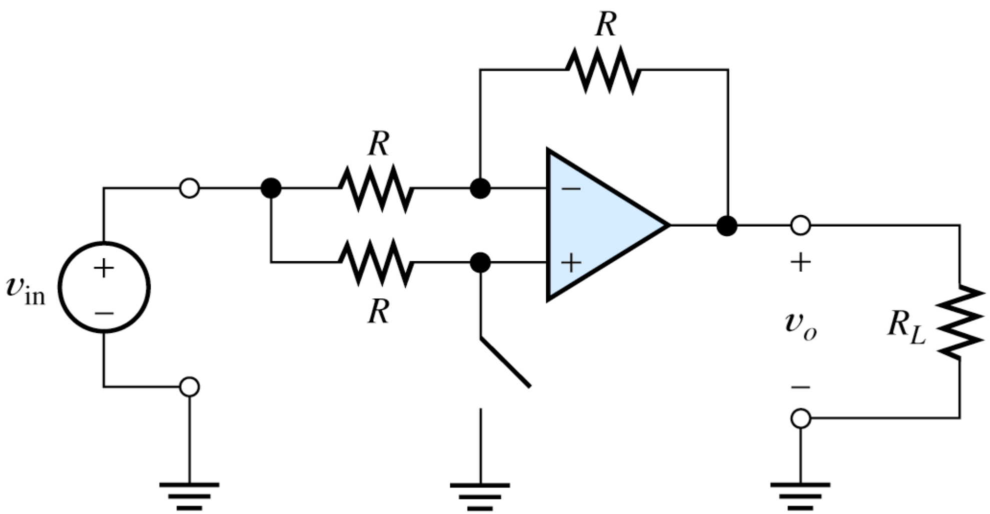
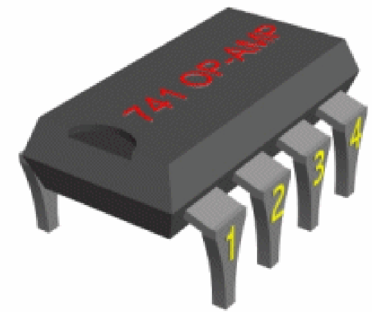
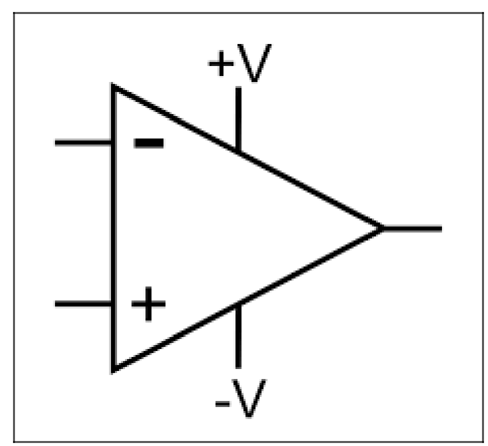
Amplifiers &
Operational Amplifiers

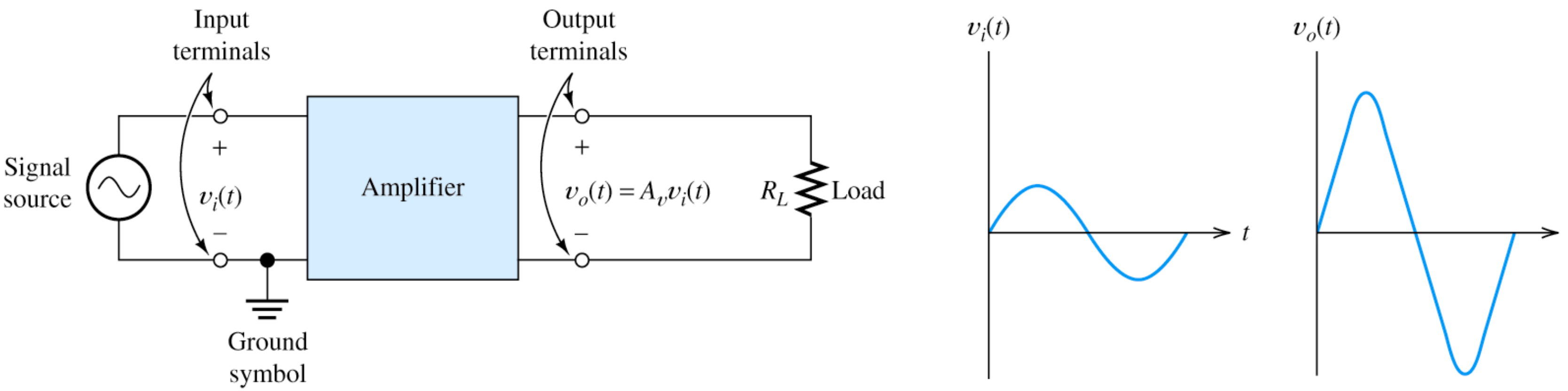






Amplifiers – Basic Amplifier Concepts

Ideally, an *amplifier* produces an output signal with identical wave-shape as the input signal but with a larger amplitude.



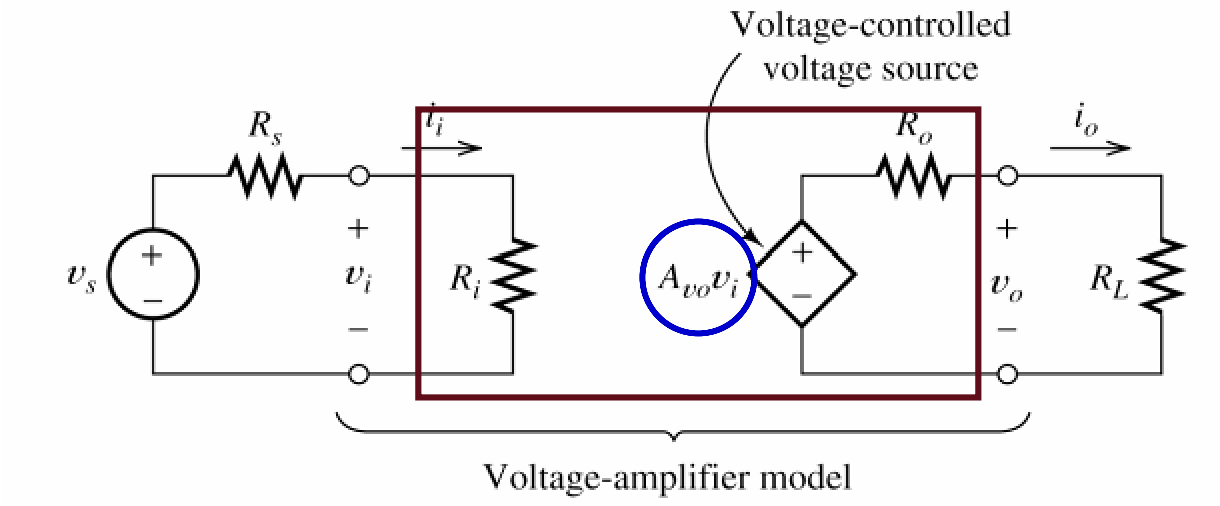
Electronic amplifier.

$$\boxed{v_o(t) = A_v\, v_i(t)}$$

A_v is the Voltage Gain

Amplifiers – Amplifier Model

Amplification can be modeled by a controlled source.



R_i : *the input resistance (or impedance), is the equivalent resistance seen when looking into the input terminals.*

R_o : *in series with the output terminals, is the output resistance (or impedance)*

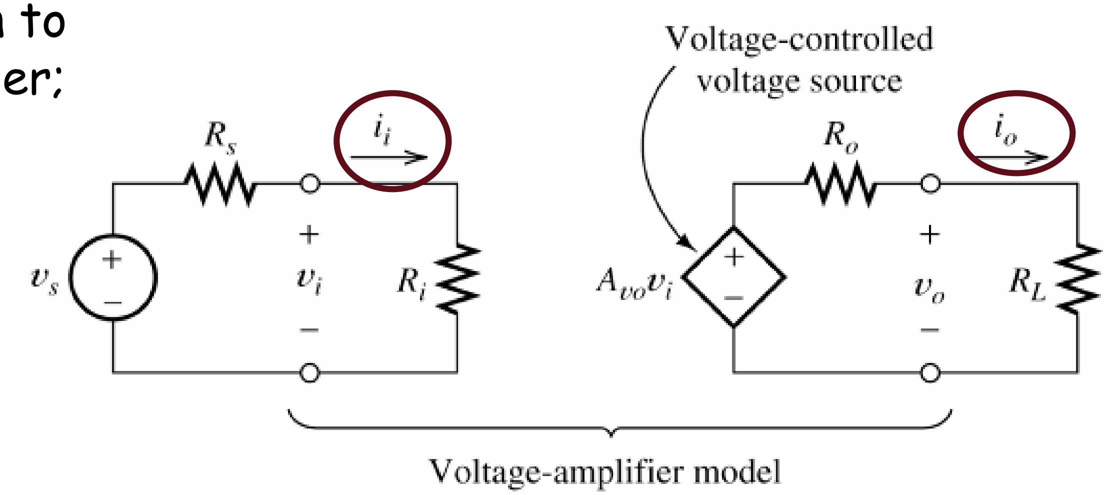
$\boxed{A_{vo} = v_{oc}/v_i : \text{ the open-circuit voltage gain}}$

(note : the real gain is smaller tham A_{vo} *)*

Amplifiers – Current and Voltage Gains

i_i is the current delivered in to the terminals of the amplifier;

i_0 is the current flowing through the load



$$\textit{The current gain } A_i \textit{ is the ratio between output and input currents}: A_i = \frac{i_o}{i_i}$$

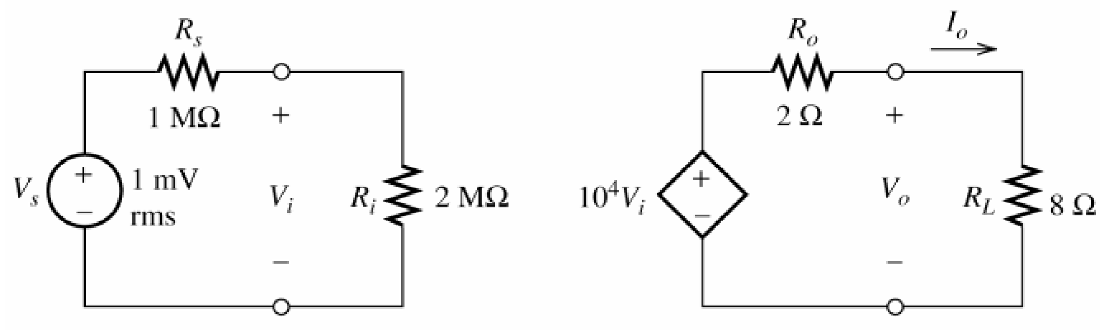
$$\textit{Furthermore, } A_i = \frac{i_o}{i_i} = \frac{v_o/R_L}{v_i/R_i} = A_v \frac{R_i}{R_L}, \textit{ where } A_v = \frac{v_o}{v_i} \textit{ is the voltage gain}$$

$$\textit{The voltage gain } A_v \textit{ is usually smaller than the open-circuit voltage gain } A_{vo}$$

$$A_v = \frac{V_o}{V_i} = A_{vo}\frac{R_L}{R_o + R_L}$$

Example

Find the voltage gains
$A_{vs} = V_o/V_s$ *and* $A_v = V_o/V_i$
Also, find the current gain



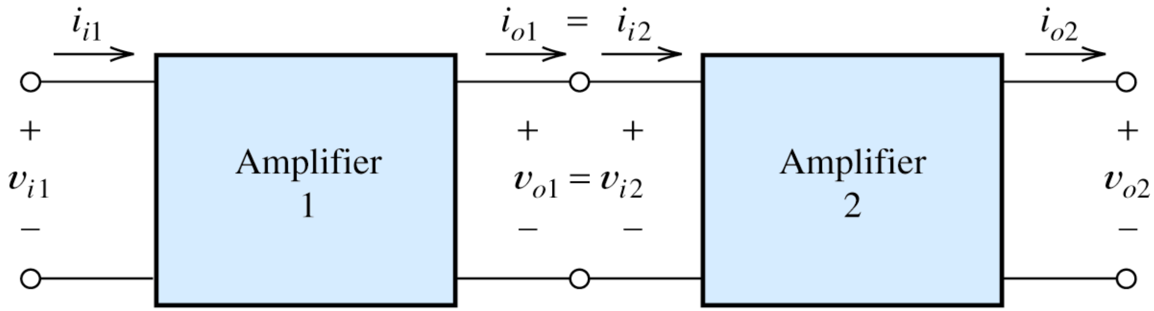
$$A_v = \frac{V_o}{V_i} = \frac{A_{vo} V_i \left(R_L / (R_o + R_L) \right)}{V_i} = A_{vo} \frac{R_L}{R_o + R_L} = 10^4 \frac{8}{2+8} = 8000$$

$$A_{vs} = \frac{V_o}{V_s} = \frac{V_o}{V_i \left((R_i + R_s)/R_i \right)} = A_v \frac{R_i}{R_i + R_s} = 5333$$

$$\left(Note : \boxed{\text{due to the loading effect, } A_{vs} < A_v < A_{vo}} \right)$$

$$A_i = A_v \frac{R_i}{R_L} = 2 \times 10^9$$

Amplifiers – Cascade Amplifiers



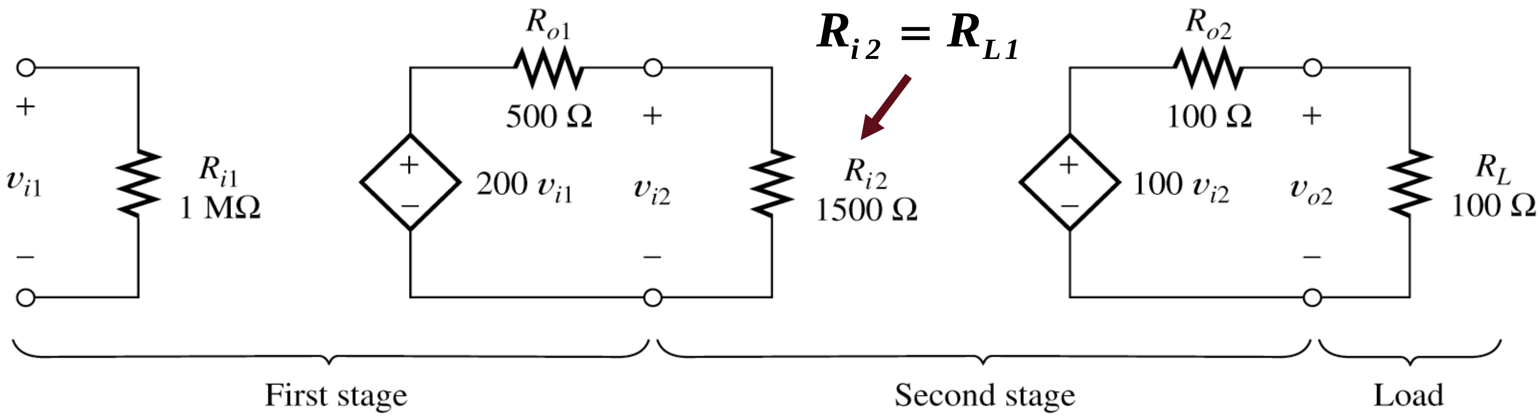
Cascade connection of two amplifiers.

$$A_v = \frac{v_{o2}}{v_{i1}} = \frac{v_{o1}}{v_{i1}} \times \frac{v_{o2}}{v_{01}} = \frac{v_{o1}}{v_{i1}} \times \frac{v_{o2}}{v_{i2}} \Rightarrow A_v = A_{v1} A_{v2}$$

$$A_{vo} = \frac{v_{oc2}}{v_{i1}} = \frac{A_{vo2} v_{i2}}{v_{i1}} = A_{vo2} \frac{v_{o1}}{v_{i2}} \Rightarrow A_{vo} = A_{v1} A_{vo2}$$

