BLG311E – FORMAL LANGUAGES AND AUTOMATA

2017 SPRING

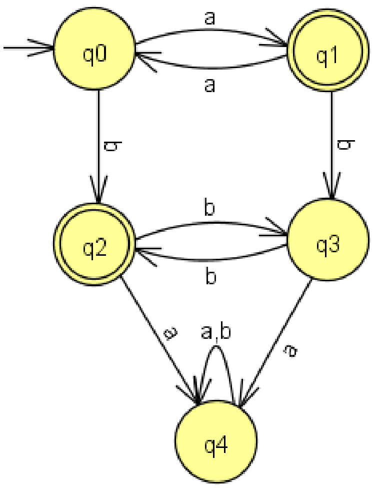
RECITEMENT 5

1) (Solution for Quiz 2) Consider the language L defined below.

$$L = \{a^n b^m \mid n + m \equiv 1 \; mod \; 2 \text{ and } n, m \geq 0\}$$

Draw the deterministic finite automaton (DFA) that accepts L as a state transition diagram.

Solution:



$a^n b^m$

q_0: n is an even number, m is zero

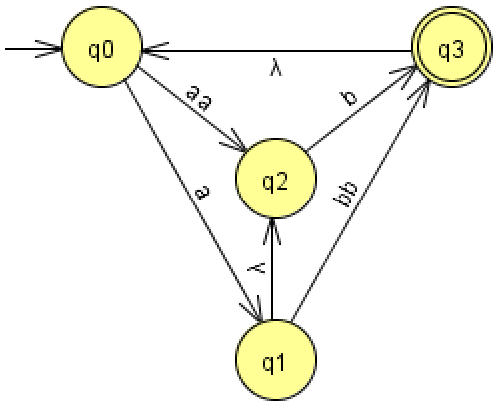
q_1: n is an odd number, m is zero

q_2: $n + m$ is an odd number

q_3: $n + m$ is an even number

q_4: death state (not in the $a^n b^m$ form)

2)



a) Heuristically derive the regular expression for the language recognized by the NFA whose state transition diagram is given aside.

b) Build the equivalent DFA for this NFA.

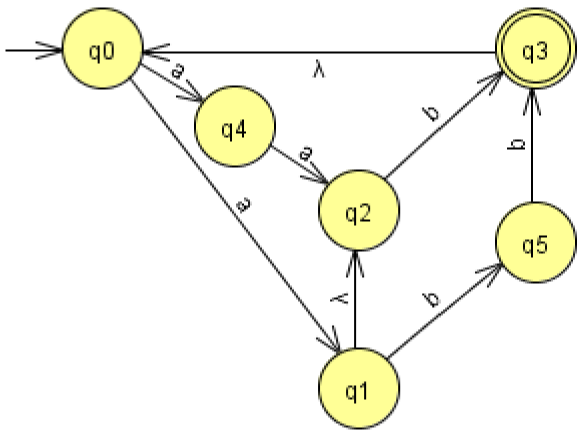
c) Produce the Type-3 grammar recognized by the DFA you found in (b).

d) Systematically derive the regular expression for the language defined by the Type-3 grammar you found in (c) and show that your answer in (a) is produced.

Solution:

a) $L = (abb \vee ab \vee aab)^{+}$

b) <u>First, we need to rearrange the NFA to make the length of each transition 1:</u>



$R(q_0) = \{q_0\}$

$R(q_1) = \{q_1, q_2\}$

$R(q_2) = \{q_2\}$

$R(q_3) = \{q_0, q_3\}$

$R(q_4) = \{q_4\}$

$R(q_5) = \{q_5\}$

$s_0 = R(q_0) = q_0$

<u>NFA→DFA:</u>

$\delta(s_0, a) = \delta(q_0, a) = \{R(q_1), R(q_4)\} = \{q_1, q_2, q_4\} \rightarrow s_1$

$\delta(s_0, b) = \delta(q_0, b) = \emptyset$

$\delta(s_1, a) = \delta(\{q_1, q_2, q_4\}, a) = R(q_2) = q_2 \rightarrow s_2$

$\delta(s_1, b) = \delta(\{q_1, q_2, q_4\}, b) = \{R(q_3), R(q_5)\} = \{q_0, q_3, q_5\} \rightarrow \boldsymbol{s_3}$

$\delta(s_2, a) = \delta(q_2, a) = \emptyset$

$\delta(s_2, b) = \delta(q_2, b) = R(q_3) = \{q_0, q_3\} \rightarrow \boldsymbol{s_4}$

