L44

Theory of MRI Reconstruction

CS 473 / CS 673
Jeff Orchard

Goal: To find out how the MR signal can be turned into tomographic images.

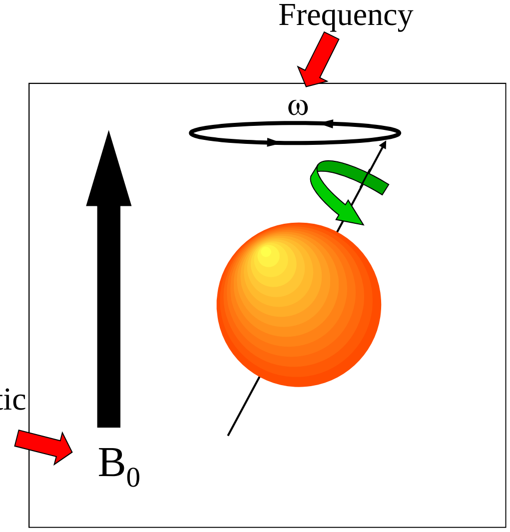
Dipole Spin

Magnetic moment

$$\frac{d\vec{\mu}}{dt} = \gamma\vec{\mu} \times \vec{B}_0$$

$$\Rightarrow \quad \omega = \gamma B_0$$

Larmour Frequency



Magnetic field

Bloch's Equation

Net magnetization vector $\vec{M}$

$$\vec{M} = \vec{M}_x + \vec{M}_y + \vec{M}_z = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$

Bloch's equation governs the behaviour of $\vec{M}$

$$\frac{d\vec{M}}{dt} = \underbrace{\boxed{\gamma\vec{M} \times \vec{B}_0}}_{\text{Larmour precession}} - \underbrace{\boxed{\frac{1}{T_2}\left(M_x\vec{i} + M_y\vec{j}\right)}}_{\text{Transverse } (x\text{-}y) \text{ decay}} - \underbrace{\boxed{\frac{1}{T_1}\left(M_z - M_0\right)\vec{k}}}_{\text{Longitudinal } (z) \text{ decay}}$$

(see page 108 of Jeff's thesis for explanation of T_1 and T_2.)

Dynamics of M_{xy}

Recall:

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \vec{B}_0 - \frac{1}{T_2}\left(M_x\vec{i} + M_y\vec{j}\right) - \frac{1}{T_1}\left(M_z - M_0\right)\vec{k}$$

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \left(\vec{B}_0 + \vec{G} \cdot \vec{x}\right) - \frac{1}{T_2}\left(M_x\vec{i} + M_y\vec{j}\right) - \frac{1}{T_1}\left(M_z - M_0\right)\vec{k}$$

We introduce a gradient in the strength of the magnetic field.
The gradient is in the direction $\vec{x}$, with no z-component.

In matrix form…

$$\frac{d\vec{M}}{dt} = -\begin{bmatrix} \frac{1}{T_2} & -\gamma\left(B_0 + \vec{G} \cdot \vec{x}\right) & 0 \\ \gamma\left(B_0 + \vec{G} \cdot \vec{x}\right) & \frac{1}{T_2} & 0 \\ 0 & 0 & \frac{1}{T_1} \end{bmatrix} \vec{M} + \frac{1}{T_1}\vec{M}_0$$

Solution for M_{xy}

$$M_{xy}(x,y,t) = c\underbrace{e^{-i\gamma B_0 t}}_{\text{Normal precession at Larmour frequency}}\overbrace{e^{\frac{-t}{T_2}}}^{\text{T2 Relaxation}}\underbrace{e^{-i(k_x x + k_y y)}}_{\text{Frequency/Phase state (the } k\text{'s depend on the gradients)}}$$

$$k_x(t) = \int_0^t G_x(\tau)d\tau \qquad k_y(t) = \int_0^t G_y(\tau)d\tau$$

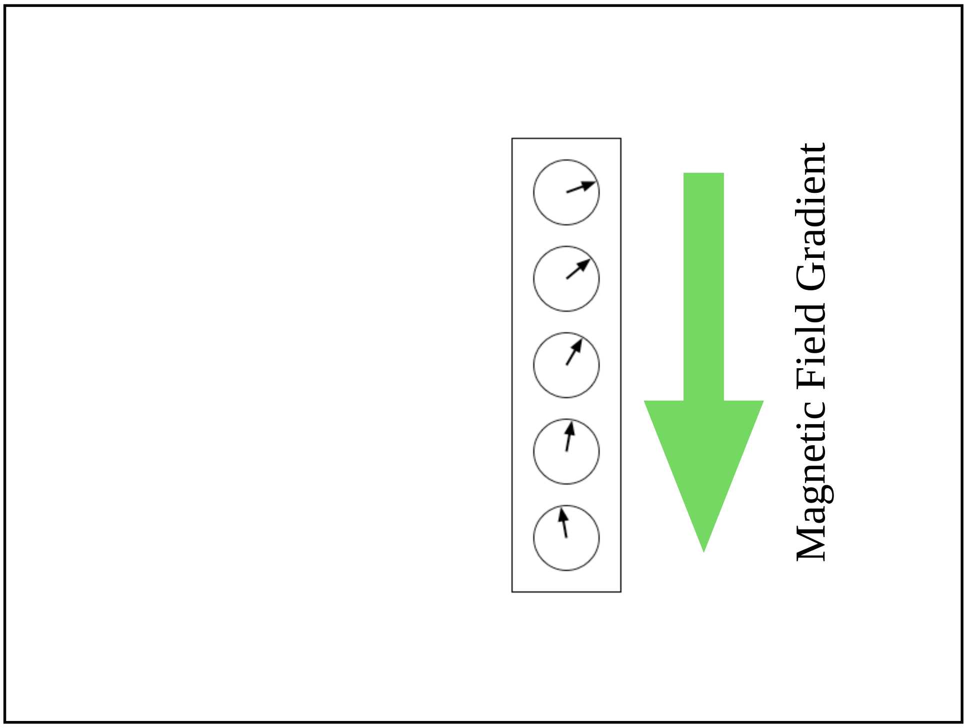
MR Signal

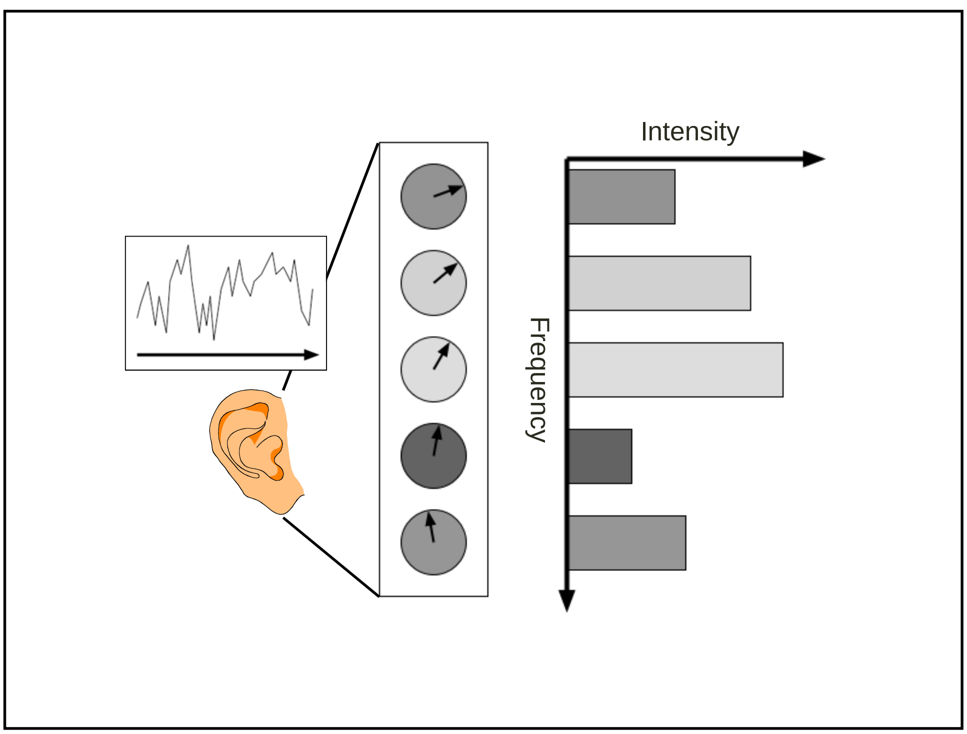
The MR signal is the sum of all the excited $\boldsymbol{M_{xy}}$'s (ignoring t now).

