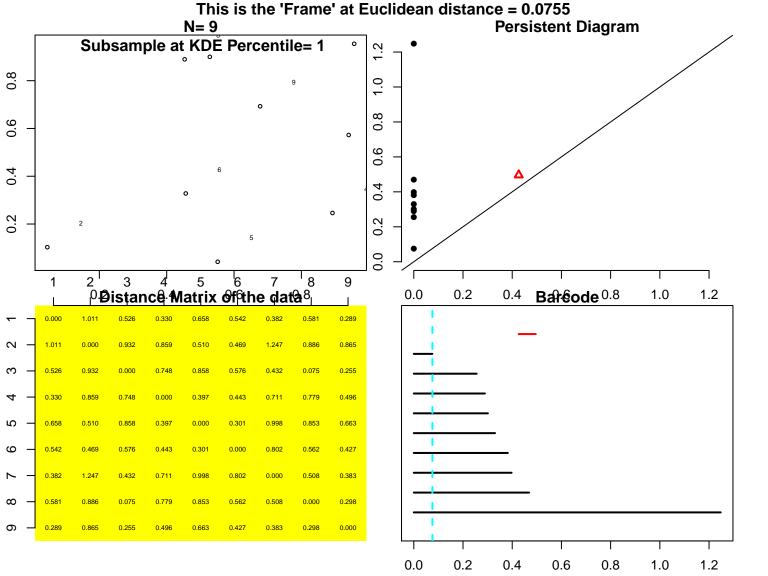
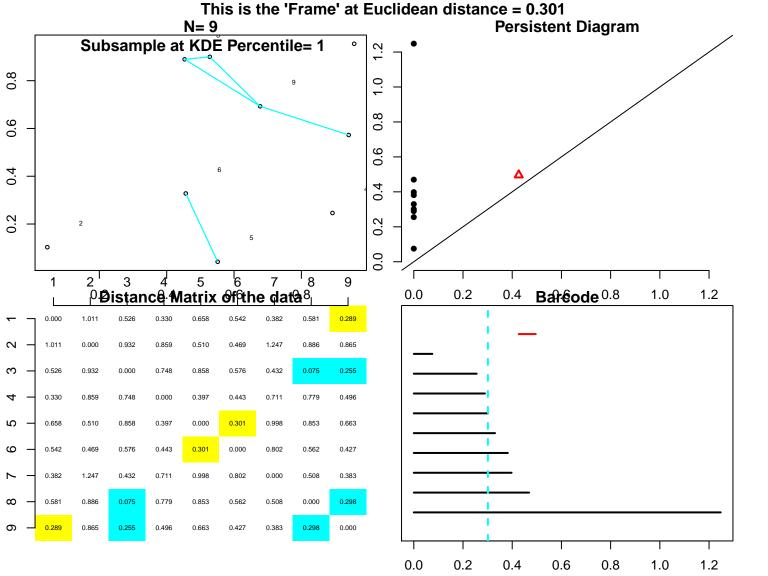


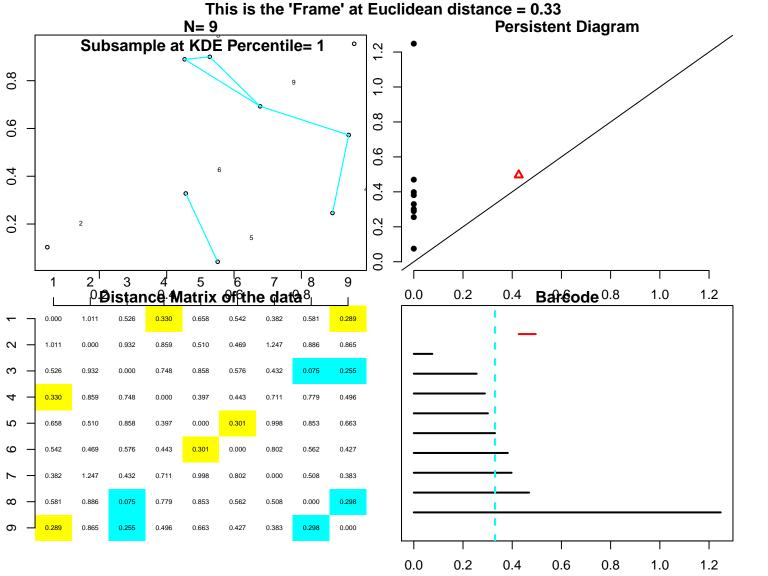
This is the 'Frame' at Euclidean distance = 0 N=9**Persistent Diagram** Subsample at KDE Percentile= 1 S 0.8 1.0 9 0.8 9.0 9.0 6 0.4 0.4 0.2 0.2 5 0 0.0 5 9 0.2 PDistance Matrix Offthe data8 0.0 0.4 Bareode 0.8 1.0 1.2 0.000 1.011 0.526 0.330 0.658 0.382 0.581 0.289 α 1.011 0.000 0.932 0.859 0.510 0.469 1.247 0.886 0.865 0.932 0.000 0.748 0.858 0.432 0.075 3 0.526 0.576 0.255 0.330 0.859 0.748 0.000 0.397 0.443 0.711 0.779 0.496 2 0.658 0.510 0.858 0.397 0.000 0.301 0.998 0.853 0.663 0.542 0.469 0.576 0.443 0.301 0.000 0.802 0.562 0.427 0.382 1.247 0.432 0.711 0.998 0.802 0.000 0.508 0.383 0.581 0.075 0.779 0.853 0.508 0.000 0.298 0.886 0.562 0.289 0.865 0.255 0.496 0.663 0.427 0.383 0.298 0.000 0.0 0.2 0.4 0.6 8.0 1.0 1.2



This is the 'Frame' at Euclidean distance = 0.255 N=9**Persistent Diagram** Subsample at KDE Percentile= 1 0 S 0.8 1.0 9 0.8 9.0 9.0 6 0.4 0.4 0 0.2 0.2 5 0 0.0 5 9 0.2 PDistance Matrix Offthe data8 0.0 0.4 Bareode 0.8 1.0 1.2 0.000 1.011 0.526 0.330 0.658 0.542 0.382 0.581 0.289 1.011 0.000 0.932 1.247 0.886 α -0.859 0.510 0.469 0.865 0.075 ი -0.526 0.932 0.000 0.748 0.858 0.576 0.432 0.255 0.330 0.859 0.748 0.000 0.397 0.443 0.711 0.779 0.496 2 0.658 0.510 0.858 0.397 0.000 0.301 0.998 0.853 0.663 0.542 0.576 0.562 0.469 0.443 0.301 0.000 0.802 0.427 0.382 1.247 0.432 0.508 0.711 0.998 0.802 0.000 0.383 0.075 0.779 0.581 0.886 0.853 0.562 0.508 0.000 0.298 0.289 0.865 0.255 0.496 0.663 0.427 0.383 0.298 0.000 0.0 0.2 0.4 0.6 8.0 1.0 1.2

This is the 'Frame' at Euclidean distance = 0.289 N=9**Persistent Diagram** Subsample at KDE Percentile= 1 S 0.8 1.0 9 0.8 9.0 9.0 6 0.4 0.4 0 0.2 0.2 5 0 0.0 5 9 0.2 PDistance Matrix Offthe data8 0.0 0.4 Bareode 0.8 1.0 1.2 0.000 1.011 0.526 0.330 0.658 0.542 0.382 0.581 0.289 1.011 0.000 0.932 0.886 0.865 α -0.859 0.510 0.469 1.247 0.075 ი -0.526 0.932 0.000 0.748 0.858 0.576 0.432 0.255 0.330 0.859 0.748 0.000 0.397 0.443 0.711 0.779 0.496 2 0.658 0.510 0.858 0.397 0.000 0.301 0.998 0.853 0.663 0.542 0.576 0.562 0.469 0.443 0.301 0.000 0.802 0.427 0.382 0.432 0.508 1.247 0.711 0.998 0.802 0.000 0.383 0.075 0.779 0.581 0.886 0.853 0.562 0.508 0.000 0.298 0.289 0.865 0.255 0.496 0.663 0.427 0.383 0.298 0.000 0.0 0.2 0.4 0.6 8.0 1.0 1.2





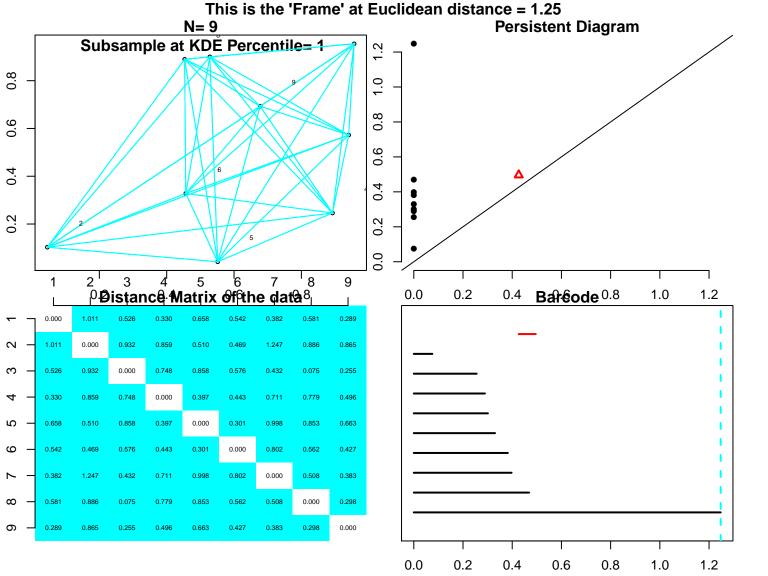
This is the 'Frame' at Euclidean distance = 0.382 N=9**Persistent Diagram** Subsample at KDE Percentile= 1 S 0.8 1.0 9 0.8 9.0 9.0 6 0.4 0.4 0.2 0.2 5 0 0.0 2 9 0.2 PDistance Matrix Offthe data8 0.0 0.4 Bareode 0.8 1.0 1.2 0.000 1.011 0.526 0.330 0.658 0.542 0.382 0.581 0.289 1.011 0.000 0.932 1.247 0.886 0.865 α -0.859 0.510 0.469 0.075 ი -0.526 0.932 0.000 0.748 0.858 0.576 0.432 0.255 0.330 0.859 0.748 0.000 0.397 0.443 0.711 0.779 0.496 0.658 0.397 0.000 0.301 0.998 0.853 2 0.510 0.858 0.663 0.301 0.542 0.469 0.576 0.443 0.000 0.802 0.562 0.427 0.382 1.247 0.432 0.711 0.998 0.508 0.383 0.802 0.000 0.075 0.779 0.000 0.298 0.581 0.886 0.853 0.562 0.508 0.289 0.865 0.255 0.496 0.663 0.427 0.383 0.298 0.000 0.0 0.2 0.4 0.6 8.0 1.0 1.2

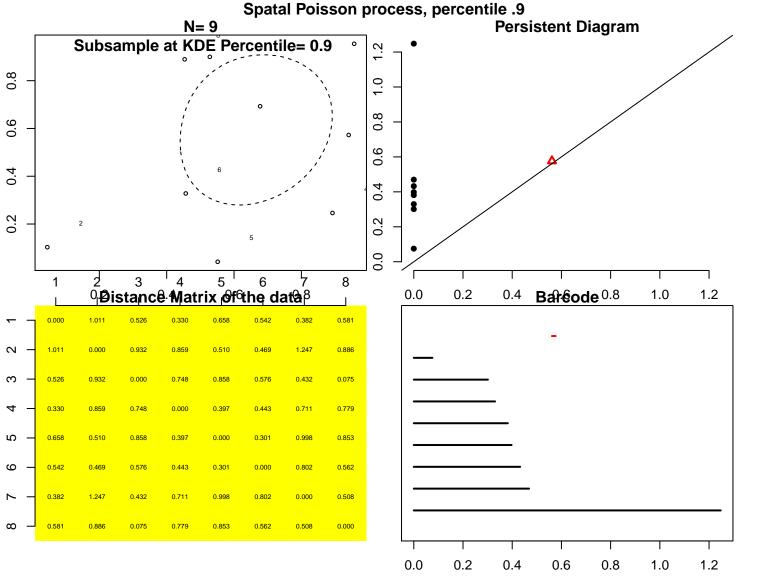
This is the 'Frame' at Euclidean distance = 0.397 N=9**Persistent Diagram** Subsample at KDE Percentile= 1 α 0.8 1.0 0.8 9.0 9.0 6 0.4 0.4 0.2 0.2 5 0 0.0 2 9 0.2 PDistance Matrix Offthe data8 0.0 0.4 Bareode 0.8 1.0 1.2 0.000 1.011 0.526 0.330 0.658 0.542 0.382 0.581 0.289 1.011 0.000 0.932 1.247 0.886 0.865 α -0.859 0.510 0.469 0.075 ი -0.526 0.932 0.000 0.748 0.858 0.576 0.432 0.255 0.330 0.859 0.748 0.000 0.397 0.443 0.711 0.779 0.496 0.658 0.397 0.000 0.301 0.998 0.853 2 0.510 0.858 0.663 0.301 0.542 0.469 0.576 0.443 0.000 0.802 0.562 0.427 0.382 1.247 0.432 0.711 0.998 0.508 0.383 0.802 0.000 0.581 0.075 0.779 0.000 0.298 0.886 0.853 0.562 0.508 0.289 0.865 0.255 0.496 0.663 0.427 0.383 0.298 0.000 0.0 0.2 0.4 0.6 8.0 1.0 1.2

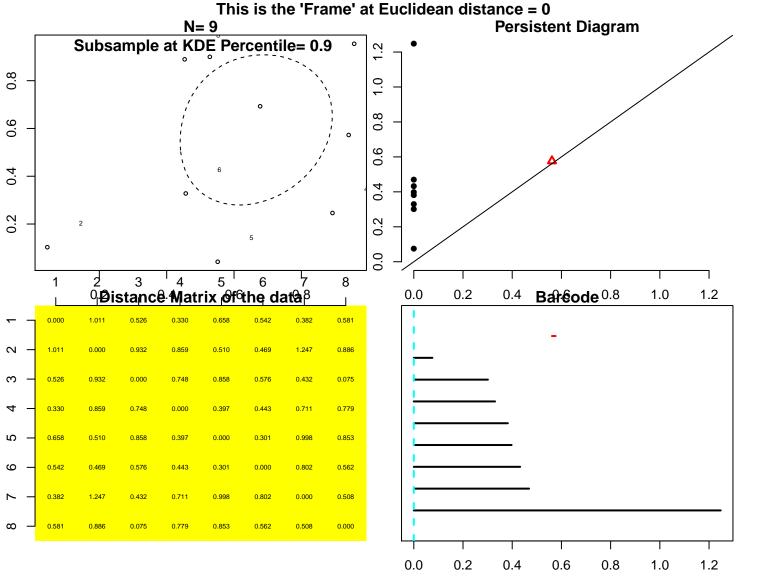
This is the 'Frame' at Euclidean distance = 0.427 N=9**Persistent Diagram** Subsample at KDE Percentile= 1 Ś 0.8 1.0 0.8 9.0 9.0 0.4 0.4 0.2 0.2 0 0.0 2 5 9 0.2 PDistance Matrix Offthe data8 0.0 0.4 Bareode 0.8 1.0 1.2 0.000 1.011 0.526 0.330 0.658 0.542 0.382 0.581 0.289 1.011 0.000 0.932 1.247 0.886 0.865 α -0.859 0.510 0.469 0.075 ი -0.526 0.932 0.000 0.748 0.858 0.576 0.432 0.255 0.330 0.859 0.748 0.000 0.397 0.443 0.711 0.779 0.496 0.397 0.000 0.301 0.998 0.853 2 0.658 0.510 0.858 0.663 0.427 0.542 0.469 0.576 0.443 0.301 0.000 0.802 0.562 0.382 1.247 0.432 0.711 0.508 0.383 0.998 0.802 0.000 0.075 0.779 0.298 0.581 0.886 0.853 0.562 0.508 0.000 0.298 0.289 0.865 0.255 0.496 0.663 0.427 0.383 0.000 0.0 0.2 0.4 0.6 8.0 1.0 1.2

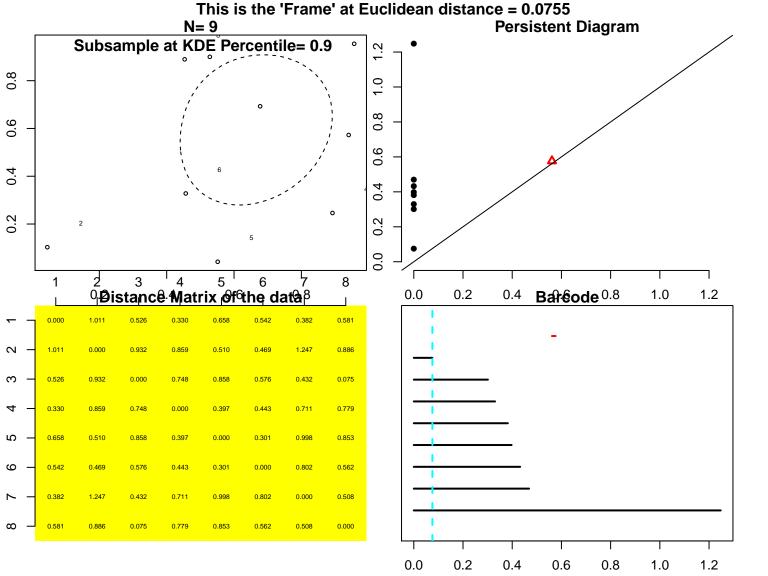
This is the 'Frame' at Euclidean distance = 0.469 N=9**Persistent Diagram** Subsample at KDE Percentile= 1 Ś 0.8 1.0 0.8 9.0 9.0 0.4 0.4 0.2 0.2 0.0 5 9 0.2 PDistance Matrix Offthe data8 0.0 0.4 Balteode 0.8 1.0 1.2 0.382 0.000 1.011 0.526 0.330 0.658 0.542 0.581 0.289 1.011 0.000 0.932 0.859 0.510 0.469 1.247 0.886 0.865 α -0.432 0.075 ი -0.526 0.932 0.000 0.748 0.858 0.576 0.255 0.330 0.859 0.748 0.000 0.397 0.443 0.711 0.779 0.496 0.658 0.510 0.858 0.397 0.000 0.301 0.998 0.853 5 0.663 0.469 0.427 0.542 0.576 0.443 0.301 0.000 0.802 0.562 0.382 1.247 0.432 0.711 0.998 0.508 0.383 0.802 0.000 0.075 0.298 0.581 0.886 0.779 0.853 0.562 0.508 0.000 0.289 0.865 0.255 0.496 0.663 0.427 0.383 0.298 0.000 0.0 0.2 0.4 0.6 8.0 1.0 1.2

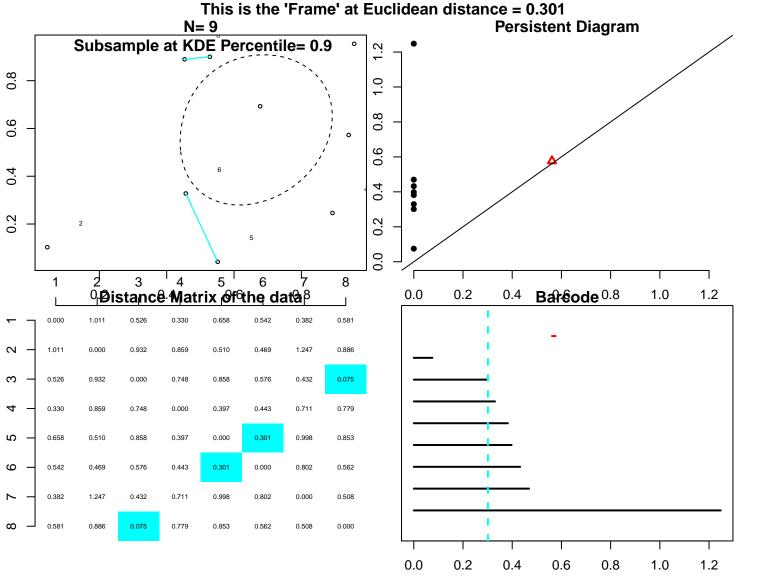
This is the 'Frame' at Euclidean distance = 0.496 N=9**Persistent Diagram** Subsample at KDE Percentile= 1 Ś 0.8 1.0 0.8 9.0 9.0 0.4 0.2 0.2 0.0 5 9 0.2 PÐistance Matrix offthe data8₁ 0.0 0.4 Balteode 0.8 1.0 1.2 0.382 0.000 1.011 0.526 0.330 0.658 0.542 0.581 0.289 1.011 0.000 0.932 0.859 0.510 0.469 1.247 0.886 0.865 α -0.432 0.075 ი -0.526 0.932 0.000 0.748 0.858 0.576 0.255 0.443 0.330 0.859 0.748 0.000 0.397 0.711 0.779 0.496 0.658 0.510 0.858 0.397 0.000 0.301 0.998 0.853 0.663 5 0.469 0.542 0.576 0.443 0.301 0.000 0.802 0.562 0.427 0.382 1.247 0.432 0.711 0.998 0.508 0.383 0.802 0.000 0.886 0.075 0.298 0.581 0.779 0.853 0.562 0.508 0.000 0.289 0.865 0.255 0.496 0.663 0.427 0.383 0.298 0.000 0.0 0.2 0.4 0.6 8.0 1.0 1.2

