Title Circular Coordinates under Different Cost Functions

Group 8 Project

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ICERM August 2019

Applied Mathematical Modeling with Topological Techniques

Problem Setup

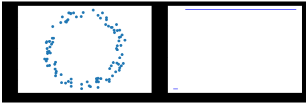
- We have a (simplicial) complex K from the dataset X, we can consider the homology and cohomology with a fixed coefficient field $\mathbb{K}$.
- Homology and boundary operator: $\partial : C_k(K) \to C_{k-1}(K)$. e.g. $[1,2,3] \mapsto [1,2] + [2,3] + [3,1]$
- Cohomology and coboundary operator: $\delta : C^k(K) \to C^{k+1}$. e.g. $\{1,2,3\} \mapsto [1,2,3]$
- Intuition: If you think of boundary operators as "differentiation", then the coboundary operator is like "anti-derivation".
- Problem (Circular Coordinates): Given a cochain $f \in C^k(K)$, can we find a cohomologous cochain z such that the norm (when the representative element function f and z is regarded as an $\mathbb{K}$-vector) $\|f - \delta z\|_{L^2}$ is minimized?

Interpretation of Different Cost Functions

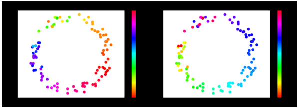
- Problem (New): How about we change the cost function $\|x\|_{L^2} := (\sum_i x_i^2)^{1/2}$ into:
  - $\|x\|_{L^1} := (\sum_i |x_i|)$ L1-norm
    It may introduce sparsity across coordinates instead of smoothness.
  - $(1-\lambda)\|x\|_{L^1} + \lambda\|x\|_{L^2}$ elastic net
    It may find a balance between L1 and L2 norms.
  - $\|x\|_{L^1} + \lambda\|x\|_{L^p}$
    It may produces some other kind of smoothness.
  - Localized penalty. Only take a penalty norm for some subvector of x.
  - In addition, we can penalized not only $x = f - \delta z$ but also
    - $x = \delta z$ (minimize edits?)
    - $x = z$ or $x = z \mod 1$ (smaller values for functions as $X \to S^1$?)

Optimization Problem: Gradient Descent

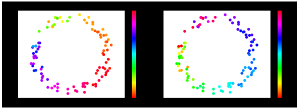
- Generic matrix optimization without Jacobian estimate: Slow and inefficient.
- Matrix optimization using Gradient Descent with Jacobian.
  - Example 1: Annulus.



    - L2 norm ($x = f - \delta z$ and $x = z \bmod 1$)

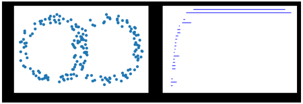


    - L1 norm ($x = f - \delta z$ and $x = z \bmod 1$)



Another Example

- Matrix optimization using Gradient Descent with Jacobian.
- Example 2: Double Annulus.



  - Mixed L2 norm $(\|f - \delta z\|_{L^2} + ||\delta z\|_{L^2})$