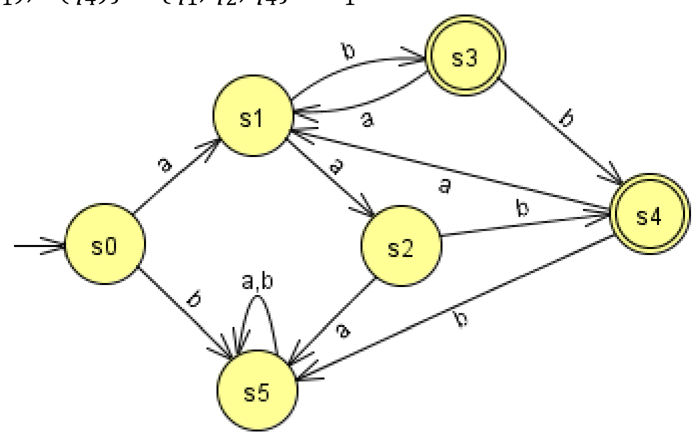
$\delta(s_3, a) = \delta(\{q_0, q_3, q_5\}, a) = \{R(q_1), R(q_4)\} = \{q_1, q_2, q_4\} \rightarrow s_1$

$\delta(s_3, b) = \delta(\{q_0, q_3, q_5\}, b) = R(q_3) = \{q_0, q_3\} \rightarrow s_4$

$\delta(s_4, a) = \delta(\{q_0, q_3\}, a) = \{R(q_1), R(q_4)\} = \{q_1, q_2, q_4\} \rightarrow s_1$

$\delta(s_4, b) = \delta(\{q_0, q_3\}, b) = \emptyset$

$\delta(\emptyset, a) = \delta(\emptyset, b) = \emptyset \rightarrow s_5$



c) $< s_0 >::= a < s_1 >$

$< s_1 >::= a < s_2 > |b < s_3 > |b$

$< s_2 >::= b < s_4 > |b$

$< s_3 >::= a < s_1 > |b < s_4 > |b$

$< s_4 >::= a < s_1 >$

$< s_0 >$ and $< s_4 >$ are the same. So production rule for $< s_4 >$ can be eliminated:

$< s_0 >::= a < s_1 >$

$< s_1 >::= a < s_2 > |b < s_3 > |b$

$< s_2 >::= b < s_0 > |b$

$< s_3 >::= a < s_1 > |b < s_0 > |b$

d) **Theorem:** $\boldsymbol{x = xa \; v \; b \; \wedge \; \Lambda \notin A \; \Rightarrow x = ba^*}$

Similarly: $\boldsymbol{x = ax \; v \; b \; \Rightarrow x = a^*b}$

$L = s_0$

Place s_0 in the expression of s_2: $s_2 = bs_0 \vee b = bas_1 \vee b$

Place s_0 in the expression of s_3: $s_3 = as_1 \vee bs_0 \vee b = as_1 \vee bas_1 \vee b$

Place s_2 and s_3 in the expression of s_1: $s_1 = as_2 \vee bs_3 \vee b$

$$s_1 = a(bas_1 \vee b) \vee b(as_1 \vee bas_1 \vee b) \vee b$$

$$s_1 = (aba \vee ba \vee bba)s_1 \vee ab \vee bb \vee b$$

Using the theorem above: $s_1 = (aba \vee ba \vee bba)^*(ab \vee bb \vee b)$

Place s_1 in the expression of s_0: $L = s_0 = as_1$

$$L = a(aba \vee ba \vee bba)^*(ab \vee bb \vee b)$$

Language defined in a was:

$L = (abb \vee ab \vee aab)^+$

$a(aba \vee ba \vee bba)^*(ab \vee bb \vee b) \stackrel{?}{=} (abb \vee ab \vee aab)^+ \rightarrow$ can be proved by induction

Induction: $a(aba \vee ba \vee bba)^n(ab \vee bb \vee b) \stackrel{?}{=} (abb \vee ab \vee aab)^{n+1}$

n=0: $a(ab \vee bb \vee b) \; = \; abb \vee ab \vee aab$ ✓

n=k: $a(aba \vee ba \vee bba)^k(ab \vee bb \vee b) \stackrel{?}{=} (abb \vee ab \vee aab)^{k+1}$ assume true

n=k+1: $a(aba \vee ba \vee bba)^{k+1}(ab \vee bb \vee b) \stackrel{?}{=} (abb \vee ab \vee aab)^{k+2}$

$$a(aba \vee ba \vee bba)(aba \vee ba \vee bba)^k(ab \vee bb \vee b)$$

$$= a(ab \vee b \vee bb)\underbrace{a(aba \vee ba \vee bba)^k(ab \vee bb \vee b)}_{(abb \vee ab \vee aab)^{k+1}}$$

$$= a(ab \vee b \vee bb)(abb \vee ab \vee aab)^{k+1}$$

$$= (aab \vee ab \vee abb)(abb \vee ab \vee aab)^{k+1}$$

$$= (abb \vee ab \vee aab)^{k+2}$$ ✓