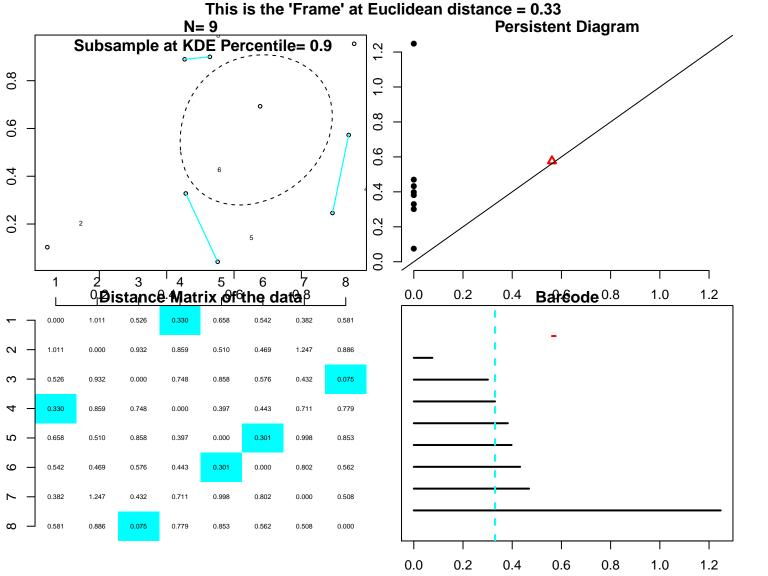


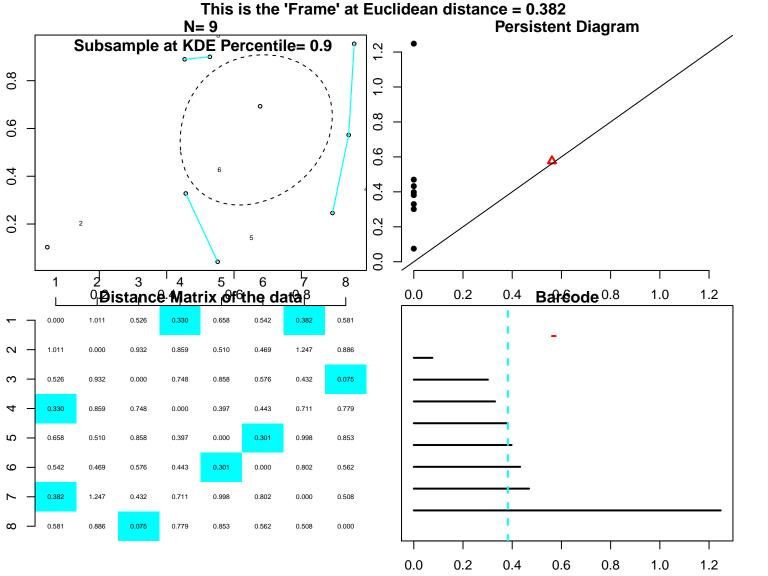


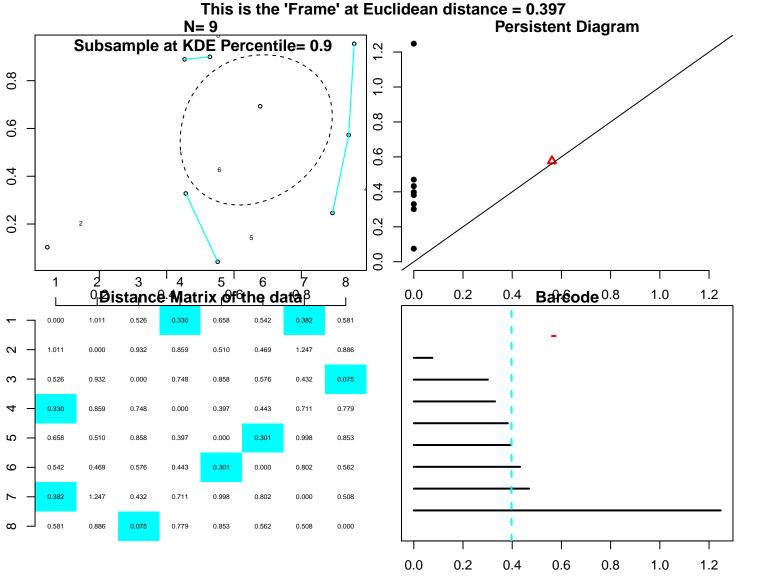


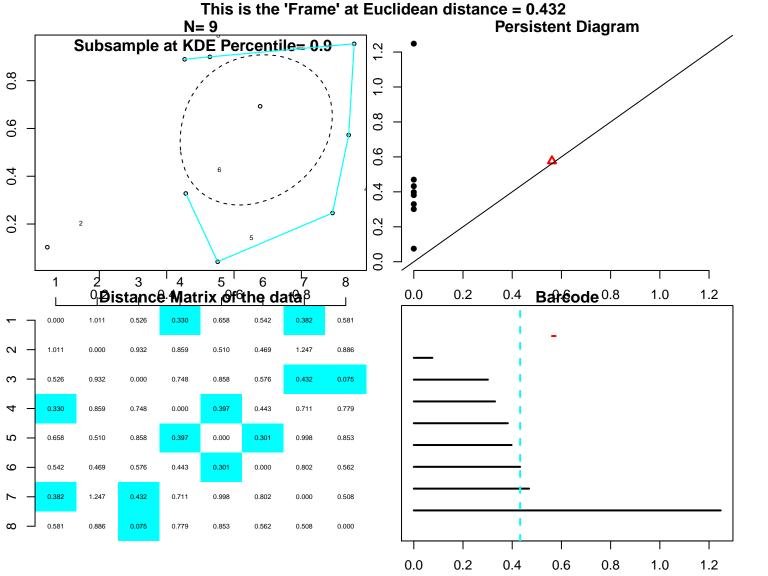


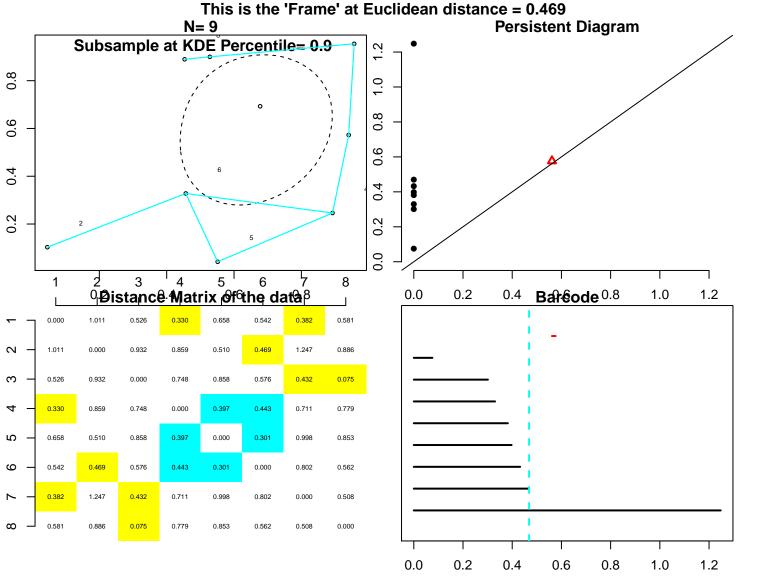


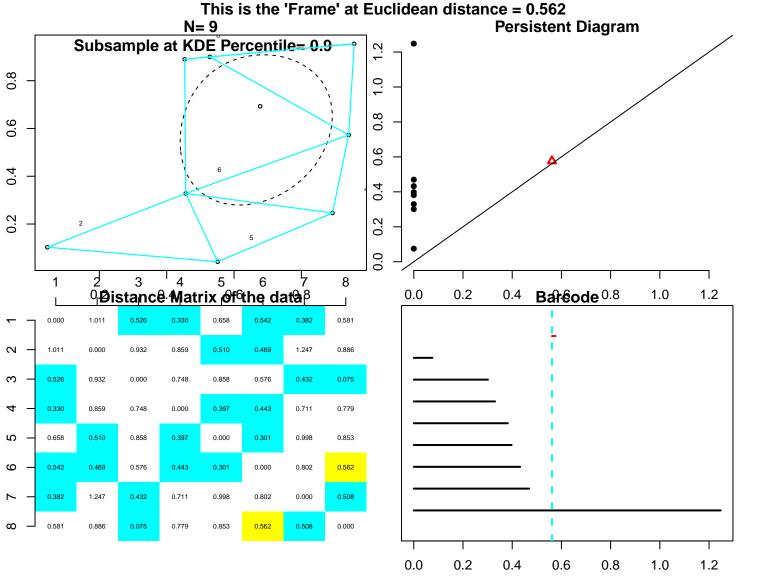


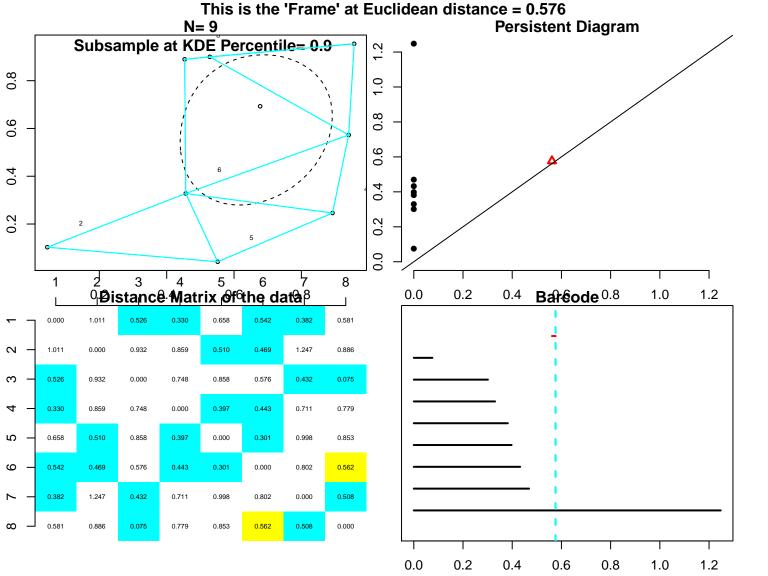


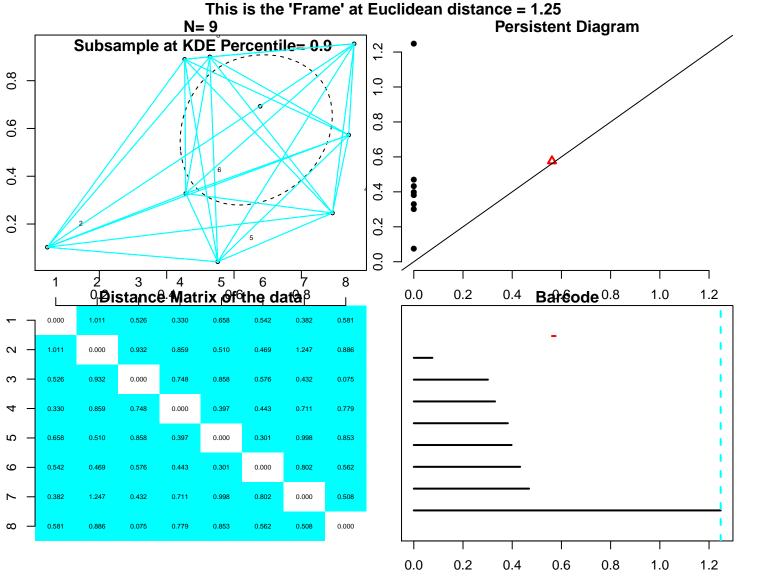


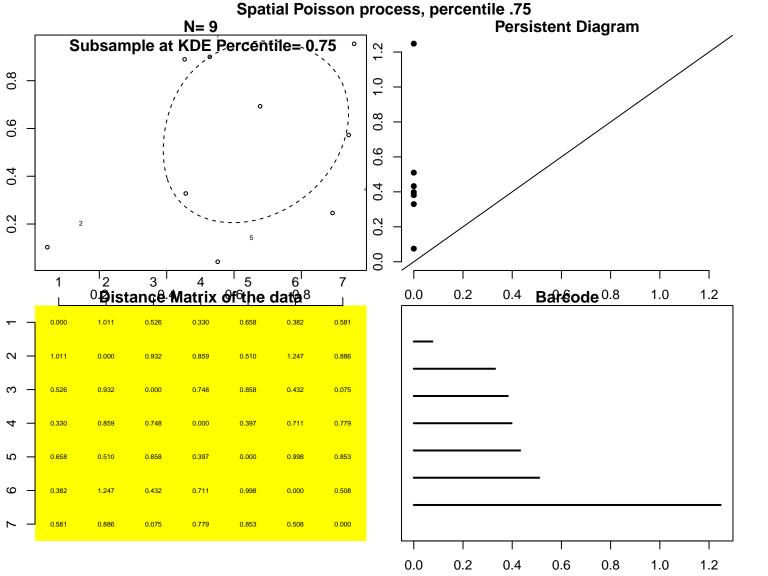


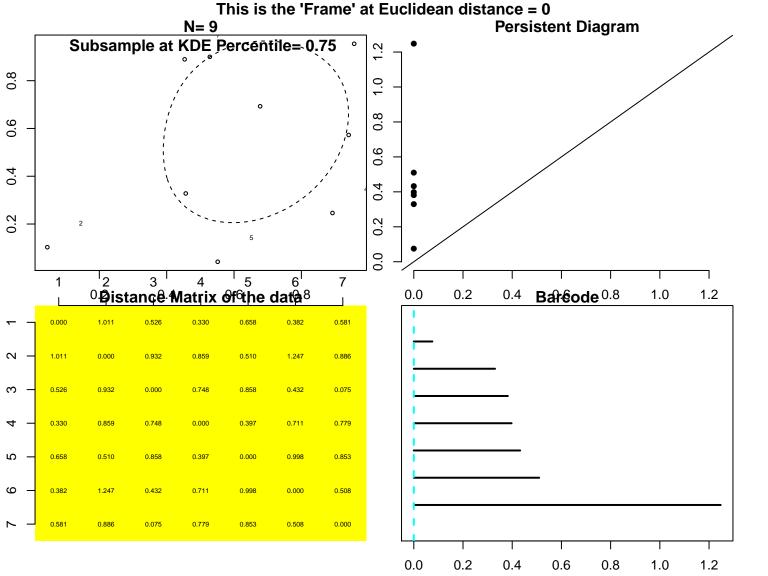


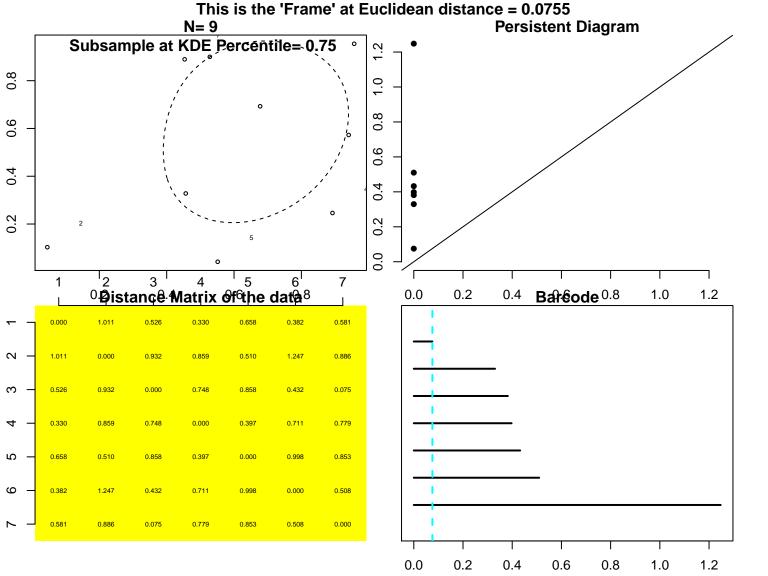


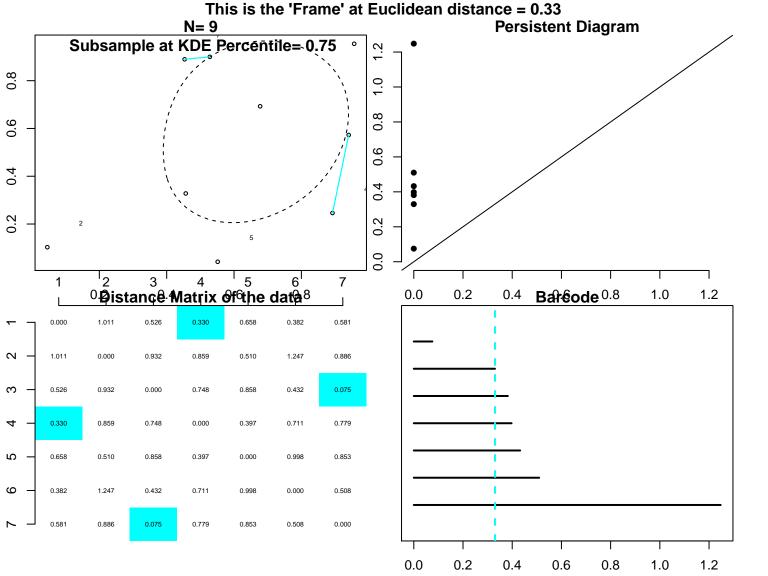


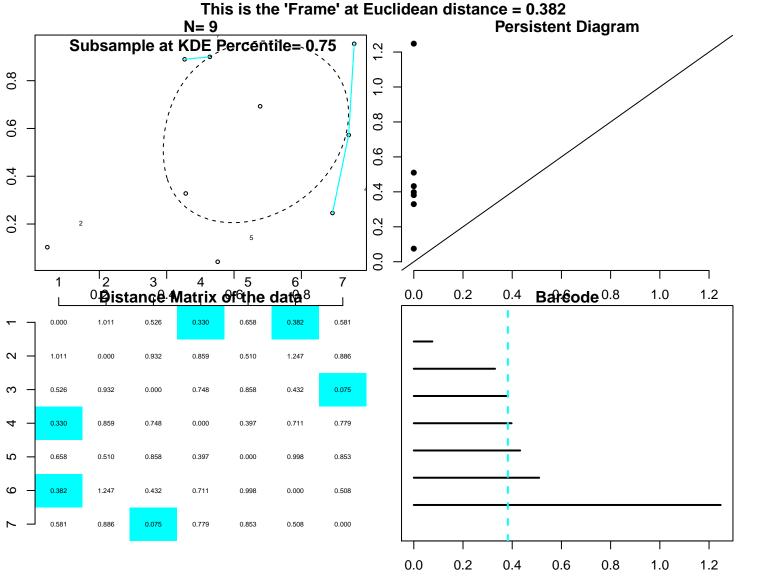


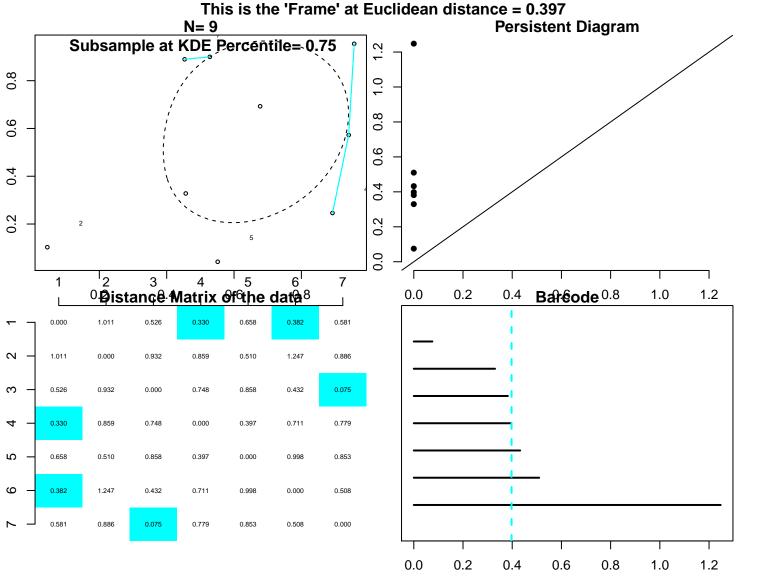


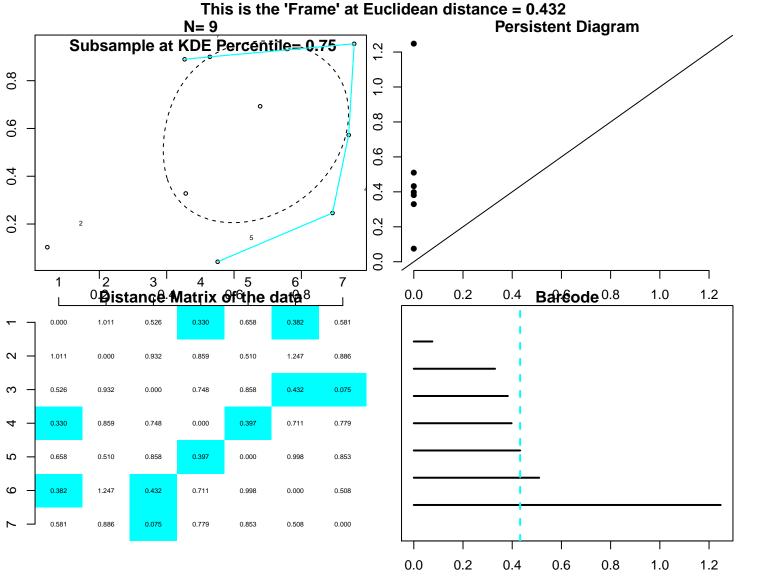


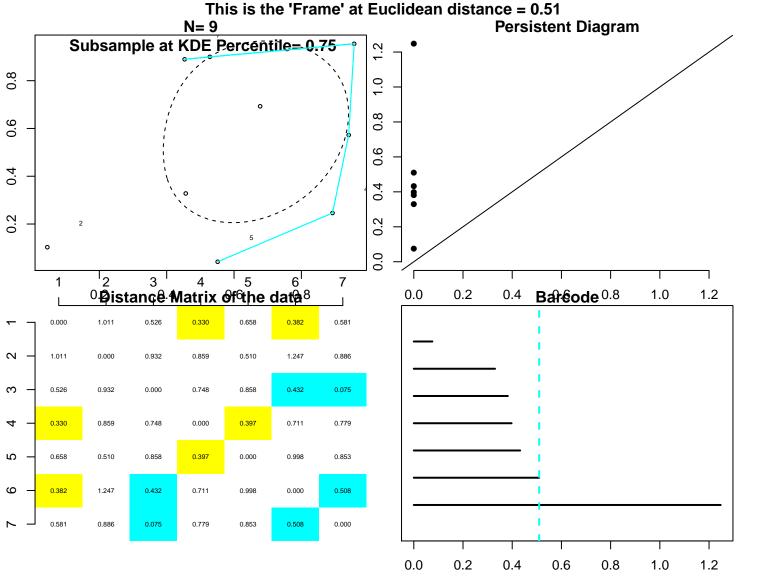


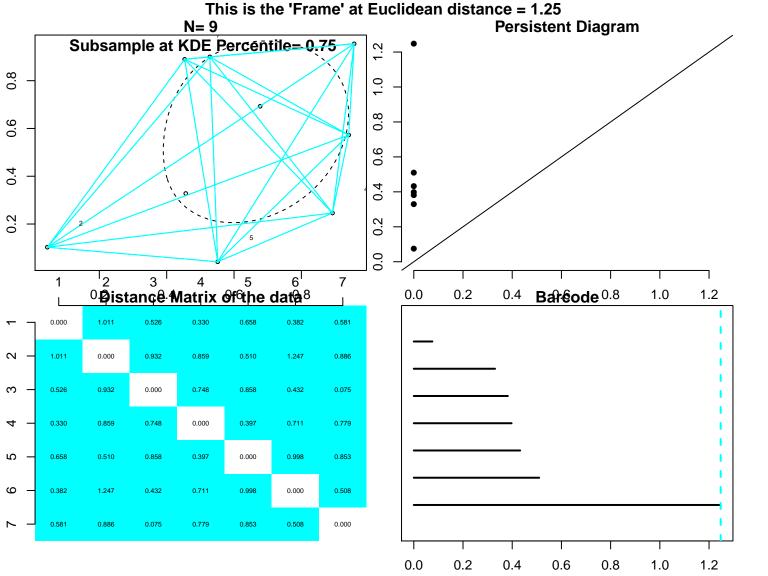


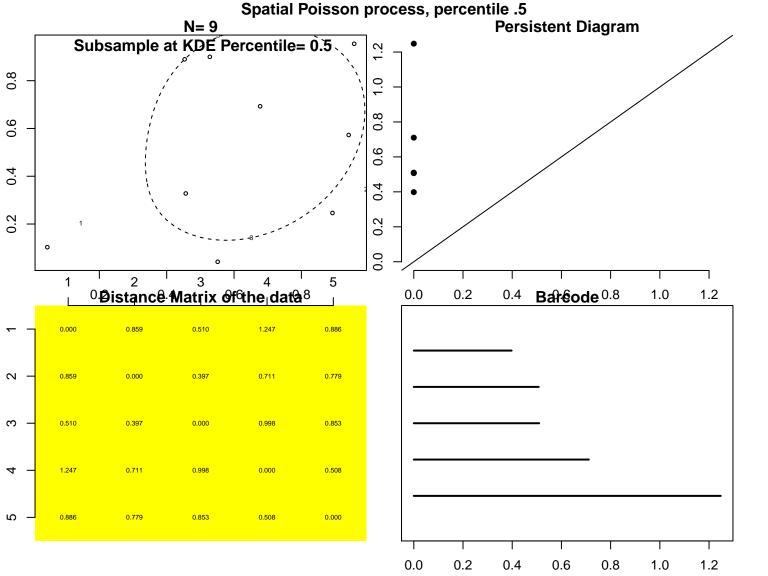


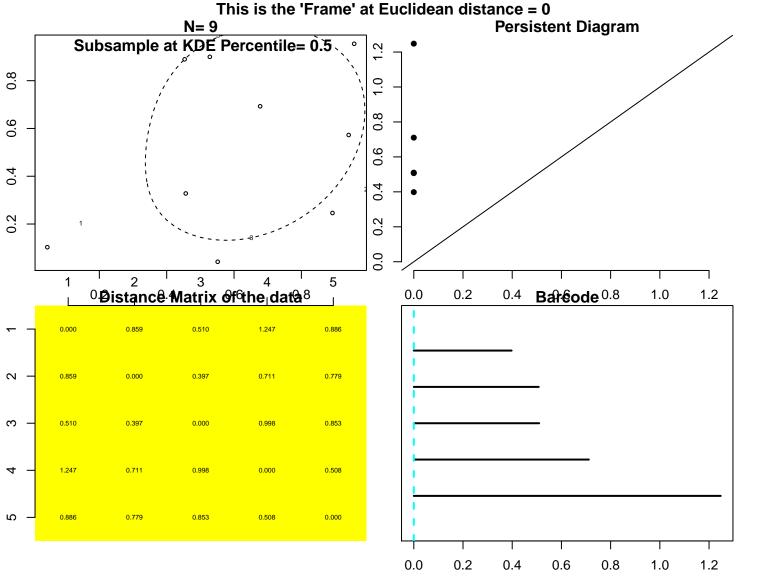


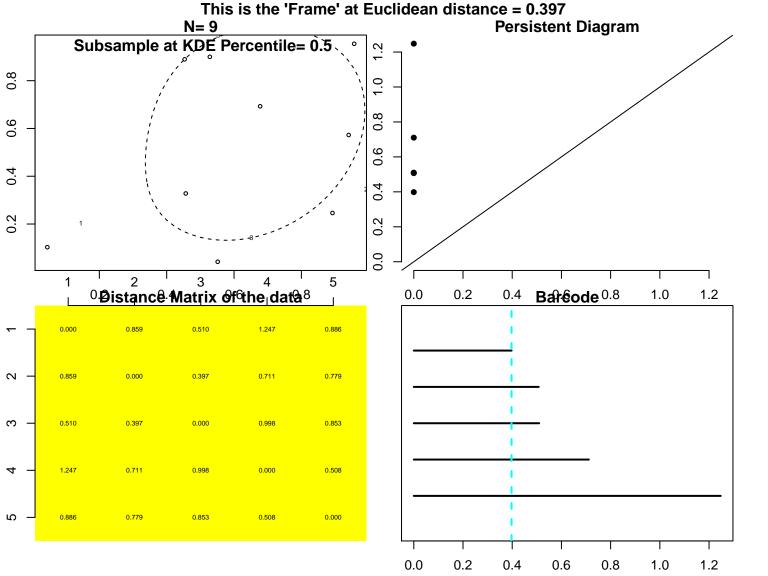


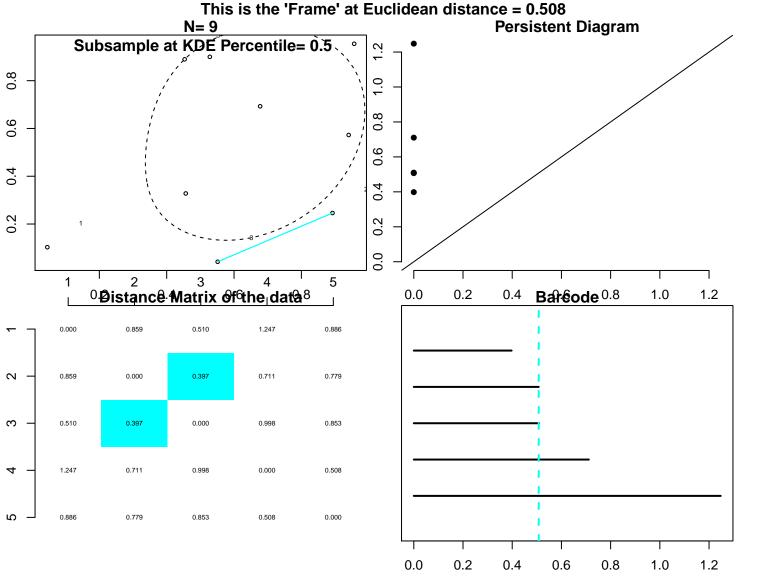


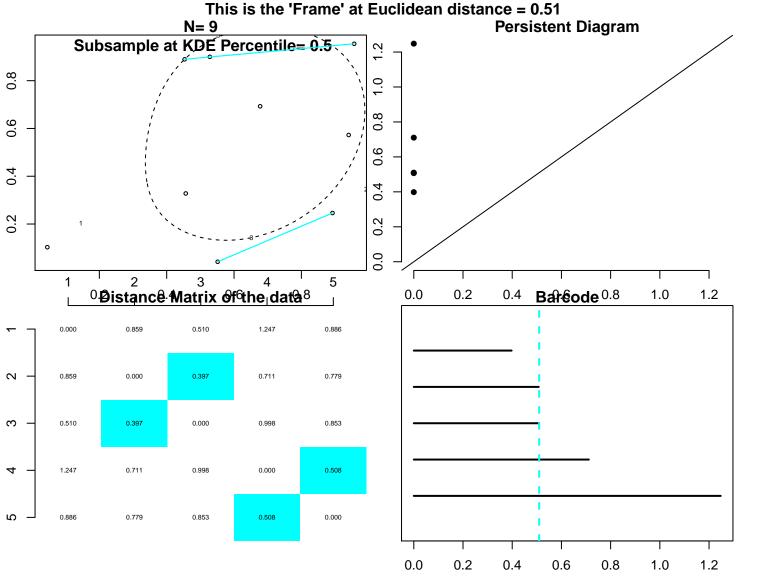


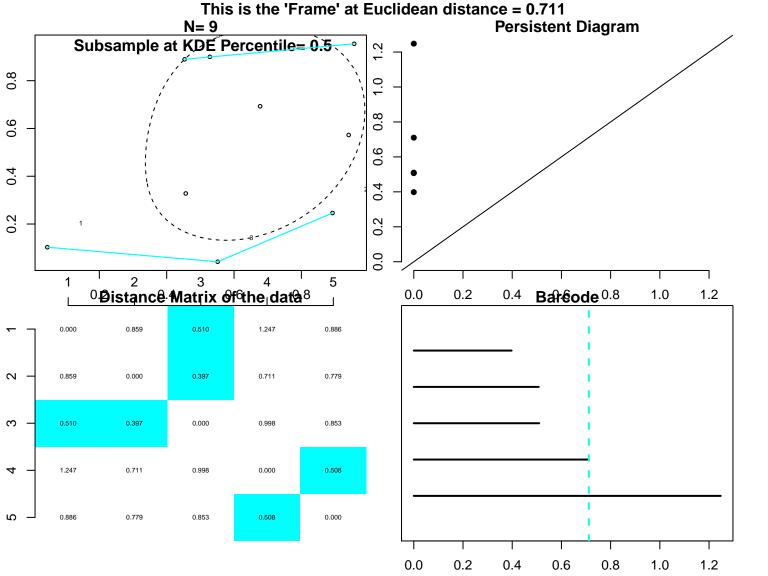


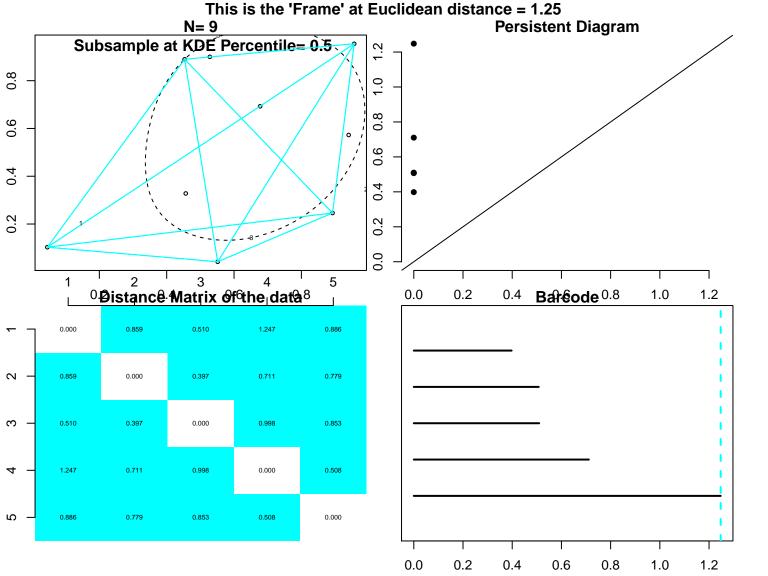


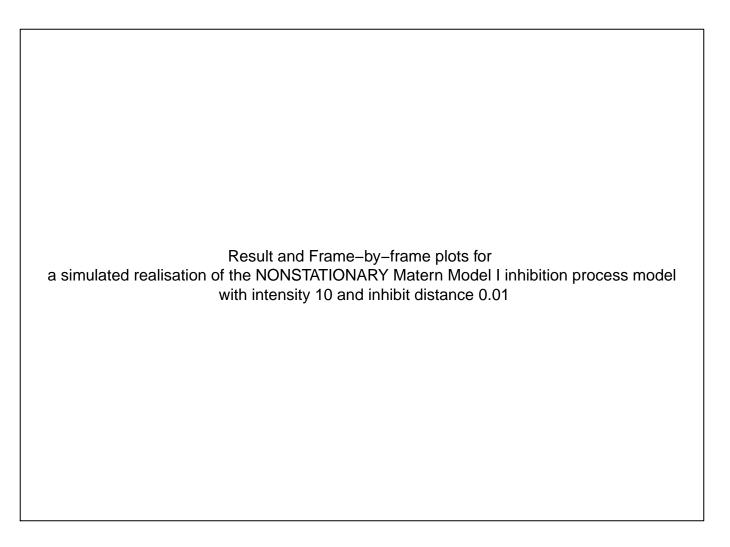


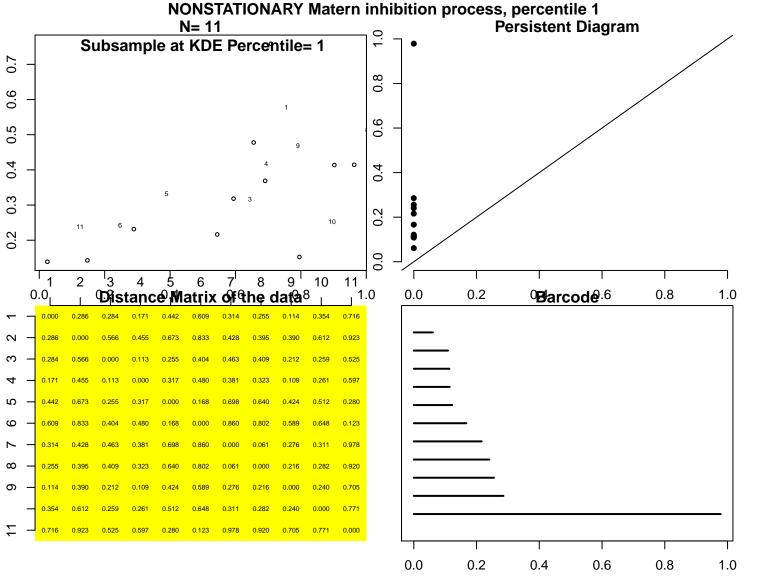












This is the 'Frame' at Euclidean distance = 0 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 0.8 9.0 9.0 0.5 0.4 0.4 0.3 0.2 0.2 0.0 9 0.0, Distance Matrix of the data8 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.428 0.923 0.000 0.255 0.463 0.212 0.284 0.525 0.317 0.381 0.113 0.109 0.597 0.280 0.609 0.168 0.860 0.589 0.123 0.860 0.314 0.463 0.381 0.698 0.000 0.276 0.978 0.640 0.061 0.216 0.409 0.920 0.212 0.424 0.276 0.000 0.705 0.771 0.525 0.280 0.978 0.705 0.000 0.0 0.2 0.4 0.6 0.8 1.0

This is the 'Frame' at Euclidean distance = 0.0607 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 0.8 9.0 9.0 0.5 0.4 0.4 0.3 0.2 10 0.2 0.0 9 0.0, Distance Matrix of the data8 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.716 0.286 0.428 0.923 0.000 0.255 0.463 0.212 0.284 0.259 0.525 0.317 0.381 0.113 0.109 0.597 0.698 0.280 0.000 0.609 0.404 0.168 0.860 0.589 0.123 0.860 0.061 0.314 0.428 0.463 0.381 0.698 0.000 0.276 0.978 0.640 0.802 0.061 0.216 0.920 0.409 0.212 0.424 0.276 0.000 0.705 0.771 0.525 0.280 0.978 0.920 0.705 0.000 0.0 0.2 0.4 0.6 0.8 1.0

This is the 'Frame' at Euclidean distance = 0.109 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 0.8 9.0 9.0 0.5 0.4 0.4 0.3 0.2 10 0.2 0.0 9 10 0.0, Distance Matrix of the data8 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.354 0.716 0.286 0.000 0.673 0.428 0.390 0.923 0.566 0.284 0.566 0.000 0.113 0.255 0.404 0.463 0.409 0.212 0.259 0.525 0.171 0.455 0.113 0.000 0.317 0.480 0.381 0.323 0.109 0.597 0.442 0.673 0.255 0.317 0.000 0.168 0.698 0.640 0.424 0.512 0.280 0.609 0.833 0.404 0.480 0.168 0.000 0.860 0.802 0.589 0.123 0.860 0.000 0.061 0.314 0.428 0.463 0.381 0.698 0.276 0.311 0.978 0.061 0.000 0.255 0.395 0.409 0.323 0.640 0.802 0.216 0.920 0.390 0.212 0.109 0.424 0.589 0.276 0.000 0.705 0.354 0.512 0.311 0.240 0.771 0.923 0.525 0.597 0.280 0.123 0.978 0.920 0.705 0.000 0.0 0.2 0.4 0.6 0.8 1.0

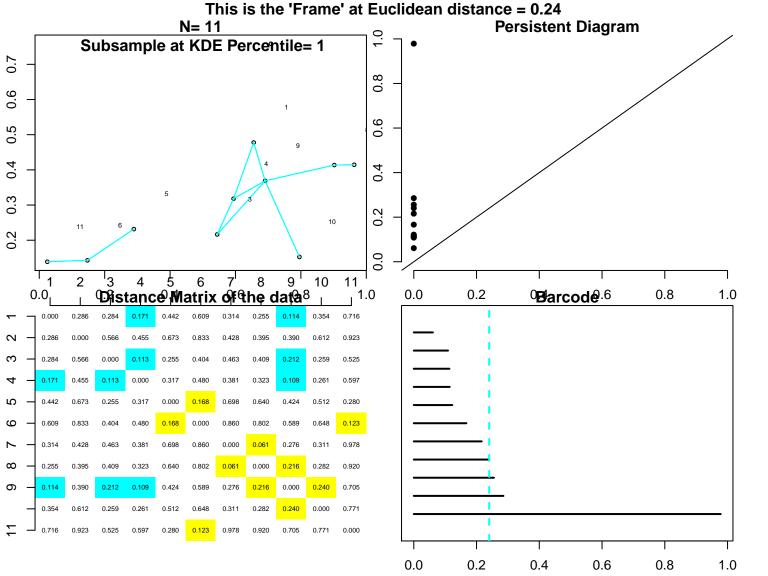
This is the 'Frame' at Euclidean distance = 0.113 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 0.8 9.0 9.0 0.5 0.4 0.4 0.3 0.2 10 0.2 0.0 9 10 0.0, Distance Matrix of the data8 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.354 0.286 0.000 0.673 0.833 0.428 0.390 0.923 0.566 0.284 0.566 0.000 0.113 0.255 0.404 0.463 0.409 0.212 0.259 0.525 0.381 0.323 0.261 0.171 0.455 0.113 0.000 0.317 0.480 0.597 0.442 0.673 0.255 0.317 0.000 0.168 0.698 0.280 0.609 0.833 0.404 0.480 0.168 0.000 0.860 0.802 0.589 0.123 0.860 0.000 0.061 0.314 0.428 0.463 0.381 0.698 0.276 0.311 0.978 0.802 0.061 0.000 0.255 0.395 0.409 0.323 0.640 0.216 0.920 0.390 0.212 0.424 0.589 0.276 0.000 0.705 0.771 0.354 0.512 0.311 0.240 0.923 0.525 0.597 0.280 0.123 0.978 0.920 0.705 0.000 0.0 0.2 0.4 0.6 0.8 1.0

