MAT271E Probability and Statistics HW #2

Instructions: Please hand in your answers toTuğba Pamay by April 4, 2016 16:00. (Use the mailbox reserved for the course in the administrative office of the Computer and Informatics faculty). Late homeworks will not be accepted. 4-5 problems will be checked in detail which will contribute 80% to the final mark. The rest will be checked for completeness which will contribute 20% to the final mark.

1. A fair coin is flipped twice and a die is rolled once.

a) Describe the sample space for this combined experiment.

b) Let X represent the number on the top face of the die minus the number of Heads. Determine the realizations of X. Determine the probability of each realization.

c) Determine $F_X(x)$ and $f_X(x)$.

2. Let $F(x) = 0.5(x(u(x+1) - u(x-1)) + 1)$. Can this function be a valid distribution function? Why or why not?

3. Determine the ranges for a and b so that $F_X(x) = (1 - \exp(-x^2 / b))u(x-a)$ is a valid distribution function. Identify the type of r.v. as discrete, continuous or hybrid for different values of a and b .

4. Let random variable X a distribution function

$$F_X(x) = 0.1xu(x+2) + 0.6u(x) - 0.1xu(x-2) + 0.2(1 + u(x-2) - u(-2-x))$$

a) Identify the type of this random variable as discrete, continuous or hybrid.

b) Determine its realizations and their probabilities.

c) What is the total probability mass of all continuous realizations of this random variable.

c) Determine $\Pr\{-\infty < X < 0\}$.

d)Determine $\Pr\{X = 0\}$, $\Pr\{X = 1\}$.

e) Determine $\Pr\{-\infty < X \le 0\}$.

5. Each egg in a package of six eggs is rotten with a probability of 0.1.

a) Draw the distribution function for X: “Number of fresh eggs”.

b) Use the distribution function to compute the probability $\Pr\{1.9 < X < 5.5\}$.

c) Up to how many rotten eggs should we allow in a package so that the percentage of such packages is no more than 10%?

6. Random variable X is used to model the time (in sec.) between two consecutive events.

$$f_X(x) = \begin{cases} \frac{1}{5} e^{-(x-10)/5} & \text{x} \geq 10 \\ 0 & \text{x} < 10 \end{cases}$$

a) What is the smallest time between two consecutive events?

b) What is the probability that time between two consecutive events is less than 30 sec.?

c) What is the probability that time between two consecutive events is 20 sec.?

7. $f_X(x) = \begin{cases} 0 & \text{x} < 0 \\ \text{Ax}^2 & 0 \leq \text{x} < 1 \\ 0 & \text{x} \geq 1 \end{cases}$

Find A such that the function given above is a valid density function.

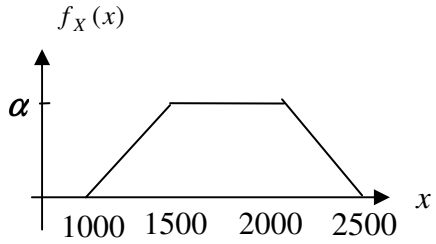
8. The Gaussian random variable X can be used to model the rainfall amount in a year (in cm). Let us assume that the annual rainfall amount in a city is distributed as N(50,10).

i) Determine the probability that the annual rainfall amount in the city is below 60% of the expected rainfall amount.

ii) Determine the probability density function for years in which the annual rainfall amount is between 40cm and 60cm.

iii) Determine the expected rainfall amount for years in which the annual rainfall amount is between 40cm and 60cm.

9. Let X represent the rental prices for apartments in a housing project



a) Determine α

b) What is the probability that an apartment has a rental price higher than 2000?

c) Determine the density function for apartments with rental prices between 1500 and 2000.

10.Let $Y = X(X-2)$ and $f_X(x) = 0.3\delta(x) + 0.2\delta(x+1) + 0.1\delta(x-1) + 0.4\delta(x-2)$. Determine $f_Y(y)$.

11. Let X be a standart Gaussian random variable. Determine the density function of the random variable $Y = \ln(X)u(x-1)$

12. One out of four days is rainy in a given city. On rainy days, the temperature T may be modeled by a Gaussian random variable with mean 15degrees Celsius and standard deviation 5 degrees Celsius. On days without rain, the temperature may be modeled by a Gaussian random variable with mean 23 degrees Celsius and standard deviation 3 degrees Celsius.

a) Determine $f_T(t \mid W)$ for i) $W = \{"rainy"\}$ ii) $W = \{"not\ rainy"\}$

b) Determine the density function for temperature $f_T(t)$. (Hint: Use a linear combination of $f_T(t \mid "rainy")$ and $f_T(t \mid "not\ rainy")$)

13. Determine the distribution function of random variable $Y = X^3 - 6X^2$ in terms of $F_X(.)$. (a is a positive real number.)