BLG456E
Robotics
Estimation for Localisation

- Intro to probabilistic frameworks.
- Intro to ML and MAP.
- ML localisation.
- MAP localisation.
- Simplified notation.
- Recursive estimation.
- Particle filtering.

Lecturer:	Damien Jade Duff
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How to deal with uncertainty in a principled way?

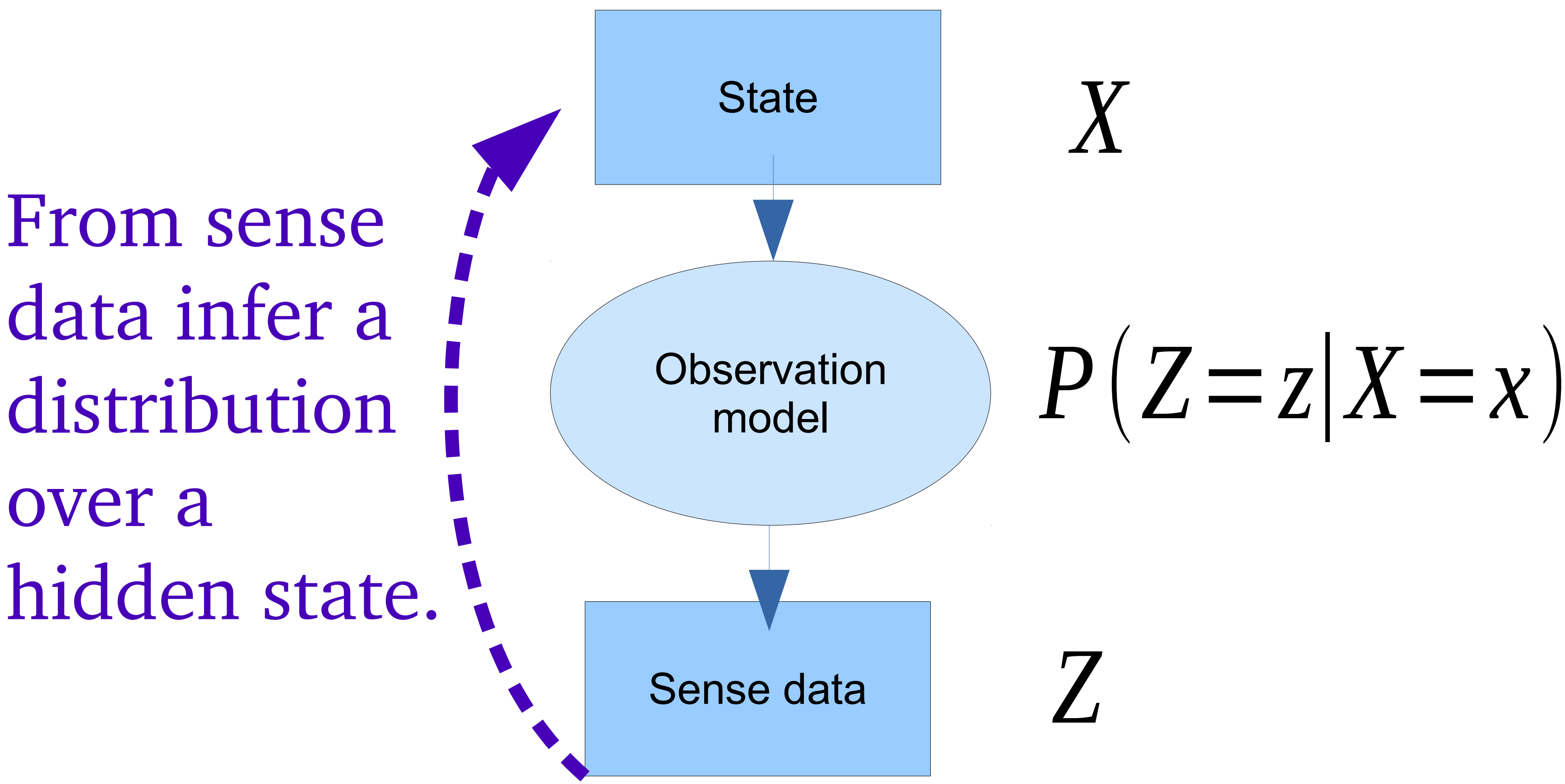
- Inaccuracy and imprecision in sensors.
- Missing/unobservable information.
- Decision making under uncertainty.
- Multiple information sources.
- Multiple hypotheses.

Odometry.
- Visual.
- Encoders.
- ...

Landmarks.
- Laser.
- Visual.
- ...

Predicted motion.
- Continuity.
- Physics.
- ...

Answer:
Probabilistic frameworks



Question: Where is the generative model and where is the discriminative model here?

MAP vs ML

$Z=z$ is the event that the observed data Z has value z.
$X=x$ is the event that the state X has value x.

Maximum Likelihood (ML):

$$\underset{x}{argmax}\, P(Z=z|X=x)$$

(easier to calculate)

Maximum A Posteriori (MAP):

$$\underset{x}{argmax}\, P(X=x|Z=z)$$

$$=\underset{x}{argmax} \frac{P(Z=z|X=x)P(X=x)}{P(Z=z)}$$

(Bayes' Law)

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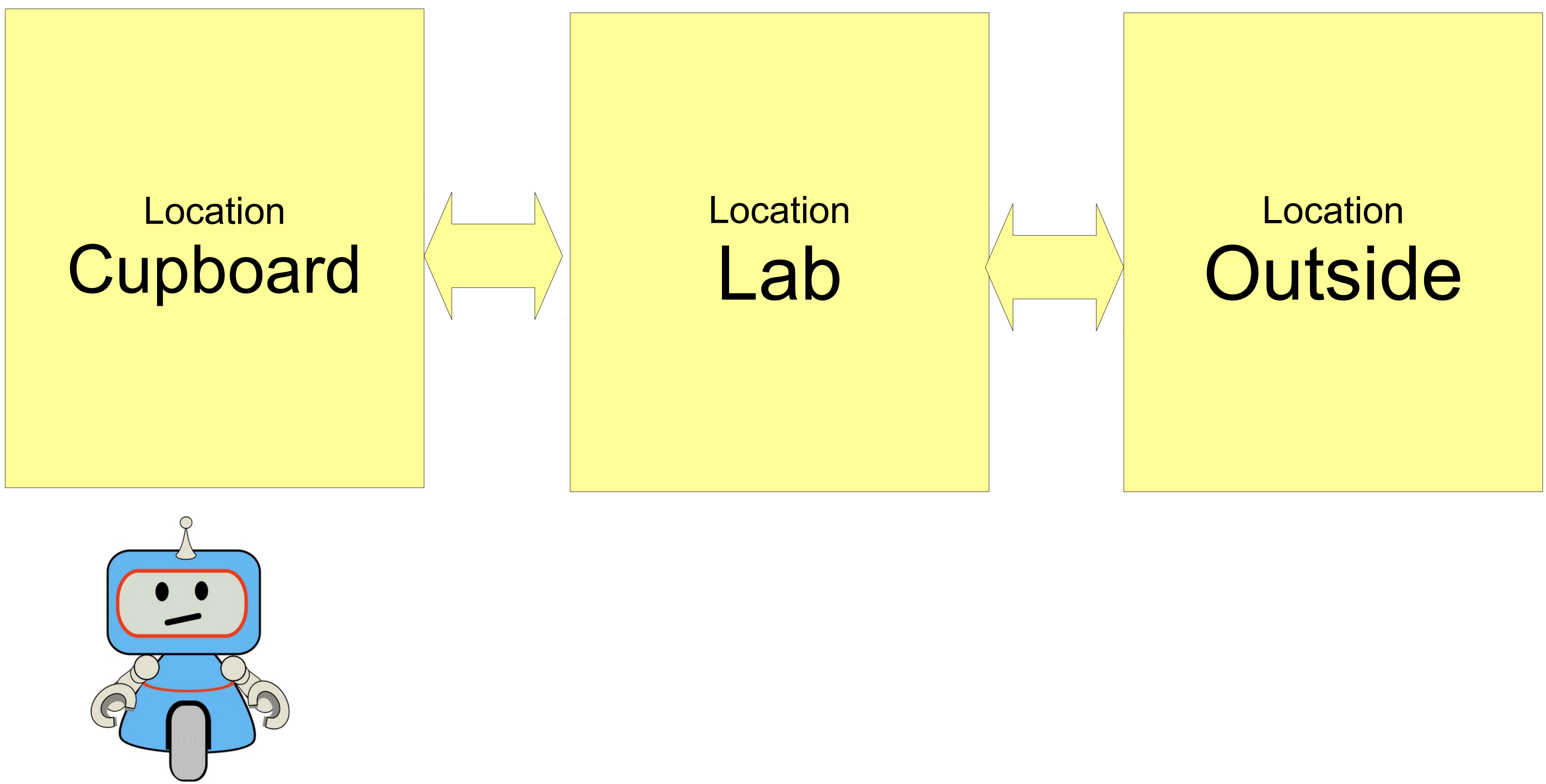
Sample localisation scenario



