BLG456E
Robotics
Wheeled locomotion & kinematics

Lecture Contents:
- Locomotion design issues.
- Wheel geometry & kinematics.
- Wheel configurations.
- Car wheels.
- Differential drive.
- Odometry (robot).
- Degrees of freedom & holonomicity.

Lecturer:	Damien Jade Duff
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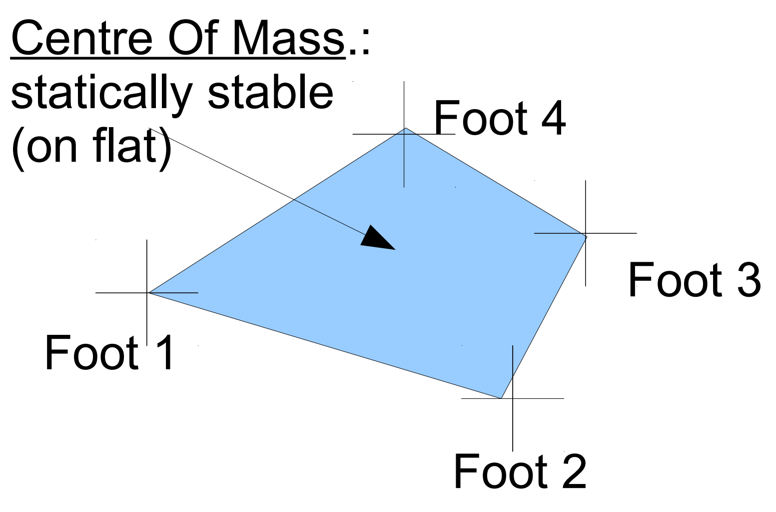
Schedule:	http://djduff.net/my-schedule
Coordination:	http://ninova.itu.edu.tr/Ders/4709

Locomotion design issues

- Required speed, strength.
- Walking? Jumping? Climbing? Flying? Swimming? Tunnelling?
- Terrain – smooth? Rough?
- Power density (never enough).
- Stability:
  - Contact points, centre of gravity (inside concave hull of contact points?).
  - Static stability: hold still, won't fall.
  - Dynamic stability: move right, won't fall.
  - Terrain inclination.
- Contact characteristics:
  - Contact point(s), friction, angle.

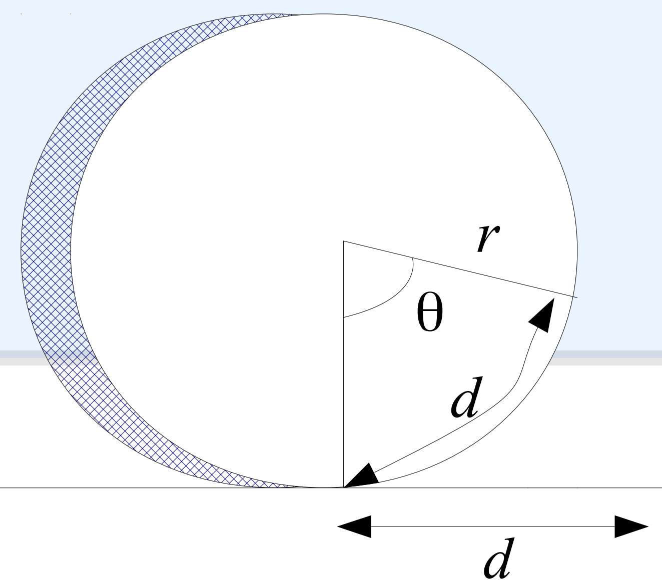


Standard wheel constraints

Rolling constraint:

Movement in direction of wheel must have accompanying wheel turning.

Side-view



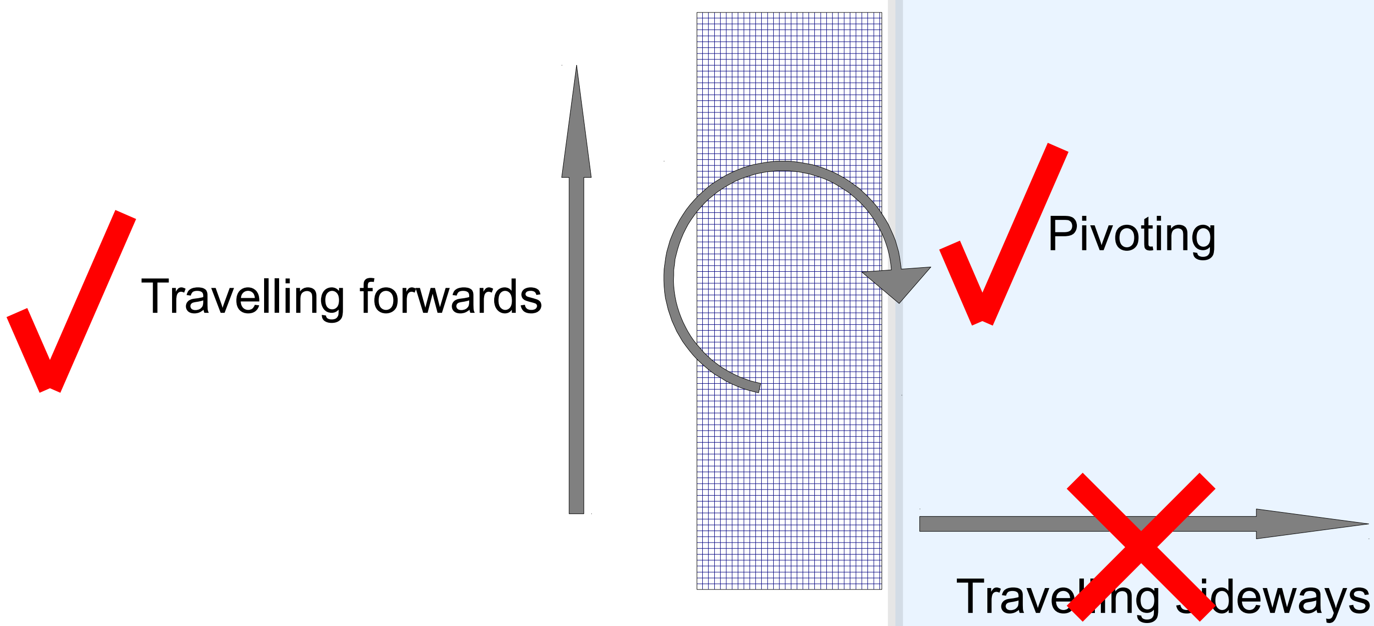
$$d = r\theta$$

Exercise: (3 minutes) A wheel of radius 0.5m turns through half a full rotation (2π radians) as it travels in a straight line. How far did it travel?

