BLG456E
Robotics
Intro to mobile robot geometry & kinematics

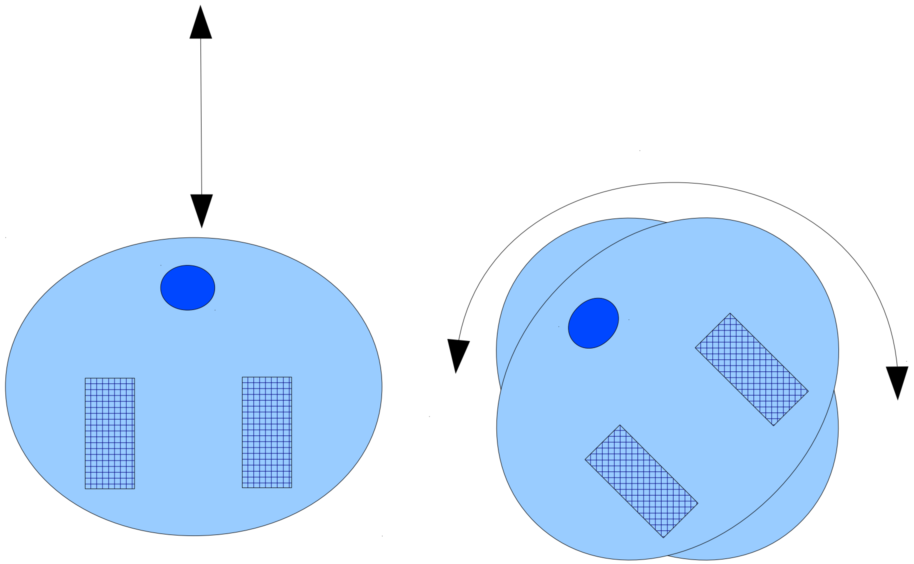
Lecture Contents:

- Differential drive robots.
- Robot abstractions:
  - Point robot.
  - Rigid robot.
    - Rotation vectors.
    - Twists.
- Introduction to reference frames.

Lecturer:	Damien Jade Duff
Email:	djduff@itu.edu.tr
Office:	EEBF 2316
Schedule:	http://djduff.net/my-schedule
Coordination:	http://ninova.itu.edu.tr/Ders/4709

