NUMERICAL METHODS

Week-10

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Integration

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Integration

- **What** is a integration?

- **Why** do we use integration?

- **How** do we solve integrals?

- **IMPORTANT NOTICE: PLEASE REFRESH YOUR CALCULUS 2 KNOWLEDGE ABOUT INTEGRALS**

What is Integration

Integration:

The process of measuring the area under a function plotted on a graph.

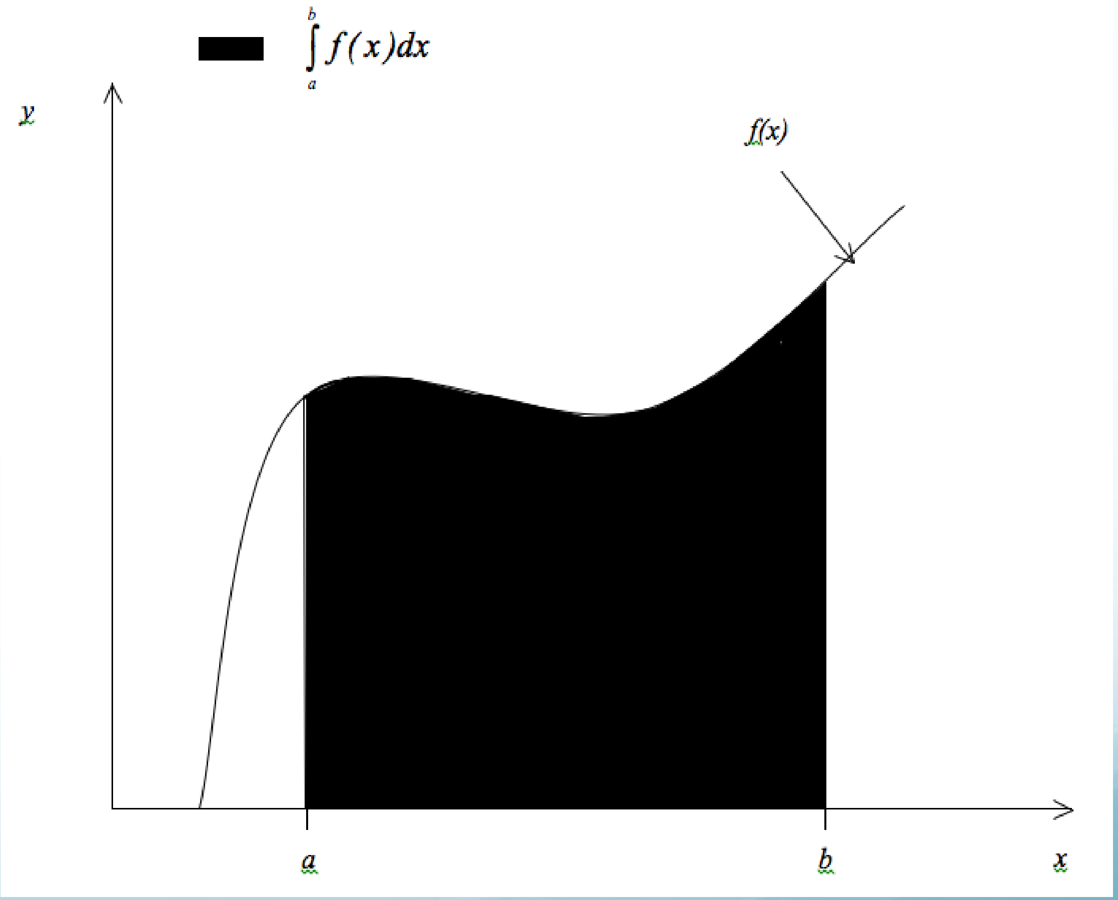
$$I = \int_a^b f(x)dx$$

Where:

f(x) is the integrand

a= lower limit of integration

b= upper limit of integration



Why to Use Integration?