In addition, $A_i = A_{i1} A_{i2}$

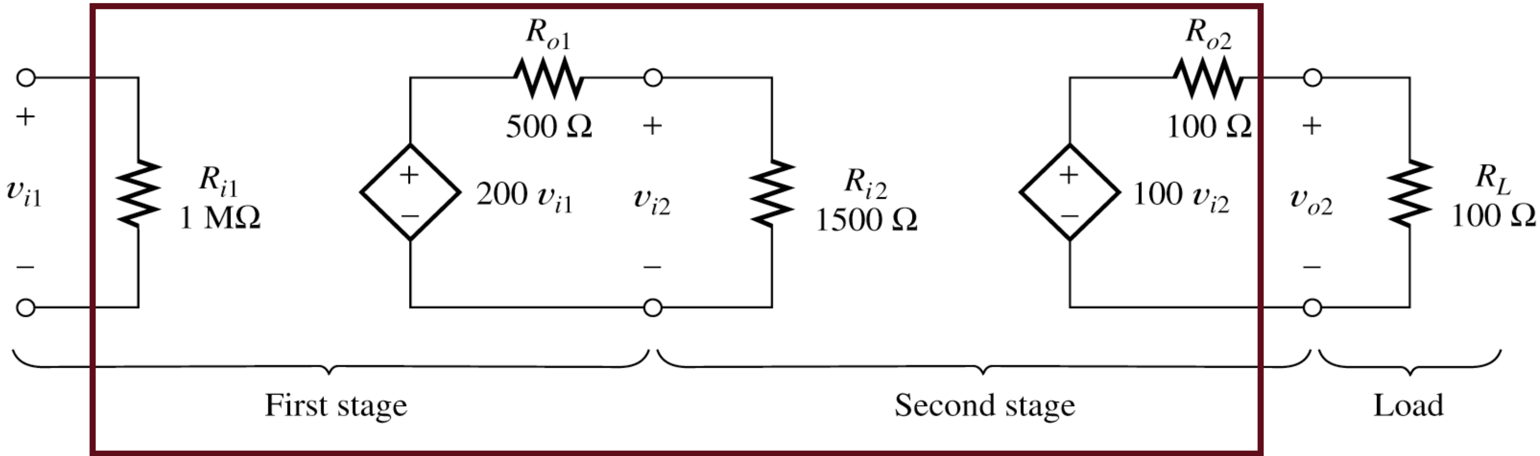
Example

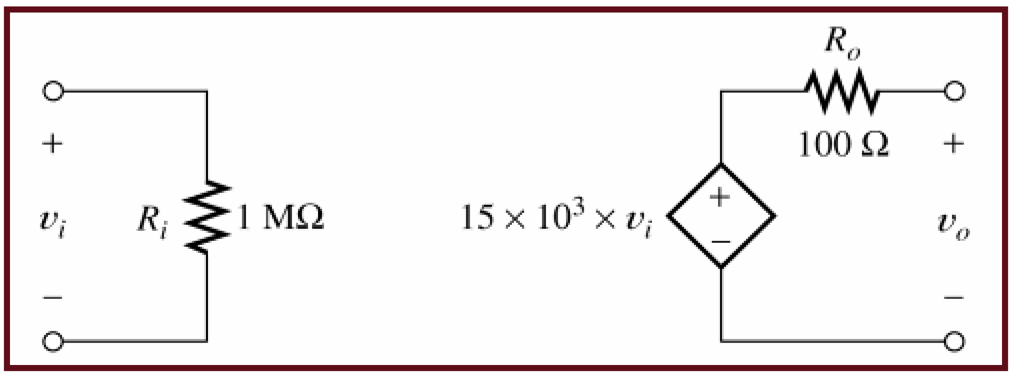


$$A_{v1} = A_{vo1}\frac{R_{i2}}{R_{i2}+R_{o1}} = 150,\ A_{v2} = A_{vo2}\frac{R_L}{R_L+R_{o2}} = 50 \quad \Rightarrow A_v = A_{v1}A_{v2} = 7500$$

$$A_{i1} = A_{v1}\frac{R_{i1}}{R_{i2}} = 10^5,\ A_{i2} = A_{v2}\frac{R_{i2}}{R_L} = 750 \quad \Rightarrow A_i = A_{i1}A_{i2} = 75\times 10^6$$

Example





$$A_{v1} = A_{vo1} \frac{R_{i2}}{R_{i2} + R_{o1}} = 150$$

$$A_{vo2} = 100$$

$$\boxed{A_{vo} = A_{v1} A_{vo2}} = 15 \times 10^3$$

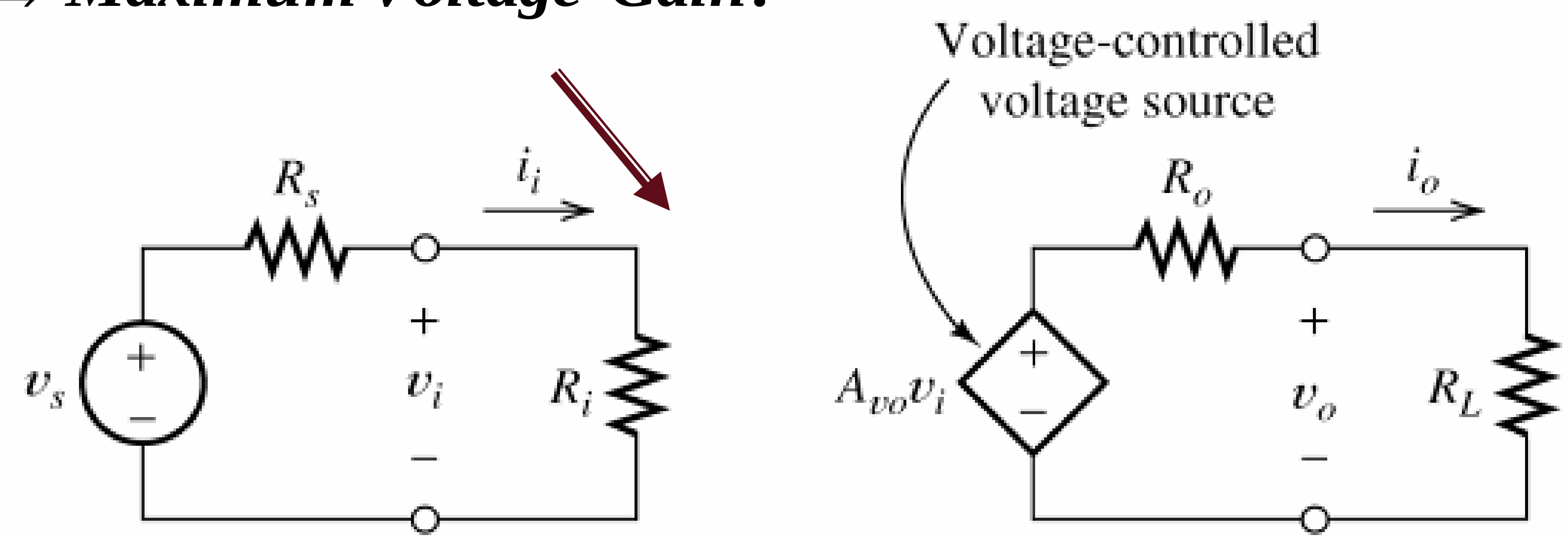
Amplifiers – Ideal Amplifiers

Voltage Amplifier :

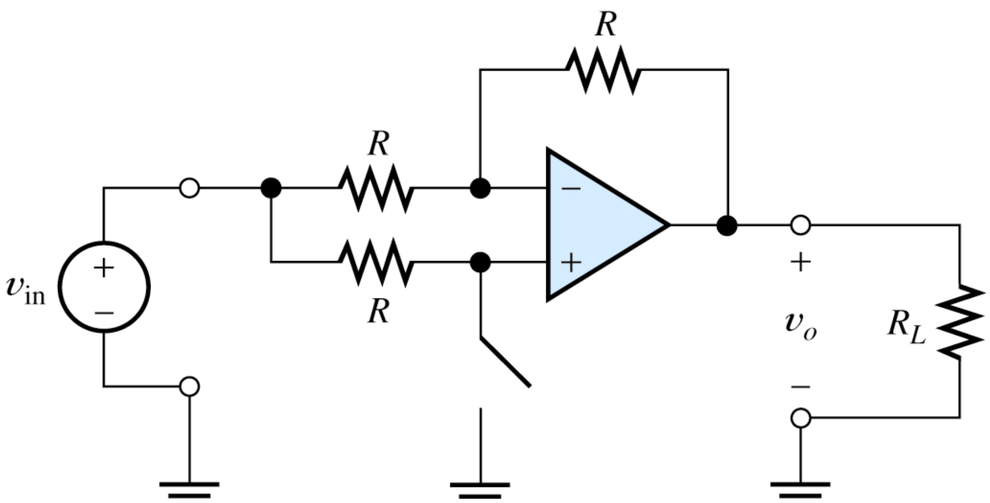
$R_i \to \infty,\ v_i \cong v_s\ \ max.$

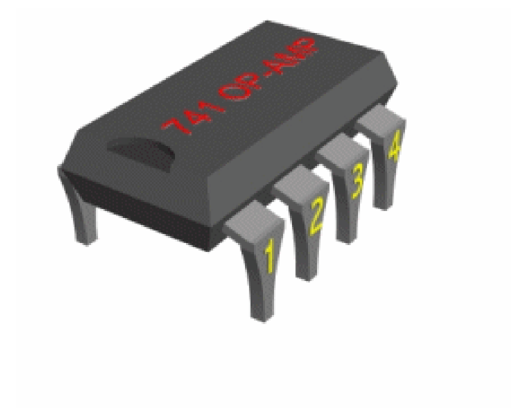
$R_o \to 0,\ v_o \cong A_{vo} v_i\ \ max$

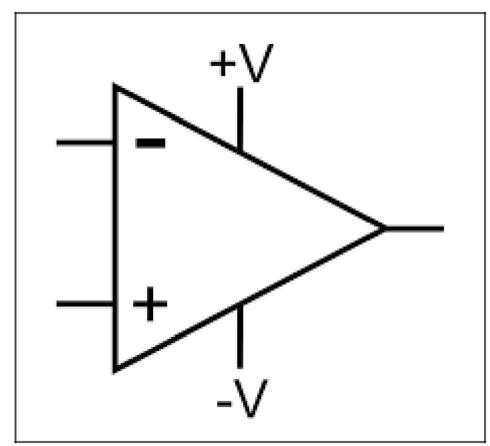
$\Rightarrow$ *Maximum Voltage Gain!*



Operational Amplifiers

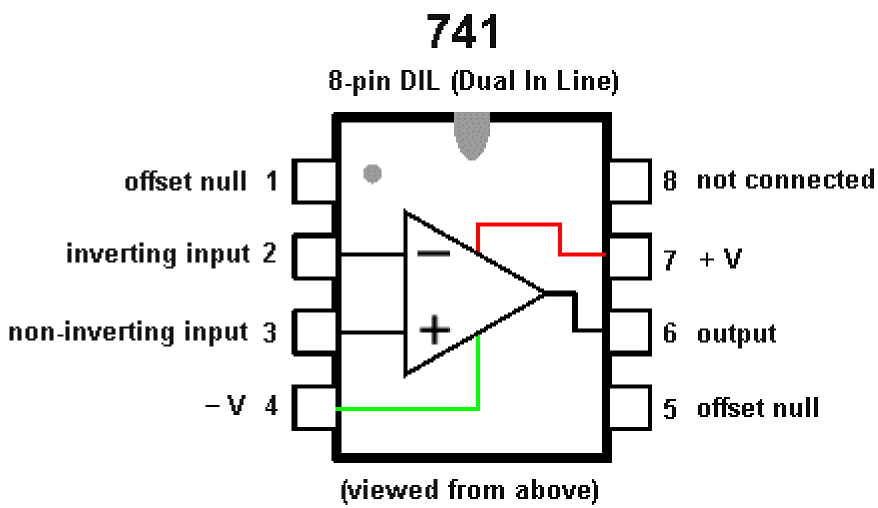


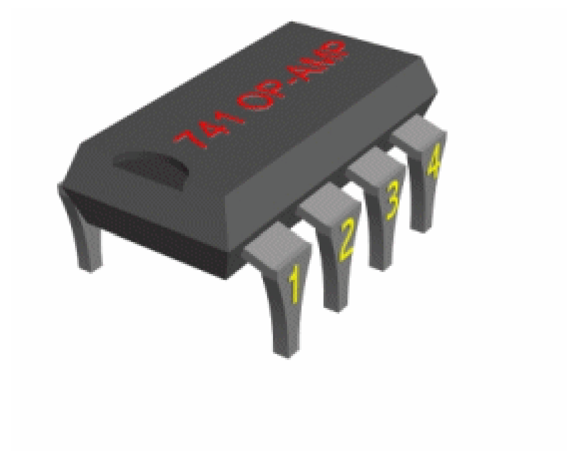




Operational amplifiers (op-amps)

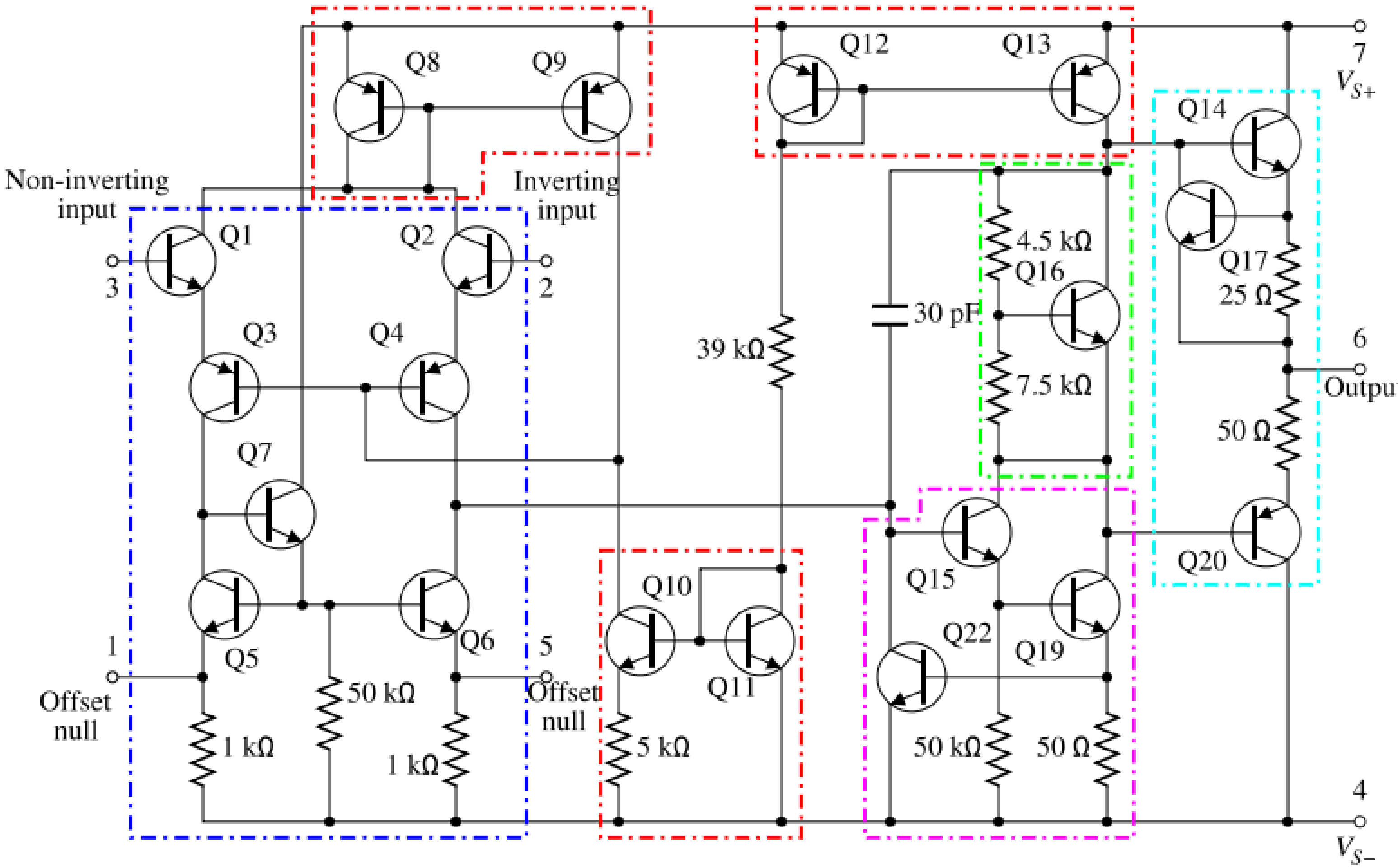
741 Amplifier is the most popular amplifier it has A_{OL}=100000 (openvoltage gain)





Operational amplifiers (op-amps)

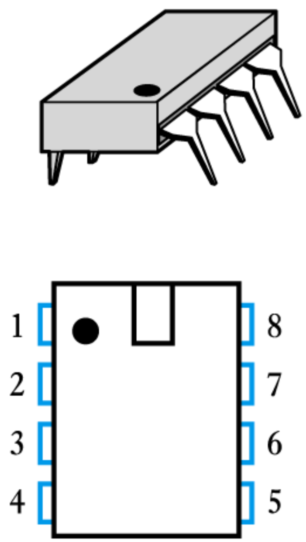
741 Amplifier BJT transistor level schematic



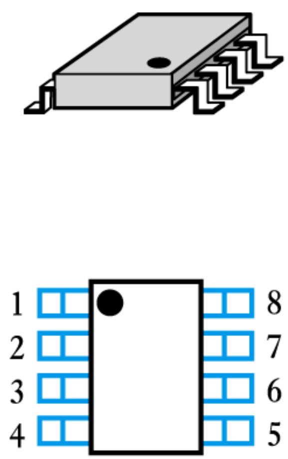
Operational amplifiers (**op-amps**) are among the most widely used building blocks in electronics.