$$S(k_x, k_y) = \iint M_{xy}(x,y)e^{-i(k_x x + k_y y)}dxdy$$

This looks just like a Fourier Transform, where (k_x, k_y) are the frequency variables. Thus, its inverse is just like the inverse FT,

$$M_{xy}(x,y) = \iint S(k_x, k_y)e^{i(k_x x + k_y y)}dk_x dk_y$$

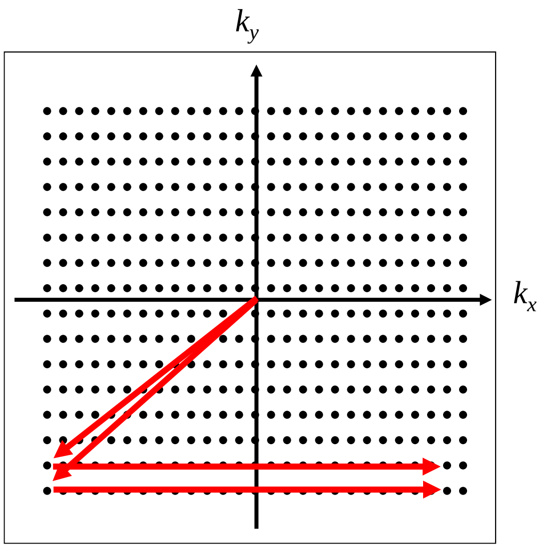

Magnetic Field Gradient


Intensity
Frequency

k-Space Traversal

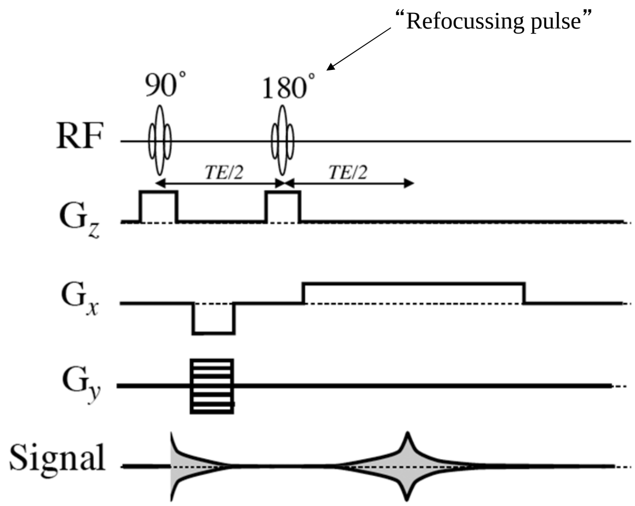
$$k_x(t) = \int_0^t G_x(\tau)d\tau$$

$$k_y(t) = \int_0^t G_y(\tau)d\tau$$



How to control the gradients

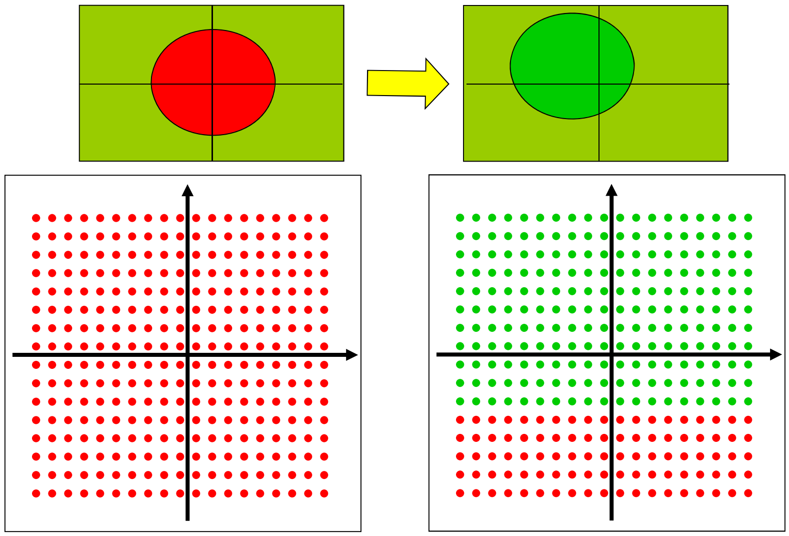
Spin-Echo Imaging



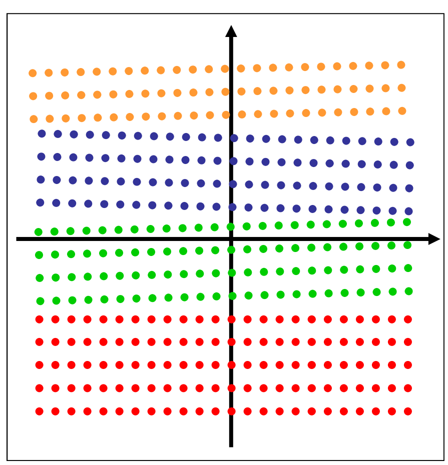
Motion Corruption

- The MR scanner collects the samples in *k*-space no matter what the patient does.
- If the patient moves part way through the scan, the data collected will not be consistent, so will not reconstruct to the correct image.

