

UNIVERSITY OF TEXAS AT EL PASO
COMPUTATIONAL SCIENCE (CPS)

COMPUTING MATRIX EXPONENTIAL FOR GENERAL SQUARE MATRICES USING OCTAVE

```
octave:1> A = [-1.01  0.86 -4.60  3.31 -4.81; 3.98  0.53 -7.04  5.29  3.55; 3.30  8.26 -3.89  8.20 -1.51;
> 4.43  4.96 -7.66 -7.33  6.18; 7.31 -6.43 -6.16  2.47  5.58]
```

A =

```
-1.01000  0.86000 -4.60000  3.31000 -4.81000
 3.98000  0.53000 -7.04000  5.29000  3.55000
 3.30000  8.26000 -3.89000  8.20000 -1.51000
 4.43000  4.96000 -7.66000 -7.33000  6.18000
 7.31000 -6.43000 -6.16000  2.47000  5.58000
```

```
octave:2> [VR lam]=eig(A)
```

VR =

```
0.10806 + 0.16865i  0.10806 - 0.16865i  0.73223 + 0.00000i  0.73223 - 0.00000i  0.46065 + 0.00000i
0.40631 - 0.25901i  0.40631 + 0.25901i -0.02646 - 0.01695i -0.02646 + 0.01695i  0.33770 + 0.00000i
0.10236 - 0.50880i  0.10236 + 0.50880i  0.19165 - 0.29257i  0.19165 + 0.29257i  0.30874 + 0.00000i
0.39863 - 0.09133i  0.39863 + 0.09133i -0.07901 - 0.07808i -0.07901 + 0.07808i -0.74385 + 0.00000i
0.53954 + 0.00000i  0.53954 - 0.00000i -0.29160 - 0.49310i -0.29160 + 0.49310i  0.15853 + 0.00000i
```

lam =

Diagonal Matrix

```
2.85813 + 10.76275i  0  0  0  0
0  2.85813 - 10.76275i  0  0  0
0  0 -0.68667 + 4.70426i  0  0
0  0  0 -0.68667 - 4.70426i  0
0  0  0  0 -10.46292 + 0.00000i
```

```
octave:3> Expm_lam = expm(lam)
```

Expm_lam =

```
-4.02132 - 16.95870i  0.00000 + 0.00000i  0.00000 + 0.00000i  0.00000 + 0.00000i  0.00000 + 0.00000i
0.00000 + 0.00000i -4.02132 + 16.95870i  0.00000 + 0.00000i  0.00000 + 0.00000i  0.00000 + 0.00000i
0.00000 + 0.00000i  0.00000 + 0.00000i -0.00409 - 0.50323i  0.00000 + 0.00000i  0.00000 + 0.00000i
0.00000 + 0.00000i  0.00000 + 0.00000i  0.00000 + 0.00000i -0.00409 + 0.50323i  0.00000 + 0.00000i
0.00000 + 0.00000i  0.00000 + 0.00000i  0.00000 + 0.00000i  0.00000 + 0.00000i  0.00003 + 0.00000i
```

```
octave:4> B = VR *Expm_lam
```

B =

```
2.42549 - 2.51083i  2.42549 + 2.51083i -0.00299 - 0.36848i -0.00299 + 0.36848i  0.00001 + 0.00000i
-6.02638 - 5.84897i -6.02638 + 5.84897i -0.00842 + 0.01339i -0.00842 - 0.01339i  0.00001 + 0.00000i
-9.04024 + 0.31021i -9.04024 - 0.31021i -0.14801 - 0.09525i -0.14801 + 0.09525i  0.00001 + 0.00000i
-3.15192 - 6.39298i -3.15192 + 6.39298i -0.03897 + 0.04008i -0.03897 - 0.04008i -0.00002 + 0.00000i
-2.16965 - 9.14981i -2.16965 + 9.14981i -0.24695 + 0.14876i -0.24695 - 0.14876i  0.00000 + 0.00000i
```

```
octave:5> IV = inv(VR)
```

IV =

```

0.215757 - 0.320571i    0.742068 + 0.348767i   -0.368529 + 0.672612i    0.347002 + 0.146447i    0.138209 - 0.434246i
0.215757 + 0.320571i    0.742068 - 0.348767i   -0.368529 - 0.672612i    0.347002 - 0.146447i    0.138209 + 0.434246i
0.560904 + 0.087316i   -0.252172 - 1.075024i    0.260476 + 0.583419i    0.299778 - 0.040266i   -0.193344 + 0.711157i
0.560904 - 0.087316i   -0.252172 + 1.075024i    0.260476 - 0.583419i    0.299778 + 0.040266i   -0.193344 - 0.711157i
0.051700 + 0.000000i    0.708900 + 0.000000i   -0.162681 + 0.000000i   -1.008618 + 0.000000i    0.231859 - 0.000000i