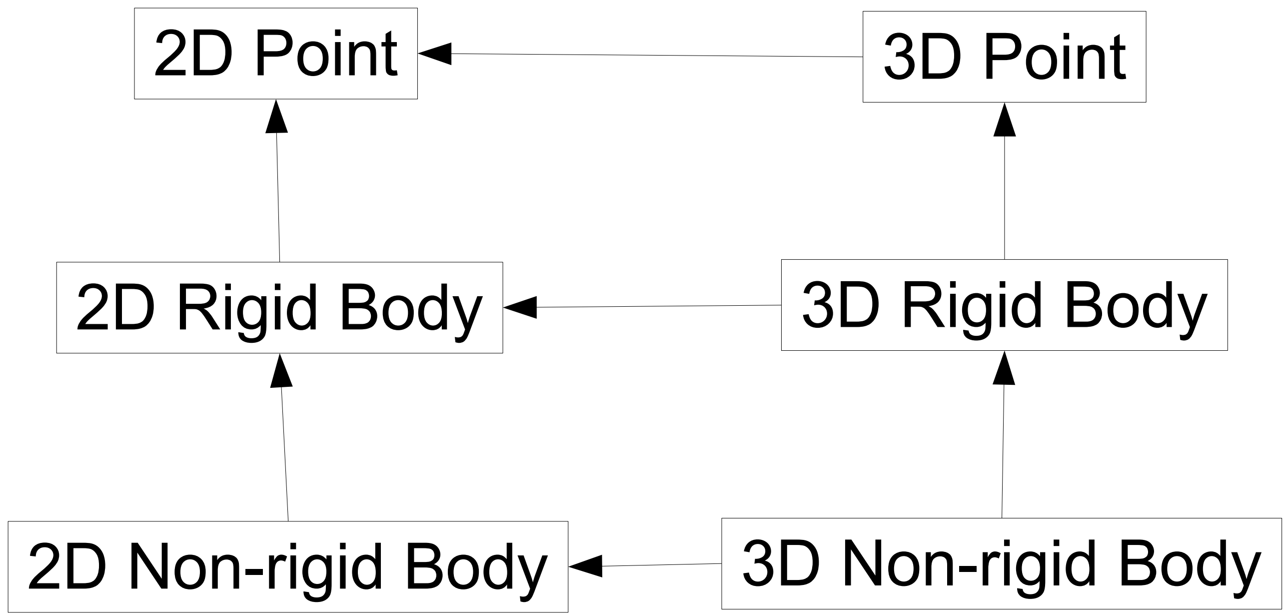
Differential Drive

- 2 wheels on common axis.
- Wheels rotate different speeds – “differentially”.
- Usually 3rd passive wheel.
- Can:
  - Rotate in-place.
  - Move forward or back.
  - Move on curve.

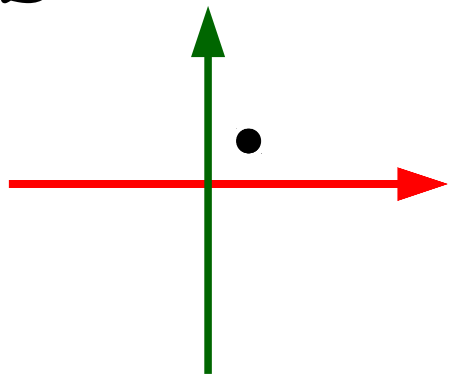


<u>Question</u>: What motions can this robot **not** make?

Abstractions



2D point geometry/kinematics



Robot point in Cartesian space.

State (position): $\chi = \begin{bmatrix} x \\ y \end{bmatrix}$

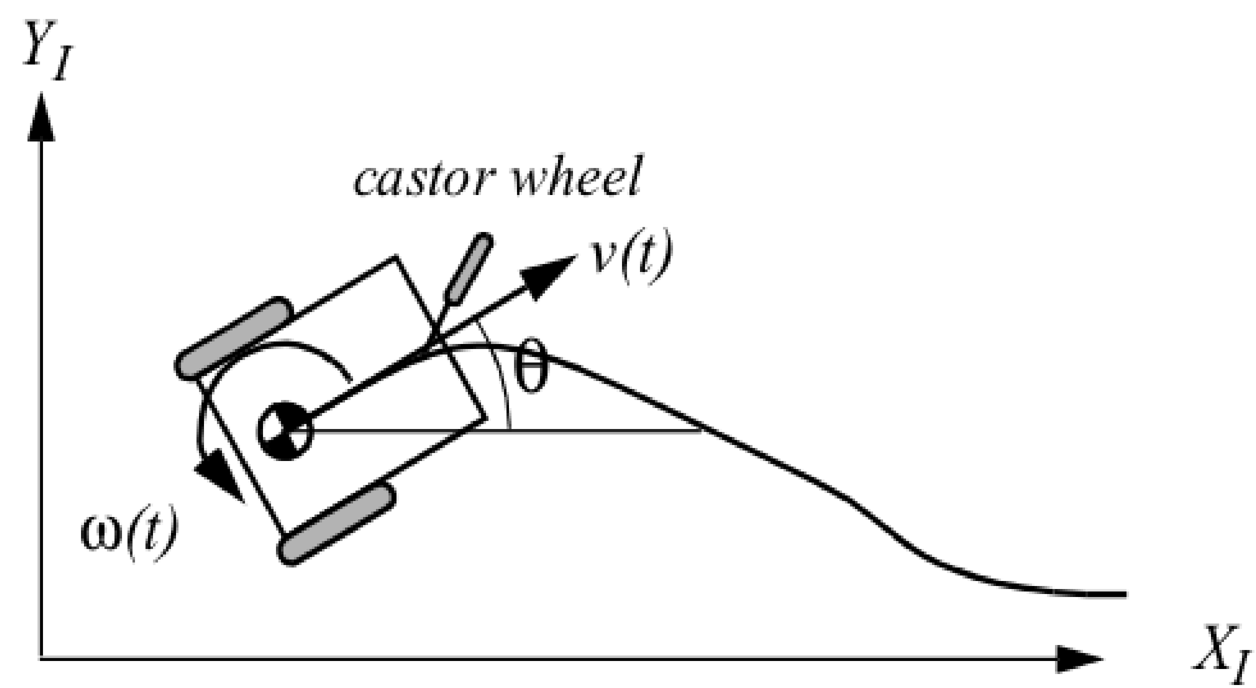
State (position & velocity): $\mathbf{X} = \begin{bmatrix} \chi \\ \dot{\chi} \end{bmatrix} = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$