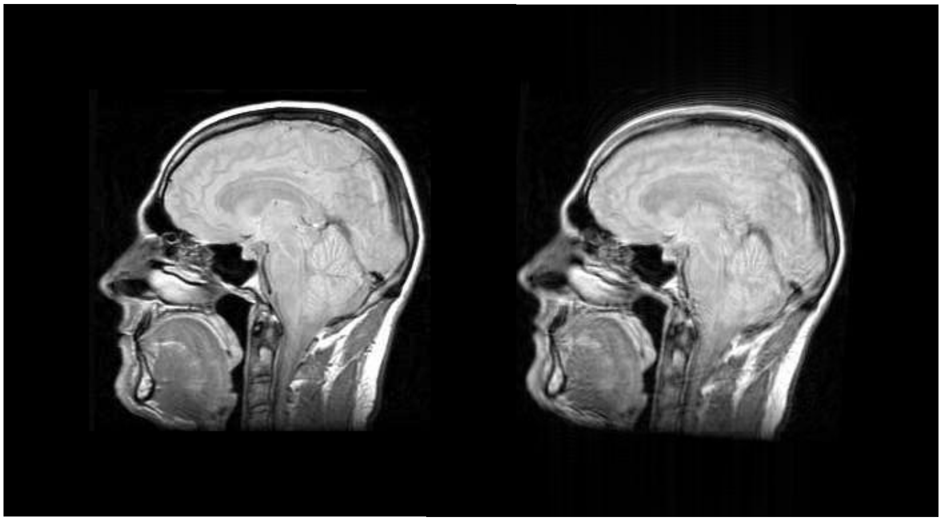
Intra-scan Motion in *k*-Space



Corrected *k*-space

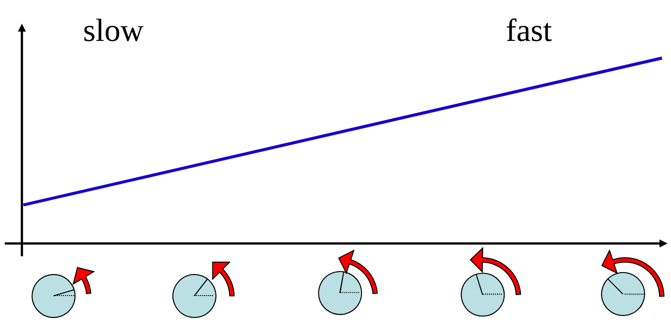


Intra-scan Motion

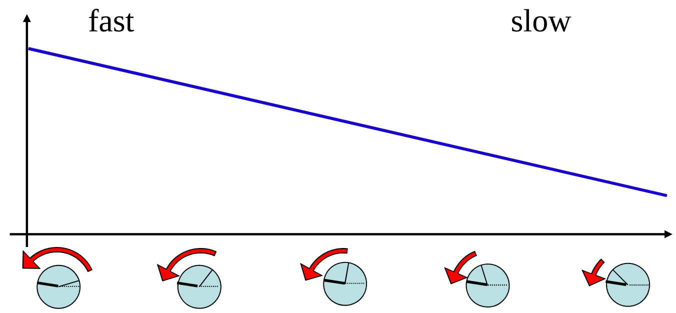


Gradient-Echo Imaging

Instead of flipping all the dipoles by 180°, reverse the phase difference to refocus the dipoles.



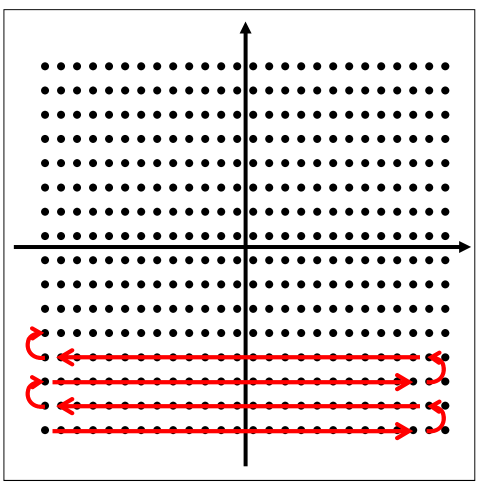
Gradient-Echo Imaging



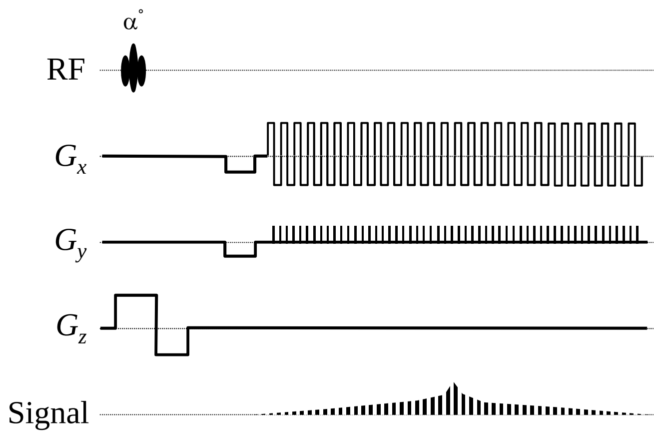
Echo-Planar Imaging (EPI)

A gradient-echo technique that allows one to collect a whole slice in one excitation.

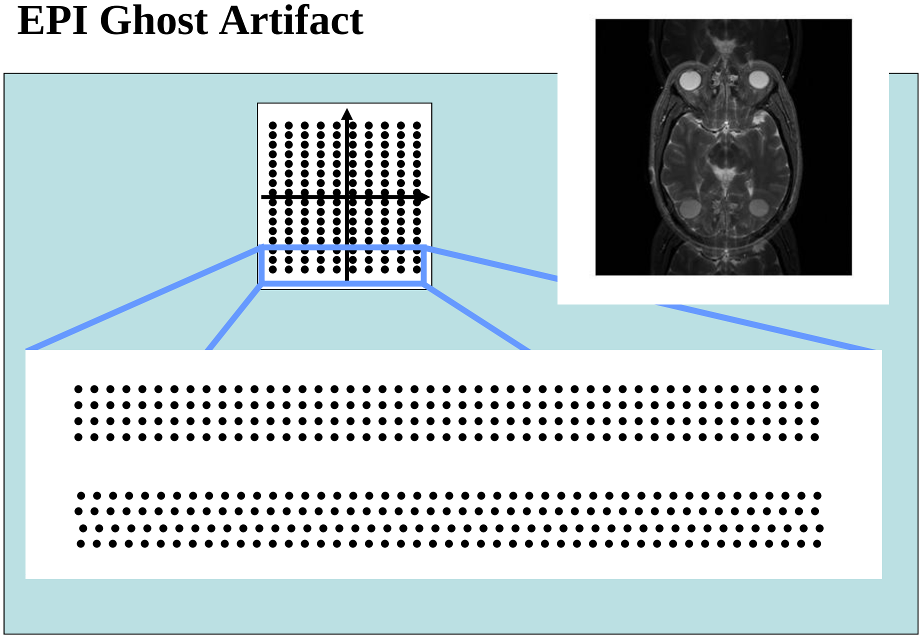
Exhibits T2* contrast, not T2 contrast.