```

```
octave:7> IV = eye(5)/VR
```

```
invVR =
```

```

0.215756 - 0.320574i    0.742081 + 0.348784i   -0.368535 + 0.672606i    0.347002 + 0.146441i    0.138208 - 0.434253i
0.215756 + 0.320574i    0.742081 - 0.348784i   -0.368535 - 0.672606i    0.347002 - 0.146441i    0.138208 + 0.434253i
0.560906 + 0.087311i   -0.252164 - 1.075043i    0.260477 + 0.583431i    0.299783 - 0.040269i   -0.193349 + 0.711158i
0.560906 - 0.087311i   -0.252164 + 1.075043i    0.260477 - 0.583431i    0.299783 + 0.040269i   -0.193349 - 0.711158i
0.051701 + 0.000000i    0.708891 - 0.000000i   -0.162684 - 0.000000i   -1.008618 + 0.000000i    0.231867 + 0.000000i

```

```
octave:8> C = B*IV
```

```
C =
```

```

-0.50217 - 0.00000i    4.56041 - 0.00000i    2.01829 + 0.00000i    2.38722 + 0.00000i   -0.98493 + 0.00000i
-6.36228 - 0.00000i   -4.83109 - 0.00000i   12.28998 - 0.00000i   -2.47319 + 0.00000i   -6.76137 - 0.00000i
-3.85152 + 0.00000i   -13.76345 + 0.00000i    6.27992 - 0.00000i   -6.46124 + 0.00000i   -2.03676 - 0.00000i
-5.50963 - 0.00000i   -0.11275 - 0.00000i   10.85609 - 0.00000i   -0.33509 + 0.00000i   -6.46545 + 0.00000i
-7.10558 - 0.00000i    3.60664 - 0.00000i   13.60548 + 0.00000i    1.03809 + 0.00000i   -8.66237 + 0.00000i

```

```
octave:9> expm(A)
```

```
ans =
```

```

-0.50217    4.56041    2.01829    2.38722   -0.98493
-6.36228   -4.83109   12.28998   -2.47319   -6.76137
-3.85152  -13.76345    6.27992   -6.46124   -2.03676
-5.50963   -0.11275   10.85609   -0.33509   -6.46545
-7.10558    3.60664   13.60548    1.03809   -8.66237

```

NOTE : Carefull exp(1) is defferent from expm(1)

```
octave:10> expm(1)
```

```
ans =
```

```

-4.02132 - 16.95870i    0.00000 + 0.00000i    0.00000 + 0.00000i    0.00000 + 0.00000i    0.00000 + 0.00000i
0.00000 + 0.00000i   -4.02132 + 16.95870i    0.00000 + 0.00000i    0.00000 + 0.00000i    0.00000 + 0.00000i
0.00000 + 0.00000i    0.00000 + 0.00000i   -0.00409 - 0.50323i    0.00000 + 0.00000i    0.00000 + 0.00000i
0.00000 + 0.00000i    0.00000 + 0.00000i    0.00000 + 0.00000i   -0.00409 + 0.50323i    0.00000 + 0.00000i
0.00000 + 0.00000i    0.00000 + 0.00000i    0.00000 + 0.00000i    0.00000 + 0.00000i    0.00003 + 0.00000i

```

```
octave:11> exp(1)
```

```
ans =
```

```

-4.02132 - 16.95870i    1.00000 + 0.00000i    1.00000 + 0.00000i    1.00000 + 0.00000i    1.00000 + 0.00000i
1.00000 + 0.00000i   -4.02132 + 16.95870i    1.00000 + 0.00000i    1.00000 + 0.00000i    1.00000 + 0.00000i
1.00000 + 0.00000i    1.00000 + 0.00000i   -0.00409 - 0.50323i    1.00000 + 0.00000i    1.00000 + 0.00000i
1.00000 + 0.00000i    1.00000 + 0.00000i    1.00000 + 0.00000i   -0.00409 + 0.50323i    1.00000 + 0.00000i
1.00000 + 0.00000i    1.00000 + 0.00000i    1.00000 + 0.00000i    1.00000 + 0.00000i    0.00003 + 0.00000i

```

C output :

Matrix A

```
-1.01000 0.86000 -4.60000 3.31000 -4.81000
3.98000 0.53000 -7.04000 5.29000 3.55000
3.30000 8.26000 -3.89000 8.20000 -1.51000
4.43000 4.96000 -7.66000 -7.33000 6.18000
7.31000 -6.43000 -6.16000 2.47000 5.58000
```