Sample observation model

The **observation** Z one of $bright$ or $dark$.
The **state** X one of $cupboard$ or lab or $outside$.

$$P(Z=bright|X=cupboard)$$

probability it is bright **if** in cupboard.

('conditional on the state of being in the cupboard').

E.g.:

$$P(Z=bright|X=cupboard)=0.2$$
$$P(Z=dark|X=cupboard)=0.8$$
$$P(Z=bright|X=lab)=0.8$$
$$P(Z=dark|X=lab)=0.2$$
$$P(Z=bright|X=outside)=0.6$$
$$P(Z=dark|X=outside)=0.4$$

Exercise

Assume we have learnt the following observation model:

$$P(Z=bright|X=cupboard)=0.2$$
$$\cancel{P(Z=dark|X=cupboard)=0.8}$$
$$P(Z=bright|X=lab)=0.8$$
$$\cancel{P(Z=dark|X=lab)=0.2}$$
$$P(Z=bright|X=outside)=0.6$$
$$\cancel{P(Z=dark|X=outside)=0.4}$$

Maximum Likelihood (ML):

$$\underset{x}{argmax}\, P(Z=z|X=x)$$

Our robot observes that it is bright:

$$Z=bright$$

- **What is the ML estimate of the current location?**

Exercise

We have learnt the following observation model:

$P(Z=bright|X=cupboard)=0.2$

$P(Z=dark|X=cupboard)=0.8$

$P(Z=bright|X=lab)=0.8$

$P(Z=dark|X=lab)=0.2$

$P(Z=bright|X=outside)=0.6$

$P(Z=dark|X=outside)=0.4$

Maximum Likelihood (ML):

$$\underset{x}{argmax}\, P(Z=z|X=x)$$

Our robot observes that it is dark:

$Z=dark$

- **What is the ML estimate of the current location?**

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MAP is more principled than ML

- To do MAP estimation need:
  - Prior model of state:
    - P(X).
- Do NOT need:
  - Prior model of observations:
    - P(Z).

Maximum A Posteriori (MAP):

$$\underset{x}{argmax}\, P\left(X=x \middle| Z=z\right)$$

$$=\underset{x}{argmax}\, \frac{P\left(Z=z \middle| X=x\right) P\left(X=x\right)}{P\left(Z=z\right)}$$

Exercise

Maximum A Posteriori (MAP):

$$\underset{x}{argmax}\, P(X=x|Z=z)$$

$$=\underset{x}{argmax} \frac{P(Z=z|X=x)P(X=x)}{P(Z=z)}$$

For the same observation model, we have the following priors:

$P(X=cupboard)=0.2$

$P(X=lab)=0.8$

$P(X=outside)=0.0$

Obs. Model:

$P(Z=bright|X=cupboard)=0.2$

$P(Z=dark|X=cupboard)=0.8$

$P(Z=bright|X=lab)=0.8$

$P(Z=dark|X=lab)=0.2$

$P(Z=bright|X=outside)=0.6$

$P(Z=dark|X=outside)=0.4$

Our robot observes brightness:

$Z=bright$

- **What is the MAP estimate of the current location?**

$$P(X=cupboard|Z=bright) = \frac{P(Z=bright|X=cupboard)P(X=cupboard)}{P(Z=bright)} = 0.04\cdot P(Z=bright)$$

$$P(X=lab|Z=bright) = \frac{P(Z=bright|X=lab)P(X=lab)}{P(Z=bright)} = 0.64\cdot P(Z=bright)$$

$$P(X=outside|Z=bright) = \frac{P(Z=bright|X=outside)P(X=outside)}{P(Z=bright)} = 0.00\cdot P(Z=bright)$$

Exercise

Maximum A Posteriori (MAP):

$$\underset{x}{argmax}\, P(X=x|Z=z)$$

$$=\underset{x}{argmax}\frac{P(Z=z|X=x)P(X=x)}{P(Z=z)}$$

For the same observation model, we have the following priors:

$P(X=cupboard)=0.2$

$P(X=lab)=0.8$

$P(X=outside)=0.0$

Obs. Model:

$P(Z=bright|X=cupboard)=0.2$

$P(Z=dark|X=cupboard)=0.8$

$P(Z=bright|X=lab)=0.8$

$P(Z=dark|X=lab)=0.2$

$P(Z=bright|X=outside)=0.6$

$P(Z=dark|X=outside)=0.4$

Our robot observes darkness:

$Z=dark$

- **What is the MAP estimate of the current location?**

Multiple observations

If $Z_1 = z_1$ and $Z_2 = z_2$ are **independent** observations,

$$P\left(Z_1 = z_1, Z_2 = z_2 \middle| X = x\right)$$
$$=$$
$$P\left(Z_1 = z_1 \middle| X = x\right) P\left(Z_2 = z_2 \middle| X = x\right)$$

Exercise

For the same observation model, same priors,
extra observation in model:

$P(Z_2 = warm | X = cupboard) = 0.8$
$P(Z_2 = cold | X = cupboard) = 0.2$
$P(Z_2 = warm | X = lab) = 0.8$
$P(Z_2 = cold | X = lab) = 0.2$
$P(Z_2 = warm | X = outside) = 0.4$
$P(Z_2 = cold | X = outside) = 0.6$

Maximum A Posteriori (MAP):

$$\underset{x}{argmax}\, P(X = x | Z = z)$$

$$= \underset{x}{argmax} \frac{P(Z = z | X = x) P(X = x)}{P(Z = z)}$$

Our robot observes that it is dark and cold:

$Z = dark$
$Z_2 = cold$

- **What is the MAP estimate of the current location?**

Exercise

For the same observation model, same priors,
extra observation in model:

$$P(Z_2 = warm | X = cupboard) = 0.8$$
$$P(Z_2 = cold | X = cupboard) = 0.2$$
$$P(Z_2 = warm | X = lab) = 0.8$$
$$P(Z_2 = cold | X = lab) = 0.2$$
$$P(Z_2 = warm | X = outside) = 0.4$$
$$P(Z_2 = cold | X = outside) = 0.6$$

Maximum A Posteriori (MAP):

$$\underset{x}{argmax}\, P(X = x | Z = z)$$
$$= \underset{x}{argmax} \frac{P(Z = z | X = x) P(X = x)}{P(Z = z)}$$

Our robot observes that it is light and cold:

$Z = light$

$Z_2 = cold$

- **What is the MAP estimate of the current location?**

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Let's simplify presentation by removing random variables

Independence Assumption

$$P(A=a|B=b)P(B=b)=P(A=a)P(B=b)$$

Same equation, random variables implicit:

$$P(a|b)P(b)=P(a)P(b)$$

Let's simplify presentation by removing random variables

Bayes' Law was:

$$P(A=a|B=b)=\frac{P(B=b|A=a)P(A=a)}{P(B=b)}$$

Bayes' Law, random variables implicit:

$$P(a|b)=\frac{P(b|a)P(a)}{P(b)}$$

Let's simplify presentation by removing random variables

Joint probabilities rule was:

$$P(A=a)=\sum_b P(A=a,B=b)$$

Joint probabilities rule becomes:

$$P(a)=\sum_b P(a,b)$$

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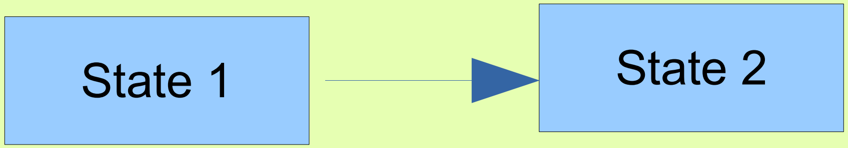
The importance of time



