Circular Coordinates under Different Cost Functions

Group 8 Project

ICERM August 2019

Applied Mathematical Modeling with Topological Techniques

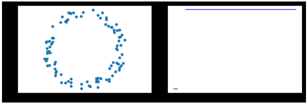
Problem Setup

- We have a (simplicial) complex K from the dataset X, we can consider the homology and cohomology with a fixed coefficient field $\mathbb{K}$.
- Homology and boundary operator: $\partial : C_k(K) \to C_{k-1}(K)$. e.g. $\partial[a,b,c] = [a,b] + [b,c] + [c,a]$
- Cohomology and coboundary operator: $\delta : C^k(K) \to C^{k+1}$.

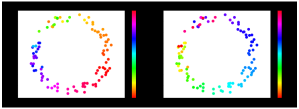
  e.g. $\delta \begin{bmatrix} a \mapsto 1 \\ b \mapsto 0 \\ c \mapsto 0 \end{bmatrix} = -[a,b]^* - [a,c]^*$ where

  $[a,b]^* = \begin{bmatrix} ab \mapsto 1 \\ bc \mapsto 0 \\ ca \mapsto 0 \end{bmatrix}$.
- Intuition: If you think of boundary operators as "derivation", then the coboundary operator is like "anti-derivation".
- Theorem (Circular Coordinates): Given a $[f] \in H^1(K)$, f can be made into a function $X \to S^1$. This **circular coordinate** can be found through the optimization problem

- Generic matrix optimization without Jacobian estimate: Slow and inefficient.
- Matrix optimization using Gradient Descent with Jacobian.
  - Example 1: Annulus.



    - L2 norm ($x = f - \delta z$ and $x = z \bmod 1$)



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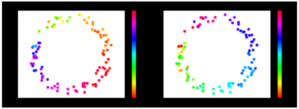
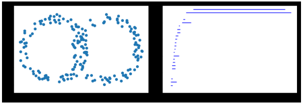


Another Example

- Matrix optimization using Gradient Descent with Jacobian.
- Example 2: Double Annulus.



  - Mixed L2 norm $(\|f - \delta z\|_{L^2} + ||\delta z\|_{L^2})$