BLG 354E Homework - 1

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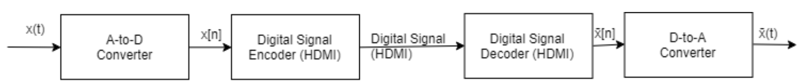
March 2018

1 Answers

This homework only includes answers to given questions

1) 

2)According to our class book *Signal Processing First* (McCellan, Schafer, Yoder, 2003) Signals are patterns of variations that represent or encode information. Each given figure has an information with different represantation. Therefore a heartbeat record and voice record are 1 dimensional signals and an image is 2 dimensinoal signal.

3)

$$z^4 = j$$

$$z = -\sqrt{j}$$

$$z = \sqrt{j}$$

$$z = -\sqrt{j}^5$$

$$z = \sqrt{j}^5$$

4)

$$e^{j\theta} = cos\theta + jsin\theta$$

$$e^{j\theta} = \sum_{n=0}^{\infty} (j\theta)^n/n! = 1 + j\theta + (j\theta)^2/2! + (j\theta)^3/3! + (j\theta)^4/4! + ...$$

$$j\theta^2 = -\theta$$

$$j\theta^3 = -j\theta$$

$$js\theta^4 = +\theta$$

$$j\theta^5 = j\theta^5$$

$$e^{j\theta} = \sum 1 + j\theta - (\theta)^2/2! - j(\theta)^3/3! + (\theta)^4/4! + ...$$

$$e^{j\theta} = \sum 1 + j\theta - (\theta)^2/2! + (\theta)^4/4! + ... - j(\theta)^3/3! + j(\theta)^5/5! + ...$$

$$e^{j\theta} = \sum 1 + j\theta - (\theta)^2/2! + (\theta)^4/4! + ...j(\theta - (\theta)^3/3! + (\theta)^5/5!) + ...$$

$$e^{j\theta} = cos\theta + jsin\theta$$

5) a) A function said tob e an odd function if function shows symetry on origin.

$$-x = f(-x)$$

Ex:$f(x) = x^3$, $f(x) = sin(x)$

b) A function said to be an even function if function shows symetry on origin.

$$x = f(-x)$$

Ex: : $f(x) = x^2$, $f(x) = cos(x)$

c)

$$sin\theta = cos(\theta - \pi/2)$$

$$cos(\theta + 2\pi k) = cos\theta \text{ , when } k \text{ is integer}$$

$$cos(-\theta) = cos\theta$$

$$sin(-\theta) - sin(\theta)$$

$$sin(\pi k) = 0 \text{ , when } k \text{ is integer}$$

$$cos(2\pi k) = 1 \text{ , when } k \text{ is integer}$$

$$cos[2\pi(k + 1/2)] = -1 \text{ , when } k \text{ is integer}$$

d)i.

$$\frac{d}{d\theta}(sin^2\theta + cos^2\theta) = \frac{d}{d\theta}1$$

$$2sin\theta cos\theta - 2cos\theta sin\theta = 0$$

ii.

$$\frac{d}{d\theta}(cos(2\theta)) = \frac{d}{d\theta}(cos^2\theta - sin^2\theta)$$

$$-2sin2\theta = -2cos\theta sin\theta - 2sin\theta cos\theta$$

$$sin2\theta = 2cos\theta sin\theta$$

iii.

$$\frac{d}{d\theta}(sin2\theta) = \frac{d}{d\theta}(2cos\theta sin\theta)$$

$$2cos2\theta = -2sin^2\theta + 2cos^2\theta$$

$$cos2\theta = cos^2\theta - sin^2\theta$$

$$x = cos(\theta) + jsin(\theta)$$

Using De Moivre's theorem:

$$x^2 = cos(2\theta) + jsin(2\theta)$$

$$x^2 = (cos(\theta) + jsin(\theta))^2$$

$$cos(2\theta) + jsin(2\theta) = cos^2(\theta) - sin^2(\theta) + 2jcos(\theta)sin(\theta)$$

