1

1) a) S. { TT1, TH1, HT1, HH1,
TT2, TH2, HT2, HH2,
TT3, TH3, HT3, HH3,
TT4, TH4, HT4, HH4,
TT5, TH5, HT5, HH5,
TT6, TH6, HT6, HH6 }

b)

X	
$-1 \to$ HH1	
$0 \to$ TH1, HT1, HH2	
$1 \to$ TT1, TH2, HT2, HH3	
$2 \to$ TT2, TH3, HT3, HH4	
$3 \to$ TT3, TH4, HT4, HH5	
$4 \to$ TT4, TH5, HT5, HH6	
$5 \to$ TT5, TH6, HT6	
$6 \to$ TT6	

P(x)

$P(-1) = 1/24$

$P(0) = 3/24$

$P(1) = 4/24$

$P(2) = 4/24$

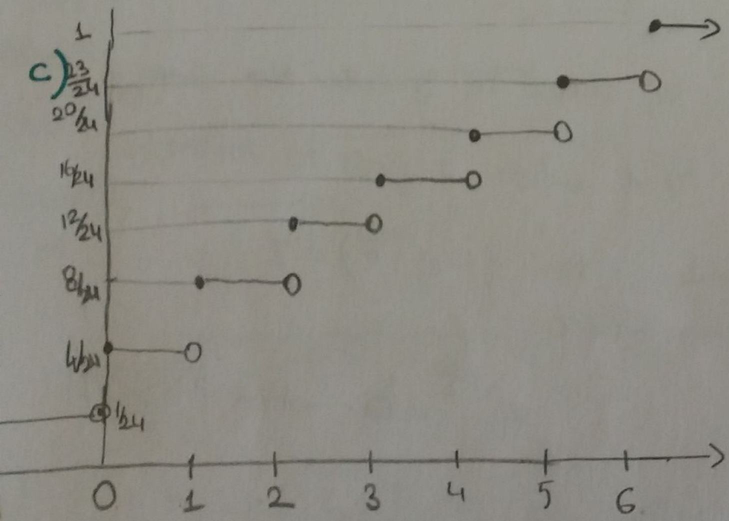
$P(3) = 4/24$

$P(4) = 4/24$

$P(5) = 3/24$

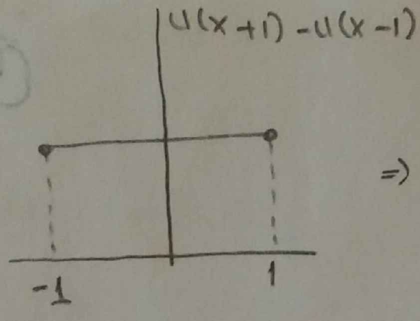
$P(6) = 1/24$

c)

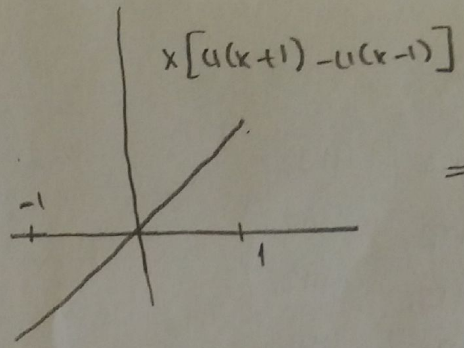


$$F(x) = \begin{cases} 0, & x < -1 \\ 1/24, & -1 \le x < 0 \\ 4/24, & 0 \le x < 1 \\ 8/24, & 1 \le x < 2 \\ 12/24, & 2 \le x < 3 \\ 16/24, & 3 \le x < 4 \\ 20/24, & 4 \le x < 5 \\ 23/24, & 5 \le x < 6 \\ 1, & x \ge 6 \end{cases}$$

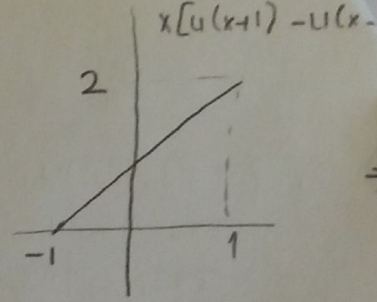
(2)



