

In The Name of Allah
Pattern Recognition (Autumn 2018)
Instructor: Mahdi Yazdian
Practical Exercise#3: Density Estimation

Due Date: 97.09.04

Perform this homework using MATLAB or Python.

[Part1: Parametric Methods]

A) ML Parameter Estimate

- a) Suppose a Normal Distribution $N(\mu_1, \Sigma_1)$ with the following mean and variance.

Note that in reality we do not access to this information.

$$\mu_1 = [0 \quad 5]^T \quad \Sigma_1 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

- b) Generate N samples with N=10 from this density;

Now, we are going to estimate this density using an ML estimate. For this, we have to suppose a specific distribution for these samples. Based on the above simulation, we assume the density of the samples is Gaussian;

Estimate the parameter of the Gaussian using ML estimate; report the estimated parameters and compare them with the true values.

- c) Plot the true density and the estimated density.
d) Repeat Section a and b for 20 times; Compute and report the bias and variance; Compare and discuss about the results.
e) Repeat Section a to d for N=100, N=1000. Discuss the effect of sample size in the performance of the estimation.
f) Repeat all the previous sections with the following mean and covariance; Compare the results with the previous ones.

$$\mu_2 = [5 \quad 0]^T \quad \Sigma_2 = \begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$$

B) [optional] MAP Parameter Estimate

Consider a Normal Distribution $N(\mu_1, \Sigma_1)$ as defined in Section A. Assume Σ_1 is known and μ_1 is unknown; Estimate the mean parameter of this density using MAP estimate. Suppose the prior of the mean is as follows:

$$P(\mu) = N(\mu_\mu, \Sigma_\mu) = \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \right)$$

Note: repeat this section for N=10, 100, 1000

- a) Complete the following table; Compare and discuss about the results.

#samples	True mean	Estimated mean		
		10	100	1000
μ_{ML}	$[0 \ 5]^T$			
μ_{MAP}				

[Part2: Non-Parametric Methods]

C) Non-Paramete

- a) Suppose a Mixture Normal Distribution $p(x) = 0.5 N(\mu_1, \Sigma_1) + 0.5 N(\mu_2, \Sigma_2)$ with the following means and variances.

$$\mu_1 = [0 \ 5] \quad \Sigma_1 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \quad \mu_2 = [5 \ 0] \quad \Sigma_2 = \begin{bmatrix} 1 & -1 \\ -1 & 4 \end{bmatrix}$$

Note that in reality we do not access to this information.

Generate N samples with N=200.

- b) Using the generated samples in Section (a), estimate their density via the Parzen Window method for $h=0.2, 0.4, 0.8, 1.6$; plot the true density and the estimated densities for different h values.
- c) Estimate their density via the k-nearest neighbors (KNN) method for $k=1, 10, 30$; plot the true density and the estimated densities for different k values.
- d) Compare and discuss about the results.

Note:

- Prepare a report in PDF format including the figures, answer to the questions and discussions mentioned in the homework.
- Put your report and your codes within a folder named as “YourNameYourFamily-Practical”+ “Exercise Number”.
(Note that your code is needed to be self-comment)
- Submit all things in a zipped folder.

Good Luck