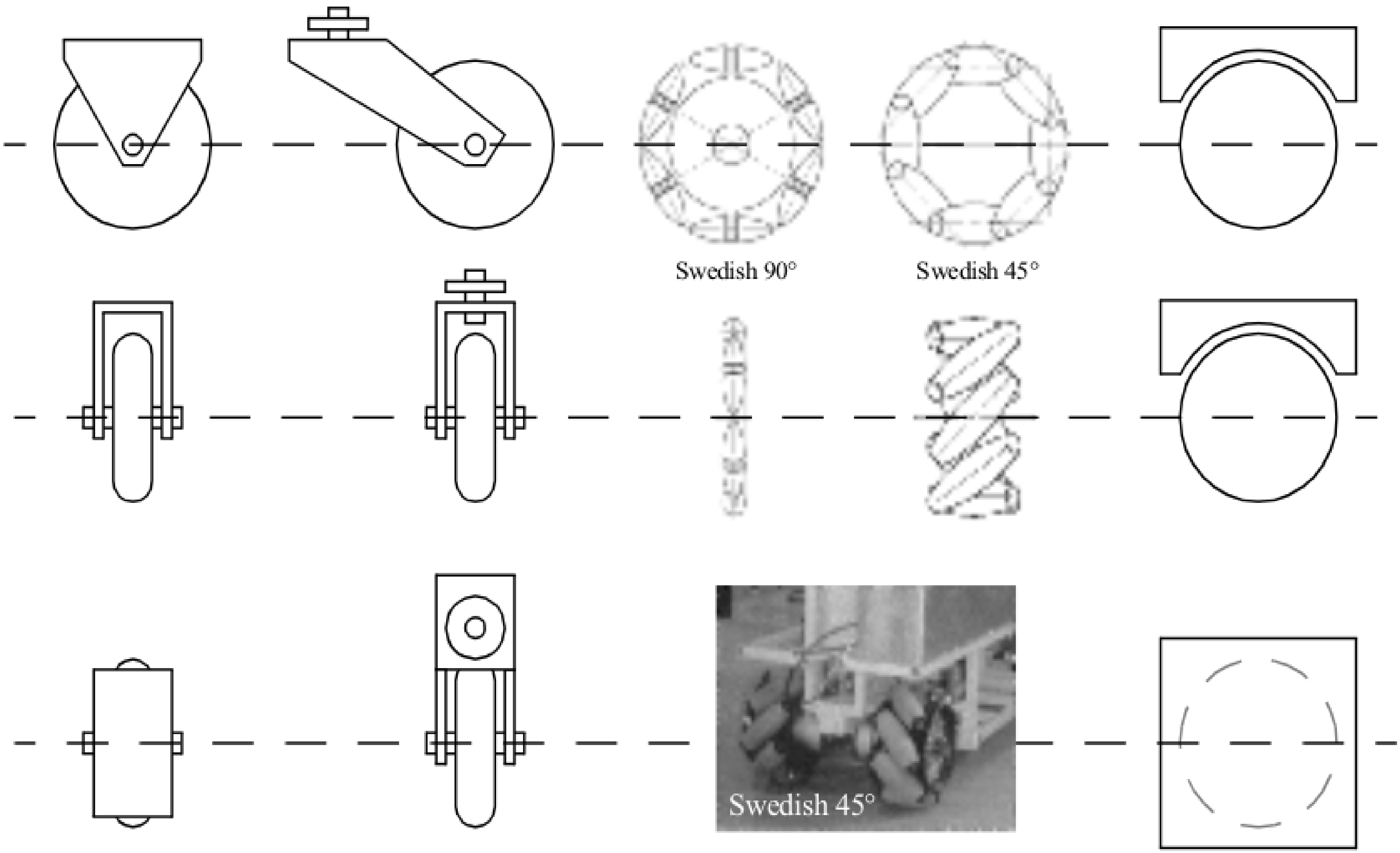
Standard wheel constraints

Sliding constraint:

Wheel can't go sideways.



Other kinds of wheel



From Siegwart & Nourbakhsh 3rd ed

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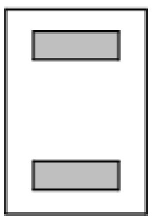
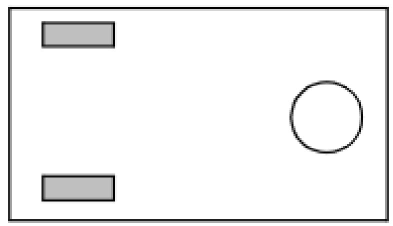
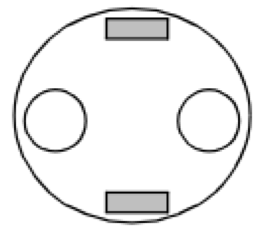
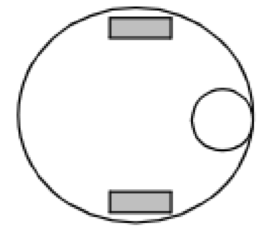
Robot wheel configurations