This is the 'Frame' at Euclidean distance = 0.114 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 0.8 9.0 9.0 0.5 0.4 0.4 0.3 0.2 10 0.2 0.0 9 10 0.0, Distance Matrix of the data8 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.354 0.716 0.286 0.000 0.566 0.455 0.673 0.833 0.428 0.390 0.923 0.113 0.255 0.404 0.284 0.566 0.000 0.463 0.409 0.212 0.259 0.525 0.171 0.113 0.000 0.317 0.381 0.323 0.261 0.480 0.597 0.442 0.673 0.255 0.317 0.000 0.168 0.698 0.280 0.609 0.833 0.404 0.480 0.168 0.000 0.860 0.802 0.589 0.123 0.860 0.000 0.061 0.314 0.428 0.463 0.381 0.698 0.276 0.311 0.978 0.640 0.802 0.061 0.000 0.255 0.395 0.409 0.323 0.216 0.920 0.390 0.212 0.424 0.589 0.276 0.216 0.000 0.354 0.512 0.311 0.240 0.771 0.923 0.525 0.597 0.280 0.123 0.978 0.920 0.705 0.000 0.0 0.2 0.4 0.6 0.8 1.0

This is the 'Frame' at Euclidean distance = 0.123 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 0.8 9.0 9.0 0.5 0.4 0.4 0.3 0.2 10 0.2 0.0 9 10 0.0, Distance Matrix of the data8 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.314 0.354 0.716 0.286 0.000 0.566 0.455 0.673 0.833 0.428 0.390 0.923 0.113 0.255 0.404 0.284 0.566 0.000 0.463 0.409 0.212 0.259 0.525 0.171 0.113 0.000 0.317 0.381 0.323 0.261 0.480 0.597 0.442 0.673 0.255 0.317 0.000 0.168 0.698 0.280 0.609 0.833 0.404 0.480 0.168 0.000 0.860 0.802 0.589 0.123 0.860 0.000 0.061 0.314 0.428 0.463 0.381 0.698 0.276 0.311 0.978 0.802 0.061 0.000 0.255 0.395 0.409 0.323 0.640 0.216 0.920 0.114 0.390 0.212 0.424 0.589 0.276 0.216 0.000 0.354 0.512 0.311 0.240 0.771 0.923 0.525 0.597 0.280 0.123 0.978 0.920 0.705 0.000 0.0 0.2 0.4 0.6 0.8 1.0

This is the 'Frame' at Euclidean distance = 0.168 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 9.0 9.0 0.5 0.4 0.3 0.2 10 0.2 0.0 9 10 0.0, Distance Matrix of the data8 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.314 0.354 0.716 0.286 0.000 0.566 0.455 0.673 0.833 0.428 0.390 0.923 0.113 0.255 0.404 0.284 0.566 0.000 0.463 0.409 0.212 0.259 0.525 0.171 0.113 0.000 0.317 0.381 0.323 0.261 0.480 0.597 0.442 0.673 0.255 0.317 0.000 0.168 0.698 0.512 0.280 0.609 0.833 0.404 0.480 0.168 0.000 0.860 0.802 0.589 0.648 0.123 0.860 0.000 0.061 0.314 0.428 0.463 0.381 0.698 0.276 0.311 0.978 0.802 0.061 0.000 0.216 0.255 0.395 0.409 0.323 0.640 0.920 0.114 0.390 0.212 0.424 0.589 0.276 0.000 0.261 0.354 0.512 0.311 0.240 0.771 0.923 0.525 0.597 0.280 0.978 0.920 0.705 0.000 0.0 0.2 0.4 0.6 0.8 1.0

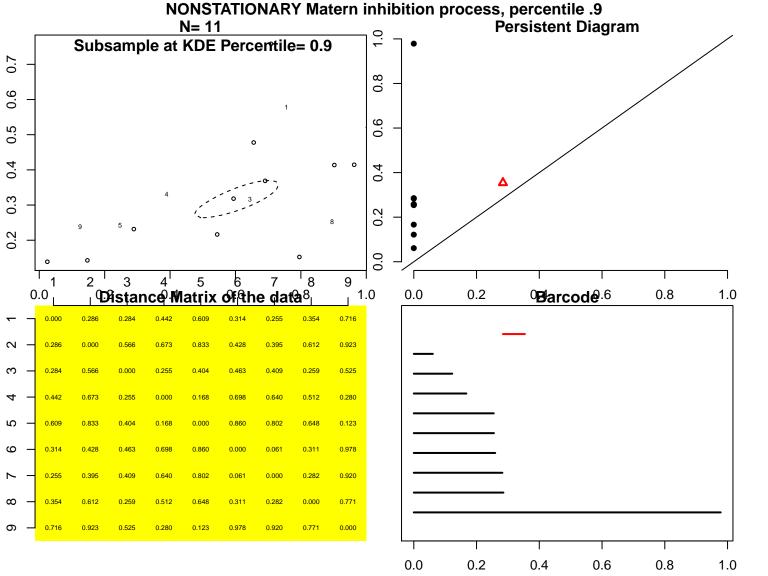
This is the 'Frame' at Euclidean distance = 0.216 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 9.0 9.0 0.5 0.4 0.3 0.2 10 11 0.2 0.0 9 6 10 0.0, Distance Matrix of the data8 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.314 0.354 0.716 0.286 0.000 0.566 0.455 0.673 0.833 0.428 0.390 0.923 0.113 0.255 0.404 0.463 0.409 0.212 0.284 0.566 0.000 0.259 0.525 0.171 0.113 0.000 0.317 0.480 0.381 0.323 0.109 0.261 0.597 0.442 0.317 0.168 0.673 0.255 0.000 0.698 0.424 0.512 0.280 0.168 0.000 0.589 0.648 0.123 0.609 0.833 0.404 0.480 0.860 0.802 0.698 0.860 0.000 0.061 0.314 0.428 0.463 0.381 0.276 0.311 0.978 0.802 0.061 0.000 0.255 0.395 0.409 0.323 0.640 0.216 0.920 0.114 0.390 0.109 0.424 0.589 0.276 0.000 0.259 0.261 0.354 0.612 0.512 0.311 0.240 0.771 0.923 0.525 0.597 0.280 0.123 0.978 0.920 0.705 0.000 0.0 0.2 0.4 0.6 0.8 1.0



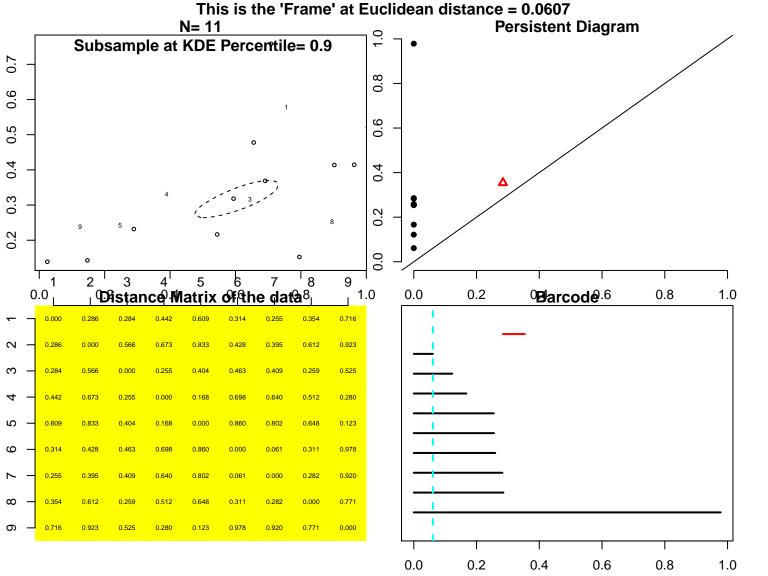
This is the 'Frame' at Euclidean distance = 0.255 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 9.0 9.0 0.5 0.4 0.3 0.2 10 11 0.2 0.0 9 6 10 0.0, Distance Matrix of the data8 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.314 0.354 0.716 0.286 0.000 0.566 0.455 0.673 0.428 0.390 0.923 0.113 0.255 0.404 0.463 0.409 0.212 0.284 0.566 0.000 0.259 0.525 0.171 0.113 0.000 0.317 0.480 0.381 0.323 0.109 0.261 0.597 0.442 0.317 0.673 0.255 0.000 0.698 0.424 0.512 0.280 0.168 0.000 0.589 0.648 0.609 0.833 0.404 0.480 0.860 0.802 0.123 0.698 0.860 0.000 0.061 0.276 0.314 0.428 0.463 0.381 0.311 0.978 0.255 0.802 0.061 0.000 0.216 0.395 0.409 0.323 0.640 0.282 0.920 0.114 0.390 0.109 0.424 0.589 0.276 0.216 0.000 0.240 0.705 0.261 0.282 0.240 0.354 0.612 0.512 0.311 0.000 0.771 0.978 0.923 0.525 0.597 0.280 0.123 0.920 0.705 0.000 0.0 0.2 0.4 0.6 0.8 1.0

