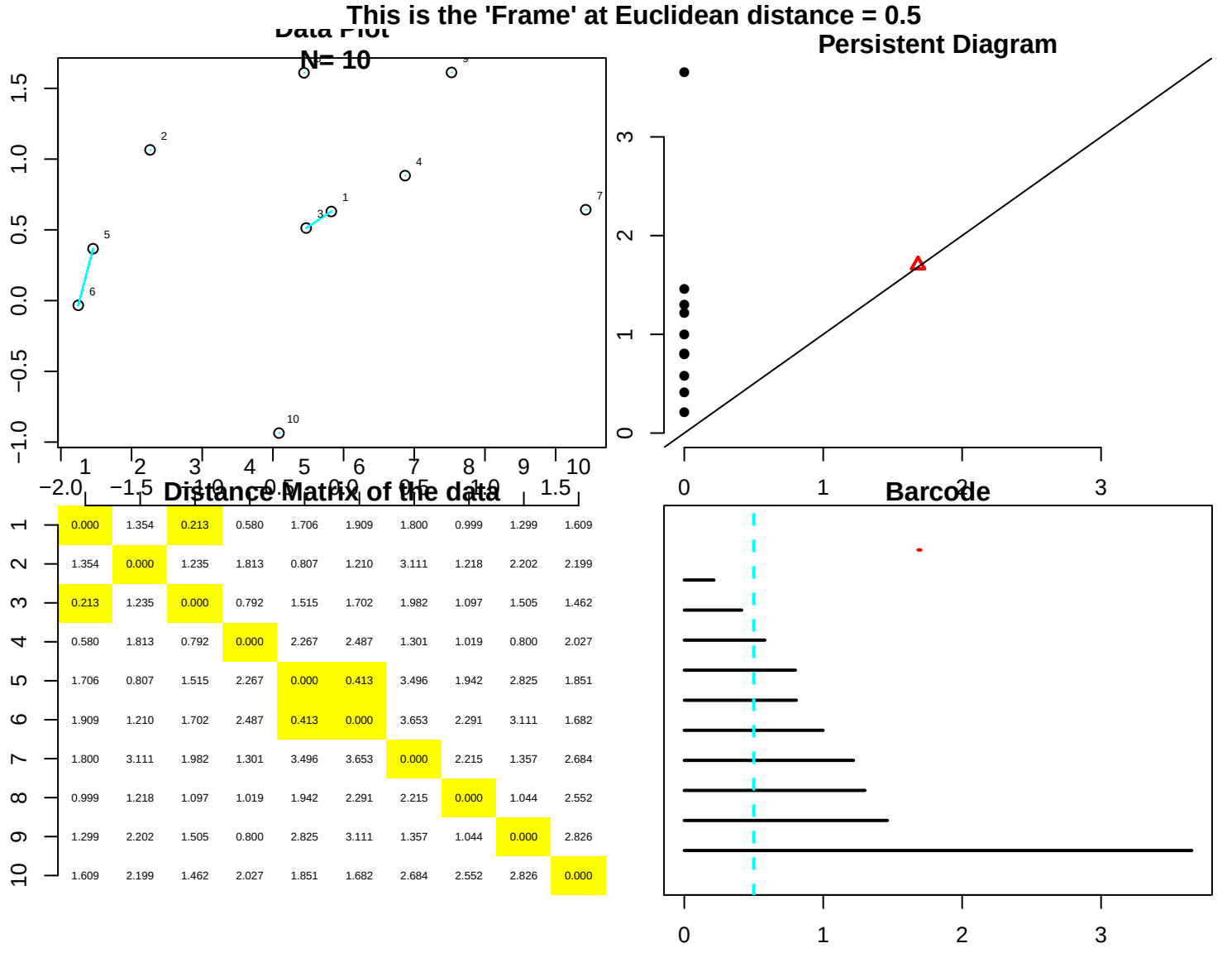


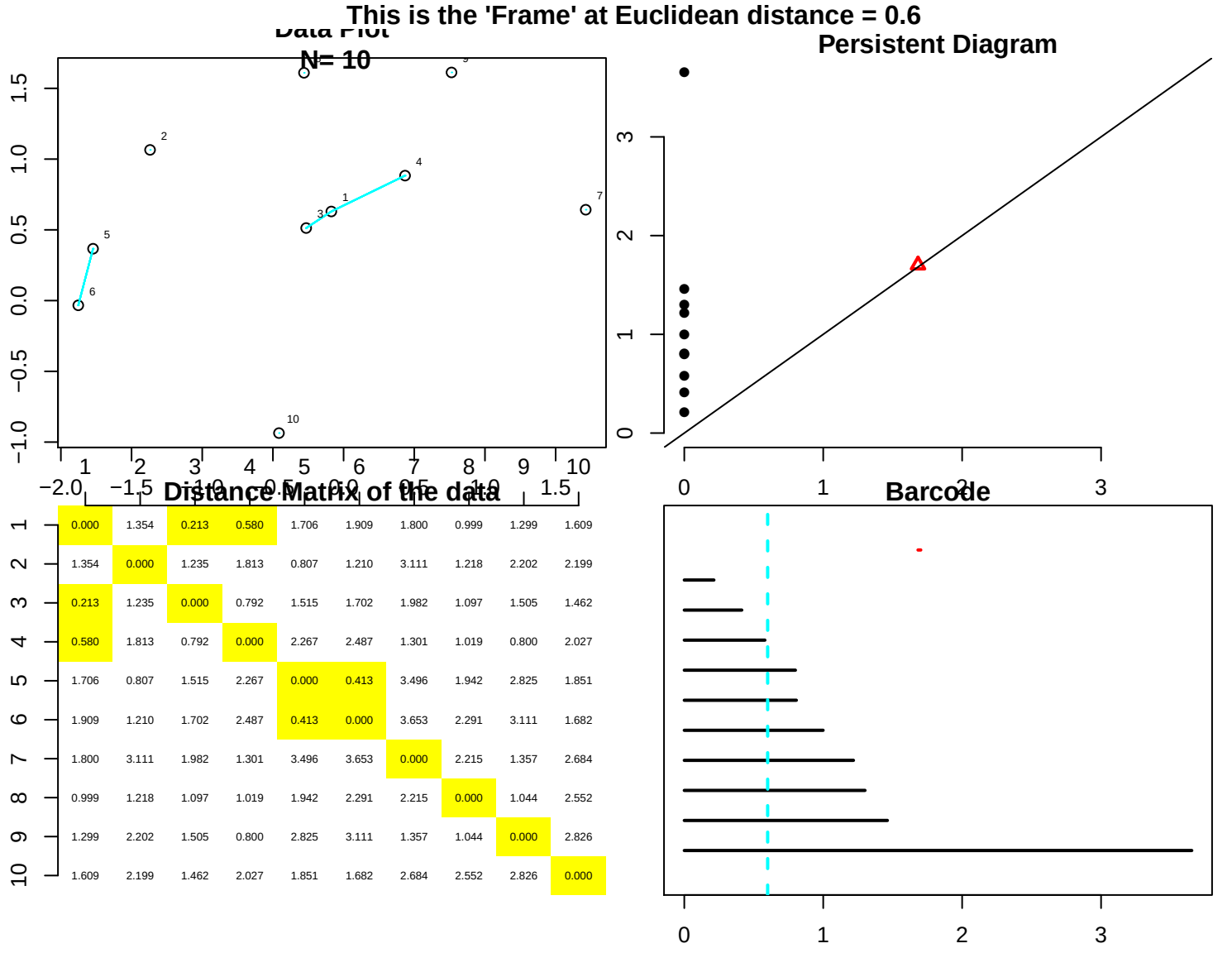


Distance Matrix of the data

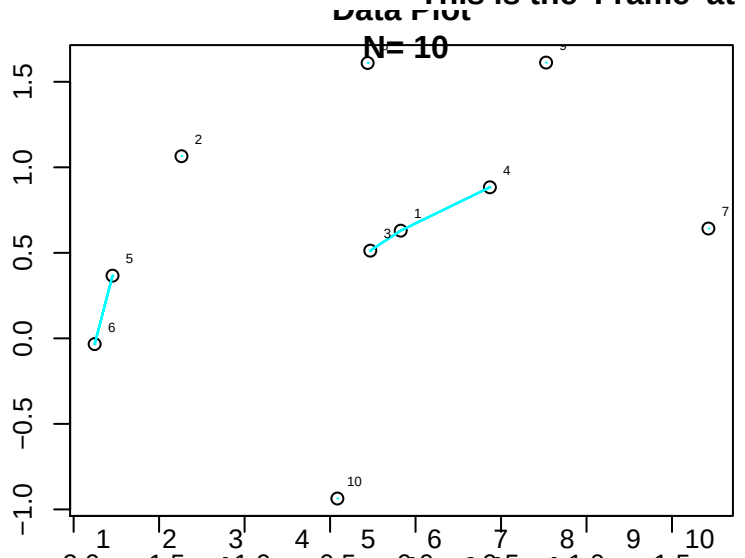
	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000

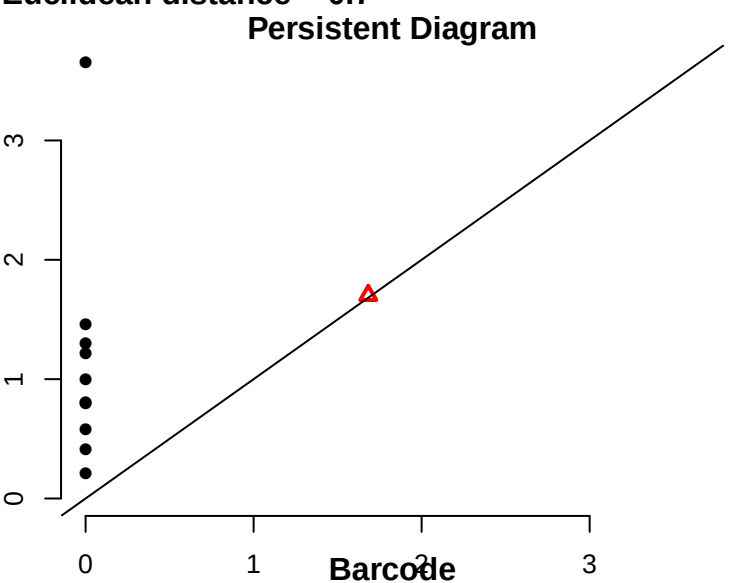



This is the 'Frame' at Euclidean distance = 0.5
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode


This is the 'Frame' at Euclidean distance = 0.6
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode

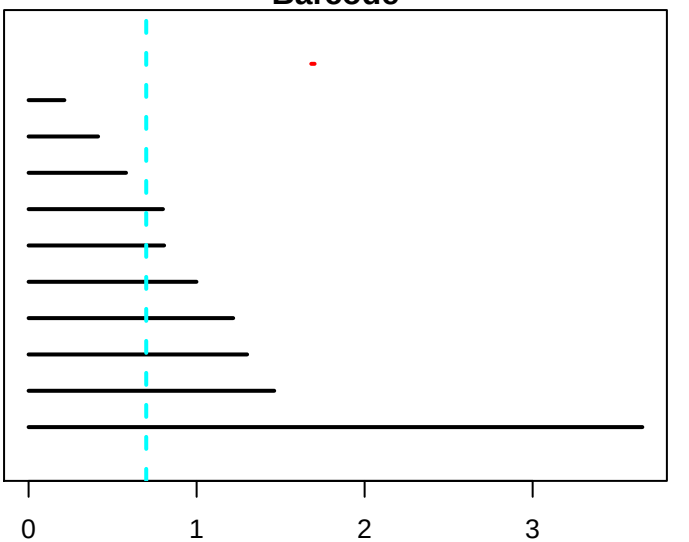
This is the 'Frame' at Euclidean distance = 0.7



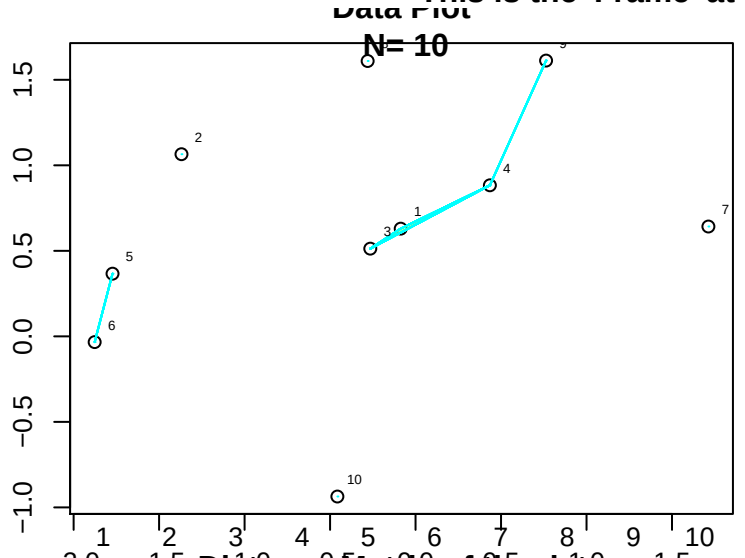


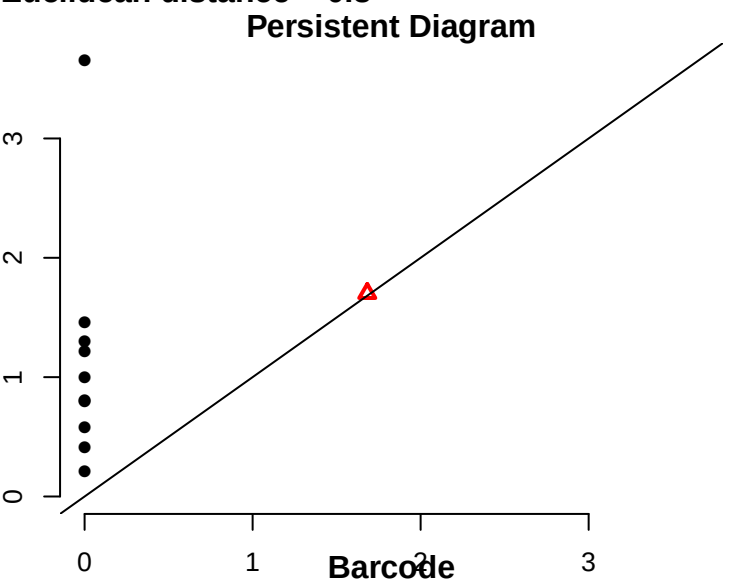
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



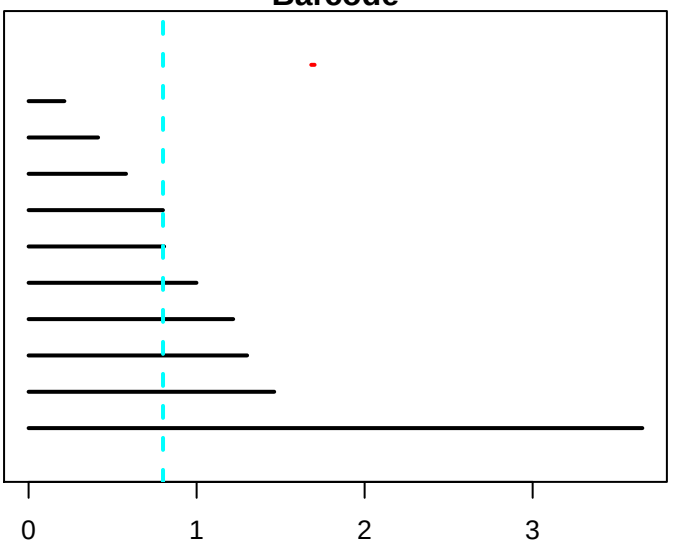
This is the 'Frame' at Euclidean distance = 0.8



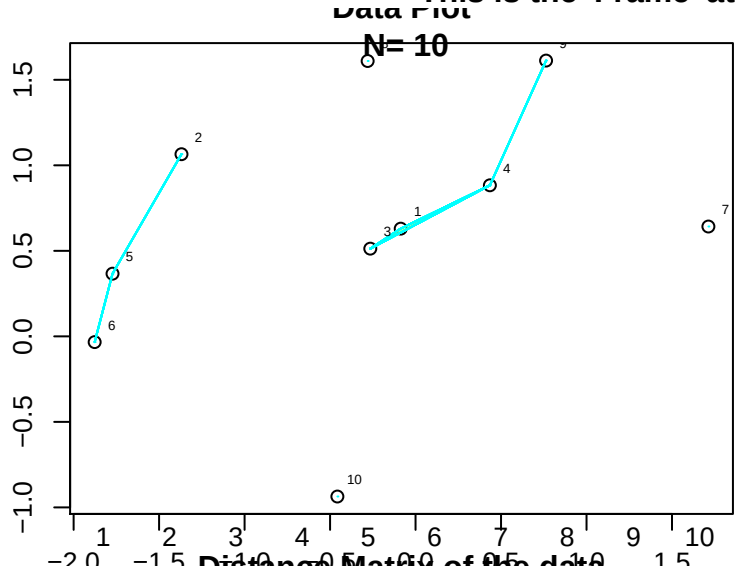


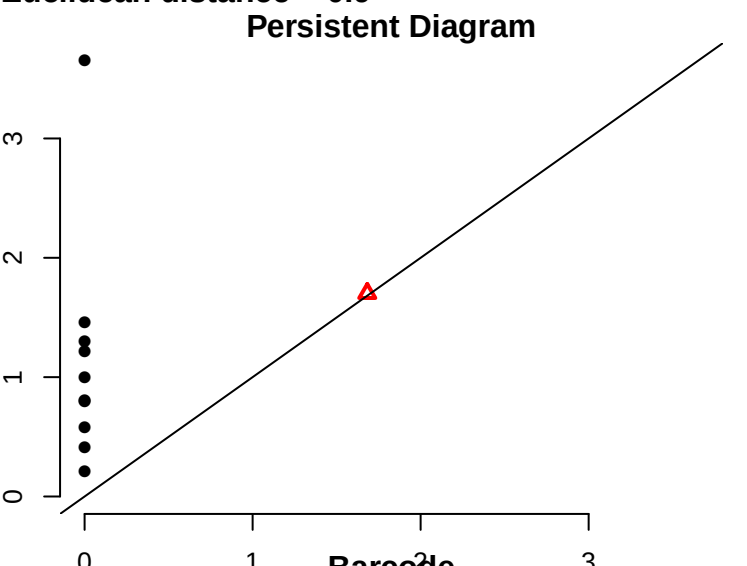
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



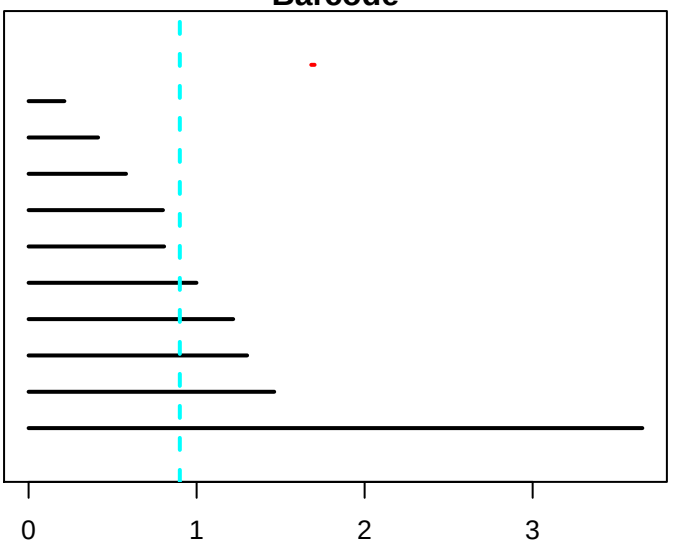
This is the 'Frame' at Euclidean distance = 0.9

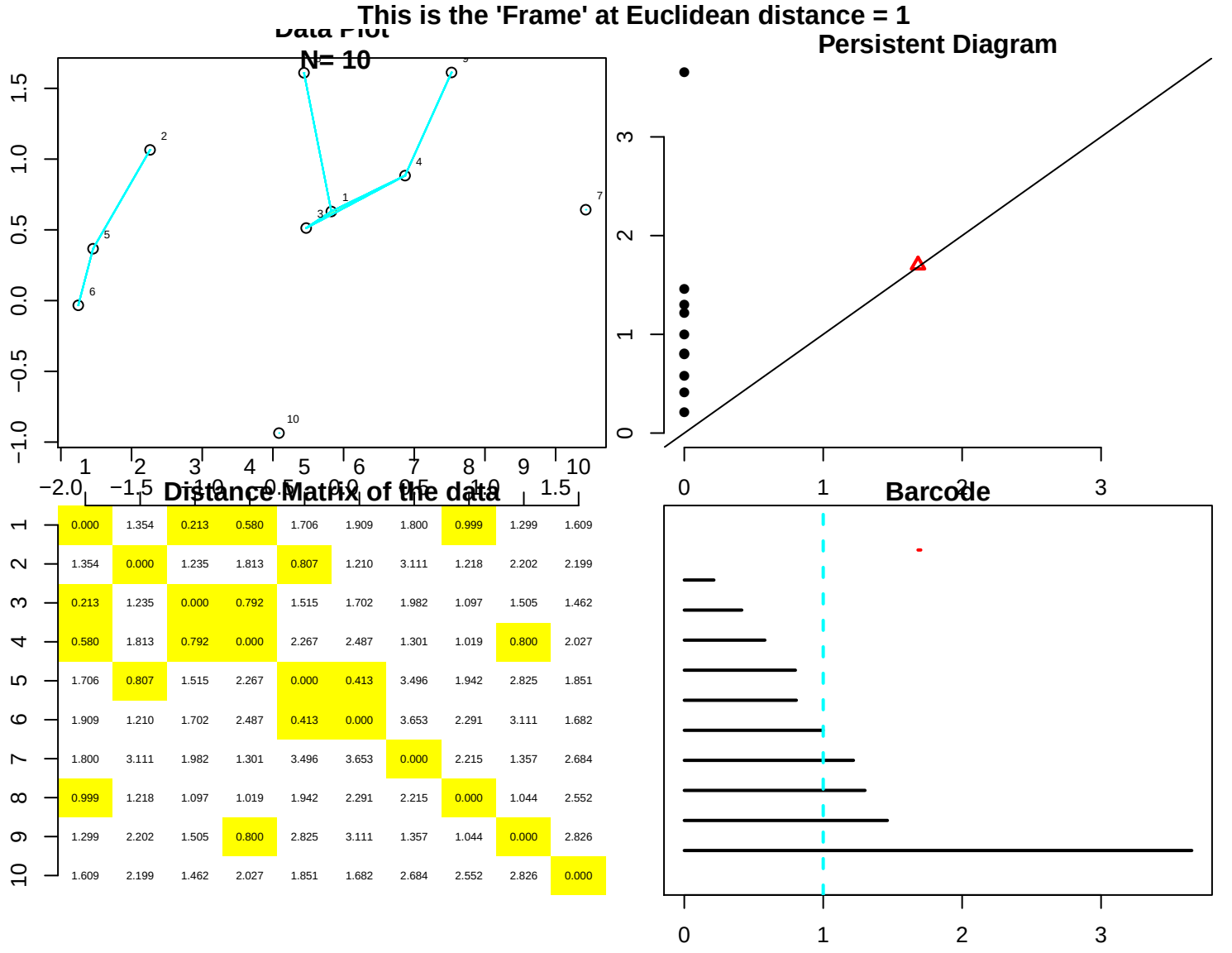




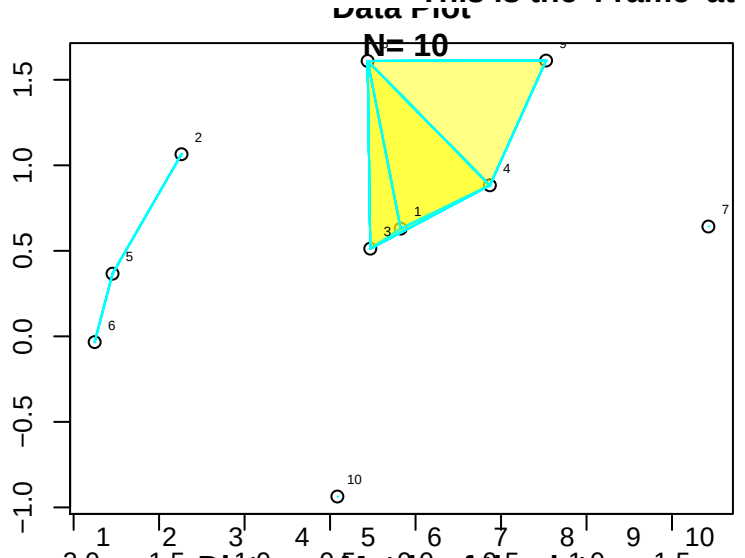
Distance Matrix of the data

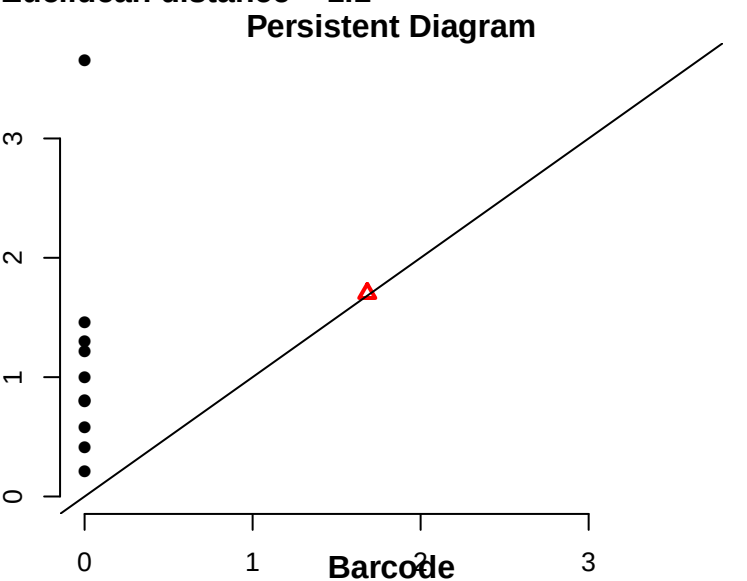
	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000




This is the 'Frame' at Euclidean distance = 1
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.354 0.213 0.580 1.706 1.909 1.800 0.999 1.299 1.609
1.354 0.000 1.235 1.813 0.807 1.210 3.111 1.218 2.202 2.199
0.213 1.235 0.000 0.792 1.515 1.702 1.982 1.097 1.505 1.462
0.580 1.813 0.792 0.000 2.267 2.487 1.301 1.019 0.800 2.027
1.706 0.807 1.515 2.267 0.000 0.413 3.496 1.942 2.825 1.851
1.909 1.210 1.702 2.487 0.413 0.000 3.653 2.291 3.111 1.682
1.800 3.111 1.982 1.301 3.496 3.653 0.000 2.215 1.357 2.684
0.999 1.218 1.097 1.019 1.942 2.291 2.215 0.000 1.044 2.552
1.299 2.202 1.505 0.800 2.825 3.111 1.357 1.044 0.000 2.826
1.609 2.199 1.462 2.027 1.851 1.682 2.684 2.552 2.826 0.000

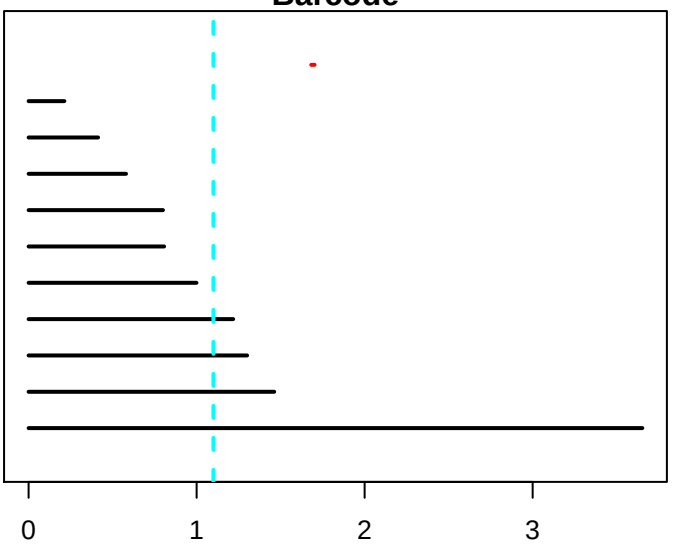
This is the 'Frame' at Euclidean distance = 1.1