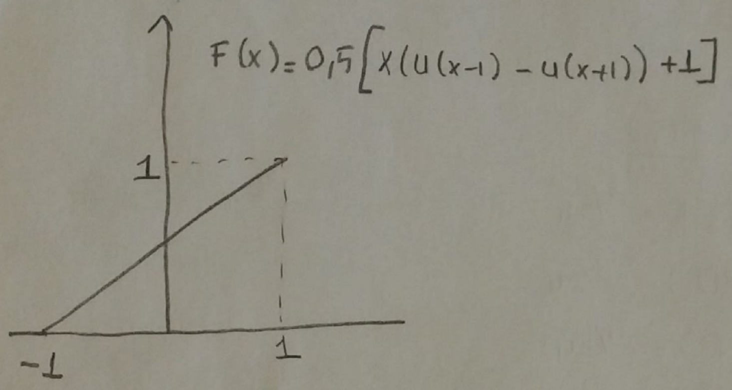
$\Rightarrow$



$\Rightarrow$



$\Rightarrow$

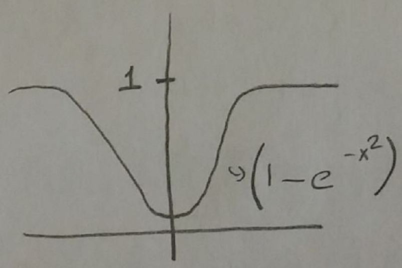


✓ Yes this is valid CDF because

$F(x) = 0$ for $x = -1$

$F(x) = 1$ for $x = 1$ this is continuous CDF

(3) $F_x(x) = \left(1 - e^{-\frac{x^2}{b}}\right) \cdot u(x-a)$



From this graph 'a' should be greater then zero.

Also, $F_x(x)$ goes to 1 when x goes to infinity.

$$\lim_{x \to \infty} F_x(x) = 1 \qquad \lim_{x \to \infty} \left(1 - e^{-\frac{x^2}{b}}\right) = 1$$

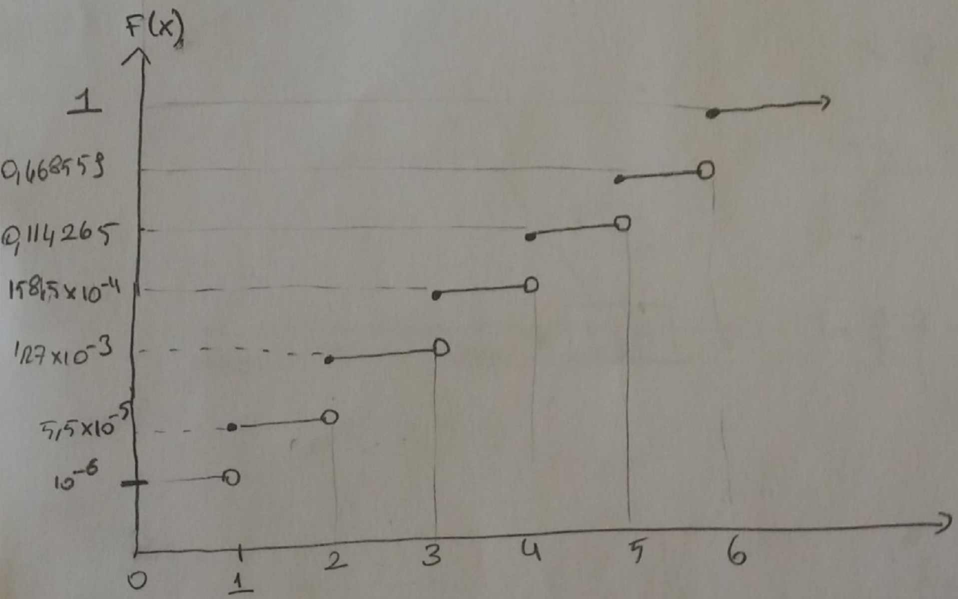
$$e^{-\frac{\infty}{b}} = 0 \text{ when } \boxed{b > 0}!$$

(2)