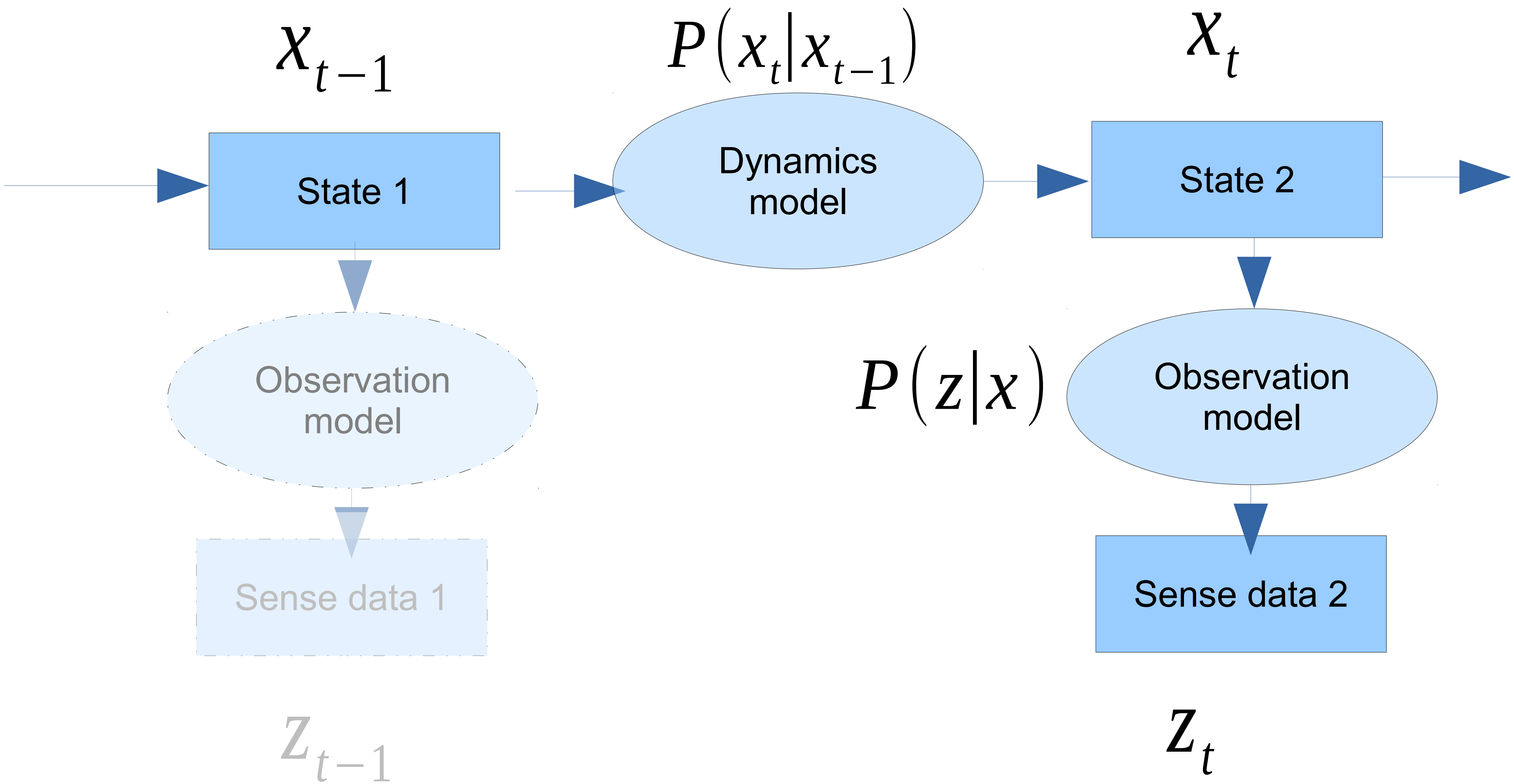
Patterns of change as you move.

- Only a single photo-cell?
- Travelling in a bus, eyes closed.

Moving to disambiguate.

- Eye movements.
- Craning to look.
- Searching.

Incorporating dynamics (change over time)

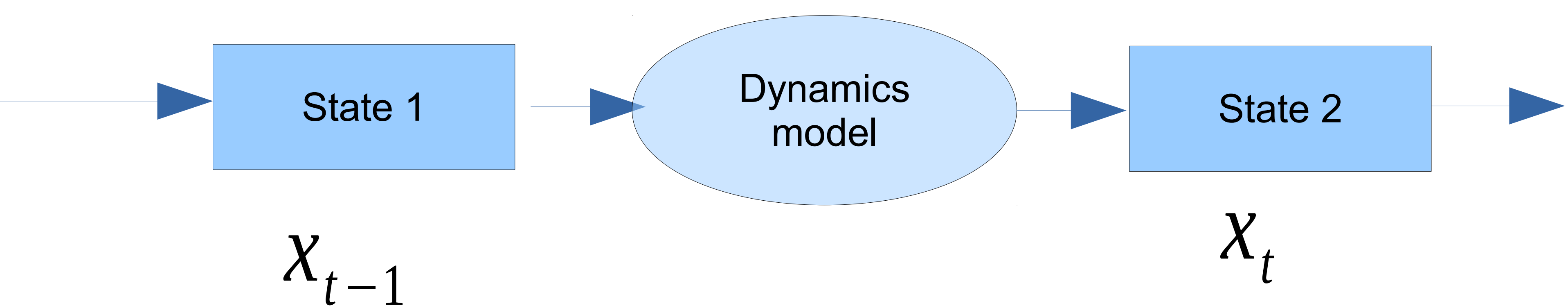


"Hidden Markov Model"

Independence assumptions (from diagram)

- **Markov Assumption:** State at time *t* is independent of state at earlier times if state at *t-1* is known.

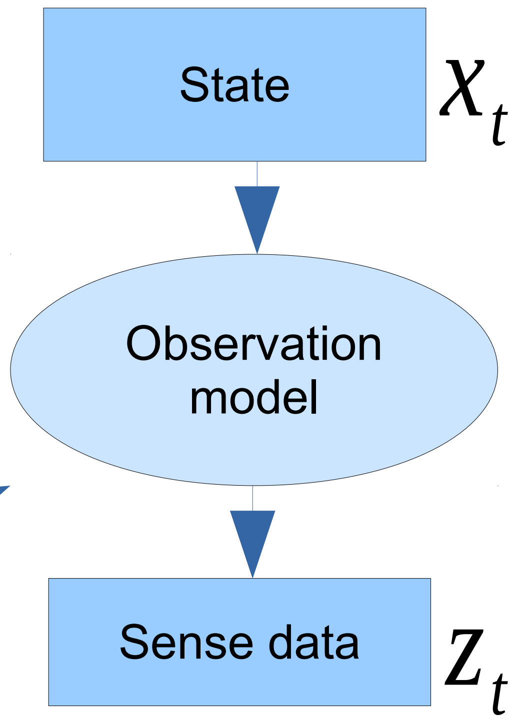
$$P(x_t|x_{t-1},\ldots,x_0,z_{t-1},\ldots,z_0) = P(x_t|x_{t-1})$$



Independence assumptions (from diagram)