- Area
- Volume
- Work
- Mass
- Distance, Velocity and acceleration
- Fluid pressure
- Accumulated Financial values
- ---

Some Rules to Solve Integrals

- Trapezoidal Rule
  - One Segment
  - Multiple Segment
- Gaussian Quadrature Rule
- Simpson's 1/3rd Rule
  - One Segment
  - Multiple Segment

Trapezoidal Rule

Basis of Trapezoidal Rule

Trapezoidal Rule is based on the Newton-Cotes Formula that states if one can approximate the integrand as an n^{th} order polynomial…

$$I = \int_a^b f(x)dx$$ where $$f(x) \approx f_n(x)$$

and $$f_n(x) = a_0 + a_1 x + \ldots + a_{n-1}x^{n-1} + a_n x^n$$

Basis of Trapezoidal Rule

Then the integral of that function is approximated by the integral of that n^{th} order polynomial.

$$\int_a^b f(x) \approx \int_a^b f_n(x)$$

Trapezoidal Rule assumes n=1, that is, the area under the linear polynomial,

$$\int_a^b f(x)dx \quad = (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

Derivation of the Trapezoidal Rule

Method Derived From Geometry

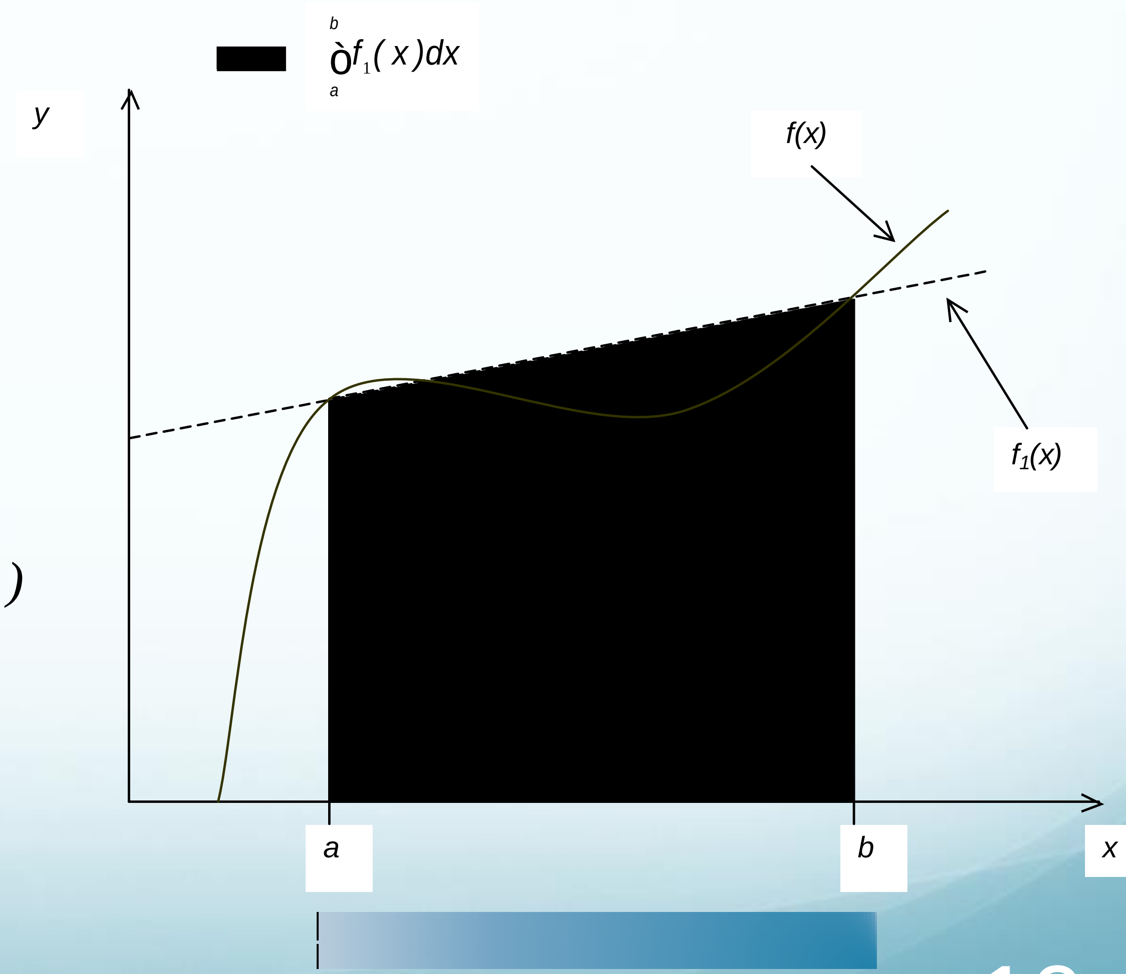
The area under the curve is a trapezoid. The integral

$$\int_a^b f(x)dx \approx Area\ of\ \ trapezoid$$

$$= \frac{1}{2}(Sum\ of\ \ parallel\ sides)(height)$$

$$= \frac{1}{2}\big(f(b)+f(a)\big)(b-a)$$

$$= (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$



Example 1

The vertical distance covered by a rocket from t=8 to t=30 seconds is given by:

$$x = \int_{8}^{30} \left(2000 ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

a) Use single segment Trapezoidal rule to find the distance covered.
b) Find the true error, E_t for part (a).
c) Find the absolute relative true error, $|\epsilon_a|$ for part (a).

