Integral de una constante

La **integral de una constante** es igual a la constante por la variable x.

$$\int k\,dx = k \cdot x + C$$

Integral de cero

La **integral de cero** es igual a la constante C.

$$\int 0\,dx = C$$

Integral de una potencia

La **integral de una potencia** es igual a la variable elevada a la potencia $n+1$ sobre $n+1$ sumando una constante.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad n \neq -1$$

$$\int u^n \cdot u'\,du = \frac{u^{n+1}}{n+1} + C \qquad n \neq -1$$

Ejemplos de integrales

1. $\int 7\,dx$

$$\int 7\,dx = 7x + C$$

2. $\int x^6 dx$

$$\int x^6 dx = \frac{x^{6+1}}{6+1} + C = \frac{x^7}{7} + C$$

3. $\int 7x^3 dx$

$$\int 7x^3 dx = \frac{7x^{3+1}}{3+1} + C = \frac{7x^4}{4} + C$$

4. $\int x^{\frac{2}{3}} dx$

$$\int x^{\frac{2}{3}}dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + C = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{3\sqrt[3]{x^5}}{5} + C = \frac{3\sqrt[3]{x^3 \cdot x^2}}{5} + C = \frac{3x \cdot \sqrt[3]{x^2}}{5} + C$$

5 $\quad\displaystyle\int \frac{3}{x^4}dx$

$$\int \frac{3}{x^4}dx = \int 3x^{-4}dx = \frac{3x^{-4+1}}{-4+1} + C = \frac{3x^{-3}}{-3} + C = -x^{-3} + C = -\frac{1}{x^3} + C$$

6 $\quad\displaystyle\int \sqrt[3]{x}\,dx$

$$\int \sqrt[3]{x}\,dx = \int x^{\frac{1}{3}}dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3}{4}x^{\frac{4}{3}} + C = \frac{3}{4}x\sqrt[3]{x} + C$$

7 $\quad\displaystyle\int \frac{1}{\sqrt[4]{x}}dx$

$$\int \frac{1}{\sqrt[4]{x}}dx = \int x^{\frac{-1}{4}}dx = \frac{x^{\frac{-1}{4}+1}}{\frac{-1}{4}+1} + C = \frac{x^{\frac{3}{4}}}{\frac{3}{4}} + C = \frac{4}{3}x^{\frac{3}{4}} + C = \frac{4}{3}\sqrt[4]{x^3} + C$$

8 $\quad\displaystyle\int \frac{1}{\sqrt[3]{x^2}}dx$

$$\int \frac{1}{\sqrt[3]{x^2}}dx = \int x^{\frac{-2}{3}}dx = \frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} + C = \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C = 3\sqrt[3]{x} + C$$

9 $\quad\displaystyle\int \frac{1}{x^2\sqrt[5]{x^2}}dx$

$$\int \frac{1}{x^2\sqrt[5]{x^2}}dx = \int x^{-2}x^{\frac{-2}{5}}dx = \int x^{\frac{-12}{5}}dx = \frac{x^{\frac{-7}{5}}}{\frac{-7}{5}} + C = -\frac{5}{7\sqrt[5]{x^7}} + C$$

10 $\quad\displaystyle\int (x^4 - 6x^2 - 2x + 4)dx$

$$\int (x^4 - 6x^2 - 2x + 4)dx = \frac{x^5}{5} - \frac{6x^3}{3} - \frac{2x^2}{2} + 4x + C = \frac{x^5}{5} - 2x^3 - x^2 + 4x + C$$

11 $\quad\displaystyle\int (3\sqrt{x} + \frac{10}{x^6})dx$

$$\int (3\sqrt{x} + \frac{10}{x^6})dx = \int (3x^{\frac{1}{2}} + 10x^{-6})dx = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{10x^{-5}}{5} + C = 2x\sqrt{x} - \frac{2}{x^5} + C$$

12
$$\int (\frac{x^2 + \sqrt[3]{x^2}}{\sqrt{x}})dx$$

$$\int (\frac{x^2 + \sqrt[3]{x^2}}{\sqrt{x}})dx = \int (\frac{x^2}{\sqrt{x}} + \frac{\sqrt[3]{x^2}}{\sqrt{x}})dx = \int (x^{\frac{3}{2}} + x^{\frac{1}{6}})dx =$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^{\frac{7}{6}}}{\frac{7}{6}} + C = \frac{2\sqrt{x^5}}{5} + \frac{6\sqrt[6]{x^7}}{7} + C = \frac{2x^2\sqrt{x}}{5} + \frac{6x\sqrt[6]{x}}{7} + C$$

13
$$\int (\sqrt{5x} + \sqrt{\frac{5}{x}})dx$$

$$\int (\sqrt{5x} + \sqrt{\frac{5}{x}})dx = \int (\sqrt{5}x^{\frac{1}{2}} + \sqrt{5}x^{\frac{-1}{2}})dx = \sqrt{5}\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \sqrt{5}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C =$$

$$\frac{2\sqrt{5}x\sqrt{x}}{3} + 2\sqrt{5}\sqrt{x} + C = \frac{2x\sqrt{5x}}{3} + 2\sqrt{5x} + C$$

14
$$\int (\frac{3\sqrt{x} - 5\sqrt[3]{x^2}}{2\sqrt[4]{x}})dx$$

$$\int (\frac{3\sqrt{x} - 5\sqrt[3]{x^2}}{2\sqrt[4]{x}})dx = \int (\frac{3\sqrt{x}}{2\sqrt[4]{x}} - \frac{5\sqrt[3]{x^2}}{2\sqrt[4]{x}})dx = \int (\frac{3}{2}x^{\frac{1}{4}} - \frac{5}{2}x^{\frac{5}{12}})dx =$$

$$= \frac{3}{2}\frac{x^{\frac{5}{4}}}{\frac{5}{4}} - \frac{5}{2}\frac{x^{\frac{17}{12}}}{\frac{17}{12}} + C = \frac{6}{5}\sqrt[4]{x^5} - \frac{30}{17}\sqrt[12]{x^{17}} + C$$

15 $\int \sin x \cos x\, dx$

$$\int \sin x \cos x\, dx = \frac{1}{2}\sin^2 x + C$$

16
$$\int \sin^2 \frac{x}{2} \cos \frac{x}{2}dx$$

$$\int \sin^2 \frac{x}{2} \cos \frac{x}{2}dx = 2\int \sin^2 \frac{x}{2} \cos \frac{x}{2} \cdot \frac{1}{2}dx = \frac{2}{3}\sin^3(\frac{x}{2}) + C$$