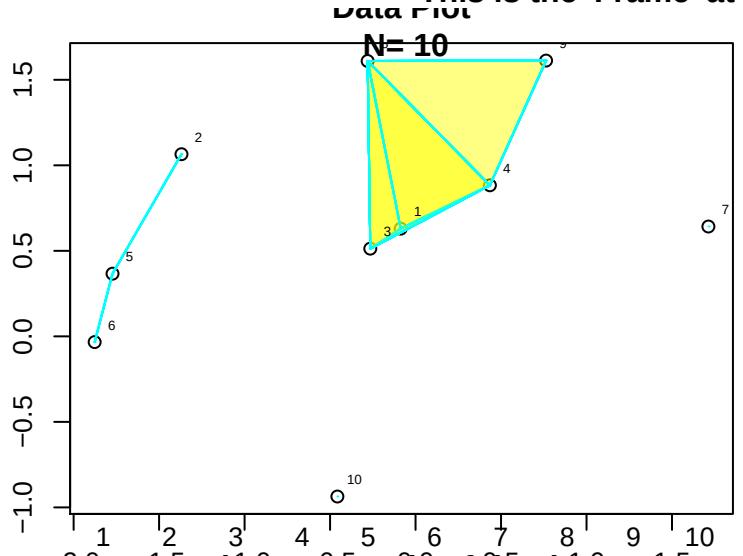


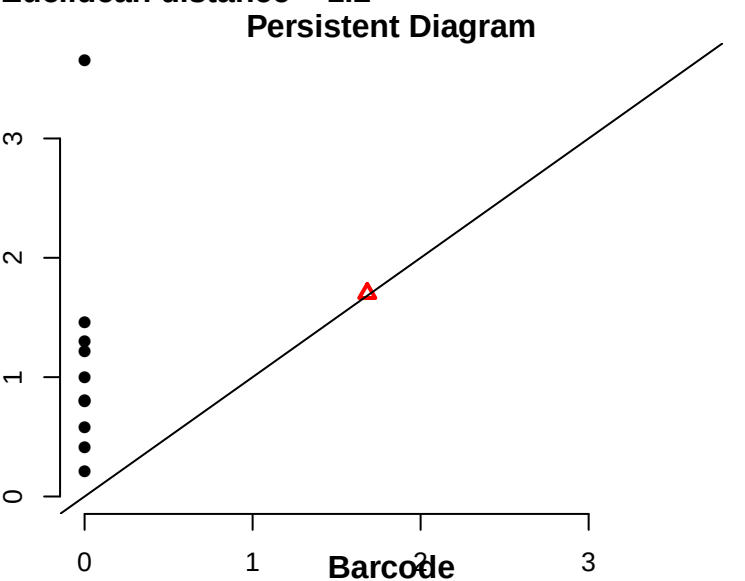
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



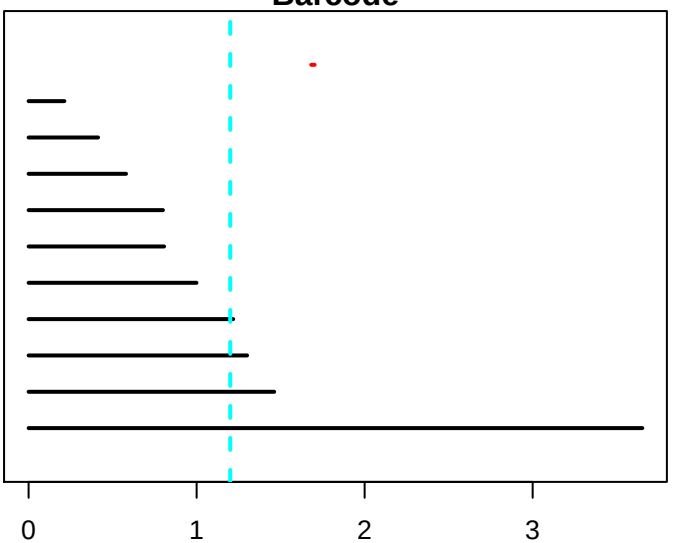
This is the 'Frame' at Euclidean distance = 1.2



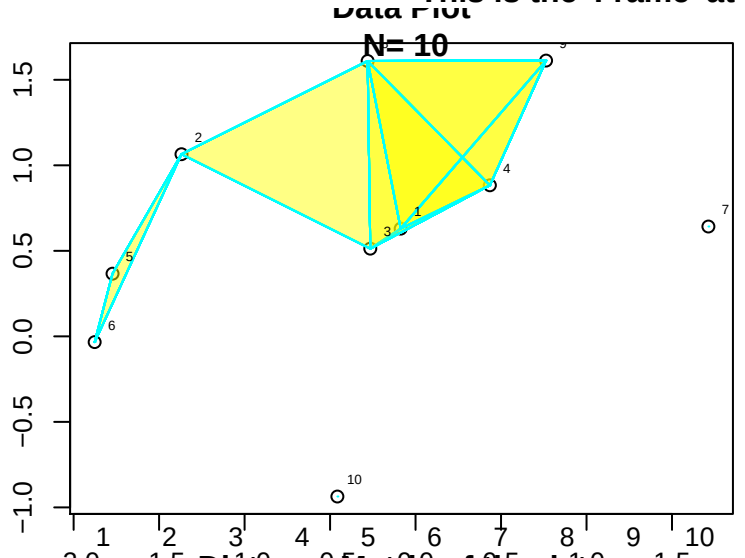


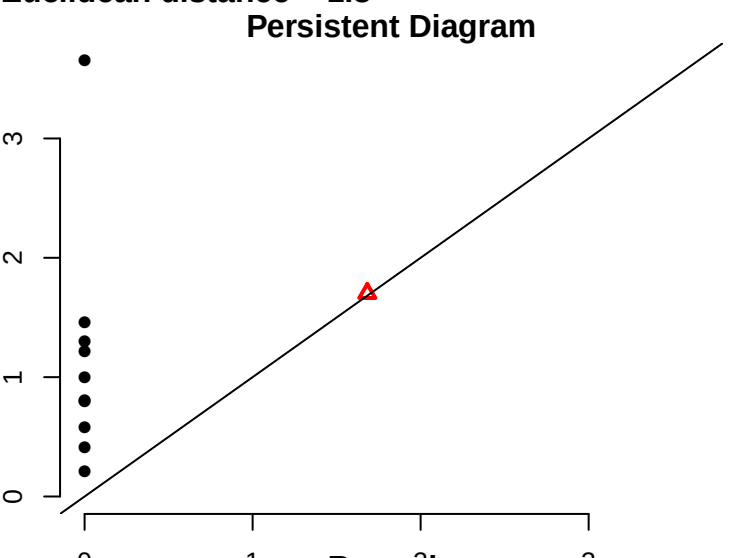
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



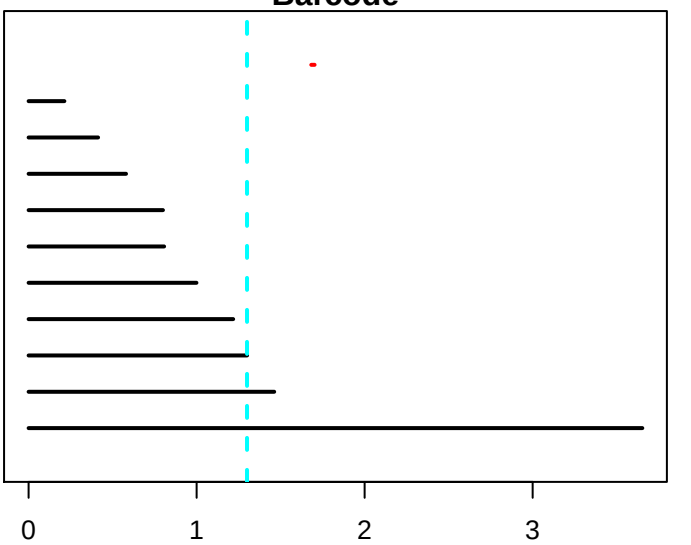
This is the 'Frame' at Euclidean distance = 1.3



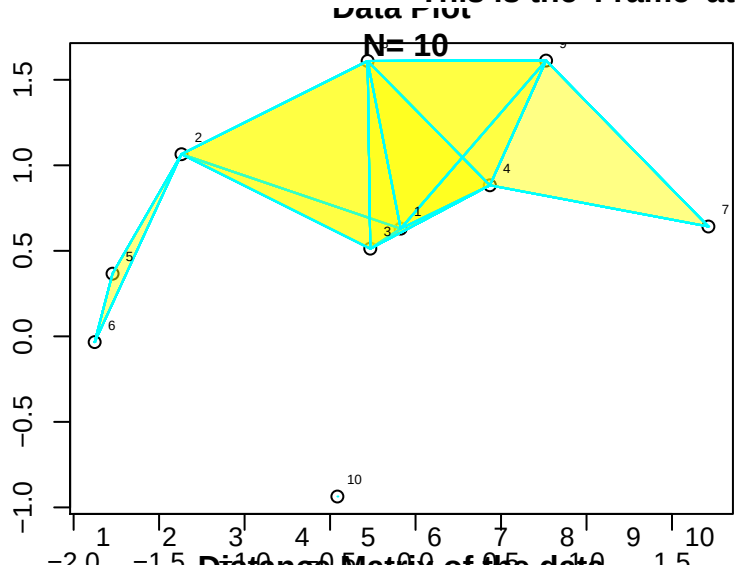


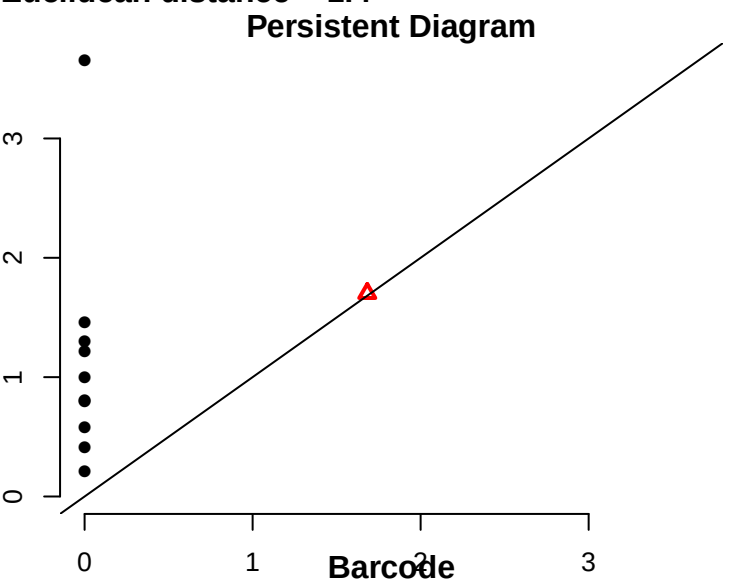
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



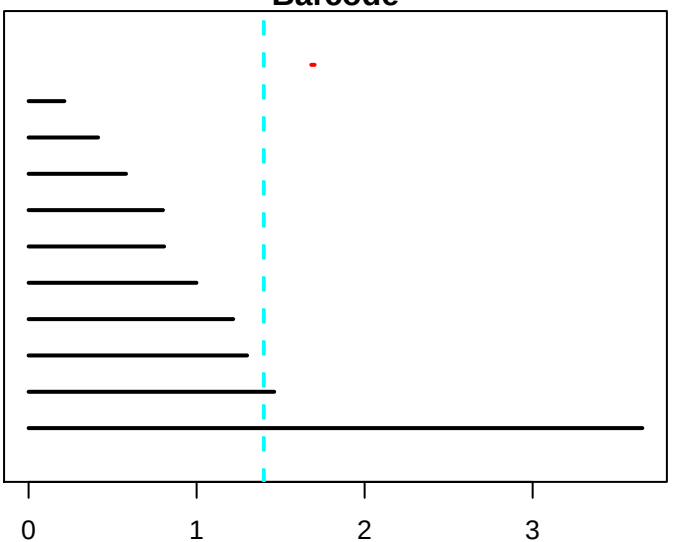
This is the 'Frame' at Euclidean distance = 1.4

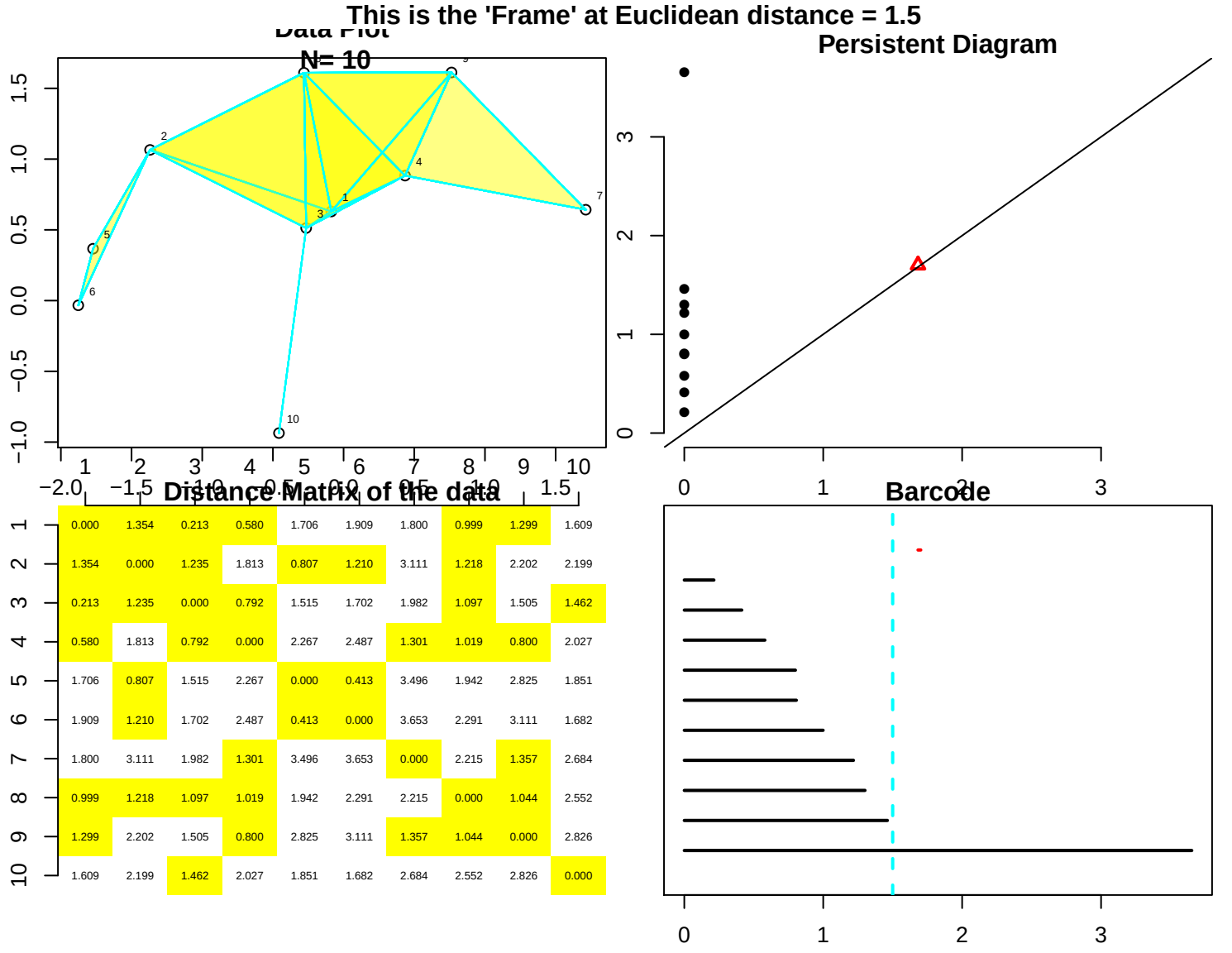




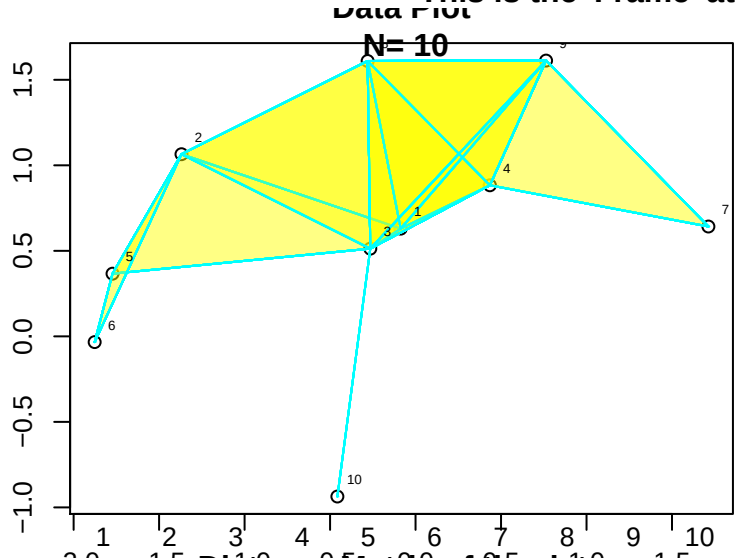
Distance Matrix of the data

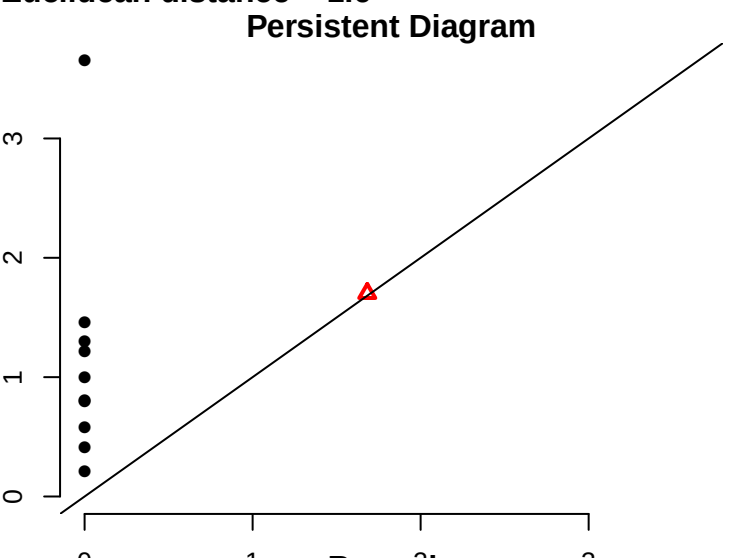
	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000




This is the 'Frame' at Euclidean distance = 1.5
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.354 0.213 0.580 1.706 1.909 1.800 0.999 1.299 1.609
1.354 0.000 1.235 1.813 0.807 1.210 3.111 1.218 2.202 2.199
0.213 1.235 0.000 0.792 1.515 1.702 1.982 1.097 1.505 1.462
0.580 1.813 0.792 0.000 2.267 2.487 1.301 1.019 0.800 2.027
1.706 0.807 1.515 2.267 0.000 0.413 3.496 1.942 2.825 1.851
1.909 1.210 1.702 2.487 0.413 0.000 3.653 2.291 3.111 1.682
1.800 3.111 1.982 1.301 3.496 3.653 0.000 2.215 1.357 2.684
0.999 1.218 1.097 1.019 1.942 2.291 2.215 0.000 1.044 2.552
1.299 2.202 1.505 0.800 2.825 3.111 1.357 1.044 0.000 2.826
1.609 2.199 1.462 2.027 1.851 1.682 2.684 2.552 2.826 0.000

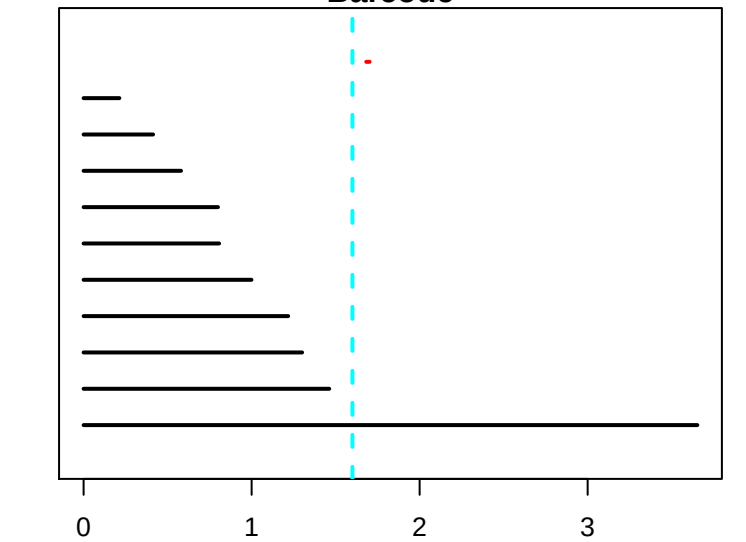
This is the 'Frame' at Euclidean distance = 1.6

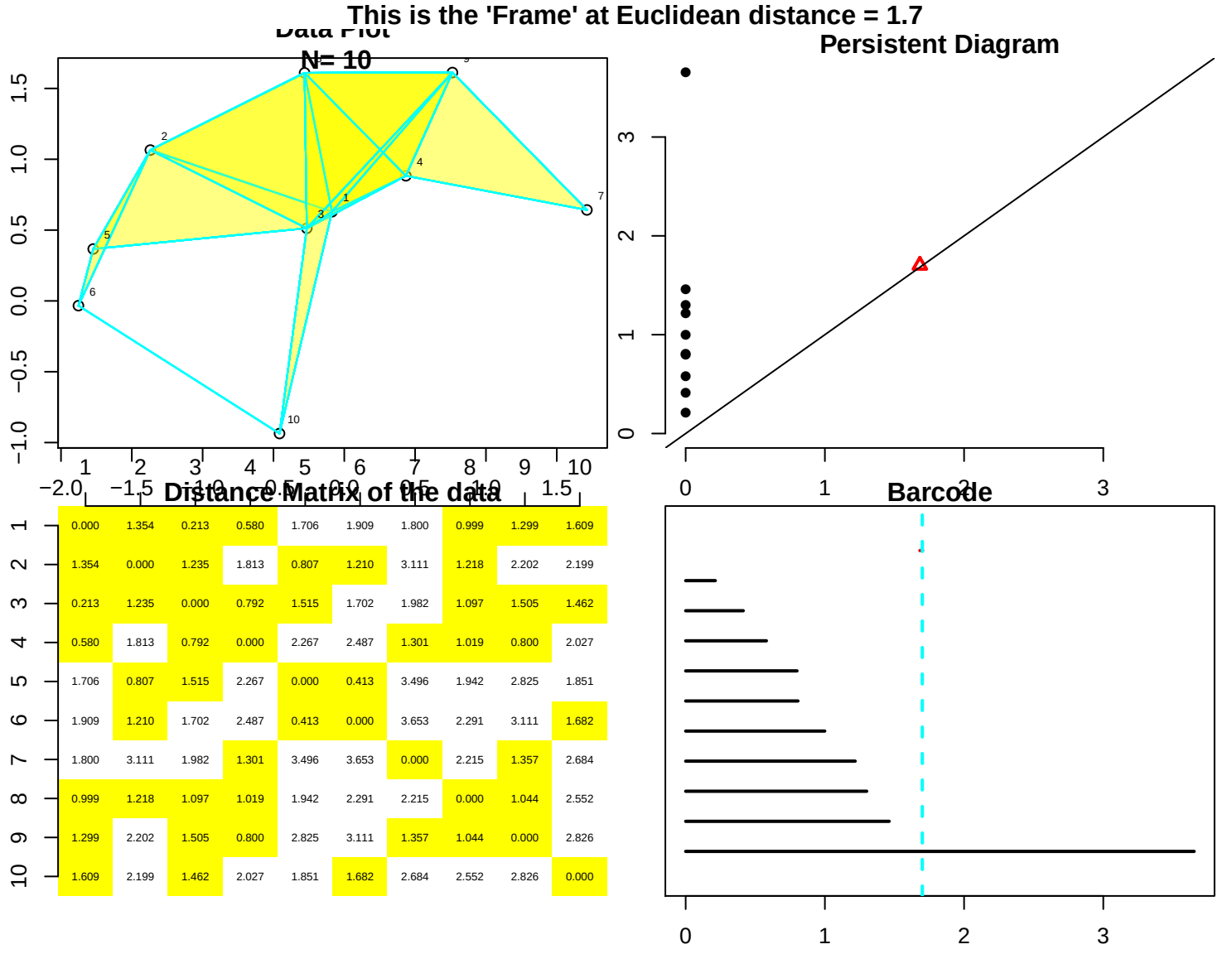




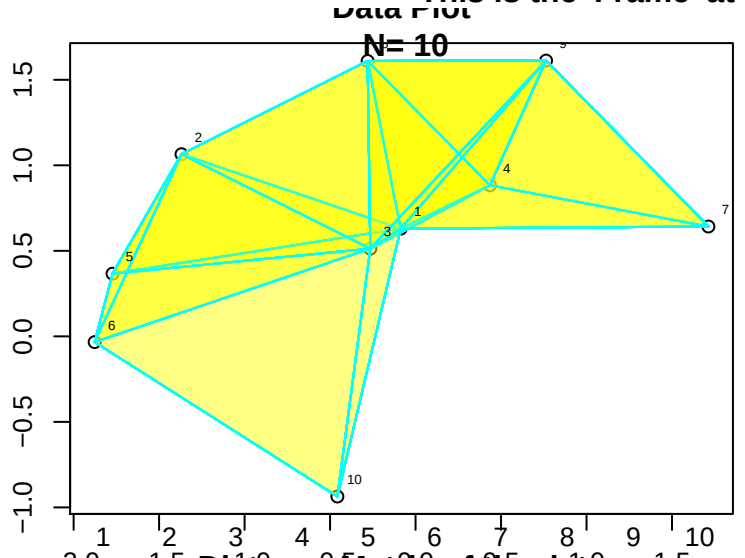
Distance Matrix of the data

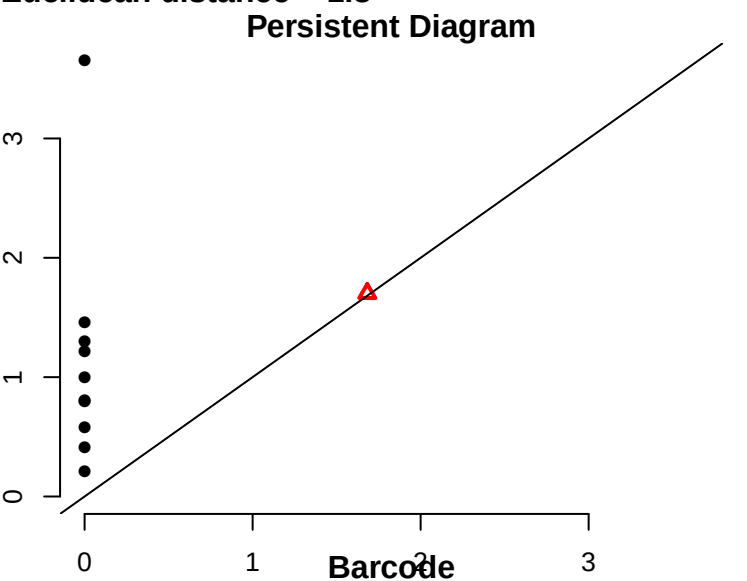
	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000




This is the 'Frame' at Euclidean distance = 1.7
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.354 0.213 0.580 1.706 1.909 1.800 0.999 1.299 1.609
1.354 0.000 1.235 1.813 0.807 1.210 3.111 1.218 2.202 2.199
0.213 1.235 0.000 0.792 1.515 1.702 1.982 1.097 1.505 1.462
0.580 1.813 0.792 0.000 2.267 2.487 1.301 1.019 0.800 2.027
1.706 0.807 1.515 2.267 0.000 0.413 3.496 1.942 2.825 1.851
1.909 1.210 1.702 2.487 0.413 0.000 3.653 2.291 3.111 1.682
1.800 3.111 1.982 1.301 3.496 3.653 0.000 2.215 1.357 2.684
0.999 1.218 1.097 1.019 1.942 2.291 2.215 0.000 1.044 2.552
1.299 2.202 1.505 0.800 2.825 3.111 1.357 1.044 0.000 2.826
1.609 2.199 1.462 2.027 1.851 1.682 2.684 2.552 2.826 0.000

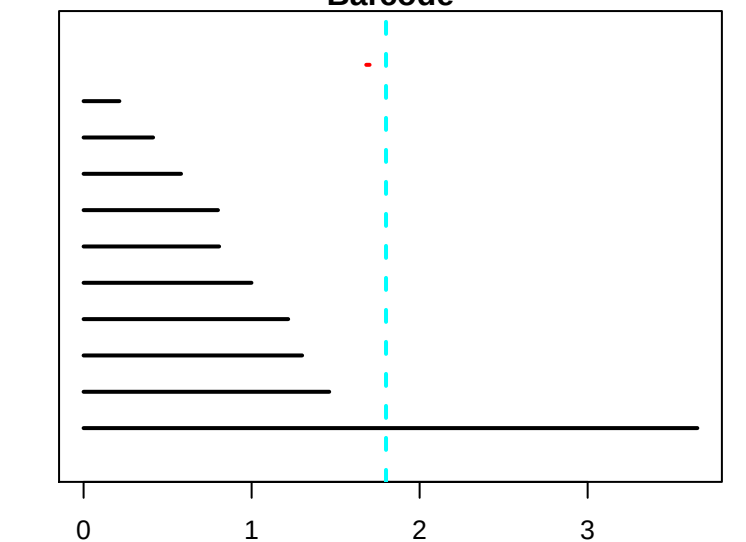
This is the 'Frame' at Euclidean distance = 1.8