5) a)

x	Rotton (0,1)	P(x)
0	6	$P(0) = \binom{6}{0} \cdot (0{,}9)^0 (0{,}1)^6 = 0{,}000001$
1	5	$P(1) = \binom{6}{1} \cdot 0{,}9 \cdot (0{,}1)^5 = 0{,}000054$
2	4	$P(2) = \binom{6}{2} \cdot 0{,}9^2\ 0{,}1^4 = 0{,}001215$
3	3	$P(3) = \binom{6}{3} \cdot 0{,}9^3\ 0{,}1^3 = 0{,}01458$
4	2	$P(4) = \binom{6}{4} \cdot 0{,}9^4 \cdot 0{,}1^2 = 0{,}098415$
5	1	$P(5) = \binom{6}{5} \cdot 0{,}9^5\ 0{,}1 = 0{,}354294$
6	0	$P(6) = 0{,}9^6 = 0{,}531441$

$+$ ——— 1



b) $P(1{,}9 < x < 5{,}5) = F_x(5{,}5) - F_x(1{,}9)$

$= 0{,}468559 - 5{,}5 \times 10^{-5} = 0{,}468504$

c) $P(X < 2) = 0{,}354294 + 0{,}531441 = 0{,}885735$ (not include 2)

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6) a)

b) $P(x<30) = \int_{-\infty}^{30} f_x(x) = \int_{-\infty}^{10} 0\,dx + \frac{1}{5}\int_{10}^{30} e^{\frac{-(x-10)}{5}}\,dx$

$= -e^{-\frac{(x-10)}{5}}\Big|_{10}^{30} = -e^{-20} \cdot e^{0} = \underline{1-e^{-20}}$

c) $P(x=20) = \int_{20}^{20} f_x(x) \cdot dx = 0$ ✓

7) $\int_0^1 Ax^2\,dx = 1$

$A \cdot \frac{x^3}{3}\Big|_0^1 = 1 \Rightarrow A \cdot \frac{1}{3} - A \cdot \frac{0}{3} = 1 \Rightarrow \boxed{A=3}$ ✓

8) $N(\mu, \sigma^2) = N(50, 10)$

a) $\frac{50 \times 60}{100} = 30$ ✓ $z = \frac{x-\mu}{\sigma}$

$P(x<30) = P\left(z < \frac{30-50}{\sqrt{10}}\right) = P(z < -6{,}32) = 0$ ✓

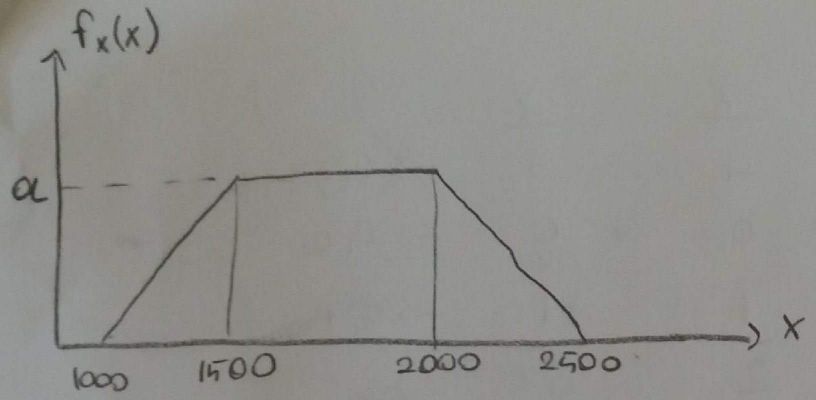
b) $P(40<x<60) = P\left(\frac{40-50}{\sqrt{10}} < z < \frac{60-50}{\sqrt{10}}\right)$

$= P(-3{,}16 < z < 3{,}16) = 0{,}9992 - (1 - 0{,}9992)$

$= 0{,}9984$ ✓

c) $E[x] = 0{,}9984 \cdot 50 = \underline{49{,}92}$

9) a)