They are integrated circuits (ICs).

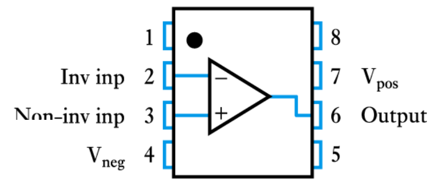
A single package will often contain several op-amps



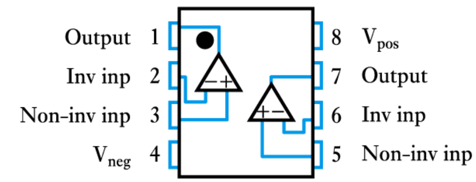
(a) A DIL package



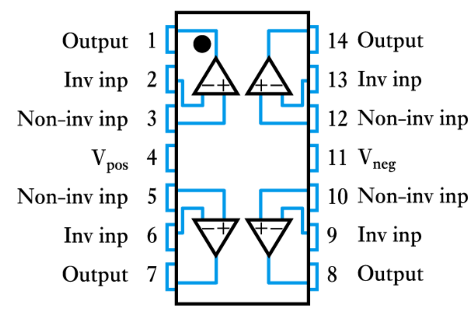
(b) An SMT package



(a) A single op-amp



(b) A dual op-amp

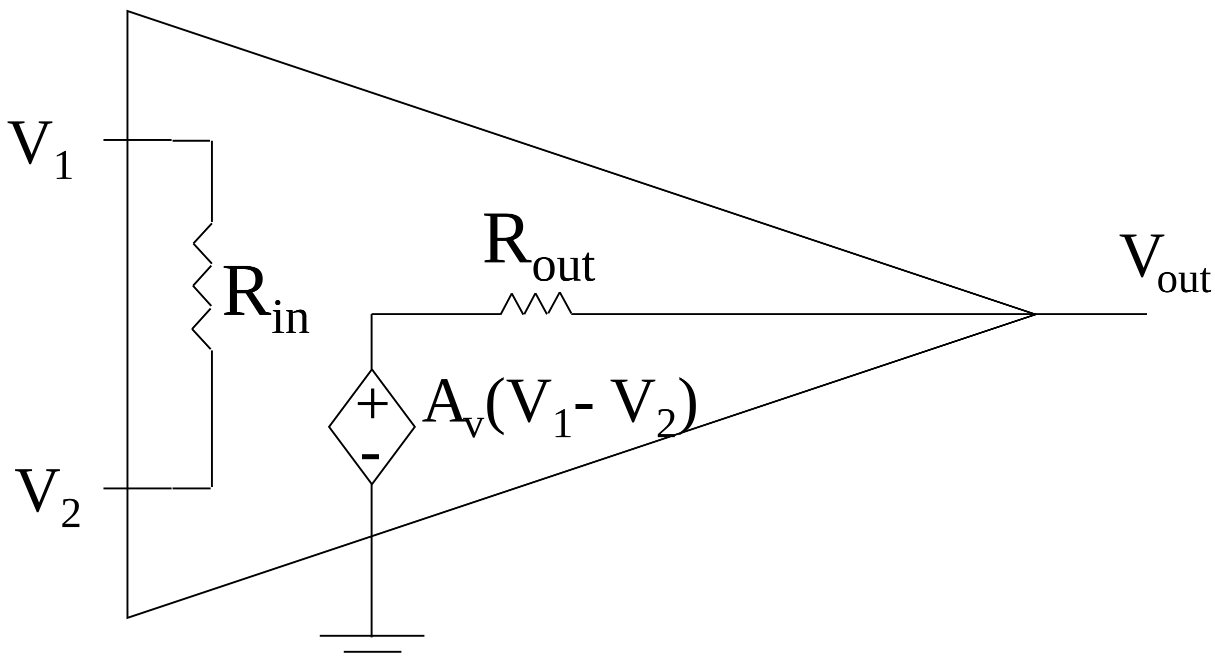


(c) A quad op-amp

Operational Amplifier Model

An operational amplifier circuit is designed so that

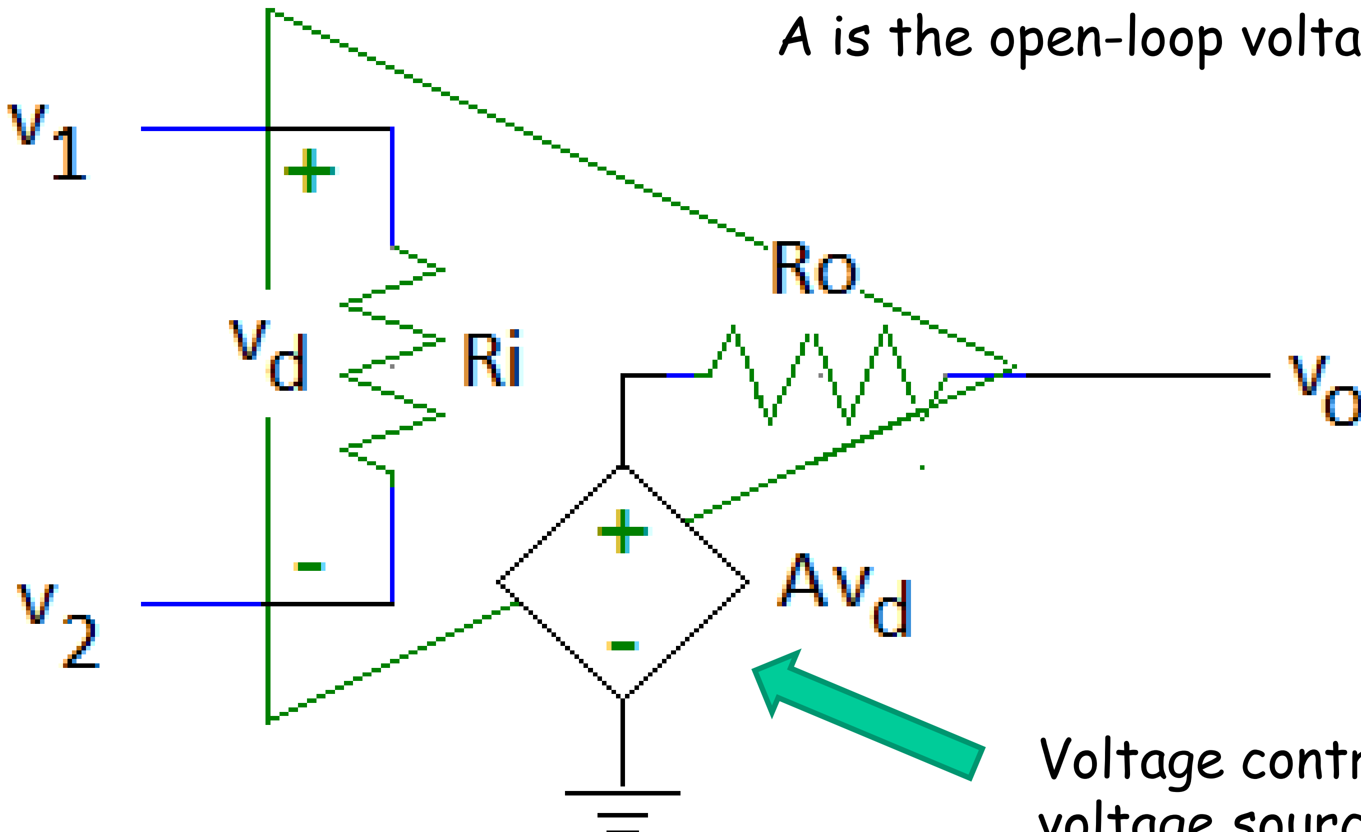
1) $V_{out} = A_v (V_1 - V_2)$ (**A_v** is a very large gain)
2) Input resistance (**R_{in}**) is very large
3) Output resistance (**R_{out}**) is very low



Op Amp Equivalent Circuit

$v_d = v_2 - v_1$

A is the open-loop voltage gain



Voltage controlled voltage source

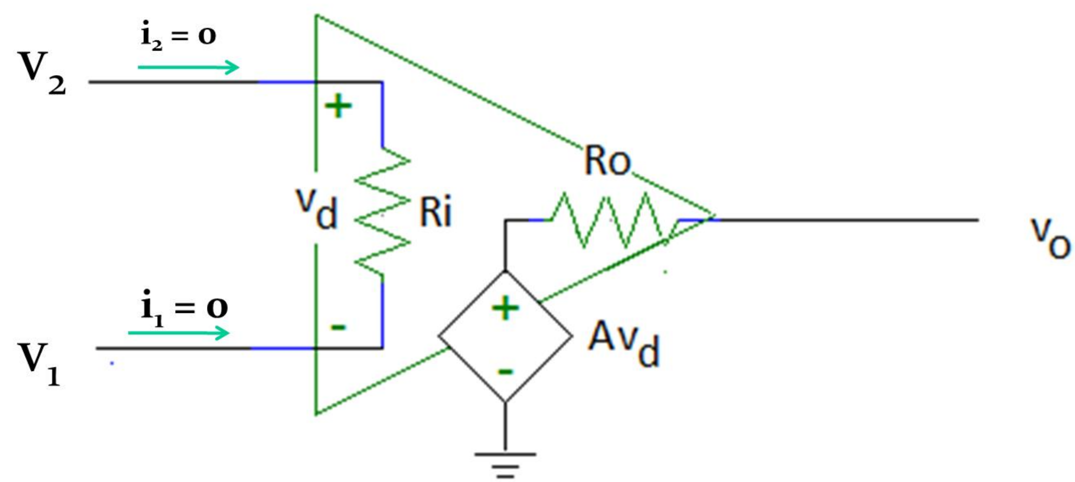
Typical Op Amp Parameters

Parameter	Variable	Typical Ranges	Ideal Values
Open-Loop Voltage Gain	A	10^5 to 10^8	∞
Input Resistance	Ri	10^5 to 10^{13} Ω	∞ Ω
Output Resistance	Ro	10 to 100 Ω	0 Ω
Supply Voltage	Vcc/V^+ -Vcc/V^-	5 to 30 V -30V to 0V	N/A N/A

An Ideal Operational Amplifier

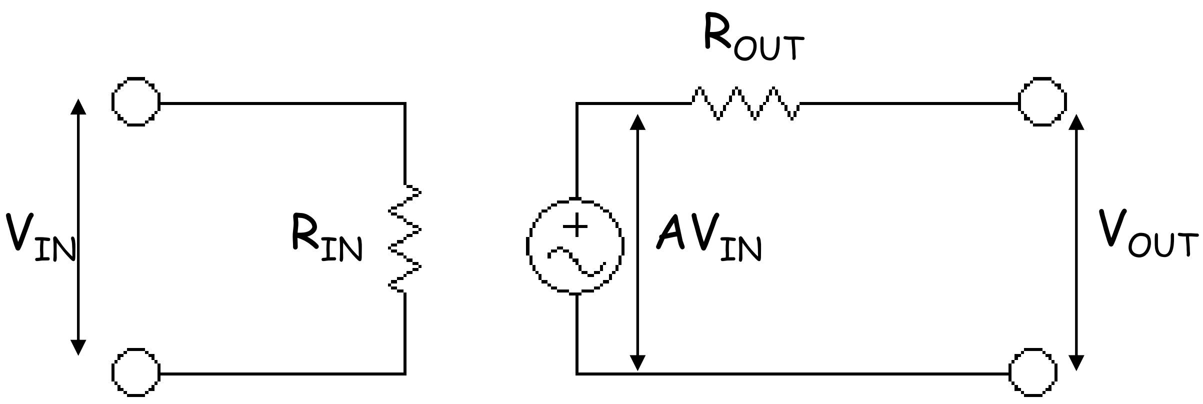
- $Ri = \infty$
- Therefore, $i_1 = i_2 = 0A$

- $Ro = 0$



Impedances

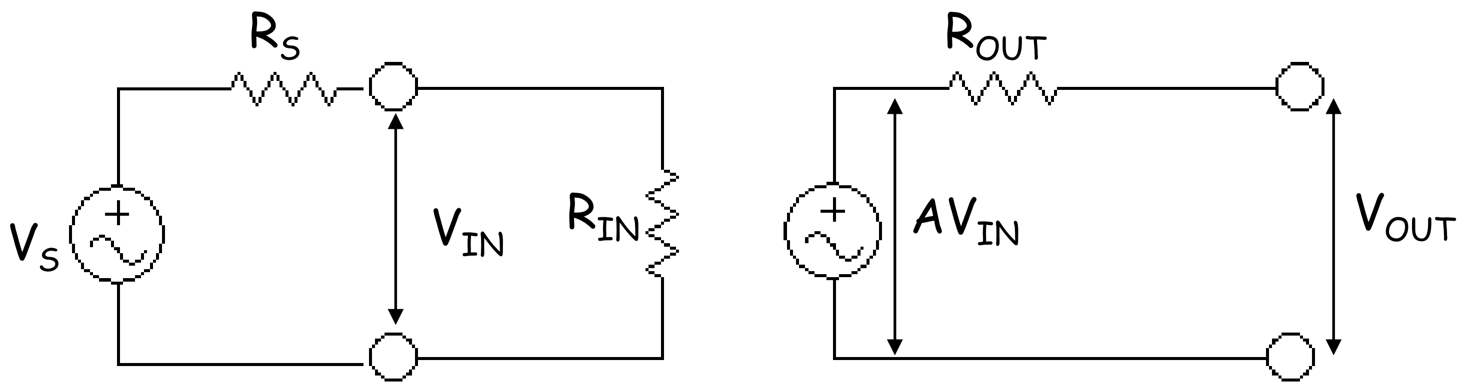
- **Why do we care about the input and output impedance?**
- Simplest "black box" amplifier model:



- The amplifier measures voltage across R_{IN}, then generates a voltage which is larger by a factor ***A***
- This voltage generator, in series with the output resistance R_{OUT}, is connected to the output port.
- ***A*** should be a constant (i.e. gain is linear)

Impedances

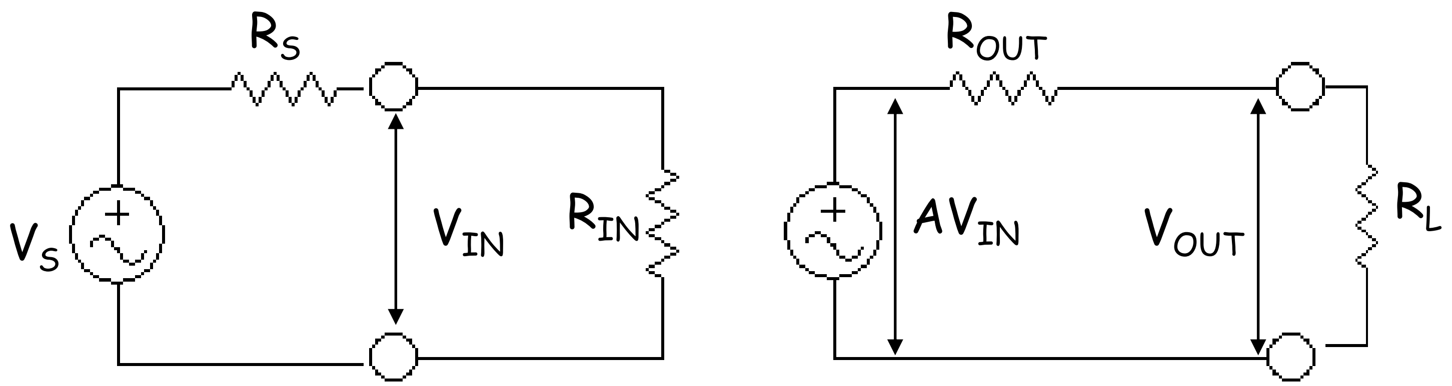
- Attach an input - a source voltage V_S plus source impedance R_S



- Note the voltage divider $R_S + R_{IN}$.
  - $V_{IN}=V_S(R_{IN}/(R_{IN}+R_S))$
  - We want $V_{IN} = V_S$ regardless of source impedance
  - So want R_{IN} to be large.
- The ideal amplifier has an infinite input impedance