Differential drive

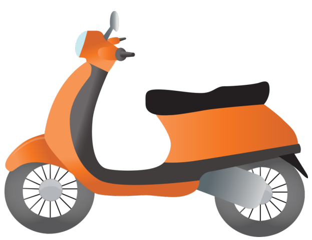


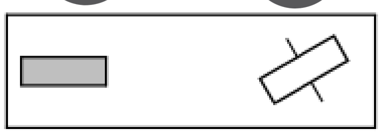


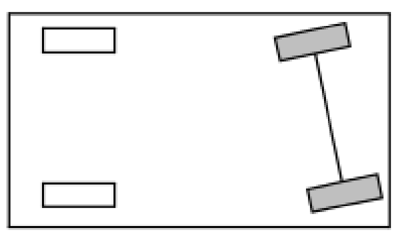


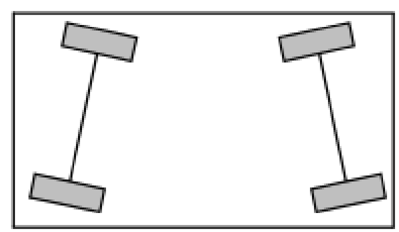


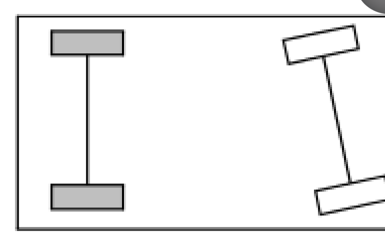
Steered wheels



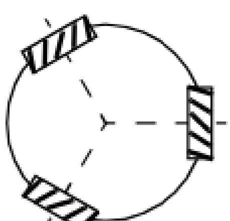


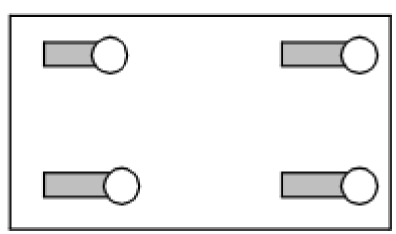


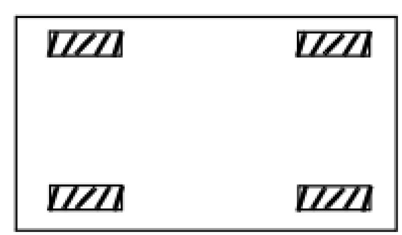




With powered omnidirectional or castor wheels



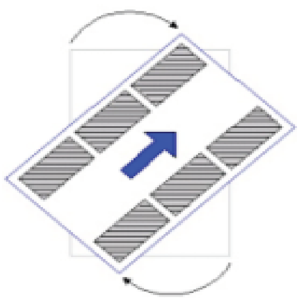




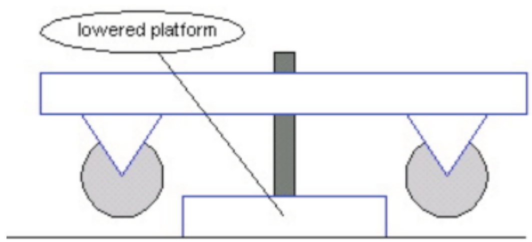
See Siegwart & Nourbakhsh for more

More kinds of wheeled locomotion

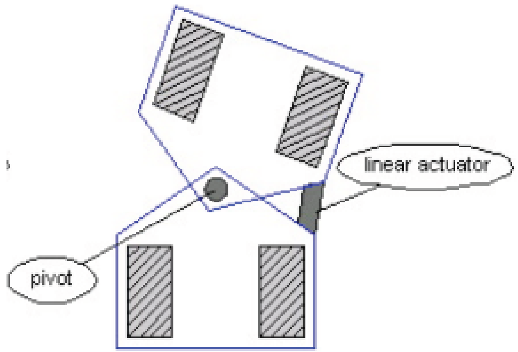
Skid-steer (tanks).



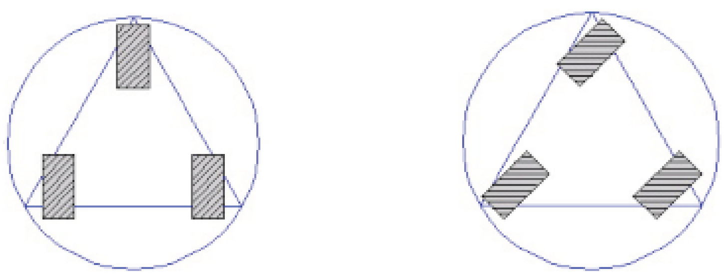
Pivot drive.



Articulated drive.



Synchronous drive.

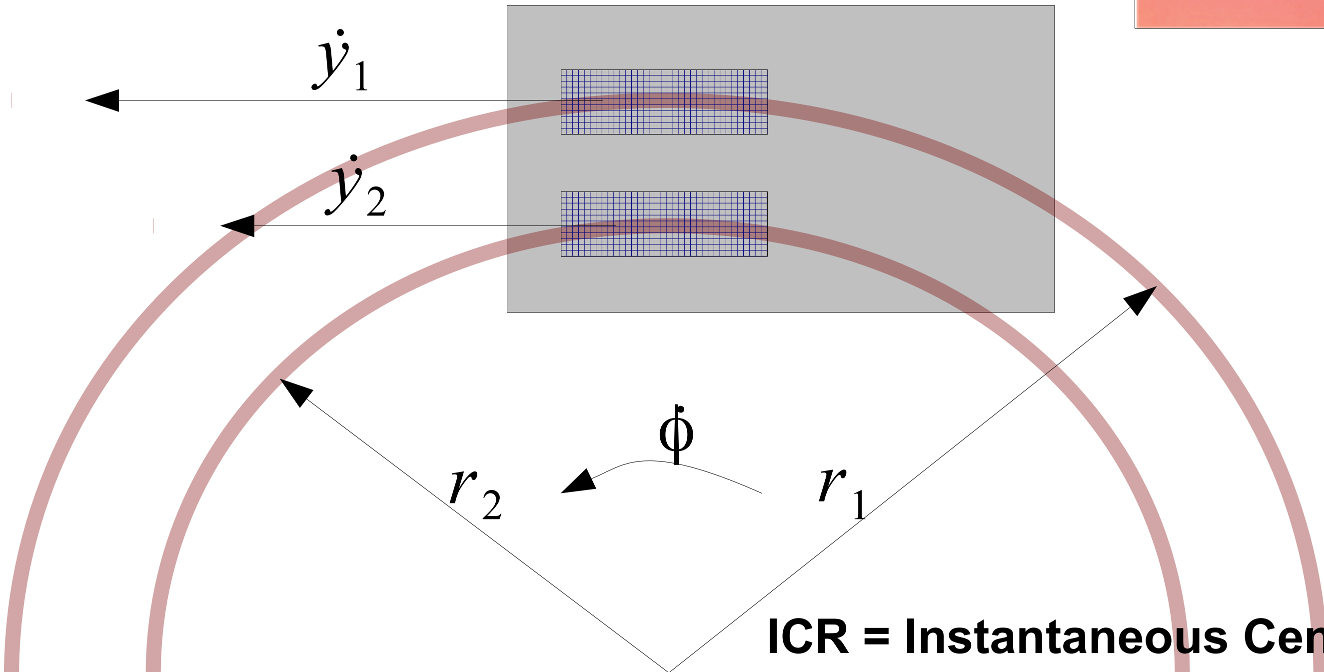


Hybrid walkers.

