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Q1 C: Citizen (Terrorist: 1 / Not a terrorist: 0)
S: Scanner (Terrorist: 1 / Not a terrorist: 0)

$P(S=1/C=1)=0.95$

$P(S=0/C=0)=0.95 \Rightarrow P(S=1/C=0)=0.05$

$P(C=1)=0.1 \Rightarrow P(C=0)=0.9$

$P(C=1/S=1)=?$

$$P(C=1/S=1)=\frac{P(S=1/C=1).P(C=1)}{P(S=1/C=1).P(C=1)+P(S=1/C=0).P(C=0)}$$

$$P(C=1/S=1)=\frac{0.95\times0.1}{0.95\times0.1+0.05\times0.9}=\frac{0.095}{0.095+0.045}=\frac{0.095}{0.14}=0.67857=67.857\%$$

Q2 $p(c=1|x)=0.7;\ p(c=2|x)=0.2;\ p(c=3|x)=0.1;$

$$U(c^{true},c^{pred})=\begin{bmatrix}5&3&1\\0&4&-2\\-3&0&10\end{bmatrix}\qquad C=\begin{bmatrix}0.7&0.2&0.1\end{bmatrix}$$

$$EU=\begin{bmatrix}0.7&0.2&0.1\end{bmatrix}\begin{bmatrix}5&3&1\\0&4&-2\\-3&0&10\end{bmatrix}=\begin{bmatrix}\underline{3.2}&2.9&1.3\end{bmatrix}$$

Best decision: $c=1$

Q3 Random sample:

115 122 130 127 149 160 152 138 149 180 $\rightarrow x_1$ to x_{10}, $N=10$

μ,σ: unknown

$\rightarrow$ Maximum likelihood function:

$$\mathcal{L}(\mu,\sigma)=\sum_{i=1}^{N}\left(\log\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)+\left(-\frac{1}{2\sigma^2}\right)(x_i-\mu)^2\right)$$

$$\mathcal{L}(\mu,\sigma)=\sum_{i=1}^{N}\left(-\log\sigma-\frac{\log2\pi}{2}\right)-\frac{1}{2\sigma^2}(x_i-\mu)^2)$$

• $\mathcal{L}(\mu,\sigma)=-N\log\sigma-\frac{N}{2}\log2\pi-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(x_i-\mu)^2$

$\rightarrow$ Maximum Likelihood Estimator of μ:

$$\frac{\partial\mathcal{L}(\mu,\sigma)}{\partial\mu}=-\frac{1}{2\sigma^2}\sum_{i=1}^{N}-2(x_i-\mu)=\frac{1}{\sigma^2}\sum_{i=1}^{N}(x_i-\mu)$$

$\frac{1}{\sigma^2}\sum_{i=1}^{N}(x_i-\mu)=0 \quad \sum_{i=1}^{N}(x_i-\mu)=0$

$\left(\sum_{i=1}^{N}x_i\right)-N\mu=0$

• $\hat{\mu}=\frac{\sum_{i=1}^{N}x_i}{N}$

$\rightarrow$ Maximum Likelihood Estimator of σ:

$$\mathcal{L}(\mu,\sigma)=-N\log\sigma-\frac{N}{2}\log2\pi-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(x_i-\mu)^2$$

$$\frac{\partial\mathcal{L}(\mu,\sigma)}{\partial\sigma}=\frac{-N}{\sigma}+\sigma^{-3}\sum_{i=1}^{N}(x_i-\mu)^2=0$$

$$\frac{N}{\sigma}=\sigma^{-3}\sum_{i=1}^{N}(x_i-\mu)^2\rightarrow\sigma^2=\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$$

• $\hat{\sigma}^2=\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}$, $\hat{\sigma}=\left(\frac{\sum_{i=1}^{N}(x_i-\mu)^2}{N}\right)^{1/2}$

★ For this sample:

$$\hat{\mu}=\frac{\sum_{i=1}^{10}x_i}{10}=\frac{115+122+130+127+149+160+152+138+149+180}{10}=\frac{1422}{10}$$

$\hat{\mu}=142.2$

BLG457E-Learning From Data HW1-Question 4

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Given data set has 2 classes and 2 features. Code written by us, selects approximately 300 elements from class 1 and another approximately 300 elements from class 2 randomly. Then using leftover data as training, it tries to estimate chosen data's classes by KNN classifier. Homework code (code.m) only requires d matrix from hw1.mat, then produces output with success rates of KNN2 and KNN5. After that it plots example data and KNN2, KNN5 data. Class 1 is red and class 2 is blue in plot. Example data has '+' shape, KNN2 data has 'o' shape and KNN5 data has '◇' shape.

Produced output:

KNN2 Success Rate: 0.870242214532872

KNN 5 Success Rate: 0.885813148788927

KNN5 has a little bit higher success rate than KNN2 in most of the cases.

Plotted data: