BLG335E ANALYSIS OF ALGORITHMS I
MIDTERM - NOVEMBER 16, 2016, 12:30-14:30 (2 hours)

1 (10 pt)	2 (20 pt)	3 (20 pt)	4 (25 pt)	5 (10 pt)	6 (15 pt)	Total (100 pt)

On my honor, I declare that I neither give nor receive any unauthorized help on this exam.

Student Signature:___________________________

Write your name on each sheet.
Write your answers neatly (in English) in the space provided for them.
Show & justify your work clearly –no partial credits for just stating the results.
Books and notes are closed.
Good Luck!

Q1) (10 points) Asymptotic notation
Explain whether the statement, "The running time of algorithm A is at least $O(n^2)$" makes sense or not.

Q2) (20 pts) Insertion sort / Mergesort

Although merge sort runs in $\Theta(n\lg n)$ worst-case time and insertion sort runs in $\Theta(n^2)$ worst-case time, the constant factors in insertion sort make it faster for small *n*. Thus, it makes sense to use insertion sort within merge sort when subproblems become sufficiently small. Consider a modification to merge sort in which n/k sublists of length *k* are sorted using insertion sort and then merged using the standard merging mechanism, where *k* is a value to be determined.

a) (10 pts) Calculate the worst-case running time for sorting the n/k sublists, each of length *k*, and sorted by insertion sort.

b) (10 pts) Calculate the worst-case running time to merge the sublists.

Q3) (20 pts) Probabilistic Analysis and Randomized Algorithms / Indicator Random Variables

Calculate the expected running time of RAND-SELECT.

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RANDOMIZED-SELECT(A, p, r, i)
if p = r
   then return A[p]
q ← RANDOMIZED-PARTITION(A, p, r)
k ← q − p + 1
if i = k          ▷ the pivot value is the answer
   then return A[q]
elseif i < k
   then return RANDOMIZED-SELECT(A, p, q − 1, i)
else return RANDOMIZED-SELECT(A, q + 1, r, i − k)
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Hint 1: Use indicator random variables, "X_k"s such that:

$$X_k = \begin{cases} 1 & \textit{if PARTITION generates a } \boldsymbol{k : n - k - 1} \textit{ split,} \\ 0 & \textit{otherwise} \end{cases}$$

Hint 2: Benefit from substitution method.

Hint 3: Use fact:

$$\sum_{k=\lfloor n/2 \rfloor}^{n-1} k \leq \frac{3}{8}n^2$$

Q4) (25 pts) Recurrences

Solve the following recurrences by giving tight Θ-notation bounds. You need to provide your justification for your answer. Your solutions should be asymptotically tight.

a) (5 pts) Use Master Method to solve the following recursion:

$$T(n) = 2T\left(\frac{n}{2}\right) + n^3$$

b) (10 pts) Use Recursion Trees Method to solve the following recursion:

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n$$

c) (10 pts) Solve the recurrence $T(n) = 2T(\sqrt{n}) + \log n$ by making a change of variables.

Master theorem:
Let a≥1 and b>1 be constants, let f(n) be a function, and let T(n) be defined on the nonnegative integers by the recurrence

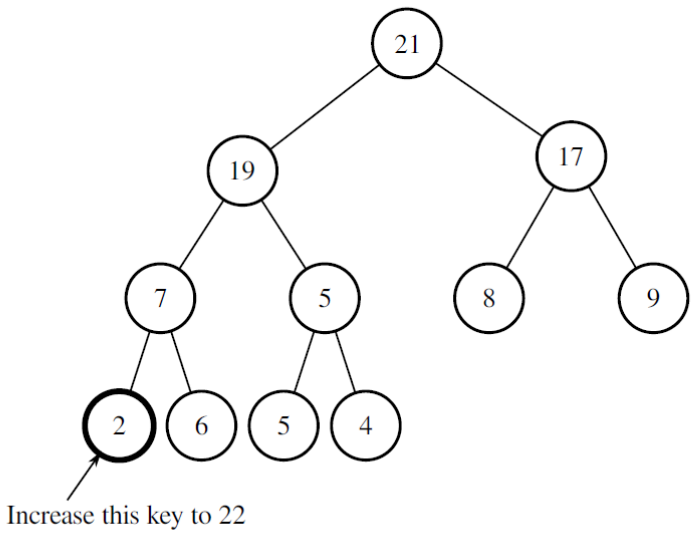
$$T(n) = aT(n/b) + f(n)$$

where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$. Then *T(n)* can be bounded asymptotically as follows.

1. If $f(n) = O(n^{\log_b a-\epsilon})$ for some constant ϵ>0, then $T(n) = \theta(n^{\log_b a})$.
2. If $f(n) = \theta(n^{\log_b a})$, then $T(n) = \theta(n^{\log_b a} \lg n)$.
3. If $f(n) = \Omega(n^{\log_b a+\epsilon})$ for some constant ϵ>0, and if $af(n/b) \leq cf(n)$ for some constant *c<1* and all sufficiently large n, then $T(n) = \theta(f(n))$.

Q5) (10 points) Heapsort

For the heap shown below, show what happens when you use HEAP-INCREASE-KEY to increase key 2 to 22. Give justification on why this operation is O(log *n*).



Q6) (15 pts) Radix sort

a) (10 pts) Use radix sort to sort *n* dates. A date is represented as month-day-year and all from 20^{th} century. Analyze the running time.

b) (5 pts) Does Radix Sort work correctly if we sort each individual digit using Insertion Sort instead of Counting Sort? Explain why.