This is the 'Frame' at Euclidean distance = 0.286 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 9.0 9.0 0.5 0.4 5 0.3 0.2 10 11 0.2 0.0 6 8 9 10 0.0, Distance Matrix of the data 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.314 0.354 0.716 0.286 0.000 0.566 0.455 0.673 0.833 0.428 0.390 0.612 0.923 0.284 0.566 0.000 0.113 0.255 0.404 0.463 0.409 0.212 0.259 0.525 0.171 0.455 0.113 0.000 0.317 0.480 0.381 0.323 0.109 0.597 0.442 0.255 0.317 0.000 0.168 0.424 0.512 0.673 0.698 0.640 0.280 0.404 0.168 0.000 0.860 0.609 0.833 0.480 0.802 0.589 0.648 0.123 0.698 0.860 0.000 0.061 0.276 0.978 0.314 0.428 0.463 0.381 0.311 0.216 0.255 0.640 0.802 0.061 0.000 0.282 0.920 0.395 0.409 0.323 0.114 0.390 0.109 0.424 0.589 0.276 0.000 0.240 0.705 0.512 0.648 0.311 0.354 0.612 0.000 0.771 0.597 0.978 0.920 0.923 0.525 0.280 0.705 0.000 0.0 0.2 0.4 0.6 0.8 1.0

This is the 'Frame' at Euclidean distance = 0.978 N = 11**Persistent Diagram** Subsample at KDE Percentile= 1 0.7 0.8 9.0 9.0 0.5 0.4 0.3 0.2 0.2 9 10 11 0.0, Distance Matrix of the data 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.000 0.923 0.000 0.113 0.255 0.463 0.212 0.284 0.171 0.000 0.317 0.381 0.109 0.000 0.698 0.168 0.000 0.860 0.589 0.609 0.123 0.698 0.000 0.061 0.276 0.314 0.978 0.061 0.000 0.216 0.920 0.276 0.000 0.240 0.705 0.000 0.771 0.000 0.280 0.978 0.0 0.2 0.4 0.6 0.8 1.0



This is the 'Frame' at Euclidean distance = 0 N= 11 **Persistent Diagram** Subsample at KDE Percentile= 0.9 0.7 0.8 9.0 9.0 0.5 0.4 0 0.4 0.3 0.2 8 0.2 0.0 9 Distance Matrix of the datas, 0.2 0.0 1.0 0.0 ⁰Barcode^{0.6} 8.0 1.0 0.716 0.000 0.286 0.284 0.442 0.609 0.255 0.354 2 0.286 0.000 0.566 0.673 0.833 0.428 0.395 0.612 0.923 0.284 0.566 0.000 0.255 0.404 0.259 0.525 0.409 0.442 0.673 0.255 0.000 0.168 0.698 0.640 0.512 0.280 2 0.609 0.833 0.404 0.168 0.000 0.860 0.802 0.648 0.123 0.314 0.428 0.463 0.698 0.860 0.000 0.061 0.311 0.978 0.255 0.395 0.409 0.640 0.802 0.061 0.000 0.282 0.920 0.354 0.612 0.259 0.648 0.311 0.282 0.000 0.771 0.512 0.716 0.923 0.525 0.280 0.123 0.978 0.920 0.771 0.000 0.0 0.2 0.4 0.6 0.8 1.0

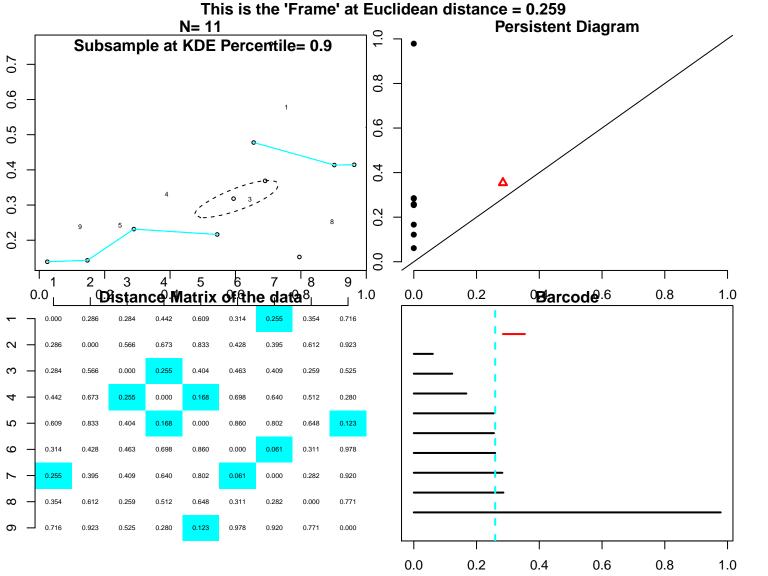


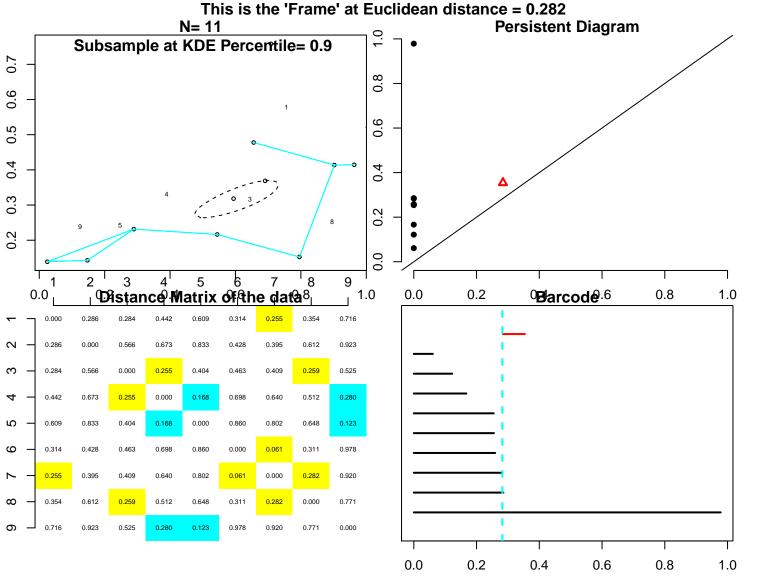
This is the 'Frame' at Euclidean distance = 0.123 N = 11**Persistent Diagram** Subsample at KDE Percentile= 0.9 0.7 0.8 9.0 9.0 0.5 0.4 0.4 0.3 0.2 8 0.2 0.0 5 9 Distance Matrix of the data8 0.2 0.0 1.0 0.0 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.442 0.609 0.314 0.255 0.354 0.716 0.286 0.000 0.833 0.395 0.612 0.923 α 0.566 0.673 0.428 3 0.284 0.566 0.000 0.255 0.404 0.463 0.409 0.259 0.525 0.442 0.673 0.255 0.000 0.168 0.698 0.640 0.512 0.280 2 0.609 0.648 0.833 0.404 0.168 0.000 0.860 0.802 0.123 0.314 0.428 0.463 0.860 0.000 0.061 0.311 0.978 0.698 0.255 0.802 0.061 0.000 0.282 0.395 0.409 0.640 0.920 0.354 0.612 0.259 0.512 0.648 0.311 0.282 0.000 0.771 0.716 0.923 0.525 0.280 0.123 0.978 0.920 0.771 0.000 0.0 0.2 0.4 0.6 0.8 1.0

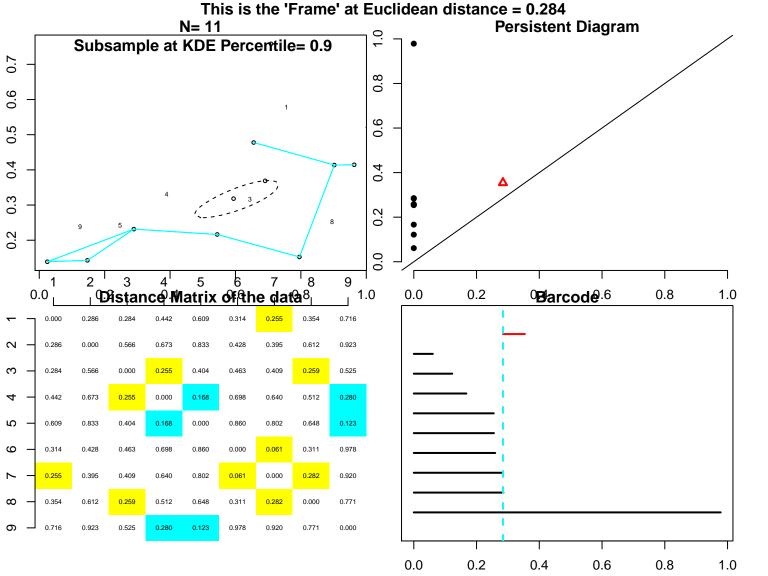
This is the 'Frame' at Euclidean distance = 0.168 N = 11**Persistent Diagram** Subsample at KDE Percentile= 0.9 0.7 0.8 9.0 9.0 0.5 0.4 0.4 0.3 0.2 8 0.2 0.0 5 9 Distance Matrix of the datas, 0.0 1.0 0.0 0.2 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.442 0.609 0.314 0.255 0.354 0.716 0.286 0.000 0.833 0.395 0.612 0.923 α 0.566 0.673 0.428 3 0.284 0.566 0.000 0.255 0.404 0.463 0.409 0.259 0.525 0.442 0.673 0.255 0.000 0.168 0.698 0.640 0.512 0.280 2 0.609 0.648 0.123 0.833 0.404 0.168 0.000 0.860 0.802 0.314 0.428 0.463 0.860 0.000 0.061 0.311 0.978 0.698 0.255 0.802 0.061 0.000 0.282 0.920 0.395 0.409 0.640 0.354 0.612 0.259 0.512 0.648 0.311 0.282 0.000 0.771 0.716 0.923 0.525 0.280 0.123 0.978 0.920 0.771 0.000 0.0 0.2 0.4 0.6 0.8 1.0

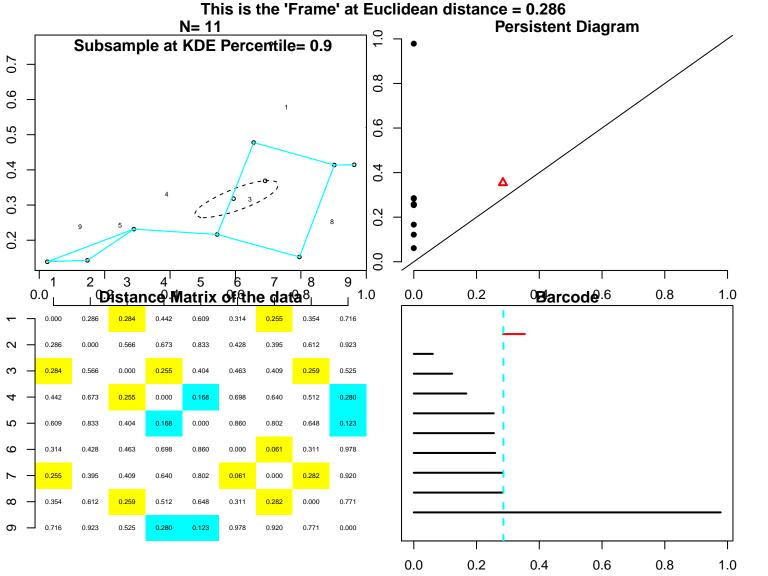
This is the 'Frame' at Euclidean distance = 0.255 N = 11**Persistent Diagram** Subsample at KDE Percentile= 0.9 0.7 0.8 9.0 9.0 0.5 0.4 0.4 0.3 0.2 8 0.2 0.0 5 9 Distance Matrix of the data8 0.2 0.0 1.0 0.0 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.442 0.609 0.314 0.255 0.354 0.716 0.286 0.000 0.395 0.612 0.923 α 0.566 0.673 0.833 0.428 3 0.284 0.566 0.000 0.255 0.404 0.463 0.409 0.259 0.525 0.442 0.673 0.255 0.000 0.168 0.698 0.640 0.512 0.280 0.609 0.404 0.168 0.000 0.648 0.123 2 0.833 0.860 0.802 0.314 0.428 0.463 0.698 0.860 0.000 0.061 0.311 0.978 0.255 0.802 0.061 0.000 0.282 0.395 0.409 0.640 0.920 0.354 0.612 0.259 0.512 0.648 0.311 0.282 0.000 0.771 0.716 0.923 0.525 0.280 0.123 0.978 0.920 0.771 0.000 0.0 0.2 0.4 0.6 0.8 1.0

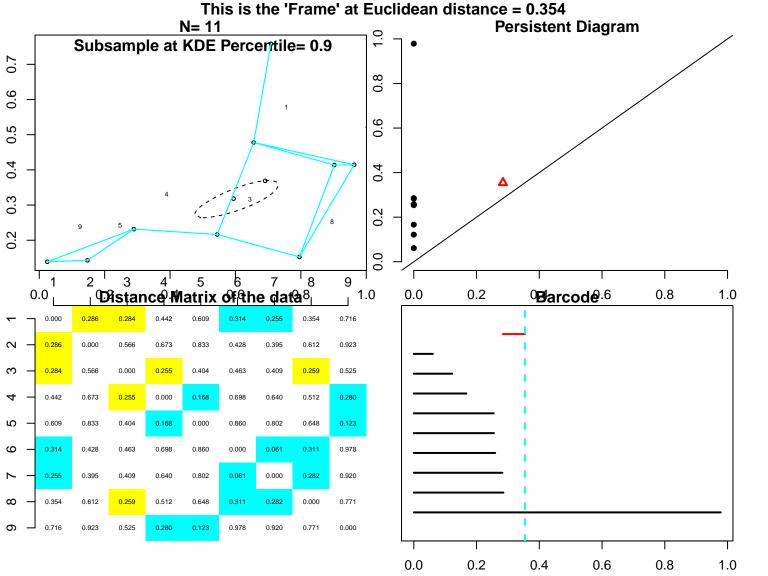
This is the 'Frame' at Euclidean distance = 0.255 N = 11**Persistent Diagram** Subsample at KDE Percentile= 0.9 0.7 0.8 9.0 9.0 0.5 0.4 0.4 0.3 0.2 8 0.2 0.0 5 9 Distance Matrix of the datas, 0.2 0.0 1.0 0.0 ⁰Barcode^{0.6} 8.0 1.0 0.000 0.286 0.284 0.442 0.609 0.314 0.255 0.354 0.716 0.286 0.000 0.395 0.612 0.923 α 0.566 0.673 0.833 0.428 3 0.284 0.566 0.000 0.255 0.404 0.463 0.409 0.259 0.525 0.442 0.673 0.255 0.000 0.168 0.698 0.640 0.512 0.280 0.609 0.404 0.168 0.000 0.648 0.123 2 0.833 0.860 0.802 0.314 0.428 0.463 0.698 0.860 0.000 0.061 0.311 0.978 0.255 0.395 0.802 0.061 0.000 0.282 0.409 0.640 0.920 0.354 0.612 0.259 0.512 0.648 0.311 0.282 0.000 0.771 0.716 0.923 0.525 0.280 0.123 0.978 0.920 0.771 0.000 0.0 0.2 0.4 0.6 0.8 1.0