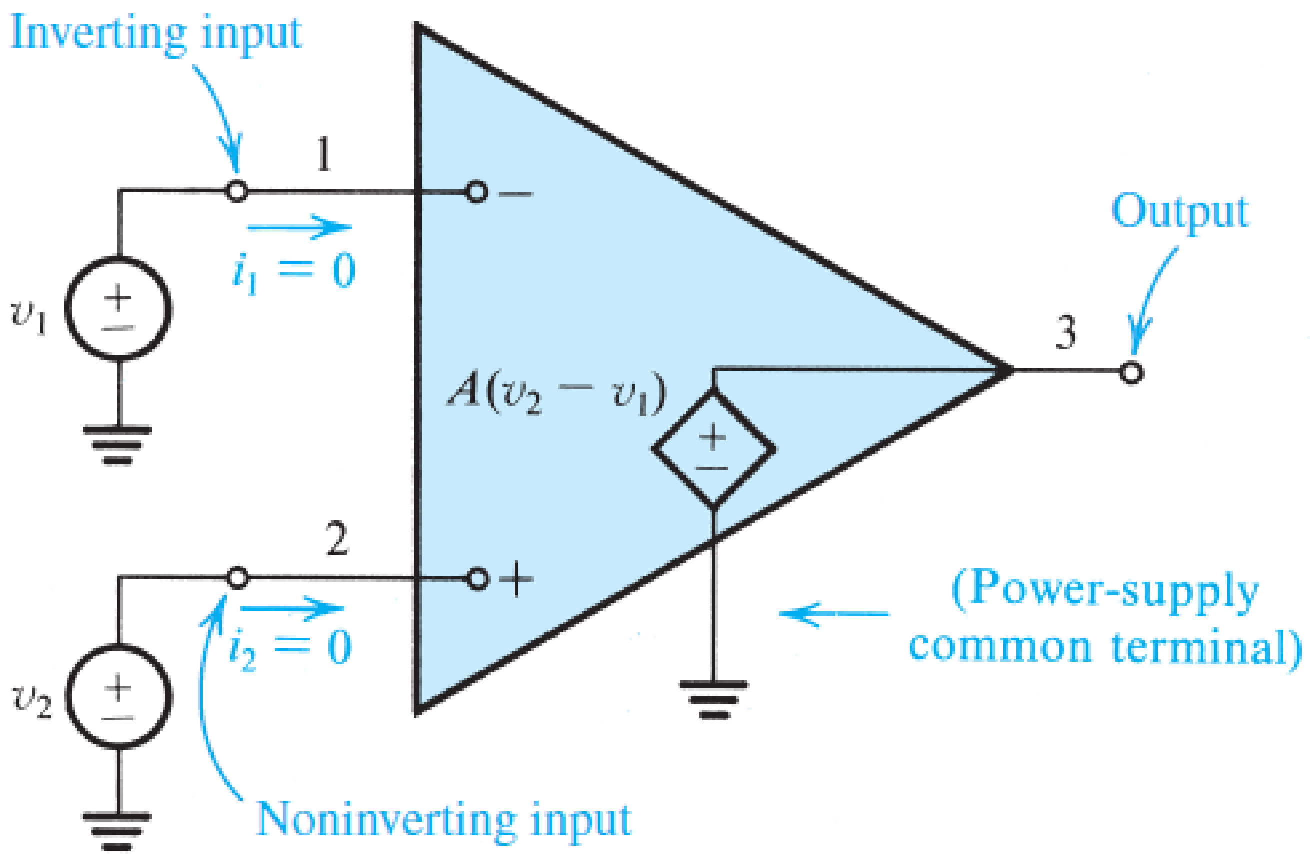
Impedances

- Attach a load - an output circuit with a resistance R_L



- Note the voltage divider R_{OUT} + R_L.
  - $V_{OUT}=AV_{IN}(R_L/(R_L+R_{OUT}))$
  - Want $V_{OUT}=AV_{IN}$ regardless of load
  - We want R_{OUT} to be small.
- The ideal amplifier has zero output impedance

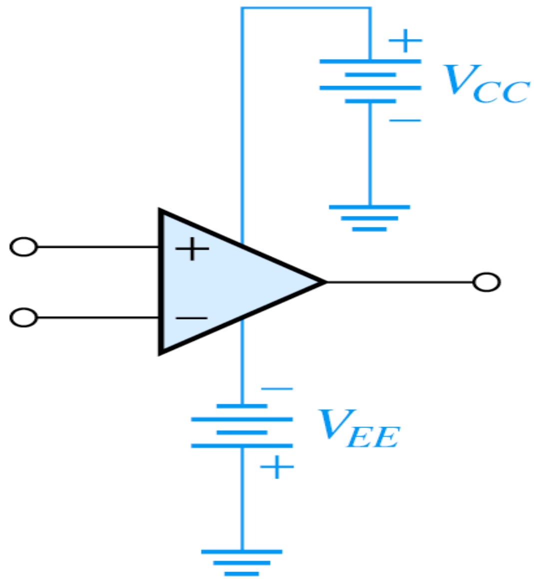
An Ideal Operational Amplifier



Equivalent circuit of the ideal op amp.

An Ideal Operational Amplifier

A real op-amp must have a DC supply voltage which is often not shown on the schematics

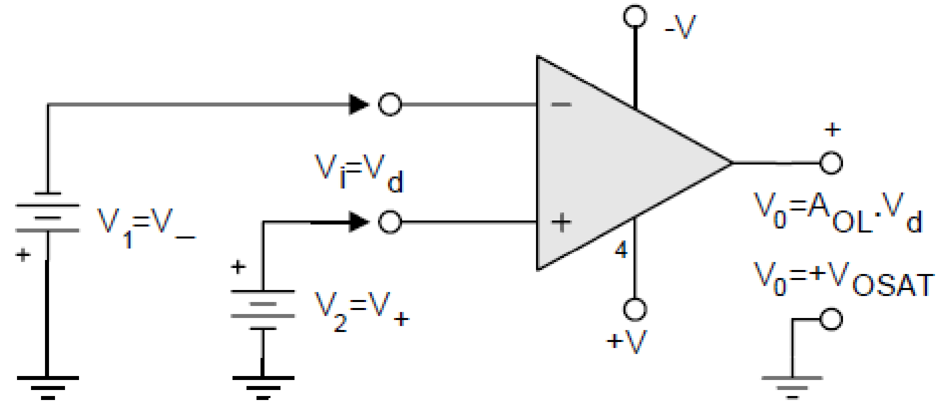


Op-amp symbol showing the dc power supplies, V_{CC} and V_{EE}.

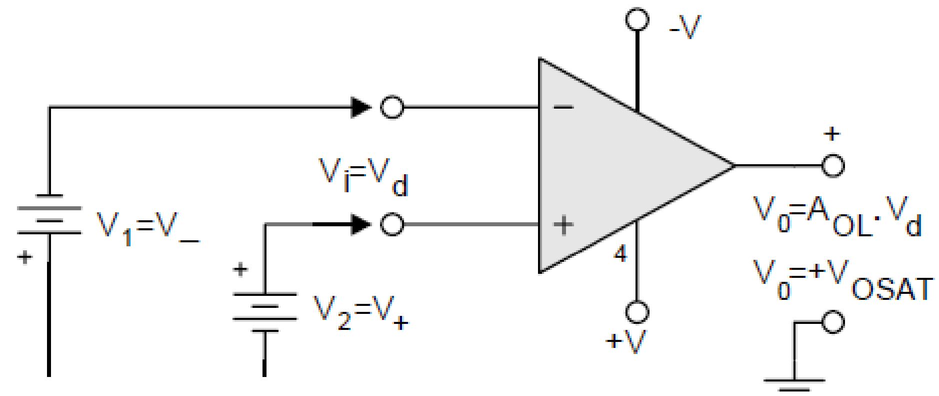
Op-amps

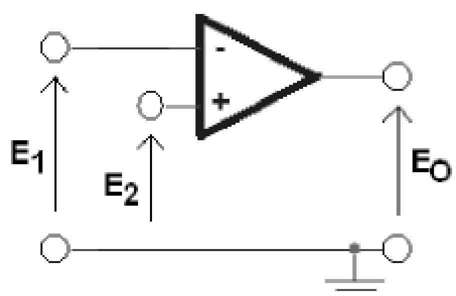
Output voltage of an Op-amp can not be greater than DC supply voltage.

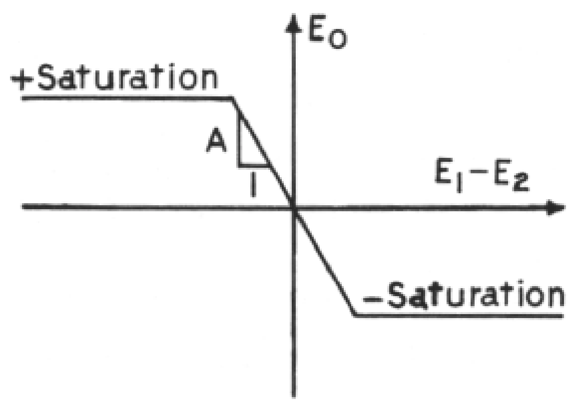
Maximum output voltage is usually slightly lower than the DC supply voltage and called as saturation voltage.



$V_2 < V_1$ ise V_d=- $V_O=-V_{SAT}$'dir

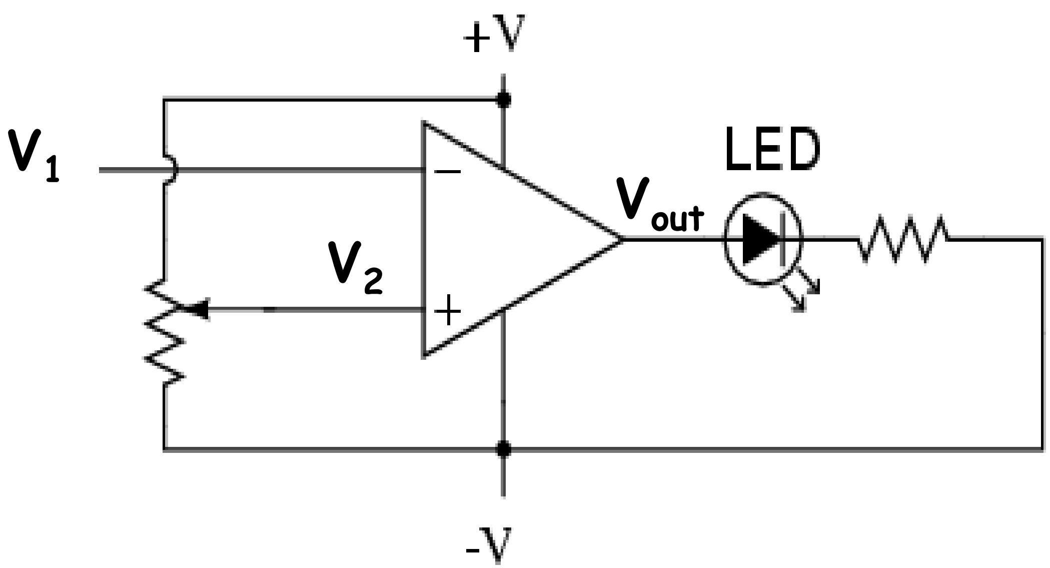






=+ $V_O=+V_{SAT}$'dir

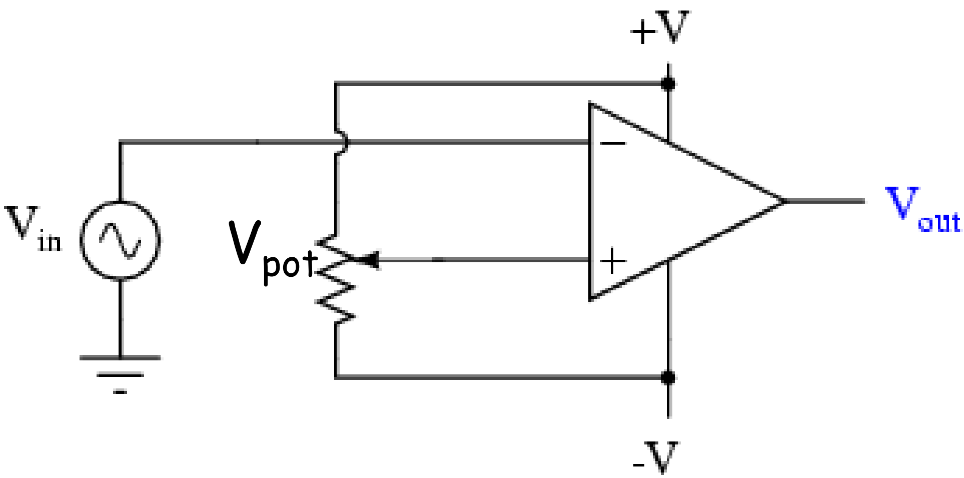
Comparator



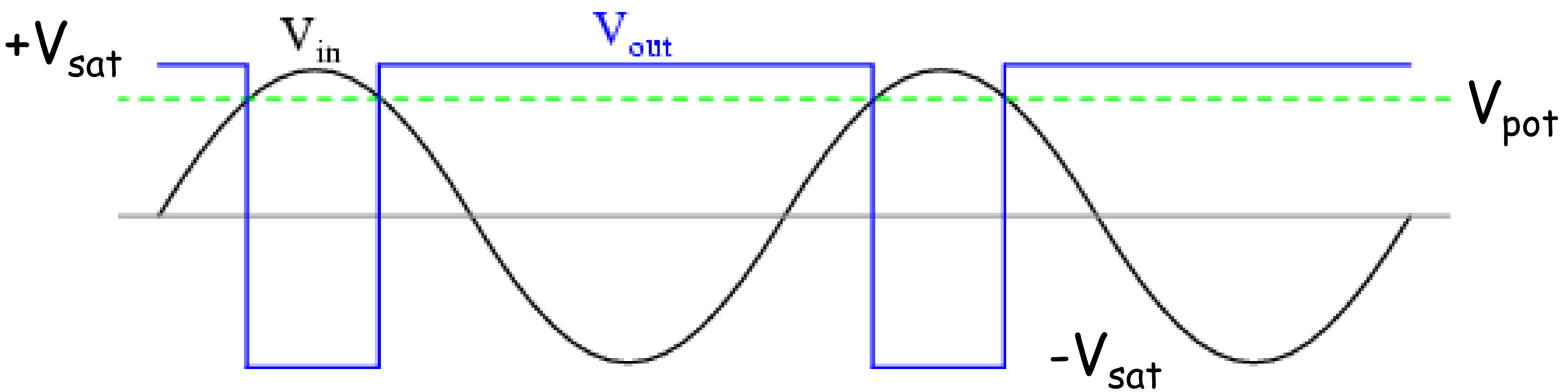
$$V_{\text{out}} = \begin{cases} V_{S+} & V_1 > V_2 \\ V_{S-} & V_1 < V_2 \end{cases}$$

Applications: Low-voltage alarms, night light controller

Pulse Width Modulator



- Output changes when $V_{in} \sim= V_{pot}$
- Potentiometer used to vary duty cycle



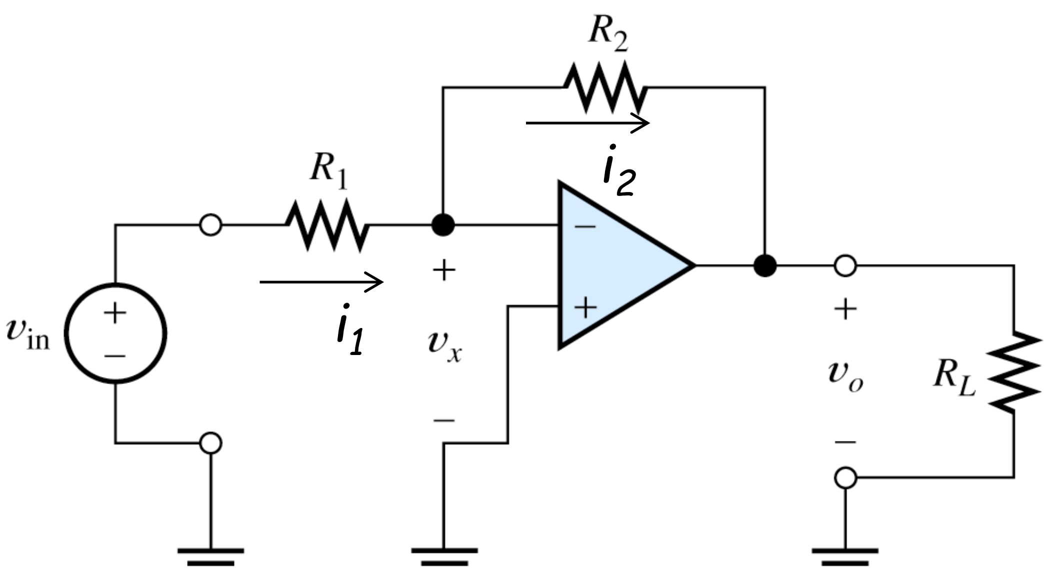
Application: Motor controllers

Negative feedback

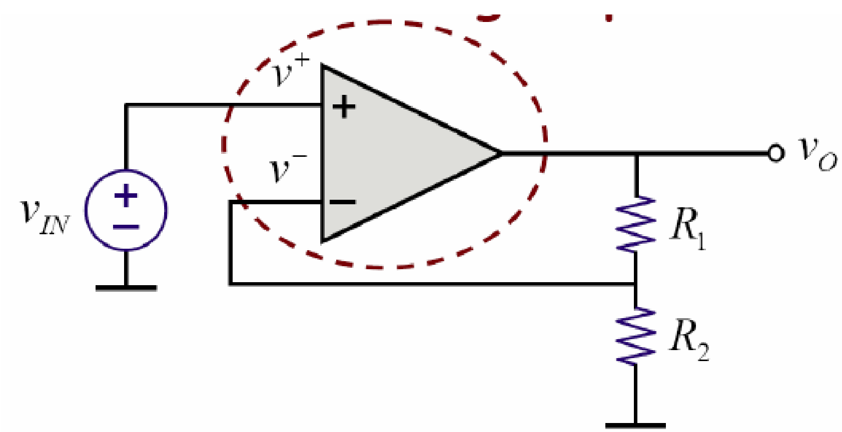
Op-amp are almost always used with a negative feedback:

- Part of the output signal is returned to the input
- Feedback reduces the gain of op-amp.