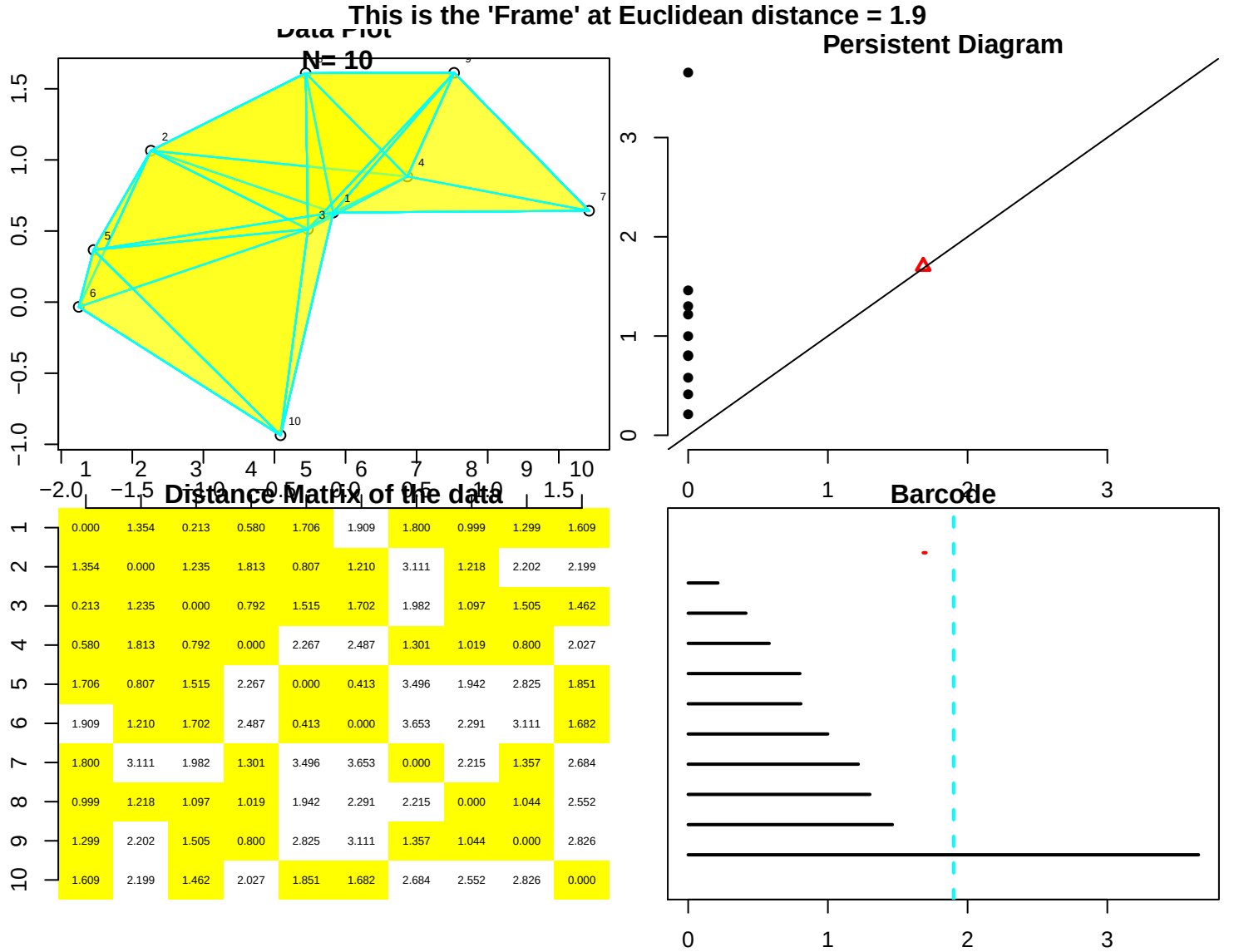




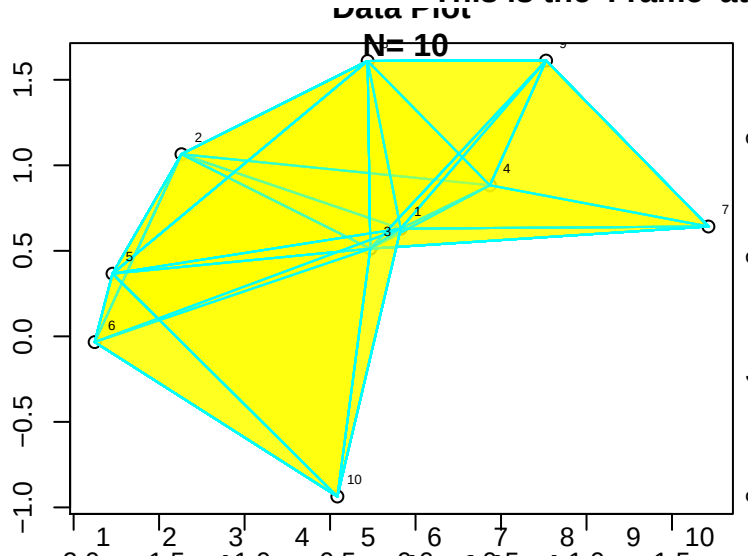
Distance Matrix of the data

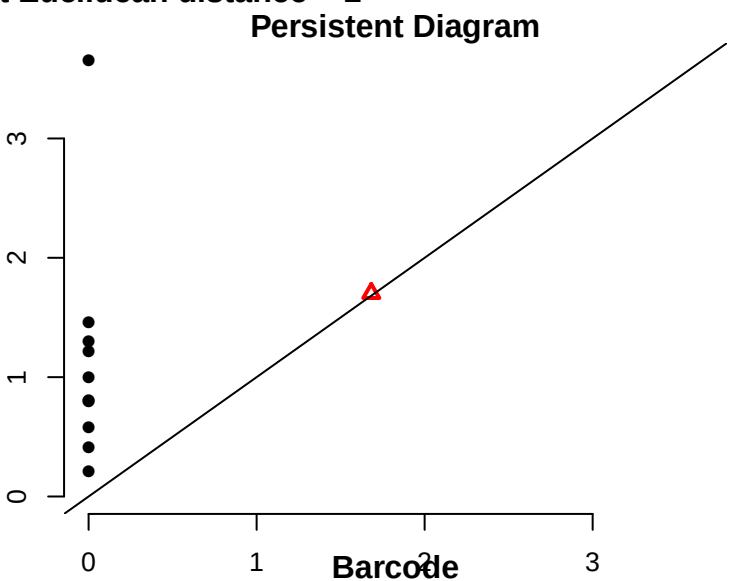
	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000




This is the 'Frame' at Euclidean distance = 1.9
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.354 0.213 0.580 1.706 1.909 1.800 0.999 1.299 1.609
1.354 0.000 1.235 1.813 0.807 1.210 3.111 1.218 2.202 2.199
0.213 1.235 0.000 0.792 1.515 1.702 1.982 1.097 1.505 1.462
0.580 1.813 0.792 0.000 2.267 2.487 1.301 1.019 0.800 2.027
1.706 0.807 1.515 2.267 0.000 0.413 3.496 1.942 2.825 1.851
1.909 1.210 1.702 2.487 0.413 0.000 3.653 2.291 3.111 1.682
1.800 3.111 1.982 1.301 3.496 3.653 0.000 2.215 1.357 2.684
0.999 1.218 1.097 1.019 1.942 2.291 2.215 0.000 1.044 2.552
1.299 2.202 1.505 0.800 2.825 3.111 1.357 1.044 0.000 2.826
1.609 2.199 1.462 2.027 1.851 1.682 2.684 2.552 2.826 0.000

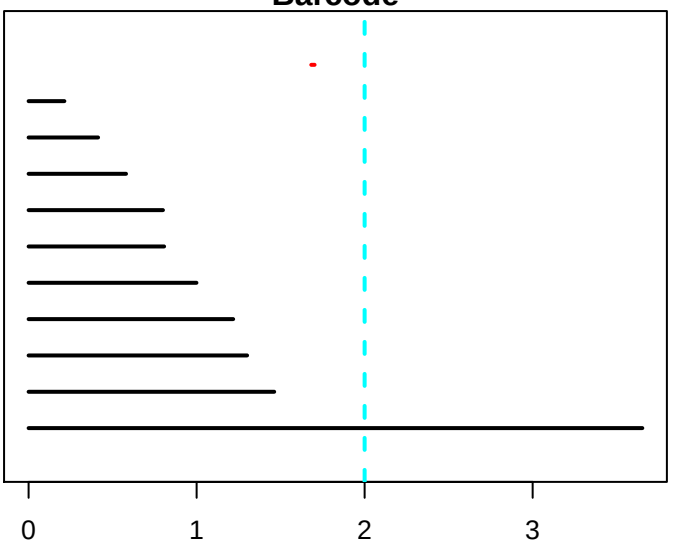
This is the 'Frame' at Euclidean distance = 2



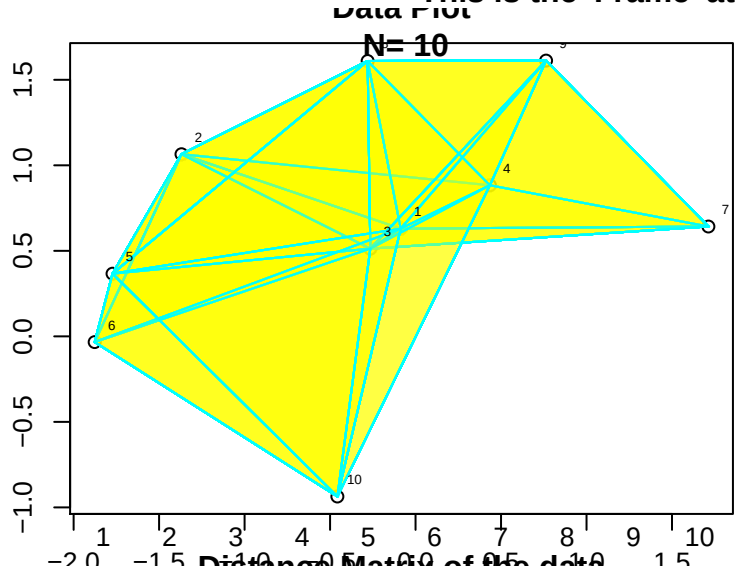


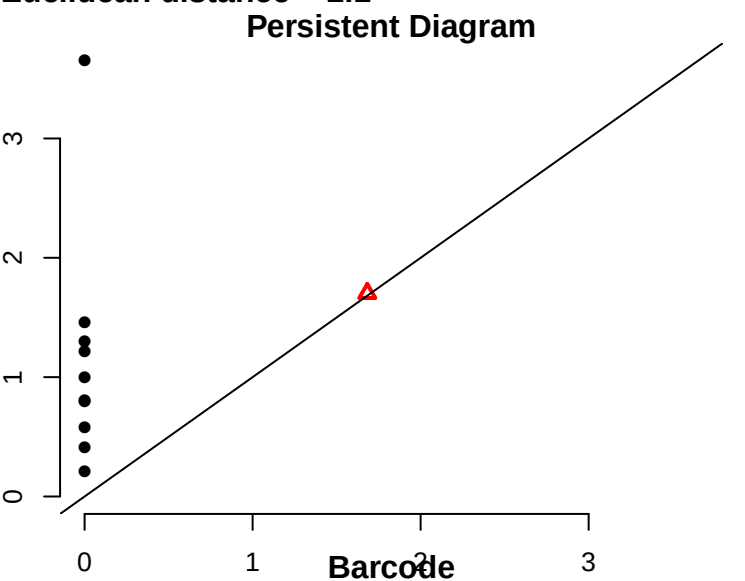
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



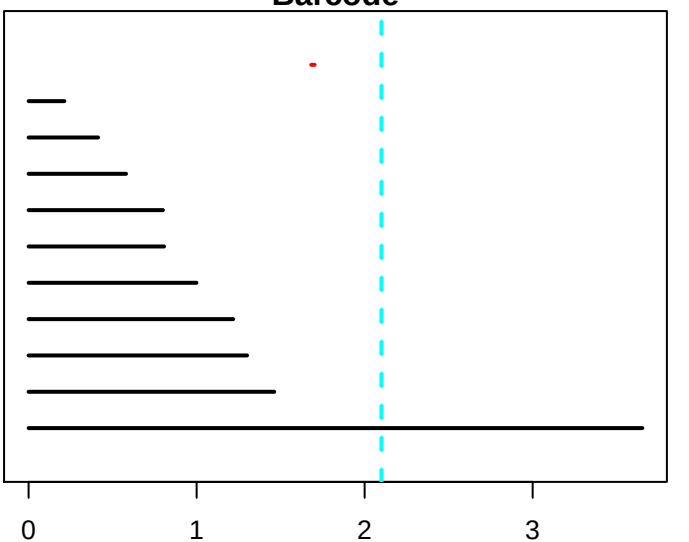
This is the 'Frame' at Euclidean distance = 2.1

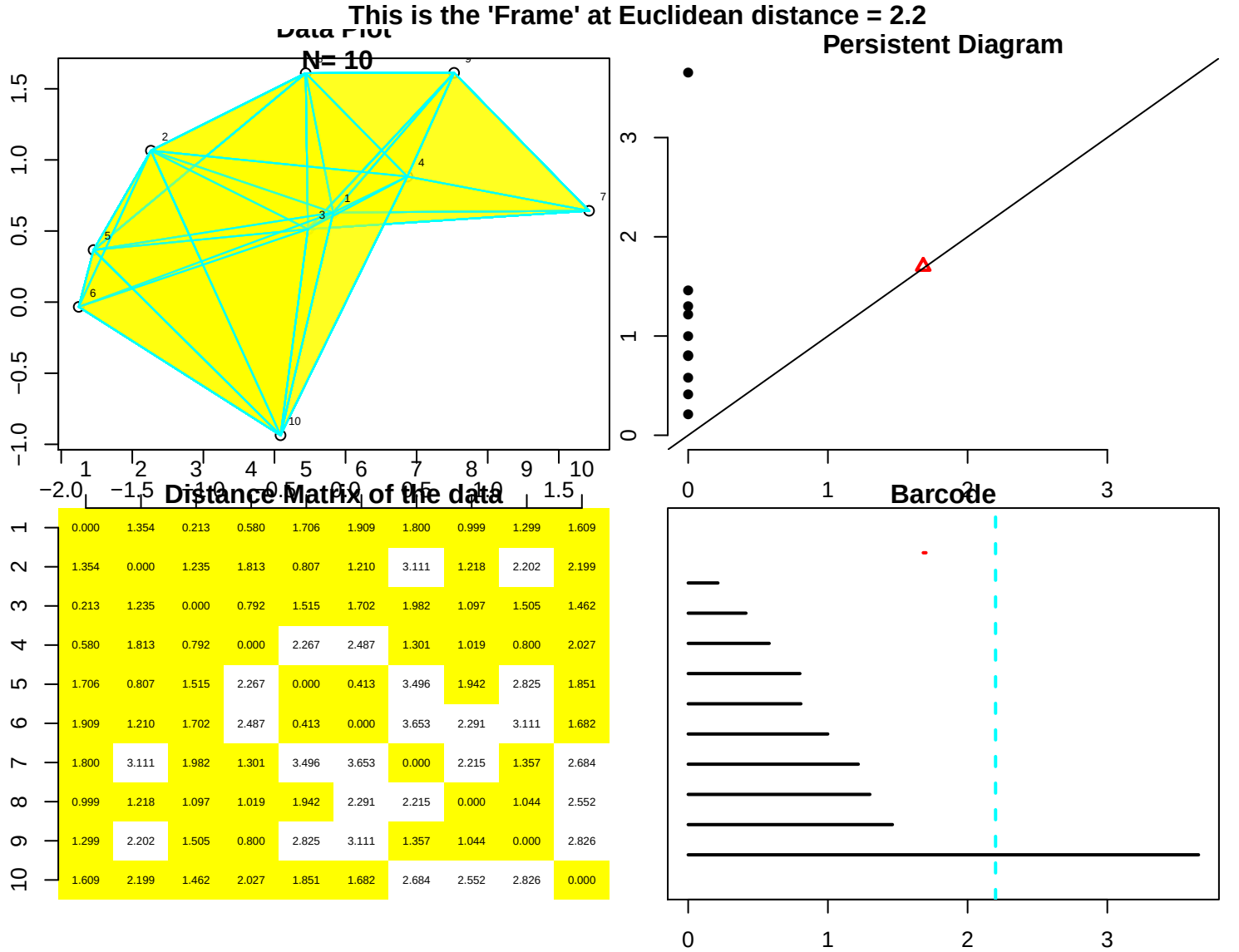




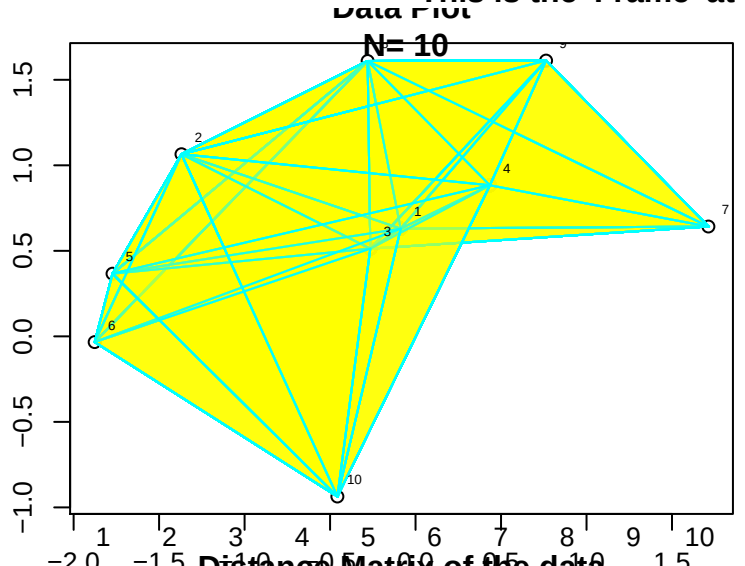
Distance Matrix of the data

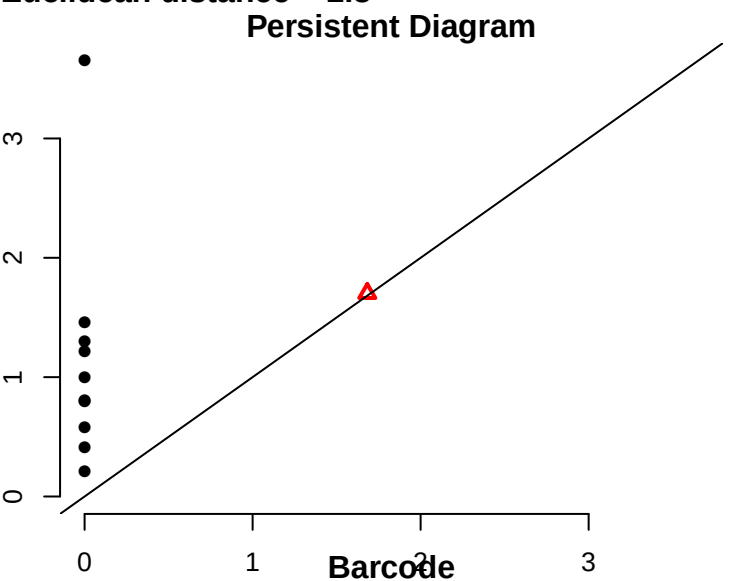
	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000




This is the 'Frame' at Euclidean distance = 2.2
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.354 0.213 0.580 1.706 1.909 1.800 0.999 1.299 1.609
1.354 0.000 1.235 1.813 0.807 1.210 3.111 1.218 2.202 2.199
0.213 1.235 0.000 0.792 1.515 1.702 1.982 1.097 1.505 1.462
0.580 1.813 0.792 0.000 2.267 2.487 1.301 1.019 0.800 2.027
1.706 0.807 1.515 2.267 0.000 0.413 3.496 1.942 2.825 1.851
1.909 1.210 1.702 2.487 0.413 0.000 3.653 2.291 3.111 1.682
1.800 3.111 1.982 1.301 3.496 3.653 0.000 2.215 1.357 2.684
0.999 1.218 1.097 1.019 1.942 2.291 2.215 0.000 1.044 2.552
1.299 2.202 1.505 0.800 2.825 3.111 1.357 1.044 0.000 2.826
1.609 2.199 1.462 2.027 1.851 1.682 2.684 2.552 2.826 0.000

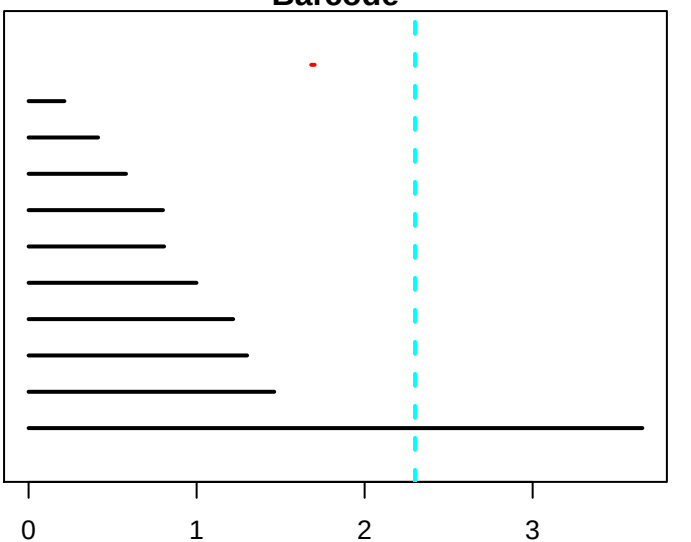
This is the 'Frame' at Euclidean distance = 2.3

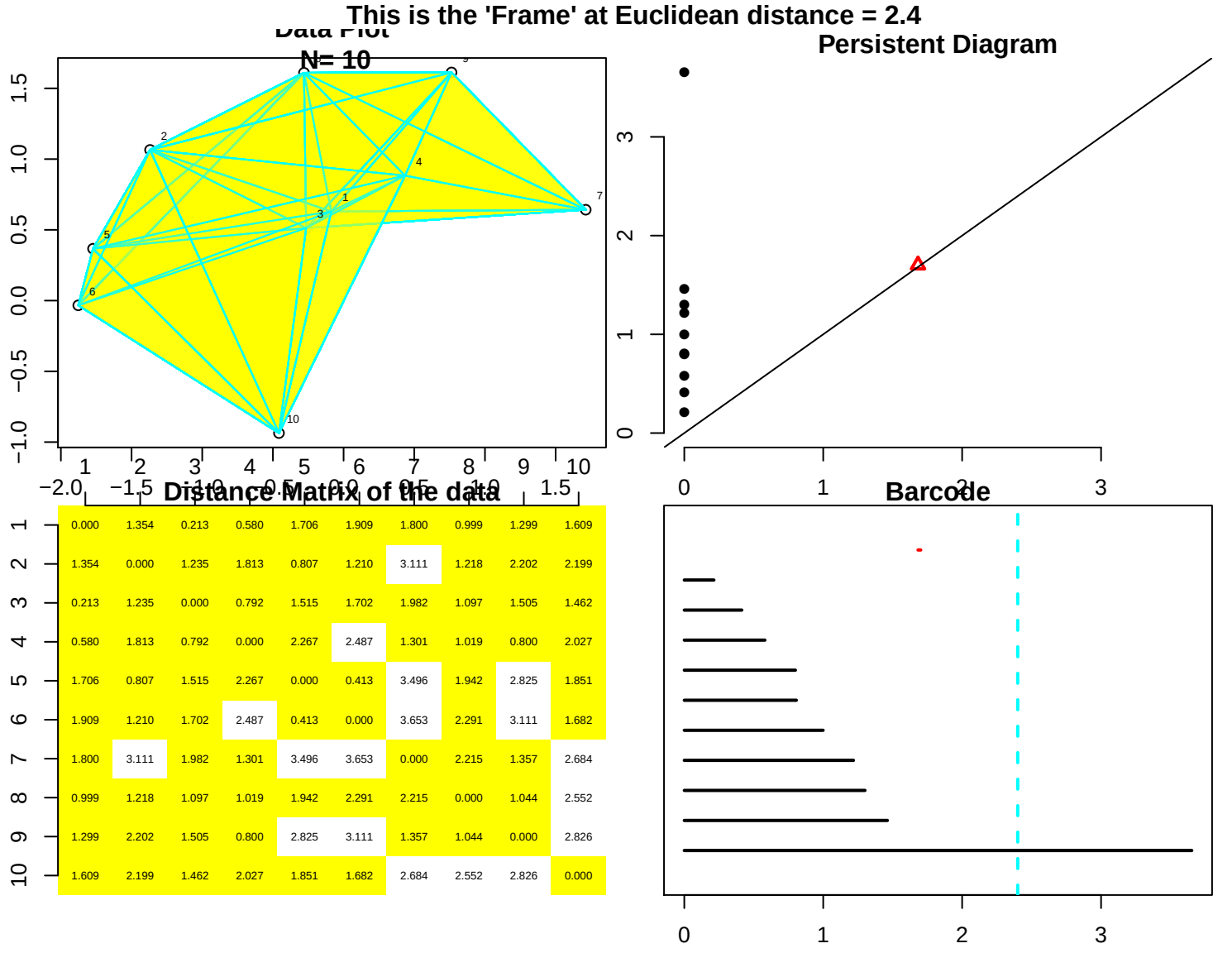


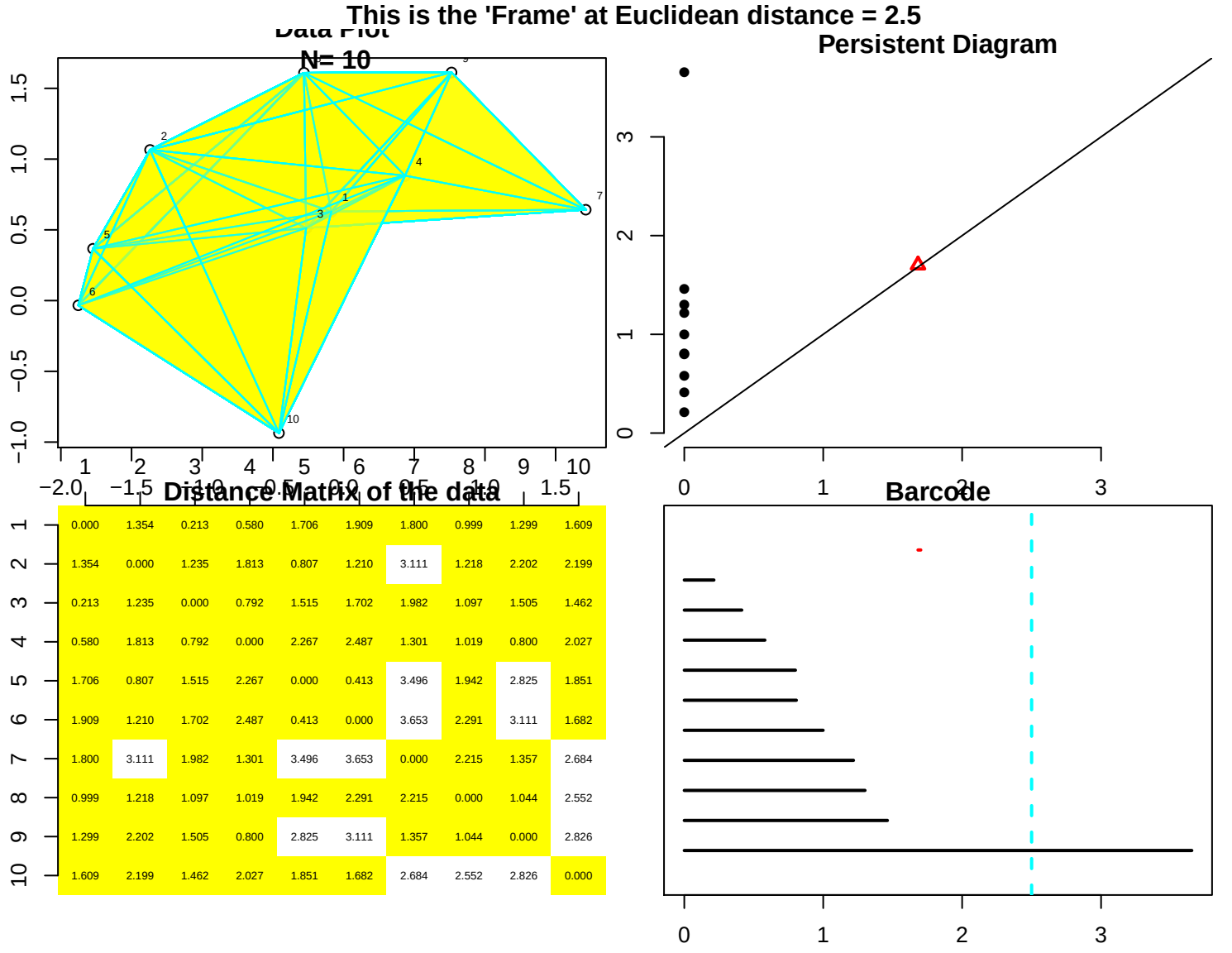


Distance Matrix of the data

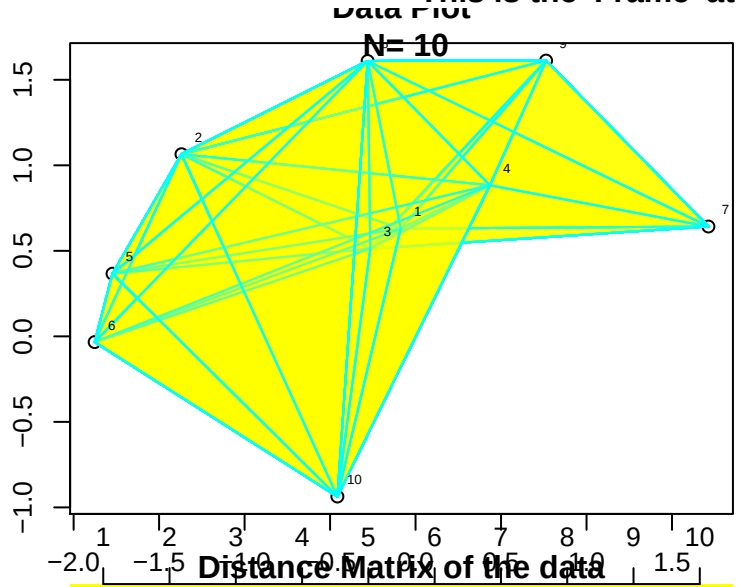
	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000

