HW #4 Solutions

1) a) $\mu_1 = \dfrac{x_1 + 2x_2}{6} + 9$

$\mu_1 = \dfrac{E[x_1] + 2E[x_2]}{6} + 9 = \dfrac{E[x_1]}{2} + 9$

$Var(\mu_1) = \left(\frac{1}{6}\right)^2 Var(x_1) + \left(\frac{2}{6}\right)^2 \cdot Var(x_2)$

$= \frac{1}{36} \cdot 10 + \frac{4}{36} \cdot 15 = \frac{70}{36}$ //

2) a) $bias(\hat{p}) = E[\hat{p}] - p$

$= \dfrac{E[x]}{10} - p = \dfrac{12p}{10} - p = \dfrac{p}{5}$ //

b) $Var(\hat{p}) = E\left[(\hat{p} - E[\hat{p}])^2\right]$

$= E\left[\left(\frac{x}{10} - \frac{12p}{10}\right)^2\right] = \dfrac{E[x^2]}{100} - \dfrac{24}{100} p \cdot \underbrace{E[x]}_{12p} + \dfrac{144}{100} p^2$

$= \dfrac{12p(1-p) + 144p^2}{100} - \dfrac{24}{100} \cdot 12p^2 + \dfrac{144}{100} p^2$

$= \dfrac{12p - 12p^2 + 144p^2 - 288p^2 + 144p^2}{100} = \dfrac{12p(1-p)}{100}$

c) $\tilde{p} \sim N\left(\dfrac{6p}{5}, \dfrac{12p(1-p)}{100}\right)$

d) $MSE(\hat{p}) = \dfrac{p^2}{25} + \dfrac{12p - 12p^2}{100} = \dfrac{12p - 8p^2}{100}$

$bias\left(\frac{x}{12}\right) = \emptyset$

$MSE\left(\frac{x}{12}\right) = Var\left(\frac{x}{12}\right) = \dfrac{Var(x)}{144} = \dfrac{12p(1-p)}{144}$

$\dfrac{12p - 8p^2}{100} > \dfrac{12p - 12p^2}{100} \Rightarrow$ So, YES!

4) $p = \frac{250}{450} = \frac{5}{9}$

$$S.E. = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{\frac{5}{9}\cdot\frac{4}{9}}{450}} = \frac{1}{27}\cdot\frac{\sqrt{2}}{\sqrt{5}}$$ ✓

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5) a) 15

$$CI_t = \left(\bar{x} - \frac{s}{\sqrt{n}} t_{0,025,14},\ \bar{x} + \frac{s}{\sqrt{n}} t_{0,025,14}\right)$$

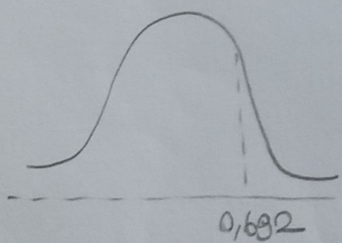
$$= (7,91 - 0,2\cdot(2,14),\ 7,91 + 0,2\cdot(2,14)) = (7,482,\ 8,338)$$

b) 10 $CI_0 = \left(\bar{x} - \frac{s}{\sqrt{n}} t_{0,1,14},\ \infty\right)$

$$= (7,91 - (0,2)\cdot(1,76),\ \infty) = (7,558,\ \infty)$$

c) 10 $H_0 = \mu \geq 8,05$

$p = Pr\{\text{This or worse} \mid H_0\} = Pr\{T < t\}$



$$t = \frac{7,81 - 8,05}{0,2} = \frac{-0,14}{0,2} = -0,7$$

$t_{0,2,14} = 0,692$ $\qquad 0,7 > 0,692 \Rightarrow$ Ho is rejected!

6) $C = \bar{x} - \frac{\sigma}{\sqrt{n}} \cdot t_{0,05,99} = 0,7680 - \frac{0,042}{10} \cdot 1,66$

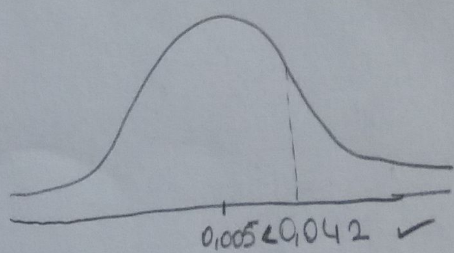
$\cong 0,761$

Ho : $\mu < 0,7675$

$p = Pr\{T \geq t\}$ $\qquad t = \frac{0,768 - 0,7675}{0,042/\sqrt{100}} = 0,005$

$t_{0,05,99} = 0,042$

$t < t_0$ so Accept Ho!