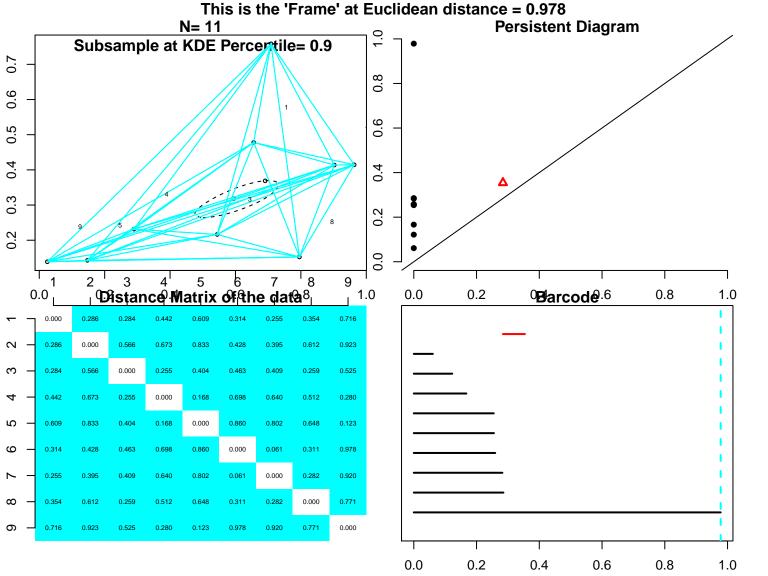


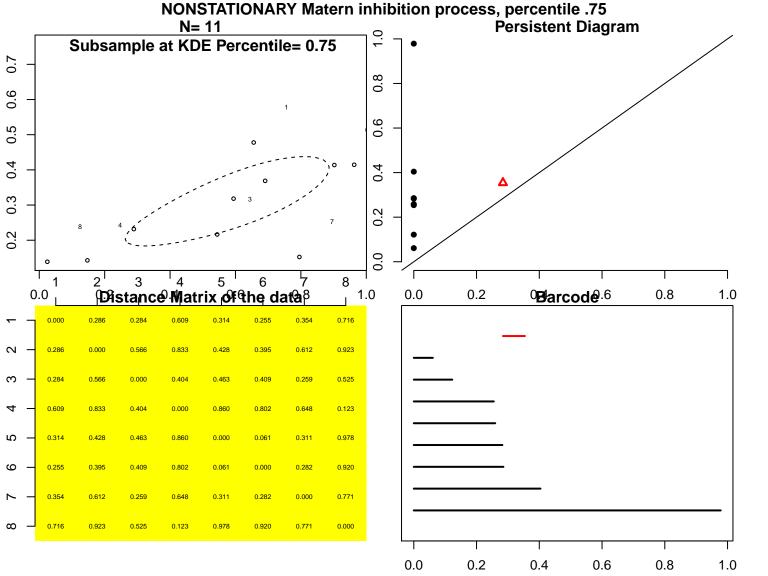


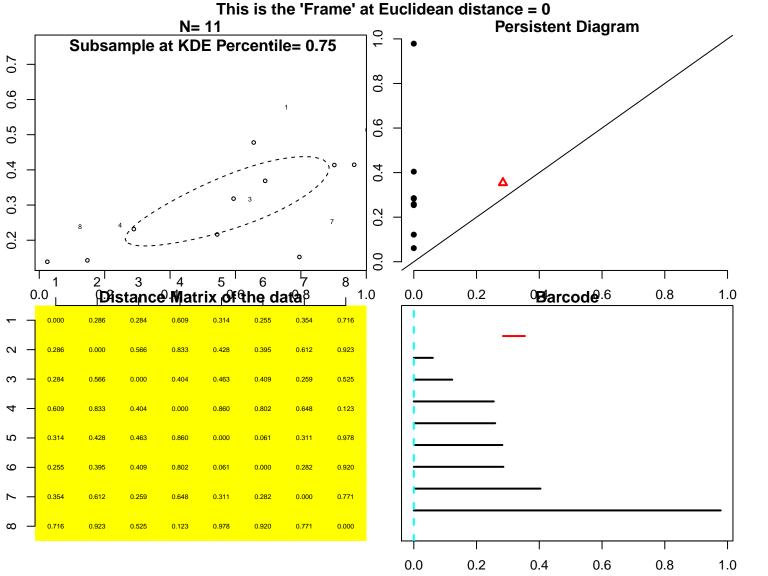


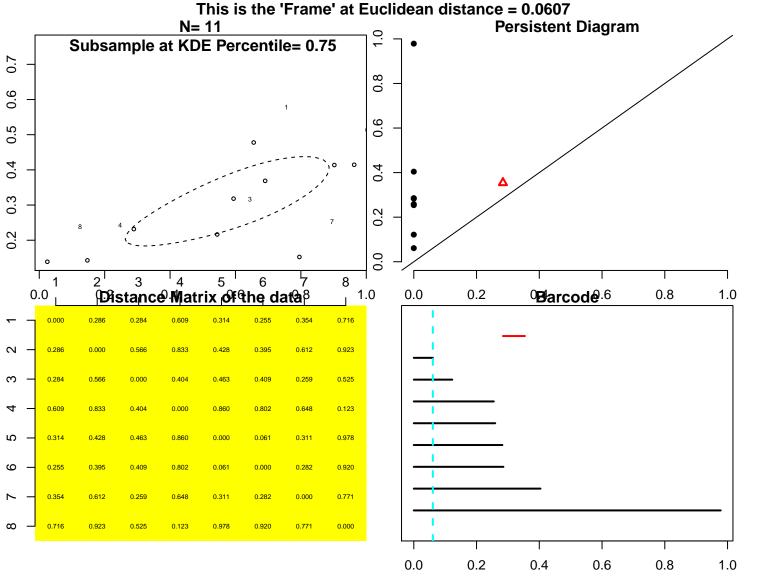


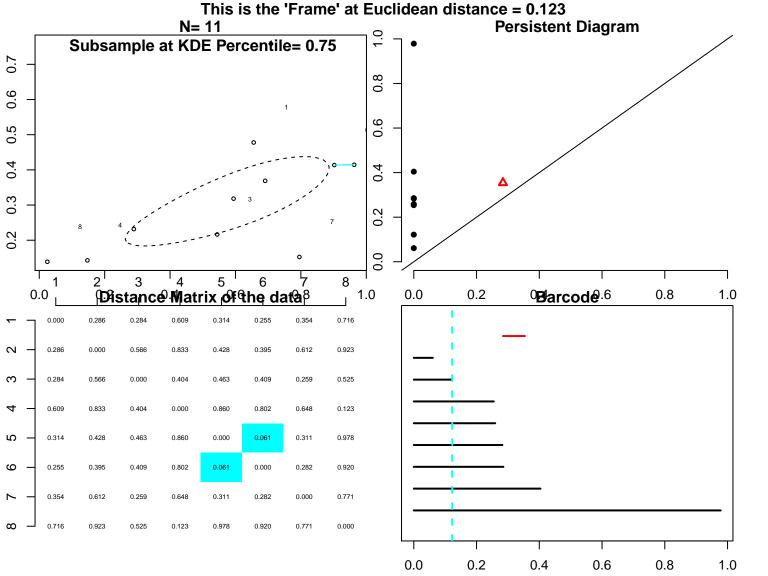




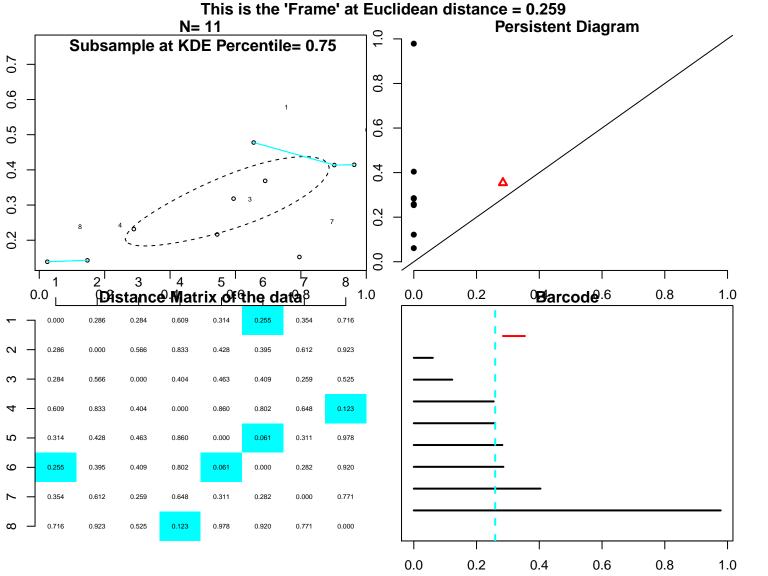


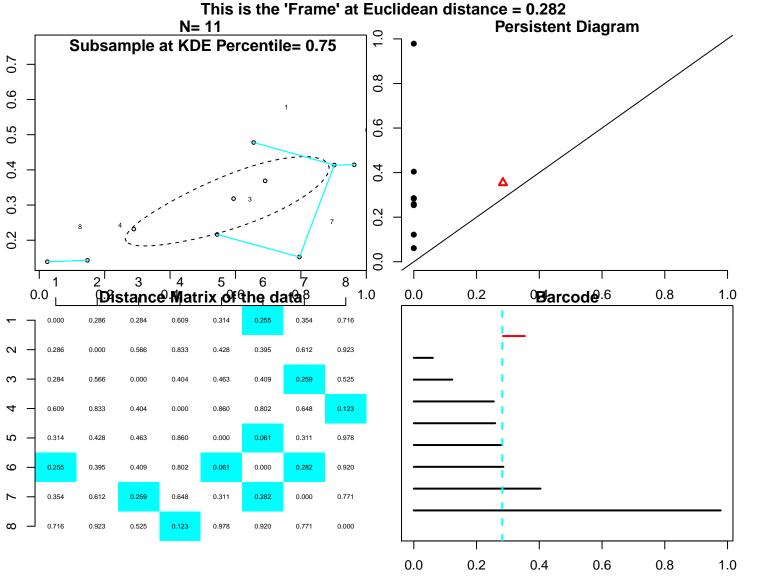


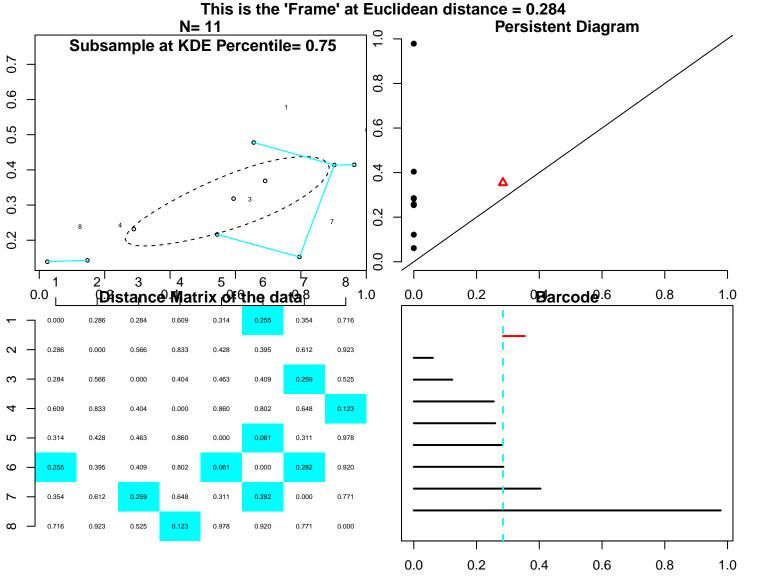


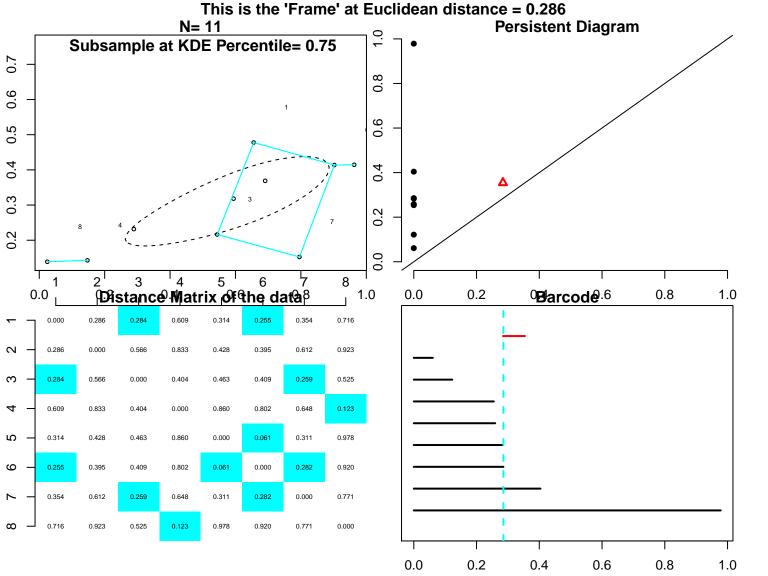


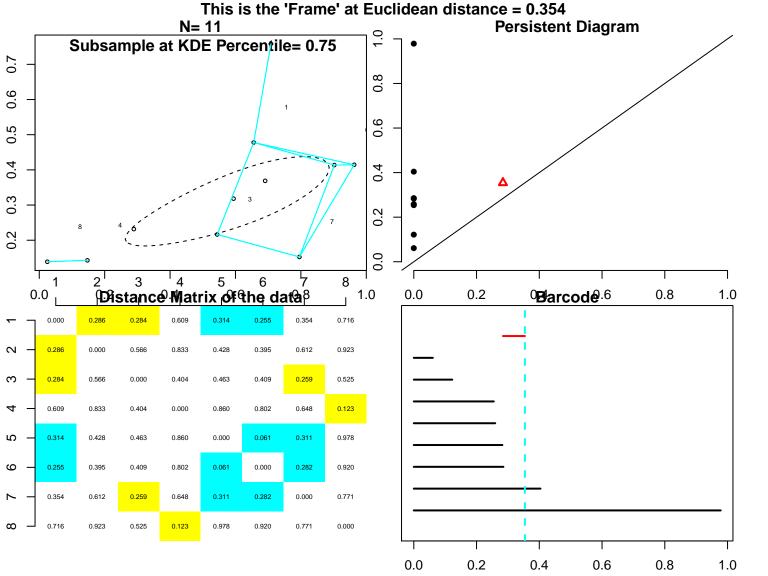
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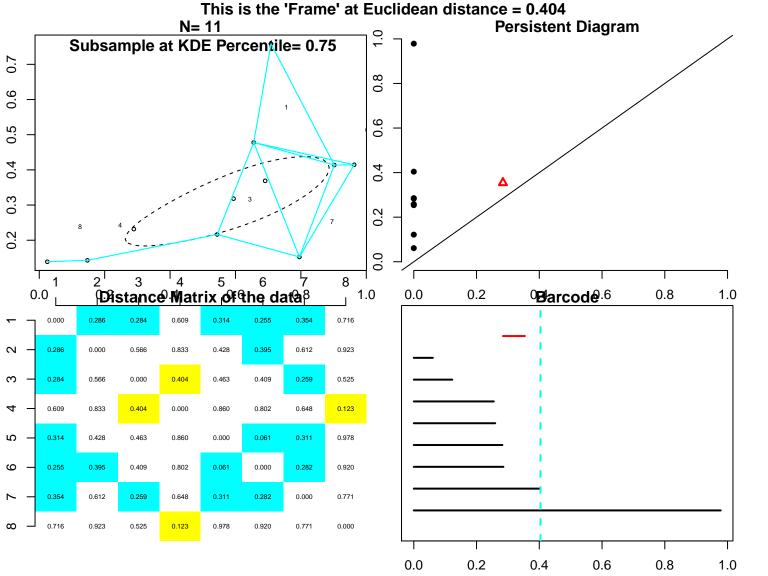


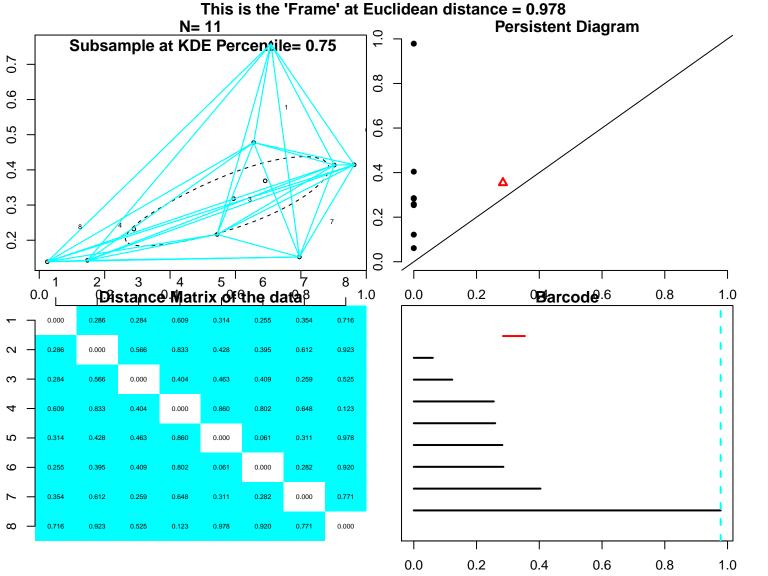


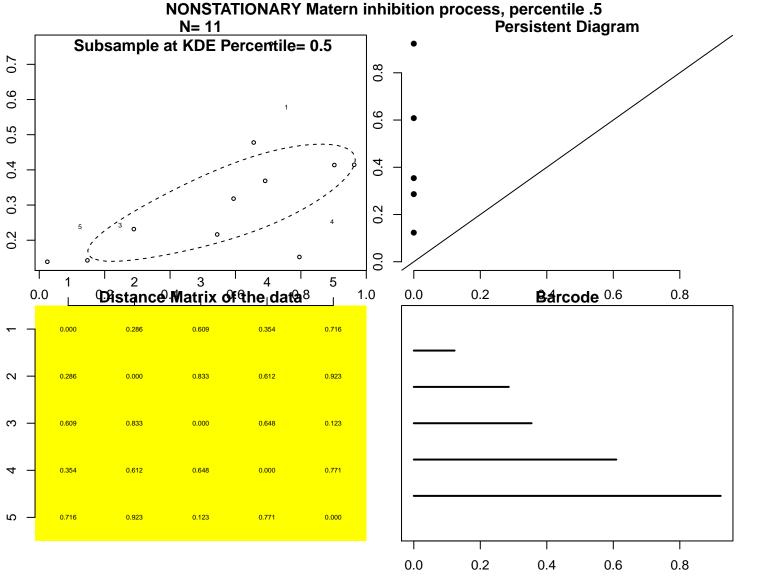


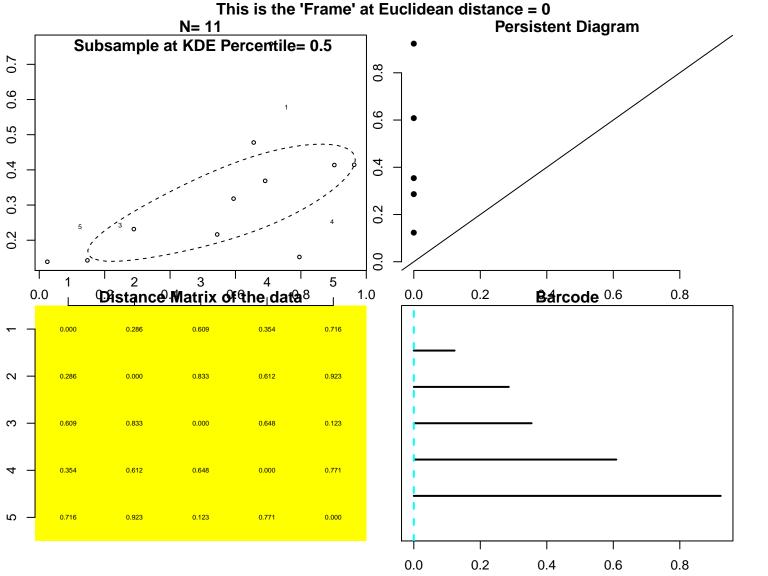


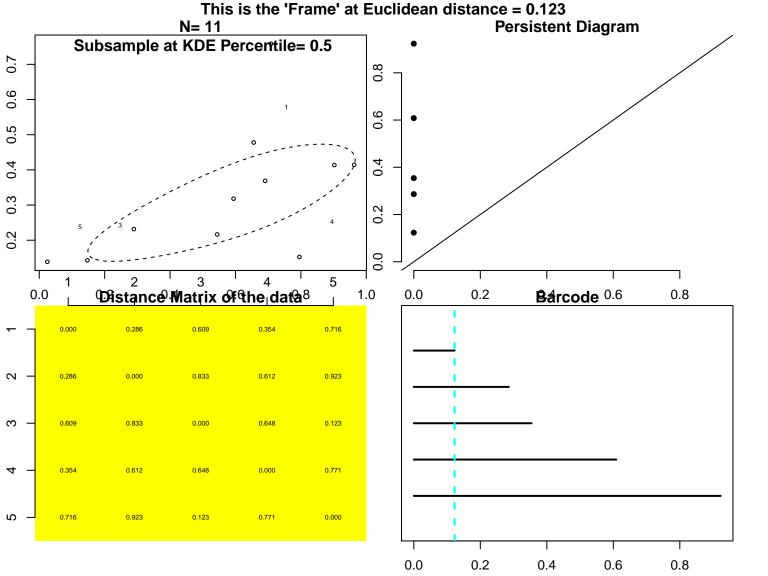


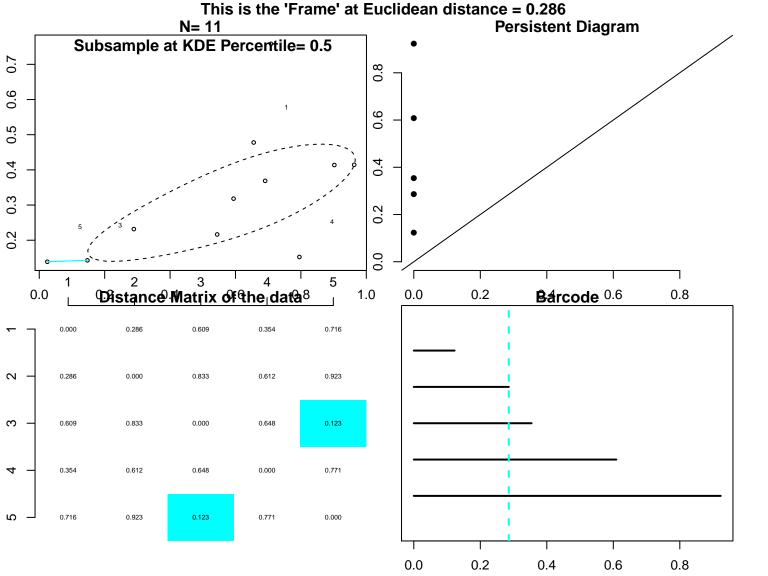


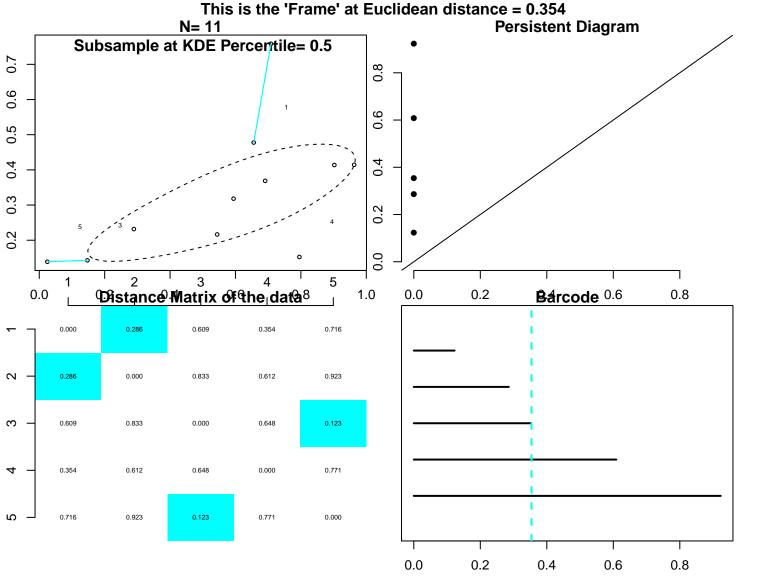


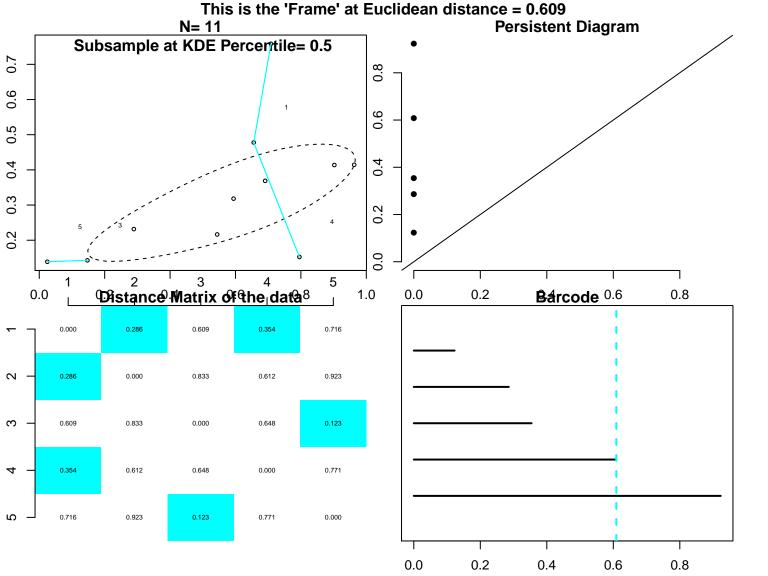


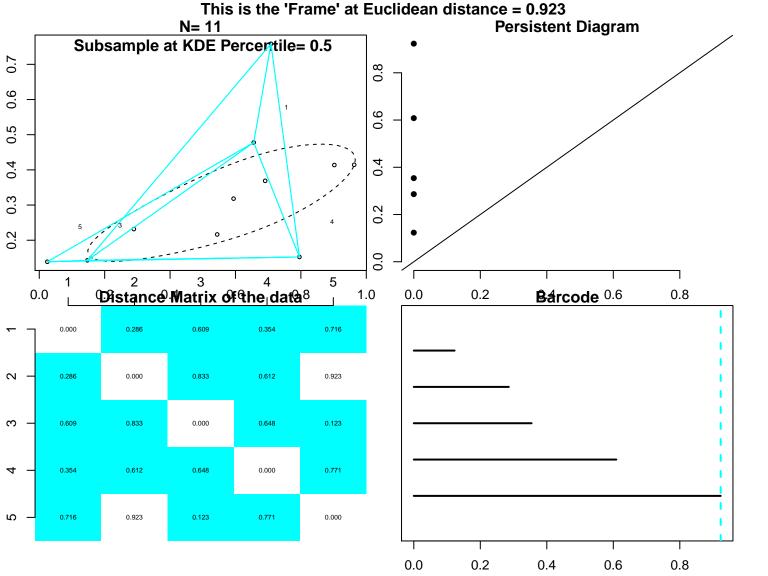


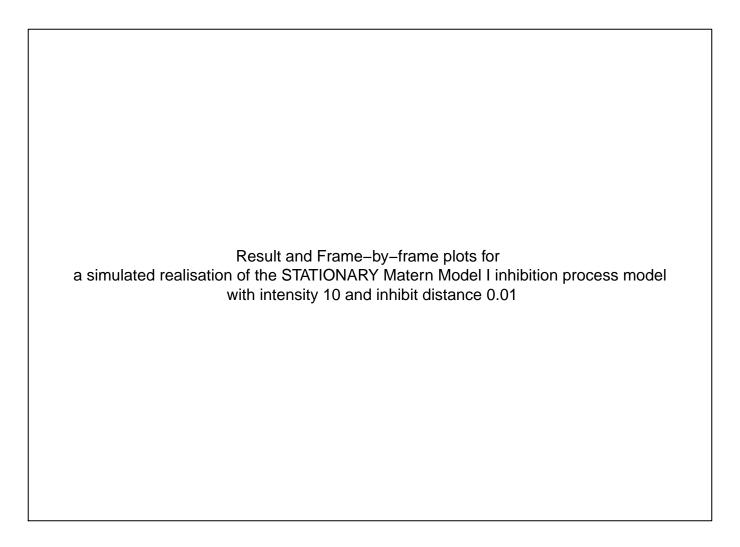


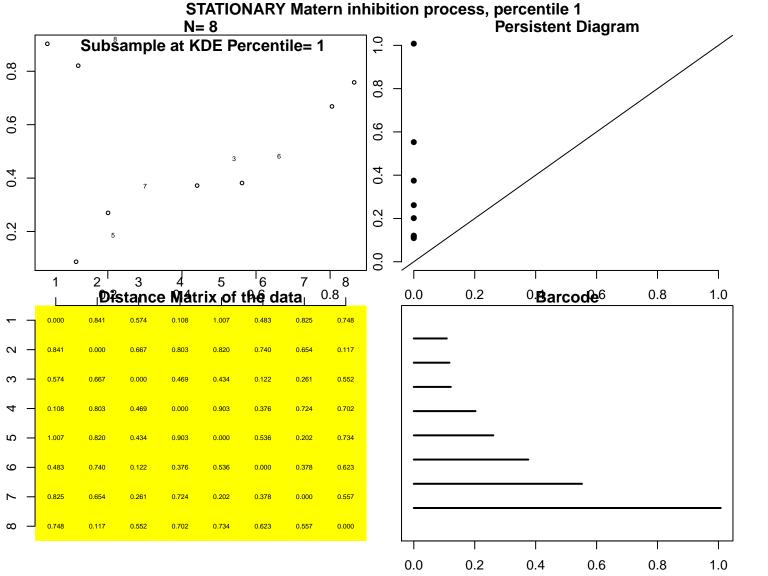






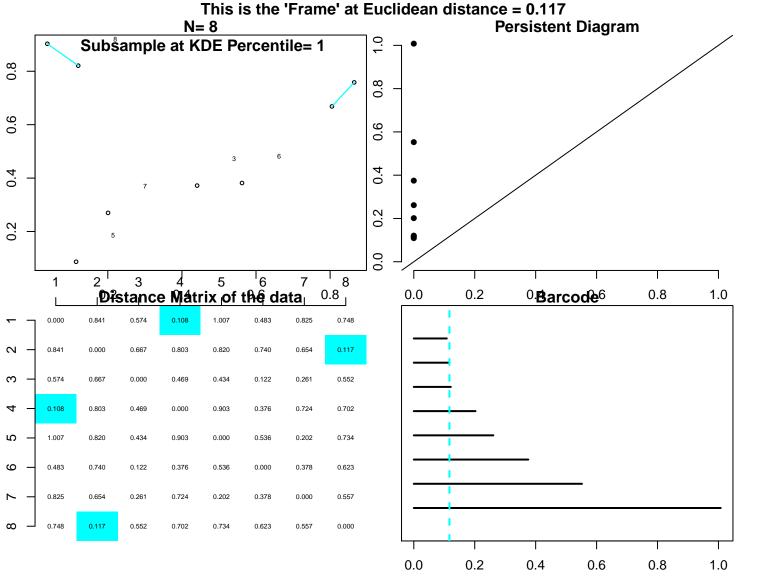




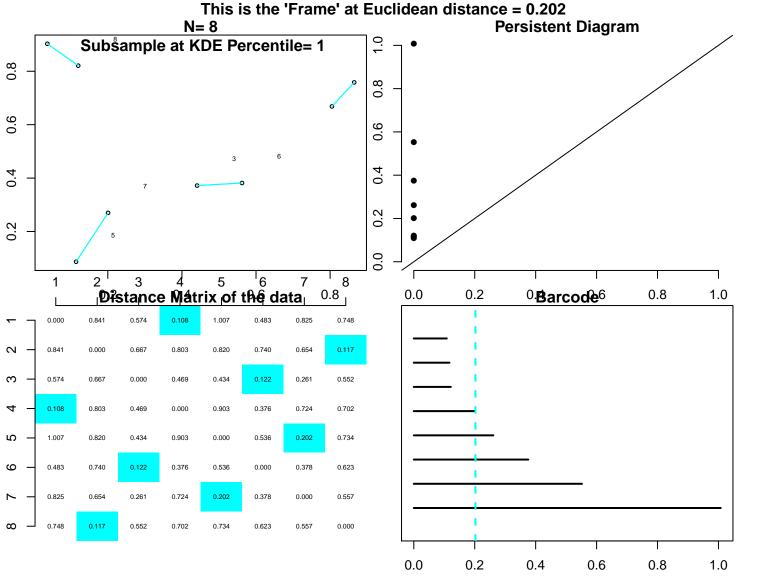


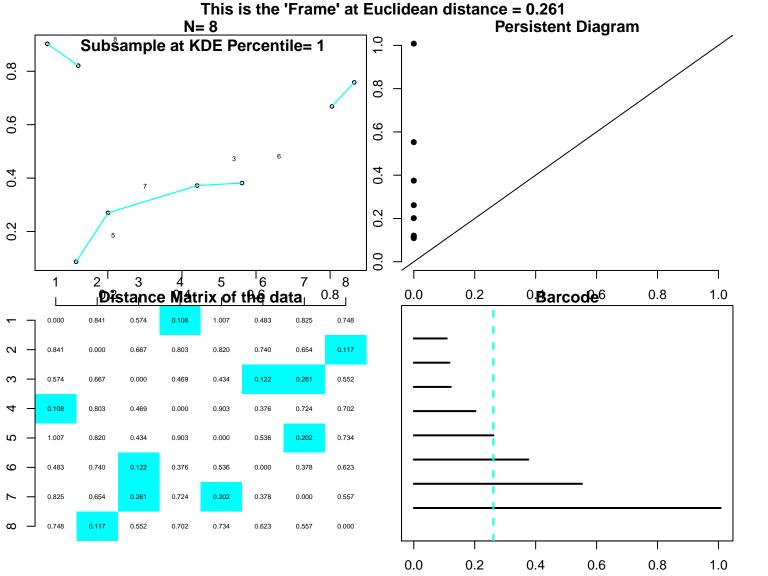
This is the 'Frame' at Euclidean distance = 0 N=8**Persistent Diagram** Subsample at KDE Percentile= 1 0.8 0 0.8 9.0 9.0 3 0.4 0.4 0.2 0.2 0.0 3 6 5 0.2 ⁰Barcode⁶ Distance Matrix of the data 0.8 0.0 8.0 1.0 0.574 0.108 1.007 0.483 0.825 0.748 0.000 0.841 0.841 0.000 0.667 0.803 0.820 0.740 0.654 0.117 0.574 0.667 0.000 0.469 0.434 0.122 0.261 0.552 0.108 0.803 0.469 0.000 0.903 0.376 0.724 0.702 2 1.007 0.820 0.434 0.903 0.000 0.536 0.202 0.734 0.483 0.740 0.122 0.376 0.536 0.000 0.378 0.623 0.825 0.654 0.261 0.724 0.202 0.378 0.000 0.557 0.748 0.117 0.552 0.702 0.734 0.623 0.557 0.000 0.0 0.2 0.4 0.6 0.8 1.0