Solution

a) $$I \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

$$a = 8 \qquad b = 30$$

$$f(t) = 2000ln\left[\frac{140000}{140000-2100t}\right] - 9.8t$$

$$f(8) = 2000ln\left[\frac{140000}{140000-2100(8)}\right] - 9.8(8) \qquad = 177.27\ m/s$$

$$f(30) = 2000ln\left[\frac{140000}{140000-2100(30)}\right] - 9.8(30) \quad = 901.67\ m/s$$

Solution (cont)

a) $$I = (30-8)\left[\frac{177.27+901.67}{2}\right]$$

$$= 11868\, m$$

b) The exact value of the above integral is

$$x = \int_{8}^{30}\left(2000 ln\left[\frac{140000}{140000-2100t}\right]-9.8t\right)dt \quad = 11061\, m$$

Solution (cont)

b) $$E_t = True\ Value - Approximate\ Value$$

$$= 11061 - 11868$$

$$= -807\ m$$

c) The absolute relative true error, $|\in_t|$, would be

$$|\in_t| = \left|\frac{11061 - 11868}{11061}\right| \times 100 = 7.2959\%$$

Multiple Segment Trapezoidal Rule

In Example 1, the true error using single segment trapezoidal rule was large. We can divide the interval [8,30] into [8,19] and [19,30] intervals and apply Trapezoidal rule over each segment.

$$f(t) = 2000\ln\left(\frac{140000}{140000 - 2100t}\right) - 9.8t$$

$$\int_{8}^{30} f(t)dt = \int_{8}^{19} f(t)dt + \int_{19}^{30} f(t)dt$$

$$= (19-8)\left[\frac{f(8) + f(19)}{2}\right] + (30-19)\left[\frac{f(19) + f(30)}{2}\right]$$

Multiple Segment Trapezoidal Rule

With

$$f(8)=177.27\ m/s$$

$$f(30)=901.67\ m/s$$

$$f(19)=484.75\ m/s$$

Hence:

$$\int_{8}^{30} f(t)dt=(19-8)\left[\frac{177.27+484.75}{2}\right]+(30-19)\left[\frac{484.75+901.67}{2}\right]$$

$$=11266\ m$$

Multiple Segment Trapezoidal Rule

The true error is:

$$E_t = 11061 - 11266$$
$$= -205\ m$$

The true error now is reduced from -807 m to -205 m.

Extending this procedure to divide the interval into equal segments to apply the Trapezoidal rule; the sum of the results obtained for each segment is the approximate value of the integral.

Multiple Segment Trapezoidal Rule

Divide into equal segments as shown in Figure 4. Then the width of each segment is:

$$h = \frac{b-a}{n}$$

The integral I is:

