Métodos de solución EDO 1er. orden:

Variables Separables $\frac{dy}{dx} = g(x)h(y)$

Ecuaciones Lineales (Variación de Parámetros) $\frac{dy}{dx} + P(x)y = f(x)$

$$e^{\int P(x)dx} \quad ye^{\int P(x)dx} = c + \int e^{\int P(x)dx} f(x)dx$$

Ecuaciones Exactas

$$M(x,y)dx + N(x,y)dy = 0$$

$$\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$$

$$\frac{\delta f}{\delta x} = M(x,y), \quad \frac{\delta f}{\delta y} = N(x,y)$$

$$f(x,y) = c$$

$$f(x,y) = \int M(x,y)dx + g(y)$$

$$g'(y) = N(x,y) - \frac{\delta}{\delta y}\int M(x,y)dx$$

$$\mu(x) = e^{\int \frac{M_y - N_x}{N} dx}$$

$$\mu(y) = e^{\int \frac{N_x - M_y}{M} dy}$$

Ecuación de Bernoulli

$$\frac{dy}{dx} + P(x)y = f(x)y^n$$

$$u = y^{1-n} \Rightarrow \frac{dy}{dx} = \frac{1}{1-n} u^{\frac{n}{1-n}} \frac{du}{dx}$$

$$\frac{du}{dx} + P(x)u = f(x)$$

Identidades:

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

$$\sin 2u = 2\sin u \cos u$$

$$\log x + \log y = \log xy$$

$$\log x - \log y = \log \frac{x}{y}$$

$$n\log x = \log x^n$$

$$a^n \cdot a^m = a^{n+m}$$

$$(a^n)^m = a^{n\cdot m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

Integrales

$$\int u^n du = \frac{u^{n+1}}{n+1} + C, \, n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u du = e^u + C$$

$$\int u dv = uv - \int v du$$

Fracciones parciales:

Lineales: $\frac{M(x)}{(px+q)^n} = \frac{A_1}{px+q} + \frac{A_2}{(px+q)^2} + \frac{A_3}{(px+q)^3} + \cdots + \frac{A_n}{(px+q)^n}$

Cuadráticos: $\frac{M(x)}{(ax^2+bx+c)^n} = \frac{B_1x+C_1}{ax^2+bx+c} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \frac{B_3x+C_3}{(ax^2+bx+c)^3} + \cdots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}$

Integrales y Derivadas Trigonométricas:

$$\frac{d}{dx}[\sin u] = \cos u \cdot u'$$

$$\frac{d}{dx}[\cos u] = -\sin u \cdot u'$$

$$\frac{d}{dx}[\tan u] = \sec^2 u \cdot u'$$

$$\frac{d}{dx}[\sec u] = \sec u \cdot \tan u \cdot u'$$

$$\frac{d}{dx}[\cot u] = -\csc^2 u \cdot u'$$

$$\frac{d}{dx}[\csc u] = -\csc u * \cot u \cdot u'$$

$$\int \cos u \cdot du = \sin u + C$$

$$\int \sin u \cdot du = -\cos u + C$$

$$\int \sec^2 u \cdot du = \tan u + C$$

$$\int \sec u \cdot \tan u \cdot du = \sec u + C$$

$$\int \csc^2 u \cdot du = -\cot u + C$$

$$\int \csc u \cdot \cot u \cdot du = -\csc u + C$$

$$\int \tan u \cdot du = -\ln|\cos u| + C$$

$$\int \cot u \cdot du = \ln|\sin u| + C$$

$$\int \sec u \cdot du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \cdot du = -\ln|\csc u + \cot u| + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$$

Método de Euler

$$\frac{dy}{dt} = f(t,y)$$

$$y(a) = \alpha = w_0$$

$$a \le t \le b$$

$$h = \frac{b-a}{N}$$

$$t_i = a + ih$$

$$w_{i+1} = w_i + hf(t_i, w_i)$$

Aplicaciones

Poblacional: $\frac{dP}{dt} = kP(t)$

Decaimiento Radiactivo: $\frac{dA}{dt} = kA(t)$

Enf/Cal Newton: $\frac{dT}{dt} = k(T - T_m)$

Propagación Enfermedades: $\frac{dx}{dt} = kxy$

Circuitos: $V_L = L\frac{di}{dt}$, $V_R = Ri$, $V_C = \frac{q(t)}{C}$, $i = \frac{dq}{dt}$

Ecuaciones Lineales Homogéneas Coeficientes Constantes

1. Raíces $\mathbb{R}$ y diferentes: $y(x) = C_1 e^{m_1 x} + C_2 e^{m_2 x}$
2. Raíces Repetidas: $y(x) = C_1 e^{m_1 x} + C_2 x e^{m_1 x} + \dots$
3. Raíces $\mathbb{C}$ conjugadas $\alpha \pm i\beta$: $y(x) = C_1 e^{\alpha x} cos(\beta x) + C_2 e^{\alpha x} sen(\beta x)$