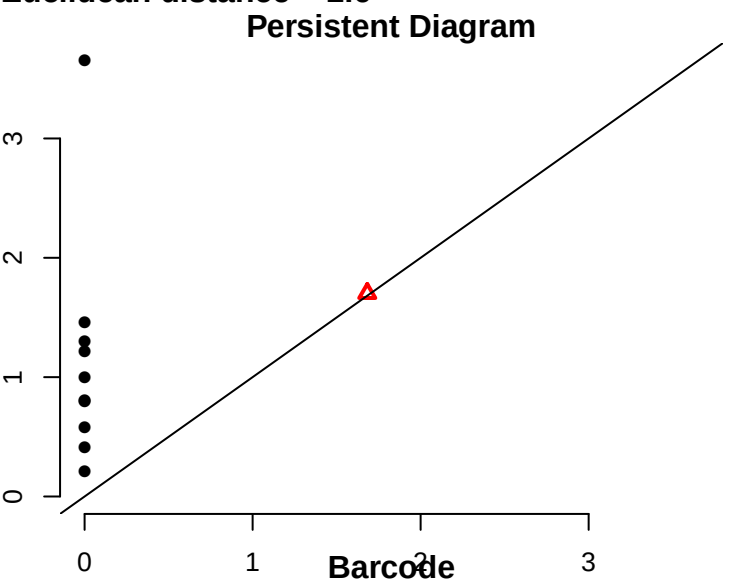



This is the 'Frame' at Euclidean distance = 2.4
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.354 0.213 0.580 1.706 1.909 1.800 0.999 1.299 1.609
1.354 0.000 1.235 1.813 0.807 1.210 3.111 1.218 2.202 2.199
0.213 1.235 0.000 0.792 1.515 1.702 1.982 1.097 1.505 1.462
0.580 1.813 0.792 0.000 2.267 2.487 1.301 1.019 0.800 2.027
1.706 0.807 1.515 2.267 0.000 0.413 3.496 1.942 2.825 1.851
1.909 1.210 1.702 2.487 0.413 0.000 3.653 2.291 3.111 1.682
1.800 3.111 1.982 1.301 3.496 3.653 0.000 2.215 1.357 2.684
0.999 1.218 1.097 1.019 1.942 2.291 2.215 0.000 1.044 2.552
1.299 2.202 1.505 0.800 2.825 3.111 1.357 1.044 0.000 2.826
1.609 2.199 1.462 2.027 1.851 1.682 2.684 2.552 2.826 0.000


This is the 'Frame' at Euclidean distance = 2.5
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode

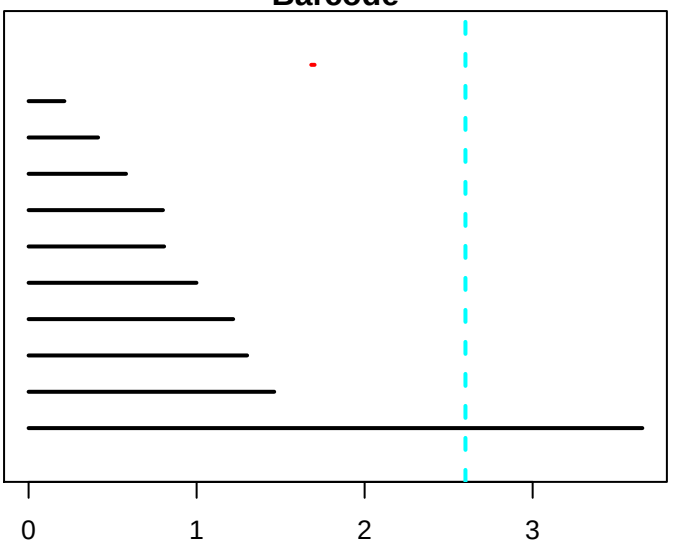
This is the 'Frame' at Euclidean distance = 2.6

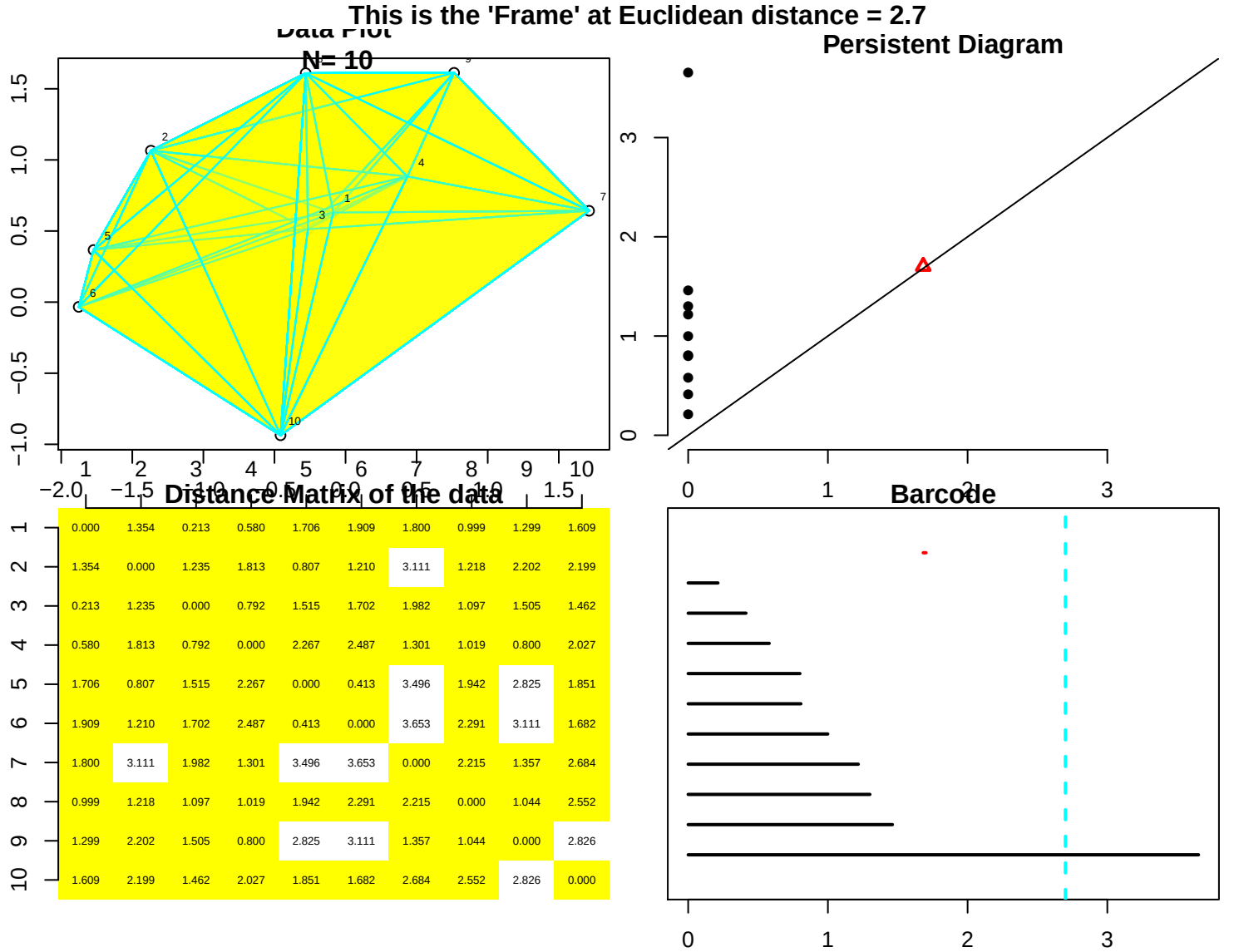




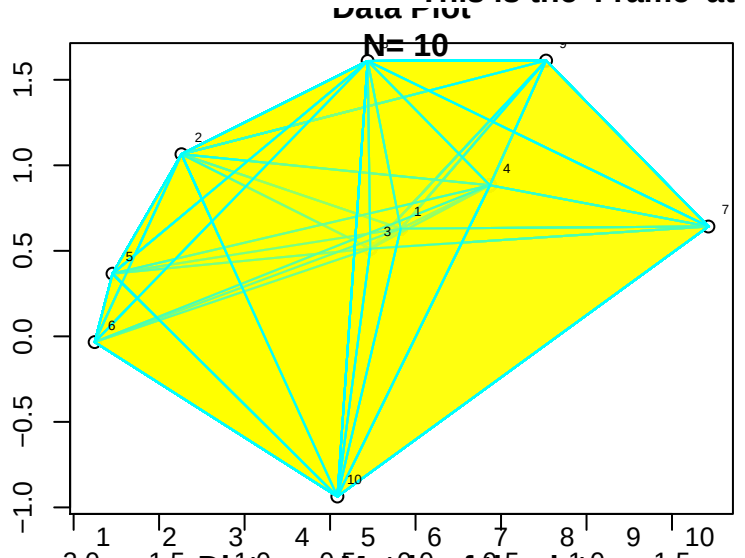
Distance Matrix of the data

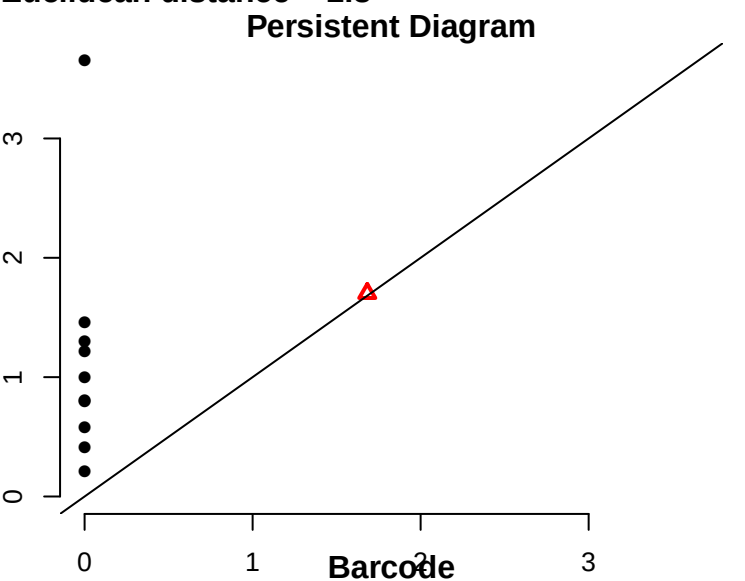
	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000




This is the 'Frame' at Euclidean distance = 2.7
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.354 0.213 0.580 1.706 1.909 1.800 0.999 1.299 1.609
1.354 0.000 1.235 1.813 0.807 1.210 3.111 1.218 2.202 2.199
0.213 1.235 0.000 0.792 1.515 1.702 1.982 1.097 1.505 1.462
0.580 1.813 0.792 0.000 2.267 2.487 1.301 1.019 0.800 2.027
1.706 0.807 1.515 2.267 0.000 0.413 3.496 1.942 2.825 1.851
1.909 1.210 1.702 2.487 0.413 0.000 3.653 2.291 3.111 1.682
1.800 3.111 1.982 1.301 3.496 3.653 0.000 2.215 1.357 2.684
0.999 1.218 1.097 1.019 1.942 2.291 2.215 0.000 1.044 2.552
1.299 2.202 1.505 0.800 2.825 3.111 1.357 1.044 0.000 2.826
1.609 2.199 1.462 2.027 1.851 1.682 2.684 2.552 2.826 0.000

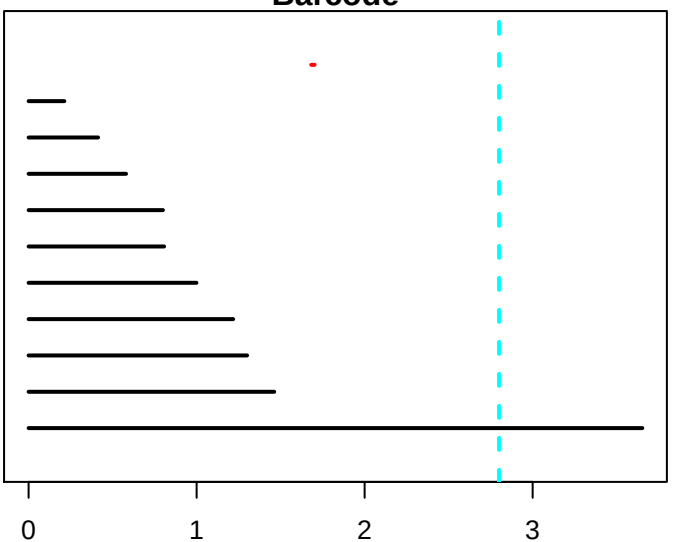
This is the 'Frame' at Euclidean distance = 2.8

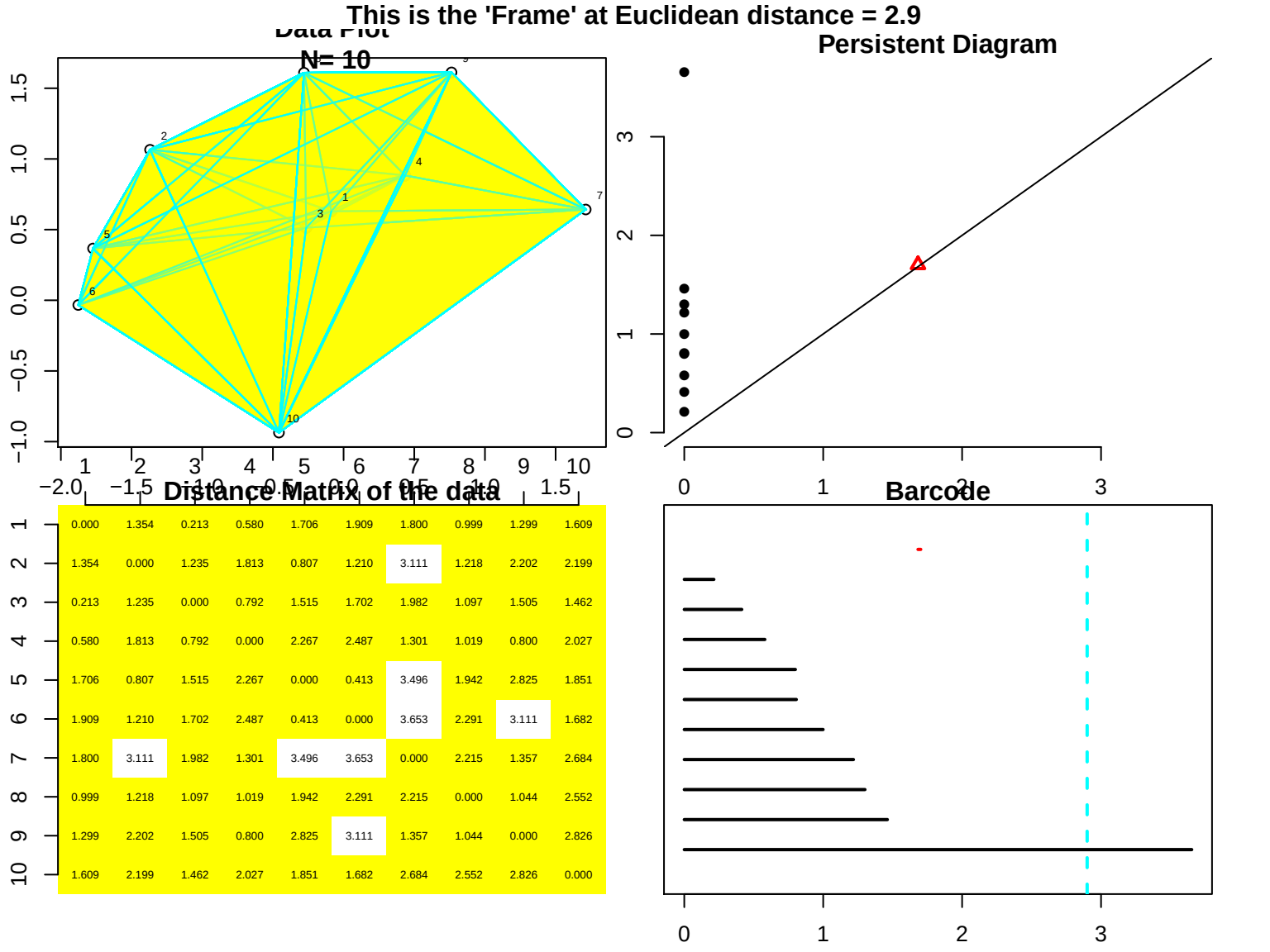




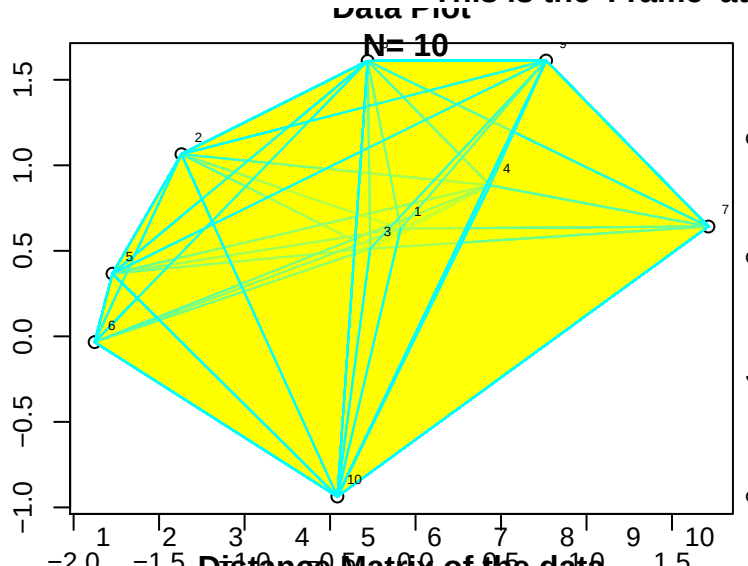
Distance Matrix of the data

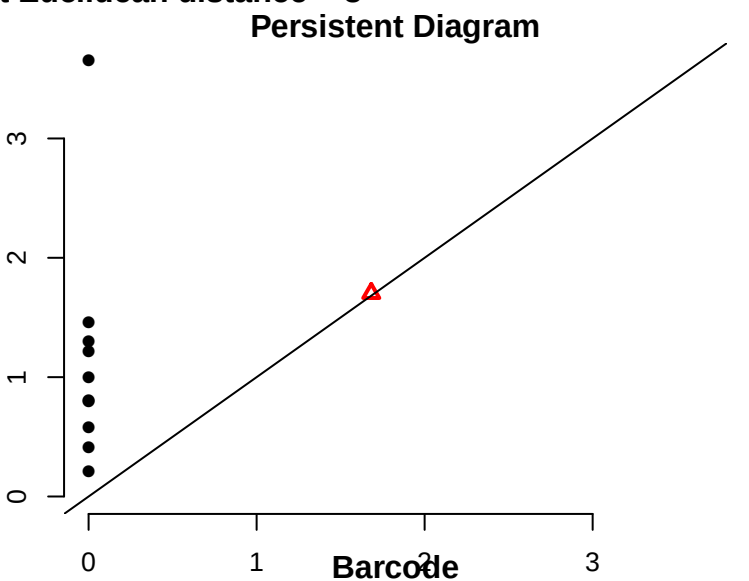
0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000




This is the 'Frame' at Euclidean distance = 2.9
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode

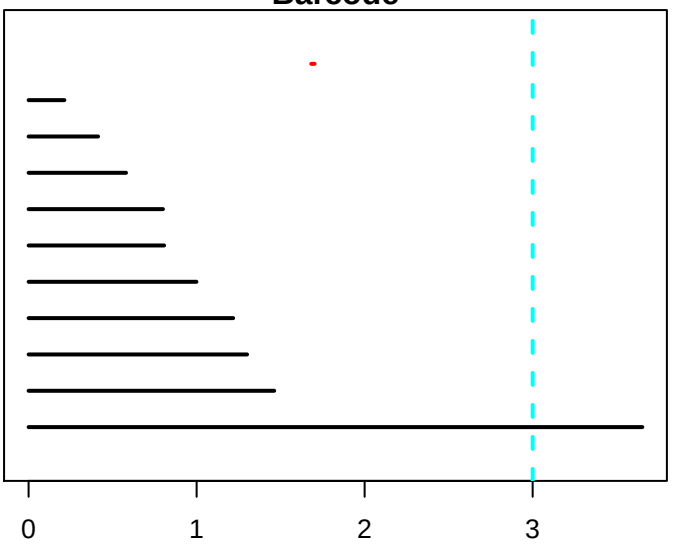
This is the 'Frame' at Euclidean distance = 3



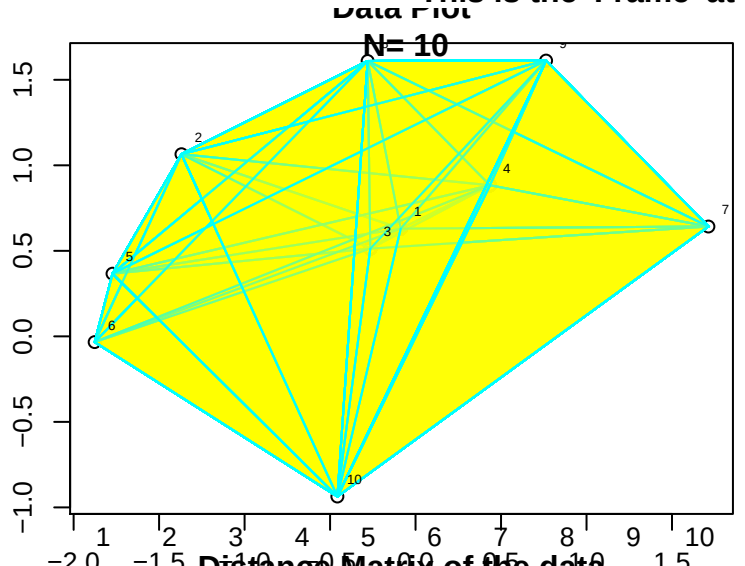


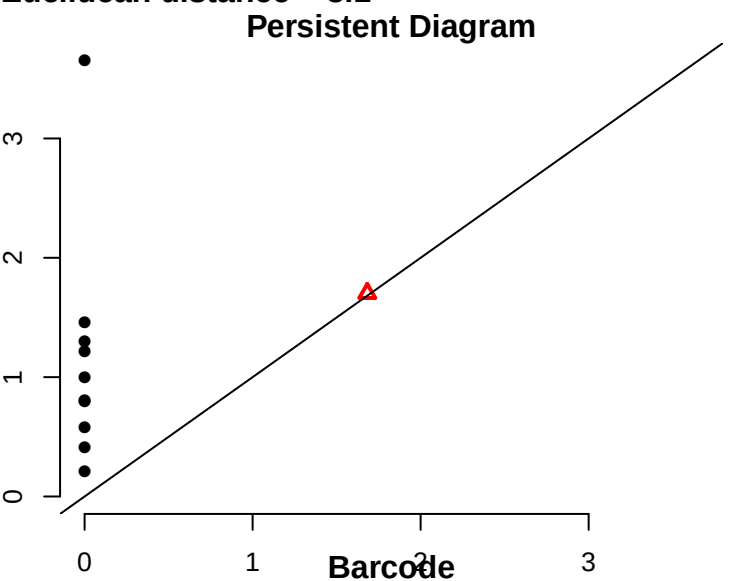
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



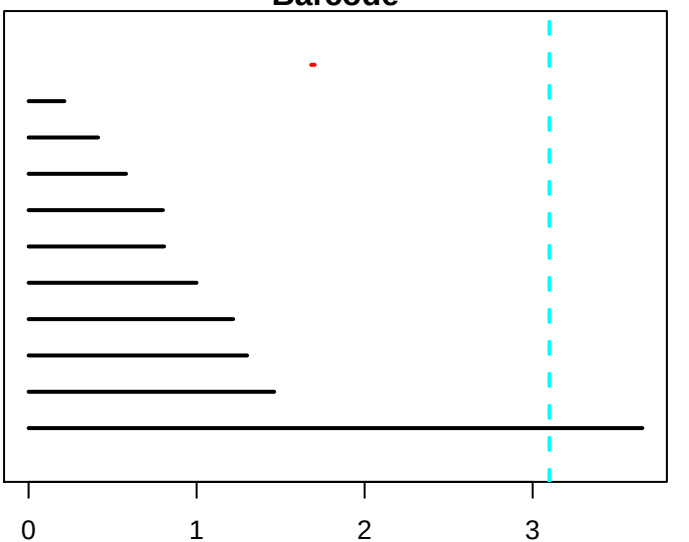
This is the 'Frame' at Euclidean distance = 3.1



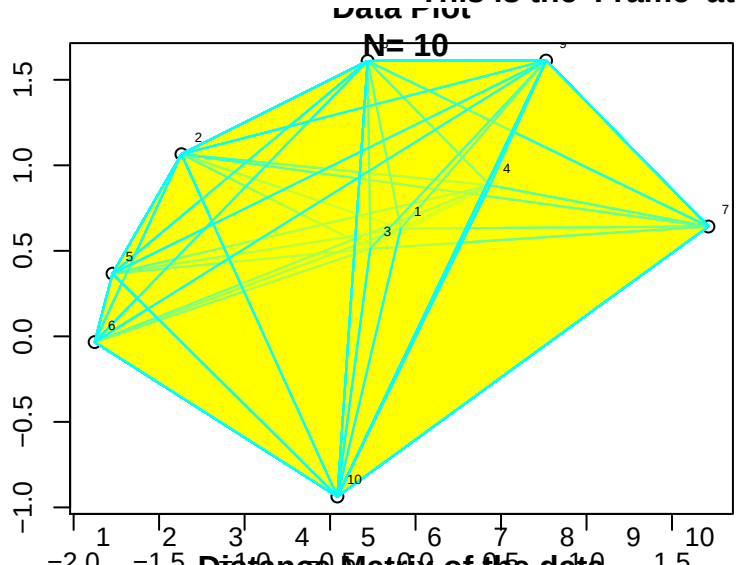


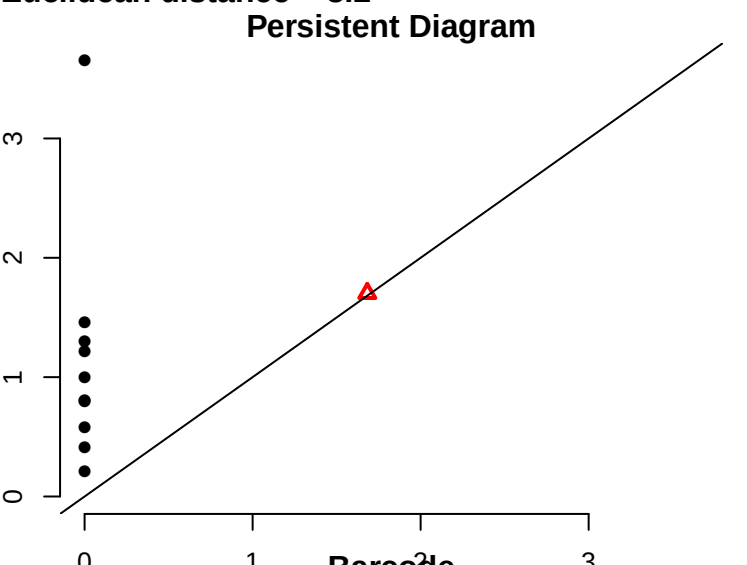
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



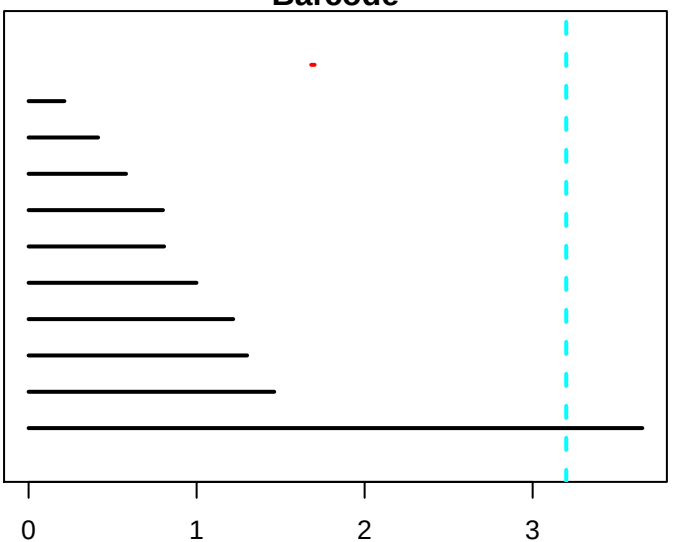
This is the 'Frame' at Euclidean distance = 3.2



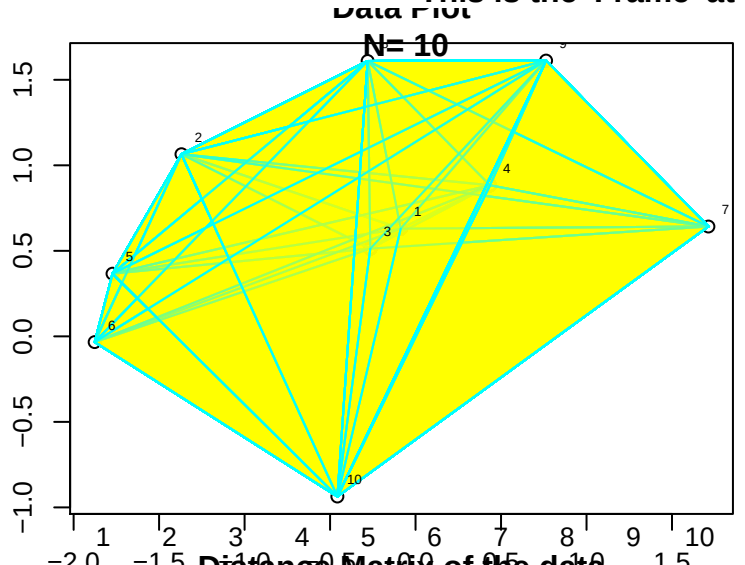


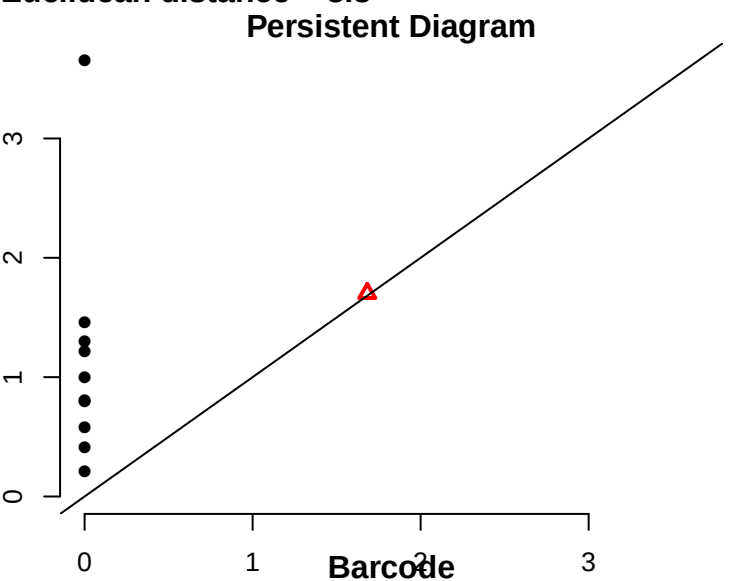
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