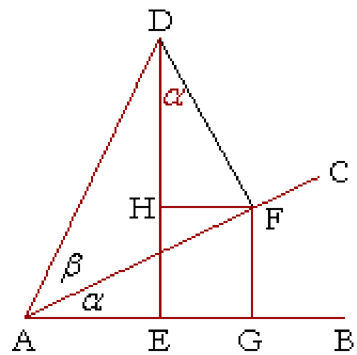
$$cos(2\theta) = cos^2(\theta) - sin^2(\theta)$$

$$sin(2\theta) = 2sin(\theta)cos(\theta)$$

iv. Using figure 1 we can derivate given formulas.

$$sin(\alpha + \beta) = \frac{ED}{DA}$$

Figure 1: Set of triangles. $EHFG$ is a rectangle. Copyrigth Lawrance Spector



Using rectangle's properties

$$GF = EH$$

$$HF = EG$$

$$sin(\alpha + \beta) = \frac{FG}{AF}\frac{AF}{DA} + \frac{HD}{FD}\frac{FD}{DA}$$

$$sin(\alpha + \beta) = sin(\alpha)cos(\beta) + cos(\alpha)sin(\beta)$$

V. Using figure 1 we can derivate given formulas.

$$cos(\alpha + \beta) = \frac{AE}{DE}$$

Using rectangle's properties

$$GF = EH$$

$$HF = EG$$

$$cos(\alpha + \beta) = \frac{AG}{AF}\frac{AF}{DA} + \frac{HF}{FD}\frac{FD}{DA}$$

$$cos(\alpha + \beta) = cos(\alpha)cos(\beta) + sin(\alpha)sin(\beta)$$

6)

$$\sum_{k=1}^{N} A_k cos(w_o t + \phi_k) = Re(\sum_{k=1}^{N} A_k e^{jw_0 t} e^{j\phi_k})$$

$$= Re((\sum_{k=1}^{N} A_k e^{j\phi_k}) e^{jw_0 t})$$

$$= Re((Ae^j\phi)e^{jw_0t})$$

$$= Acos(w_0t + \phi)$$

7)

$$z_1(t) = cos(wt - \frac{1}{3}\pi)$$

$$z_2(t) = 3cos(wt - \frac{7}{4}\pi)$$

$$z_3(t) = 2cos(wt - \frac{3}{2}\pi)$$

$$x(t) = cos(wt - \frac{1}{3}\pi) + 3cos(wt - \frac{7}{4}\pi) + 2cos(wt - \frac{3}{2}\pi)$$

a)

$$x(t) = \sum_{k=1}^{N} A_k cos(w_o t + \phi_k) = Re((\sum_{k=1}^{N} A_k e^{j\phi_k})e^{jw_0t})$$

$$= Re((e^{-j\frac{1}{3}\pi} + 3e^{-j\frac{7}{4}\pi} + 2e^{-j\frac{3}{2}\pi})e^{jwt})$$

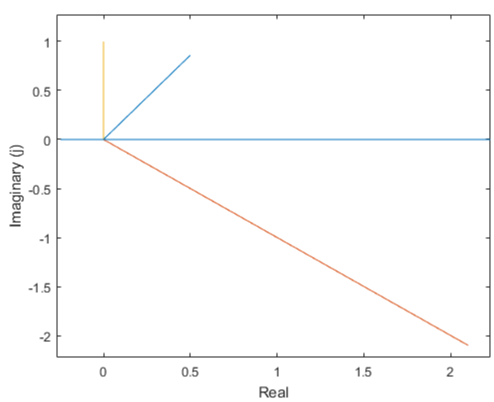
$$= Re((0.5 + 0.86j + 2.1 - 2.1j + j)e^{jwt})$$