$$I = \int_a^b f(x)dx$$

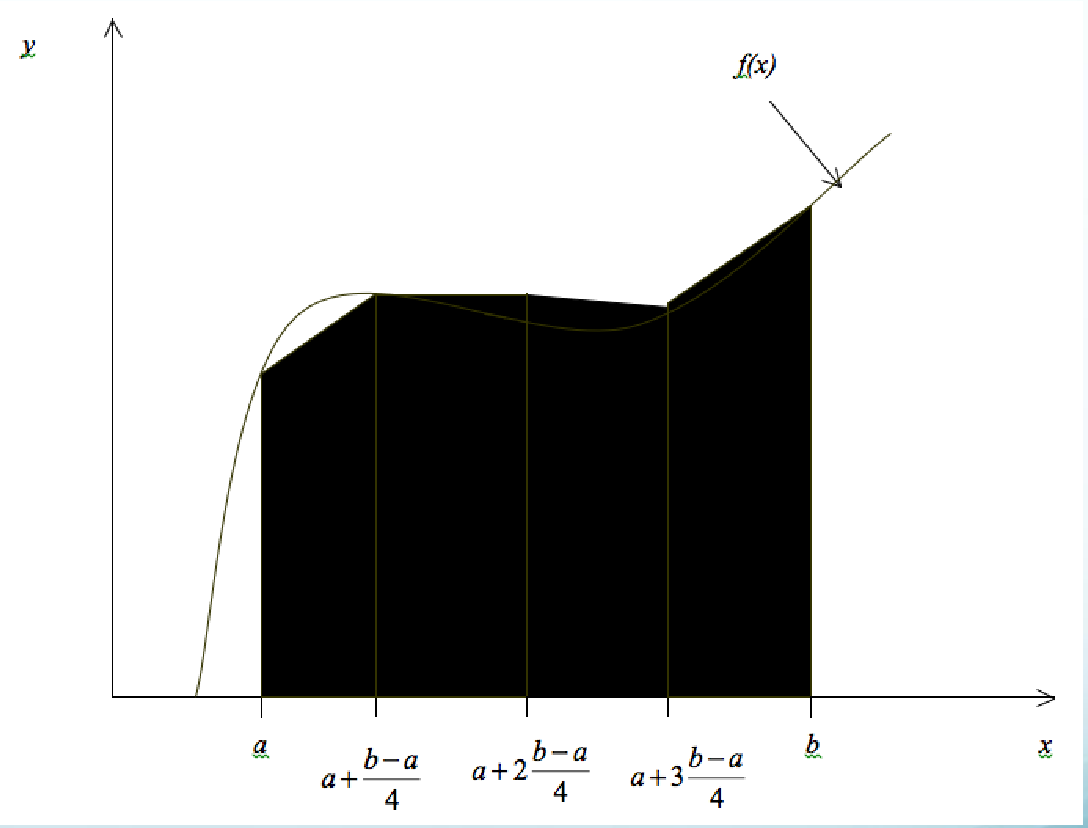


Figure: Multiple (n=4) Segment Trapezoidal Rule

Multiple Segment Trapezoidal Rule

The integral I can be broken into h integrals as:

$$\int_a^b f(x)dx = \int_a^{a+h} f(x)dx + \int_{a+h}^{a+2h} f(x)dx + \ldots + \int_{a+(n-2)h}^{a+(n-1)h} f(x)dx + \int_{a+(n-1)h}^{b} f(x)dx$$

Applying Trapezoidal rule on each segment gives:

$$\int_a^b f(x)dx = \frac{b-a}{2n}\left[f(a) + 2\left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right]$$

Example 2

The vertical distance covered by a rocket from to seconds is given by:

$$x = \int_{8}^{30} \left(2000\, ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t \right) dt$$

a) Use two-segment Trapezoidal rule to find the distance covered.
b) Find the true error, E_t for part (a).
c) Find the absolute relative true error, $|\in_a|$ for part (a).

Solution

a) The solution using 2-segment Trapezoidal rule is

$$I = \frac{b-a}{2n}\left[f(a) + 2\left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right]$$

$$n = 2 \qquad a = 8 \qquad b = 30$$

$$h = \frac{b-a}{n} = \frac{30-8}{2} = 11$$

Solution (cont)

Then:

$$I = \frac{30-8}{2(2)}\left[f(8) + 2\left\{ \sum_{i=1}^{2-1} f(a+ih) \right\} + f(30) \right]$$

$$= \frac{22}{4}\left[f(8) + 2f(19) + f(30) \right]$$

$$= \frac{22}{4}\left[177.27 + 2(484.75) + 901.67 \right]$$

$$= 11266\ m$$

Solution (cont)

b) The exact value of the above integral is

$$x = \int_{8}^{30}\left(2000\,ln\left[\frac{140000}{140000-2100t}\right]-9.8t\right)dt \quad = 11061\,m$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$

$$= 11061 - 11266$$

Solution (cont)

The absolute relative true error, $|\in_t|$, would be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100$$

$$= \left|\frac{11061 - 11266}{11061}\right| \times 100$$

$$= 1.8534\%$$

Solution (cont)

Table 1 gives the values obtained using multiple segment Trapezoidal rule for:

$$x = \int_{8}^{30} \left(2000 \ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t \right) dt$$

n	Value	E_t	$\lvert\in_t\rvert\%$	$\lvert\in_a\rvert\%$
1	11868	-807	7.296	---
2	11266	-205	1.853	5.343
3	11153	-91.4	0.8265	1.019
4	11113	-51.5	0.4655	0.3594
5	11094	-33.0	0.2981	0.1669
6	11084	-22.9	0.2070	0.09082
7	11078	-16.8	0.1521	0.05482
8	11074	-12.9	0.1165	0.03560