Note:
$\dot{a} \equiv \frac{da}{dt}$

Kinematics: $\chi_{t_1} = \chi_{t_0} + \int_{t_0}^{t_1} \dot{\chi}_t \, dt$

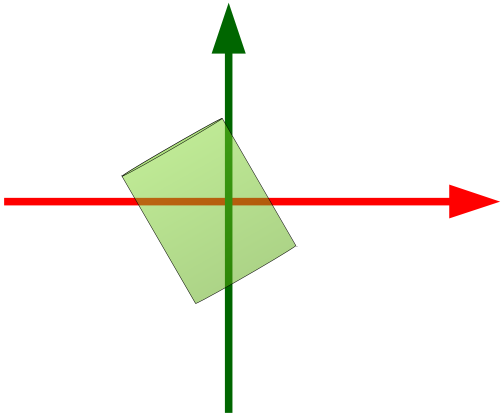
Constant velocity kinematics: $\chi_{t_1} = \chi_{t_0} + t \cdot \dot{\chi}_{t_1}$

Motion of all points in a rigid body is captured by its linear and angular velocities.



From Siegwart & Nourbaksh.

2D geometry with rotations



Robot has many points - is **extended**.

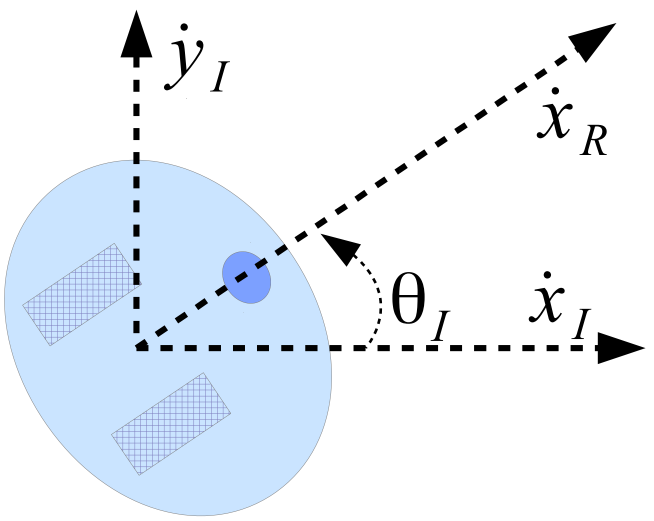
State (pose): $$\chi = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

State (position & velocity): $$\mathbf{x} = \begin{bmatrix} \chi \\ \dot{\chi} \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

2D kinematics with rotations: differential drive robot

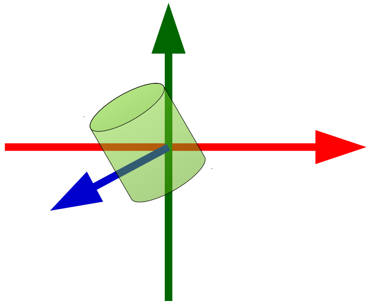
Translational velocity depends on current orientation.

$$\dot{\chi}_I = \begin{bmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta}_I \end{bmatrix} = \begin{bmatrix} \dot{x}_R \cos\theta_I \\ \dot{x}_R \sin\theta_I \\ \dot{\theta}_I \end{bmatrix}$$



3D geometry/kinematics with rotations

3D rotation is stranger.