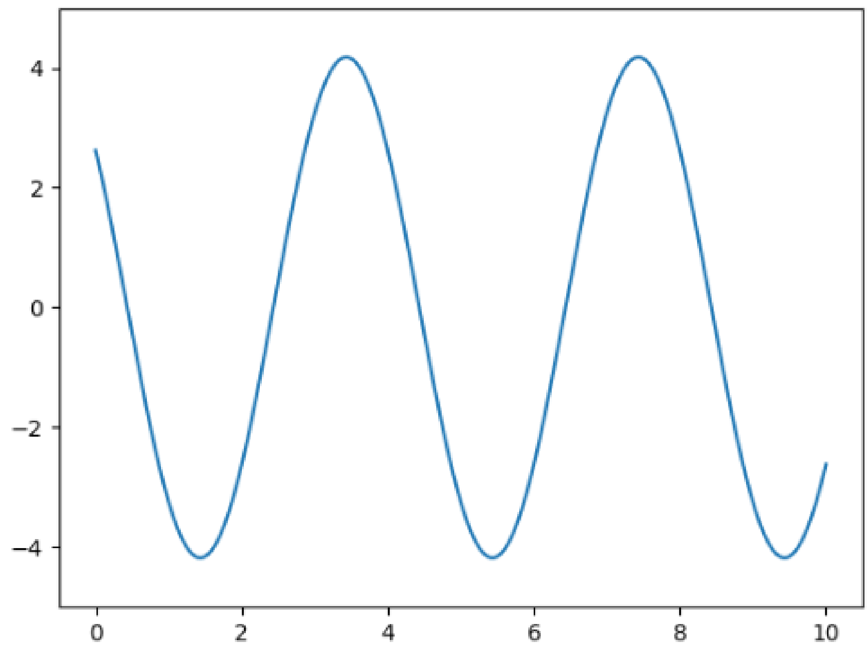
$$= Re((2.6 - 0.24j)e^{jwt})$$

$$= Re((2.6 - 0.24j)e^{jwt})$$

$$= Re(2.612e^{0.557j})e^{jwt}) = 2.612cos(wt + 0.177\pi)$$

b)





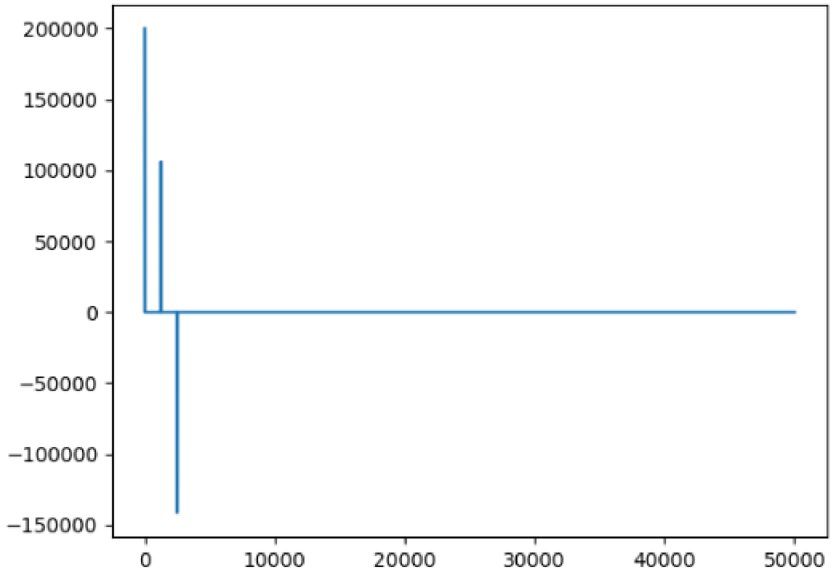
c)

Script to plot given graphic can be found in attachment (hw1q7c.py)

8)

$$x(t) = 2 + 4cos(500\pi t + \frac{5}{4}\pi) - 3cos(60\pi t - \frac{1}{2}\pi) + 3cos(250\pi t - \frac{1}{4})$$

a)



Script to plot given graphic can be found in attachment (hw1q8a.py)

b) x(t) is periodic. Period is least common mmultiple of each sinusoid's period. Those periods are respectivly, $\frac{1}{250}$, $\frac{1}{30}$, $\frac{1}{125}$. Therefore $x(t)$ has period of 3.

$$\frac{1}{250}750 = 3$$
$$\frac{1}{30}90 = 3$$
$$\frac{1}{125}375 = 3$$

c)It has fundamental frequency of 30 Hertz. It has 0th and 1st harmonic

9)

