Application of Monte-Carlo Simulations in Estimation of Pi

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Abstract. The term Pi can be defined as the share of the boundary of any circle to the diameter of that circle. Despite the circle's dimension, this ratio is the same for all circles. Sometimes it is approximated as 22/7. In decimal form, the value of Pi is approximately 3.14. These approximations result in an error during precise calculations because actually, it is an irrational number. Its decimal representation is non-repeating & non-terminating. In this paper, an effort has been made to estimate the value of Pi using Monte Carlo Simulations.

Keywords: *Simulation; Random numbers; MATLAB, Modeling; Probability*.

1. Introduction & Literature Review

Archimedes of Syracuse (287–212 BC), did the first calculation of Pi. [1] applied Monte Carlo simulations in measuring the statistical uncertainties in numerical simulations. Pi does not equal 22/7. The fraction 22/7 is only a convenient approximation. Pi is not an integer. Pi cannot be defined as even or odd because it is not an integer. It is always seen in mathematical formulae of areas, volumes, etc of curved boundaries. Monte Carlo is the name of a city in France famous for gambling and casino. A processor for specified applications based on Monte Carlo simulations was designed by [2].

An application of Monte Carlo simulations was observed in handling the logistic operations of ceramics by[3]. [4] used by chance generated numbers or actions to reproduce stochastic processes in the estimation of the value of Pi. Monte Carlo simulations are based on sport of probability [5], which is the cause for using the name of the Mediterranean town familiar for casino making a bet. The Monte Carlo method describes a replication in which a structure factor worth is approximated by performing an original random stochastic experiment [6]. Monte Carlo reenactments are utilized to show monetary frameworks, to mimic media transmission organizations, to figure results for high-dimensional integrals, or to inexact qualities, for example, constants or numeric integrals [7, 8]. The Monte Carlo procedure is likewise utilized in displaying a wide scope of actual frameworks at the front line of consistent examination today, in light of a run of irregular numbers [9].

Strictly speaking, Monte Carlo estimation is based on the relative occurrence understanding of probability [10, 11]. Uncertainty in inventory and back orders was handled by [12-17]. The leftover of the research paper is structured as follows. The methodology and Mathematical modeling of the event of interest along with the suitability of MATLAB in carrying simulations are described in section 2. MAT Lab program for different simulation runs is given Appendix at the last. Results are discussed in section 3. Section 4 displays the conclusions derived from the total work.

2. Methodology Mathematical modeling of the event of interest

MATLAB (Matrix Laboratory) software is utilized for performing the computer simulations in the determination of the value of Pi. MATLAB has an extensive library of more than 1000 predefined functions. Its advantages are ease of use, outstanding tool for visualizing data. First, a quarter circle of unit radius is drawn within a square as shown in Figure 1. Two random numbers are generated both between 0 to 1. If the sum of the squares is less than 1 then the dot is considered within the quarter circle and square both else it is considered within the square only.

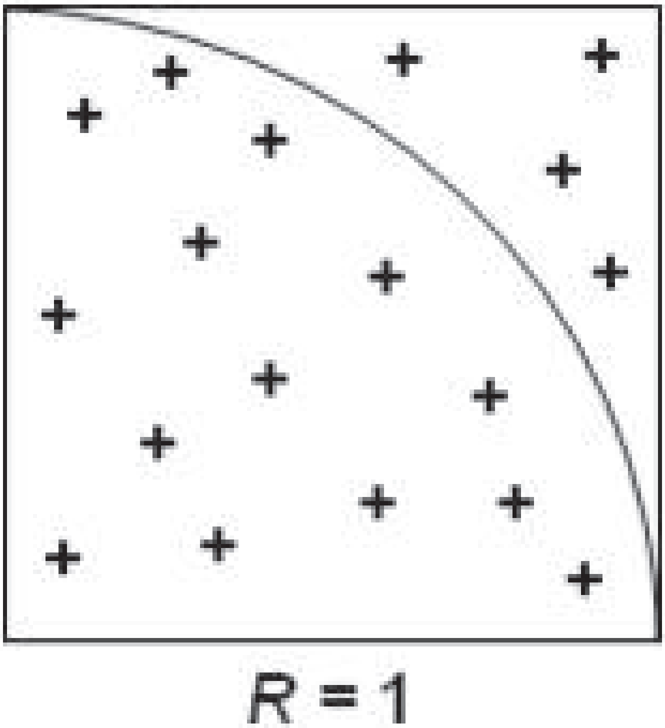

Figure 1: Event of interest

The following formula is utilized in the estimation of Pi.