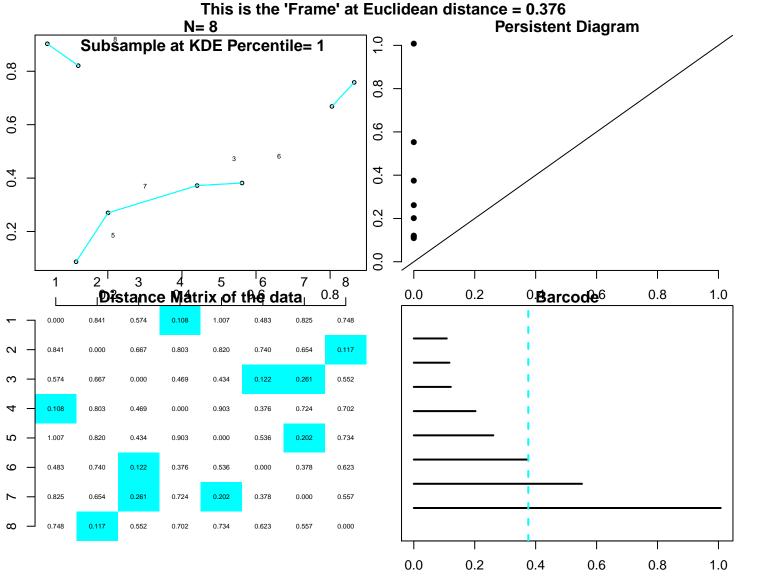
This is the 'Frame' at Euclidean distance = 0.108 N=8**Persistent Diagram** Subsample at KDE Percentile= 1 0.8 0.8 9.0 9.0 3 0.4 0.4 0.2 0.2 0.0 6 5 Distance Matrix of the data 0.2 0.8 0.0 ⁰Barcode⁶ 8.0 1.0 0.108 0.000 0.841 0.574 1.007 0.483 0.825 0.748 0.841 0.000 0.667 0.803 0.820 0.740 0.654 0.117 0.574 0.667 0.000 0.469 0.434 0.122 0.261 0.552 0.108 0.803 0.469 0.000 0.903 0.376 0.724 0.702 1.007 0.820 0.434 0.903 0.000 0.536 0.202 0.734 0.483 0.740 0.122 0.376 0.536 0.000 0.378 0.623 0.825 0.724 0.202 0.378 0.000 0.557 0.654 0.261 0.748 0.117 0.552 0.702 0.734 0.623 0.557 0.000 0.0 0.2 0.4 0.6 0.8 1.0

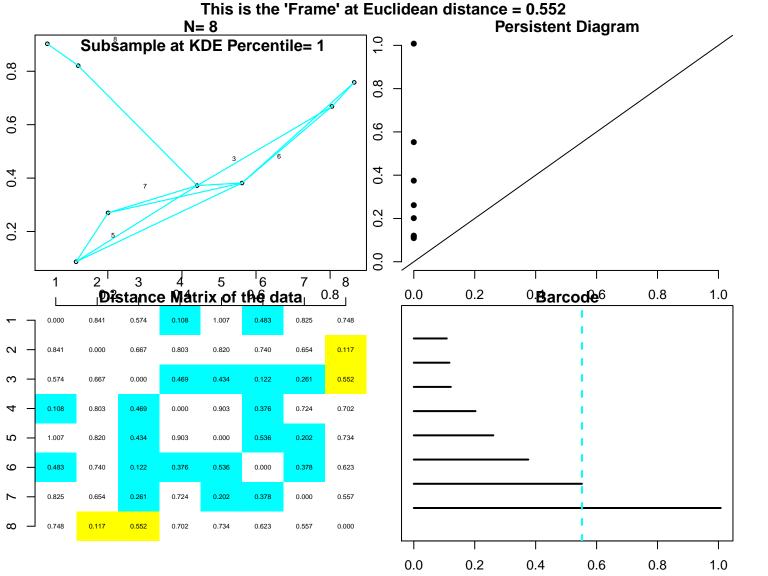


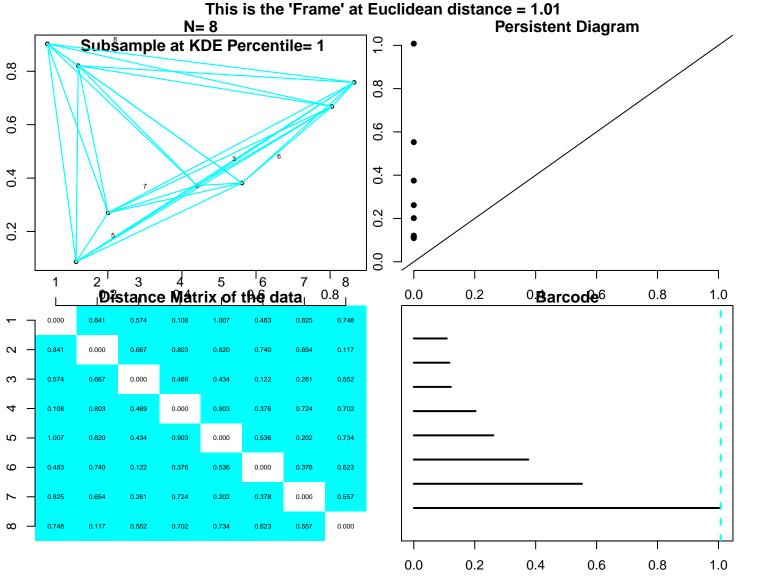
This is the 'Frame' at Euclidean distance = 0.122 N=8**Persistent Diagram** Subsample at KDE Percentile= 1 0.8 0.8 9.0 9.0 3 0.4 0.4 0.2 0.2 0.0 6 5 Distance Matrix of the data 0.2 0.8 0.0 ⁰Barcode⁶ 8.0 1.0 0.108 0.000 0.841 0.574 1.007 0.483 0.825 0.748 0.117 0.841 0.000 0.667 0.803 0.820 0.740 0.654 0.574 0.667 0.000 0.469 0.434 0.122 0.261 0.552 0.108 0.803 0.469 0.000 0.903 0.376 0.724 0.702 1.007 0.820 0.434 0.903 0.000 0.536 0.202 0.734 0.483 0.740 0.122 0.376 0.536 0.000 0.378 0.623 0.825 0.261 0.724 0.202 0.378 0.000 0.654 0.557 0.748 0.552 0.702 0.734 0.623 0.557 0.000 0.0 0.2 0.4 0.6 0.8 1.0