EPI Scan Sequence



EPI Ghost Artifact

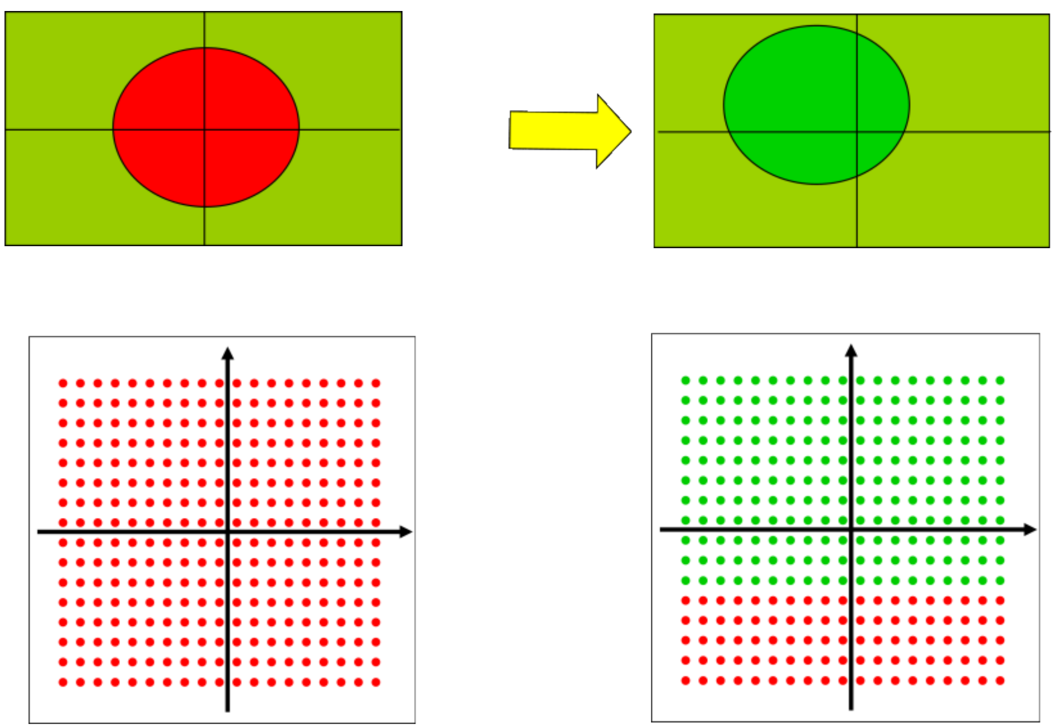


MRI Motion Compensation

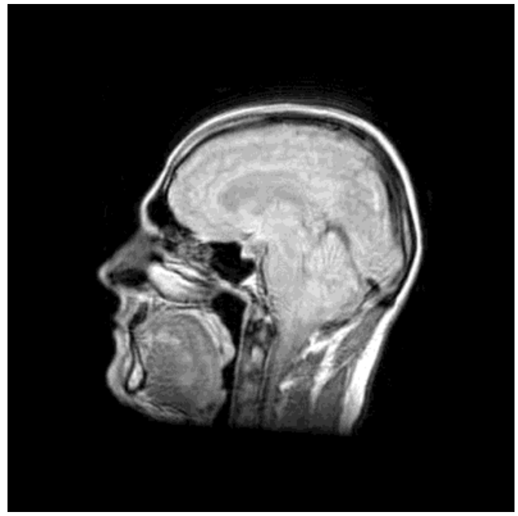
L45

Goal: To investigate how patient motion can corrupt MR images, and some methods for correcting it.

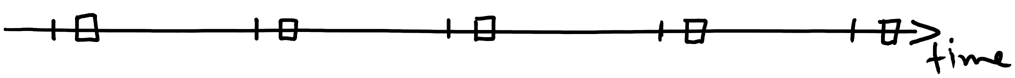
The MR scanner collects the samples in *k*-space no matter what the patient does.
If the patient moves part way through the scan, the data collected will not be consistent, so will not reconstruct to the correct image. This motion that happens during the acquisition of a single image is called   motion.

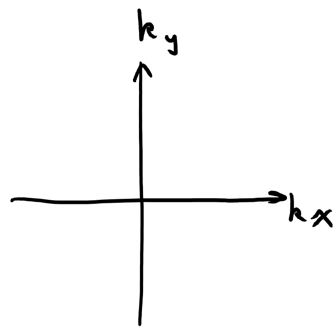


These inconsistencies cause   in the reconstructed images.



Much of the effects of motion can be corrected by **processing.** Most methods assume that motion is enough that motion within a single "phase encode" (row in *k*-space) is

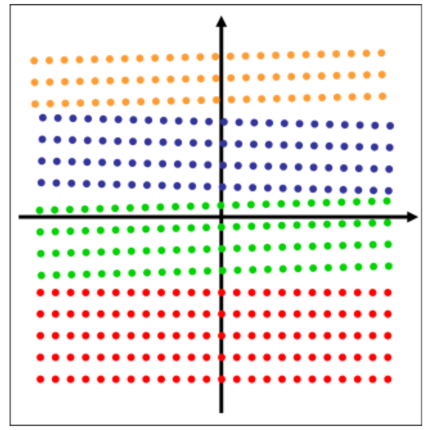




A translation in the patient causes the signal (the anatomy) to shift, which causes a phase ramp in its Fourier coefficients.

If you know the translation, you can undo it by simply changing the of the *k*-space samples using the appropriate

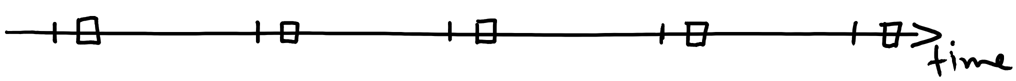
Rotation of the patient will   the corresponding content in the frequency domain.
If you know the rotations, you can compensate (to some extent) by   the samples.



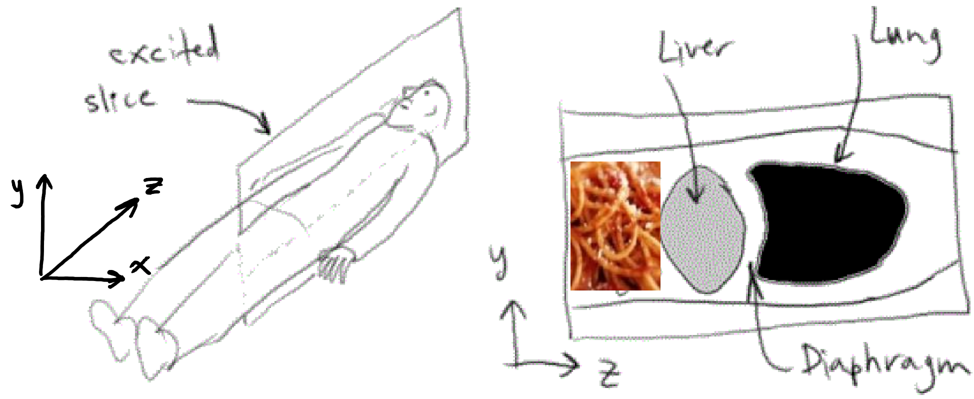
Information is lost if there are   regions.

Navigator Echoes

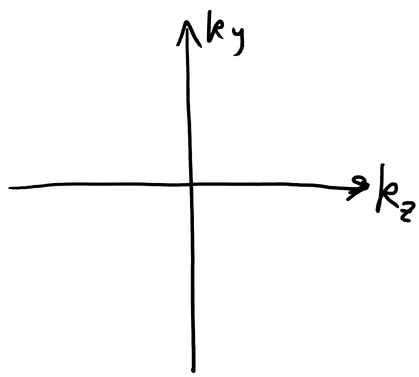
Navigator echoes, or   are small sets of *k*-space samples that are acquired between imaging excitations. Their purpose is to gain an estimate of the patient's position. The echoes commonly consist of a few lines in *k*-space. These ultra-fast acquisitions are   throughout the scanning sequence, and are useful in intra-scan motion correction because they offer a   set of measurements that can be used to



For these NAVs to be super-fase, naturally they are not full images.
Consider the saggital slice below.

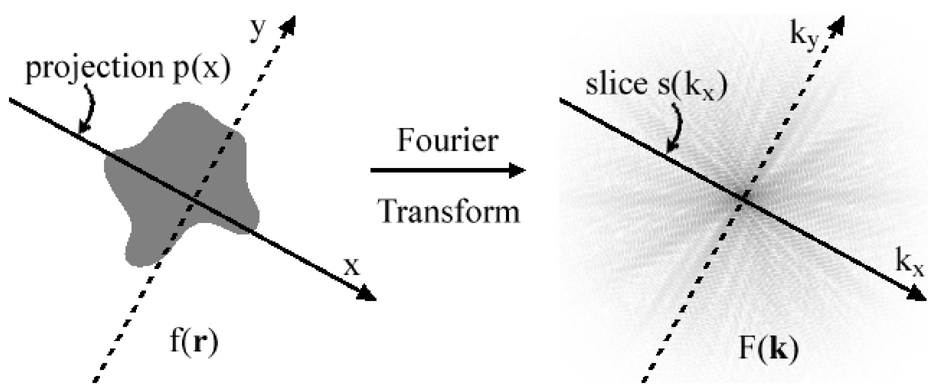


One could use all 3 gradients to acquire an image like above, but that would take too long (many excitation cycles). Instead, what would happen if, after exciting the slice, only the   was used for spatial encoding?
Ie.