Eigenvalues (lambda)

```
(2.85813,10.76275) (2.85813,-10.76275) (-0.68667,4.70426) (-0.68667,-4.70426) -10.46292
```

Left eigenvectors (VI)

```
(0.04441,0.28792) (0.04441,-0.28792) (-0.13256,-0.32729) (-0.13256,0.32729) 0.04084
(0.61816,0.00000) (0.61816,-0.00000) (0.68687,0.00000) (0.68687,-0.00000) 0.55995
(-0.03576,-0.57711) (-0.03576,0.57711) (-0.39033,-0.07487) (-0.39033,0.07487) -0.12850
(0.28373,0.01135) (0.28373,-0.01135) (-0.01820,-0.18727) (-0.01820,0.18727) -0.79670
(-0.04495,0.34061) (-0.04495,-0.34061) (-0.40322,0.21812) (-0.40322,-0.21812) 0.18314
```

Right eigenvectors (VR)

```
(0.10806,0.16865) (0.10806,-0.16865) (0.73223,0.00000) (0.73223,-0.00000) 0.46065
(0.40631,-0.25901) (0.40631,0.25901) (-0.02646,-0.01695) (-0.02646,0.01695) 0.33770
(0.10236,-0.50880) (0.10236,0.50880) (0.19165,-0.29257) (0.19165,0.29257) 0.30874
(0.39863,-0.09133) (0.39863,0.09133) (-0.07901,-0.07808) (-0.07901,0.07808) -0.74385
(0.53954,0.00000) (0.53954,-0.00000) (-0.29160,-0.49310) (-0.29160,0.49310) 0.15853
```

expm(lambda)

```
(-4.02132,-16.95870) (-4.02132,16.95870) (-0.00409,-0.50323) (-0.00409,0.50323) 0.00003
```

Matrix B = VR * expm(lambda)

```
(2.42549, -2.51083) (2.42549, 2.51083) (-0.00299, -0.36848) (-0.00299, 0.36848) (0.00001, 0.00000)
(-6.02638, -5.84897) (-6.02638, 5.84897) (-0.00842, 0.01339) (-0.00842, -0.01339) (0.00001, 0.00000)
(-9.04024, 0.31021) (-9.04024, -0.31021) (-0.14801, -0.09525) (-0.14801, 0.09525) (0.00001, 0.00000)
(-3.15192, -6.39298) (-3.15192, 6.39298) (-0.03897, 0.04008) (-0.03897, -0.04008) (-0.00002, 0.00000)
(-2.16965, -9.14981) (-2.16965, 9.14981) (-0.24695, 0.14876) (-0.24695, -0.14876) (0.00000, 0.00000)
```

Righth Eigenvector Inverse array VR⁽⁻¹⁾

```
(0.21576, -0.32057) (0.74207, 0.34877) (-0.36853, 0.67261) (0.34700, 0.14645) (0.13821, -0.43425)
(0.21576, 0.32057) (0.74207, -0.34877) (-0.36853, -0.67261) (0.34700, -0.14645) (0.13821, 0.43425)
(0.56090, 0.08732) (-0.25217, -1.07502) (0.26048, 0.58342) (0.29978, -0.04027) (-0.19334, 0.71116)
(0.56090, -0.08732) (-0.25217, 1.07502) (0.26048, -0.58342) (0.29978, 0.04027) (-0.19334, -0.71116)
(0.05170, 0.00000) (0.70890, 0.00000) (-0.16268, 0.00000) (-1.00862, 0.00000) (0.23186, -0.00000)
```

C = expm(A) = BB*VR⁽⁻¹⁾

```
(-0.50217, -0.00000) (4.56041, -0.00000) (2.01829, 0.00000) (2.38722, 0.00000) (-0.98493, 0.00000)
(-6.36228, -0.00000) (-4.83109, -0.00000) (12.28998, -0.00000) (-2.47319, 0.00000) (-6.76137, -0.00000)
(-3.85152, 0.00000) (-13.76345, 0.00000) (6.27992, -0.00000) (-6.46124, 0.00000) (-2.03676, -0.00000)
(-5.50963, -0.00000) (-0.11275, -0.00000) (10.85609, -0.00000) (-0.33509, 0.00000) (-6.46545, 0.00000)
(-7.10558, -0.00000) (3.60664, -0.00000) (13.60548, 0.00000) (1.03809, 0.00000) (-8.66237, 0.00000)
```