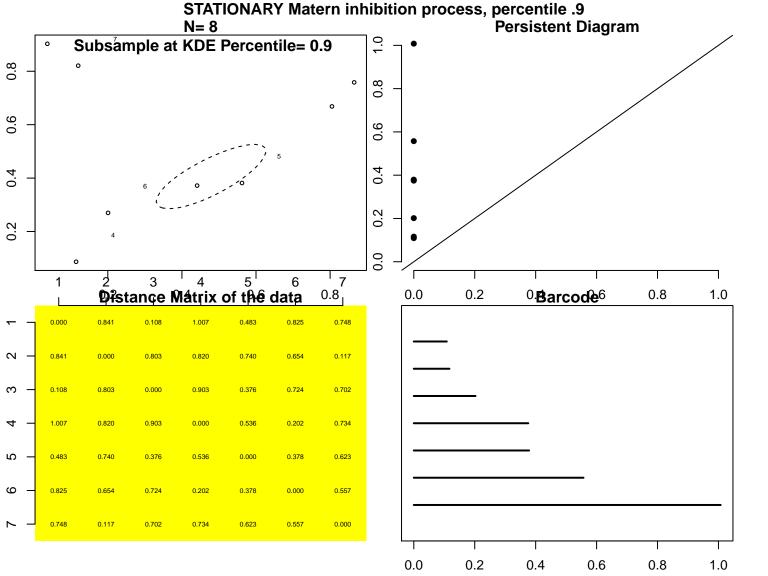


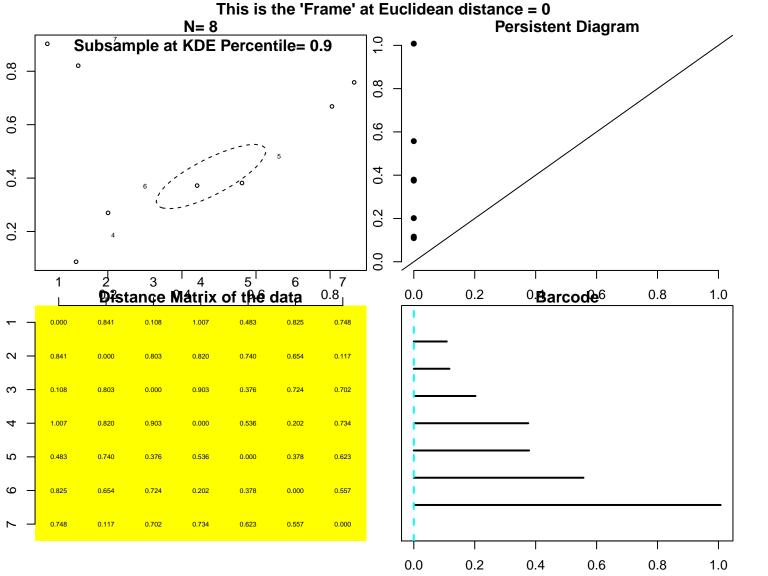


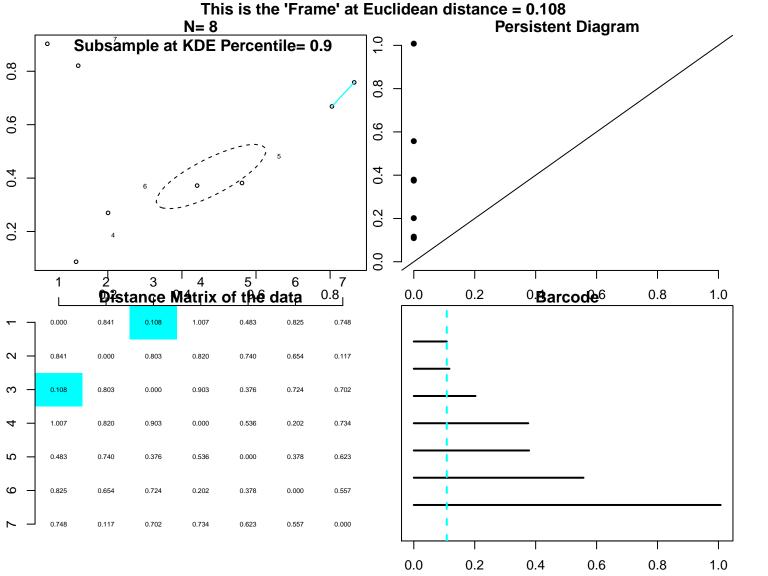


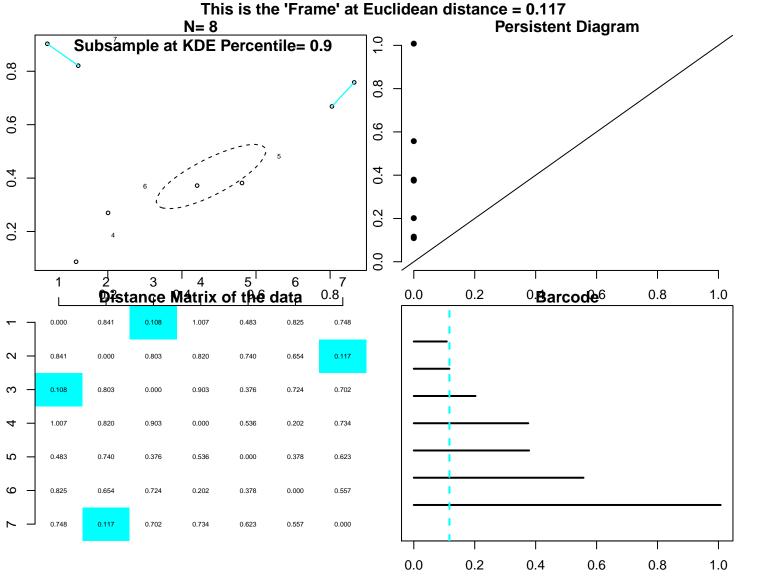


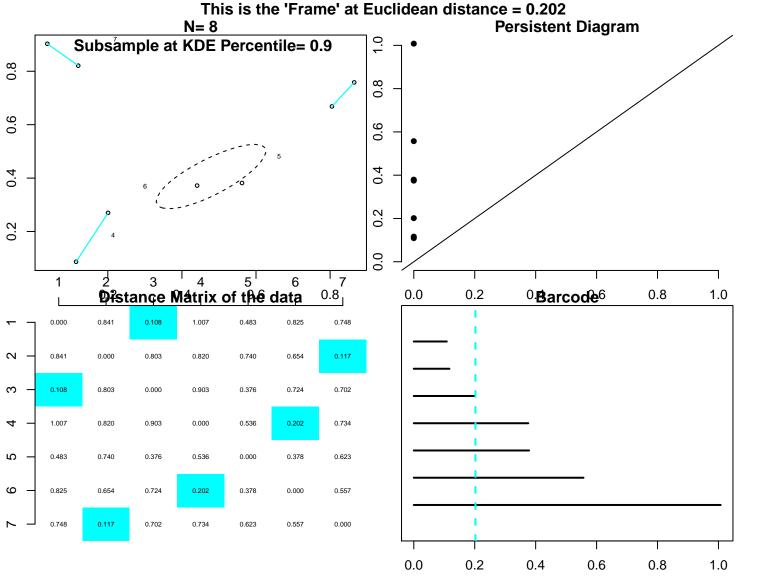


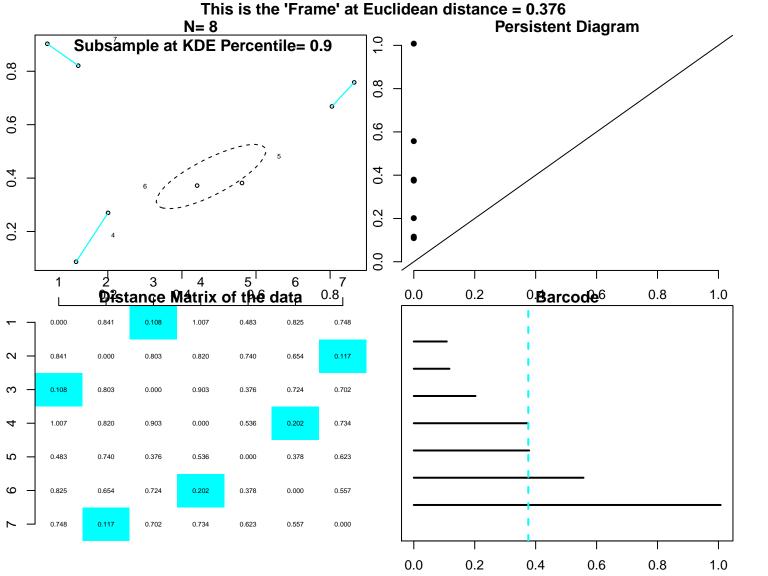


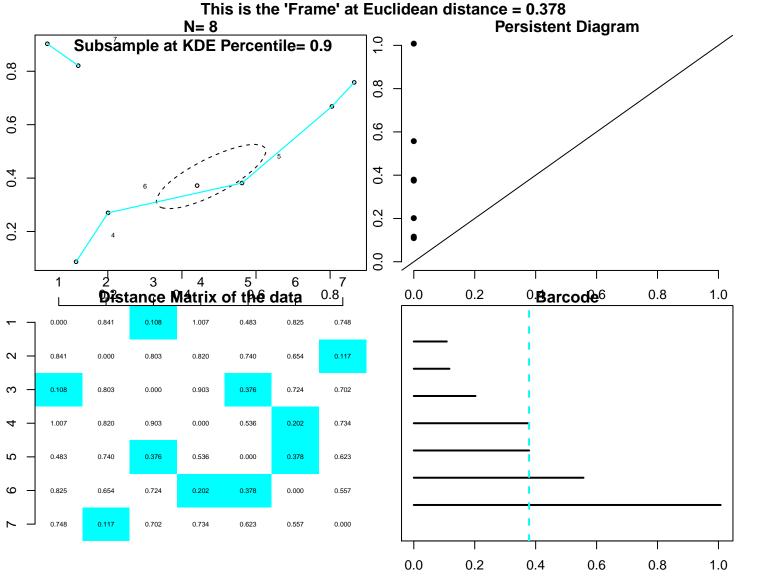


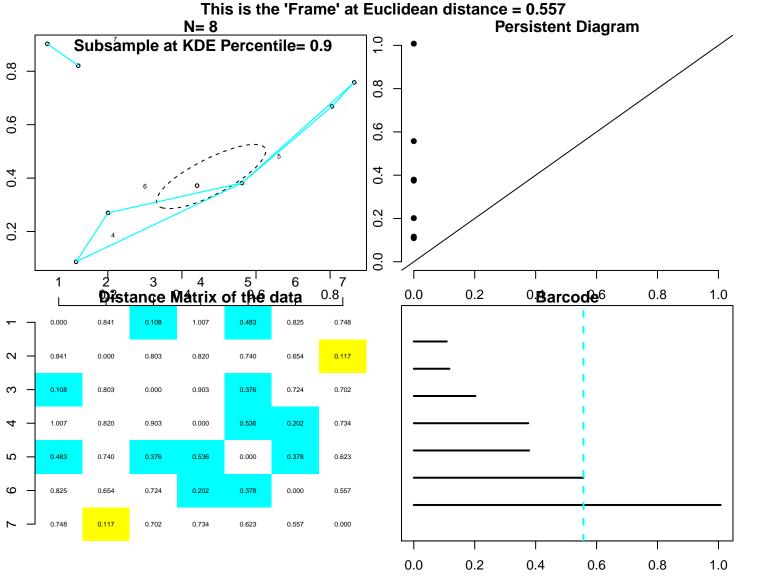


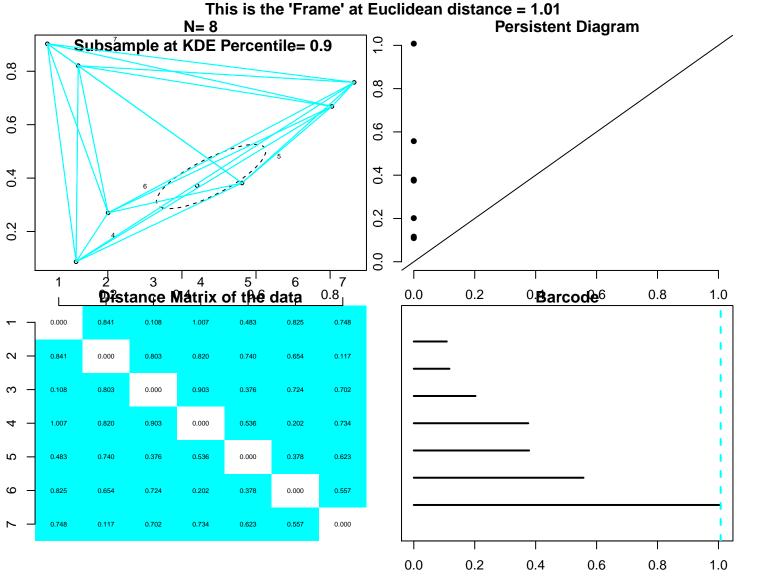


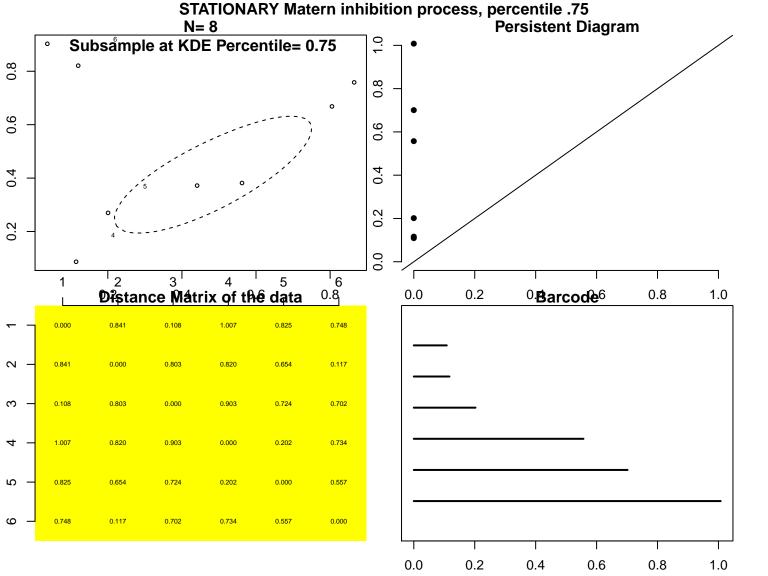


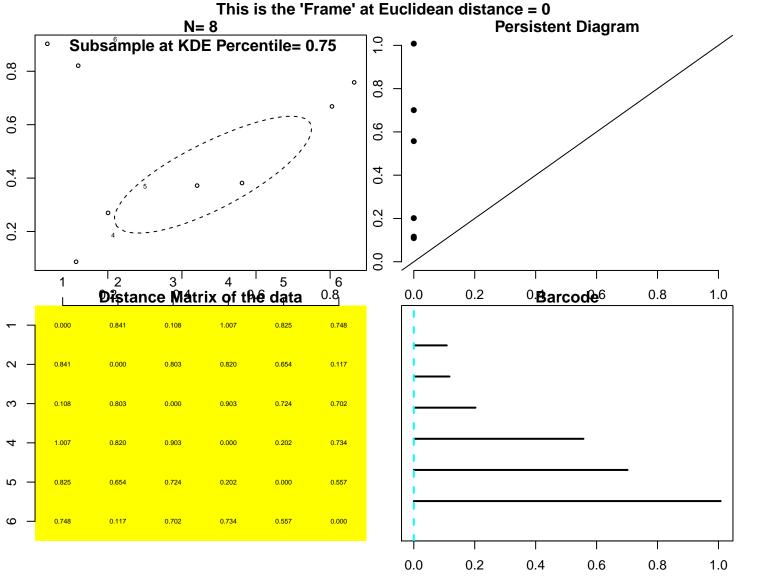


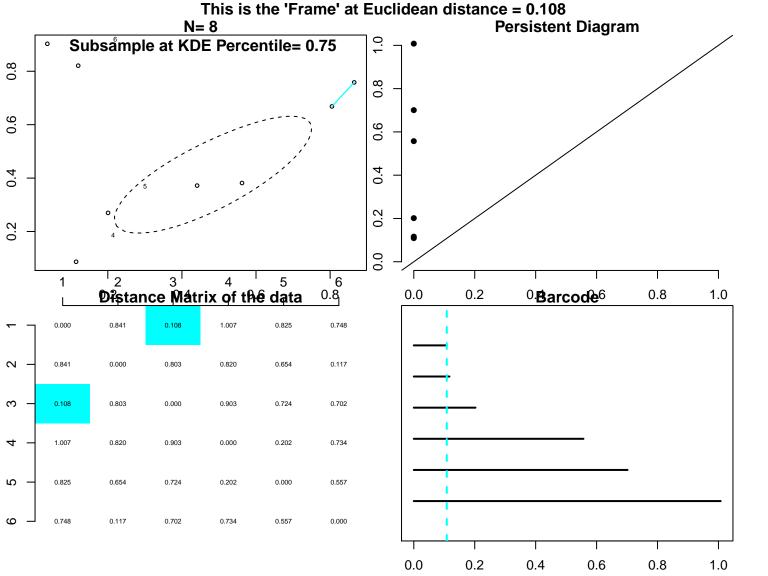


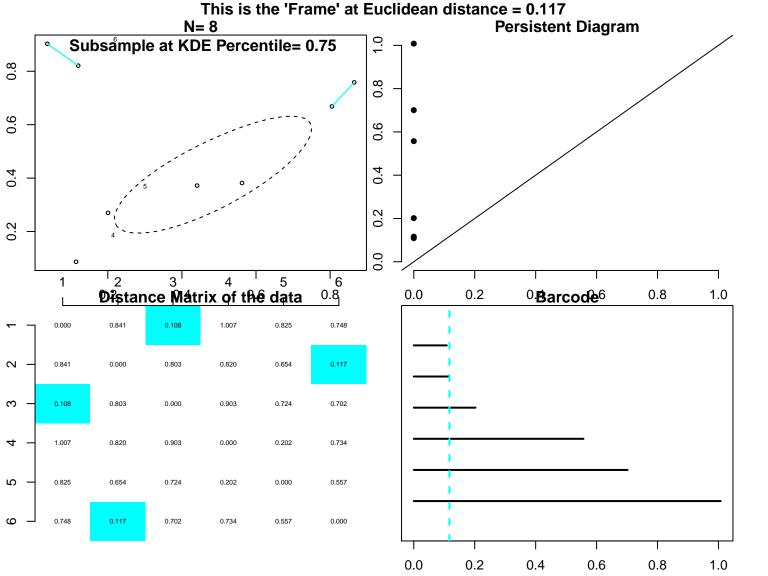


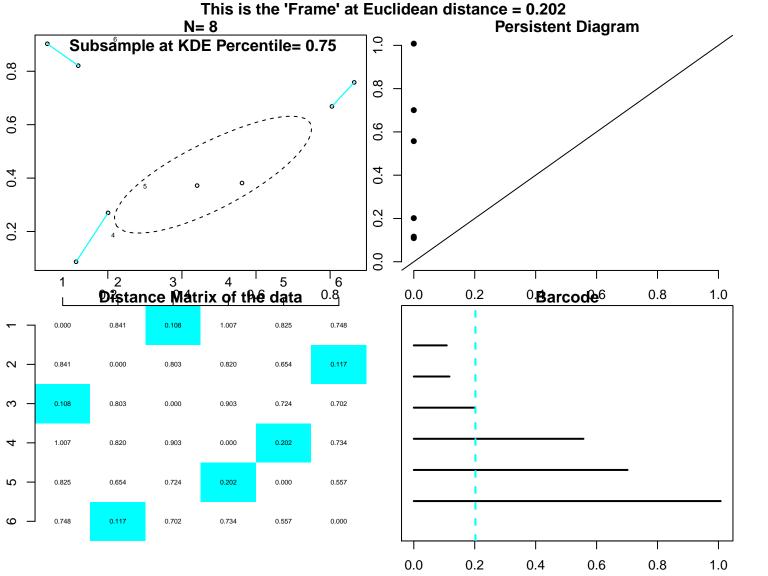


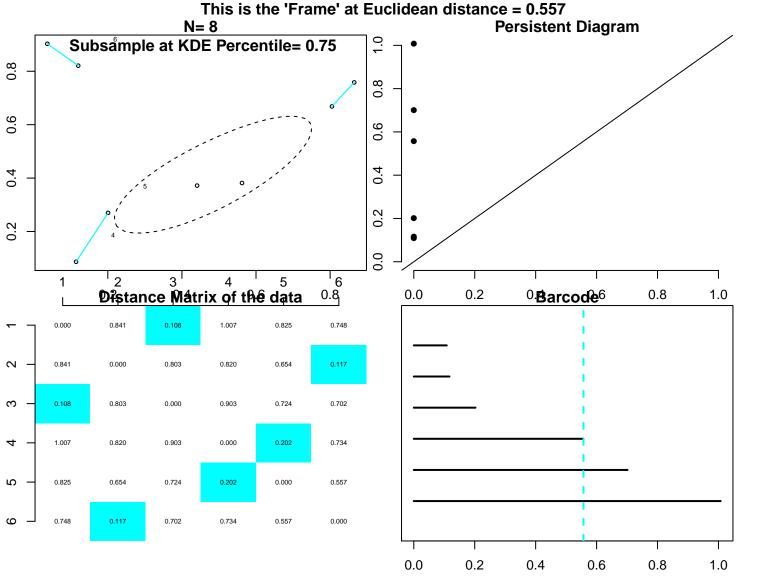


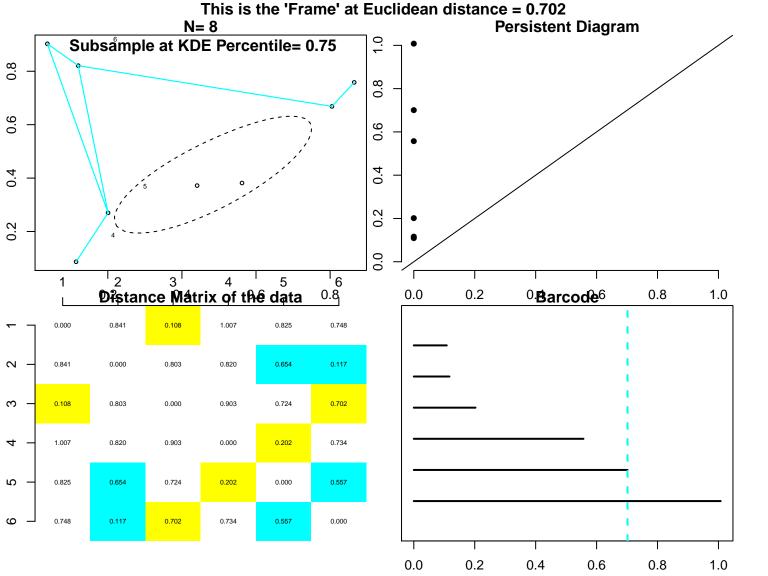


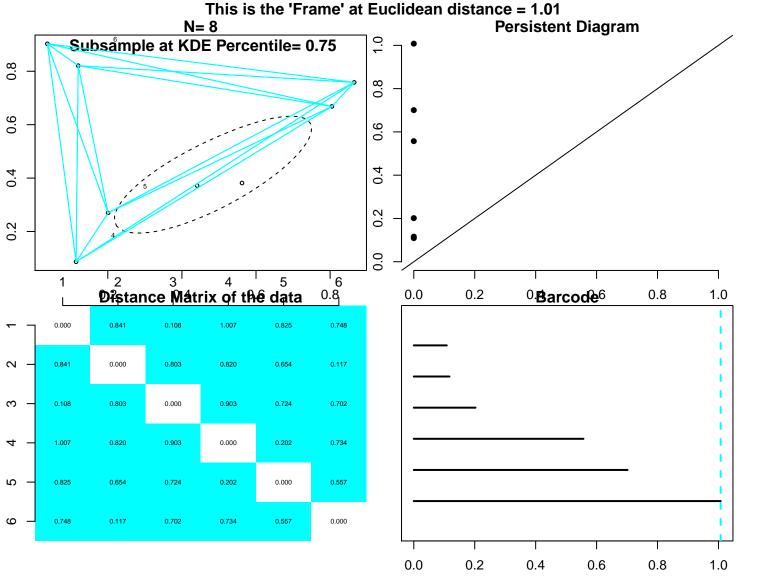


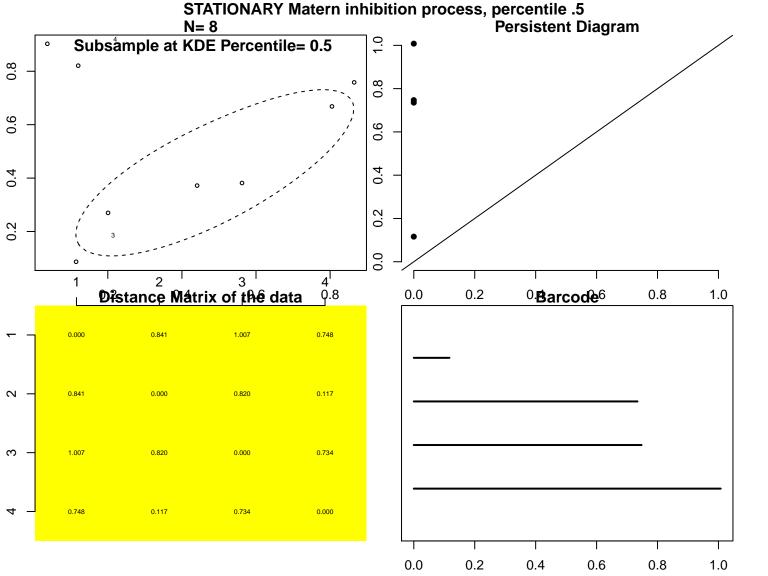


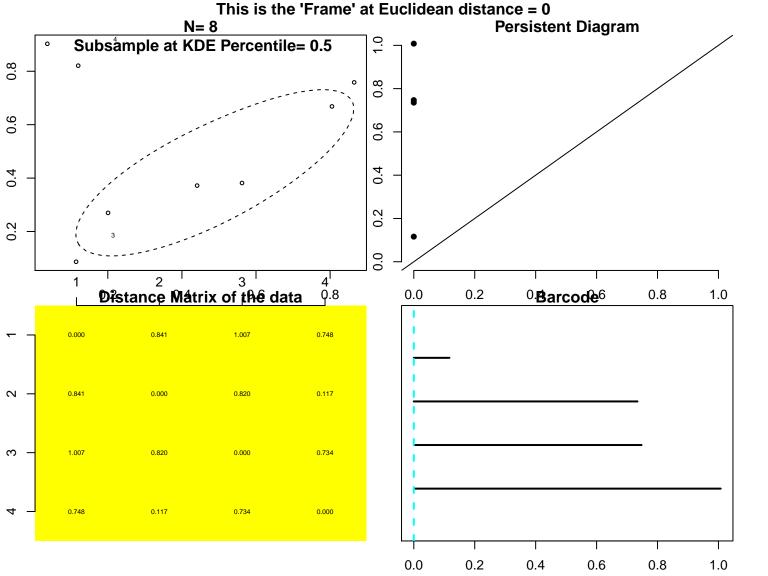


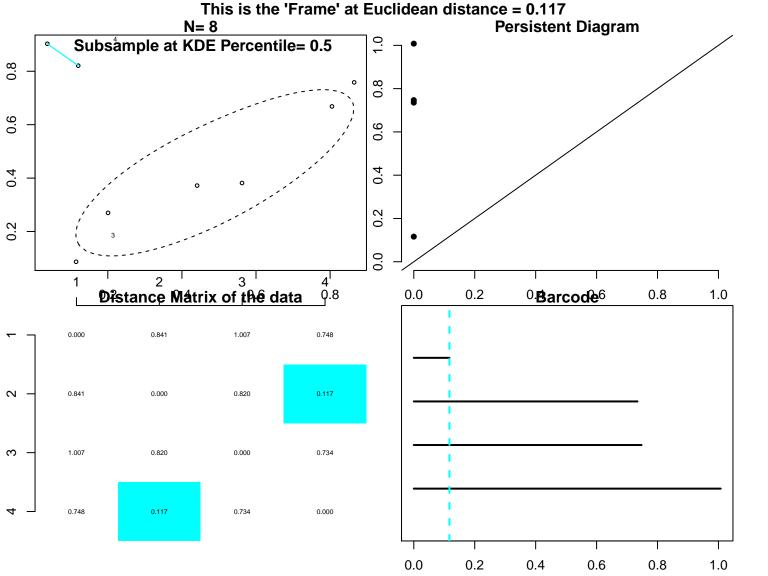


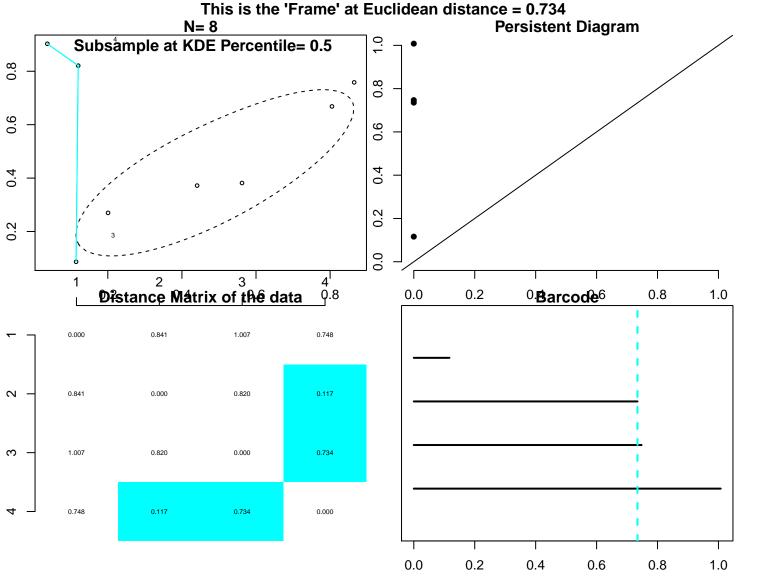


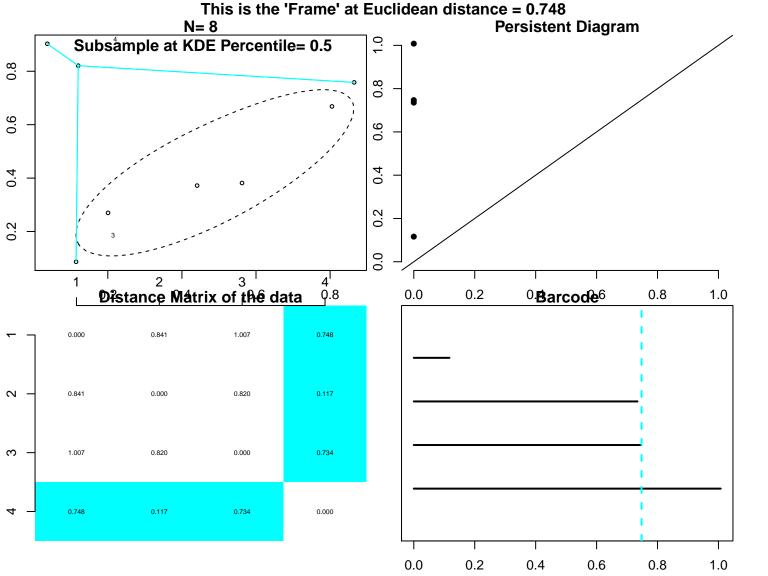


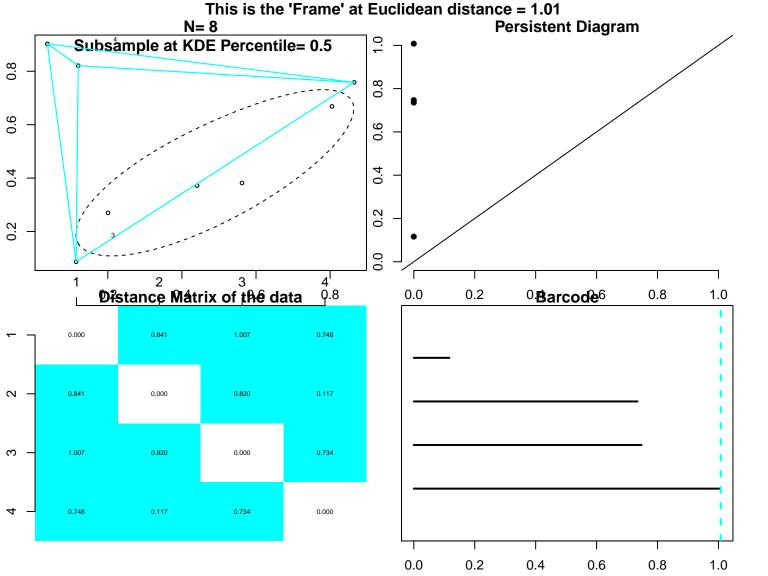


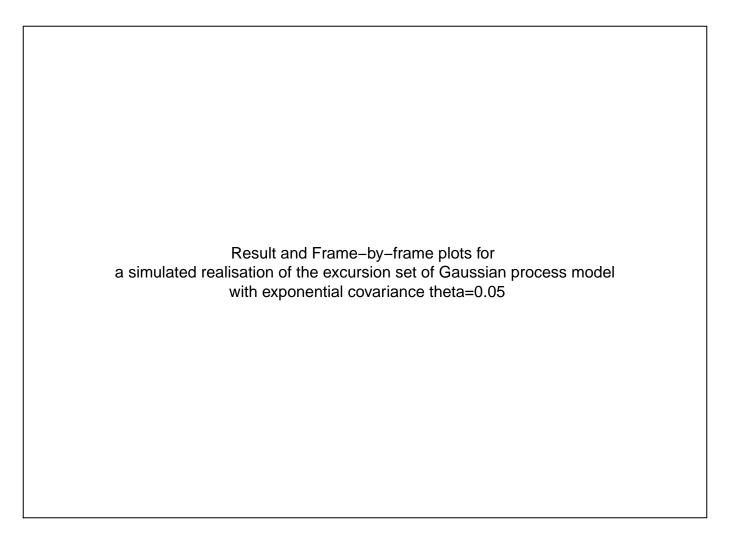












excursion set of Gaussian process, percentile 1 **Persistent Diagram** Subsample at KDE Percentile= 1 $^{\circ}$ ω^{12} Distance Matrix of the data ² Barcôde

This is the 'Frame' at Euclidean distance = 0 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 3 $^{\circ}$ ω^{12} 10 Distance Matrix of the data ² Barcôde

This is the 'Frame' at Euclidean distance = 0.0505 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 က 3 $^{\circ}$ $^{\circ}$ ω^{12} 10 Distance Matrix of the data ² Barcôde 5.311 0.051 0.000 0.051 5.226 5.268 0.101 0.051 0.000 0.071 0.272 0.101 0.385 0.071 0.000 5.675 5.718 0.364 0.407 0.452 0.385 5.541 5.578 0.995 0.975 0.958 0.929 0.910 0.981

This is the 'Frame' at Euclidean distance = 0.0505 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 က 3 $^{\circ}$ $^{\circ}$ ω^{12} 10 12 Distance Matrix of the data ² Barcôde 5.311 5.354 0.000 5.226 5.268 0.101 0.000 0.071 0.071 0.000 0.208 0.071 $5.541 \quad 5.578 \quad 0.995 \quad 0.975 \quad 0.958 \quad 0.929 \quad 0.910 \quad 0.981$

This is the 'Frame' at Euclidean distance = 0.0505 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 က 3 $^{\circ}$ $^{\circ}$ ω^{12} 10 12 Distance Matrix of the data ² Barcôde 5.311 5.354 0.000 5.226 5.268 0.101 0.000 0.071 0.071 0.000 0.208 0.071 $5.541 \quad 5.578 \quad 0.995 \quad 0.975 \quad 0.958 \quad 0.929 \quad 0.910 \quad 0.981$

This is the 'Frame' at Euclidean distance = 0.0505 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 က 3 $^{\circ}$ $^{\circ}$ ω^{12} 10 12 Distance Matrix of the data ² Barcôde 5.311 5.354 0.000 5.226 5.268 0.101 0.000 0.071 0.071 0.000 0.208 0.071 5.675 5.718 0.364 0.407 0.452 0.385 $5.541 \quad 5.578 \quad 0.995 \quad 0.975 \quad 0.958 \quad 0.929 \quad 0.910 \quad 0.981$

This is the 'Frame' at Euclidean distance = 0.0505 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 က 3 $^{\circ}$ 7 **c**¹² 9 10 Distance Matrix of the data ² Barcôde 0.000 5.354 5.311 5.268 5.338 5.311 5.354 0.000 5.226 5.268 0.101 0.000 0.071 0.071 0.000 0.208 0.071 0.319 0.000 0.258 → 5.281 5.323 0.143 0.113 0.101 0.071 0.253 0.000 5.675 5.718 0.364 0.407 0.452 0.385 $5.541 \quad 5.578 \quad 0.995 \quad 0.975 \quad 0.958 \quad 0.929 \quad 0.910 \quad 0.981$

This is the 'Frame' at Euclidean distance = 0.0714 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 က 3 $^{\circ}$ 7 **c**¹² 9 10 Distance Matrix of the data ² Barcôde 0.051 0.000 5.354 5.311 5.268 5.338 5.295 5.536 5.311 5.354 0.000 0.101 0.051 0.000 0.051 5.226 5.268 0.101 0.051 0.000 0.071 0.051 0.272 0.101 0.385 0.071 0.000 0.208 0.071 0.319 0.000 0.258 **○** - 5.281 5.323 0.143 0.113 0.101 0.071 0.051 0.253 5.675 5.718 0.364 0.407 0.452 0.385

This is the 'Frame' at Euclidean distance = 0.0714 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 က 3 $^{\circ}$ 7 **c**¹² 9 10 Distance Matrix of the data ² Barcôde 0.051 0.000 5.354 5.311 5.268 5.338 5.295 5.536 5.311 5.354 0.000 0.101 0.051 0.000 0.051 5.226 5.268 0.101 0.051 0.000 0.071 0.051 0.272 0.101 0.385 0.071 0.000 0.208 0.071 0.319 0.000 0.258 **○** - 5.281 5.323 0.143 0.113 0.101 0.071 0.051 0.253 5.675 5.718 0.364 0.407 0.452 0.385