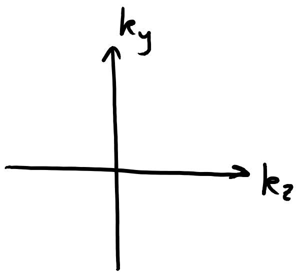
Performing the reconstruction (via a 1D IFFT), this gives a 1D signal. What does it represent?
Hint: Unsampled coefs are all zeros.
What would the 2D IFFT give you?

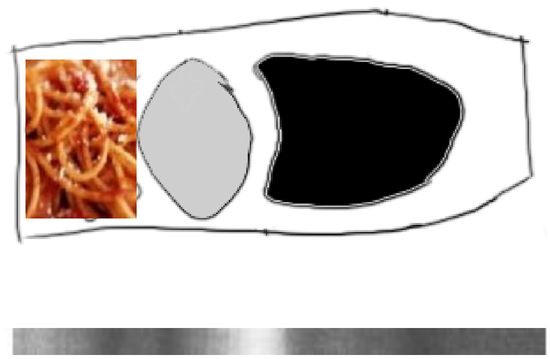
Recall the Projection Slice Theorem:



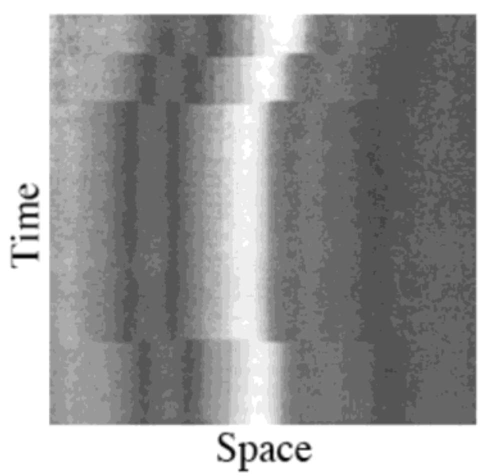
http://en.wikipedia.org/wiki/Projection-slice_theorem

One line of *k*-space samples gives you a   of the spatial-domain signal.

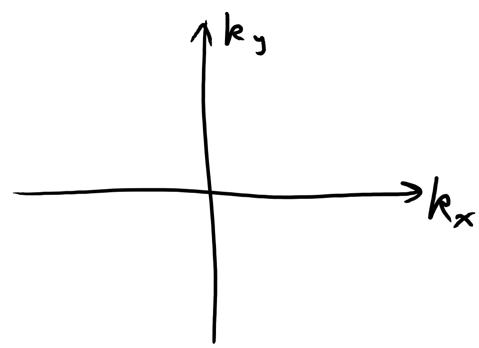




Collecting these projections during the scan can give use information about motion during the scan.



An   collects a circle of samples in *k*-space, and allows one to determine both



Entropy Focussing

If you do not know the patient motion, there are some methods to try to guess the motion.

IEEE TRANSACTIONS ON MEDICAL IMAGING, VOL. 16, NO. 6, DECEMBER 1997 903

Automatic Correction of Motion Artifacts in Magnetic Resonance Images Using an Entropy Focus Criterion

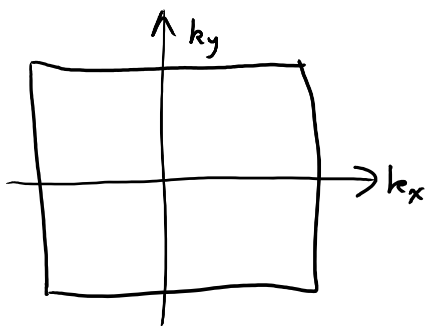
David Atkinson, Derek L. G. Hill,* *Member, IEEE*, Peter N. R. Stoyle, Paul E. Summers, and Stephen F. Keevil

This approach is based on the fact that most movements cause artefacts to appear in the background and other dark parts of the image. These additional structures tend to the of the image.

If we assume the motion is fairly slow, we can use a multi-resolution approach in time.

Here is the idea:

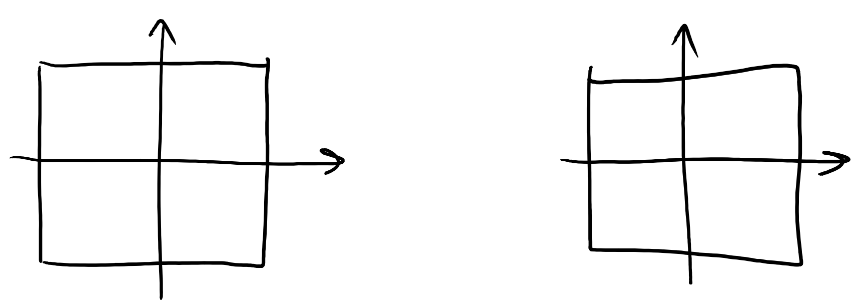
1) Group phase encodes into chunks, and assume motion only occurs between these chunks.



2) Choose motion parameters, and apply the corresponding correction to the each chunk.
3) Reconstruct the image (IFFT).
4) Compute the   of the reconstructed image.
5) Adjust the motion parameters to reduce the image

Optimization: minimize the image   by iterating the steps above.

Once it's done, subdivide the temporal chunks into smaller chunks and repeat the optimization.

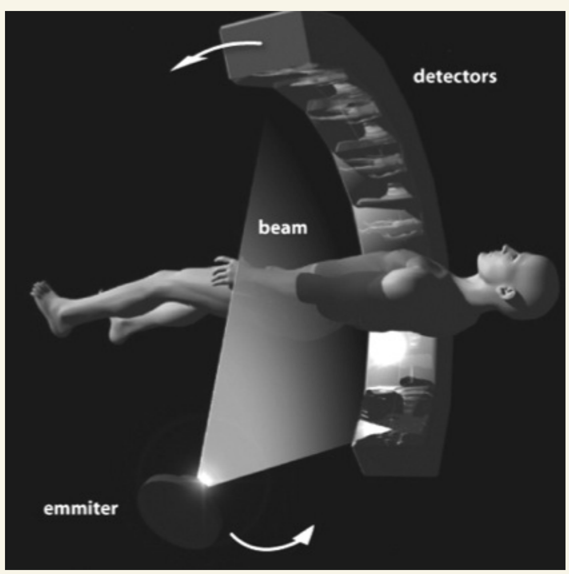


The translation corrections can be applied using Fourier Shift Theorem, and rotation corrections are implemented by rotating the *k*-space samples.
Note that the original *k*-space data is already in the   domain, so these corrections are being applied