Table 1: Multiple Segment Trapezoidal Rule Values

Example 3

Use Multiple Segment Trapezoidal Rule to find the area under the curve

$$f(x)=\frac{300x}{1+e^{x}}$$ from $x=0$ to $x=10$

Using two segments, we get $h=\frac{10-0}{2}=5$ and

$$f(0)=\frac{300(0)}{1+e^{0}}=0 \qquad f(5)=\frac{300(5)}{1+e^{5}}=10.039 \qquad f(10)=\frac{300(10)}{1+e^{10}}=0.136$$

Solution

Then:

$$I = \frac{b-a}{2n}\left[f(a) + 2\left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right]$$

$$= \frac{10-0}{2(2)}\left[f(0) + 2\left\{ \sum_{i=1}^{2-1} f(0+5) \right\} + f(10) \right]$$

$$= \frac{10}{4}\left[f(0) + 2f(5) + f(10) \right] = \frac{10}{4}\left[0 + 2(10.039) + 0.136 \right]$$

$$= 50.535$$

Solution (cont)

So what is the true value of this integral?

$$\int_0^{10} \frac{300x}{1+e^x}\, dx = 246.59$$

Making the absolute relative true error:

$$\left|\epsilon_t\right| = \left|\frac{246.59 - 50.535}{246.59}\right| \times 100\%$$

$$= 79.506\%$$

Solution (cont)

Table 2: Values obtained using Multiple Segment Trapezoidal Rule for:

$$\int_0^{10} \frac{300x}{1+e^x}\,dx$$

n	Approximate Value	E_t	$\lvert \in_t \rvert$
1	0.681	245.91	99.724%
2	50.535	196.05	79.505%
4	170.61	75.978	30.812%
8	227.04	19.546	7.927%
16	241.70	4.887	1.982%
32	245.37	1.222	0.495%
64	246.28	0.305	0.124%

Gaussian Quadrature Rule

Basis of the Gaussian Quadrature Rule

Previously, the Trapezoidal Rule was developed by the method of undetermined coefficients. The result of that development is summarized below.

$$\int_a^b f(x)dx \approx c_1 f(a) + c_2 f(b)$$

$$= \frac{b-a}{2} f(a) + \frac{b-a}{2} f(b)$$

Basis of the Gaussian Quadrature Rule

The two-point Gauss Quadrature Rule is an extension of the Trapezoidal Rule approximation where the arguments of the function are not predetermined as a and b but as unknowns x_1 and x_2. In the two-point Gauss Quadrature Rule, the integral is approximated as

$$I = \int_a^b f(x)dx \approx c_1 f(x_1) + c_2 f(x_2)$$

Basis of the Gaussian Quadrature Rule

The four unknowns x_1, x_2, c_1 and c_2 are found by assuming that the formula gives exact results for integrating a general third order polynomial, $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$.

Hence

$$\int_a^b f(x)dx = \int_a^b \left(a_0 + a_1x + a_2x^2 + a_3x^3\right)dx$$

$$= \left[a_0x + a_1\frac{x^2}{2} + a_2\frac{x^3}{3} + a_3\frac{x^4}{4}\right]_a^b$$

$$= a_0(b-a) + a_1\left(\frac{b^2-a^2}{2}\right) + a_2\left(\frac{b^3-a^3}{3}\right) + a_3\left(\frac{b^4-a^4}{4}\right)$$

Basis of the Gaussian Quadrature Rule

It follows that

$$\int_a^b f(x)dx = c_1\left(a_0 + a_1x_1 + a_2x_1^{\ 2} + a_3x_1^{\ 3}\right) + c_2\left(a_0 + a_1x_2 + a_2x_2^{\ 2} + a_3x_2^{\ 3}\right)$$

Equating Equations the two previous two expressions yield

$$a_0(b-a) + a_1\left(\frac{b^2-a^2}{2}\right) + a_2\left(\frac{b^3-a^3}{3}\right) + a_3\left(\frac{b^4-a^4}{4}\right)$$

$$= c_1\left(a_0 + a_1x_1 + a_2x_1^{\ 2} + a_3x_1^{\ 3}\right) + c_2\left(a_0 + a_1x_2 + a_2x_2^{\ 2} + a_3x_2^{\ 3}\right)$$

$$= a_0(c_1 + c_2) + a_1(c_1x_1 + c_2x_2) + a_2\left(c_1x_1^{\ 2} + c_2x_2^{\ 2}\right) + a_3\left(c_1x_1^{\ 3} + c_2x_2^{\ 3}\right)$$