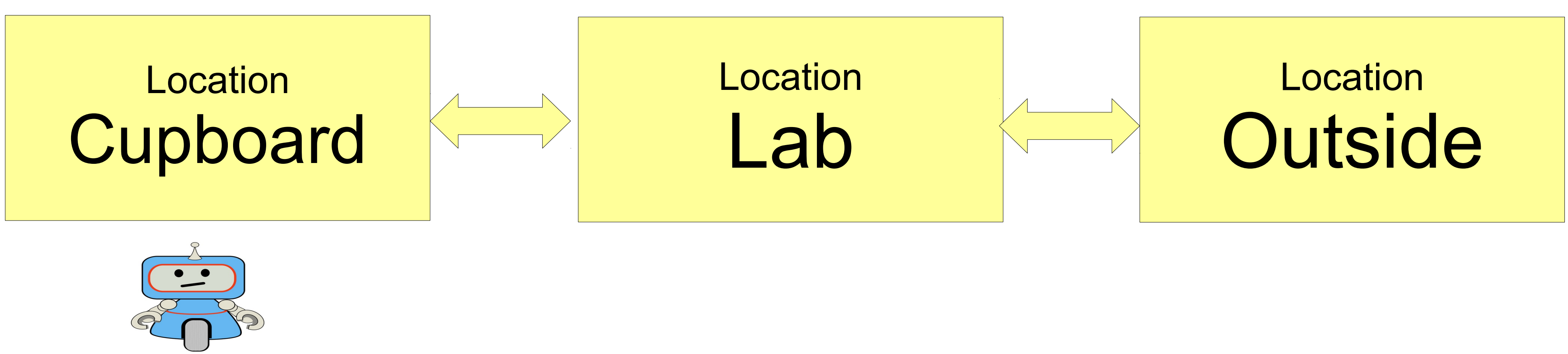
- **Observation Independence:** Observations at time t independent of all other variables except state at time t.

$$P\left(z_t \middle| x_t, \ldots, x_0, z_{t-1}, \ldots, z_0\right) = P\left(z_t \middle| x_t\right)$$



Example dynamics model

$$P(X_t = cupboard \mid X_{t-1} = cupboard) = 0.8$$
$$P(X_t = lab \mid X_{t-1} = cupboard) = 0.2$$
$$P(X_t = outside \mid X_{t-1} = cupboard) = 0.0$$
$$P(X_t = cupboard \mid X_{t-1} = lab) = 0.1$$
$$P(X_t = lab \mid X_{t-1} = lab) = 0.8$$
$$P(X_t = outside \mid X_{t-1} = lab) = 0.1$$
$$P(X_t = cupboard \mid X_{t-1} = outside) = 0.0$$
$$P(X_t = lab \mid X_{t-1} = outside) = 0.8$$
$$P(X_t = outside \mid X_{t-1} = outside) = 0.2$$



Importance of Markov Models

- Used for:
  - Localisation.
  - Mapping.
  - SLAM.
  - Object tracking.
  - **Planning under uncertainty.**
  - **Robot action learning.**

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Recursive estimation (simple picture)

- From
  - Probability distribution over previous state. $P\left(x_{t-1} \middle| z_{t-1}, \ldots, z_0\right)$
  - Current observation. z_t
- Calculate:
  - Distribution over current state.

$$\Downarrow$$

$$P\left(x_t \middle| z_t, \ldots, z_0\right)$$

Recursive estimation

- Makes use of:
  - A dynamics model. $P\left(x_t | x_{t-1}\right)$
  - An observation model. $P\left(z_t | x_t\right)$

Recursive estimation (simple picture)

- From
  - Probability distribution over previous state. $P\left(x_{t-1} \middle| z_{t-1},\ldots,z_0\right)$
  - Current observation. z_t
  - Dynamics & observation models. $P\left(x_t \middle| x_{t-1}\right)$ $P\left(z_t \middle| x_t\right)$
- Calculate:
  - Distribution over current state. $P\left(x_t \middle| z_t,\ldots,z_0\right)$

Recursive estimation (with predict-update)

$$P\left(x_{t-1} \middle| z_{t-1}, \ldots, z_0\right)$$

$$P\left(x_t \middle| x_{t-1}\right)$$

Intermediate step:

- Prior over current state.

$$P\left(x_t \middle| z_{t-1}, \ldots, z_0\right)$$

$$P\left(z_t \middle| x_t\right) \qquad z_t$$

$$P\left(x_t \middle| z_t, \ldots, z_0\right)$$

The filtering equations (I: Predict)

Dynamics prediction:

(From Markov property + joint distribution)

$$P\left(x_t \middle| z_{t-1} \ldots z_0\right)$$

Current time-step prior

$$=$$

$$\sum_{x_{t-1}} P\left(x_t \middle| x_{t-1}\right) P\left(x_{t-1} \middle| z_{t-1} \ldots z_0\right)$$

Dynamics model

Previous time-step posterior

Exercise: recursion

Dynamics model:

$P(X_t = cupboard | X_{t-1} = cupboard) = 0.8$

$P(X_t = lab | X_{t-1} = cupboard) = 0.2$

$P(X_t = outside | X_{t-1} = cupboard) = 0.0$

$P(X_t = cupboard | X_{t-1} = lab) = 0.1$

$P(X_t = lab | X_{t-1} = lab) = 0.8$

$P(X_t = outside | X_{t-1} = lab) = 0.1$

$P(X_t = cupboard | X_{t-1} = outside) = 0.0$

$P(X_t = lab | X_{t-1} = outside) = 0.8$

$P(X_t = outside | X_{t-1} = outside) = 0.2$

Initial state:

$P(X_0 = cupboard) = 0.5$

$P(X_0 = lab) = 0.5$

$P(X_0 = outside) = 0.0$

Calculate:

$P(X_1 = cupboard)$

$P(X_1 = lab)$

$P(X_1 = outside)$

$$P(x_t | z_{t-1} \ldots z_0)$$
$$=$$
$$\sum_{x_{t-1}} P(x_t | x_{t-1}) P(x_{t-1} | z_{t-1} \ldots z_0)$$

Exercise: recursion

Dynamics model:

$P(X_t = cupboard | X_{t-1} = cupboard) = 0.8$
$P(X_t = lab | X_{t-1} = cupboard) = 0.2$
$P(X_t = outside | X_{t-1} = cupboard) = 0.0$
$P(X_t = cupboard | X_{t-1} = lab) = 0.1$
$P(X_t = lab | X_{t-1} = lab) = 0.8$
$P(X_t = outside | X_{t-1} = lab) = 0.1$
$P(X_t = cupboard | X_{t-1} = outside) = 0.0$
$P(X_t = lab | X_{t-1} = outside) = 0.8$
$P(X_t = outside | X_{t-1} = outside) = 0.2$

Initial state:

$P(X_0 = cupboard) = 1.0$
$P(X_0 = lab) = 0.0$
$P(X_0 = outside) = 0.0$

Calculate:

$P(X_1 = cupboard)$
$P(X_1 = lab)$
$P(X_1 = outside)$

$$P(x_t | z_{t-1} \ldots z_0) = \sum_{x_{t-1}} P(x_t | x_{t-1}) P(x_{t-1} | z_{t-1} \ldots z_0)$$

The filtering equations (II: Update)

Observation update:

(Observation independence + Bayes')

$$P\left(x_t \mid z_t \ldots z_0\right) = \frac{P\left(z_t \mid x_t\right) P\left(x_t \mid z_{t-1} \ldots z_0\right)}{P\left(z_t\right)}$$

Current time-step posterior

Observation model

Observations prior (often) not calculated

Current time-step prior

Example: update

Use same observation model as before:

$P(Z=bright|X=cupboard)=0.2$
$P(Z=dark|X=cupboard)=0.8$
$P(Z=bright|X=lab)=0.8$
$P(Z=dark|X=lab)=0.2$
$P(Z=bright|X=outside)=0.6$
$P(Z=dark|X=outside)=0.4$

Robot observes that it is ***bright.***

Pre-update estimate of state at time 1:

$P(X_1=cupboard)=0.8$
$P(X_1=lab)=0.2$
$P(X_1=outside)=0.0$

1. Calculate:

$P(X_1=cupboard|Z_1=bright)$
$P(X_1=lab|Z_1=bright)$
$P(X_1=outside|Z_1=bright)$

2. Calculate MAP estimate of current location.

$$P(x_t|z_t\ldots z_0)=\frac{P(z_t|x_t)P(x_t|z_{t-1}\ldots z_0)}{P(z_t)}$$

Exercise: update

Use same observation model as before:

$P(Z=bright|X=cupboard)=0.2$
$P(Z=dark|X=cupboard)=0.8$
$P(Z=bright|X=lab)=0.8$
$P(Z=dark|X=lab)=0.2$
$P(Z=bright|X=outside)=0.6$
$P(Z=dark|X=outside)=0.4$

Robot observes that it is ***dark.***

Pre-update estimate of state at time 1:

$P(X_1=cupboard)=0.8$
$P(X_1=lab)=0.2$
$P(X_1=outside)=0.0$

1. Calculate:

$P(X_1=cupboard|Z_1=bright)$
$P(X_1=lab|Z_1=bright)$
$P(X_1=outside|Z_1=bright)$

2. Calculate MAP estimate of current location.

$$P(x_t|z_t\ldots z_0)=\frac{P(z_t|x_t)P(x_t|z_{t-1}\ldots z_0)}{P(z_t)}$$

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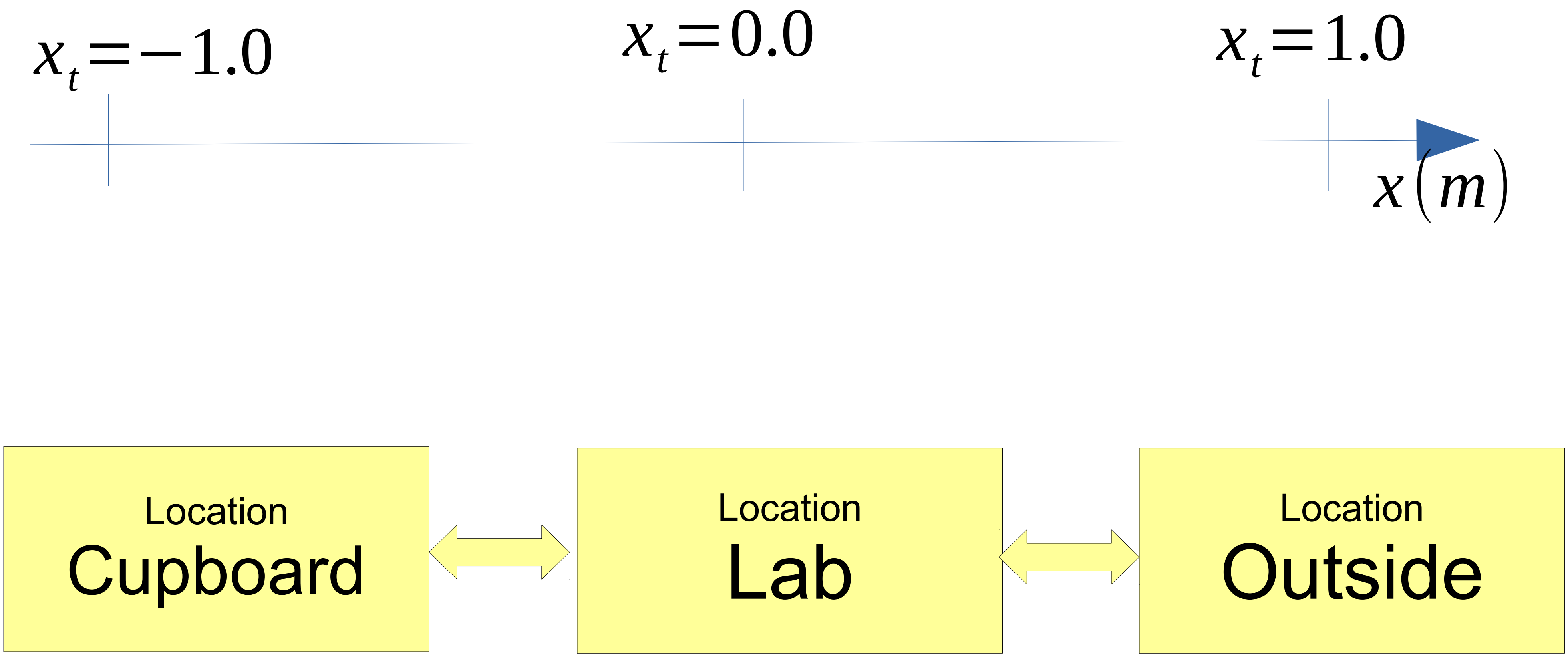
Some possible state representations

- X = node in topological map (discrete).
- X = grid entry (discrete).
- X = pose (continuous).
- X = dynamic configuration (continuous).

Need:
- Dynamics model.
- Observation model.
- Observations.

A continuous representation of location



Can extend Bayes' filter to continuous domains

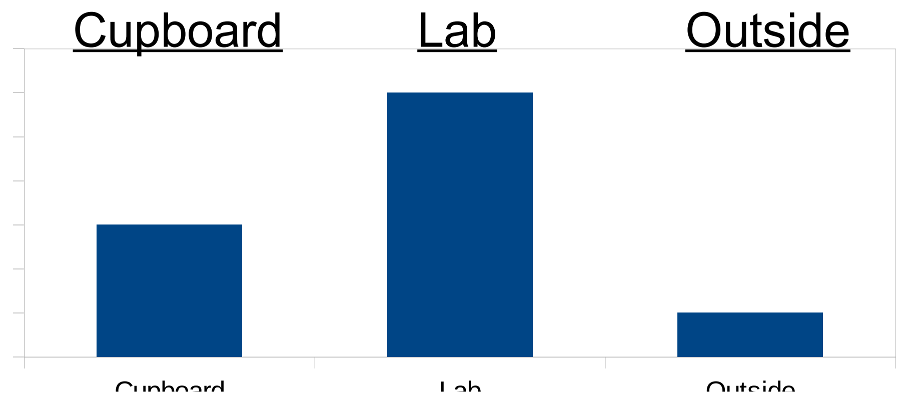
Continuous domains:

- Infinite possible states.
- Can't update all of them!

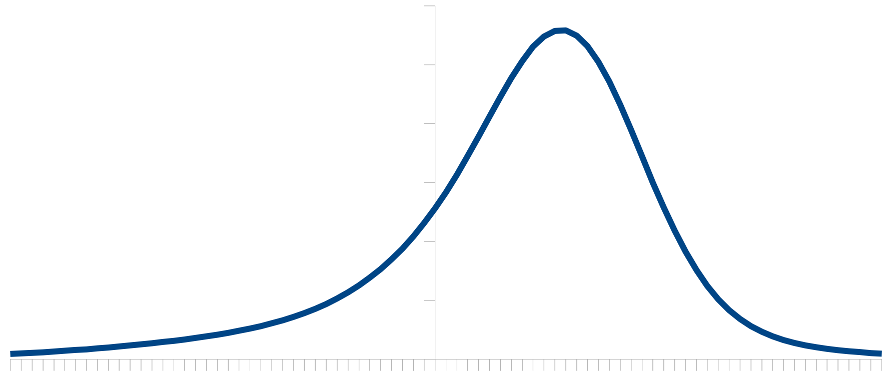
Some possible solutions:

- Parametric distributions:
  - e.g. Kalman filter (normally distributed data).
- Sampling of continuous states:
  - e.g. Particle filter (any kind of distribution).

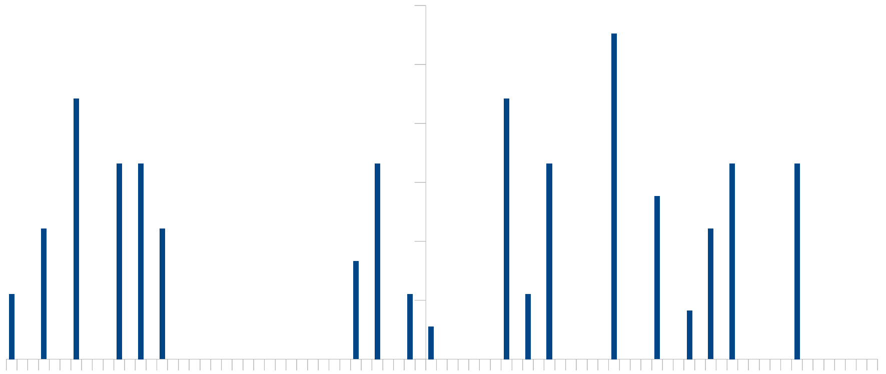
Representations of continuous distributions



- Discrete.



