BLG 202E Homework - 1

Due 29.02.2016 23:59

- An e-report should be prepared individually. The written MATLAB codes should be included in the submitted report.
- Plagiarized assignments will be given a negative mark.
- No late submissions will be accepted.

Submissions: Please submit your report and your MATLAB codes through Ninova e-Learning System.

1. (20 pt.) The function $f_1(x_0, h) = sin(x_0 + h) - sin(x_0)$ can be transformed into another form, $f_2(x_0, h)$, using the trigonometric formula,

$$sin(\phi) - sin(\psi) = 2cos(\frac{\phi + \psi}{2})cos(\frac{\phi - \psi}{2}).$$

   Thus, f_1 and f_2 have the same values, in exact arithmetic, for any given argument values x_0 and h.

   (a) Derive $f_2(x_0, h)$

   (b) Suggest a formula that avoids cancellation errors for computing the approximation $(f(x_0 + h) - f(x))/h$ to the derive of $f(x) = sin(x)$ at $x = x_0$. Write a MATLAB program that implements your formula and computes an approximation of $f'(1.2)$, for $h = 1e-20, 1e-19, ..., 1$.

2. (20 pt.) Show that nested evaluation of a polynomial of degree n requires only $2n$ elementary operations and hence has $O(n)$ complexity.

3. (20 pt.) Is there a finite floating point system (i.e., some finite integer base β and precision t) in which numbers e and π have an exact representation? If yes, then describe such a system.

4. (20 pt.) Consider the function $g(x) = x^2 + \frac{3}{16}$.

   (a) This function has two fixed points. What are they?

   (b) Consider the fixed point iteration $x_{k+1} = g(x_k)$ for this g. For which of the points you have found in (a) can you be sure that the iterations will converge to that fixed point? Briefly justify your answer. You may assume that the initial guess is sufficiently close to the fixed point.

   (c) For the point or points you found in (b), roughly how many iterations will be required to reduce the convergence error by a factor of 10?

5. (20 pt.) You are working for a start-up computer assembly company and have been asked to determine the minimum number of computers that the shop will have to sell to make a profit. Assume that the number of computers to be sold after considering the total costs and the total sales, is n. The equation that gives the minimum n is,

$$f(n) = 40n^{1.5} - 875n + 35000$$

   (a) Use the Newton-Raphson Method to find the minimum number of computers that is needed to be sold to make a profit, by conducting sufficient iterations to reach an absolute relative approximate error less than 1%.

   (b) Write MATLAB code to solve.