The cost function $c(z)$ is

$$c(z) = (f - \delta z)^\top (f - \delta z) + \lambda \mathcal{P}(f - \delta z).$$

Then the gradient of the cost function becomes

$$\nabla c(z) = 2\delta^\top (\delta z - f) + \lambda \delta^\top \nabla \mathcal{P}(f - \delta z).$$

If the penalization term is $\mathcal{P}(x) = \|x\|_p$, then

$$\frac{\partial \mathcal{P}(x)}{\partial x_i} = \frac{\partial}{\partial x_i} \left(\sum_j |x_j|^p \right)^{1/p} = \operatorname{sign}(x_i) \cdot |x_i|^{p-1} \left(\sum_j |x_j|^p \right)^{1/p-1},$$

hence

$$\nabla \mathcal{P}(x) = \frac{(\operatorname{sign}(x_i) \cdot |x_i|^{p-1})_i}{\|x\|_p^{p-1}}.$$

and hence the gradient of the cost function becomes

$$\nabla c(z) = 2\delta^\top (\delta z - f) + \lambda \delta^\top \frac{(\operatorname{sign}((\delta z - f)_i) \cdot |(\delta z - f)_i|^{p-1})_i}{\|\delta z - f\|_p^{p-1}}.$$

In particular, when $p = 1$, then

$$\nabla c(z) = 2\delta^\top (\delta z - f) + \lambda \delta^\top \operatorname{sign}(\delta z - f),$$

and when $p = 2$, then

$$\nabla c(z) = 2\delta^\top (\delta z - f) + \lambda \delta^\top \frac{\delta z - f}{\|\delta z - f\|_2}.$$