Pi=4*M/N Eq.1

Where,

M= Number of dots within the quarter circle

N= Number of dots within the square

3. Result & Discussion

The plan is to mimic irregular (x, y) focuses in a level surface with a field as a square of side 1 unit. Dream a quarter circle engraved into the square. We at that point compute the proportion of number focuses that lied inside the circle and the whole number of created specks. Refer to Figure 2. MATLAB Programme is made to find the value of Pi using Monte Carlo simulations as shown in the appendix.

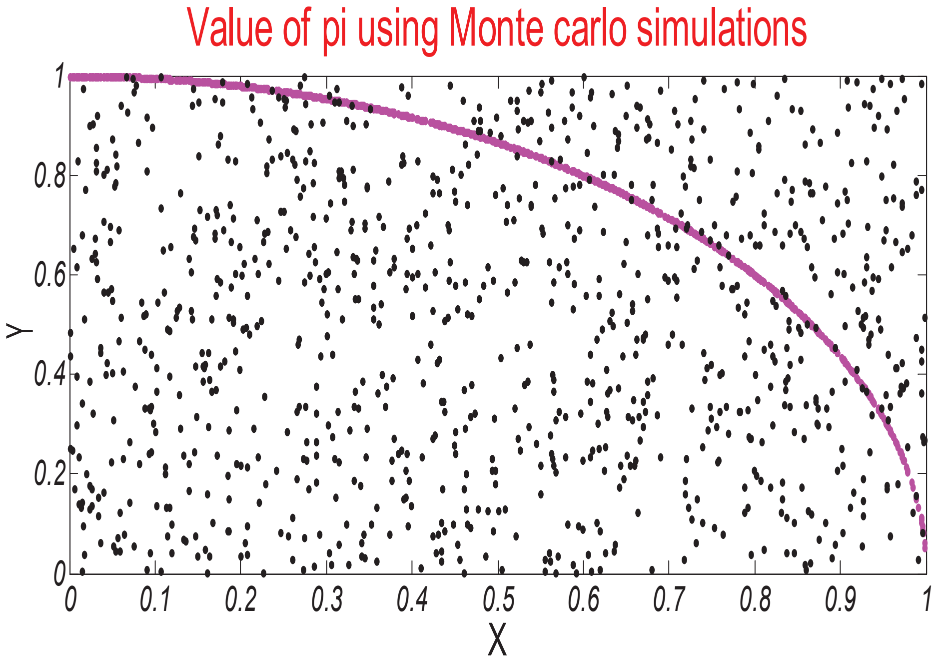

Figure 2: Monte-Carlo Simulations in a 2-D Plane

Table 1 shows the reduction in error in the estimation of Pi concerning an increase in the simulation runs. For 10 simulations 6 dots were found inside the quarter circle and the error is very high around 23.61%. For 50 simulations 38 dots were found inside the quarter circle and the error is significantly reduced to 3.23%. For 100 simulations 81 dots were found inside the quarter circle and the error is again reduced by a small amount to 3.13%. For 500 and 1000 simulations, 401 and 797 dots were found inside the quarter circle and further decrement in the error(i.e. 2.11% & 1.48% is observed. For the last case i.e. for 10000 simulations 7852 dots were found inside the quarter circle and the error is very small i.e. 0.0252 % which can be neglected. Hence simulations are stopped here. A further length of simulation can be increased as the demand for the accuracy required in a particular application.

Table 1. Error Analysis in the Estimation of Pi

Simulation runs	M	N	Value of Pi	Absolute error	%Error
10	6	10	2.4000	0.7416	23.6056
50	38	50	3.0400	0.1016	3.2338
100	81	100	3.2400	0.0984	3.1324
500	401	500	3.2080	0.0664	2.1138
1000	797	1000	3.1880	0.0464	1.4772
10000	7852	10000	3.1408	0.0008	0.0252

4. Conclusion

The Monte Carlo method uses randomly generated facts or events to simulate random processes and calculate approximately complicated results. Monte Carlo simulations are applied to model monetary systems, to compute results for high-dimensional integrals, or to approximate values such as numeric integrals or constants. In this paper the approximate value of Pi is estimated using Monte Carlo simulations, by generating random points in a square around a quarter circle, using the connection between the region of the square and the circle. MATLAB's fixed random number generation functions are used.

Appendix

MATLAB Programme to find the value of pie using Monte Carlo simulations is given below.

```
clear
clc
M=0
N=0
for i=1:1000
        r1(i)=rand();
    r2(i)=rand();

    y(i)=(1-r1(i)^2)^(0.5);

    if(r2(i)<y(i))
        M=M+1;
    else
        N=N+1;
    end
end
M=M
N=N
p=4*M/(M+N)
 plot(r1,y,'m.')
 hold on
 plot(r1,r2,'k.')
fprintf('The value of Pi using Monte Carlo simulation=%d',p)
```

Outpt of MATLAB Programme at 1000 Runs

M = 797
N = 203
pi = p=3.188

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