Negative feedback forces the voltage at the inverting input terminal to be equal to the voltage at the noninverting input terminal.



When A is very large:



$$v_O = \frac{A v_{IN}}{\cancel{1+}\boxed{\frac{AR_2}{R_1+R_2}}}$$

>>1

$$v_O = \frac{\cancel{A} v_{IN}}{\cancel{A}\cdot \frac{R_2}{R_1+R_2}}$$

$$v_O \approx v_{IN} \frac{R_1+R_2}{R_2}$$

Take $A=10^6$, $R_1=9R$, $R_2=R$

$$v_O = \frac{10^6 v_{IN}}{1+10^6 R/(9R+R)}$$

$$v_O = \frac{\cancel{10^6} v_{IN}}{\cancel{1+10^6}\cdot \frac{1}{10}}$$

$$v_O \approx v_{IN} \cdot 10$$

- Gain now determined only by resistance ratio
- Doesn't depend on A, (or temperature, frequency, variations in fabrication)

Why use Negative feedback?:

- Helps to overcome distortion and non-linearity

- Improves the frequency response

- Makes properties predictable - independent of temperature, manufacturing differences or other properties of the opamp

- Circuit properties only depend upon the external feedback network and so can be easily controlled

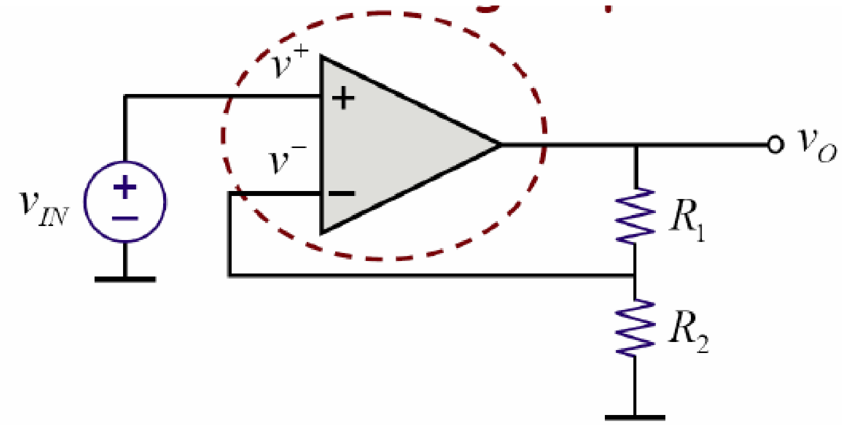
- Simplifies circuit design - can concentrate on circuit function (as opposed to details of operating points, biasing etc.)

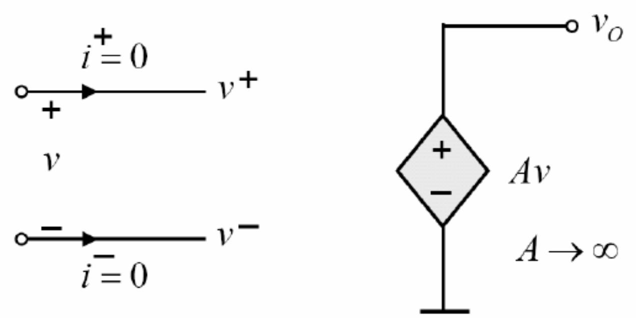
More insight

- Under negative feedback:

$$v^{+} - v^{-} = \frac{v_O}{A} = \frac{\left(\frac{R_1 + R_2}{R_1}\right) v_{IN}}{A} \rightarrow 0$$

$$v^{+} \approx v^{-}$$



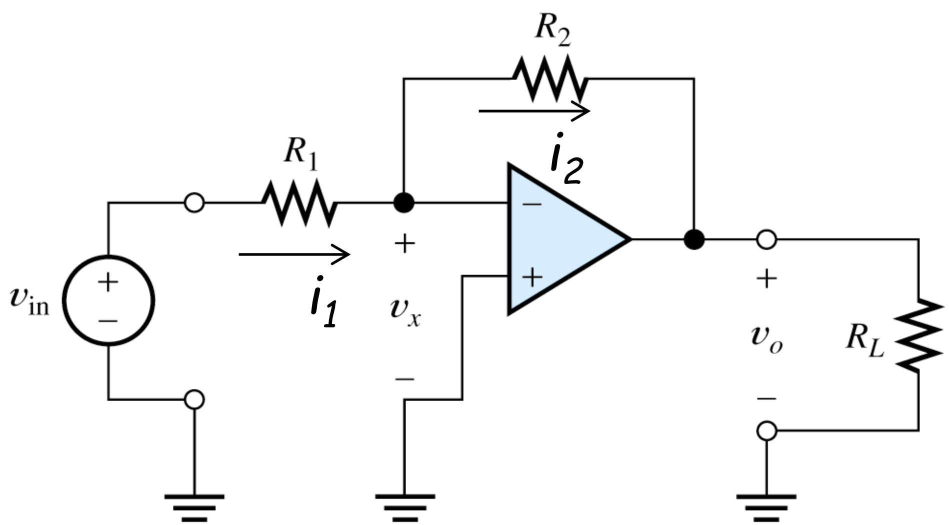


- We also know
  - $i^+ \approx 0$
  - $i^- \approx 0$
- Helpful for analysis (under negative feedback)
- Two "Golden Rules«

1) No current flows into the op-amp
2) $v^+ \approx v^-$

Inverting Amplifier

Since negative feedback forces the voltage at the inverting input terminal (v_-) to be equal to the voltage at the noninverting input terminal (v_+), v_x is equal to zero.



$v_- = v_+ = 0V$

$i_1 = v_{in} / R_1$

$i_2 = i_1$ and

$v_o = -i_2 R_2 = -v_{in} R_2 / R_1$

So

$A_v = v_o / v_{in} = -R_2 / R_1$

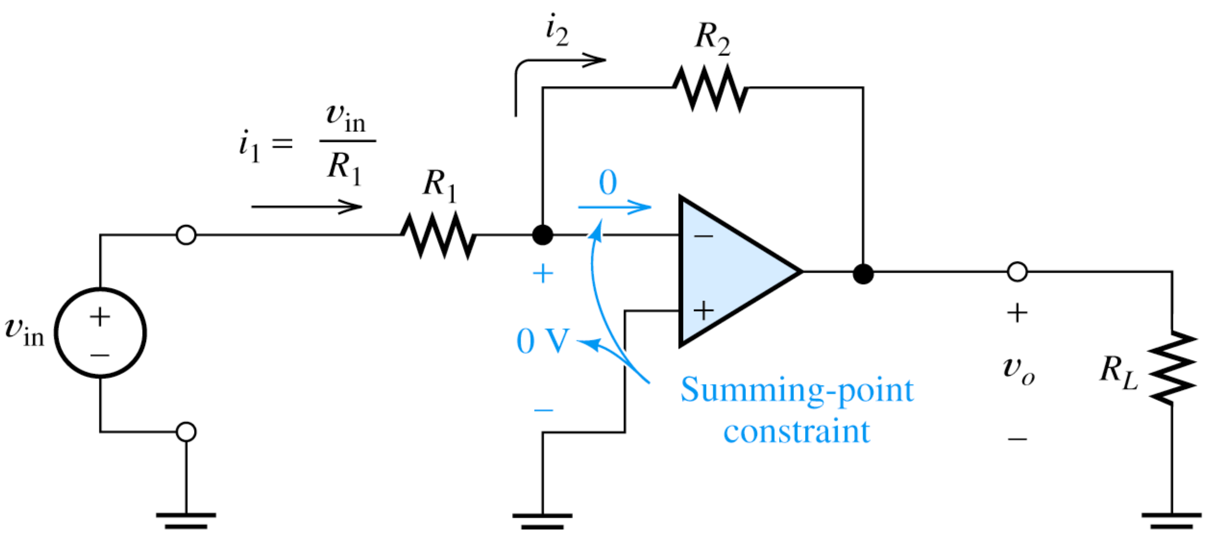
Inverting Amplifier

Since $v_o = -i_2 R_2 = -v_{in} R_2 / R_1$

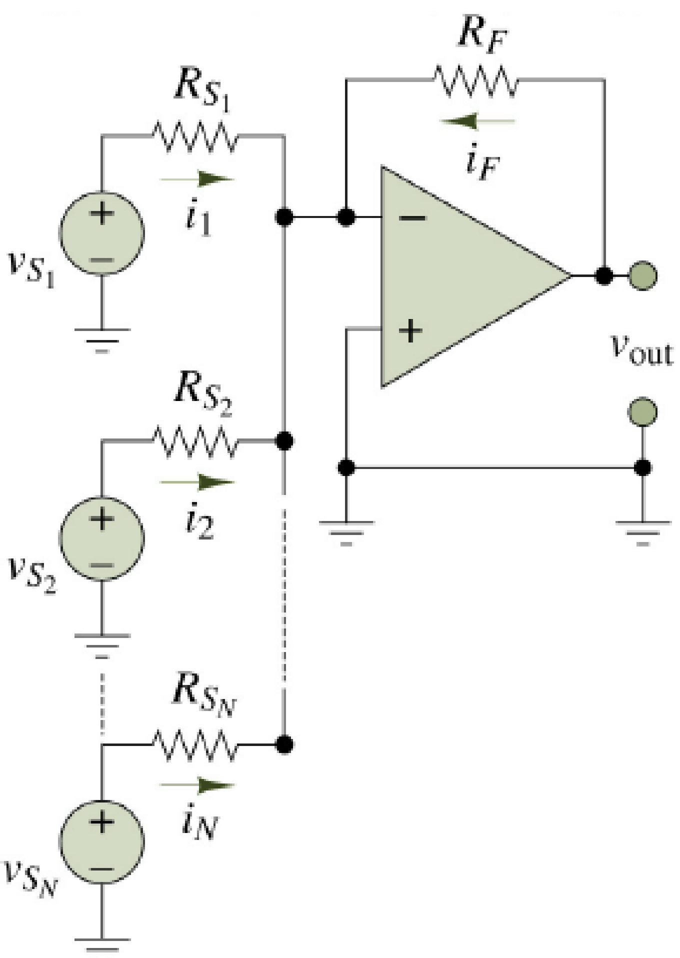
We see that the output voltage does not depend on the load resistance and behaves as voltage source.

The output impedance of the inverting amplifier is zero.

The input impedance is: $Z_{in} = v_{in}/i_1 = R_1$



Summing Amplifier



The output voltage in summing amplifier is $v_{out}=i_f*R_f$ since $v_-=v_+=0$

$$i_1+i_2+.....+i_N=-i_F$$

$$\frac{v_{S1}}{R_{S1}}+\frac{v_{S2}}{R_{S2}}+.....+\frac{v_{SN}}{R_{SN}}=-\frac{v_{out}}{R_F}$$

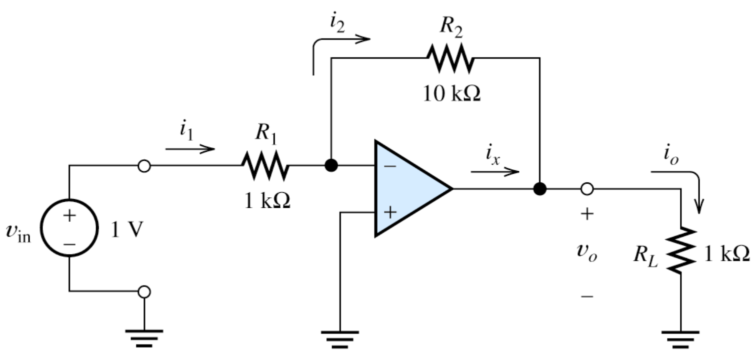
$$v_{out}=-\left(\frac{R_F}{R_{S1}}v_{S1}+\frac{R_F}{R_{S2}}v_{S2}+.....+\frac{R_F}{R_{SN}}v_{SN}\right)$$

If $R_{S1}=R_{S2}=...=R_{SN}=R_S$

$$v_{out}=-\frac{R_F}{R_S}(v_{S1}+v_{S2}+...+v_{SN})$$

Exercise

Find the currents and voltages in these two circuits:



(a)

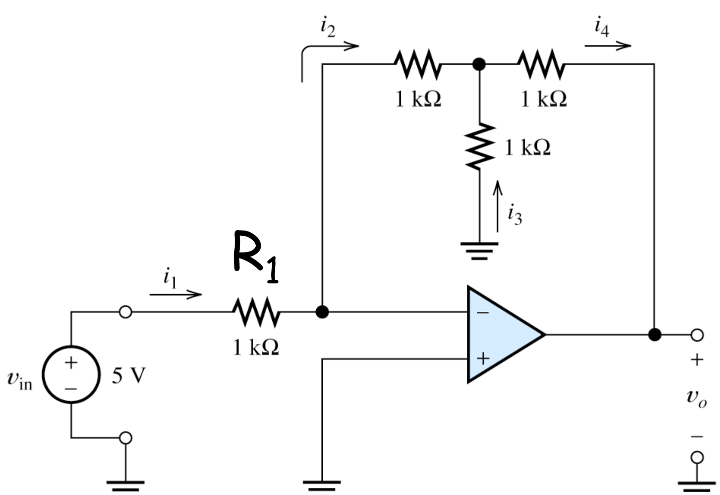
a) $i_1=v_{in}/R_1=1V/1k\,\Omega=1mA$

$i_2=i_1=1mA$ from KCL

$v_o=-i_2*R_2=-10V$ from KVL

$i_o=v_o/R_L=-10mA$ from Ohms law

$i_x=i_o-i_2=-10mA-1mA=-11mA$



(b)

b) $i_1=v_{in}/R_1=5mA$

$i_2=i_1=5mA$

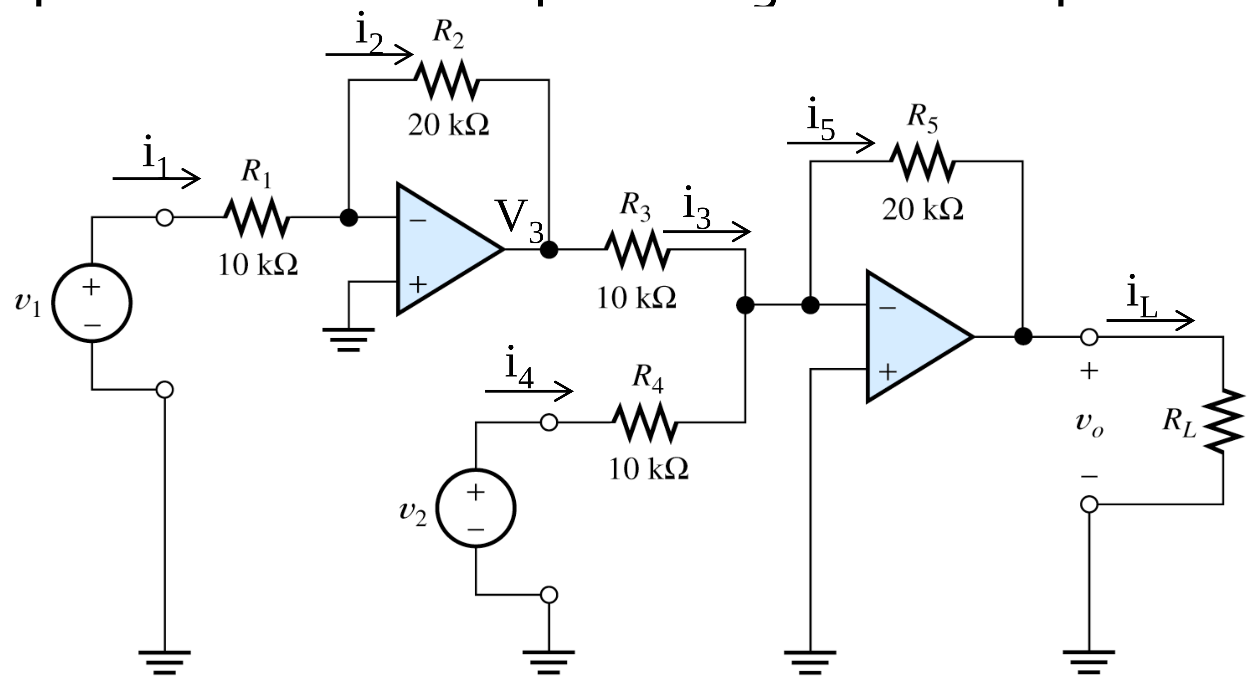
$i_2*1k\Omega= i_3*1k\Omega \Rightarrow i_3=5mA$