Basis of the Gaussian Quadrature Rule

Since the constants a_0, a_1, a_2, a_3 are arbitrary

$$b-a=c_1+c_2$$

$$\frac{b^2-a^2}{2}=c_1x_1+c_2x_2$$

$$\frac{b^3-a^3}{3}=c_1x_1^{\ 2}+c_2x_2^{\ 2}$$

$$\frac{b^4-a^4}{4}=c_1x_1^{\ 3}+c_2x_2^{\ 3}$$

Basis of Gauss Quadrature

The previous four simultaneous nonlinear Equations have only one acceptable solution,

$$x_1 = \left(\frac{b-a}{2}\right)\left(-\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2} \qquad x_2 = \left(\frac{b-a}{2}\right)\left(\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}$$

$$c_1 = \frac{b-a}{2} \qquad c_2 = \frac{b-a}{2}$$

Basis of Gauss Quadrature

Hence Two-Point Gaussian Quadrature Rule

$$\int_a^b f(x)dx \approx c_1 f(x_1) + c_2 f(x_2)$$

$$= \frac{b-a}{2} f\left(\frac{b-a}{2}\left(-\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}\right) + \frac{b-a}{2} f\left(\frac{b-a}{2}\left(\frac{1}{\sqrt{3}}\right) + \frac{b+a}{2}\right)$$

Example

For an integral $\int_a^b f(x)dx,$ derive the one-point Gaussian Quadrature Rule.

Solution

The one-point Gaussian Quadrature Rule is

$$\int_a^b f(x)dx \approx c_1 f\left(x_1\right)$$

Solution

The two unknowns x_1, and c_1 are found by assuming that the formula gives exact results for integrating a general first order polynomial,

$$f(x) = a_0 + a_1 x.$$

$$\int_a^b f(x)dx = \int_a^b (a_0 + a_1 x)dx$$

$$= \left[a_0 x + a_1 \frac{x^2}{2} \right]_a^b$$

$$= a_0(b-a) + a_1\left(\frac{b^2 - a^2}{2}\right)$$

Solution

It follows that

$$\int_a^b f(x)dx = c_1\left(a_0 + a_1 x_1\right)$$

Equating Equations, the two previous two expressions yield

$$a_0(b-a) + a_1\left(\frac{b^2 - a^2}{2}\right) = c_1\left(a_0 + a_1 x_1\right) = a_0(c_1) + a_1(c_1 x_1)$$

Basis of the Gaussian Quadrature Rule

Since the constants a_0, and a_1 are arbitrary

$$b - a = c_1$$

$$\frac{b^2 - a^2}{2} = c_1 x_1$$

giving

$$c_1 = b - a$$

$$x_1 = \frac{b + a}{2}$$

Solution

Hence One-Point Gaussian Quadrature Rule

$$\int_a^b f(x)dx \approx c_1 f\left(x_1\right) = (b-a)\ f\left(\frac{b+a}{2}\right)$$

Simpson's 1/3rd Rule

Basis of Simpson's 1/3rd Rule

Trapezoidal rule was based on approximating the integrand by a first order polynomial, and then integrating the polynomial in the interval of integration. Simpson's 1/3rd rule is an extension of Trapezoidal rule where the integrand is approximated by a second order polynomial.

Hence $$I = \int_a^b f(x)dx \approx \int_a^b f_2(x)dx$$

Where $f_2(x)$ is a second order polynomial.

$$f_2(x) = a_0 + a_1 x + a_2 x^2$$

Basis of Simpson's 1/3rd Rule

Choose

$$(a, f(a)), \left(\frac{a+b}{2}, f\left(\frac{a+b}{2}\right)\right), \text{ and } (b, f(b))$$

as the three points of the function to evaluate a_0, a_1 and a_2.

$$f(a) = f_2(a) = a_0 + a_1 a + a_2 a^2$$

$$f\left(\frac{a+b}{2}\right) = f_2\left(\frac{a+b}{2}\right) = a_0 + a_1\left(\frac{a+b}{2}\right) + a_2\left(\frac{a+b}{2}\right)^2$$

$$f(b) = f_2(b) = a_0 + a_1 b + a_2 b^2$$

Basis of Simpson's 1/3rd Rule

Solving the previous equations for a_0, a_1 and a_2 give

$$a_0 = \frac{a^2 f(b) + abf(b) - 4abf\left(\frac{a+b}{2}\right) + abf(a) + b^2 f(a)}{a^2 - 2ab + b^2}$$

$$a_1 = -\frac{af(a) - 4af\left(\frac{a+b}{2}\right) + 3af(b) + 3bf(a) - 4bf\left(\frac{a+b}{2}\right) + bf(b)}{a^2 - 2ab + b^2}$$

$$a_2 = \frac{2\left(f(a) - 2f\left(\frac{a+b}{2}\right) + f(b)\right)}{a^2 - 2ab + b^2}$$