Cars are more complicated 1: Differentials

Problem: <u>Powered</u> wheels while going through a turn must either skid or turn at different rates.

Solution: A "differential".



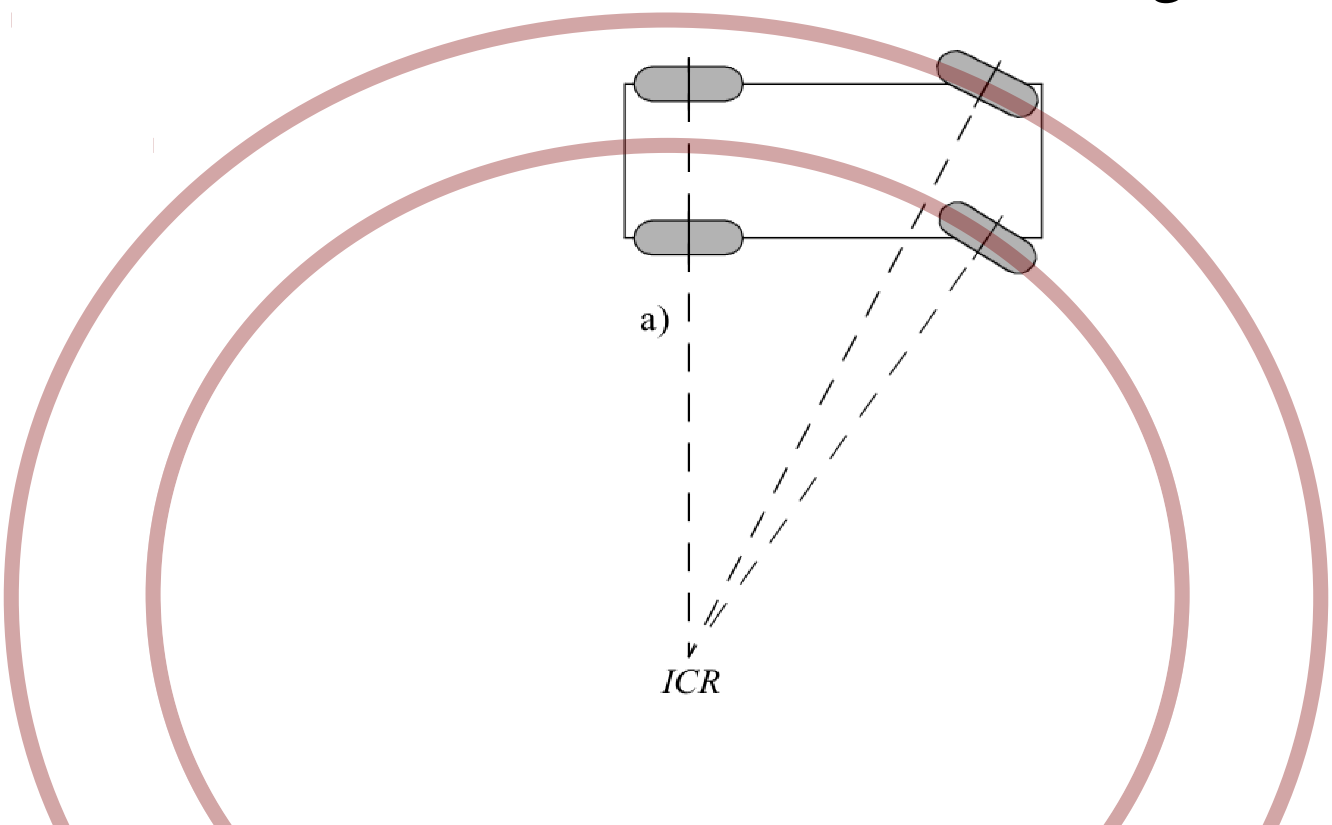
$$\dot{y}_1 = r_1 \dot{\phi}$$
$$\dot{y}_2 = r_2 \dot{\phi}$$

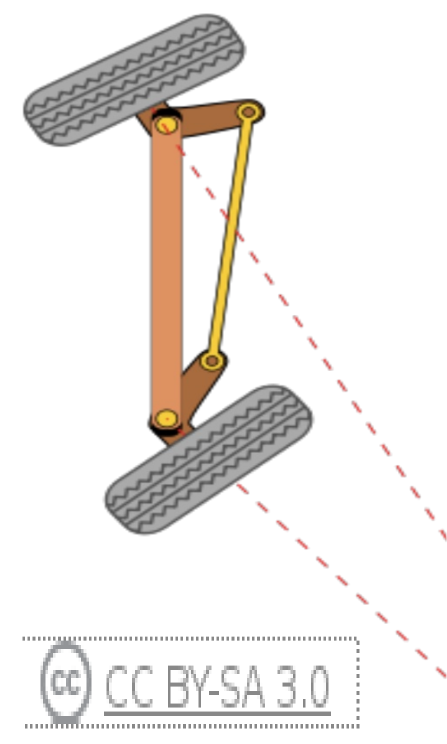
ICR = Instantaneous Centre of Rotation

Cars are more complicated 2: Ackermann Steering

Problem: Parallel wheels during steering are on circles of different radii and so skid.