$i_4=i_2+i_3=10mA$

$v_o=-\,i_2*1k\,\Omega-i_4*1k\,\Omega=-15\ V$

Exercise

Find expression for the output voltage in the amplifier circuit:



$i_1=v_1/R_1=v_1/10k\,\Omega$

$i_2=i_1=v_1/10mA$

$v_3 =- i_2*R_2=- v_1/10k\,\Omega *20k\,\Omega =-2v_1$

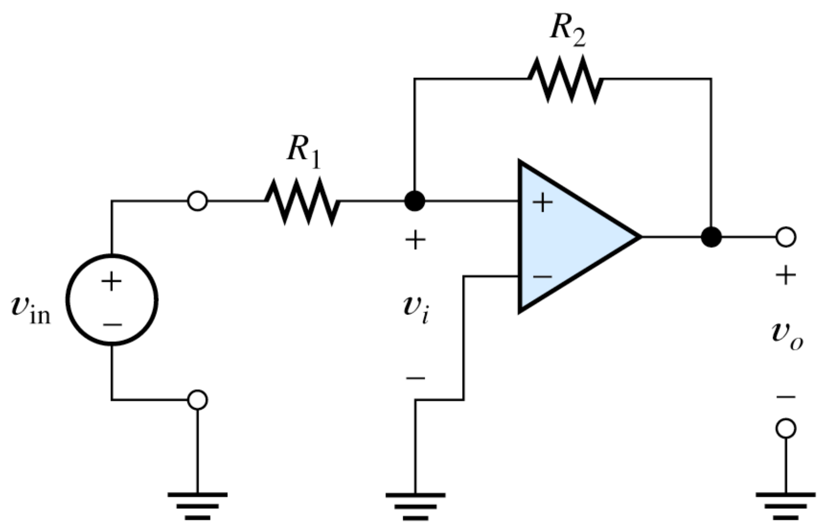
$i_5=i_3+i_4=v_3/10k\,\Omega +v_2/10k\,\Omega$

$v_o =- i_5*R_5=-(v_3/10k\,\Omega +v_2/10k\,\Omega)*20k\,\Omega =-2v_3 -2v_2 =4v_1 -2v_2$

Positive Feedback

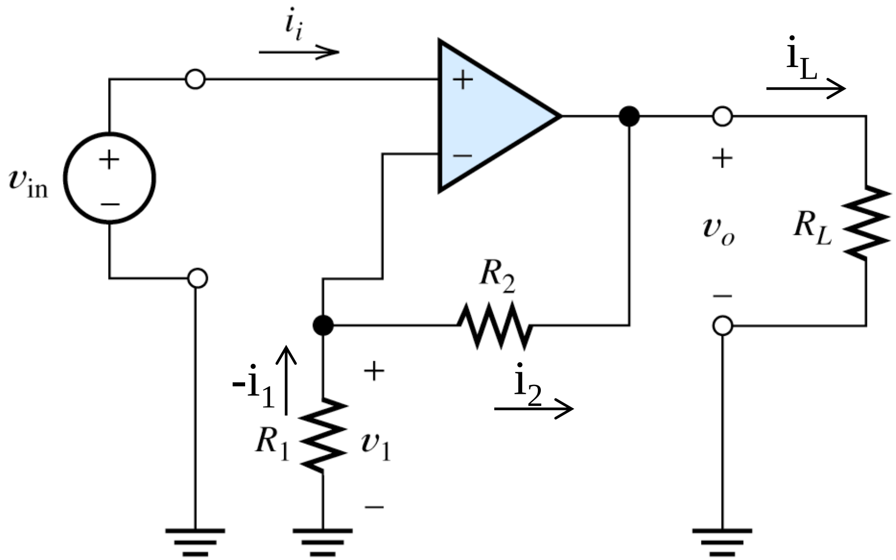
When we flip the polarization of the op-amp as shown on the figure we will get a positive feedback that **saturates** the amplifier output.

This is not a good idea.



Circuit with positive feedback.

Noninverting Amplifier



Noninverting amplifier.

$v_1 = v_{in}$

$i_1 = v_1/R_1$

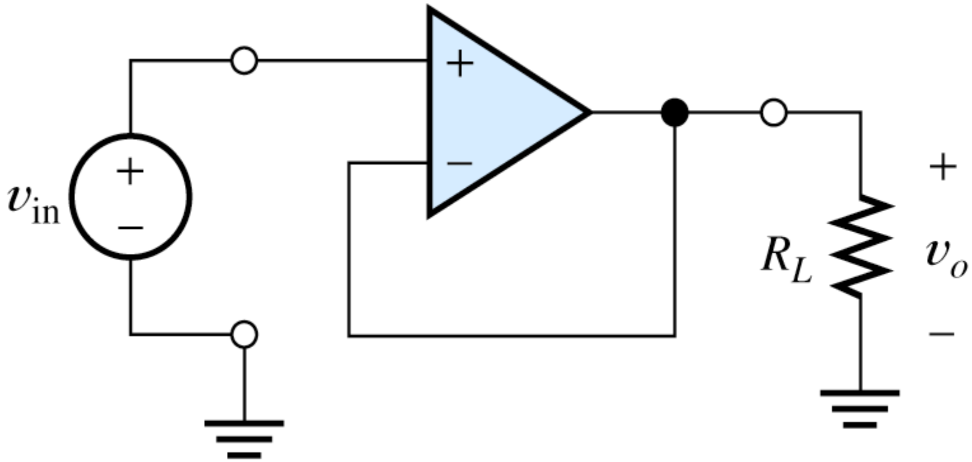
$i_2 = -i_1$

$v_o = v_1 - i_2 * R_2 = v_1 + i_1 * R_2 =$

$= v_1 + v_1/R_1 * R_2 = v_1 * (1 + R_2/R_1)$

Thus the voltage gain of noninverting amplifier is:
$A_v = v_o / v_{in} = 1 + R_2 / R_1$

Voltage Follower



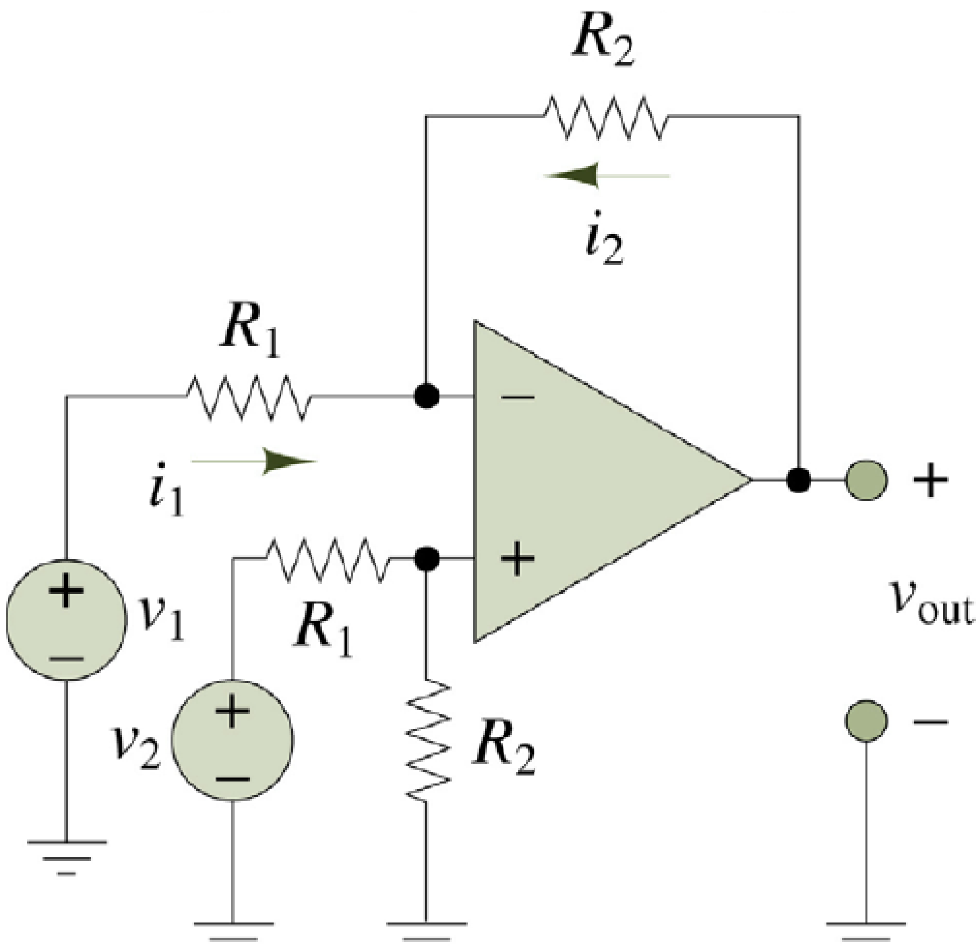
The voltage follower which has $A_v = 1$.

- What's the application of this circuit?
- Buffer

  voltage gain = 1
  input impedance=∞
  output impedance=0

  Useful interface between different circuits:
  Has minimum effect on previous and next circuit in signal chain

Differential Amplifier (subtractor)



$$\boldsymbol{i}_1 + \boldsymbol{i}_2 = 0$$

$$\frac{v_1 - v^-}{R_1} = -\frac{v_{out} - v^-}{R_2}$$

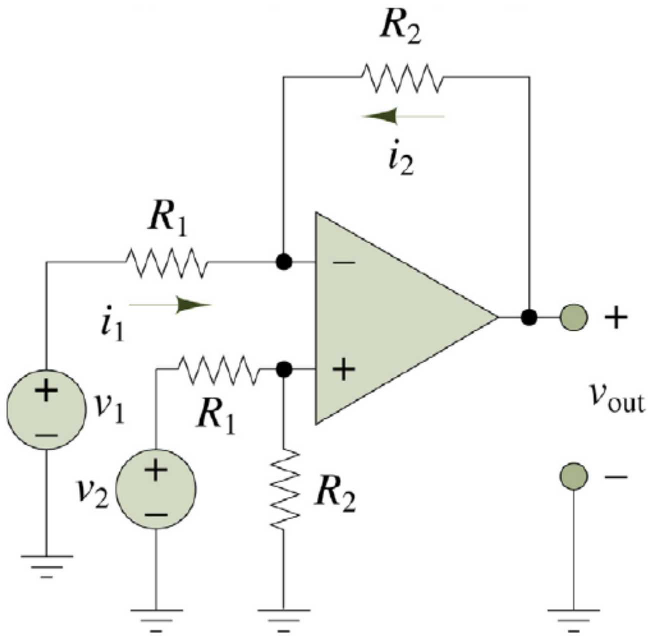
$$v^- = v^+$$

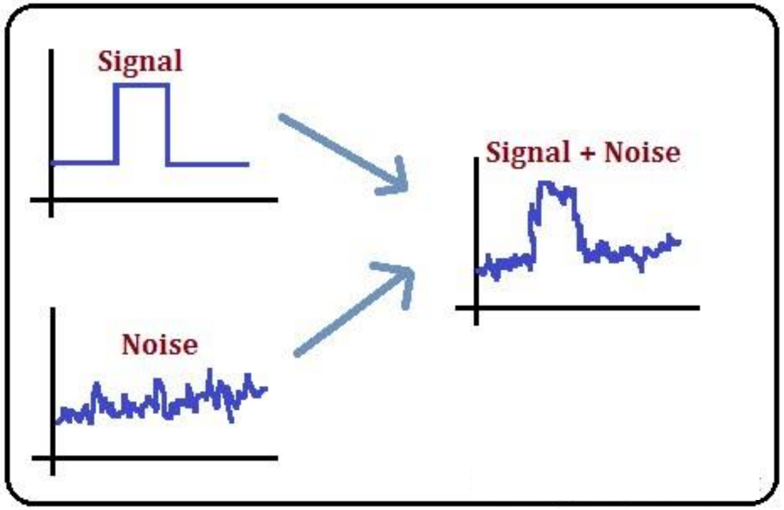
$$v^+ = \frac{R_2}{R_1 + R_2} v_2 = v^-$$

$$v_{out} = \frac{R_2}{R_1}(v_2 - v_1)$$

Differential Amplifier applications

- Very useful if you have two inputs corrupted with the same noise
- Subtract one from the other to remove noise, remainder is signal



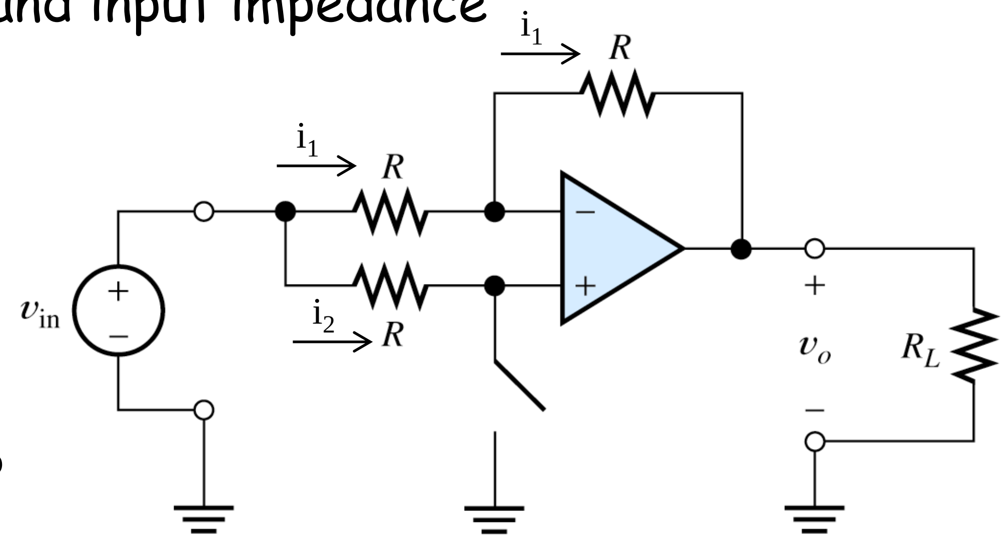


Exercise

Find voltage gain $A_v=v_o/v_{in}$ and input impedance

a. With the switch open

b. With the switch closed



a. From KVL: $v_{in}=i_1*R+i_1*R+v_o$

$i_2=0$ and $i_1*R=i_2*R$ => $i_1=0$

so $v_{in}=v_o$ and $A_v=v_o/v_{in}=1$

Input impedance:

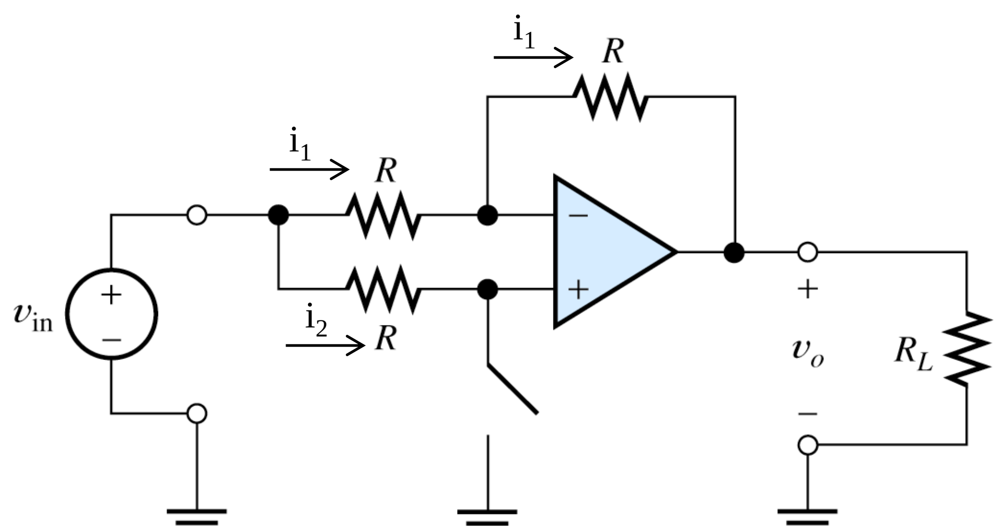
$Z_{in}=v_{in}/i_{in} =v_{in}/0 =\text{inf}$