✓

7) $\sigma = 100$

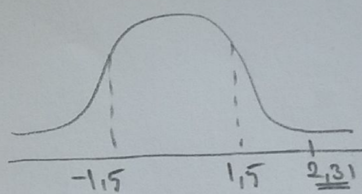
$$\bar{X} = \frac{1552 + 1475 + 1505 + 1462 + 1531 + 1538 + 1511 + 1432}{8} \cong 1500 \text{ hours}$$

$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right) = N\left(\mu, \frac{10^4}{8}\right)$$

$$\mu \in \left(\bar{x} - \frac{\sigma}{\sqrt{n}} \cdot z_{0,05}\right) = \left(1500 - \frac{100}{\sqrt{8}}(1,645), \infty\right) \cong (91,84, \infty)$$

For quality control purposes, it might be desired to specify a lower bound to lifetime. Below the lower bound, the fraction of lightbulbs is small (5%)

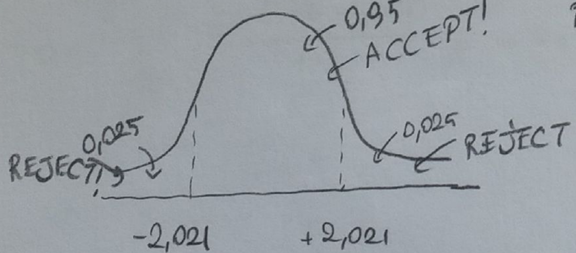
8) $H_0: \mu = 2,6$ cm
$H_1: \mu \neq 2,6$ cm
$\alpha = 0,05 \Rightarrow t_{\frac{0,05}{2}} = t_{0,025} = 2,31$



$$t = \frac{2,55 - 2,6}{0.1/\sqrt{9}} = \frac{-3 \cdot 0,05}{0,1} = -1,5$$

So, Ho is rejected!

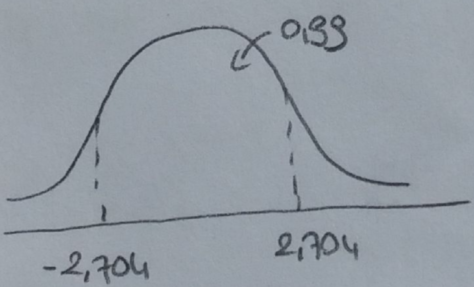
9) a) $t_{0,05,\,40} = 2,02$



if $-2,021 < t < 2,021 \Rightarrow$ Ho accepted!

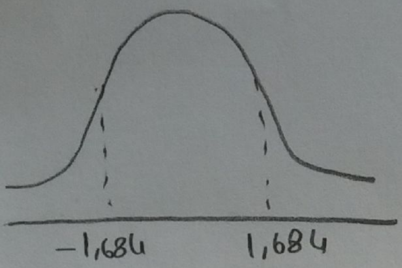
b) $t_{0,01,\,40} = 2,704$

if $|t| > 2,704 \Rightarrow$ Ho rejected!



9) c) $t = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{3{,}04 - 3}{\frac{0{,}06}{\sqrt{41}}} = 4{,}27$ $t_{0{,}1,\ 40} = 1{,}684$

$4{,}27 > 1{,}684 \Rightarrow$ so, reject Ho for $\alpha = 0{,}$



$t_{0{,}01,\ 40} = 2{,}704$

$4{,}27 > 2{,}704 \Rightarrow$ so, reject Ho for $\alpha = 0{,}01$

P_value $= 2P(|T| > 4{,}27) = P(T > 4{,}27 \text{ or } T < -4{,}27)$

read P-value from "Critical values of t-Distribution" table.

$= 2 * \underbrace{P(T \geq 4{,}27)}_{< 0{,}005} < 0{,}01$ (no need to exactly determine p-value

12) $\bar{x} = 3{,}28$

$n = 28$

$\sigma = 1{,}5$ (deviation)

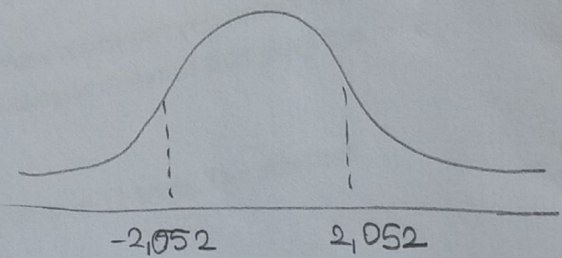
$\alpha = 0{,}05$ (0,95 C.I.)

$\bar{x} \pm t_{\frac{\alpha}{2}} \dfrac{s}{\sqrt{n}}$

$3{,}28 \pm 2{,}052 \cdot \dfrac{1{,}5}{\sqrt{28}}$

$-2{,}69 < \mu < 3{,}86$

$t_0 = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \dfrac{3{,}28 - 4{,}00}{\frac{1{,}5}{\sqrt{28}}} = -2{,}54$



IF $\mu = 4{,}00$, it is not in the interval of $-2{,}69 < \mu < 3{,}86$ so 4.00 is out of range!

$-2{,}54 = t_0 < t_{0{,}025} \Rightarrow$ Reject Ho!