- Parametric continuous.



- Sampled.

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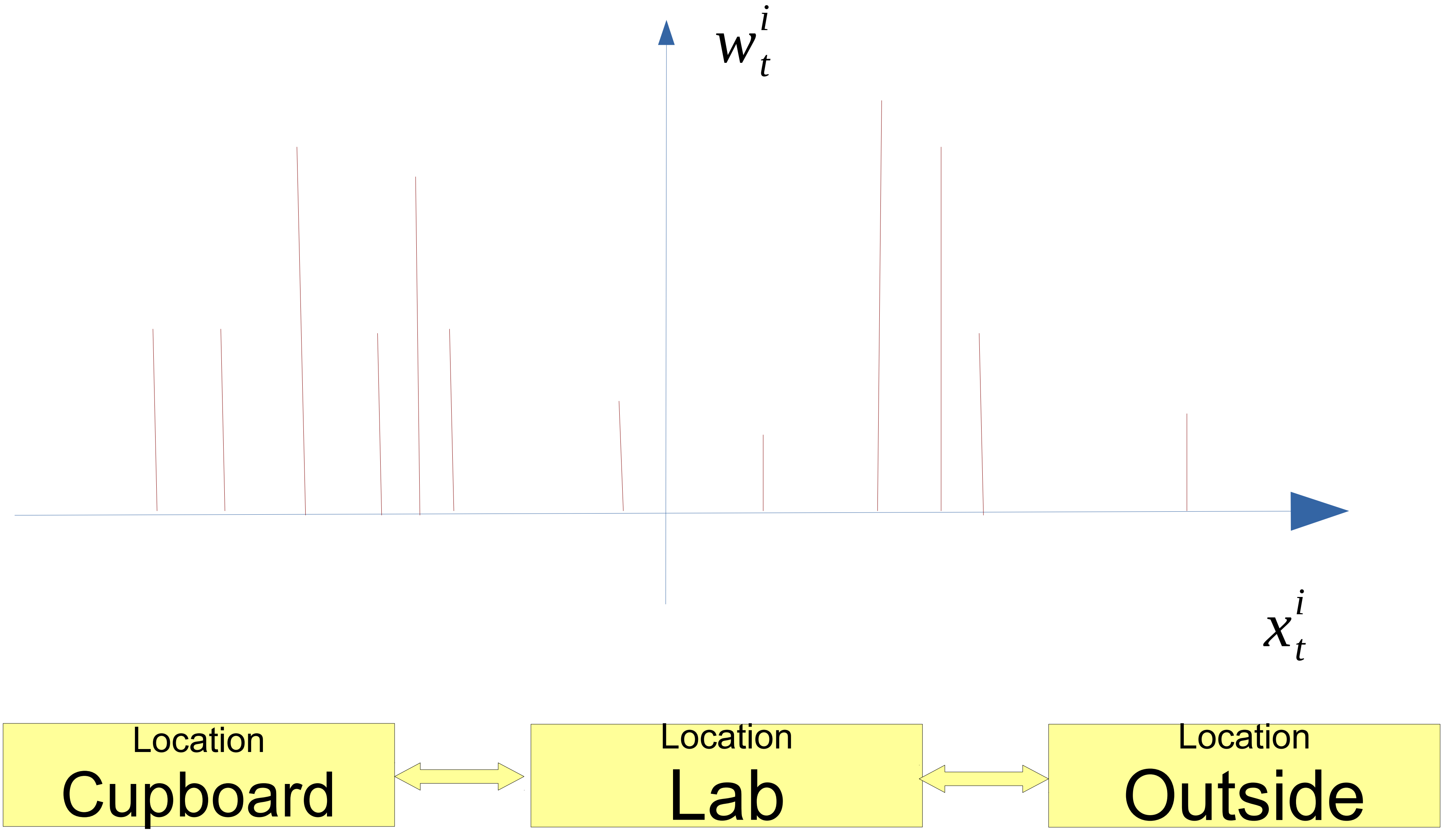
Representation of uncertainty using particle filter



Particle filter: the key ideas I

- **Sample states randomly** from continuous domain.

  (now have **finite list** of states)

- For each state keep a *weight* representing state's probability.

$$P\left(x_t \middle| z_t \ldots z_0\right)$$

$$\sim$$

$$\left\{\left(x_t^0, w_t^0\right), \ldots, \left(x_t^i, w_t^i\right), \ldots, \left(x_t^N, w_t^N\right)\right\}$$

Particle filter: the key ideas II

Predict:

Sample state from dynamics model (usually).

Update:

Update weight from observation model (usually).

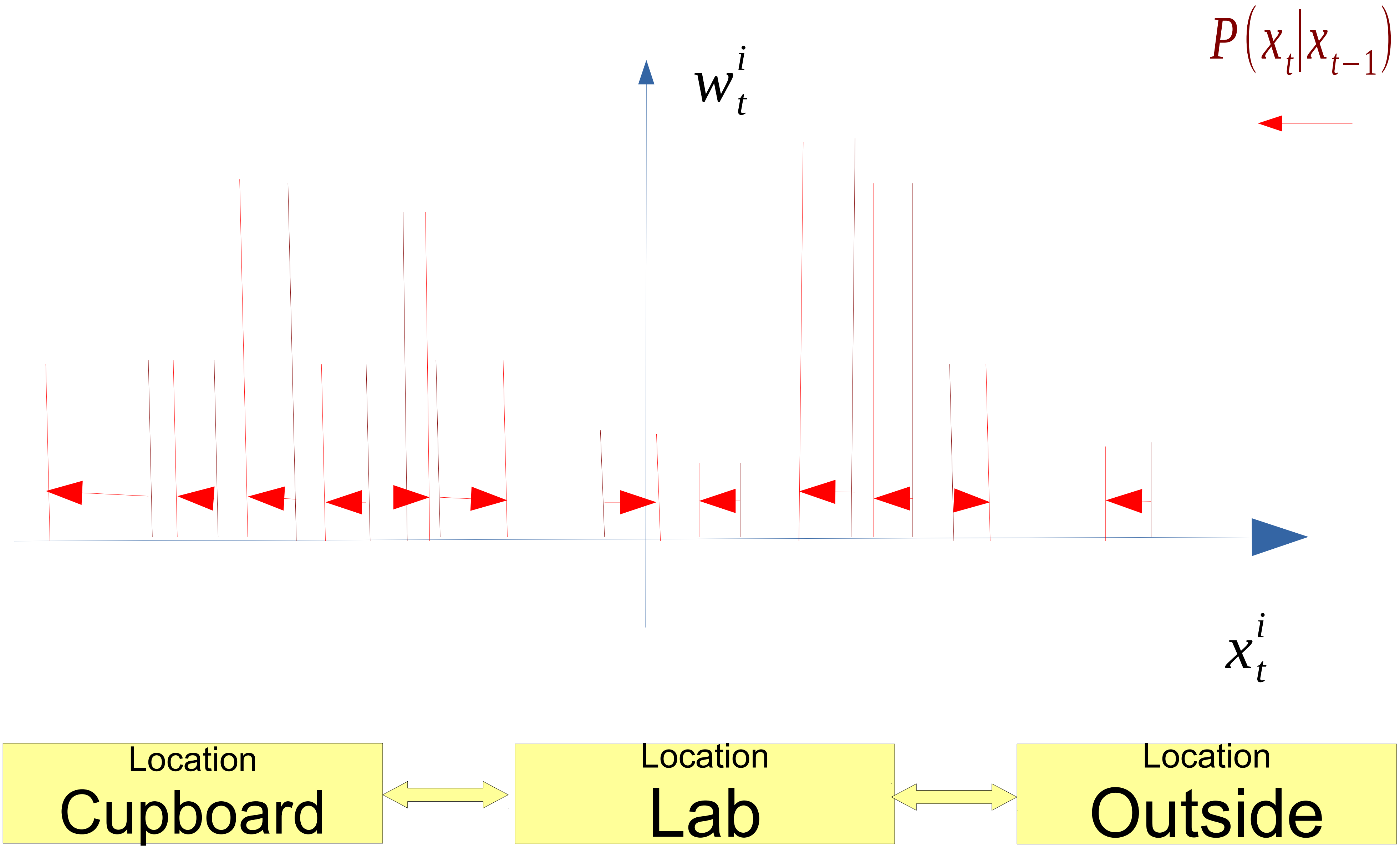
Resample:

Normalise weights, adding and removing particles (usually).