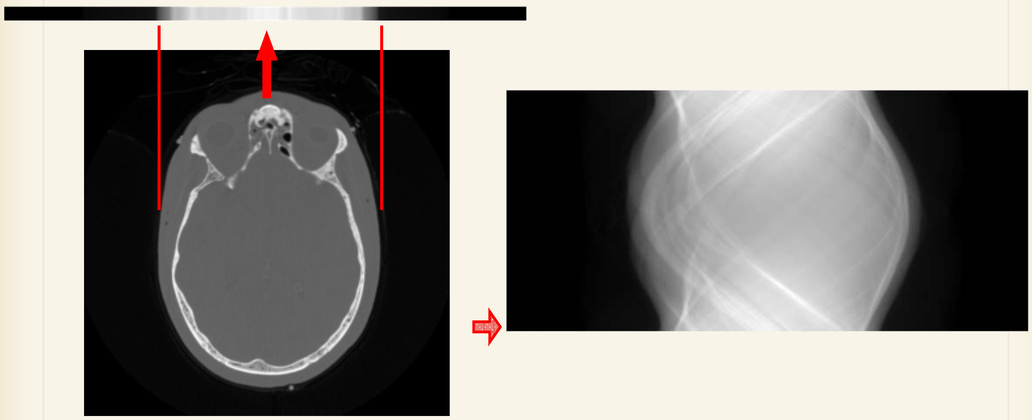
to the raw samples.

http://www.auh.dk/sks/afd/billeddi/dk/

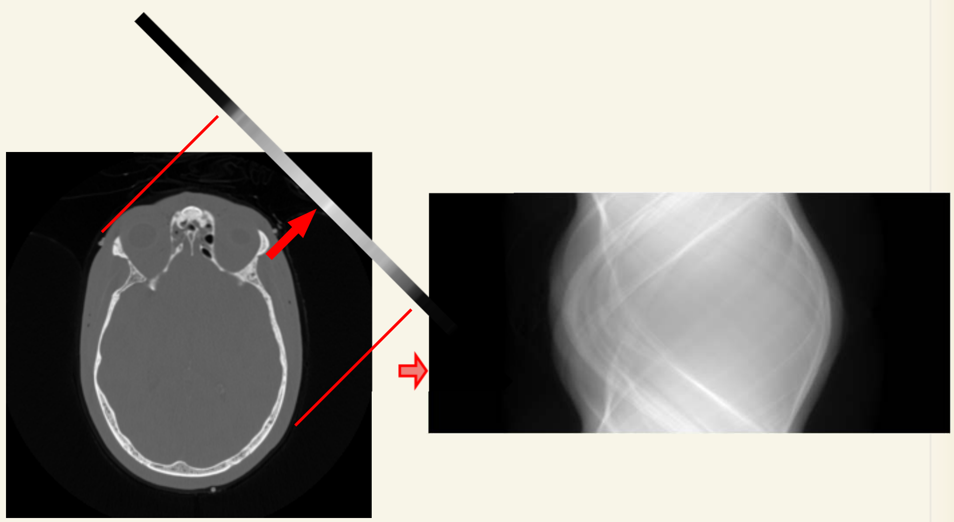
CT Reconstruction



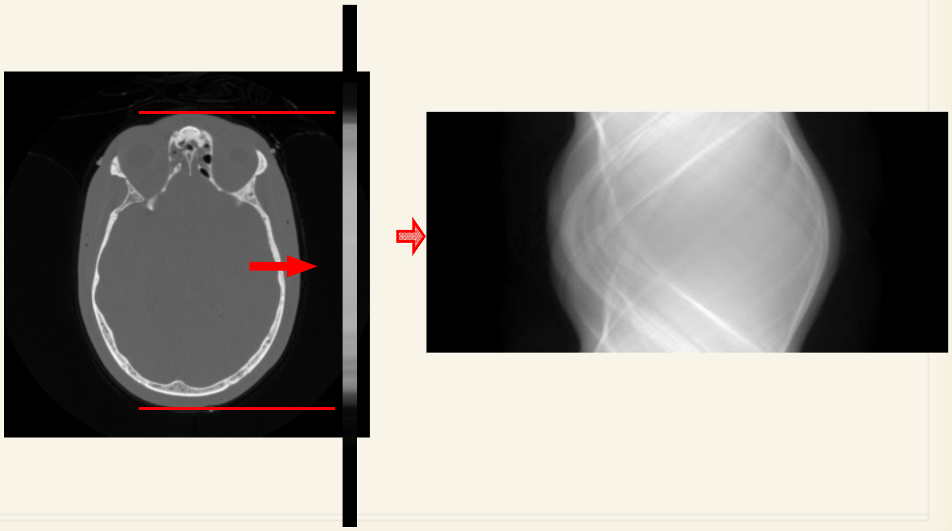
A slice from many small x-rays



A slice from many small x-rays



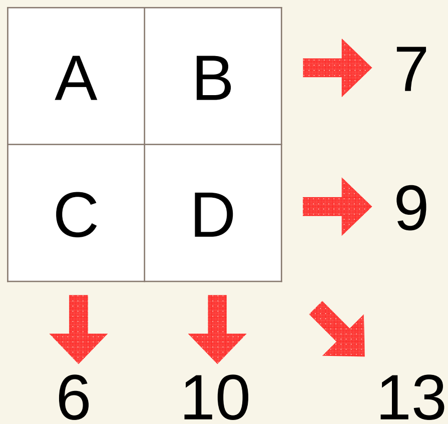
A slice from many small x-rays



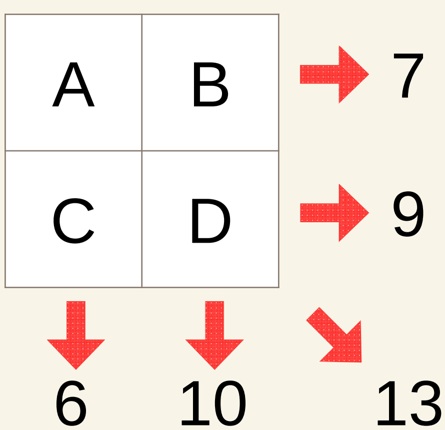
Arithmetic Reconstruction

5	2	7
1	8	9
6	10	13

Arithmetic Reconstruction



Arithmetic Reconstruction



A+B=7

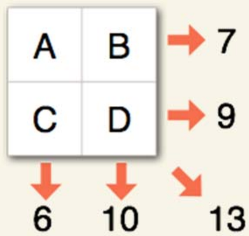
C+D=9

A+D=13

A+C=6

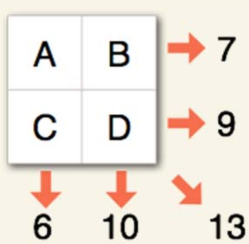
B+D=10

Arithmetic Reconstruction



$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 13 \\ 6 \end{bmatrix}$$

Arithmetic Reconstruction



$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \approx \begin{bmatrix} 7 \\ 9 \\ 13 \\ 6 \\ 10 \end{bmatrix} ?$$

$$\mathbf{MX} \approx \mathbf{P}$$

Least-Squares Solution

$$\mathbf{MX} = \mathbf{P}$$

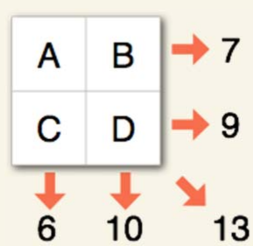
$$\mathbf{M}^{\mathrm{T}}\mathbf{MX} = \mathbf{M}^{\mathrm{T}}\mathbf{P}$$

$$\mathbf{X} = \left(\mathbf{M}^{\mathrm{T}}\mathbf{M}\right)^{-1}\mathbf{M}^{\mathrm{T}}\mathbf{P}$$

$$\text{Minimizes} \quad \|\mathbf{MX} - \mathbf{P}\|_2^2$$

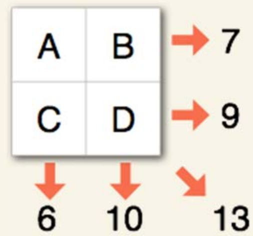
Iterative Reconstruction

A	B
C	D



Iterative Reconstruction

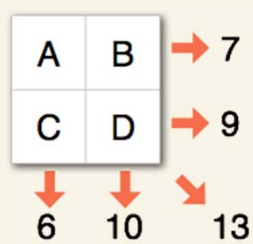
3.5	3.5	→ 7
4.5	4.5	→ 9



Distribute row-sums across each row.