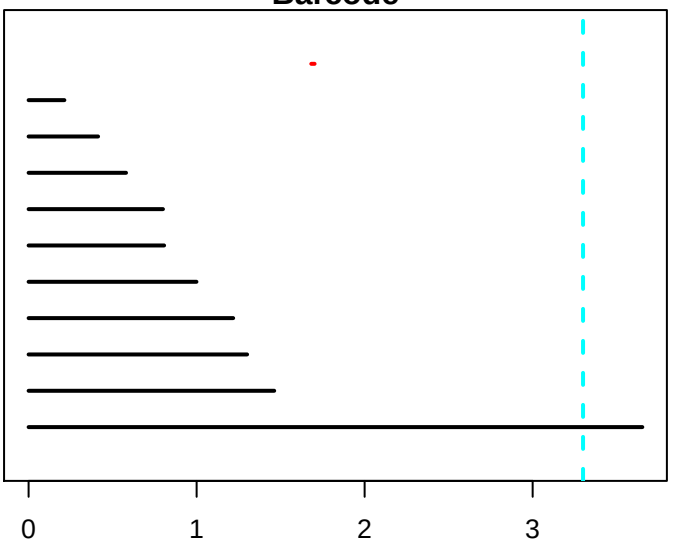
This is the 'Frame' at Euclidean distance = 3.3



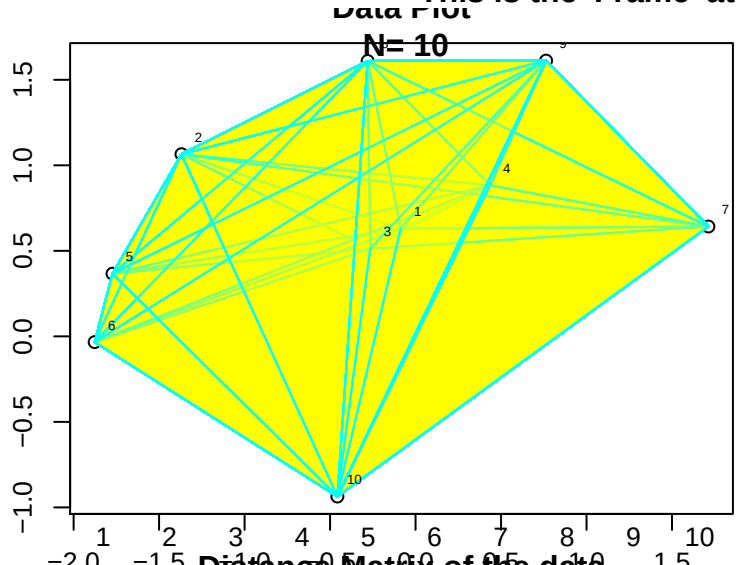


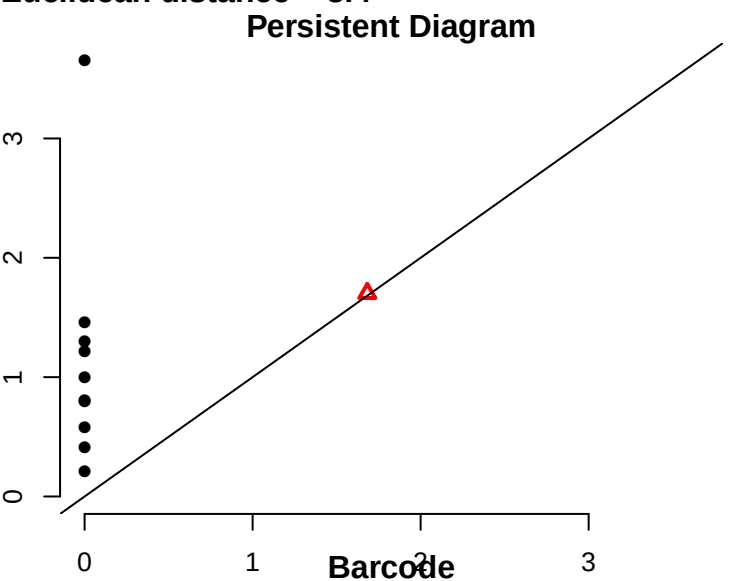
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



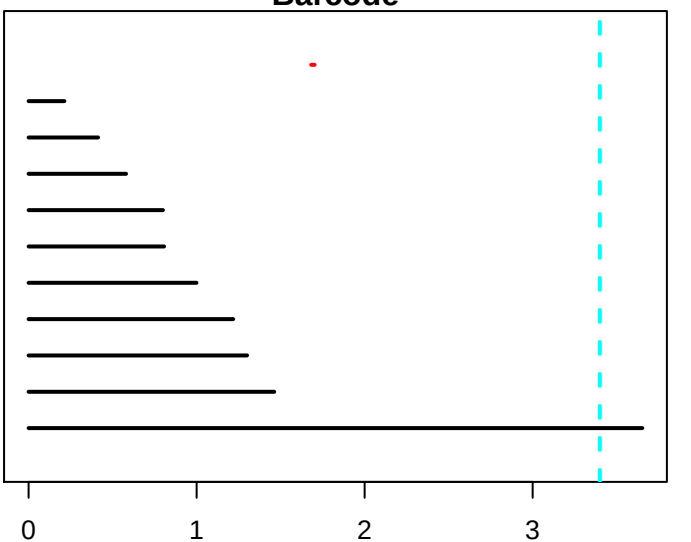
This is the 'Frame' at Euclidean distance = 3.4



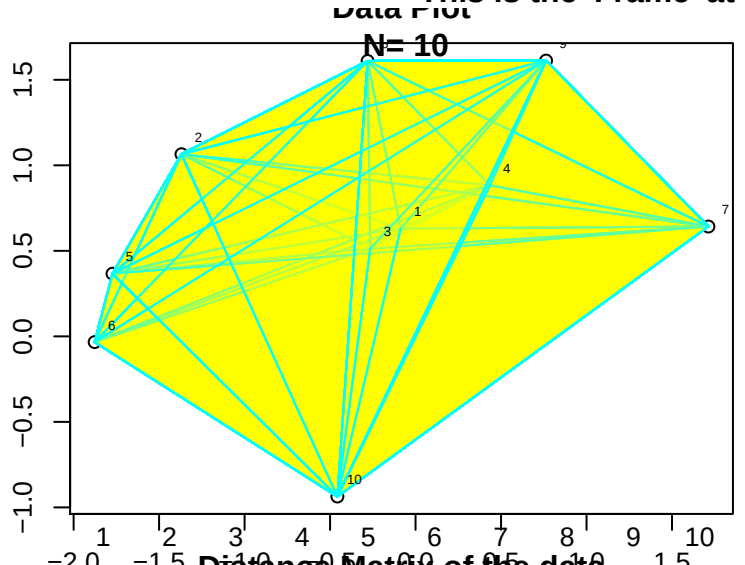


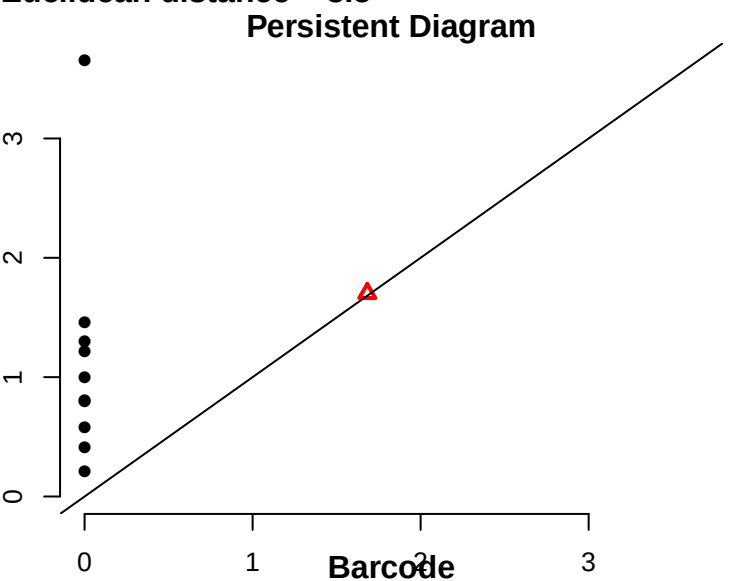
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



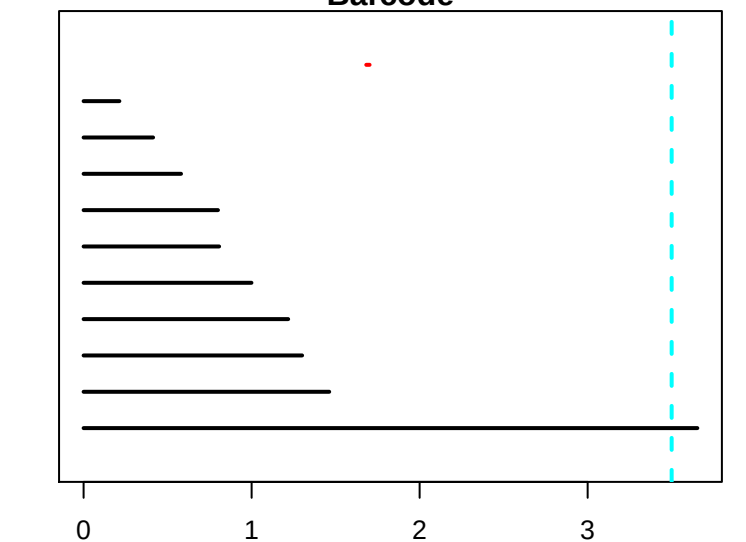
This is the 'Frame' at Euclidean distance = 3.5



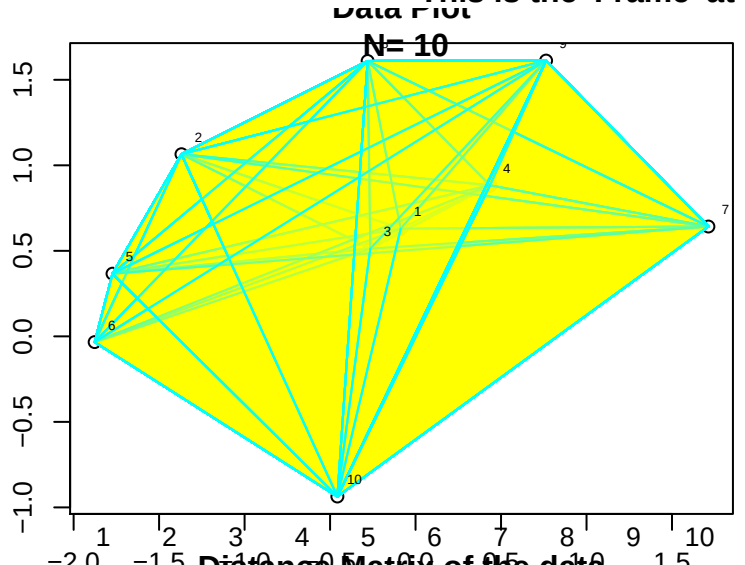


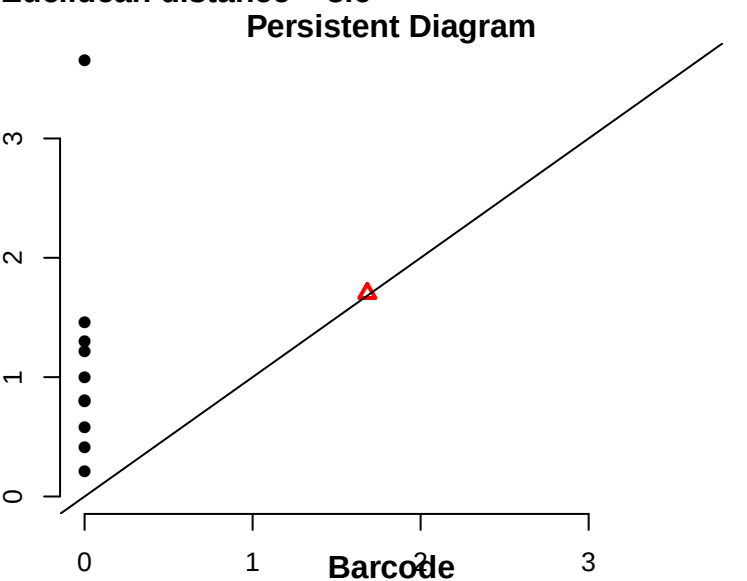
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
2	1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
3	0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
4	0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
5	1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
6	1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
7	1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
8	0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
9	1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
10	1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



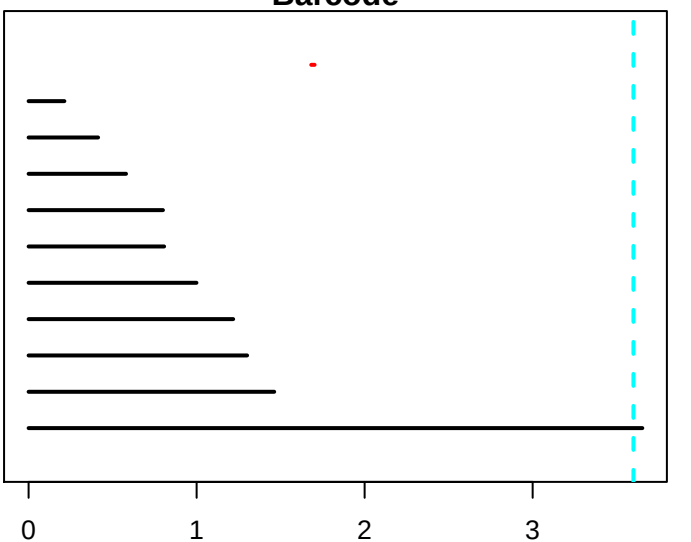
This is the 'Frame' at Euclidean distance = 3.6





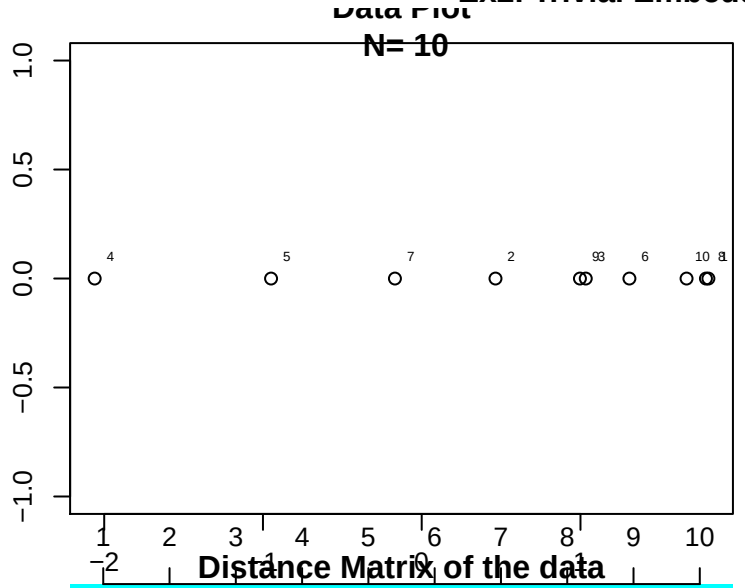
Distance Matrix of the data

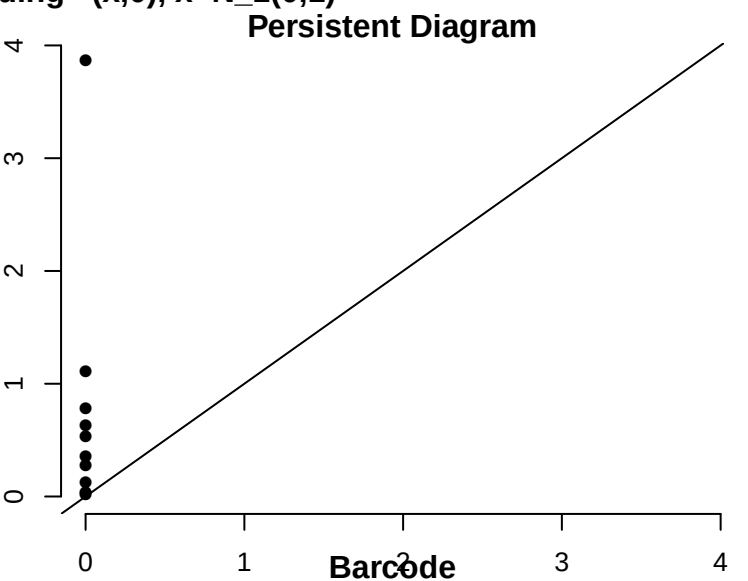
0.000	1.354	0.213	0.580	1.706	1.909	1.800	0.999	1.299	1.609
1.354	0.000	1.235	1.813	0.807	1.210	3.111	1.218	2.202	2.199
0.213	1.235	0.000	0.792	1.515	1.702	1.982	1.097	1.505	1.462
0.580	1.813	0.792	0.000	2.267	2.487	1.301	1.019	0.800	2.027
1.706	0.807	1.515	2.267	0.000	0.413	3.496	1.942	2.825	1.851
1.909	1.210	1.702	2.487	0.413	0.000	3.653	2.291	3.111	1.682
1.800	3.111	1.982	1.301	3.496	3.653	0.000	2.215	1.357	2.684
0.999	1.218	1.097	1.019	1.942	2.291	2.215	0.000	1.044	2.552
1.299	2.202	1.505	0.800	2.825	3.111	1.357	1.044	0.000	2.826
1.609	2.199	1.462	2.027	1.851	1.682	2.684	2.552	2.826	0.000



Result and Frame−by−frame plots for Example 2

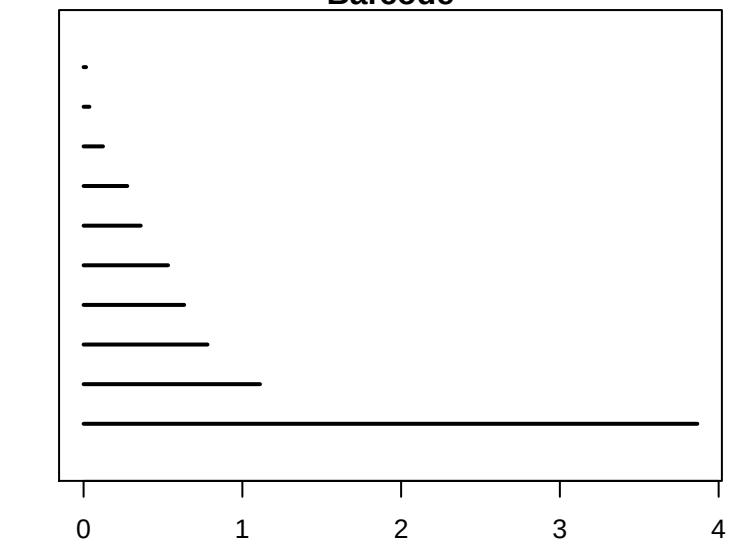
Ex2. Trivial Embedding (x,0), x~N_1(0,1)