Basis of Simpson's 1/3rd Rule

Then

$$I \approx \int_a^b f_2(x)dx$$

$$= \int_a^b \left(a_0 + a_1 x + a_2 x^2\right)dx$$

$$= \left[a_0 x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3}\right]_a^b$$

$$= a_0(b-a) + a_1 \frac{b^2 - a^2}{2} + a_2 \frac{b^3 - a^3}{3}$$

Basis of Simpson's 1/3rd Rule

Substituting values of a_0, a_1, a_2 give

$$\int_a^b f_2(x)dx = \frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

Since for Simpson's 1/3rd Rule, the interval [a, b] is broken into 2 segments, the segment width

$$h = \frac{b-a}{2}$$

Basis of Simpson's 1/3rd Rule

Hence

$$\int_a^b f_2(x)dx = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

Because the above form has 1/3 in its formula, it is called Simpson's 1/3rd Rule.

Example

The distance covered by a rocket from t=8 to t=30 is given by

$$x = \int_{8}^{30} \left(2000\, ln\left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

a) Use Simpson's 1/3rd Rule to find the approximate value of x

b) Find the true error, E_t

c) Find the absolute relative true error, $\left| \in_t \right|$

Solution

a)

$$x = \int_{8}^{30} f(t)dt$$

$$x = \left(\frac{b-a}{6}\right)\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$= \left(\frac{30-8}{6}\right)[f(8) + 4f(19) + f(30)]$$

$$= \left(\frac{22}{6}\right)[177.2667 + 4(484.7455) + 901.6740]$$

$$= 11065.72\ m$$

Solution (cont)

b) The exact value of the above integral is

$$x = \int_{8}^{30} \left(2000\, ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

$$= 11061.34\ m$$

True Error

$$E_t = 11061.34 - 11065.72$$

$$= -4.38\ m$$

Solution (cont)

a)c) Absolute relative true error,

$$\left|\in_t\right| = \left|\frac{11061.34 - 11065.72}{11061.34}\right| \times 100\%$$

$$= 0.0396\%$$

Multiple Segment Simpson's 1/3rd Rule

Multiple Segment Simpson's 1/3rd Rule

Just like in multiple segment Trapezoidal Rule, one can subdivide the interval

[a, b] into n segments and apply Simpson's 1/3rd Rule repeatedly over every two segments. Note that n needs to be even. Divide interval [a, b] into equal segments, hence the segment width

$$h = \frac{b-a}{n} \qquad \int_a^b f(x)dx = \int_{x_0}^{x_n} f(x)dx$$

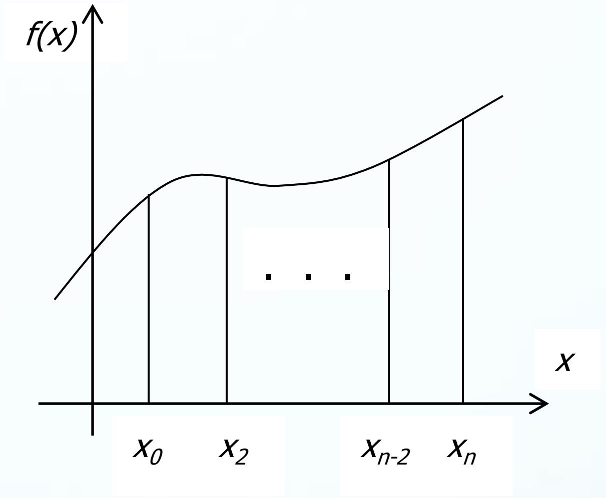
where

$$x_0 = a \qquad x_n = b$$

Multiple Segment Simpson's 1/3rd Rule

$$\int_a^b f(x)dx = \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \ldots..$$

$$\ldots. + \int_{x_{n-4}}^{x_{n-2}} f(x)dx + \int_{x_{n-2}}^{x_n} f(x)dx$$



Apply Simpson's 1/3rd Rule over each interval,

$$\int_a^b f(x)dx = (x_2 - x_0)\left[\frac{f(x_0)+4f(x_1)+f(x_2)}{6}\right] + \ldots$$