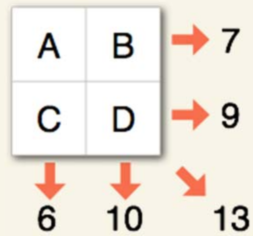
Iterative Reconstruction

3.5	3.5
4.5	4.5
↓ 8	↓ 8



Split new residuals evenly over columns.

Iterative Reconstruction

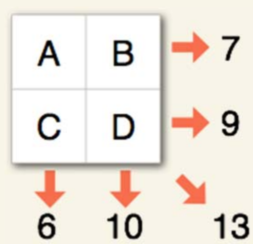


2.5	4.5
3.5	5.5

6 10

Split new residuals evenly over columns.

Iterative Reconstruction



2.5	4.5
3.5	5.5

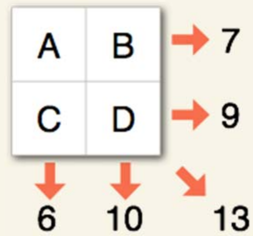
8

And now the diagonal vs. off-diagonals.

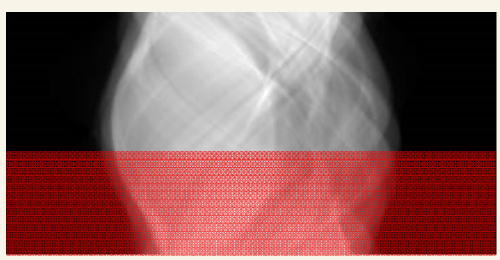
Iterative Reconstruction

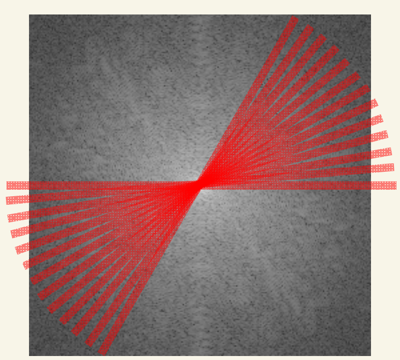
5	2
1	8

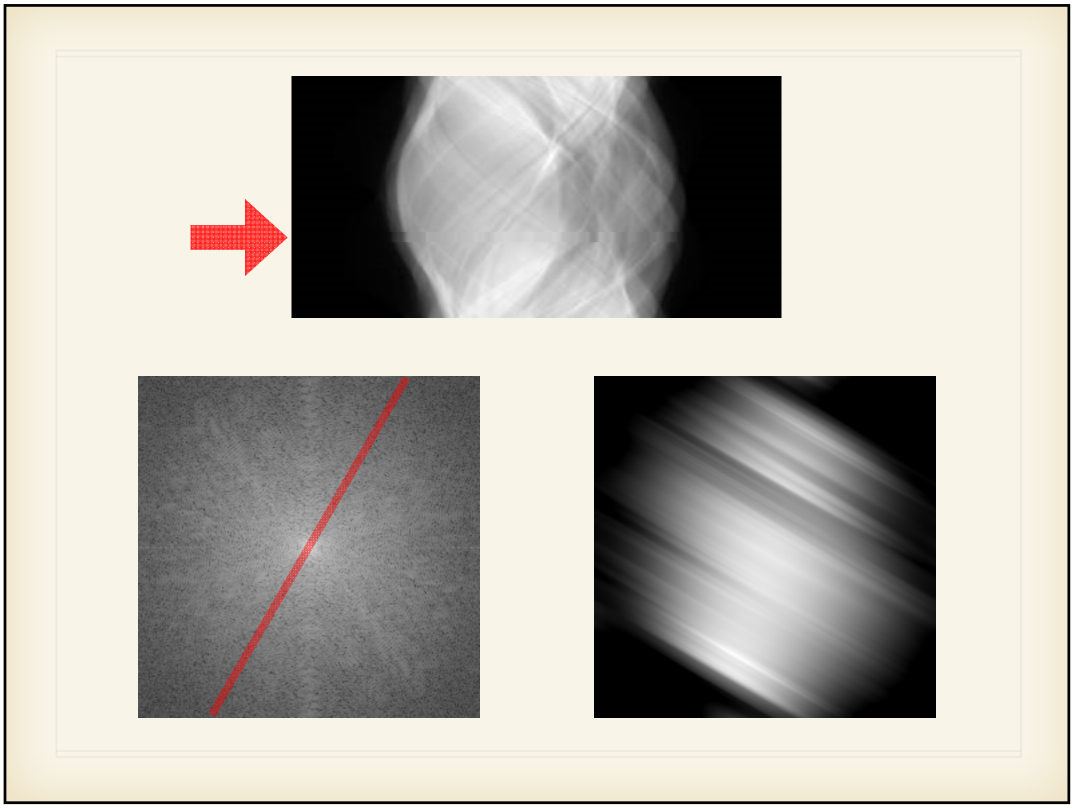
13



And now the diagonal vs. off-diagonals.







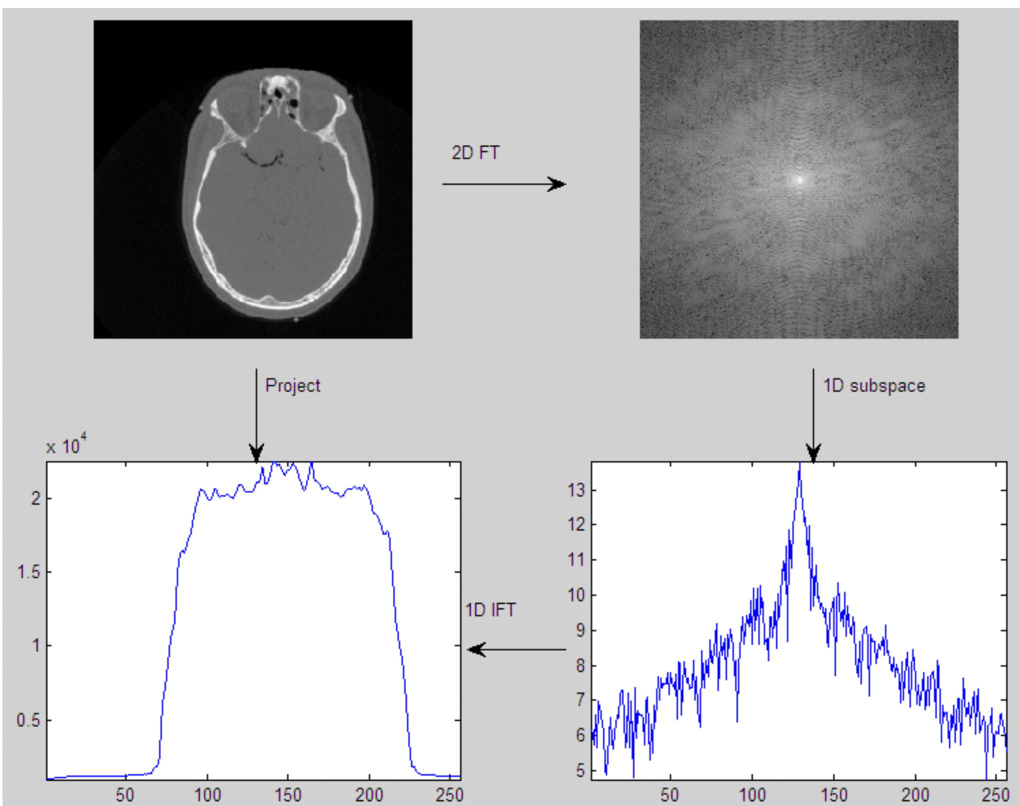
CT Back Projection

L47

Goal: To see how the Fourier Projection Theorem can help us in CT reconstruction.