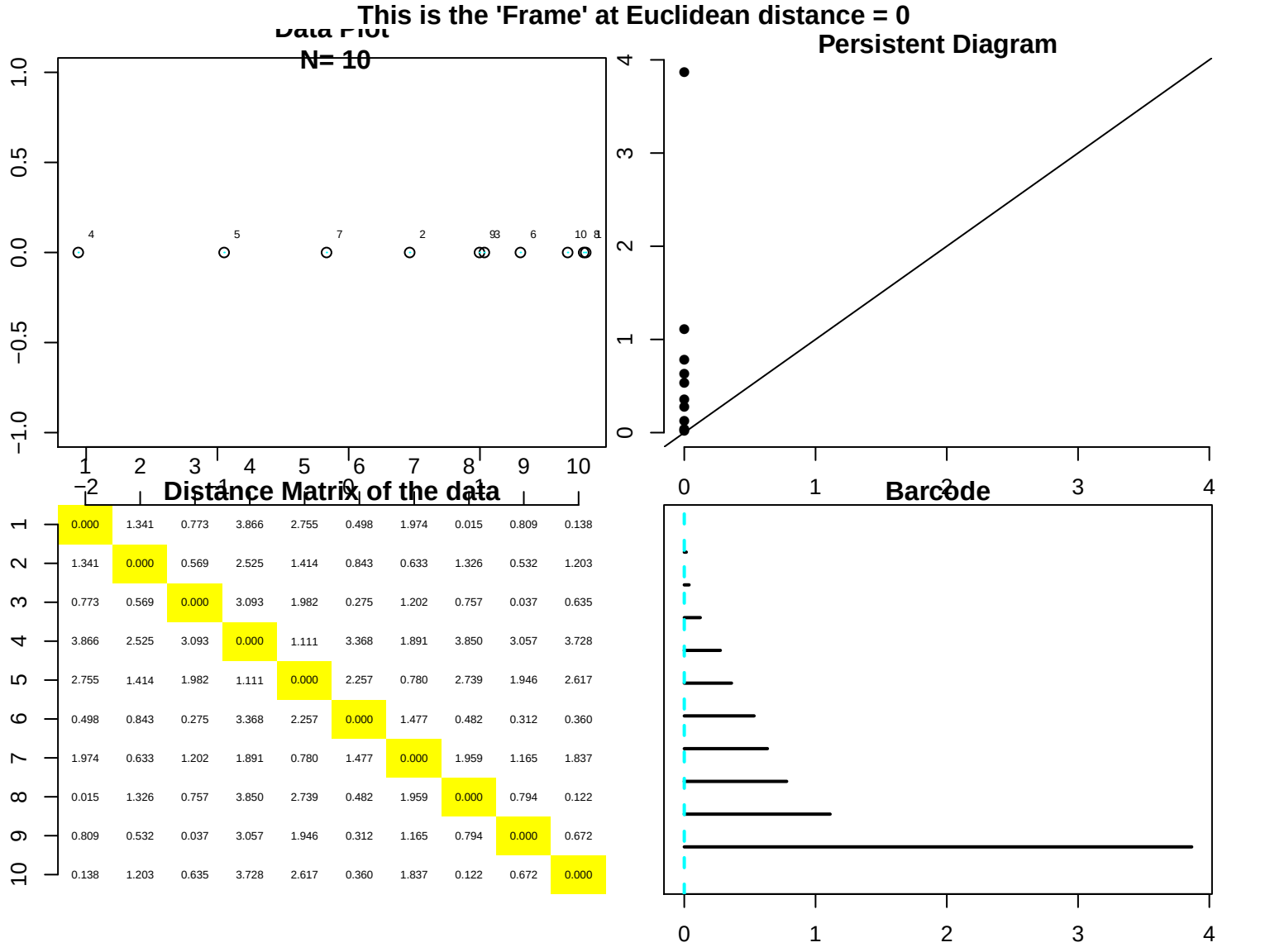


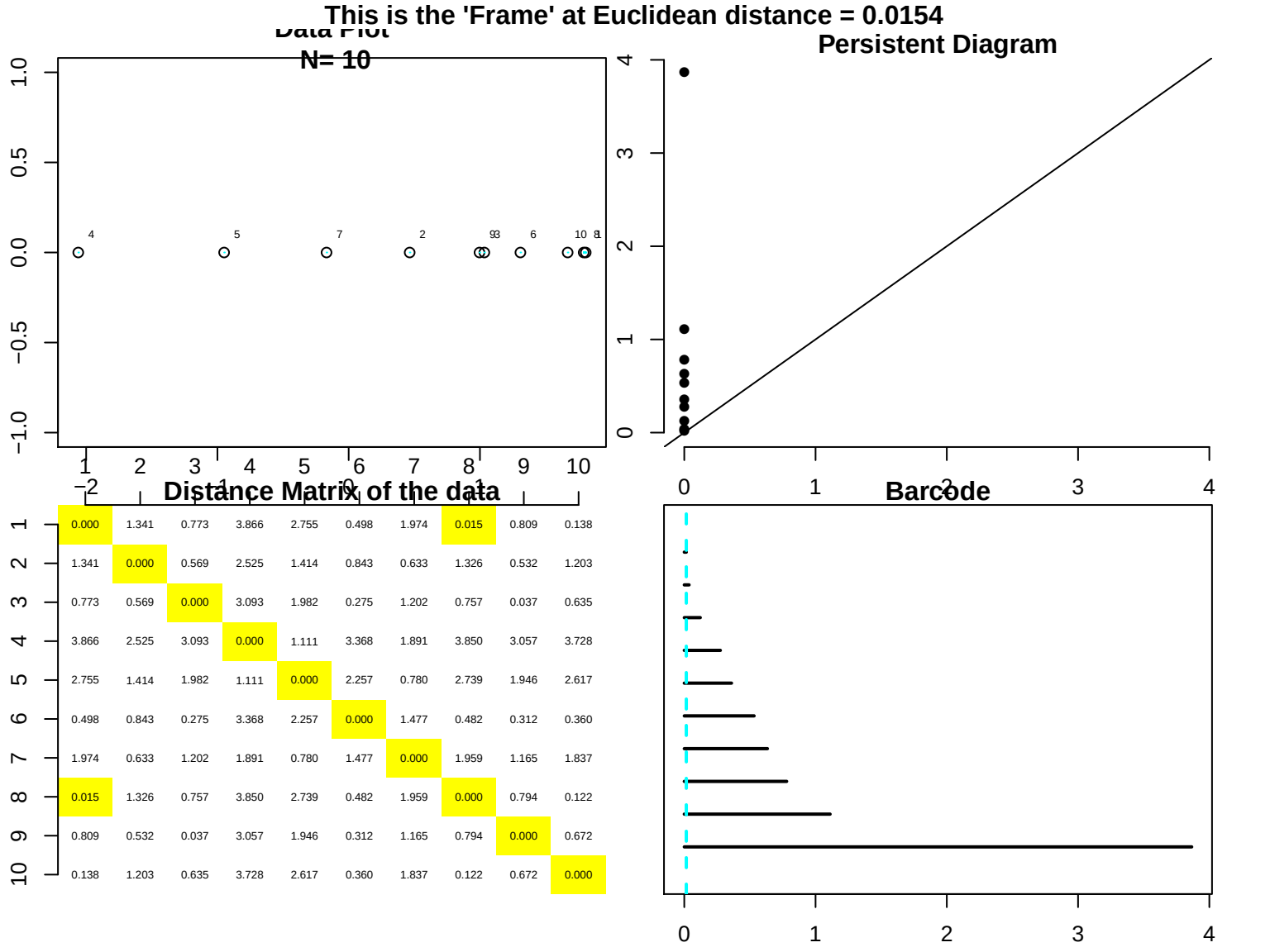


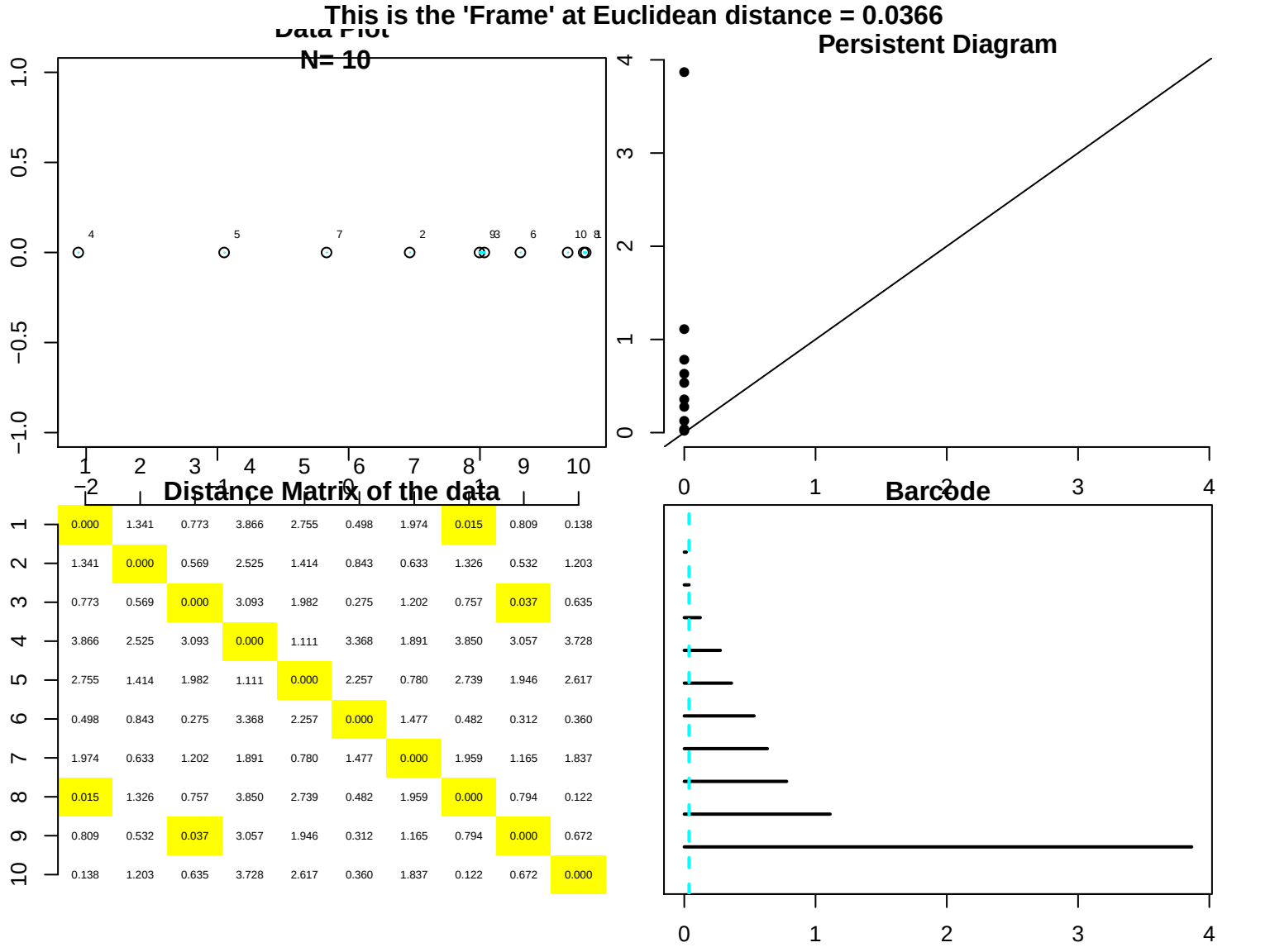
Distance Matrix of the data

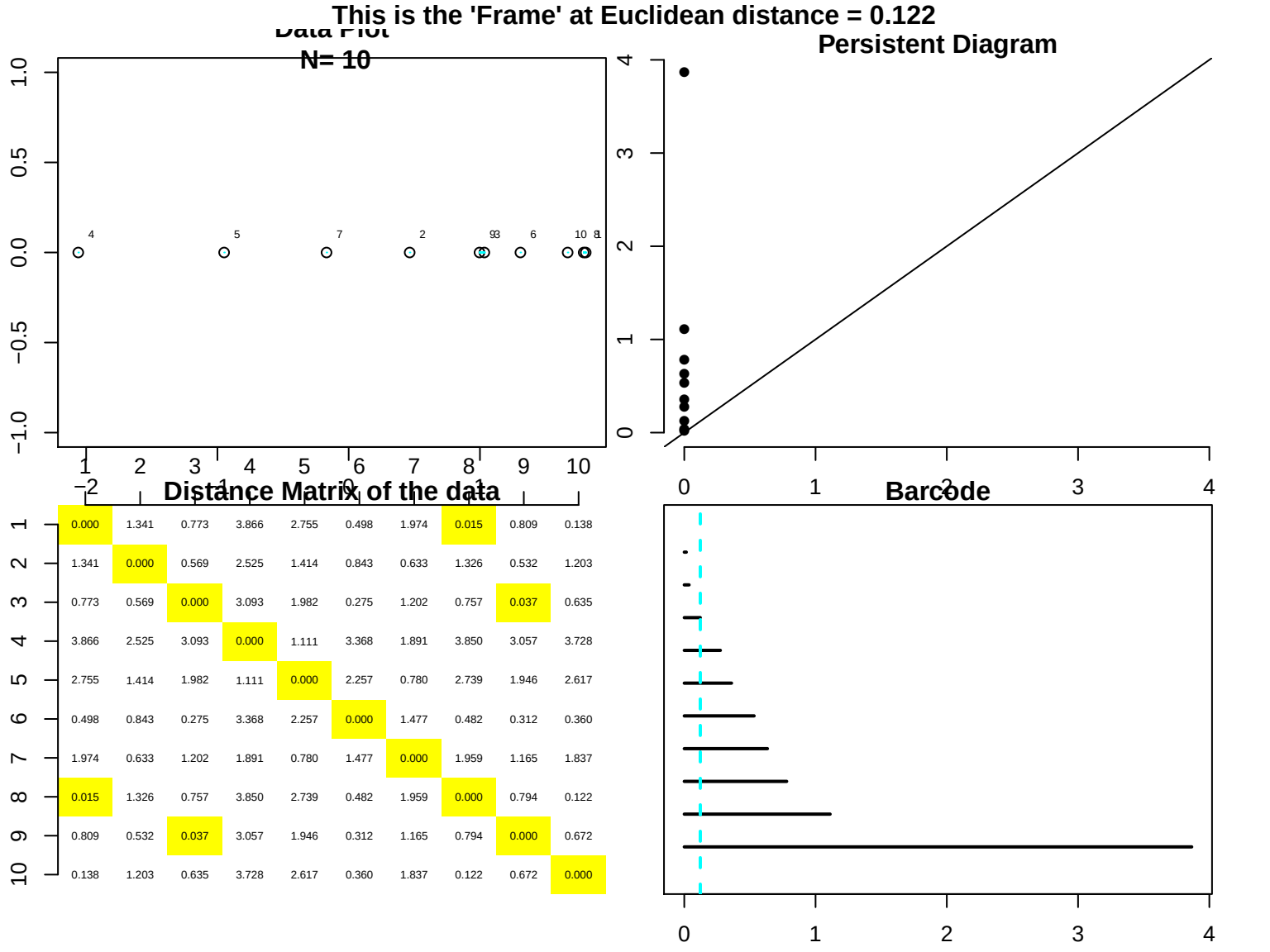
0.000	1.341	0.773	3.866	2.755	0.498	1.974	0.015	0.809	0.138
1.341	0.000	0.569	2.525	1.414	0.843	0.633	1.326	0.532	1.203
0.773	0.569	0.000	3.093	1.982	0.275	1.202	0.757	0.037	0.635
3.866	2.525	3.093	0.000	1.111	3.368	1.891	3.850	3.057	3.728
2.755	1.414	1.982	1.111	0.000	2.257	0.780	2.739	1.946	2.617
0.498	0.843	0.275	3.368	2.257	0.000	1.477	0.482	0.312	0.360
1.974	0.633	1.202	1.891	0.780	1.477	0.000	1.959	1.165	1.837
0.015	1.326	0.757	3.850	2.739	0.482	1.959	0.000	0.794	0.122
0.809	0.532	0.037	3.057	1.946	0.312	1.165	0.794	0.000	0.672
0.138	1.203	0.635	3.728	2.617	0.360	1.837	0.122	0.672	0.000

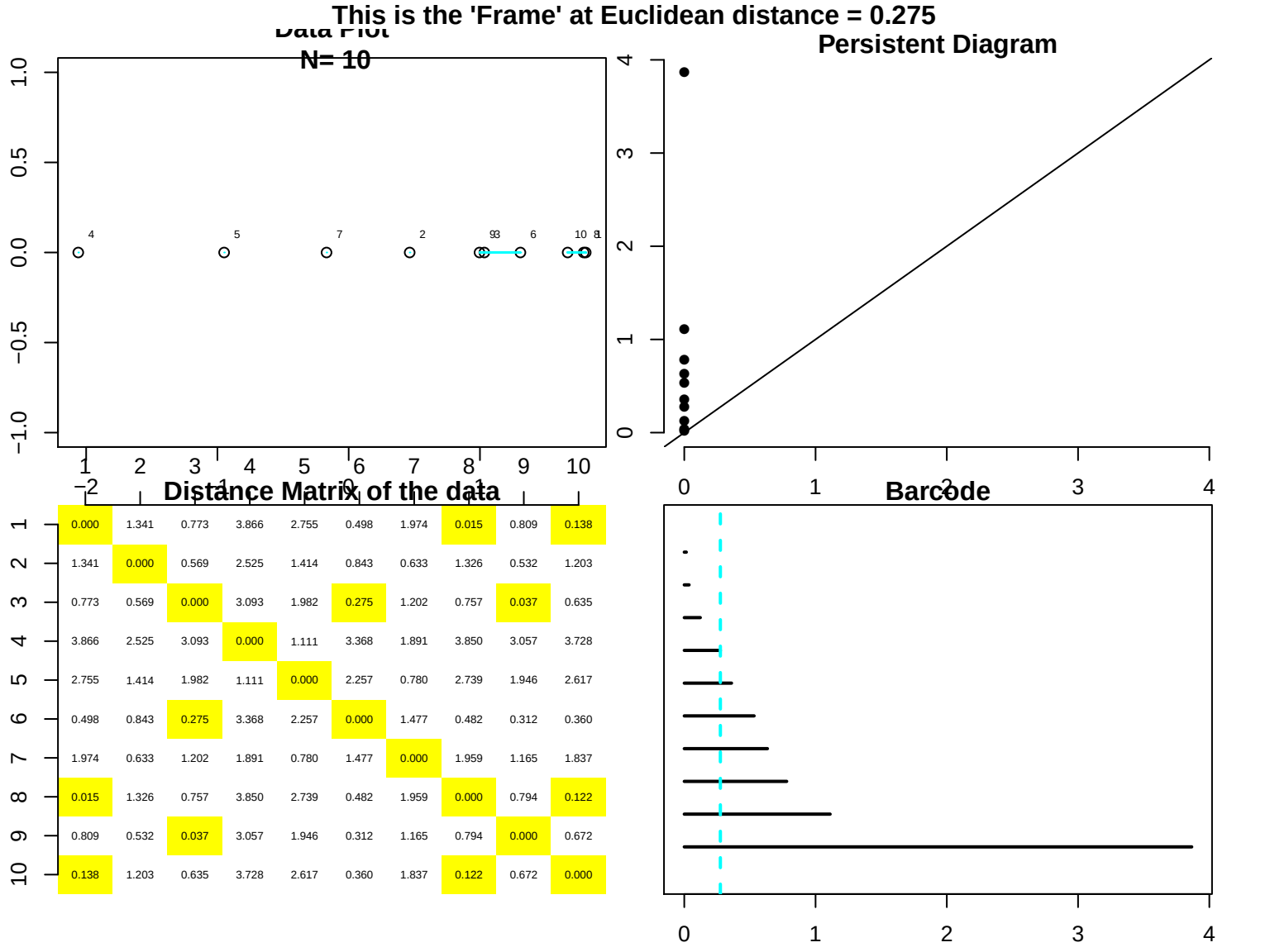


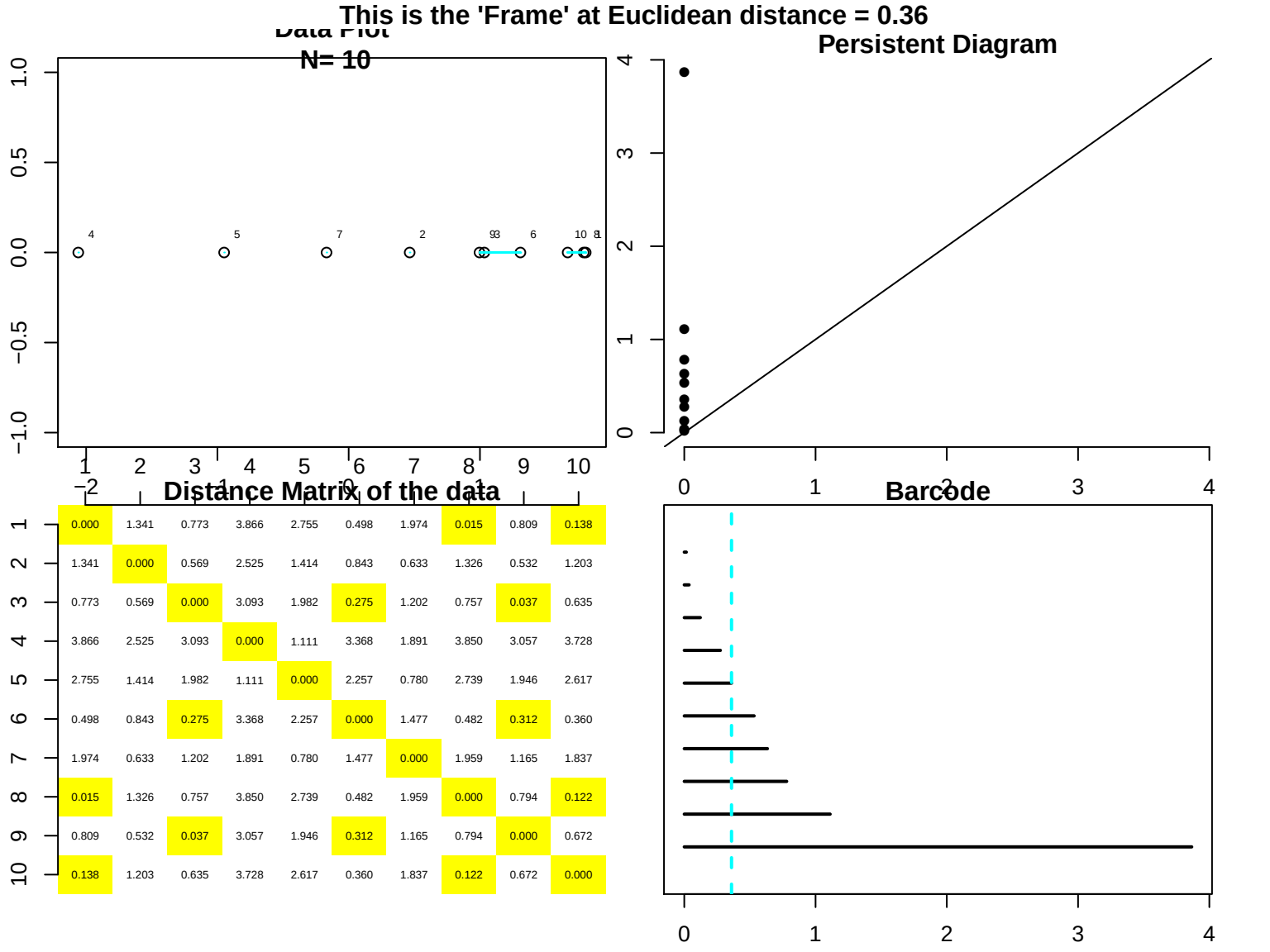

This is the 'Frame' at Euclidean distance = 0
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode

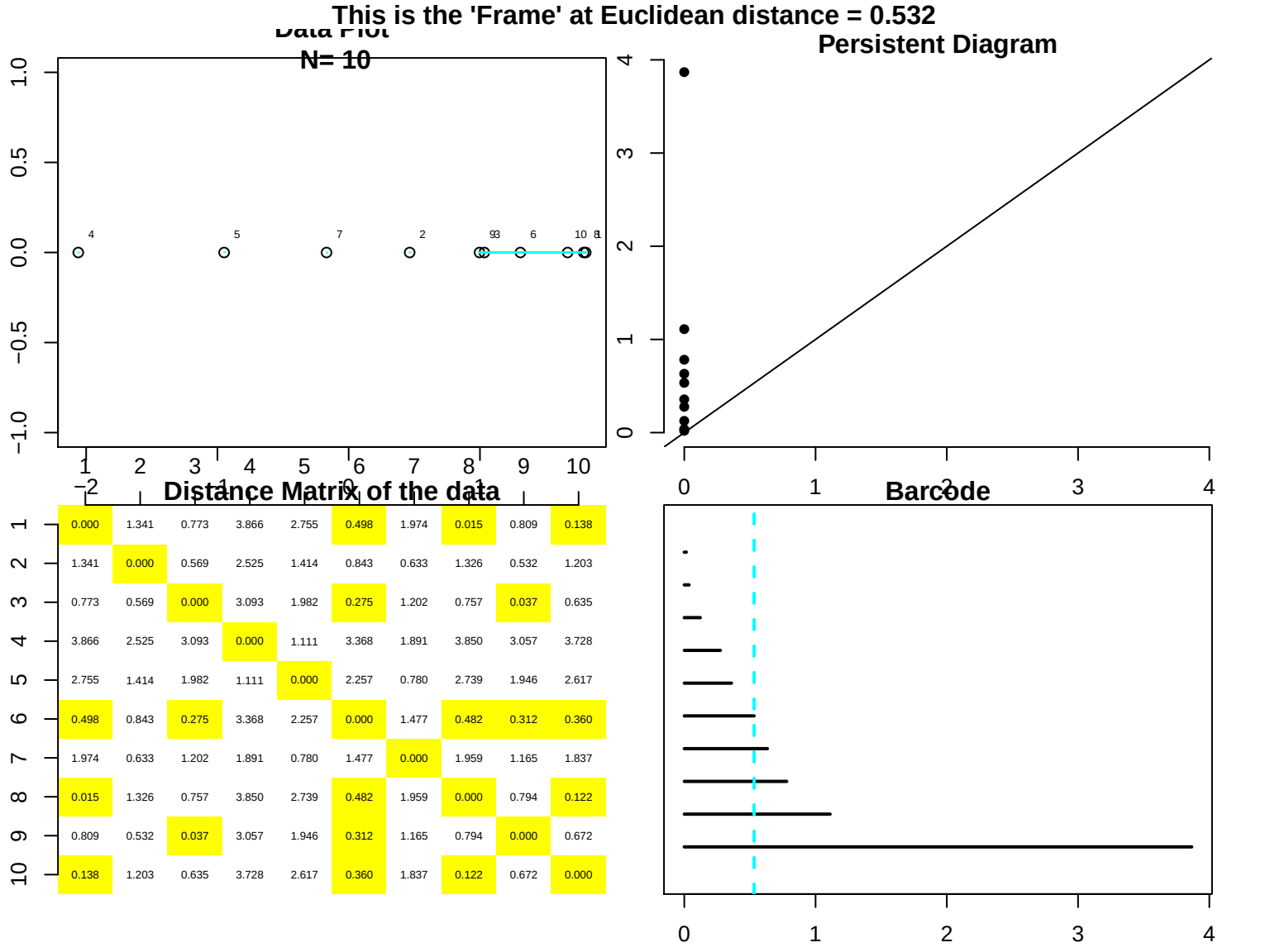

This is the 'Frame' at Euclidean distance = 0.0154
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.341 0.773 3.866 2.755 0.498 1.974 0.015 0.809 0.138
1.341 0.000 0.569 2.525 1.414 0.843 0.633 1.326 0.532 1.203
0.773 0.569 0.000 3.093 1.982 0.275 1.202 0.757 0.037 0.635
3.866 2.525 3.093 0.000 1.111 3.368 1.891 3.850 3.057 3.728
2.755 1.414 1.982 1.111 0.000 2.257 0.780 2.739 1.946 2.617
0.498 0.843 0.275 3.368 2.257 0.000 1.477 0.482 0.312 0.360
1.974 0.633 1.202 1.891 0.780 1.477 0.000 1.959 1.165 1.837
0.015 1.326 0.757 3.850 2.739 0.482 1.959 0.000 0.794 0.122
0.809 0.532 0.037 3.057 1.946 0.312 1.165 0.794 0.000 0.672
0.138 1.203 0.635 3.728 2.617 0.360 1.837 0.122 0.672 0.000

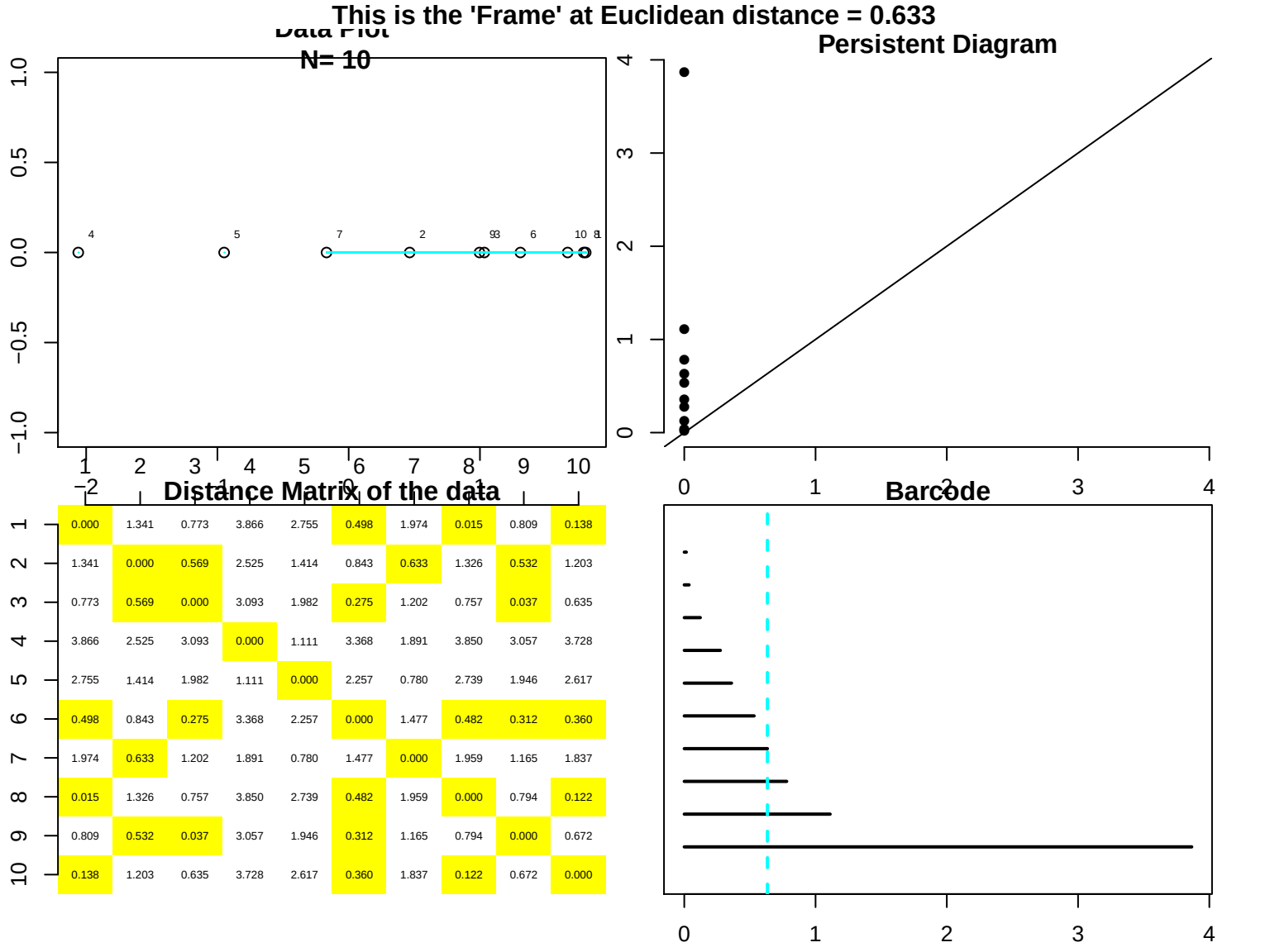

This is the 'Frame' at Euclidean distance = 0.0366
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.341 0.773 3.866 2.755 0.498 1.974 0.015 0.809 0.138
1.341 0.000 0.569 2.525 1.414 0.843 0.633 1.326 0.532 1.203
0.773 0.569 0.000 3.093 1.982 0.275 1.202 0.757 0.037 0.635
3.866 2.525 3.093 0.000 1.111 3.368 1.891 3.850 3.057 3.728
2.755 1.414 1.982 1.111 0.000 2.257 0.780 2.739 1.946 2.617
0.498 0.843 0.275 3.368 2.257 0.000 1.477 0.482 0.312 0.360
1.974 0.633 1.202 1.891 0.780 1.477 0.000 1.959 1.165 1.837
0.015 1.326 0.757 3.850 2.739 0.482 1.959 0.000 0.794 0.122
0.809 0.532 0.037 3.057 1.946 0.312 1.165 0.794 0.000 0.672
0.138 1.203 0.635 3.728 2.617 0.360 1.837 0.122 0.672 0.000

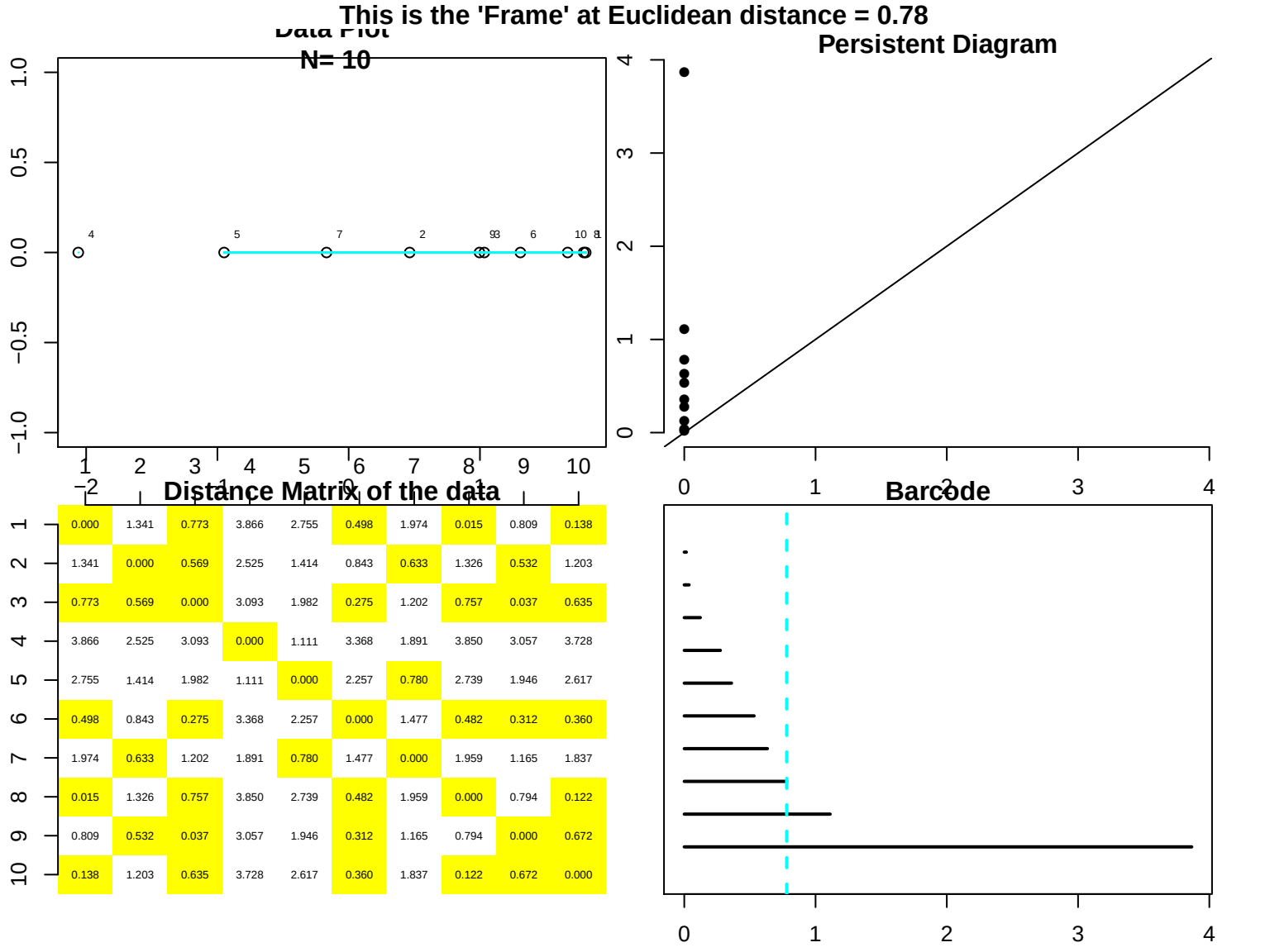

This is the 'Frame' at Euclidean distance = 0.122
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode

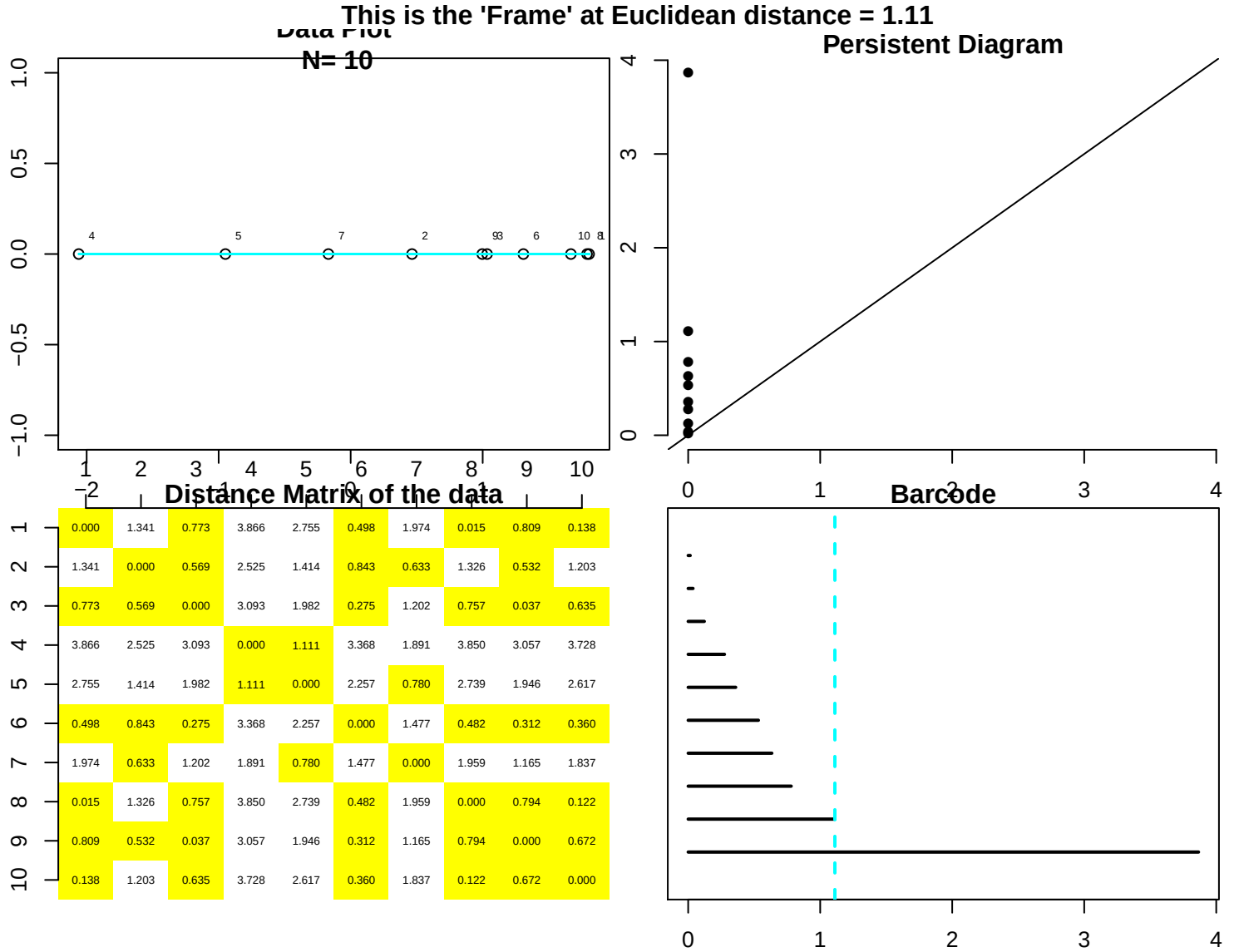

This is the 'Frame' at Euclidean distance = 0.275
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.341 0.773 3.866 2.755 0.498 1.974 0.015 0.809 0.138
1.341 0.000 0.569 2.525 1.414 0.843 0.633 1.326 0.532 1.203
0.773 0.569 0.000 3.093 1.982 0.275 1.202 0.757 0.037 0.635
3.866 2.525 3.093 0.000 1.111 3.368 1.891 3.850 3.057 3.728
2.755 1.414 1.982 1.111 0.000 2.257 0.780 2.739 1.946 2.617
0.498 0.843 0.275 3.368 2.257 0.000 1.477 0.482 0.312 0.360
1.974 0.633 1.202 1.891 0.780 1.477 0.000 1.959 1.165 1.837
0.015 1.326 0.757 3.850 2.739 0.482 1.959 0.000 0.794 0.122
0.809 0.532 0.037 3.057 1.946 0.312 1.165 0.794 0.000 0.672
0.138 1.203 0.635 3.728 2.617 0.360 1.837 0.122 0.672 0.000