Exercise

Find voltage gain $A_v = v_o/v_{in}$ and input impedance

a. With the switch open
b. With the switch closed

b. for closed switch: $i_2 = v_{in}/R$
and $i_1 * R = i_2 * R \Rightarrow i_1 = i_2 \Rightarrow v_{in} = i_1 * R + i_1 * R + v_o$
so $v_{in} = v_{in}/R * R + v_{in}/R * R + v_o \Rightarrow -v_{in} = v_o$
and $A_v = v_o/v_{in} = -1$



Exercise

Find voltage gain $A_v = v_o / v_{in}$ and input impedance

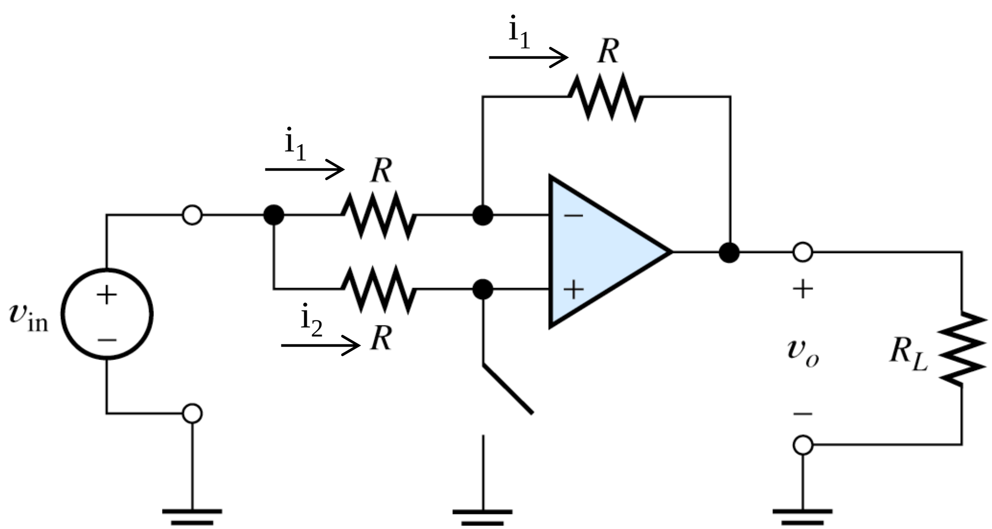
a. With the switch open
b. With the switch closed

b. $i_2 = v_{in}/R$

Input impedance: $Z_{in} = v_{in}/i_{in} = v_{in}/(i_1 + i_2)$

and $i_1 = i_2$ =>

$Z_{in} = v_{in}/i_{in} = v_{in}/(2 * v_{in}/R) = R/2$



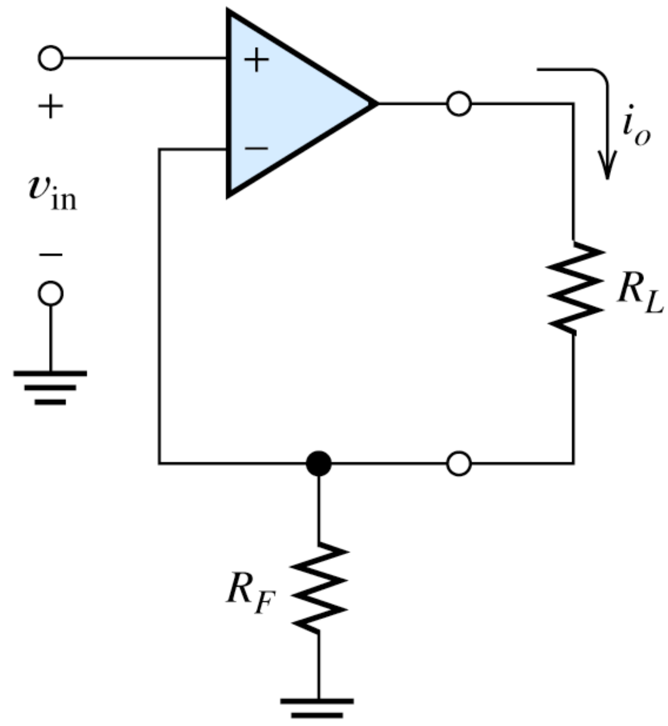
Voltage to Current Converter

Find the output current i_o as a function of v_{in}

$v_{in} = i_o * R_f$

so

$i_o = v_{in} / R_f$

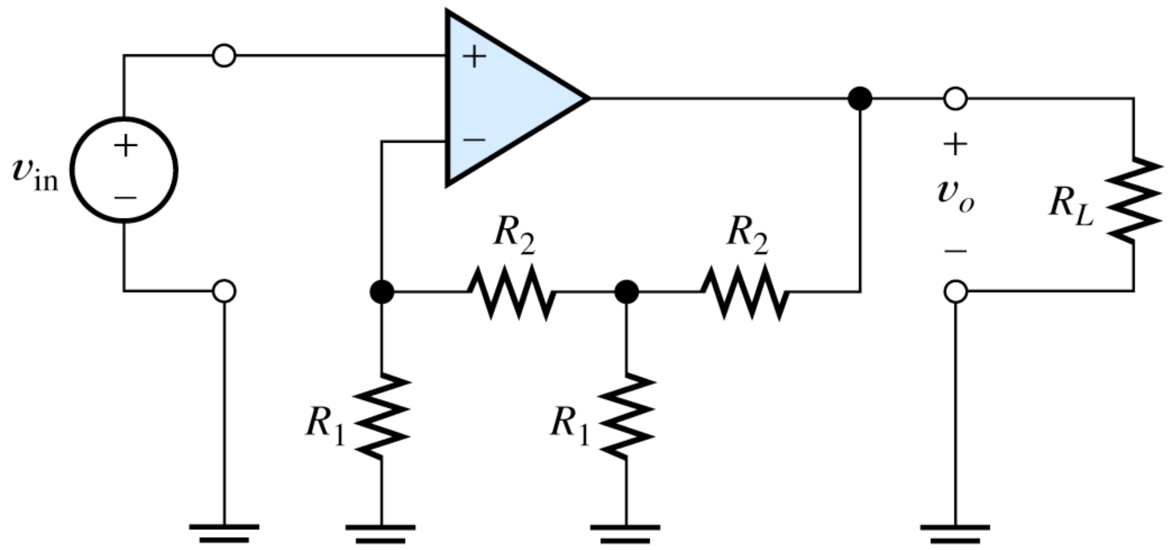


Exercise

a) Find the voltage gain v_o/v_{in}

b) Calculate the voltage gain v_o/v_{in} for R_1=10 kW, R_2 = 100 kW

c) Find the input resistance



Exercise

a) Find the voltage gain v_o/v_{in}

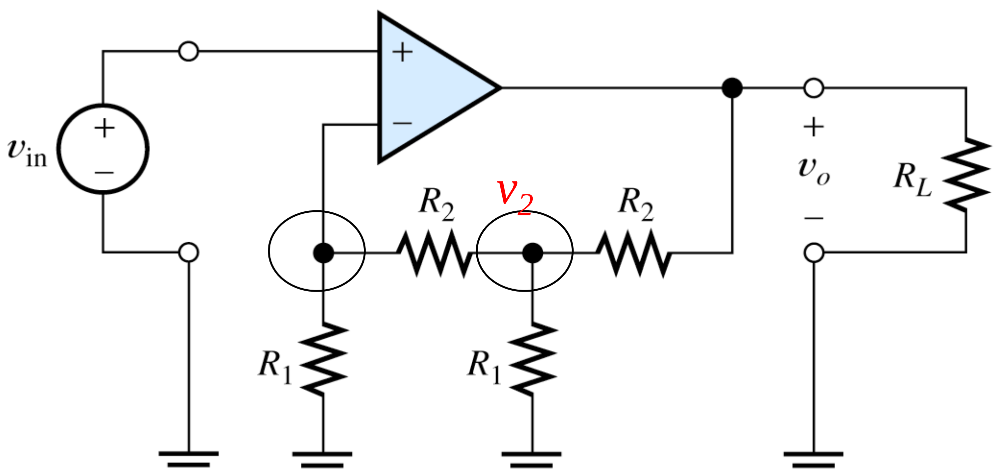
b) Calculate the voltage gain v_o/v_{in} for R_1=10 kW, R_2 = 100 kW

c) Find the input resistance

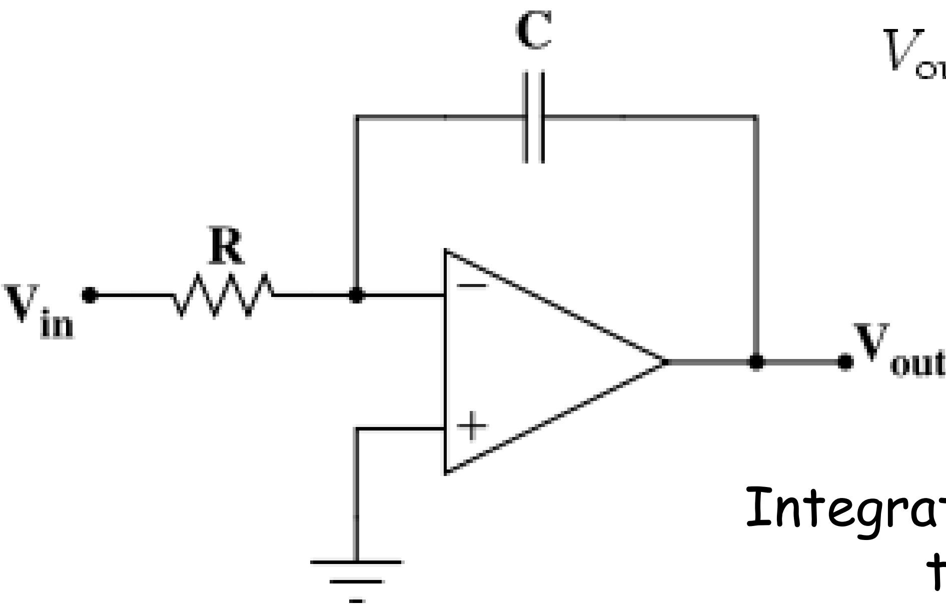
From KCL1: $v_{in}/R_1 = (v_2 - v_{in})/R_2 \Rightarrow v_2 = v_{in}(1 + R_2/R_1)$

From KCL2: $(v_2 - v_{in})/R_2 + v_2/R_1 + (v_2 - v_0)/R_2 = 0 \Rightarrow$

$v_0 = (v_2 - v_{in}) + v_2 R_2/R_1 + v_2 \Rightarrow$ $\boxed{v_0 / v_{in} = 131}$

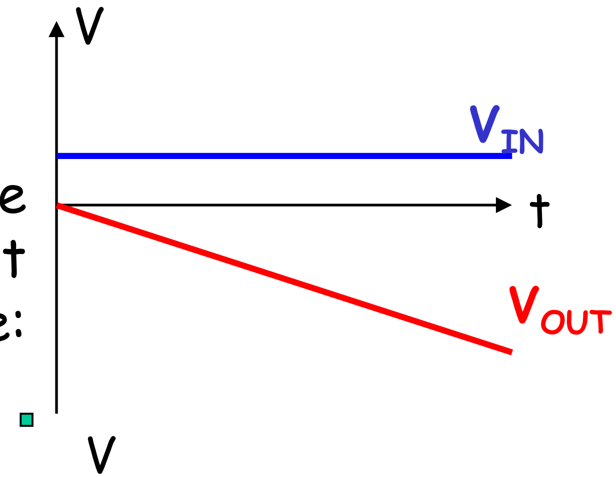


Op-amp integrator

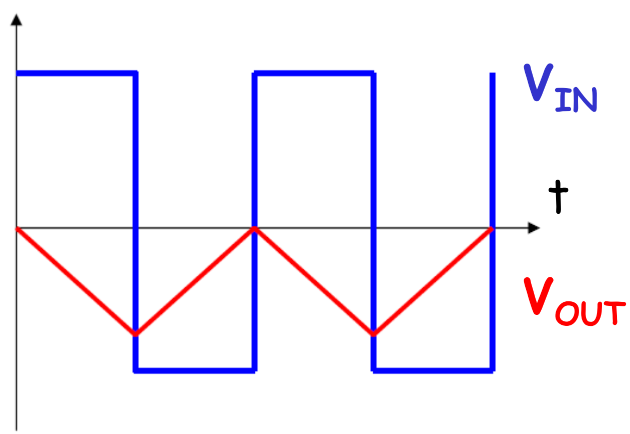


$$V_{\text{out}} = \int_0^t -\frac{V_{\text{in}}}{RC}\, dt + V_{\text{initial}}$$

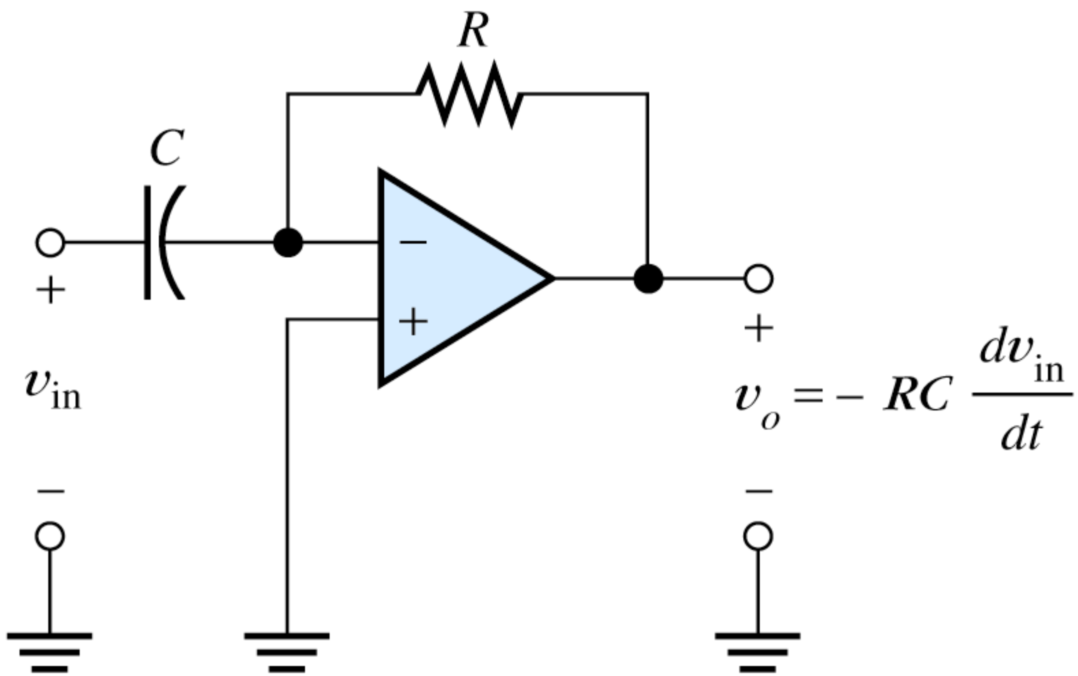
Integrator response to a constant voltage:



What's the integrator response to a square wave?



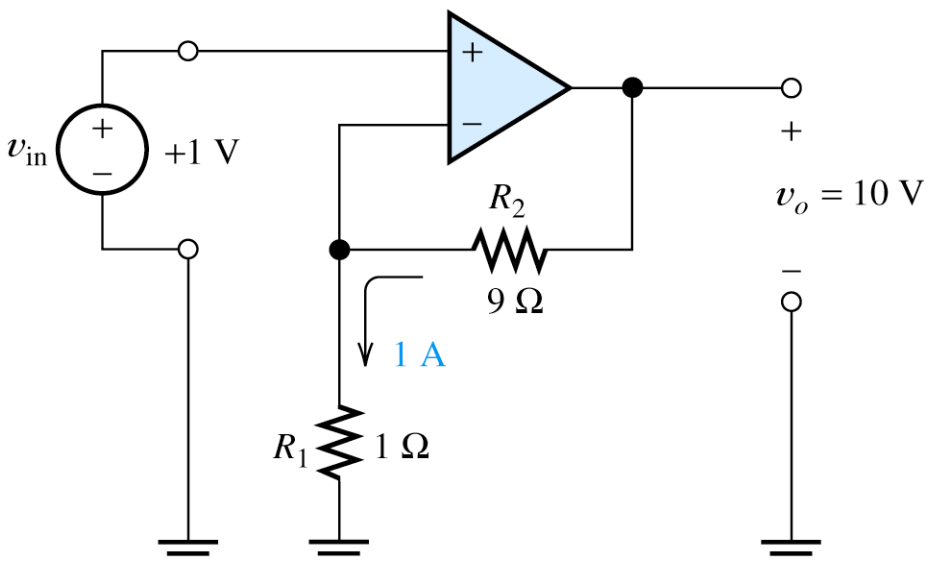
Differentiating Op-Amp



Design of Simple Amplifiers

Practical amplifiers can be designed using op-amp with feedback.