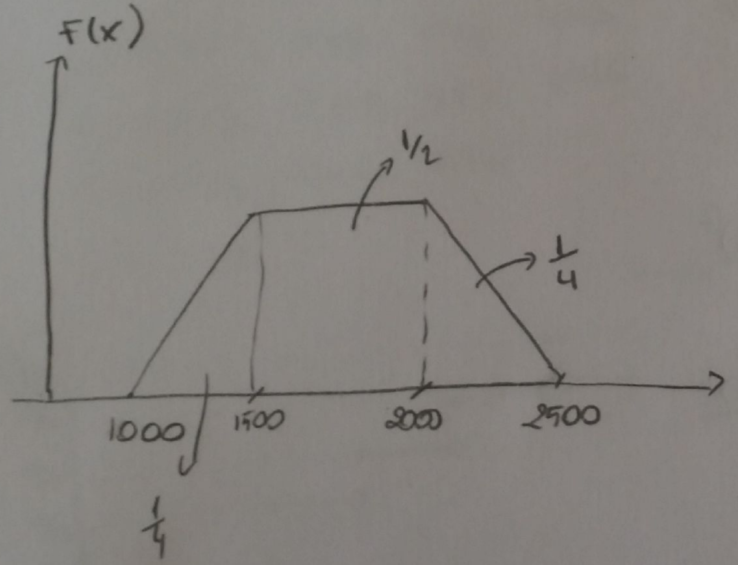


$$A = \frac{(2000-1500)+(2500-1000)}{2} \cdot \alpha = 1$$

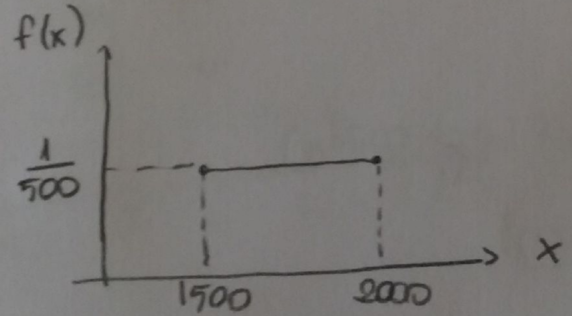
$$1000\,\alpha = 1 \quad \Rightarrow \quad \alpha = \frac{1}{1000}$$ //

b) $$P(2000 < x < 2500) = \frac{(2500-2000) \cdot \frac{1}{1000}}{2} = \frac{1}{4}$$ //

c)



$$\frac{\frac{1}{1000}}{\frac{1}{2}} = \frac{1}{500}$$



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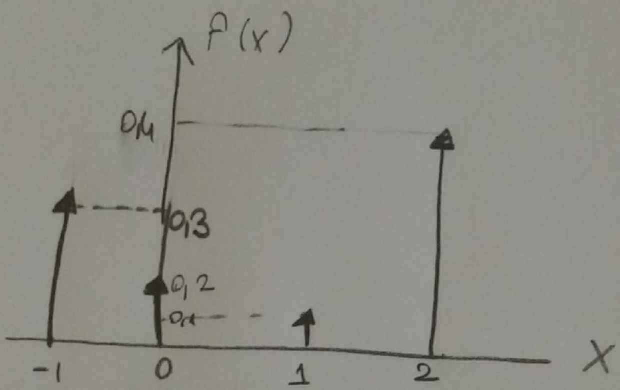
10) $Y = X(X-2) = X^2 - 2X$



X	Y	
-1	3	$\to 0{,}3$
0	0	$\to 0{,}2$
1	-1	$\to 0{,}1$
2	0	$\to 0{,}4$

Y	-1	0	3
P(Y)	0,1	0,6	0,3

$$f(Y) = 0{,}1\,\delta(Y+1) + 0{,}6\,\delta(Y) + 0{,}3\,\delta(Y-3)$$

12) $$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

a) $$f_T(t \mid \text{"rainy"}) = \frac{1}{\sqrt{2\pi}\cdot 5} \cdot e^{\frac{-(x-15)^2}{50}}$$

$$f_T(t \mid \text{"not rainy"}) = \frac{1}{\sqrt{2\pi}\cdot 3} \cdot e^{\frac{-(x-23)^2}{18}}$$

b) $P(A) = P(A \mid B_1) \cdot P(B_1) + P(A \mid B_2) \cdot P(B_2)$

$F_T(t) = f(t \mid \text{"rainy"}) \cdot f(\text{"rainy"}) + f(t \mid \text{"not rainy"}) \cdot f(\text{not rainy})$

$$= \underbrace{\frac{1}{4}}_{\text{rainy day}} \cdot \frac{1}{\sqrt{2\pi}\cdot 5} \cdot e^{\frac{-(x-15)^2}{50}} + \underbrace{\frac{3}{4}}_{\text{not rainy day}} \frac{1}{\sqrt{2\pi}\cdot 3} e^{\frac{-(x-23)^2}{18}}$$