BLG311E – FORMAL LANGUAGES AND AUTOMATA

2017 SPRING

RECITEMENT 4 (MIDTERM 1 SOLUTIONS)

Q1) (30p) Consider the following state transition table of machine M.

	1	2	3	4
A	F/1	-/-	H/-	E/0
B	E/0	E/1	H/1	D/-
C	-/1	H/1	D/0	D/1
D	A/1	-/1	G/0	C/1
E	E/0	B/1	A/1	C/1
F	A/1	A/0	F/-	-/0
G	A/1	-/1	C/0	C/1
H	A/1	H/0	H/0	E/0

a) Simplify machine M.

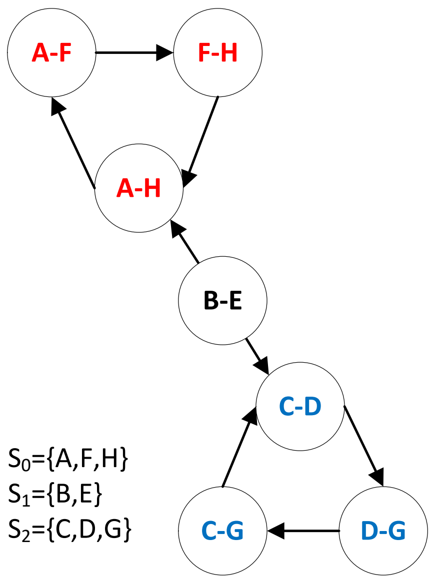
b) Draw the state diagram for the simplified machine (Me-aly model).

c) Transform the simplified machine into Moore model.

Solution:

a)

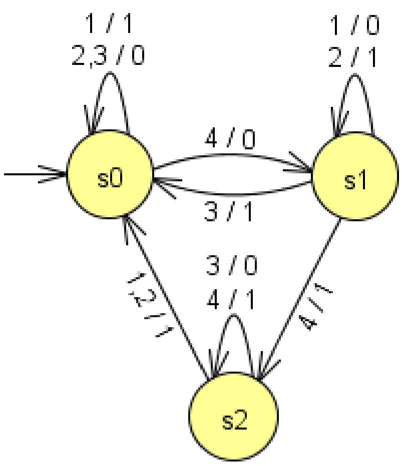
	A	B	C	D	E	F	G
B	X						
C	X	X					
D	X	X	D-G				
E	X	A-H C-D	X	X			
F	F-H	X	X	X	X		
G	X	X	C-D	C-G	X	X	
H	A-F	X	X	X	X	A-H	X



S_0={A,F,H}
S_1={B,E}
S_2={C,D,G}

	1	2	3	4
S_0	S_0/1	S_0/0	S_0/0	S_1/0
S_1	S_1/0	S_1/1	S_0/1	S_2/1
S_2	S_0/1	S_0/1	S_2/0	S_2/1

b)



c)

		1	2	3	4	Çıkış
S_0/0	Q_0	Q_1	Q_0	Q_0	Q_2	0
S_0/1	Q_1	Q_1	Q_0	Q_0	Q_2	1
S_1/0	Q_2	Q_2	Q_3	Q_1	Q_5	0
S_1/1	Q_3	Q_2	Q_3	Q_1	Q_5	1
S_2/0	Q_4	Q_1	Q_1	Q_4	Q_5	0
S_2/1	Q_5	Q_1	Q_1	Q_4	Q_5	1

Q2) (20p) Examine the erroneous proof steps given below. Let A,B and C be languages defined on the alphabet $\Sigma = \{a, b\}$.

$$A(B \cap C) = AB \cap AC$$

$$\forall z[(z = xy) \wedge (z \in AB \wedge z \in AC)] \tag{1}$$

$$xy \in AB \wedge xy \in AC \tag{2}$$

$$x \in A \wedge y \in B \wedge x \in A \wedge y \in C \tag{3}$$

$$x \in A \wedge y \in B \wedge y \in C \tag{4}$$

$$x \in A \wedge y \in B \cap C \tag{5}$$

$$xy \in A(B \cap C) \tag{6}$$

$$z \in A(B \cap C) \tag{7}$$

a) Which step transition contains the mistake?

b) Show the mistake on an example.