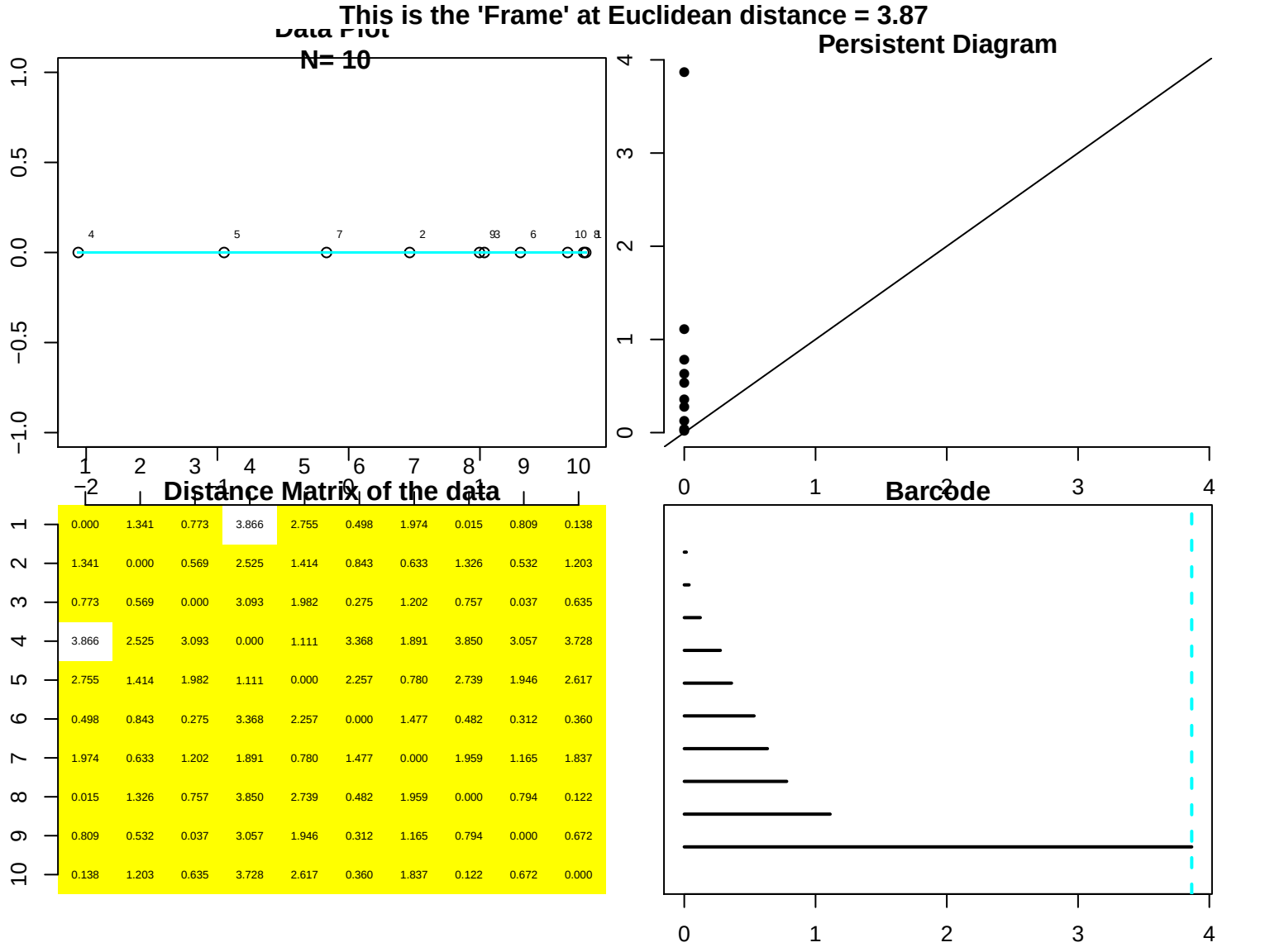

This is the 'Frame' at Euclidean distance = 0.36
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode


This is the 'Frame' at Euclidean distance = 0.532
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode


This is the 'Frame' at Euclidean distance = 0.633
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode

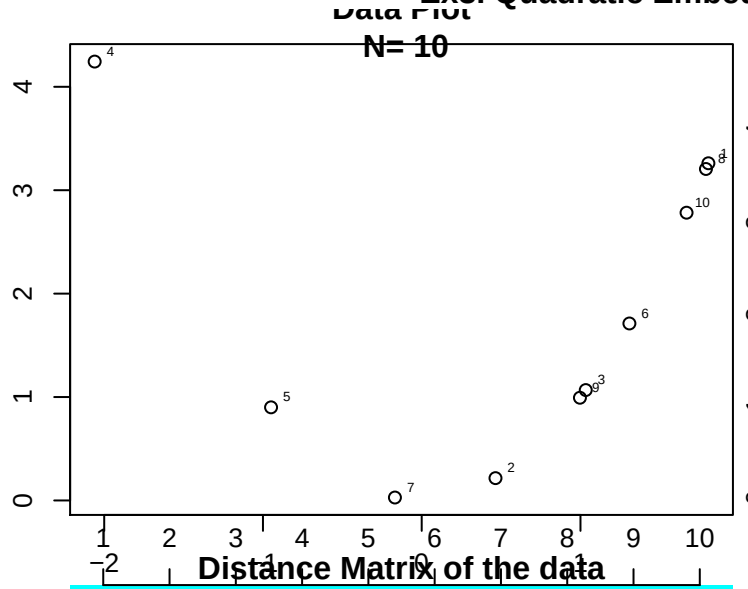

This is the 'Frame' at Euclidean distance = 0.78
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 1.341 0.773 3.866 2.755 0.498 1.974 0.015 0.809 0.138
1.341 0.000 0.569 2.525 1.414 0.843 0.633 1.326 0.532 1.203
0.773 0.569 0.000 3.093 1.982 0.275 1.202 0.757 0.037 0.635
3.866 2.525 3.093 0.000 1.111 3.368 1.891 3.850 3.057 3.728
2.755 1.414 1.982 1.111 0.000 2.257 0.780 2.739 1.946 2.617
0.498 0.843 0.275 3.368 2.257 0.000 1.477 0.482 0.312 0.360
1.974 0.633 1.202 1.891 0.780 1.477 0.000 1.959 1.165 1.837
0.015 1.326 0.757 3.850 2.739 0.482 1.959 0.000 0.794 0.122
0.809 0.532 0.037 3.057 1.946 0.312 1.165 0.794 0.000 0.672
0.138 1.203 0.635 3.728 2.617 0.360 1.837 0.122 0.672 0.000

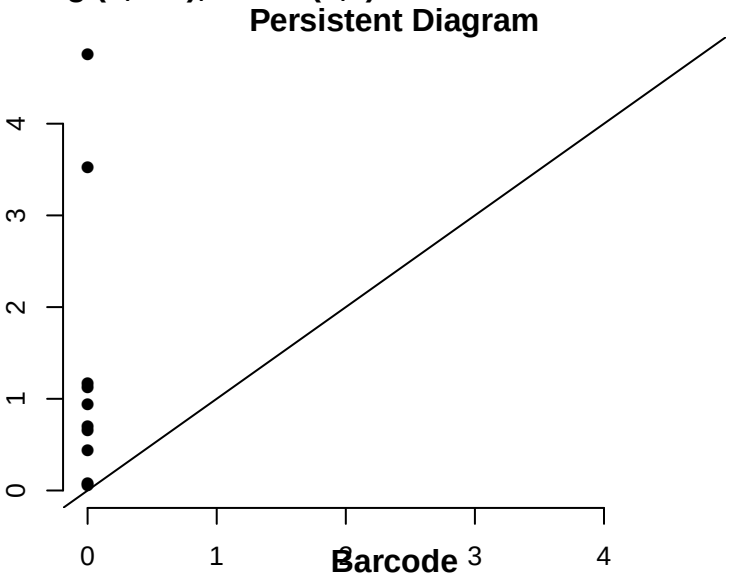

This is the 'Frame' at Euclidean distance = 1.11
Data Plot
N= 10
4
5
7
2
9
3
6
10
8
1
Persistent Diagram
Distance Matrix of the data
0.000 1.341 0.773 3.866 2.755 0.498 1.974 0.015 0.809 0.138
1.341 0.000 0.569 2.525 1.414 0.843 0.633 1.326 0.532 1.203
0.773 0.569 0.000 3.093 1.982 0.275 1.202 0.757 0.037 0.635
3.866 2.525 3.093 0.000 1.111 3.368 1.891 3.850 3.057 3.728
2.755 1.414 1.982 1.111 0.000 2.257 0.780 2.739 1.946 2.617
0.498 0.843 0.275 3.368 2.257 0.000 1.477 0.482 0.312 0.360
1.974 0.633 1.202 1.891 0.780 1.477 0.000 1.959 1.165 1.837
0.015 1.326 0.757 3.850 2.739 0.482 1.959 0.000 0.794 0.122
0.809 0.532 0.037 3.057 1.946 0.312 1.165 0.794 0.000 0.672
0.138 1.203 0.635 3.728 2.617 0.360 1.837 0.122 0.672 0.000
Barcode


This is the 'Frame' at Euclidean distance = 3.87
Data Plot
N= 10
4
5
7
2
9
3
6
10
8
1
Persistent Diagram
Distance Matrix of the data
0.000 1.341 0.773 3.866 2.755 0.498 1.974 0.015 0.809 0.138
1.341 0.000 0.569 2.525 1.414 0.843 0.633 1.326 0.532 1.203
0.773 0.569 0.000 3.093 1.982 0.275 1.202 0.757 0.037 0.635
3.866 2.525 3.093 0.000 1.111 3.368 1.891 3.850 3.057 3.728
2.755 1.414 1.982 1.111 0.000 2.257 0.780 2.739 1.946 2.617
0.498 0.843 0.275 3.368 2.257 0.000 1.477 0.482 0.312 0.360
1.974 0.633 1.202 1.891 0.780 1.477 0.000 1.959 1.165 1.837
0.015 1.326 0.757 3.850 2.739 0.482 1.959 0.000 0.794 0.122
0.809 0.532 0.037 3.057 1.946 0.312 1.165 0.794 0.000 0.672
0.138 1.203 0.635 3.728 2.617 0.360 1.837 0.122 0.672 0.000
Barcode

Result and Frame−by−frame plots for Example 3

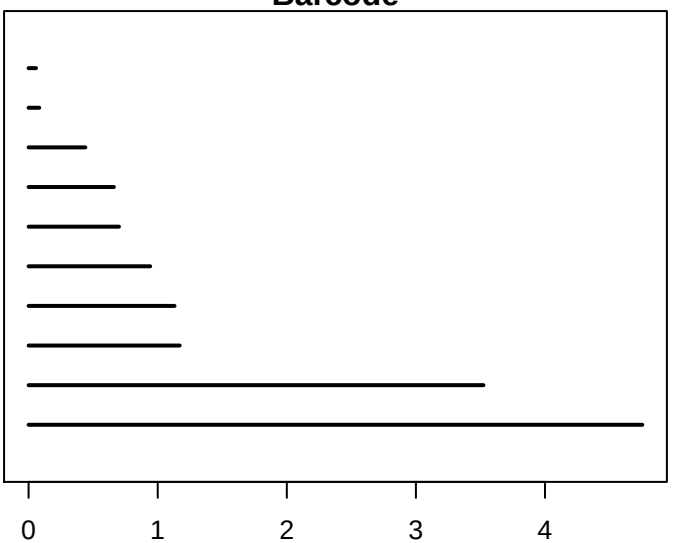
Ex3. Quadratic Embedding (x,x^2), x~N_1(0,1)



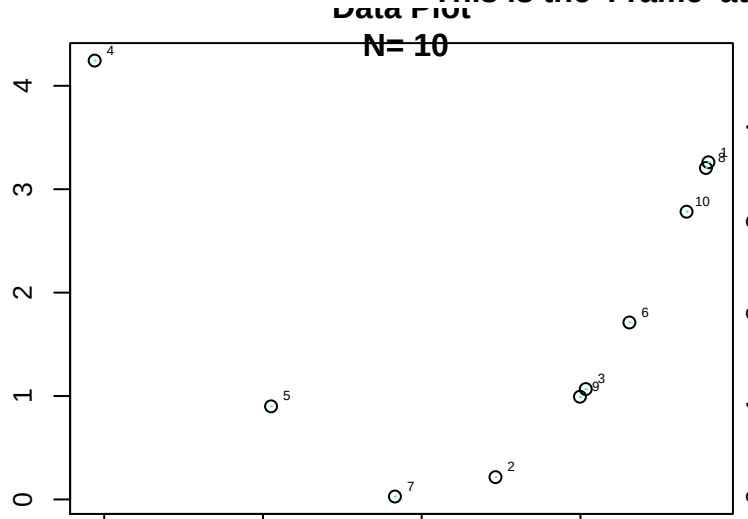


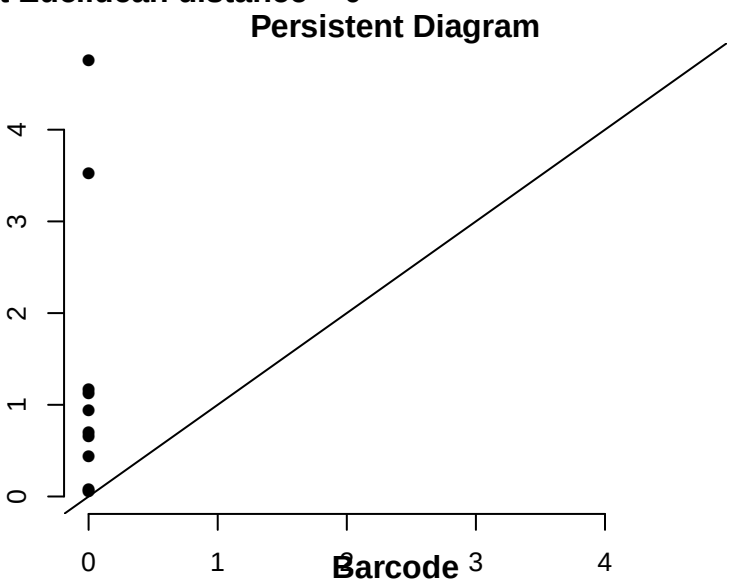
Distance Matrix of the data

0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000



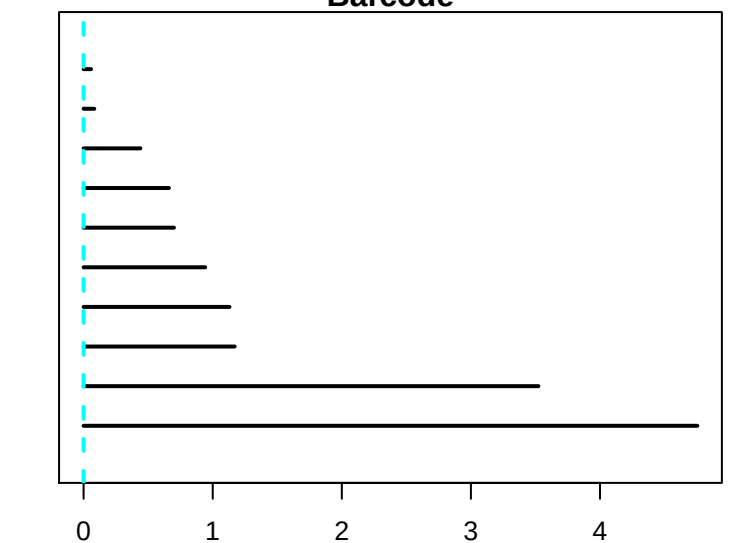
This is the 'Frame' at Euclidean distance = 0



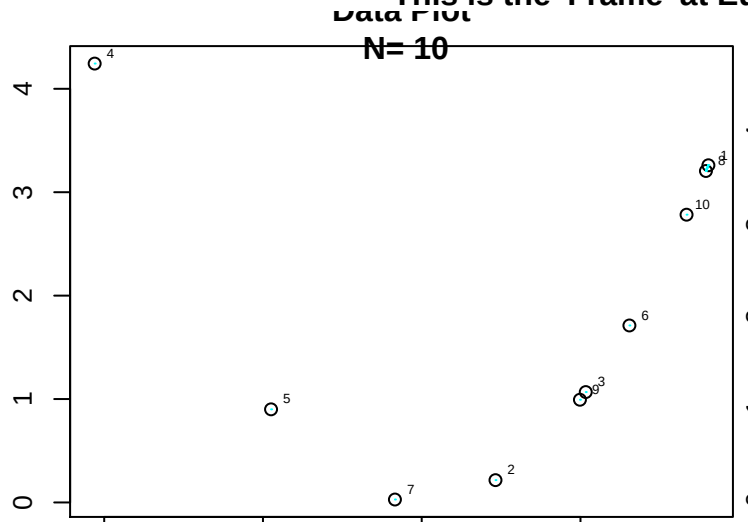


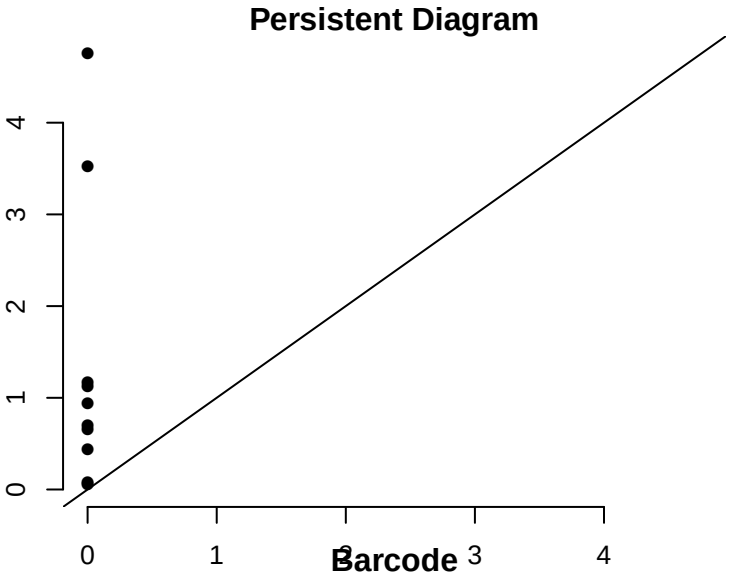
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
2	3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
3	2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
4	3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
5	3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
6	1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
7	3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
8	0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
9	2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
10	0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000



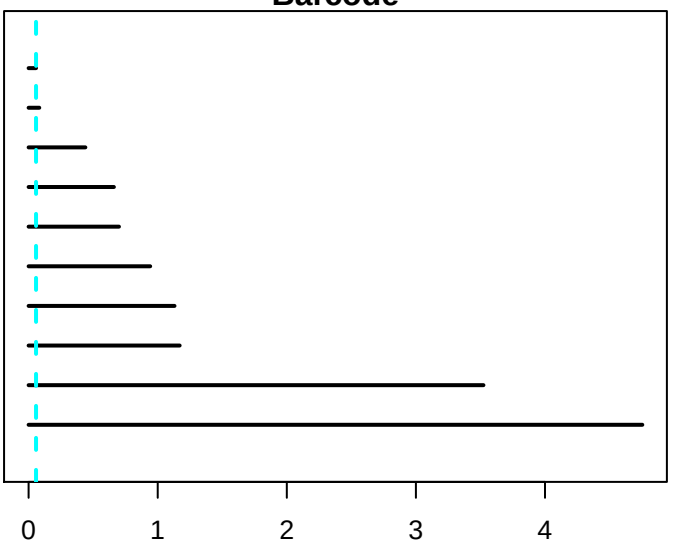
This is the 'Frame' at Euclidean distance = 0.0577



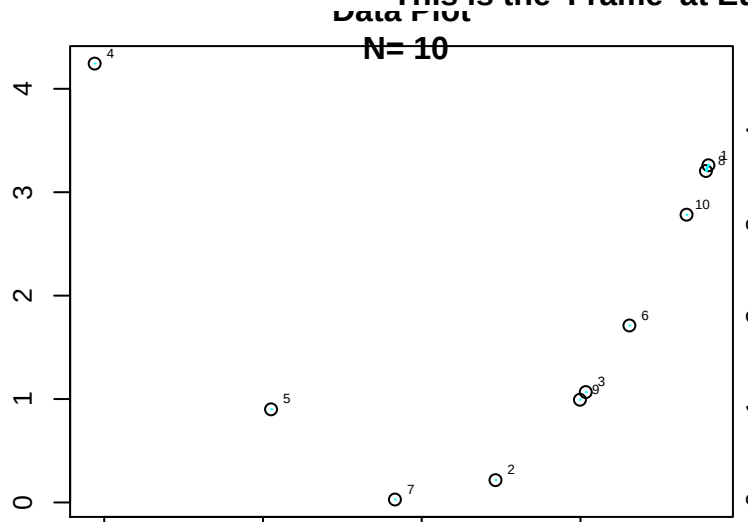


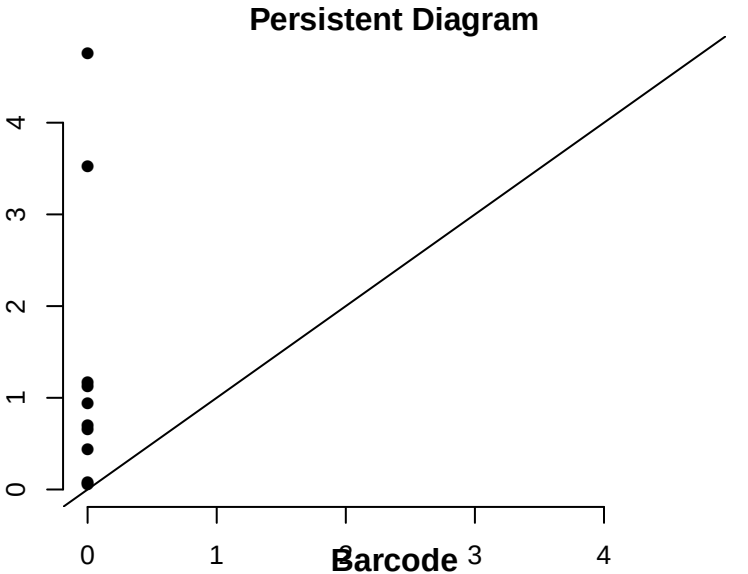
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
2	3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
3	2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
4	3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
5	3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
6	1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
7	3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
8	0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
9	2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
10	0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000



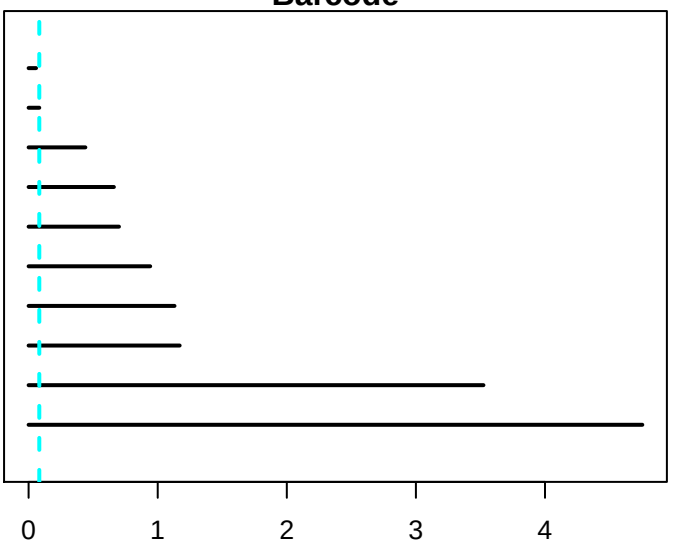
This is the 'Frame' at Euclidean distance = 0.0828



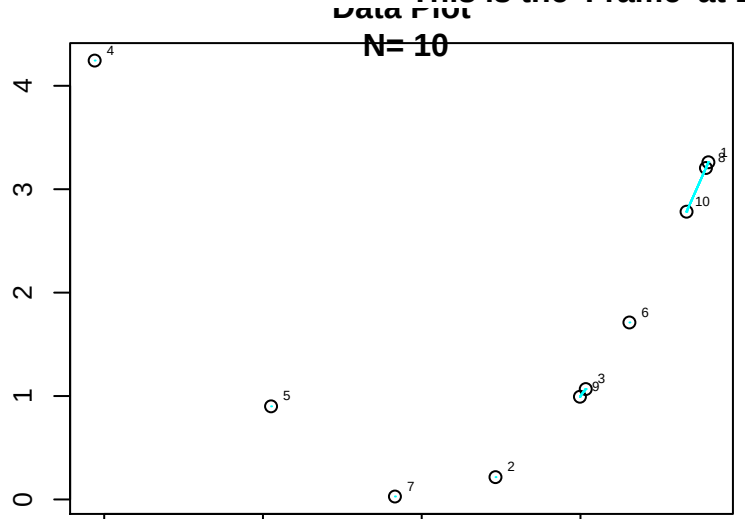


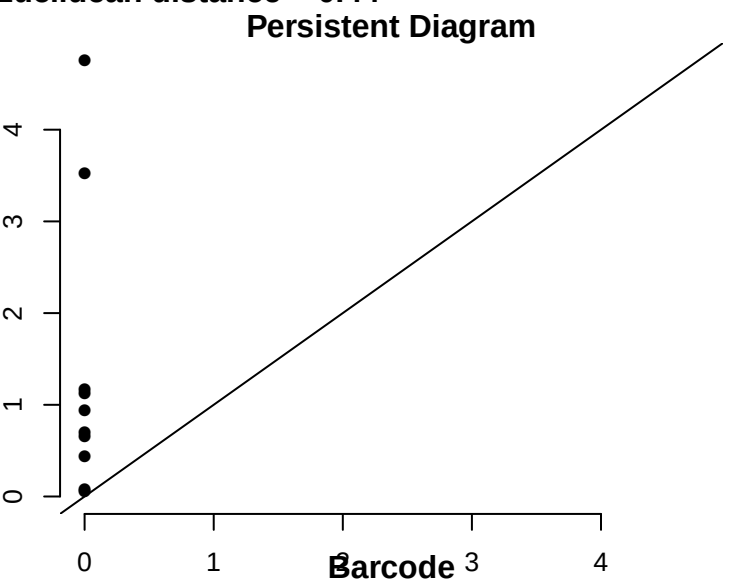
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
2	3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
3	2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
4	3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
5	3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
6	1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
7	3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
8	0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
9	2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
10	0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000



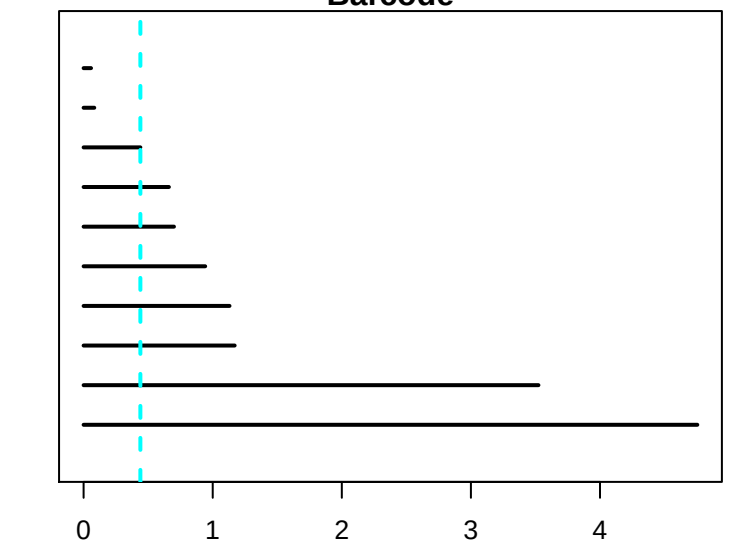
This is the 'Frame' at Euclidean distance = 0.44

