BLG202E Numerical Methods:

Learning Objectives: Students will be able to:

1. Define errors in numerical analysis; define discretization error and compare to round-off errors in computing; Construct floating point representation and relate to precision; connect it to round-off errors;
2. Roughly identify computational complexity of algorithms via O(.) notation
3. Define condition number of a matrix and describe its utility in assessing conditioning of a given problem such as solution of a linear system of equations
4. Construct an iterative solution to find the zeros of a nonlinear equation through techniques such as Fixed Point Iteration, Newton's method, Secant method; describe its connection to optimization; analyze convergence of iterative methods
5. Construct algorithms for solution of linear system of equations through both Gaussian elimination, forward/backward substitution, and LU decomposition; define their complexities; describe the advantage of LU decomposition in solution of linear systems
6. Formulate a linear least squares problem to solve an overdetermined system of equations; derive its solution via normal equations ; apply to data fitting problems
7. Construct the spectral decomposition (Eigen Value Decomposition-EVD) and Singular Value Decomposition (SVD) of matrices, and truncated SVD. Describe and use Power method to estimate the dominant eigen value/vector of a given matrix; Apply eigen (singular) value /vector and truncated SVD methods to real life problems;
8. Describe principles of data interpolation and approximation; Expand the interpolating/approximating function over a set of basis functions; construct polynomial interpolation solution using different polynomial bases such as monomial, Lagrange, Newton polynomials; apply those to real problems

Learning Objectives (continued):

9. Derive formulae for approximation of numerical derivatives at different orders using Taylor series method and Richardson extrapolation method; Similarly formulate numerical differentiation using polynomial interpolation method; Analyze and discuss effects of round-off errors and data errors (noise) in numerical differentiation

10. Formulate numerical integration using both basic and composite quadrature algorithms: trapezoidal rule, midpoint rule, and Simpson's rule, and how those are obtained using polynomial interpolation

11. Construct numerical solutions to initial value problems in systems of ordinary differential equations (ODEs). Apply the explicit methods such as forward Euler method, and implicit methods such as backward Euler method to solve those ODEs, and describe the stability criteria for those methods. Apply higher order methods, specifically the Runge Kutta 2nd order and 4th order methods, to solution of initial value ODE problems.

12. Implement all concepts in Learning Objectives 1-11 in a programming environment (such as MATLAB)