Pose (translation vector, rotation vector): $\chi = \begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}$

Instantaneous velocity:

$$\dot{\chi} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \end{bmatrix}$$

Velocity (**twist**):

$$V = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$$\boxed{\dot{\theta}_x \neq \omega_x}$$

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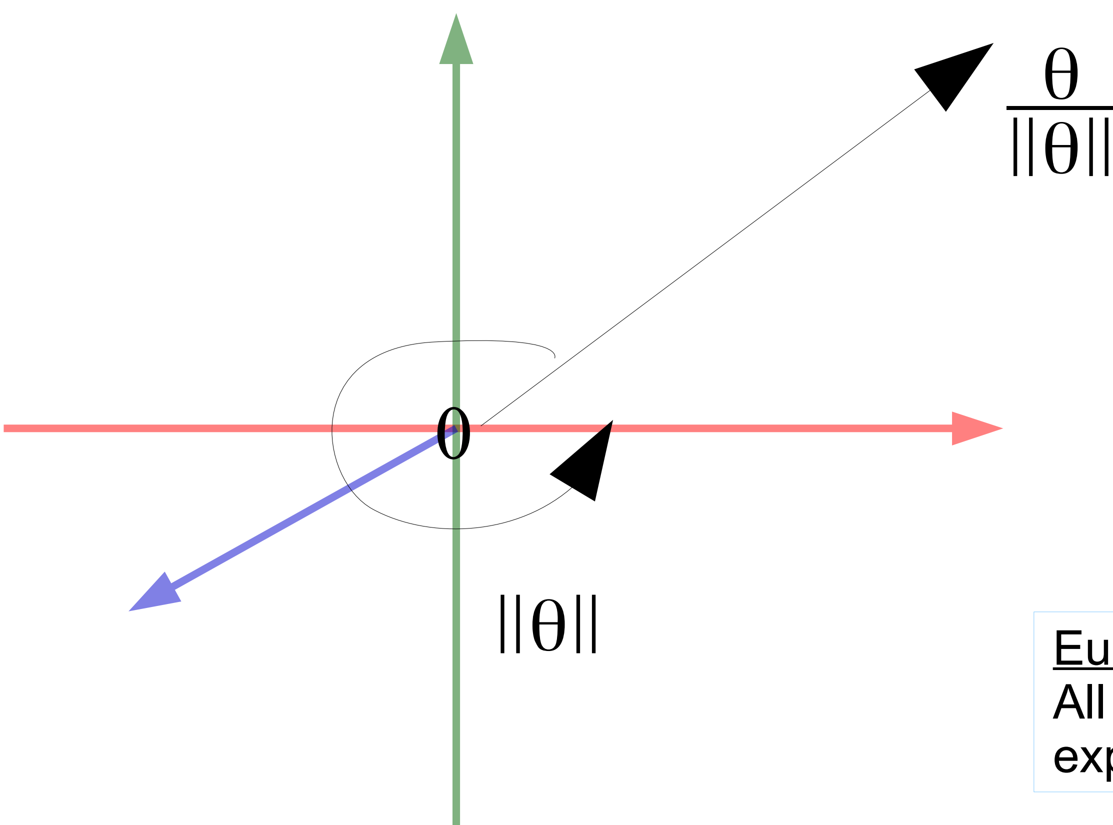
Rotation Vector $\boldsymbol{\theta}=\begin{bmatrix}\theta_x\\ \theta_y\\ \theta_z\end{bmatrix}$