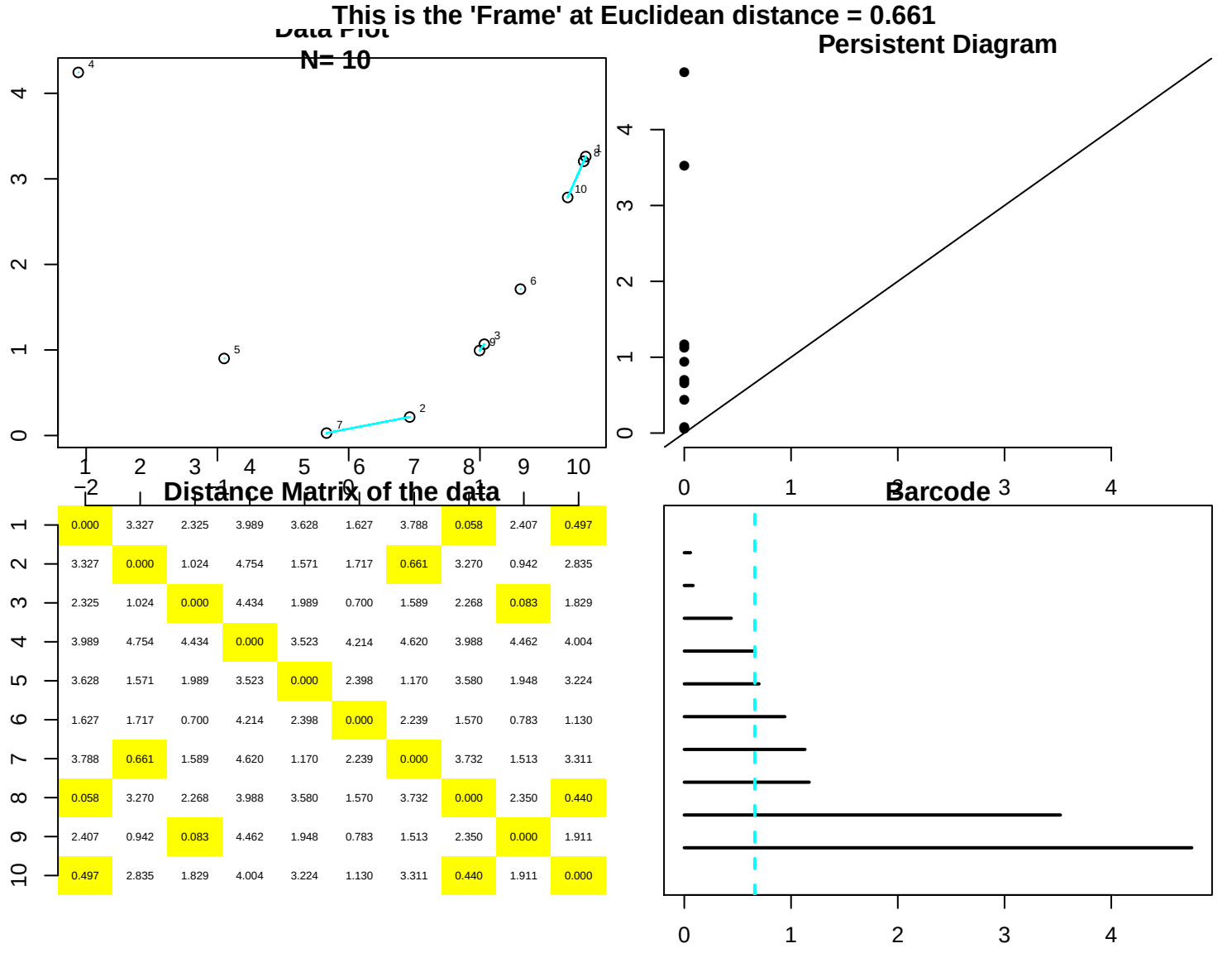




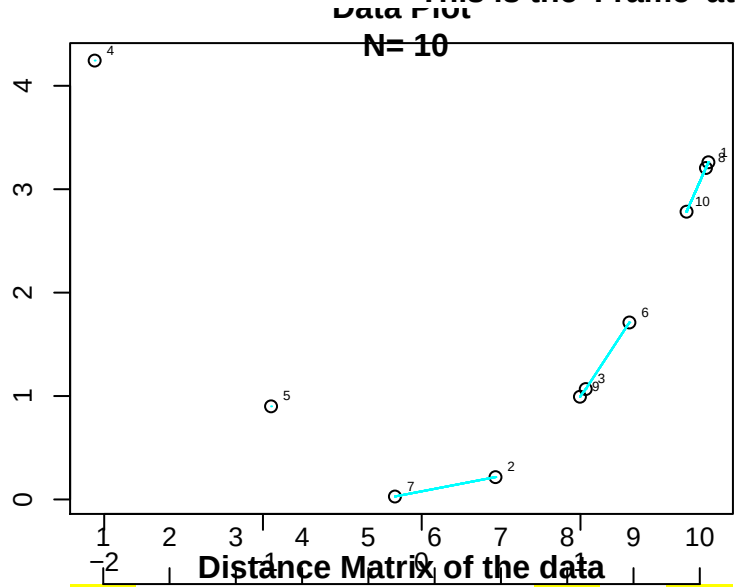
Distance Matrix of the data

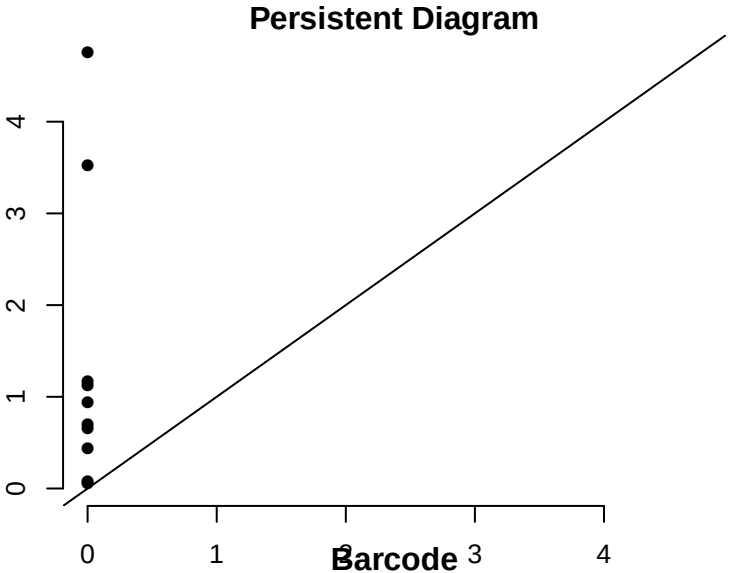
	1	2	3	4	5	6	7	8	9	10
1	0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
2	3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
3	2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
4	3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
5	3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
6	1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
7	3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
8	0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
9	2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
10	0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000




This is the 'Frame' at Euclidean distance = 0.661
Data Plot
N= 10
Persistent Diagram
Distance Matrix of the data
Barcode
0.000 3.327 2.325 3.989 3.628 1.627 3.788 0.058 2.407 0.497
3.327 0.000 1.024 4.754 1.571 1.717 0.661 3.270 0.942 2.835
2.325 1.024 0.000 4.434 1.989 0.700 1.589 2.268 0.083 1.829
3.989 4.754 4.434 0.000 3.523 4.214 4.620 3.988 4.462 4.004
3.628 1.571 1.989 3.523 0.000 2.398 1.170 3.580 1.948 3.224
1.627 1.717 0.700 4.214 2.398 0.000 2.239 1.570 0.783 1.130
3.788 0.661 1.589 4.620 1.170 2.239 0.000 3.732 1.513 3.311
0.058 3.270 2.268 3.988 3.580 1.570 3.732 0.000 2.350 0.440
2.407 0.942 0.083 4.462 1.948 0.783 1.513 2.350 0.000 1.911
0.497 2.835 1.829 4.004 3.224 1.130 3.311 0.440 1.911 0.000

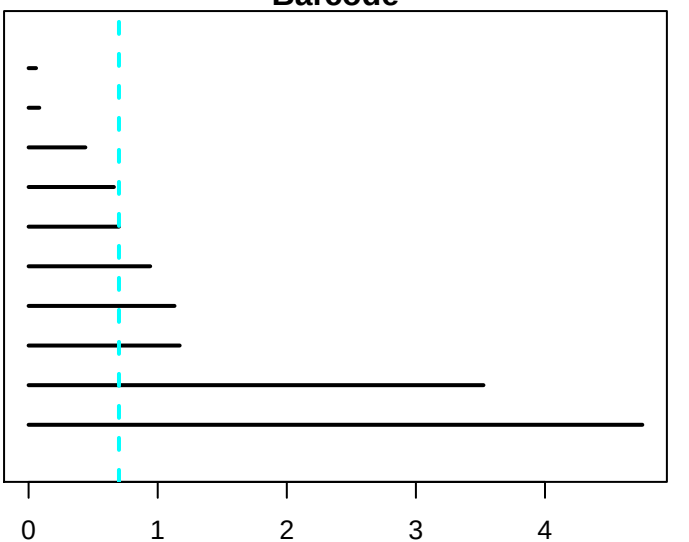
This is the 'Frame' at Euclidean distance = 0.7



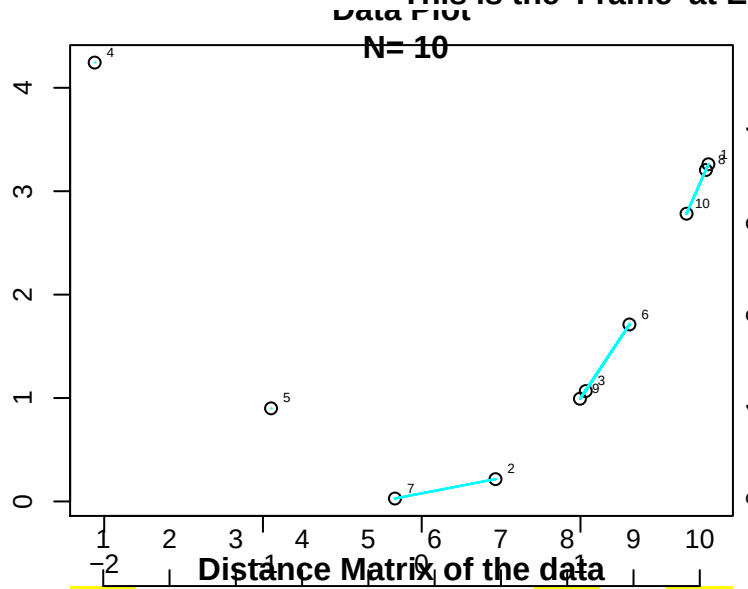


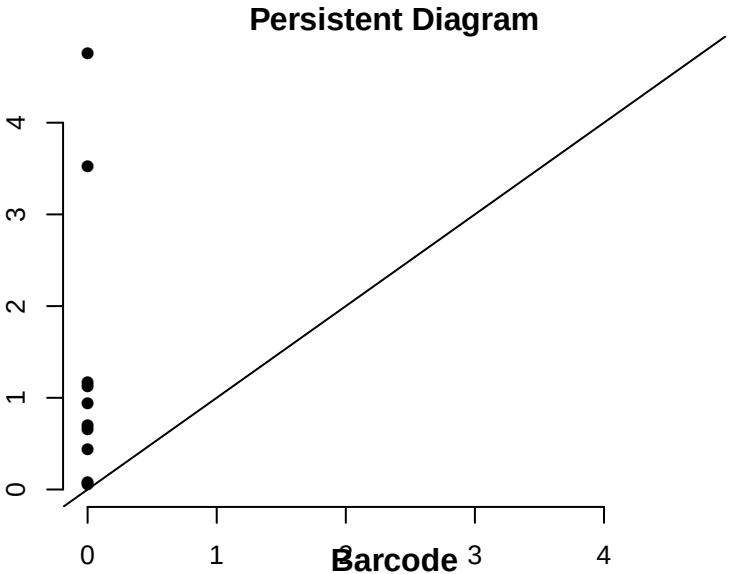
Distance Matrix of the data

0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000



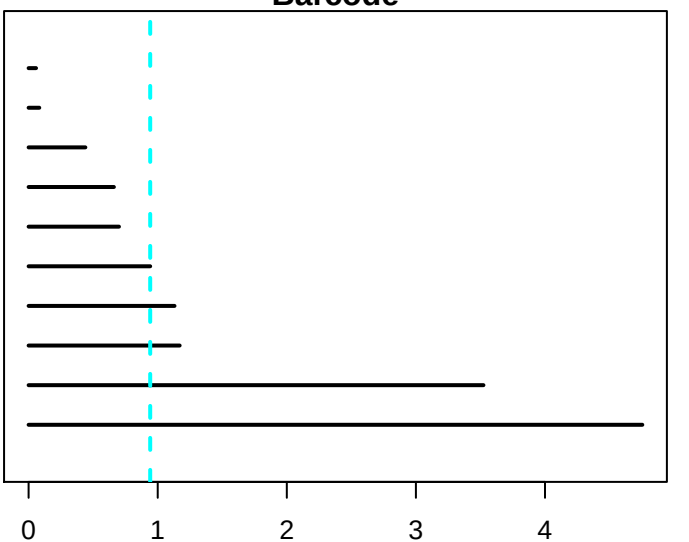
This is the 'Frame' at Euclidean distance = 0.942



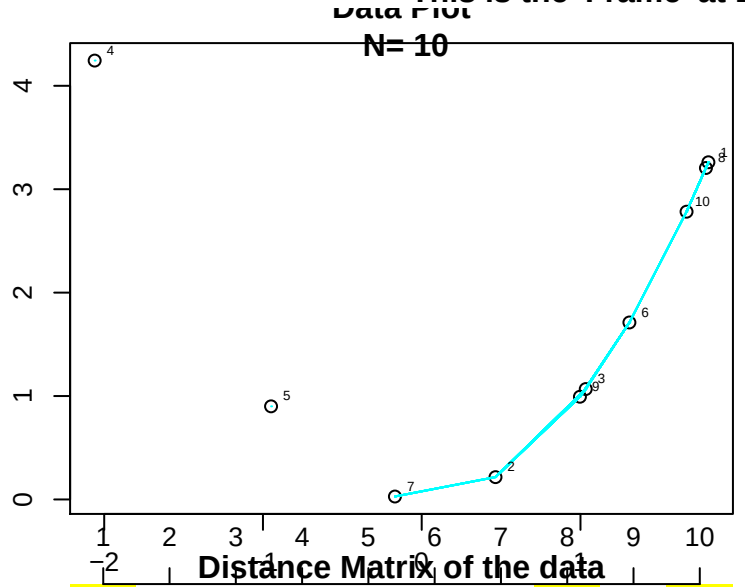


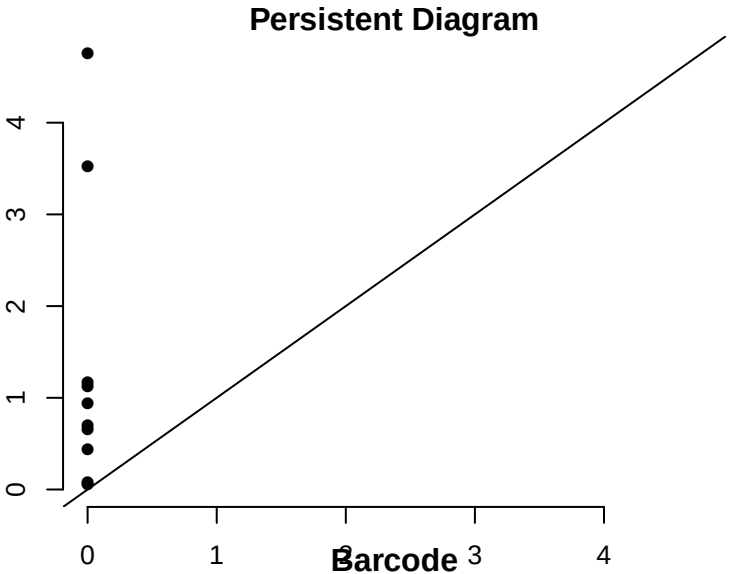
Distance Matrix of the data

0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000



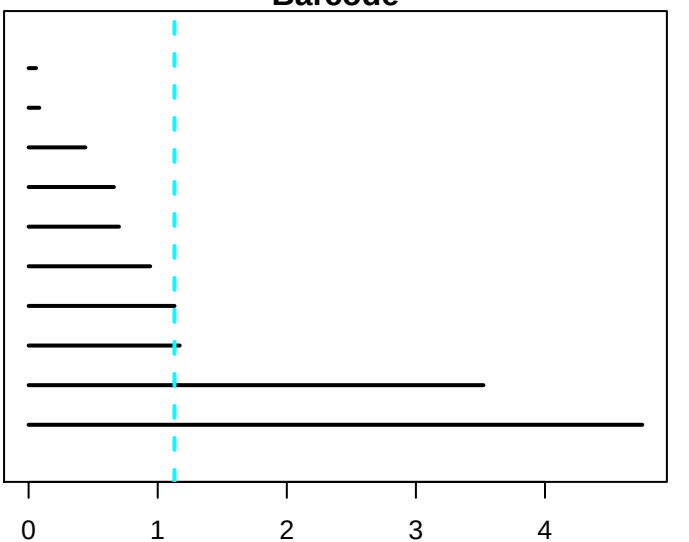
This is the 'Frame' at Euclidean distance = 1.13



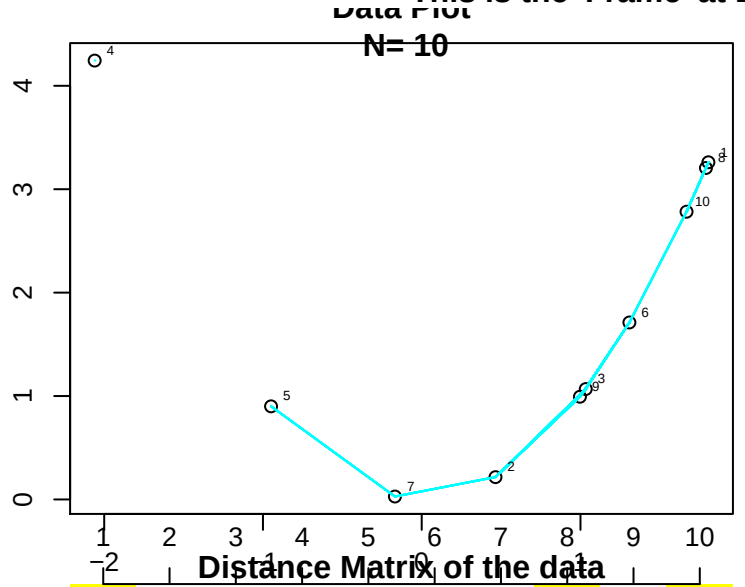


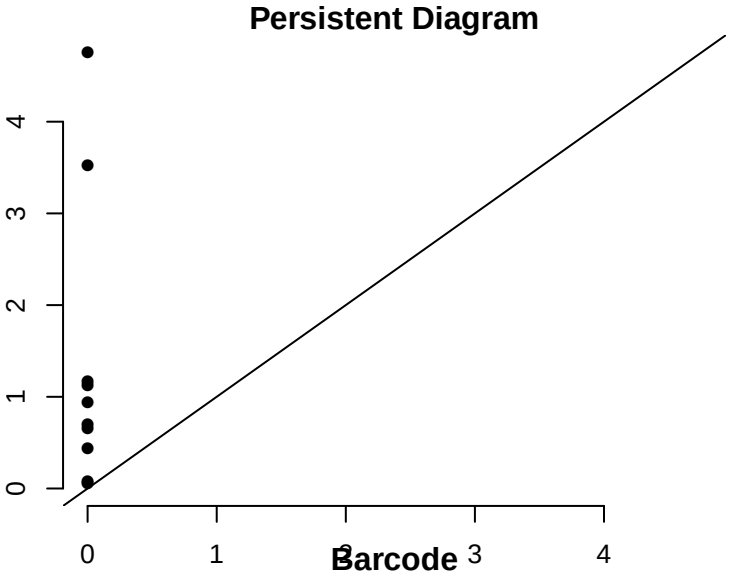
Distance Matrix of the data

0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000



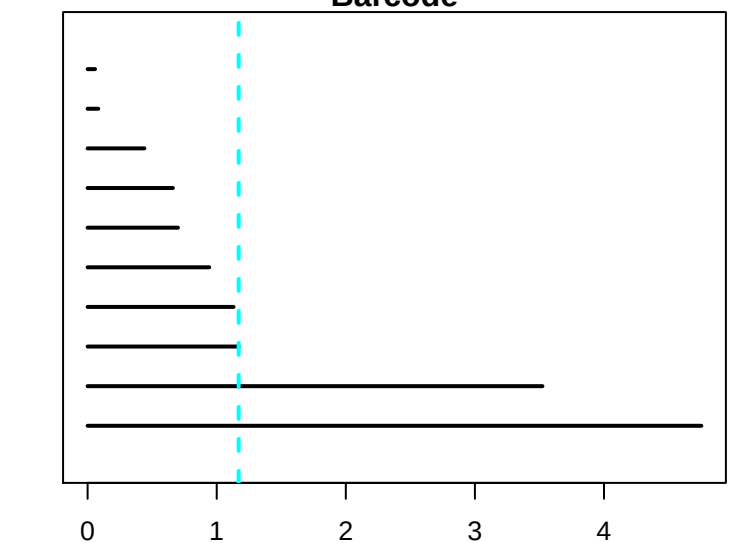
This is the 'Frame' at Euclidean distance = 1.17



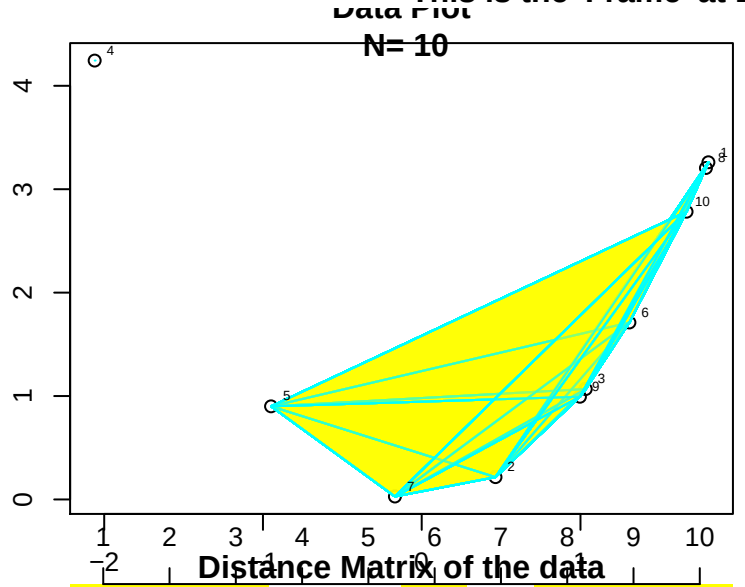


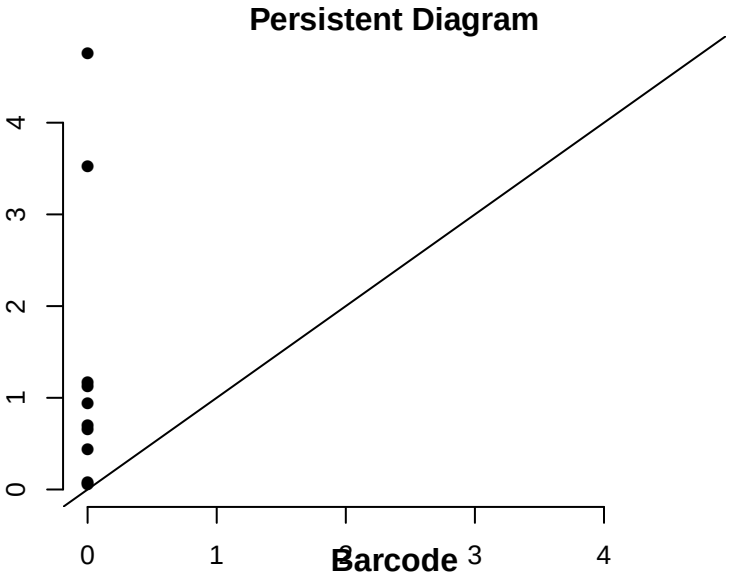
Distance Matrix of the data

	1	2	3	4	5	6	7	8	9	10
1	0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
2	3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
3	2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
4	3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
5	3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
6	1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
7	3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
8	0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
9	2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
10	0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000



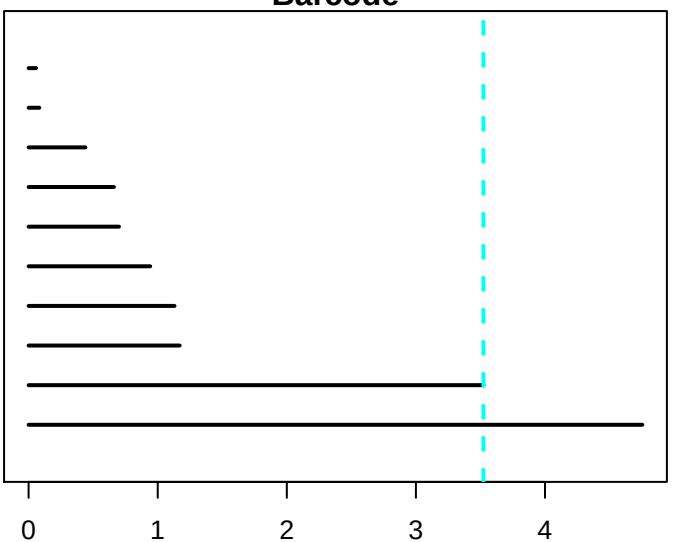
This is the 'Frame' at Euclidean distance = 3.52



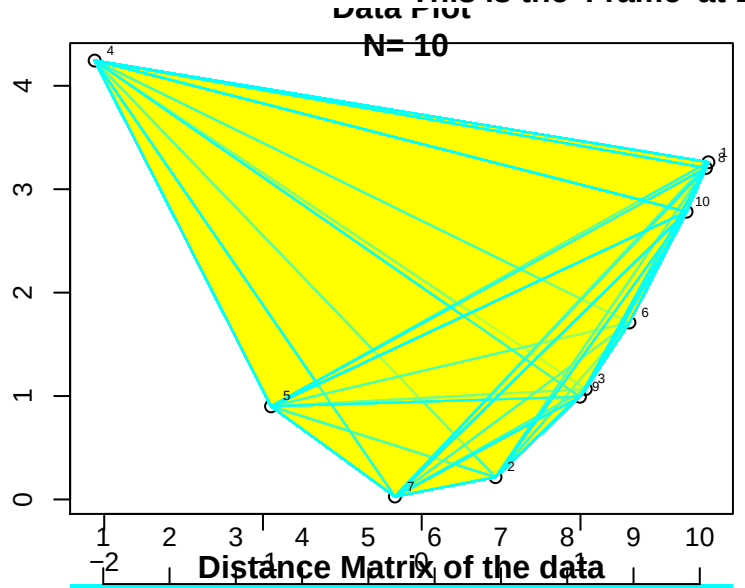


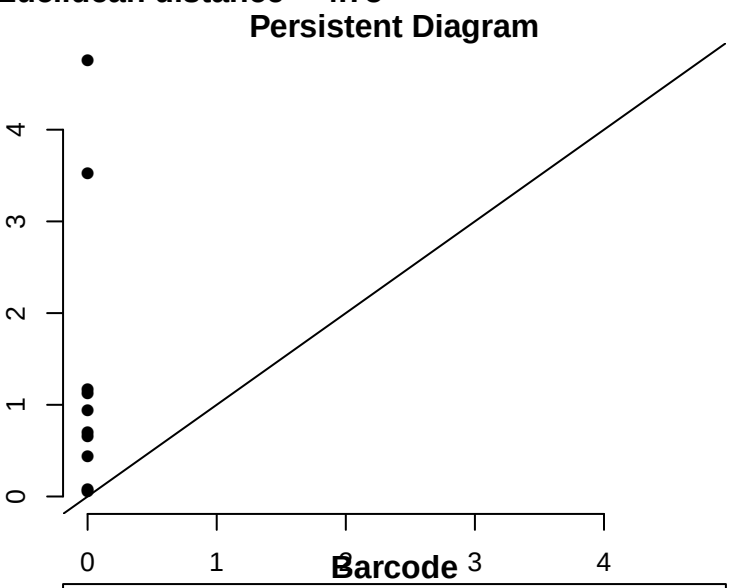
Distance Matrix of the data

0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000



This is the 'Frame' at Euclidean distance = 4.75





Distance Matrix of the data

0.000	3.327	2.325	3.989	3.628	1.627	3.788	0.058	2.407	0.497
3.327	0.000	1.024	4.754	1.571	1.717	0.661	3.270	0.942	2.835
2.325	1.024	0.000	4.434	1.989	0.700	1.589	2.268	0.083	1.829
3.989	4.754	4.434	0.000	3.523	4.214	4.620	3.988	4.462	4.004
3.628	1.571	1.989	3.523	0.000	2.398	1.170	3.580	1.948	3.224
1.627	1.717	0.700	4.214	2.398	0.000	2.239	1.570	0.783	1.130
3.788	0.661	1.589	4.620	1.170	2.239	0.000	3.732	1.513	3.311
0.058	3.270	2.268	3.988	3.580	1.570	3.732	0.000	2.350	0.440
2.407	0.942	0.083	4.462	1.948	0.783	1.513	2.350	0.000	1.911
0.497	2.835	1.829	4.004	3.224	1.130	3.311	0.440	1.911	0.000