1. For the language $L = \{a^i b^{i+j} a^j \mid i > 0, j \geq 0 \}$,

    a) Write the grammar production rules.
    b) Design a PDA for this language.
    c) Show how the strings $aabbba$ and $aaabbb$ are accepted by the PDA you designed.

2. Design a pushdown automaton (PDA) that recognizes the following language.

    $L(G) = \{a^k b^m c^n \mid k, m, n > 0 \text{ and } k = 2m + n\}$

SOLUTIONS:

1.

a) $L = \{a^i b^{i+j} a^j \mid i > 0, j \geq 0\} \rightarrow L = \{a^i b^i b^j a^j \mid i > 0, j \geq 0\}$

                    `<S> ::= <A><B>`

                    `<A> ::= a<A>b | ab`

                    `<B> ::= b<B>a | Λ`   Chomsky Type 2.

b) $M = (S, \Sigma, \Gamma, \delta, s_0, F)$

$S = \{q_0, q_1, q_2, q_3, f\}, \Sigma = \{a, b\}, \Gamma = \{a, b, c\}, s_0 = q_0, F = f$

$\delta = \{\ \underbrace{[(q_0, a, \Lambda), (q_1, ac)]}_{a},$ → push c to be able to check if the stack is empty

$\underbrace{[(q_1, a, \Lambda), (q_1, a)]}_{a^{i-1}}, \underbrace{[(q_1, b, a), (q_2, \Lambda)]}_{b}, \underbrace{[(q_2, b, a), (q_2, \Lambda)]}_{b^{i-1}}, \underbrace{[(q_2, \Lambda, c), (f, \Lambda)]}_{\text{accept } a^i b^i},$

$\underbrace{[(q_2, b, c), (q_2, bc)]}_{b}, \underbrace{[(q_2, b, b), (q_2, bb)]}_{b^{j-1}}, \underbrace{[(q_2, a, b), (q_3, \Lambda)]}_{a}, \underbrace{[(q_3, a, b), (q_3, \Lambda)]}_{a^{j-1}}, \underbrace{[(q_3, \Lambda, c), (f, \Lambda)]}_{\text{accept } a^i b^{i+j} a^j}\ \}$

c)

State	Tape	Stack	Transition Rule	State	Tape	Stack	Transition Rule
q_0	$aabbba$	Λ	$[(q_0, a, \Lambda), (q_1, ac)]$	q_0	$aaabbb$	Λ	$[(q_0, a, \Lambda), (q_1, ac)]$
q_1	$abbba$	ac	$[(q_1, a, \Lambda), (q_1, a)]$	q_1	$aabbb$	ac	$[(q_1, a, \Lambda), (q_1, a)]$
q_1	$bbba$	aac	$[(q_1, b, a), (q_2, \Lambda)]$	q_1	$abbb$	aac	$[(q_1, a, \Lambda), (q_1, a)]$
q_2	bba	ac	$[(q_2, b, a), (q_2, \Lambda)]$	q_1	bbb	$aaac$	$[(q_1, b, a), (q_2, \Lambda)]$
q_2	ba	c	$[(q_2, b, c), (q_2, bc)]$	q_2	bb	aac	$[(q_2, b, a), (q_2, \Lambda)]$
q_2	a	bc	$[(q_2, a, b), (q_3, \Lambda)]$	q_2	b	ac	$[(q_2, b, a), (q_2, \Lambda)]$
q_3	Λ	c	$[(q_3, \Lambda, c), (f, \Lambda)]$	q_2	Λ	c	$[(q_2, \Lambda, c), (f, \Lambda)]$
f	Λ	Λ		f	Λ	Λ	

2. $M = (S, \Sigma, \Gamma, \delta, s_0, F)$

$S = \{q_0, q_1, q_2, q_3, q_4, q_5\}, \Sigma = \{a, b, c\}, \Gamma = \{x, y\}, s_0 = q_0, F = q_5$

$\delta = \{\ \underbrace{[(q_0, a, \Lambda), (q_1, xy)]}_{a},$ → push y to be able to check if the stack is empty

$\underbrace{[(q_1, a, \Lambda), (q_1, x)]}_{a^{k-1}}, \underbrace{[(q_1, b, x), (q_2, \Lambda)]}_{b},$

$[(q_2, \Lambda, x), (q_3, \Lambda)],$ → needed as it is not possible to pop multiple symbols

$\underbrace{[(q_3, b, x), (q_2, \Lambda)]}_{b^{m-1}}, \underbrace{[(q_3, c, x), (q_4, \Lambda)]}_{c},$

$\underbrace{[(q_4, c, x), (q_4, \Lambda)]}_{c^{n-1}}, \underbrace{[(q_4, \Lambda, y), (q_5, \Lambda)]}_{\text{accept the word}}\ \}$