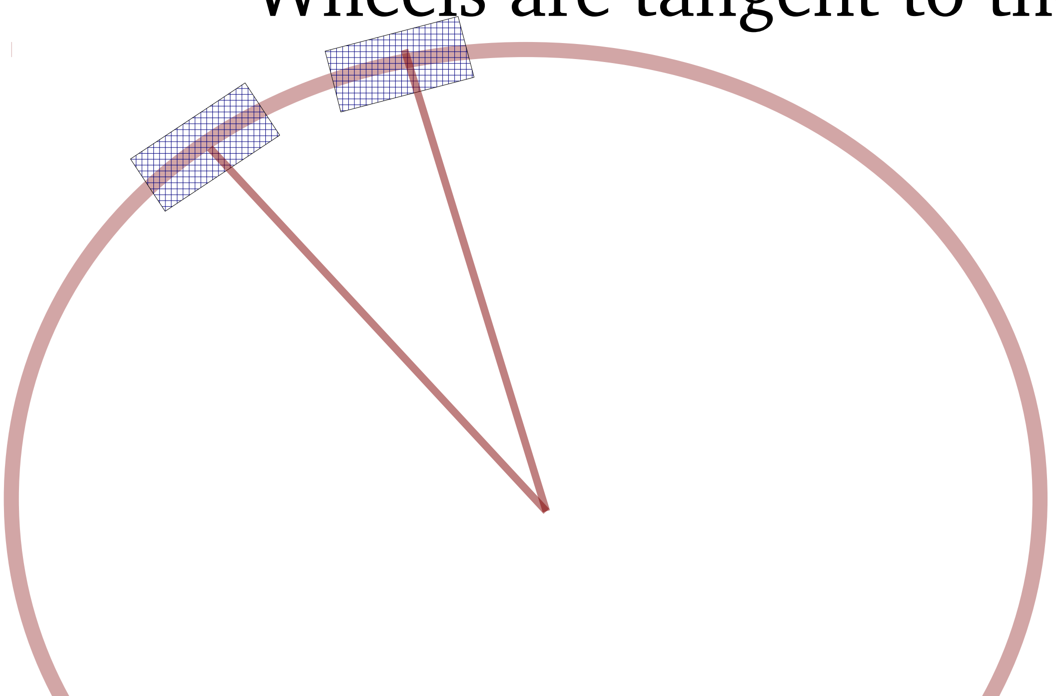
Solution: Ackermann steering.

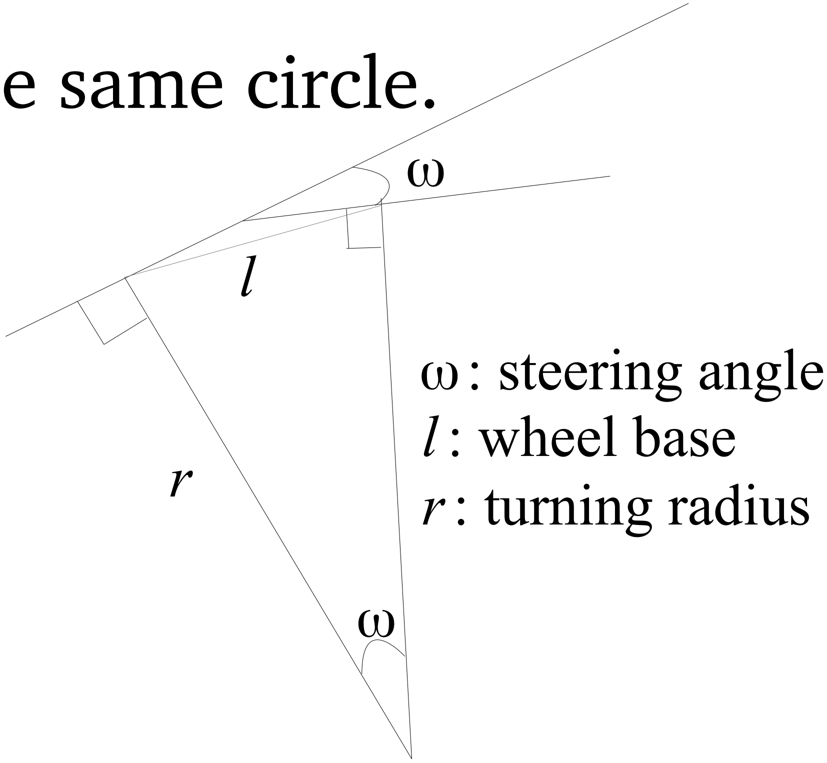




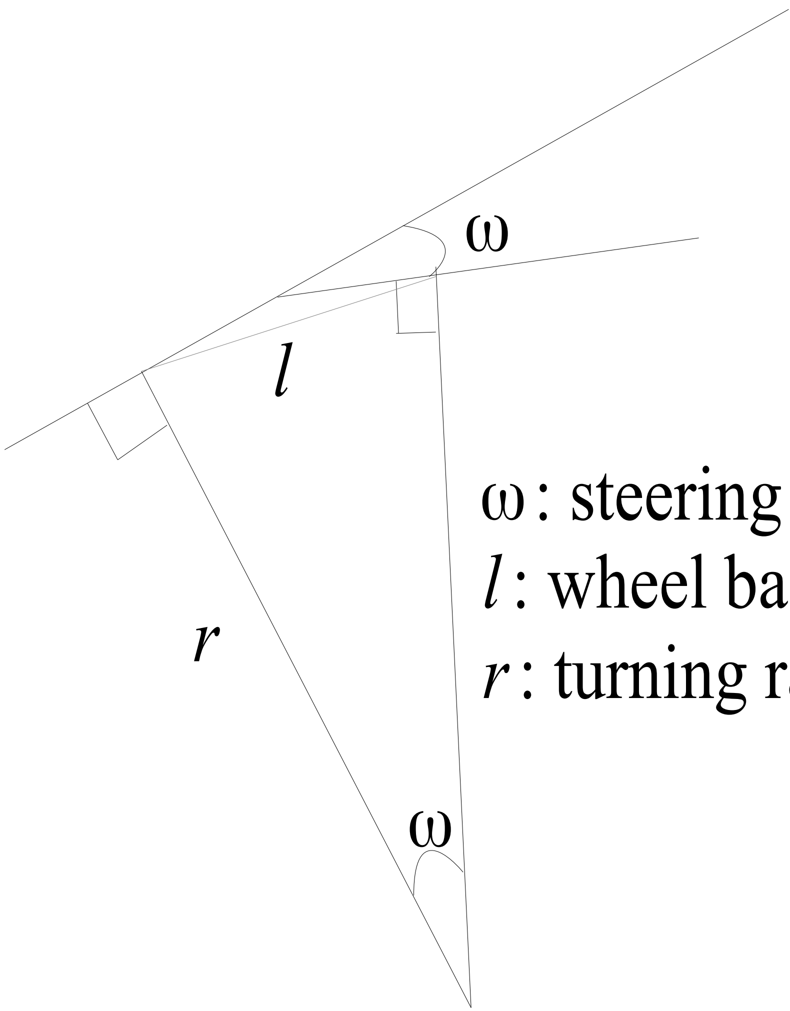
Common simplification with cars motion planning:

- Assumption 1:
  - 1 wheel in front, 1 wheel in back.
- Assumption 2:
  - Wheels are tangent to the same circle.