$$+(x_4 - x_2)\left[\frac{f(x_2)+4f(x_3)+f(x_4)}{6}\right] + \ldots$$

Multiple Segment Simpson's 1/3rd Rule

$$\ldots+(x_{n-2}-x_{n-4})\left[\frac{f(x_{n-4})+4f(x_{n-3})+f(x_{n-2})}{6}\right]+\ldots$$

$$+(x_n-x_{n-2})\left[\frac{f(x_{n-2})+4f(x_{n-1})+f(x_n)}{6}\right]$$

Since

$$x_i-x_{i-2}=2h \qquad i=2,4,\ldots,n$$

Multiple Segment Simpson's 1/3rd Rule

Then

$$\int_a^b f(x)dx = 2h\left[\frac{f(x_0)+4f(x_1)+f(x_2)}{6}\right]+\ldots$$

$$+2h\left[\frac{f(x_2)+4f(x_3)+f(x_4)}{6}\right]+\ldots$$

$$+2h\left[\frac{f(x_{n-4})+4f(x_{n-3})+f(x_{n-2})}{6}\right]+\ldots$$

$$+2h\left[\frac{f(x_{n-2})+4f(x_{n-1})+f(x_n)}{6}\right]$$

Multiple Segment Simpson's 1/3rd Rule

$$\int_a^b f(x)dx = \frac{h}{3}[f(x_0)+4\{f(x_1)+f(x_3)+\ldots+f(x_{n-1})\}+\ldots]$$

$$\ldots+2\{f(x_2)+f(x_4)+\ldots+f(x_{n-2})\}+f(x_n)\}]$$

$$=\frac{h}{3}\left[f(x_0)+4\sum_{\substack{i=1\\i=odd}}^{n-1}f(x_i)+2\sum_{\substack{i=2\\i=even}}^{n-2}f(x_i)+f(x_n)\right]$$

$$=\frac{b-a}{3n}\left[f(x_0)+4\sum_{\substack{i=1\\i=odd}}^{n-1}f(x_i)+2\sum_{\substack{i=2\\i=even}}^{n-2}f(x_i)+f(x_n)\right]$$

Example

Use 4-segment Simpson's 1/3rd Rule to approximate the distance

covered by a rocket from t= 8 to t=30 as given by

$$x = \int_{8}^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

a) Use four segment Simpson's 1/3rd Rule to find the approximate value of *x*.
b) Find the true error, E_t for part (a).
c) Find the absolute relative true error, $|\epsilon_a|$ for part (a).

Solution

a) Using n segment Simpson's 1/3rd Rule,

$$h = \frac{30-8}{4} = 5.5$$

So

$$f(t_0) = f(8)$$

$$f(t_1) = f(8+5.5) = f(13.5)$$

$$f(t_2) = f(13.5+5.5) = f(19)$$

$$f(t_3) = f(19+5.5) = f(24.5)$$

$$f(t_4) = f(30)$$

Solution (cont.)

$$x = \frac{b-a}{3n}\left[f(t_0) + 4\sum_{\substack{i=1 \\ i=odd}}^{n-1} f(t_i) + 2\sum_{\substack{i=2 \\ i=even}}^{n-2} f(t_i) + f(t_n) \right]$$

$$= \frac{30-8}{3(4)}\left[f(8) + 4\sum_{\substack{i=1 \\ i=odd}}^{3} f(t_i) + 2\sum_{\substack{i=2 \\ i=even}}^{2} f(t_i) + f(30) \right]$$

$$= \frac{22}{12}\left[f(8) + 4f(t_1) + 4f(t_3) + 2f(t_2) + f(30) \right]$$

Solution (cont.)

cont.

$$= \frac{11}{6}\left[f(8) + 4f(13.5) + 4f(24.5) + 2f(19) + f(30) \right]$$

$$= \frac{11}{6}\left[177.2667 + 4(320.2469) + 4(676.0501) + 2(484.7455) + 901.6740 \right]$$

$$= 11061.64\ m$$

Solution (cont.)

b) In this case, the true error is

$$E_t = 11061.34 - 11061.64 = -0.30\ m$$

c) The absolute relative true error

$$\left|\in_t\right| = \left|\frac{11061.34 - 11061.64}{11061.34}\right| \times 100\%$$

$$= 0.0027\%$$

Solution (cont.)

Table 1: Values of Simpson's 1/3rd Rule for Example 2 with multiple segments

n	Approximate Value	E_t	$\lvert\epsilon_t\rvert$
2	11065.72	4.38	0.0396%
4	11061.64	0.30	0.0027%
6	11061.40	0.06	0.0005%
8	11061.35	0.01	0.0001%
10	11061.34	0.00	0.0000%