This is the 'Frame' at Euclidean distance = 0.182 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 က 3 $^{\circ}$ 7 **c**¹² 10 Distance Matrix of the data ² Barcôde 0.051 0.000 0.051 0.226 0.000 0.071 0.051 0.272 0.101 0.051 0.071 0.000 0.208 0.071 0.000 0.258 **○** − 5.281 5.323 0.253 0.000 0.113 0.357 0.000 0.307 0.000 5.675 5.718 0.364 0.407

This is the 'Frame' at Euclidean distance = 0.863 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 3 $^{\circ}$ 7 **∞**¹² 8 ' 9 10 Distance Matrix of the data ² Barcôde 0.051 0.000 0.051 0.051 0.101 0.051 0.000 0.071 0.051 0.272 0.101 0.385 5.295 5.338 0.071 0.051 0.071 0.000 0.051 0.208 0.071 0.319 5.493 5.536 0.182 0.226 0.272 0.208 0.258 0.000 0.253 0.113 0.071 **○** - 5.281 5.323 0.143 0.113 0.101 0.071 0.051 0.253 0.000 0.357 0.307 - 5.605 5.648 0.294 0.339 0.385 0.319 0.368 0.113 0.357 0.000 0.051 0.071

This is the 'Frame' at Euclidean distance = 5.23 N = 13**Persistent Diagram** Subsample at KDE Percentile= 1 3 2 7 10 Distance Matrix of the data ² Barcôde 0.051 0.000 5.354 5.311 5.268 5.338 5.268 5.311 **0.051** 0.000 **0.051 5.226 5.268 0.101 0.051 0.000 0.071 0.051 0.272 0.101 0.385 0.339** 0.051 0.071 0.000 0.051 0.208 0.071 5.493 5.536 0.182 0.226 0.272 0.208 0.258 0.000 0.253 0.113 0.071 0.182 **○** - 5.281 5.323 **0.143 0.113 0.101 0.071 0.051 0.253 0.000 0.357 0.307 0.416 -** 5.605 5.648 0.294 0.339 0.385 0.319 0.368 0.113 0.357 0.000 0.051 0.071

This is the 'Frame' at Euclidean distance = 5.72 N = 13**Persistent Diagram** ຽພbsample at KDE Percentile= 1 3 2 7 8 ' 9 10 Distance Matrix of the data ² Barcôde **(7)** - 5.311 5.354 0.000 0.051 0.101 0.071 **5.268 5.311 0.051 0.000 0.051 0.051 0.071 Ω** - 5.226 5.268 0.101 0.051 0.000 0.071 0.051 0.272 0.101 0.385 0.339 **—** 5.295 5.338 0.071 0.051 0.071 0.000 0.051 0.208 0.071 0.319 0.272 **5.253 5.295 0.113 0.071 0.051 0.051 0.000 0.258 0.051 0.368 0.319** - 5.493 5.536 0.182 0.226 0.272 0.208 0.258 0.000 0.253 0.113 0.071 0.182 0.981 O - 5.281 5.323 0.143 0.113 0.101 0.071 0.051 0.253 0.000 0.357 0.307 0.416 0.863 **-** 5.605 5.648 0.294 0.339 0.385 0.319 0.368 0.113 0.357 0.000 0.051 0.071

excursion set of Gaussian process, percentile .9 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 4 က 3 2 7 ω^{12} 0 10 2 Distance Matrix of the data 0 1 ² Barcôde 4 5.541 0.000 0.051 5.675 0.000 5.338 5.295 5.323 5.718 5.578 5.354 0.000 0.071 0.113 0.143 0.364 0.995 5.338 0.071 0.000 0.051 0.071 0.319 0.385 0.929 0.113 0.000 5.253 5.295 0.051 0.258 0.051 0.368 0.432 0.910 5.493 5.536 0.182 0.208 0.258 0.000 0.253 0.113 0.182 0.981 0.051 0.357 5.281 5.323 0.071 0.000 0.416 0.863 5.605 5.648 0.294 0.319 0.368 0.113 0.357 0.000 0.071 1.002 5.675 5.718 0.364 0.385 0.432 0.182 0.416 0.071 0.000 1.000 1.002 5.578 0.929 0.910 0.863 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 98 6 4<u>₹</u> 4 က 3 \sim 7 ω^{12} 0 10 2 Distance Matrix of the data 1 ² Barcôde 4 5 0 5.675 5.541 0.000 0.051 0.000 5.338 5.295 5.323 5.718 5.578 5.354 0.000 0.071 0.113 0.143 0.364 0.995 5.338 0.071 0.000 0.051 0.071 0.319 0.385 0.929 0.113 0.000 0.368 0.432 5.253 5.295 0.051 0.258 0.051 0.910 5.493 5.536 0.182 0.208 0.258 0.000 0.253 0.113 0.182 0.981 5.281 0.143 0.071 0.051 0.357 5.323 0.000 0.416 0.863 5.605 5.648 0.294 0.319 0.368 0.113 0.357 0.000 0.071 1.002 5.675 5.718 0.364 0.385 0.432 0.182 0.416 0.071 0.000 1.000 1.002 1.000 0.000 5.578 0.929 0.910 0.863 0 3 5

This is the 'Frame' at Euclidean distance = 0.0505 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 98 6 4<u>₹</u> 4 ი . 3 2 α ω^{12} 0 3 6 10 2 Distance Matrix of the data 5 1 ² Barcôde 4 5 0 5.541 0.000 0.051 5.311 5.295 5.253 5.281 5.605 5.675 0.051 0.000 5.578 5.354 5.338 5.295 5.536 5.323 5.648 5.718 5.311 5.354 0.000 0.113 0.182 0.071 0.143 0.294 0.364 0.995 5.295 5.338 0.071 0.000 0.051 0.208 0.071 0.319 0.385 0.929 0.000 5.253 5.295 0.113 0.051 0.258 0.051 0.368 0.432 0.910 5.493 5.536 0.182 0.208 0.258 0.000 0.253 0.113 0.182 0.981 5.281 5.323 0.143 0.071 0.051 0.253 0.000 0.357 0.416 0.863 5.605 5.648 0.294 0.319 0.368 0.113 0.357 0.000 0.071 1.002 5.675 5.718 0.364 0.385 0.432 0.182 0.416 0.071 0.000 1.000 5.541 0.910 5.578 0.995 0.929 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.0505 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 98 6 4<u>₹</u> 4 ი . 3 2 α ω^{12} 0 3 6 10 2 Distance Matrix of the data 5 1 ² Barcôde 4 5 0 5.541 0.000 0.051 5.311 5.295 5.253 5.281 5.605 5.675 0.051 0.000 5.578 5.354 5.338 5.295 5.536 5.323 5.648 5.718 5.311 5.354 0.000 0.113 0.182 0.071 0.143 0.294 0.364 0.995 5.295 5.338 0.071 0.000 0.051 0.208 0.071 0.319 0.385 0.929 0.000 5.253 5.295 0.113 0.051 0.258 0.051 0.368 0.432 0.910 5.493 5.536 0.182 0.208 0.258 0.000 0.253 0.113 0.182 0.981 5.281 5.323 0.143 0.071 0.051 0.253 0.000 0.357 0.416 0.863 5.605 5.648 0.294 0.319 0.368 0.113 0.357 0.000 0.071 1.002 5.675 5.718 0.364 0.385 0.432 0.182 0.416 0.071 0.000 1.000 5.541 0.910 5.578 0.995 0.929 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.0505 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 **ઌઌ૾ૺ** ૾૽ૼૺૼૼૼૺ૾૾ૼ 4 ი . 3 \sim α **c**12 0 ¹3 6 10 2 Distance Matrix of the data 1 ² Barcôde 4 5 0 5.541 0.000 0.051 5.311 5.295 5.253 5.281 5.605 5.675 0.051 0.000 5.578 5.354 5.338 5.295 5.536 5.323 5.648 5.718 5.311 5.354 0.000 0.113 0.182 0.294 0.071 0.143 0.364 0.995 5.295 5.338 0.071 0.000 0.051 0.208 0.071 0.319 0.385 0.929 0.000 0.051 5.253 5.295 0.113 0.051 0.258 0.368 0.432 0.910 5.493 5.536 0.182 0.208 0.258 0.000 0.253 0.113 0.182 0.981 5.281 5.323 0.143 0.071 0.051 0.253 0.000 0.357 0.416 0.863 5.605 5.648 0.294 0.319 0.368 0.113 0.357 0.000 0.071 1.002 5.675 5.718 0.364 0.385 0.432 0.182 0.416 0.071 0.000 1.000 5.541 5.578 0.995 0.929 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.0714 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 98 6 4<u>₹</u> 4 ი . 3 2 α **c**12 0 ¹3 6 10 2 Distance Matrix of the data 1 ² Barcôde 4 5 0 5.541 0.000 0.051 5.311 5.295 5.253 5.281 5.605 5.675 0.051 0.000 5.354 5.323 5.578 5.338 5.295 5.536 5.648 5.718 5.311 5.354 0.000 0.071 0.113 0.182 0.143 0.294 0.364 0.995 5.295 5.338 0.071 0.000 0.051 0.208 0.071 0.319 0.385 0.929 0.113 0.000 0.051 5.253 5.295 0.051 0.258 0.368 0.432 0.910 5.493 5.536 0.182 0.208 0.258 0.000 0.253 0.113 0.182 0.981 5.281 5.323 0.143 0.071 0.051 0.253 0.000 0.357 0.416 0.863 5.605 5.648 0.294 0.319 0.368 0.113 0.357 0.000 0.071 1.002 5.675 5.718 0.364 0.385 0.432 0.182 0.416 0.071 0.000 1.000 5.541 0.910 5.578 0.995 0.929 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.0714 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 98 6 4<u>₹</u> 4 ი . 3 2 α **c**12 0 ¹3 6 10 2 Distance Matrix of the data 1 ² Barcôde 4 5 0 5.541 0.000 0.051 5.311 5.295 5.253 5.281 5.605 5.675 0.051 0.000 5.354 5.323 5.578 5.338 5.295 5.536 5.648 5.718 5.311 5.354 0.000 0.071 0.113 0.182 0.143 0.294 0.364 0.995 5.295 5.338 0.071 0.000 0.051 0.208 0.071 0.319 0.385 0.929 0.113 0.000 0.051 5.253 5.295 0.051 0.258 0.368 0.432 0.910 5.493 5.536 0.182 0.208 0.258 0.000 0.253 0.113 0.182 0.981 5.281 5.323 0.143 0.071 0.051 0.253 0.000 0.357 0.416 0.863 5.605 5.648 0.294 0.319 0.368 0.113 0.357 0.000 0.071 1.002 5.675 5.718 0.364 0.385 0.432 0.182 0.416 0.071 0.000 1.000 5.541 0.910 5.578 0.995 0.929 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.113 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 **98** 6 4 4 ი . 3 2 α **c**12 0 3 6 10 2 Distance Matrix of the data 1 ² Barcôde 4 5 0 5.541 0.000 0.051 5.311 5.295 5.253 5.281 5.605 5.675 0.051 0.000 5.295 5.323 5.578 5.354 5.338 5.536 5.648 5.718 5.311 5.354 0.000 0.071 0.113 0.182 0.143 0.294 0.364 0.995 5.295 5.338 0.071 0.000 0.051 0.208 0.071 0.319 0.385 0.929 0.113 0.051 0.000 5.253 5.295 0.258 0.051 0.368 0.432 0.910 5.493 0.182 5.536 0.208 0.258 0.000 0.253 0.113 0.182 0.981 0.051 5.281 5.323 0.143 0.071 0.253 0.000 0.357 0.416 0.863 5.605 5.648 0.294 0.319 0.368 0.113 0.357 0.000 0.071 1.002 0.071 5.675 5.718 0.364 0.385 0.432 0.182 0.416 0.000 1.000 5.541 5.578 0.995 0.929 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.182 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 98 6 4E 4 ი . 3 2 α **c**12 0 3 10 2 Distance Matrix of the data 1 ² Barcôde 4 5 0 5.541 0.000 0.051 5.311 5.295 5.253 5.281 5.605 5.675 0.051 0.000 5.295 5.536 5.323 5.578 5.354 5.338 5.648 5.718 5.311 5.354 0.000 0.071 0.113 0.182 0.143 0.294 0.364 0.995 5.295 5.338 0.071 0.000 0.051 0.208 0.071 0.319 0.385 0.929 0.113 0.051 0.000 0.051 5.253 5.295 0.258 0.368 0.432 0.910 5.493 0.182 0.208 0.253 0.113 0.182 5.536 0.258 0.000 0.981 0.051 0.357 5.281 5.323 0.071 0.253 0.000 0.416 0.863 5.605 5.648 0.294 0.319 0.368 0.113 0.357 0.000 0.071 1.002 0.071 5.675 5.718 0.364 0.385 0.432 0.182 0.416 0.000 1.000 5.541 5.578 0.995 0.929 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.863 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 و 98 و 4 3 3 2 α **c**12 0 6 10 2 Distance Matrix of the data 0 ² Barcôde 4 5 5.541 0.000 0.051 5.311 5.295 5.253 5.281 5.605 5.675 0.051 0.000 5.295 5.323 5.578 5.354 5.338 5.536 5.648 5.718 5.311 5.354 0.000 0.071 0.113 0.143 0.294 0.364 0.995 5.295 5.338 0.071 0.000 0.051 0.071 0.385 0.929 0.113 0.051 0.000 0.051 0.368 0.432 5.253 5.295 0.910 5.493 0.182 0.208 0.258 0.000 0.253 0.113 0.182 0.981 5.536 5.281 0.000 0.357 0.416 0.863 5.323 0.071 0.051 0.253 5.605 5.648 0.294 0.319 0.368 0.357 0.000 0.071 1.002 0.416 5.675 5.718 0.385 0.432 0.071 0.000 1.000 5.541 5.578 0.995 0.929 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 5.25 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.9 2 4 က 3 2 α 0 10 2 8 Distance Matrix of the data 1 ² Barcôde 4 5 0 0.000 5.541 0.051 5.311 5.295 5.281 5.605 5.675 0.051 0.000 5.295 5.323 5.578 5.354 5.338 5.536 5.718 5.311 5.354 0.000 0.071 0.113 0.143 0.294 0.364 0.995 5.295 5.338 0.071 0.000 0.051 0.071 0.385 0.929 5.253 5.295 0.113 0.051 0.000 0.051 0.432 0.910 5.493 0.182 0.208 0.258 0.000 0.253 0.113 0.182 0.981 5.536 5.281 0.071 0.000 0.357 5.323 0.051 0.863 5.605 5.648 0.294 0.319 0.368 0.357 0.000 0.071 1.002 1.000 5.675 5.718 0.385 0.432 0.416 0.071 0.000 5.541 5.578 1.000 0.000 0.863 0 3 5

This is the 'Frame' at Euclidean distance = 5.72 N = 13**Persistent Diagram** Sທີ່sample at KDE Percentile= 0.9 2 4 က 3 2 α . 10 2 8 Distance Matrix of the data ² Barcôde 4 5 0 0.051 5.541 0.000 5.295 5.675 0.051 0.000 5.338 5.323 5.718 5.578 5.311 5.354 0.000 0.071 0.143 0.364 0.995 5.338 0.071 0.000 0.051 0.071 0.385 0.929 5.295 0.051 0.000 0.051 0.432 0.910 5.253 5.536 0.208 0.258 0.000 0.253 0.113 0.182 0.981 5.281 0.051 0.000 0.357 5.323 0.071 0.863 5.605 5.648 0.319 0.368 0.357 0.000 0.071 1.002 0.416 1.000 5.675 5.718 0.000 5.541 5.578 1.000 0.000 0.863 0 3 5

excursion set of Gaussian process, percentile .75 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 90% 5 A F 4 က 3 \sim 7 $\mathbf{\omega}^{12}$ 0 3 9 2 5 Distance Matrix of the data 0 1 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.605 5.675 5.541 0.051 0.000 5.354 5.295 5.536 5.323 5.648 5.718 5.578 5.311 5.354 0.000 0.113 0.182 0.364 0.294 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 5.493 5.536 0.182 0.258 0.000 0.253 0.113 0.182 0.981 2 5.281 5.323 0.143 0.051 0.253 0.000 0.357 0.416 0.863 5.605 5.648 0.294 0.368 0.113 0.357 0.000 0.071 1.002 5.718 0.000 1.000 5.675 0.364 0.432 0.182 0.416 0.071 5.541 5.578 0.995 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 906 5 A 4 က 3 \sim α . $\mathbf{\omega}^{12}$ 0 3 9 2 5 Distance Matrix of the data 0 1 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.605 5.675 5.541 0.051 0.000 5.354 5.295 5.536 5.323 5.648 5.718 5.578 5.311 5.354 0.000 0.113 0.182 0.143 0.294 0.364 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 2 5.493 5.536 0.182 0.258 0.000 0.253 0.113 0.182 0.981 5.281 5.323 0.143 0.051 0.253 0.000 0.357 0.416 0.863 5.605 5.648 0.294 0.368 0.113 0.357 0.000 0.071 1.002 5.675 5.718 0.364 0.182 0.071 0.000 1.000 0.432 0.416 5.541 5.578 0.995 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.0505 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 906 5 A 4 က 3 \sim α $\mathbf{\omega}^{12}$ 0 3 9 2 5 Distance Matrix of the data 0 1 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.605 5.675 5.541 0.051 0.000 5.354 5.295 5.536 5.323 5.648 5.718 5.578 5.311 5.354 0.000 0.113 0.182 0.143 0.294 0.364 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 2 5.493 5.536 0.182 0.258 0.000 0.253 0.113 0.182 0.981 5.281 5.323 0.143 0.051 0.253 0.000 0.357 0.416 0.863 5.605 5.648 0.294 0.368 0.113 0.357 0.000 0.071 1.002 5.675 5.718 0.364 0.182 0.071 0.000 1.000 0.432 0.416 5.541 5.578 0.995 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.0505 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 90% 5 M 4 ი . 3 \sim α . **∞**¹² 0 3 2 5 9 0 Distance Matrix of the data 5 0 1 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.281 5.605 5.675 5.541 0.051 0.000 5.718 5.354 5.295 5.536 5.323 5.648 5.578 5.311 5.354 0.000 0.113 0.182 0.143 0.294 0.364 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 2 5.493 5.536 0.182 0.258 0.000 0.253 0.113 0.182 0.981 5.281 0.143 0.051 0.253 0.416 5.323 0.000 0.357 0.863 5.605 0.071 5.648 0.294 0.368 0.113 0.357 0.000 1.002 5.675 5.718 0.364 0.432 0.182 0.416 0.071 0.000 1.000 5.541 5.578 0.995 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.0714 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 90% 5 M 4 ი . 3 \sim α . **∞**¹² 0 3 2 5 9 0 Distance Matrix of the data 5 0 1 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.281 5.605 5.675 5.541 0.051 0.000 5.718 5.354 5.295 5.536 5.323 5.648 5.578 5.311 5.354 0.000 0.113 0.182 0.143 0.294 0.364 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 5.493 5.536 0.182 0.258 0.000 0.253 0.113 0.182 0.981 5.281 0.143 0.051 0.253 0.416 5.323 0.000 0.357 0.863 5.605 0.071 5.648 0.294 0.368 0.113 0.357 0.000 1.002 5.675 5.718 0.364 0.432 0.182 0.416 0.071 0.000 1.000 5.541 5.578 0.995 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.113 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 % 5 4 4 -4 ი . 3 2 α . **∞**¹² 0 3 2 5 9 0 Distance Matrix of the data 5 0 1 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.281 5.605 5.675 5.541 0.051 0.000 5.354 5.718 5.295 5.536 5.323 5.648 5.578 5.311 0.113 5.354 0.000 0.182 0.143 0.294 0.364 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 5.493 0.182 5.536 0.258 0.000 0.253 0.113 0.182 0.981 5.281 5.323 0.143 0.051 0.253 0.416 0.863 0.000 0.357 5.605 0.368 0.000 0.071 1.002 5.648 0.294 0.113 0.357 0.071 0.000 5.675 5.718 0.364 0.432 0.182 0.416 1.000 5.541 5.578 0.995 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.113 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 % 5 4 4 -4 ი . 3 2 α . **∞**¹² 0 3 2 5 9 0 Distance Matrix of the data 5 0 1 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.281 5.605 5.675 5.541 0.051 0.000 5.354 5.718 5.295 5.536 5.323 5.648 5.578 5.311 0.113 5.354 0.000 0.182 0.143 0.294 0.364 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 5.493 0.182 5.536 0.258 0.000 0.253 0.113 0.182 0.981 5.281 5.323 0.143 0.051 0.253 0.416 0.863 0.000 0.357 5.605 0.368 0.000 0.071 1.002 5.648 0.294 0.113 0.357 0.071 0.000 5.675 5.718 0.364 0.432 0.182 0.416 1.000 5.541 5.578 0.995 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.182 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 97 5 A 4 ი . 3 2 α **∞**¹² 3 2 5 9 Distance Matrix of the data 5 0 1 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.281 5.605 5.675 5.541 0.051 0.000 5.536 5.323 5.648 5.718 5.354 5.295 5.578 0.113 0.182 0.143 5.311 5.354 0.000 0.294 0.364 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 5.493 0.182 0.113 0.182 5.536 0.258 0.000 0.253 0.981 5.281 5.323 0.143 0.253 0.000 0.357 0.416 0.863 5.605 0.368 0.113 0.357 0.000 0.071 1.002 5.648 0.294 0.416 0.071 5.675 5.718 0.364 0.432 0.182 0.000 1.000 5.541 5.578 0.995 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 0.863 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 €⁸⁷ 5 4 ი . 3 2 α **∞**¹² 3 2 5 9 Distance Matrix of the data 5 0 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.281 5.605 5.675 5.541 0.051 0.000 5.648 5.718 5.578 5.354 5.295 5.536 5.323 0.113 0.182 5.311 5.354 0.000 0.294 0.364 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 5.493 5.536 0.182 0.258 0.000 0.253 0.113 0.182 0.981 0.416 5.281 5.323 0.143 0.051 0.253 0.000 0.357 0.863 5.605 5.648 0.294 0.368 0.357 0.000 0.071 1.002 0.432 0.182 0.416 0.071 0.000 5.675 5.718 0.364 1.000 5.541 5.578 0.995 0.910 0.981 0.863 1.002 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 5.25 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 4 က 3 2 α . 0 2 5 9 Distance Matrix of the data 5 0 1 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.281 5.605 5.675 5.541 0.051 0.000 5.648 5.718 5.578 5.354 5.295 5.536 5.323 0.113 0.182 5.311 5.354 0.000 0.294 0.364 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 5.493 5.536 0.182 0.258 0.000 0.253 0.113 0.182 0.981 0.416 5.281 5.323 0.143 0.051 0.253 0.000 0.357 0.863 5.605 5.648 0.294 0.368 0.357 0.000 0.071 1.002 0.432 0.071 0.000 1.000 5.675 5.718 0.364 0.182 0.416 5.541 5.578 0.995 1.000 0.000 0 3 5

This is the 'Frame' at Euclidean distance = 5.72 N = 13**Persistent Diagram** Subsample at KDE Percentile= 0.75 2 4 က 3 2 α . 0 9 0 Distance Matrix of the data 0 1 ² Barcôde 4 5 0.000 0.051 5.311 5.253 5.493 5.605 5.675 5.541 0.051 0.000 5.354 5.295 5.536 5.323 5.648 5.718 5.578 0.113 5.311 5.354 0.000 0.182 0.294 0.364 0.995 5.253 5.295 0.113 0.000 0.258 0.051 0.368 0.432 0.910 5.493 5.536 0.182 0.258 0.000 0.253 0.113 0.182 0.981 0.416 5.281 5.323 0.143 0.051 0.253 0.000 0.357 0.863 5.605 5.648 0.294 0.368 0.357 0.000 0.071 1.002 5.718 0.071 0.000 1.000 5.675 0.364 0.432 0.416 5.541 5.578 0.995 1.000 0.000 0 3 5