Coeficientes Indeterminados

$g(x)$	y_P
$3x^2 - 2$	$Ax^2 + Bx + C$
$x^3 - x + 1$	$Ax^3 + Bx^2 + Cx + D$
$sen(4x)$	$Acos(4x) + Bsen(4x)$
$cos(3x)$	$Acos(3x) + Bsen(3x)$
e^{5x}	Ae^{5x}
$(9x^2 - 2)e^{6x}$	$(Ax^2 + Bx + C)e^{6x}$
$x^3 e^{5x}$	$(Ax^3 + Bx^2 + Cx + D)e^{5x}$
$e^{3x} sen(4x)$	$Ae^{3x} cos(4x) + Be^{3x} sen(4x)$
$5x^2 sen(4x)$	$(Ax^2 + Bx + C)cos(4x) + (Dx^2 + Ex + F)sen(4x)$
$xe^{3x} cos(4x)$	$(Ax + B)e^{3x} cos(4x) + (Cx + D)e^{3x} sen(4x)$

Método del Anulador

$g(x)$	Anulador
x^{n-1}	D^n
$x^{n-1} e^{\alpha x}$	$(D - \alpha)^n$
$x^{n-1} e^{\alpha x} cos(\beta x)$	$[D^2 - 2\alpha D + (\alpha^2 + \beta^2)]^n$
$x^{n-1} e^{\alpha x} sen(\beta x)$	$[D^2 - 2\alpha D + (\alpha^2 + \beta^2)]^n$

Variación de Parámetros

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \qquad u_2' = \frac{y_1 f(x)}{W}$$

$$u_1' = \frac{-y_2 f(x)}{W} \qquad y_P = u_1 y_1 + u_2 y_2$$

Ley de Hooke

$$F = kx$$

Movimiento Armónico Amortiguado

$$\frac{d^2 x}{dt^2} + \left(\frac{\beta}{m}\right)\frac{dx}{dt} + \left(\frac{k}{m}\right)x = 0$$

$$\frac{d^2 x}{dt^2} + 2\lambda \frac{dx}{dt} + w^2 x = 0$$

Caso i): Sobreamortiguado

$$x(t) = e^{-\lambda t}\left(c_1 e^{\sqrt{\lambda^2 - w^2}\, t} + c_2 e^{-\sqrt{\lambda^2 - w^2}\, t}\right)$$

Caso ii): Críticamente amortiguado

$$x(t) = c_1 e^{-\lambda t} + c_2 t e^{-\lambda t}$$

Caso iii): Subamortiguado

$$x(t) = e^{-\lambda t}\left(c_1 \cos(\sqrt{w^2 - \lambda^2}\, t) + c_2 \sin(\sqrt{w^2 - \lambda^2}\, t)\right)$$

Serie de Potencias

$$y = \sum_{n=0}^{\infty} c_n x^n$$

Método de Frobenius

$$y = \sum_{n=0}^{\infty} c_n x^{n+r}$$

Sistemas Lineales Homogéneos

$X' A = X$	SEDL
$\lvert A - \lambda I \rvert = 0$	Eigenvalores
$(A - \lambda I)K = 0$	Eigenvectores

Eigenvalores reales y diferentes.

$$X = c_1 K_1 e^{\lambda_1 t} + c_2 K_2 e^{\lambda_2 t} + \cdots + c_n K_n e^{\lambda_n t}$$

Eigenvalores repetidos

m<n

$$c_1 K_1 e^{\lambda_1 t} + c_2 K_2 e^{\lambda_1 t} + \cdots + c_m K_m e^{\lambda_1 t}$$

m=n

$$X_1 = K_{11} e^{\lambda_1 t}$$

$$X_2 = K_{21} t e^{\lambda_1 t} + K_{22} e^{\lambda_1 t}$$

$$\vdots$$

$$X_m = K_{m1} \frac{t^{m-1}}{(m-1)!} e^{\lambda_1 t} + K_{m2} \frac{t^{m-2}}{(m-2)!} e^{\lambda_1 t} + \cdots + K_{mm} e^{\lambda_1 t}$$

Eigenvalores complejos

$\lambda_1 = \alpha + i\beta$

$$X_1 = \left[B_1 \cos\beta t - B_2 \sin\beta t\right] e^{\alpha t}$$

$$X_2 = \left[B_2 \cos\beta t + B_1 \sin\beta t\right] e^{\alpha t}$$

$$B_1 = \Re(K_1)$$

$$B_2 = \Im(K_1)$$

Solución General

$$X(t) = c_1 X_1 + c_2 X_2 + c_3 X_3 + \cdots c_n X_n$$