Car motion exercises



ω: steering angle
l: wheel base
r: turning radius

(1)
I set my steering angle ω to $\frac{\pi}{8}$.
My car has a wheel base of $0.25\,m$.
--- what is my turning radius r?

(2)
With my steering angle fixed,
I move the same car forward $0.5\,m$.
(car is moving on the arc of the circle)
--- What is the angle $\Delta\theta$ of the arc
that the car is moving through?

(3)
My car's intial orientation is $\theta_i = \frac{\pi}{2}$.
--- What is my car's new orientation θ_f?

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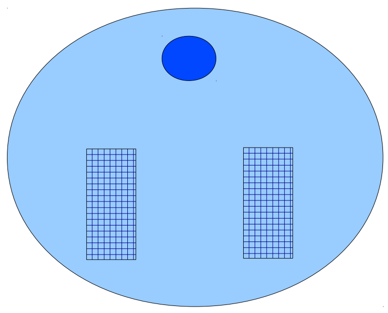
Differential Drive

- 2 wheels on common axis.
- Wheels rotate different speeds – differentially.
- Usually 3rd passive wheel (caster/omnidirectional) for balance.
- Can:
  - Rotate in-place.
  - Move forward or back.
  - Move on curve.



Differential Drive is simpler

$$\dot{\theta}\left(r+\frac{l}{2}\right)=\dot{x}_1$$

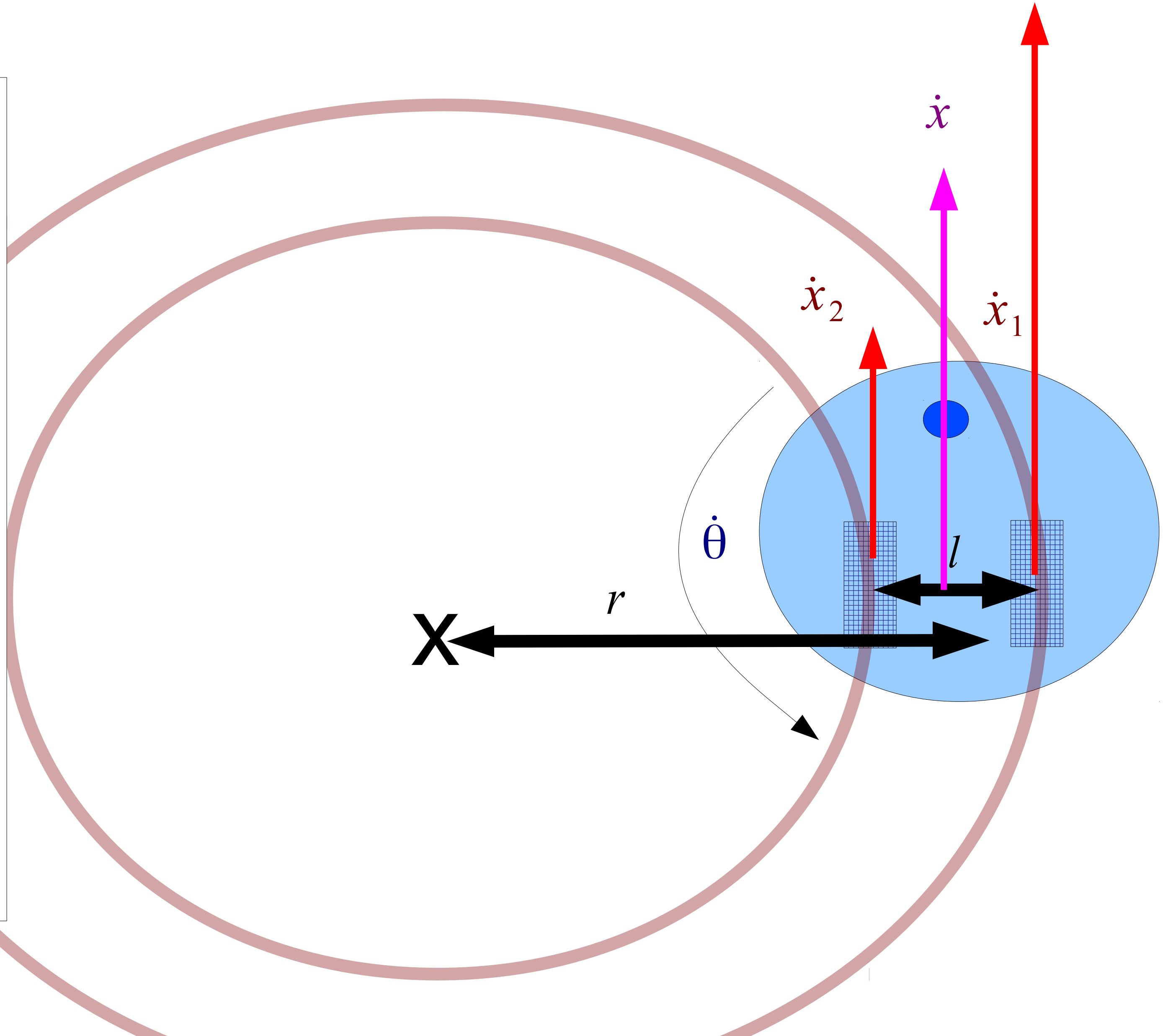
$$\dot{\theta}\left(r-\frac{l}{2}\right)=\dot{x}_2$$

$$\downarrow$$

$$\dot{\theta}=\frac{\dot{x}_1-\dot{x}_2}{l}$$

$$r=\frac{l}{2}\,\frac{\dot{x}_1+\dot{x}_2}{\dot{x}_2-\dot{x}_1}$$

$$\dot{x}=\frac{\dot{x}_1+\dot{x}_2}{2}$$



Odometry exercises (differential drive)

A large differential drive robot has wheels of 0.4m diameter (0.2m radius). The wheel encoder has 100 holes. The wheel base is 0.5m.

(1) The encoder in the left wheel measures 250 pulses, and the one in the right measures 500 pulses. How far has each wheel travelled?

(2) Assuming it took 0.5 seconds to travel this distance, what is the forward velocity of each wheel over this period?

(3) Using the equations on the right, determine the angular velocity and forward velocity of the robot.