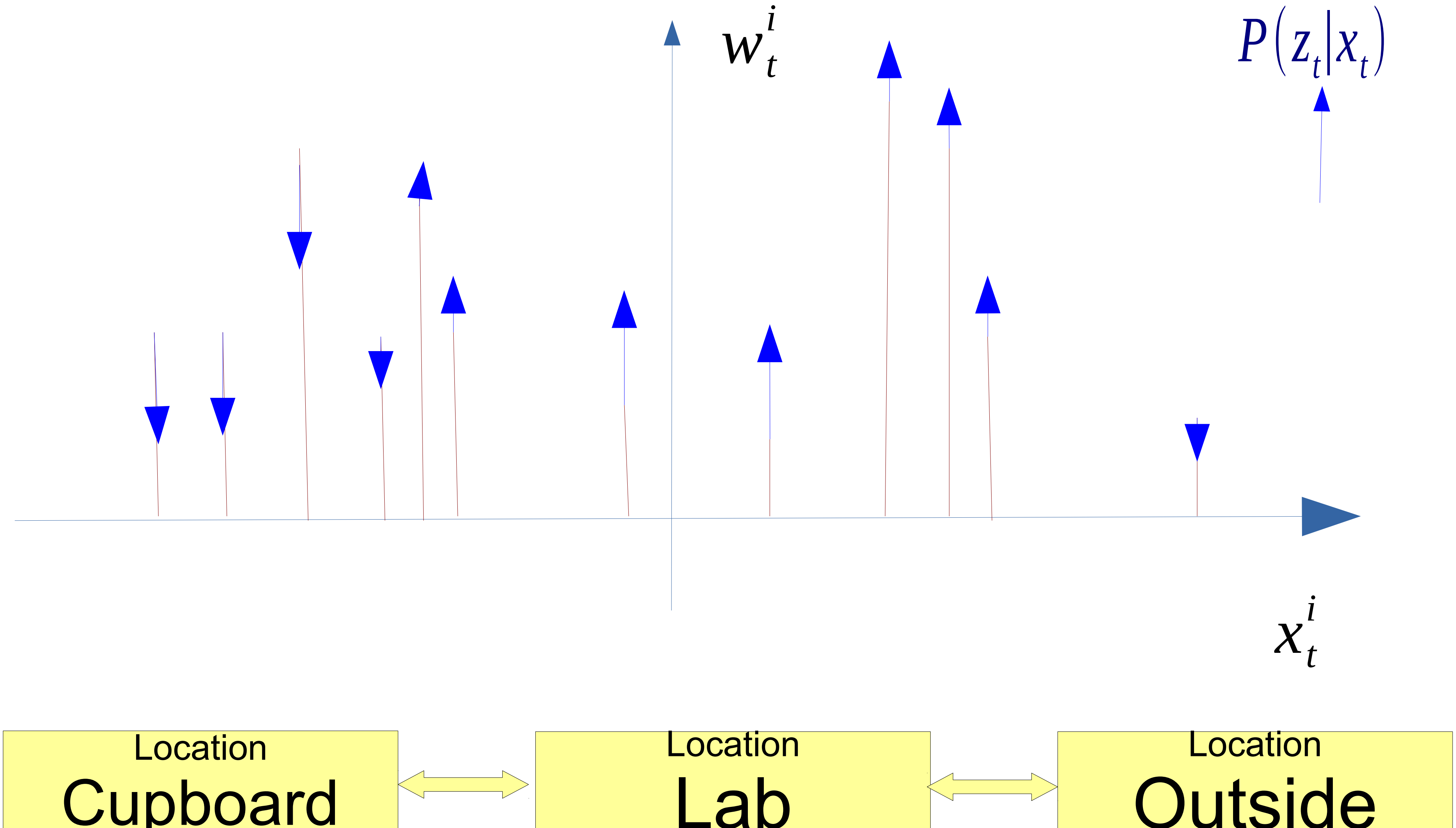
Variants:

- Importance sampling v. rejection sampling v. no resampling.
- Predict-update v. adapted particle filter v. auxiliary particle filter.
- Sampling v. no resampling.

Predict step moves samples estimate according to dynamics model



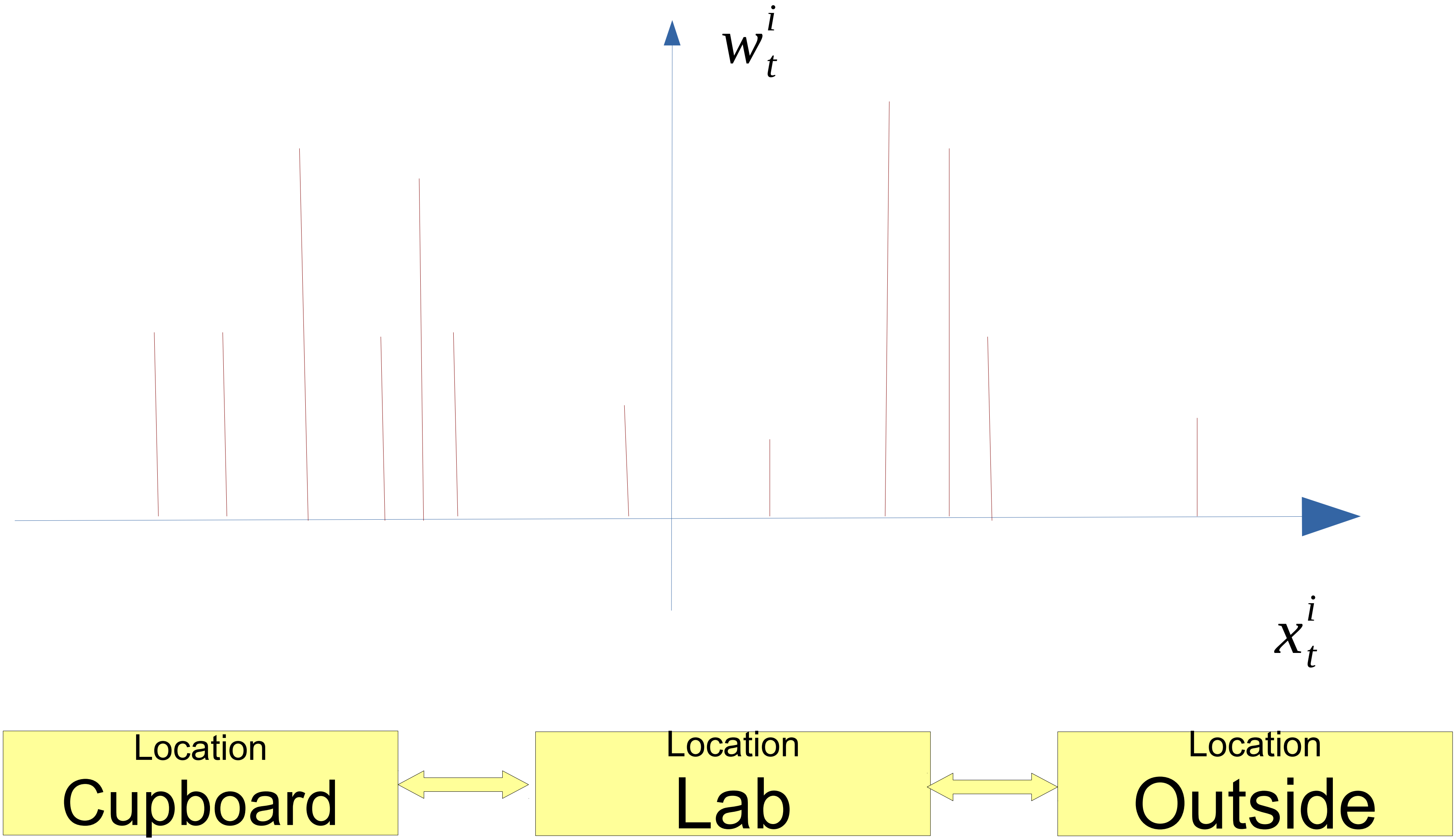
Update step changes particle weight according to observation model