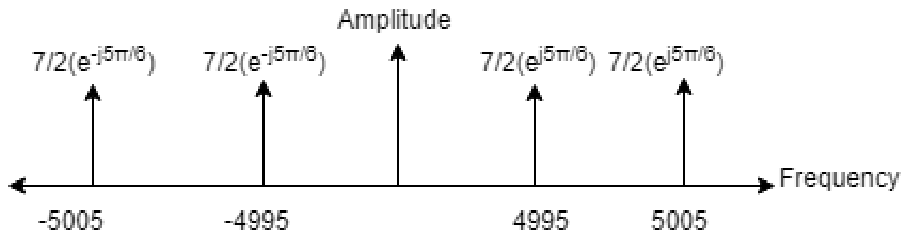
$$y_1(t) = 2cos(10\pi t)$$
$$y_2(t) = 7cos(-10000\pi t + \frac{5}{6}\pi) = 7cos(10000\pi t - \frac{5}{6}\pi$$

a)

$$x(t) = 2cos(10\pi t)7cos(10000\pi t - \frac{5}{6}\pi)$$

$$= 7/2(e^{j10\pi t} + e^{-j10\pi t})(e^{j10000\pi t - \frac{5}{6}\pi} + e^{-j10000\pi t + \frac{5}{6}\pi})$$

$$= 7/2(e^{j10010\pi t - \frac{5}{6}\pi} + e^{-j9990\pi t + \frac{5}{6}\pi} + e^{j9990\pi t - \frac{5}{6}\pi} + e^{-j10010\pi t + \frac{5}{6}\pi}$$



b)

c)

$$x(t) = 7cos(9990\pi t - \frac{5}{6}\pi) + 7cos(10010\pi t - \frac{5}{6}\pi)$$

10)Two fuunctions are said to be orthogonal functions if addition of their multiplication for every value equal to zero. This means that they are orthogonal in an infinite vector space.

$f(x)$ and $g(x)$ are orthogonal in $i < x < j$ if

$$\int_i^j f(x)g(x)dx = 0$$

a)

$$\int_{-L}^{L} sin(2\pi nft)sin(2\pi mft)dt = \frac{sin(2\pi ft(m-n))}{4\pi(m-n)} - \frac{sin(2\pi ft(m+n))}{4\pi(m+n)}|_{-L}^{L}$$

$$= \frac{sin(2\pi fL(m-n))}{4\pi(m-n)} - \frac{sin(2\pi fL(m+n))}{4\pi(m+n)} + \frac{sin(2\pi fL(m-n))}{4\pi(m-n)} - \frac{sin(2\pi fL(m+n))}{4\pi(m+n)} = 0$$

Assuming $fL(m-n)$ and $fL(m+n)$ are integers $sin(2\pi) = 0$ functions are orthogonal.

b)

$$\int_{-L}^{L} cos(2\pi nft)cos(2\pi mft)dt = \frac{sin(2\pi ft(m-n))}{4\pi(m-n)} - \frac{sin(2\pi ft(m+n))}{4\pi(m+n)}|_{-L}^{L}$$

$$= \frac{sin(2\pi fL(m-n))}{4\pi(m-n)} - \frac{sin(2\pi fL(m+n))}{4\pi(m+n)} + \frac{sin(2\pi fL(m-n))}{4\pi(m-n)} - \frac{sin(2\pi fL(m+n))}{4\pi(m+n)} = 0$$

Assuming $fL(m-n)$ and $fL(m+n)$ are integers $sin(2\pi) = 0$ functions are orthogonal.

c)

$$\int_{-L}^{L} sin(2\pi nft)cos(2\pi mft)dt = \frac{cos(2\pi ft(m-n))}{4\pi(m-n)} - \frac{cos(2\pi ft(m+n))}{4\pi(m+n)}|_{-L}^{L}$$

$$= \frac{cos(2\pi fL(m-n))}{4\pi(m-n)} - \frac{cos(2\pi fL(m+n))}{4\pi(m+n)} - \frac{cos(2\pi fL(m-n))}{4\pi(m-n)} + \frac{cos(2\pi fL(m+n))}{4\pi(m+n)} = 0$$

Functions are orthogonal.

11)Gibbs phenomenon is occurs when a discontinuous signal such as a square signal, represented as sum of sinusoidal functions such as Fourier series. Sum of sinusoidal functions osillates near the discontinuity and creates a bump in the shape of wave.