Recall the **Fourier Projection Theorem**
(see the end of L09)

$$\mathcal{F}\left\{ p_x(f(\vec{x}))(x) \right\}(\omega) = F(\omega, 0)$$



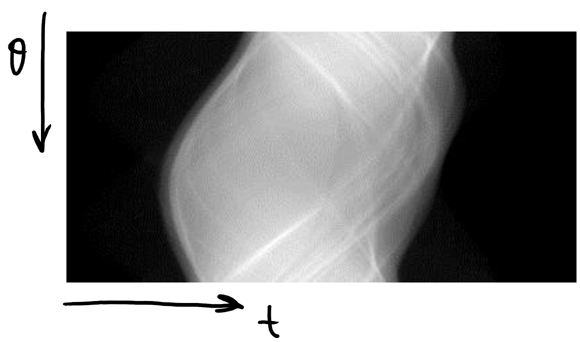
Theory of Back Projection

Since the Fourier transform is rotation invariant, we can apply the above in any direction.

$$\mathcal{F}_{1D}\left\{ p_\theta\left(f(\vec{x})\right)(t) \right\}(\rho) \ =$$

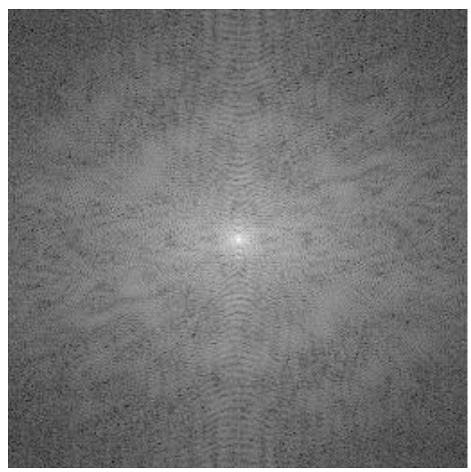
Denote this as
The scanner gives us

The spatial-domain projection at angles



We just take the 1D-FT over to get

Notice that this is simply a polar representation of



Hence, we can reconstruct f by taking the inverse FT, which itself is constructed using

We want

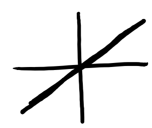
$$f(x,y) =$$

Change of variables...

$$f(x,y) =$$

Instead of $\rho \geq 0$ and $-\pi \leq \theta < \pi$

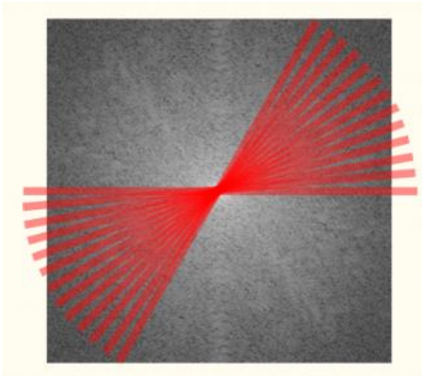
we can integrate



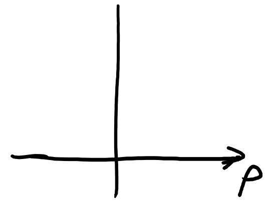
$$f(x,y) =$$

This suggests that the way to reconstruct an image is to populate the frequency domain by adding each

(astounding PowerPoint demo)



Then multiply by
the cone filter.

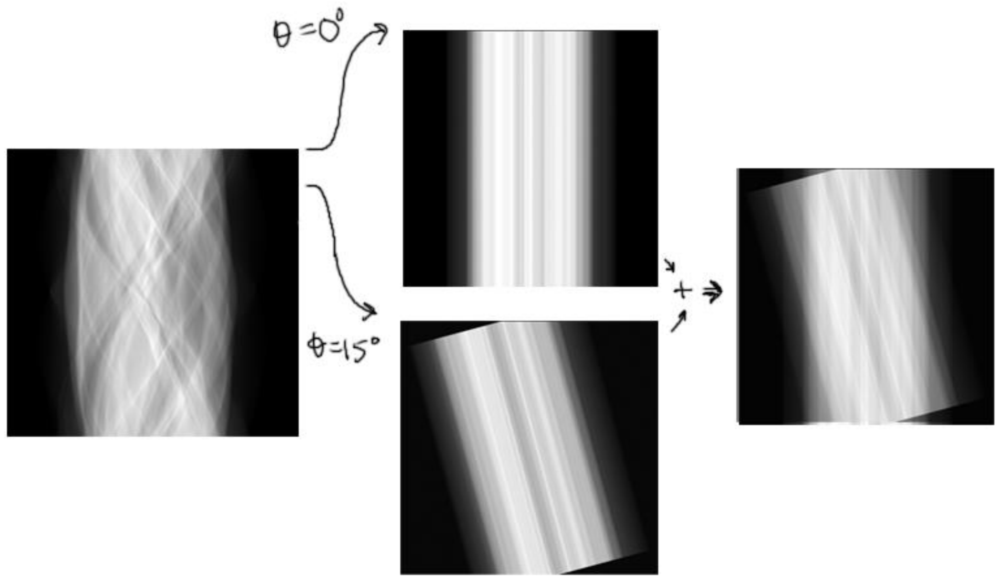


The cone filter compensates for the   of projections near the origin. It's essentially a   filter.

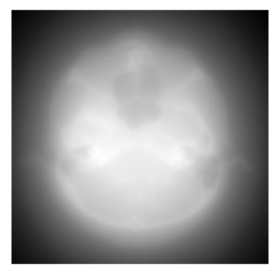
Filtered Back Projection

The method described above is called "filtered back projection".
However, while this frequency-domain method works in theory, it requires   in the frequency domain, which can sometimes be

Instead, we can do the equivalent operations in the spatial domain.
We take each projection and   it. That is, we   it back across the image in the   it was acquired (ie. the gantry angle).

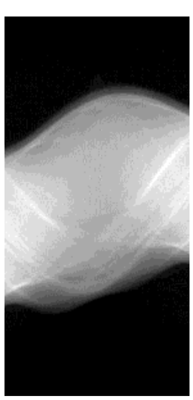


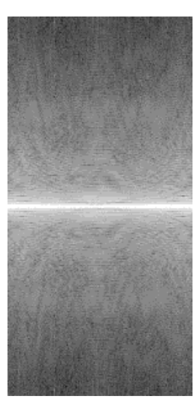
Then we simply add the backprojections together… one for each projection in our Radon Transform.

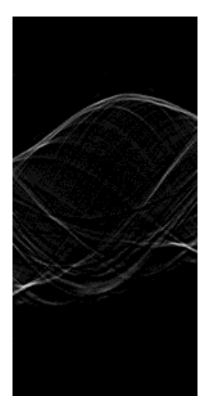


The resulting image will be blurry. That's because we haven't applied the yet We can do that by multiplying the 2D FFT of our image by a cone.

This filter can be applied in 1D to the projections themselves.







Then we do the back projection, and get a much crisper image.