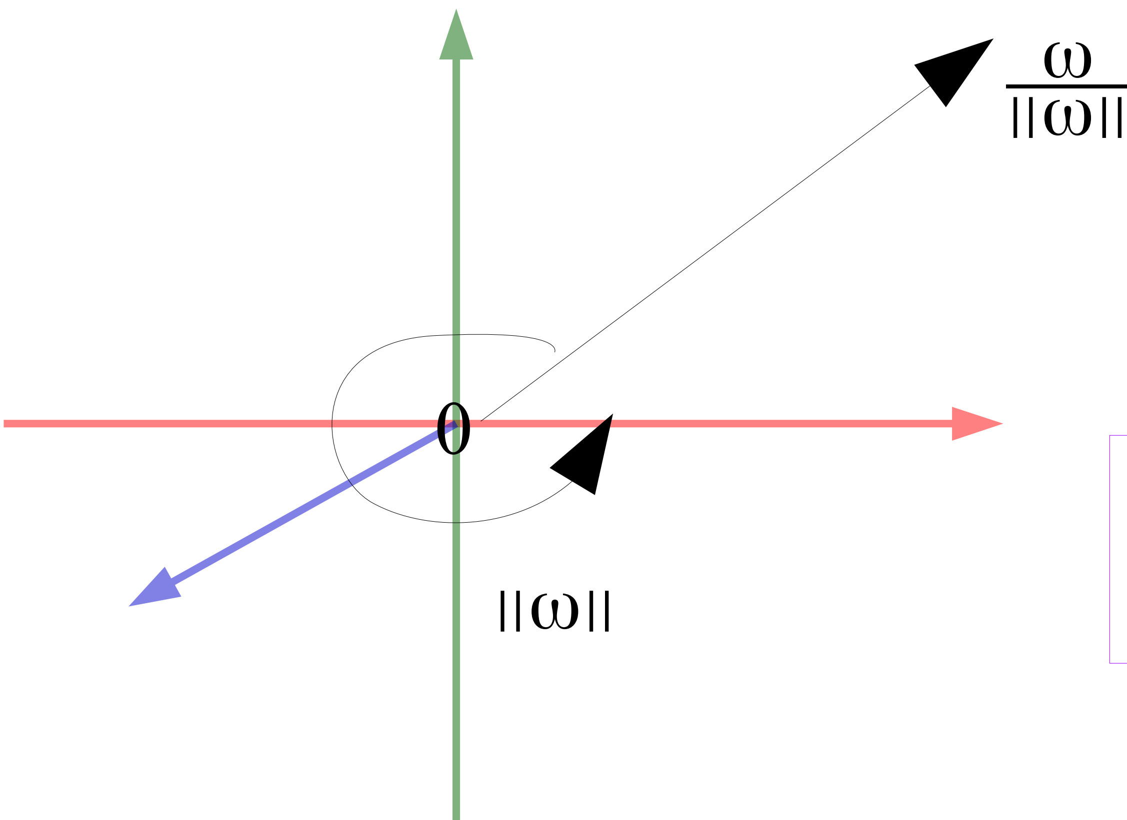
- Length is amount of rotation.
- Direction is axis of rotation.



<u>Euler's rotation theorem</u>: All rotations can be expressed like this.

Twist (Rotation Part) $\boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$

- Length is speed of rotation.
- Direction is axis of rotation.

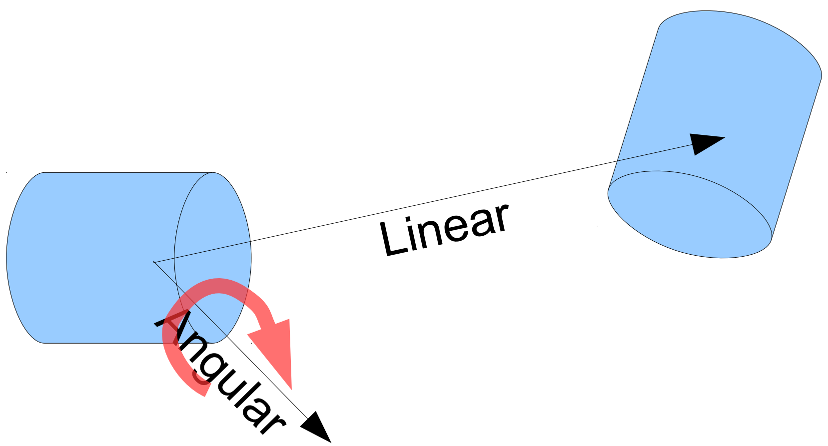


$$\dot{\theta}_x \neq \omega_x$$

Twists

Twist = a pair:

- 3D Linear Velocity (vector).
  - **Direction** = direction of travel.
  - **Magnitude** = speed of body.
- 3D Angular Velocity (vector).
  - **Direction** = axis of rotation.
  - **Magnitude** = speed of rotation (angular velocity).



Twists in two dimensions

2D case (body constrained to XY plane):

- Linear velocity vector is in the plane.
  - $\dot{z} = 0$
- Direction of angular velocity vector is vertical.
  - $\omega_x = 0$
  - $\omega_y = 0$

For differential drive robots:

- Robot cannot move sideways:
  -

$$\dot{y} = 0$$

Exercise

- My robot is at position $(x,y) = (3,5)$

  (displacement from world origin).