(4) Draw the circle through which the robot is turning.

$$\dot{\theta}\left(r+\frac{l}{2}\right)=\dot{x}_1$$

$$\dot{\theta}\left(r-\frac{l}{2}\right)=\dot{x}_2$$

$$\downarrow$$

$$\dot{\theta}=\frac{\dot{x}_1-\dot{x}_2}{l}$$

$$r=\frac{l}{2}\,\frac{\dot{x}_1+\dot{x}_2}{\dot{x}_2-\dot{x}_1}$$

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Configuration space vs phase space

	Mobile robot	4-joint arm
Configuration: Location of all parts.	$\xi = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$	$\xi = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$
Phase state: Location & velocity of parts.	$\Xi = \begin{bmatrix} x \\ y \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$	$\Xi = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix}$
Workspace state: Location (sometimes pose) of effector.	$\chi = \begin{bmatrix} x \\ y \end{bmatrix}$	$\chi = \begin{bmatrix} x \\ y \end{bmatrix}$

Non-holonmicity and degrees of freedom

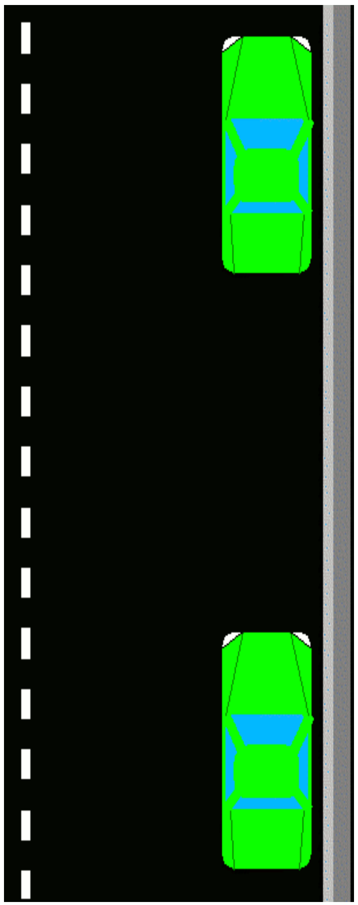
Degrees of Freedom (DOF):

Minimum number of parameters to describe body's configuration (e.g. x,y,θ).

Differentiable Degrees of Freedom (DDOF):

Minimum number of parameters that can be instantaneously changed (e.g. x,θ).

Non-holonmicity and degrees of freedom



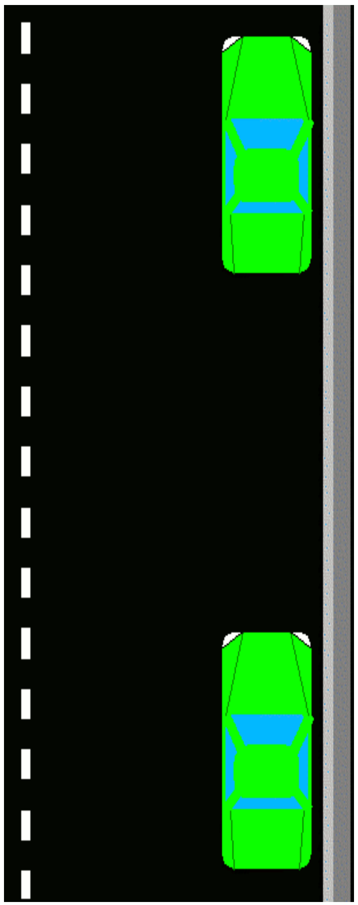
If Differentiable DOF (DDOF) < Total DOF:

- *non-holonomic robot*. e.g. Differential-drive robot, Car.

Else:

- *holonomic robot*. e.g. Swedish-wheel based robot.

Non-holonmicity and degrees of freedom



Non-holonomic constraint:

- Equality constraint.
- Applies to time-derivatives.
- Is non-integrable.

Holonomic constraint:

- Equality constraint.
- Applies to configurations.
- Derivative is integrable.

Manoeuvrability

Degree of manoeuvrability

=

DDOF + Steerability.

Only additive if degrees of freedom are mutually independent.

- Differential drive robot:
  - DDOF = 2.
  - Degree of steerability = 0.
- Basic car-type robot:
  - DDOF = 1.
  - Degree of steerability = 1.
- Omnidirectional (e.g. 3 Swedish wheels):
  - DDOF = 3.
  - Degree of steerability = 0.

Manoeuvrability = DDOF + Steerability

Robot	DOF	DDOF	Steerability	Manoeuvrability
Abstract car	3	1	1	2
Differential drive	3	2	0	2
Omnidirectional	3	3	0	3

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Differential-drive pose control with goal-centred coordinates

A simple control law:

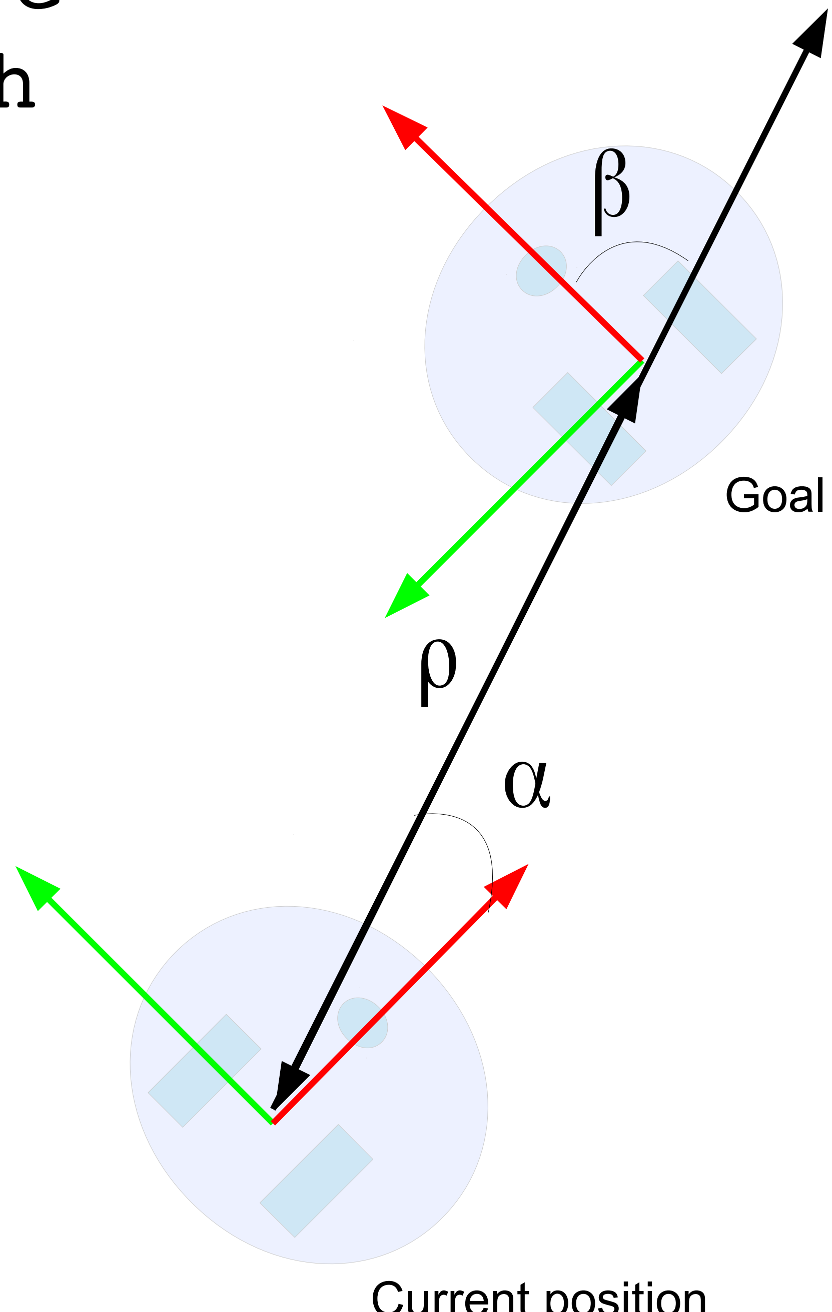
$$\dot{x}_r = k_\rho \rho$$

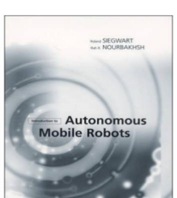
$$\dot{\theta}_r = k_\alpha \alpha + k_\beta \beta$$

<u>Question</u>: How to calculate

ρ, α, β?

Need $\Delta x, \Delta y, \Delta \theta$



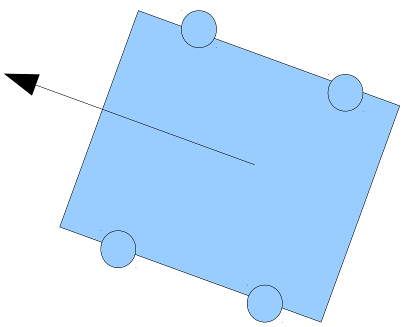


See Siegwart & Nourbakhsh section 3.6 for more details.

Review exercise

- A robot is at the following pose:

$$\begin{bmatrix} x_w \\ y_w \\ \theta_w \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ \pi/4 \end{bmatrix}$$



1) Draw a set of axes and a picture of the robot at the pose. Any picture is okay, but do indicate direction.

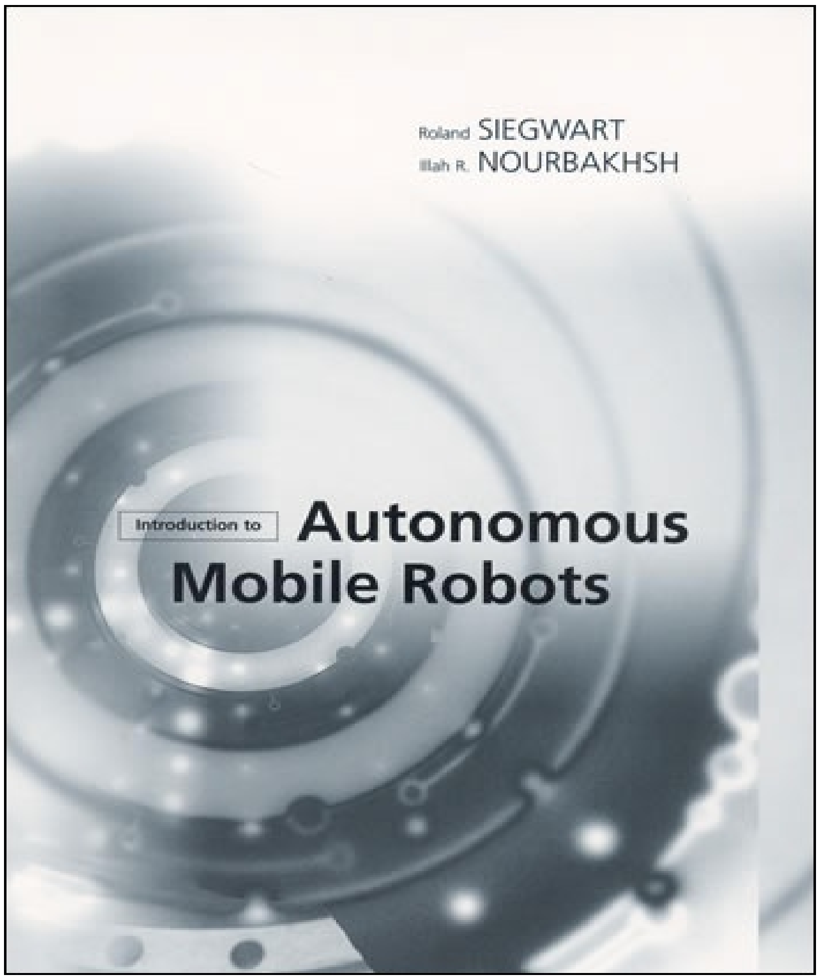
Questions: Holonomicity

- Are these robots holonomic or non-holonomic?
  - A differential drive robot?
  - A robot on Swedish / mecanum wheels?
  - A car-like robot?
  - A quadcopter robot?
- Are these constraints holonomic or non-holonomic?
  - A Swedish-wheeled robot blocked by wall?
  - A robot running on railway tracks?
  - A Swedish-wheeled robot blocked by a moving wall?
  - A Swedish-wheeled robot with an upper speed limit?

Review

- What is the difference between:
  - Differential drive/steering.
  - A differential.

Wheeled locomotion reading



Siegwart & Nourbakhsh.

Chapters 2 & 3.