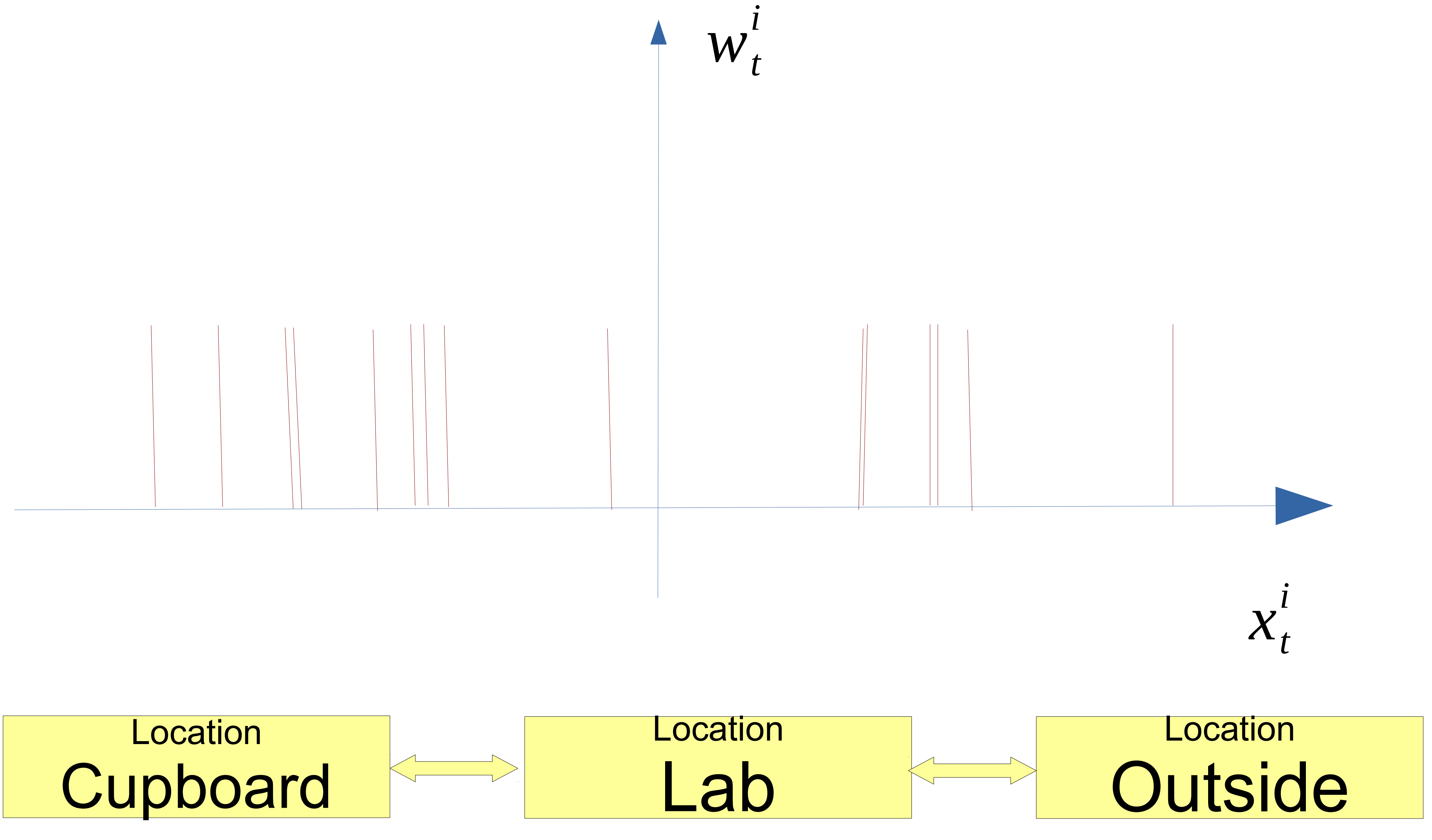
Resampling

- Maintain particles in high-probability regions.
- For each particle:
  - If high weight:
    - Split into multiple particles according to weight.
  - If low weight:
    - Possibly delete it.

Representation of uncertainty using particle filter



Post-resampling representation of uncertainty



Particle filter algorithm (without resampling)

$$Recurse\left(\left\{ \left(x_{t-1}^{0}, w_{t-1}^{0}\right), \ldots, \left(x_{t-1}^{i}, w_{t-1}^{i}\right), \ldots, \left(x_{t-1}^{N}, w_{t-1}^{N}\right) \right\} \;,\; z_t \right):$$

$\quad$ for $i \in \{0, \ldots, N\}$:

$\quad\quad$ sample x_t^i from $P\left(x_t \mid x_{t-1}^i\right)$ Predict

$\quad\quad$ let $w_t^i \leftarrow w_{t-1}^i \cdot P\left(z_t \mid x_t\right)$ Update

$\quad$ for $i \in \{0, \ldots, N\}$:

$\quad\quad$ let $w_t^i \leftarrow \dfrac{w_t^i}{\sum_j w_t^j}$ Normalise

BLG456E
Robotics
Estimation for Localisation

- Intro to probabilistic frameworks.
- Intro to ML and MAP.
- ML localisation.
- MAP localisation.
- Simplified notation.
- Recursive estimation.
- Particle filtering.

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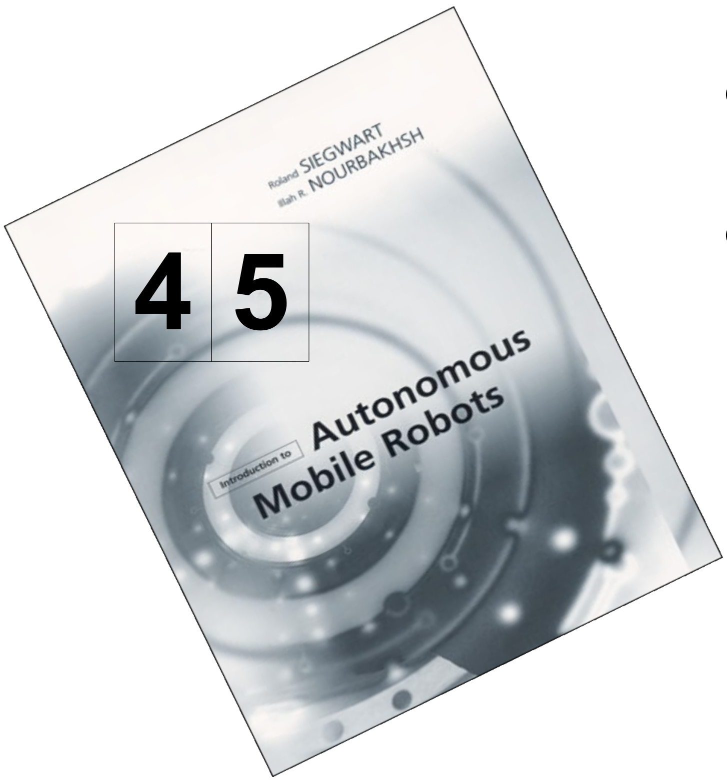
Review Questions

- How can a probabilistic observation model be acquired?
- What if we don't know the observation model?
- What is the difference between a discriminative and a generative observation model? Can you write it in probabilistic terms?
- When might you use MAP and when might you use ML?
- What is “recursive” about a “recursive estimator”?
- What does the “update” update in a predict-update cycle?

Reading



- **Chapter 4**. Perception.
- **Chapter 5**. Mobile Robot Localization.