  - 1. Plot this point against x,y axes.

- Its orientation is $\boldsymbol{\theta}_z = \boldsymbol{\pi}/2$ rad

  (rotation from the x-axis).

  - 2. Draw the robot.

- 3. From the *robot's* perspective, **what are the coordinates of the origin?**

  (forward is positive x axis, left is positive y axis).

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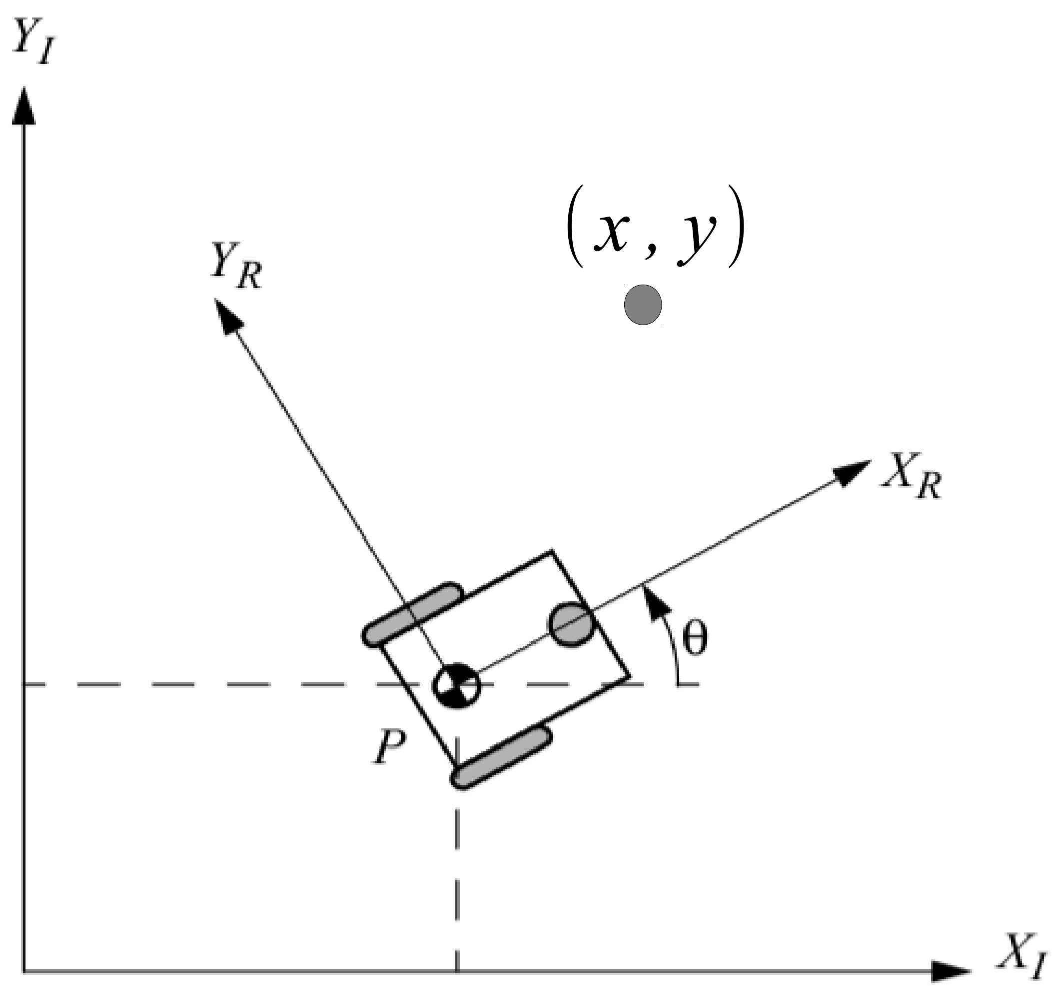
Introduction to reference frames



World reference frame:

$$x_I \quad y_I$$

Robot reference frame:

$$x_R \quad y_R$$

Mapping between:

$$x_I = x_R \cos\theta - y_R \sin\theta$$
$$y_I = x_R \sin\theta + y_R \cos\theta$$