MAT271E Probability and Statistics HW #1

Instructions: Please hand in your answers to Tuğba Pamay by March 8, 2013 16:00. (Use the mailbox reserved for the course in the administrative office of the Computer and Informatics faculty). Late homeworks will not be accepted. 4-5 problems will be checked in detail which will contribute 80% to the final mark. The rest will be checked for completeness which will contribute 20% to the final mark.

1. A universal set is given as S={a ,b,c,d,f,g,j,l,p,x}. A={a, j,p}, B={b,c,d,g,p,x} are its subsets. Find the following:

a) $\overline{A} = S - A$

b) $A - B$

c) $A \cup B$

d) $B \cap (A \cup \overline{B})$

e) $\overline{A} \cap B$

2. Use Venn diagrams to prove:

i) $\overline{A \cup \overline{B}} = \overline{A} \cap B$,

ii) $A \cup B = A \cup (B - A) = (A - B) \cup (A \cap B) \cup (B - A)$.

3. Prove or give a counter example.

a) $\overline{A \cap (B \cup C)} = (\overline{A} \cup \overline{B}) \cap (\overline{A} \cup \overline{C})$

b) $\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$

4. An experiment has a sample space that consists of 12 equally likely outcomes. $S = \{a_1, a_2, ..., a_{12}\}$. Three events are defined as $A = \{a_1, a_5, a_6, a_9, a_{10}\}$, $B = \{a_1, a_2, a_5, a_7\}$ and $C = \{a_2, a_4, a_6, a_7, a_9, a_{11}\}$. Find

a) $P(A \cup C)$

b) $P(A \cap C)$

c) $P(A \mid C)$

d) $P(C \mid A)$

e) $P(B \cup \overline{C})$

f) $P(B - C)$

g) $P(A \mid (B \cup \overline{C}))$

h) $P(\overline{A \cup B})$

5. How many different ways can you order the letters of “Baggage”? You can not distinguish between two or more letters that are the same.

6. A multiple choice exam consists of 18 questions. Each question has three answers(i.e. A, B ve C). Every group of 6 consecutive questions has exactly two answers of each type. How many permutations of answers can you come up with?

7. A shoecase contains 5 pairs of shoes. Four shoes are drawn randomly. What is the probability that they belong to two pairs?

8. r boxes are placed in a row. n balls (n<r) are randomly placed these boxes. Find the probability that a) any two boxes contain a total of k balls. b)any two adjacent boxes contain a total of k balls.

9. . Two cards are drawn from a deck of 52 cards without replacement

a) What is the probability that the first card is hearts and the second one is not.

b) If the first one is hearts, what is the probability that the second one is not hearts.

c) If the first card is a 6, what is the probability that the second one is a spade.

d) What is the probability that the two cards have a combined value that is between 10 and 16? Jack, queen, king have a value of 10 each, ace has a value of 11.

11. By using total probability theorem, show that
$\min(P(A \mid B), P(A \mid \overline{B})) \le P(A) \le \max(P(A \mid B), P(A \mid \overline{B}))$

12. A plane crashes with probability 0.9 if both of its engines (engine #1 and engine #2) fail. It crashes with probability 0.05 if one engine fails. It crashes with probability 10^{-8} if no engine failure occurs. On each flight each engine has a probability of failure of 10^{-5}. Both engines fail with a probability of 10^{-8}.

a) Are crash event and events of engine failures independent of one another? Are the engine failures independent of each other?

b) If engine #1 fails what is the probability that engine #2 also fails?

c) What is the probability that the plane crashes?

d) In 10000000 flights what is the probability that 1 or more crashes occur?

13. The pills of a pharmaceutical drug is manufactured by two companies A and B. The pills from two companies are identical in appearance. The probability that a pill is made by company A and company B are 0.6 and 0.4, respectively. The probability of finding a pill in the market made by company A during the years 2014,2015 and 2016 are 0.2,0.7,0.1, respectively . The probability of finding a pill in the market made by company B during the years 2014,2015 and 2016 are 0.3,0.5,0.2, respectively .

a)Find the probability of purchasing a pill made by company A in 2016.

b)If a pill has a manufacturing date of 2016 written on its box, determine the probability that it was made by company A.

14. In a game of chance two dice are tossed one after the other. The player wins if the first one shows 5 or more, or if the difference of the two dice is 2. What is the probability of winning?

15. A microcontroller operating the speed brake of each wheel of a racing car malfunctions with a probability of 0.01. For safety, two such microcontrollers (one being a backup of the other) operate the speed brake of each of four wheels. The microcontrollers break down independently of each other. The racing car gets involved in an accident with probability 0.01 if the brakes on at least one out of four wheels fail. What is the probability of an accident? (Assume no other cause of break failure other than microcontroller break down)

15. 400 pills are placed in a drug bottle. Each pill fails the quality control check with a probability of $4x10^{-4}$. The bottle fails the quality control check if it contains two or more pills that fail their check. What is the probability that a bottle passes the quality control check?

16. A part on a car breaks down with probability 0.001. When the part breaks down the car is taken to service to get repaired.

a) What is the probability that 3 out of 4 such cars that we see on the street have got repaired.

b) The replaced part is likely to break down with probability 0.001. What is the probability that a car is taken to service no more than once?