We know that for noninverting amplifier
$A_v = v_o / v_{in} = 1 + R_2 / R_1$
so to obtain Av=10 we could use R1=1 Ω and R2=9 Ω. But such low output resistance will draw too much current from the power supply.

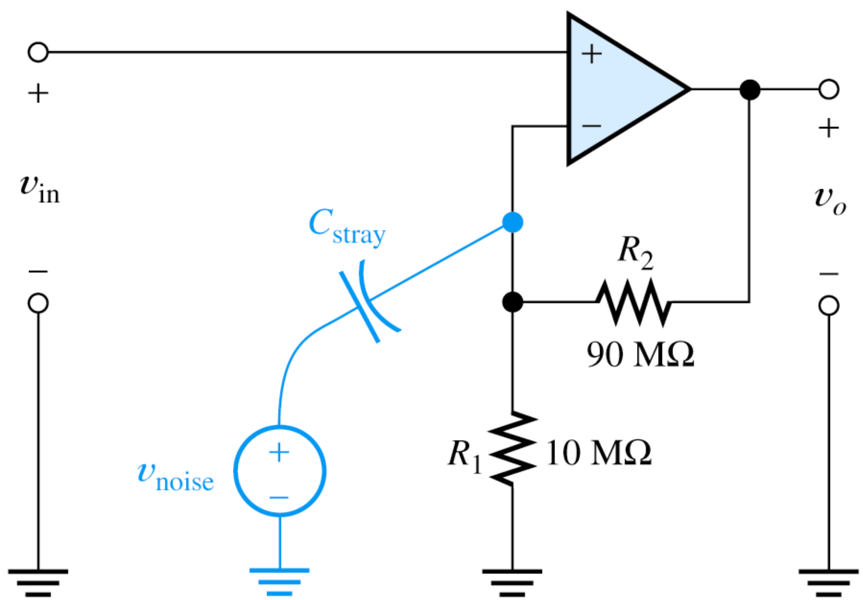


Design of Simple Amplifiers

The same gain can be obtained with large resistance values.

$$A_v = v_o / v_{in} = 1 + R_2/R_1$$

But for high output resistance are sensitive to bias current and we must use a filtering output capacitor to remove the noise.



If very high resistances are used, stray capacitance can couple unwanted signals into the circuit.

Op-Amp Imperfections in a Linear Mode

We consider the following op-amp imperfections:

Input and output impedances:

Ideal opamp $R_{in} = \infty; \quad R_{out} = 0\Omega$

Real op-amp has $R_{in} = 1M\Omega - 10^{12}\Omega;$

$R_{out} = 1\Omega - 100\Omega$

Nonlinear Limitations

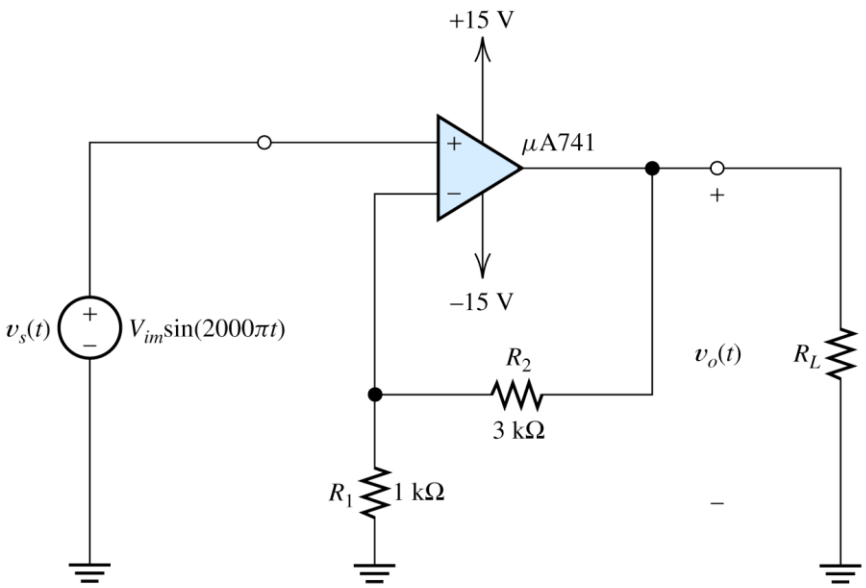
Nonlinear limitations:

Output voltage swing is limited and depend on power supply voltage for

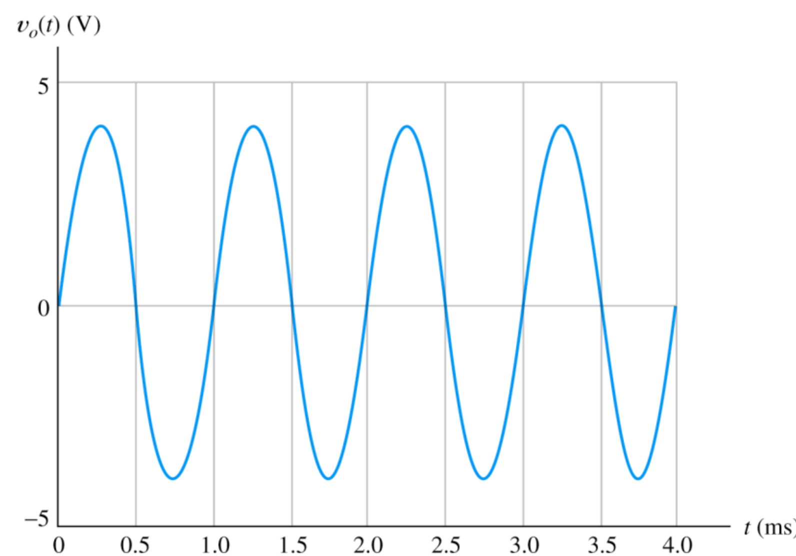
$$V_{DD} \in (-15V, +15V), \quad v_o(t) \in (-12V, +12V)$$

Maximum output current is limited

$$for \quad \mu A741 \quad amplifier \quad i_o(t) \in (-40mA, +40mA)$$



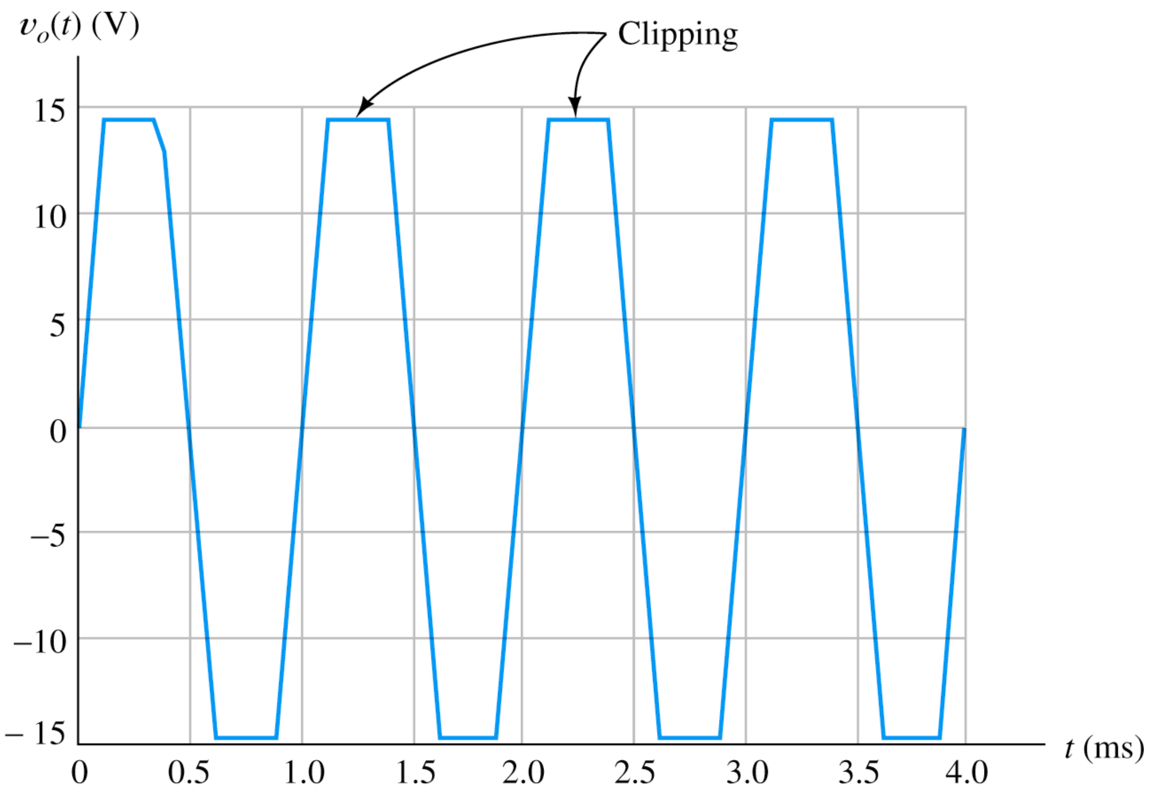
Noninverting amplifier used to demonstrate various nonlinear limitations of op amps.



Output of the circuit of Figure 14.23 for $R_L = 10$ kΩ and $V_{im} = 1$ V. None of the limitations are exceeded, and $v_o(t) = 4v_s(t)$.

Nonlinear Limitations

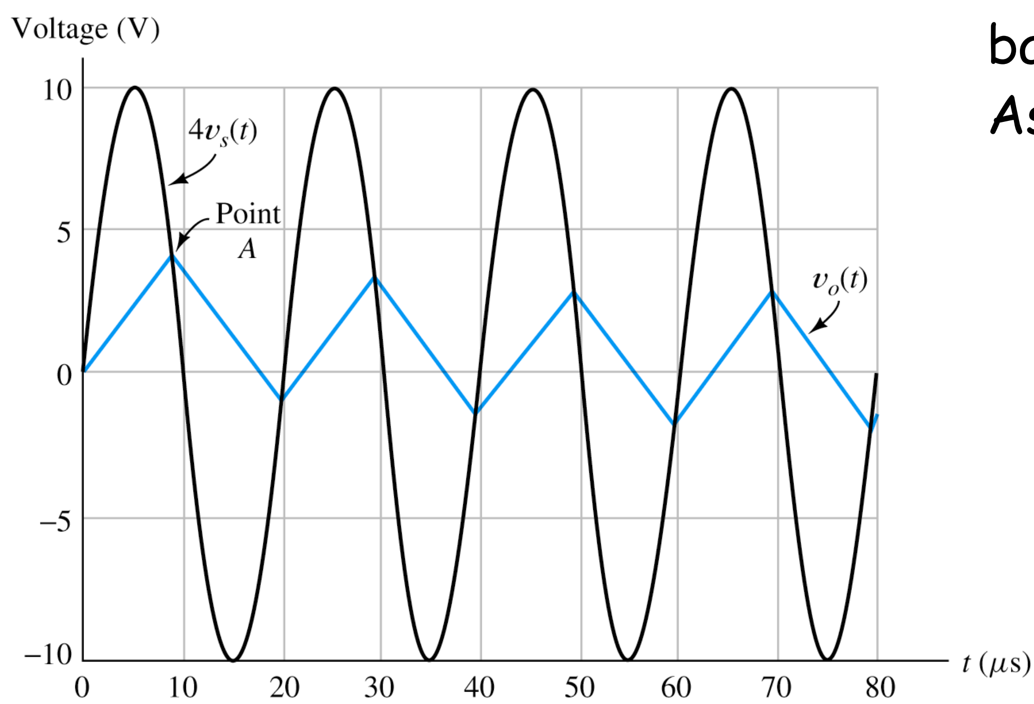
When voltage or current limits are exceeded, clipping of the output signal occurs causing large nonlinear distortions



Nonlinear Limitations

Another nonlinear limitation is limited rate of change of the output signal known as the slew-rate limit SR

$$\left|\frac{dv_o}{dt}\right| \le SR$$



Using slew rate (SR) we can find maximum frequency known as full-power bandwidth.

Assuming:

$$v_o(t) = V_{om}\sin(\omega t)$$

$$\frac{dv_o}{dt} = \omega V_{om}\cos(\omega t) \le$$

$$\le 2\pi\ f\ V_{om} \le SR$$

So the full-power bandwidth

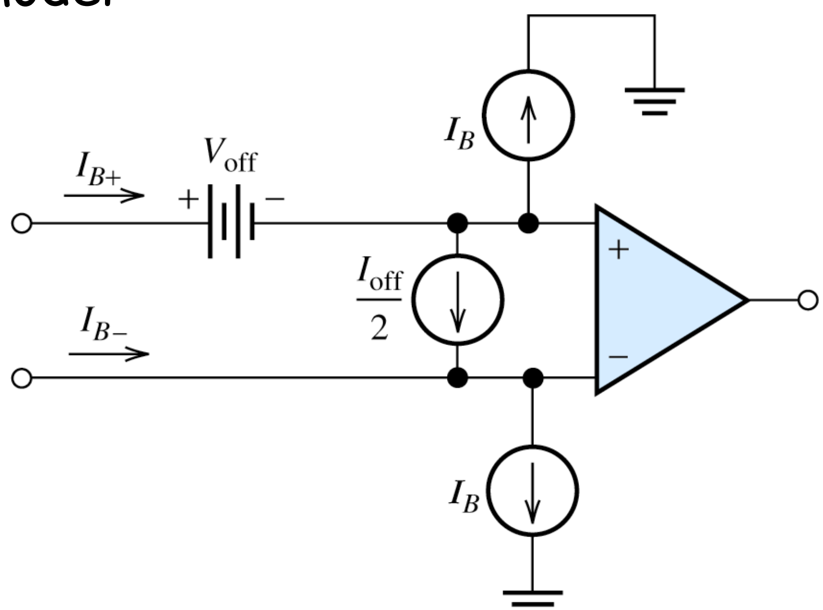
$$f_{FP} \le \frac{SR}{2\pi V_{om}}$$

DC offset values

There are three DC offset values related to op-amp design:

1) *Bias currents I_{B+}, I_{B-} – related to differential inputs*
2) *Offset current – ideally zero value*
3) *Offset voltage – results in nonzero output for zero input*

They can be represented as additional DC sources in the op-amp model

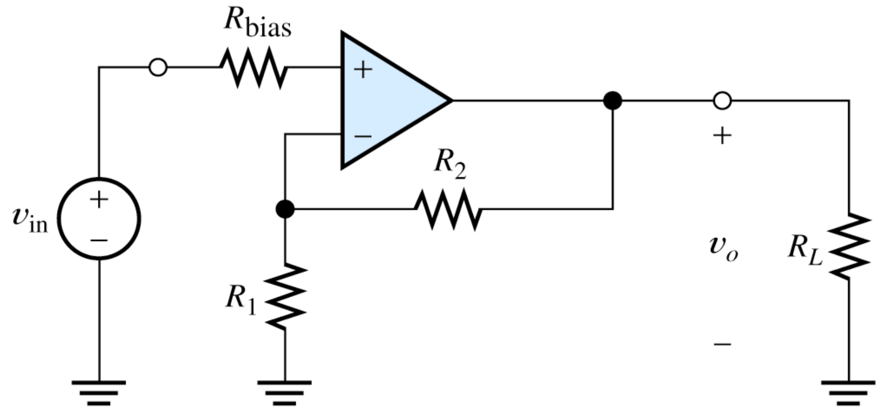


Three current sources and a voltage source model the dc imperfections of an op amp.

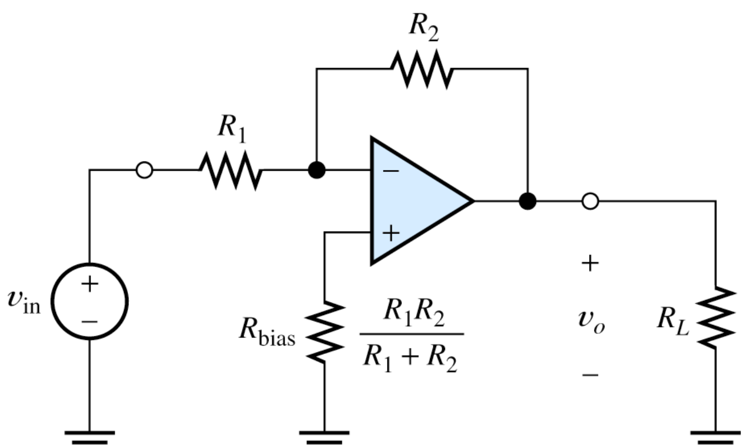
$$I_B = \frac{I_{B+} + I_{B-}}{2}$$

$$I_{off} = I_{B+} - I_{B-}$$

Resistor to balance the effect of bias currents



Noninverting amplifier, including resistor R_{bias} to balance the effects of the bias currents. See Exercise 14.15.



Adding the resistor R_{bias} to the inverting amplifier circuit causes the effects of bias currents to cancel.

Example: Calculate vo