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BLG311E – FORMAL LANGUAGES AND AUTOMATA

2016 FALL

QUIZ 6

Let $\mathbf{L} = \{\mathbf{ww} \mid \mathbf{w} \in \{\mathbf{0}, \mathbf{1}\}^*\}$.

For example: $\mathbf{0101}$, $\mathbf{00010001}, \mathbf{00110011} \in \mathbf{L}$ and 0110 $\notin$ L.

Prove that L is not a CFL language by using Pumping Lemma for CFLs.

Solution:

Suppose L is CFL

Let p be the P/L constant

Choose $z = 0^p 10^p 1 \in L,\ |z| \geq p$

$$z = uvwxy, \qquad |vwx| \leq p, \qquad |vx| \geq 1$$

$$uv^i wx^i y \in L \quad for\ all\ i \geq 0$$

If we make following division,

$$v = 0^k, \qquad w = 1, \qquad x = 0^k, \qquad k < p$$

$$for\ i \geq 0 \quad z' = 0^{p+i\cdot k} 10^{p+i\cdot k} 1 \in L$$

We could find a suitable division that satisfies the rule of PL. But this doesn't mean L is CFL. We should choose another word to be able prove that L is not CFL.

Let's choose $z = 0^p 1^p 0^p 1^p$ where $|z| \geq p$.

$$z = uvwxy, \qquad |vwx| \leq p, \qquad |vx| \geq 1$$

Since $|vwx| \leq p$, vwx can be in several parts of the word.

- Case 1: It may be found in one of contiguous 0's or 1's, e.g., first 0's, first 1's, second 0's or second 1's. When we pump it, obviously the resulting string is not in the form ww.
- Case 2: It may straddles two of the sections. There are three places where it can be in. When we pump down, we can show that the new string is not in the form ww. For example, $vy = 0^{k1} 1^{k2}$ and in the fist section of $0^p 1^p$. When we pump down, $z' = 0^{p-k1} 1^{p-k2} 0^p 1^p$. The midpoint must be somewhere in the second 0's section.
  So $z' = w_1 w_2$ where $w_1 = 0^{p-k1} 1^{p-k2} 0^m$ and $w_2 = 0^{p-m} 1^p$. Clearly two strings are not the same and z' is not in the form ww.

Finally,

L is not CFL because it violates the PL.