Solution:

a) (2) → (3) contains the mistake

b)
$$\left.\begin{array}{lll} z = abbabab & & \\ abb \in A & abab \in B & abab \notin C \\ abbab \in A & ab \notin B & ab \in C \end{array}\right\} \rightarrow \begin{array}{c} z = abbabab \ \in \ AB \cap AC \\ z = abbabab \ \notin \ A(B \cap C) \\ B \cap C = \{\Lambda\} \end{array}$$

Q3) (20p) Let α be a relation defined on $\Sigma = \{a, b, c\}$, and $t(\alpha)$ - $s(\alpha)$ be the transitive and symmetric closures of α respectively. Examine the erroneous expression given below.

$$\forall(x, y)[(x, y) \in ts(\alpha) \Leftrightarrow (x, y) \in st(\alpha)]$$

Show the mistake in this expression by providing an example for each of the 3 relations below. Afterwards, state the correct form of the expression.

- α:
- $ts(\alpha)$:
- $st(\alpha)$:

Solution:

$\boldsymbol{\alpha} = \{(\boldsymbol{a}, \boldsymbol{b}), (\boldsymbol{b}, \boldsymbol{c})\}$

$s(\alpha) = \{(a, b), (b, a), (b, c), (c, b)\}$

$\boldsymbol{ts}(\boldsymbol{\alpha}) = \{(\boldsymbol{a}, \boldsymbol{a}), (\boldsymbol{a}, \boldsymbol{b}), (\boldsymbol{a}, \boldsymbol{c}), (\boldsymbol{b}, \boldsymbol{a}), (\boldsymbol{b}, \boldsymbol{b}), (\boldsymbol{b}, \boldsymbol{c}), (\boldsymbol{c}, \boldsymbol{a}), (\boldsymbol{c}, \boldsymbol{b}), (\boldsymbol{c}, \boldsymbol{c})\}$

$t(\alpha) = \{(a, b), (b, c), (a, c)\}$

$\boldsymbol{st}(\boldsymbol{\alpha}) = \{(\boldsymbol{a}, \boldsymbol{b}), (\boldsymbol{b}, \boldsymbol{a}), (\boldsymbol{b}, \boldsymbol{c}), (\boldsymbol{c}, \boldsymbol{b}), (\boldsymbol{a}, \boldsymbol{c}), (\boldsymbol{c}, \boldsymbol{a})\}$

For example, $(a, a) \in ts(\alpha)$, $(a, a) \notin st(\alpha)$ shows that the given expression is wrong.

Corrected expression: $\forall(x, y)[(x, y) \in st(\alpha) \Rightarrow (x, y) \in ts(\alpha)]$

Q4) (30p) Consider the grammar G defined below. Let $L(G)$ be the language corresponding to this grammar G.

$$\begin{array}{c} S \rightarrow ACB \\ A \rightarrow aY \mid a \\ B \rightarrow Xa \mid a \\ X \rightarrow aYb \mid ab \\ Y \rightarrow bXa \mid ba \\ aCa \rightarrow Cba \mid cba \\ abC \rightarrow Cba \mid cba \end{array}$$

a) Which type does this grammar G belong to according to the Chomsky hierarchy?

b) Is it possible to produce abc and cba by using this grammar G? Draw the parsing sequence if it is possible for either of these strings.

c) Heuristically, find the simplest regular expression $L(G)$ for this grammar G.

d) Heuristically, find the simplest Type-3 grammar which is equivalent to this grammar G.

Solution:

a) Type-1

b) abc cannot be produced. cba can be produced as $S \rightarrow ACB \rightarrow aCB \rightarrow aCa \rightarrow cba$.

c) $X \rightarrow aYb \mid ab \rightarrow (ab)^+ \quad Y \rightarrow bXa \mid ba \rightarrow (ba)^+$

$A \rightarrow aY \mid a \rightarrow a(ba)^+ \vee a \quad B \rightarrow Xa \mid a \rightarrow (ab)^+a \vee a$

$$S \rightarrow ACB \rightarrow (a(ba)^+ \vee a)C((ab)^+a \vee a) \rightarrow \begin{array}{l} a(ba)^+C(ab)^+a \\ a(ba)^+Ca \\ aC(ab)^+a \\ aCa \end{array} \qquad \boxed{\begin{array}{l} aCa \rightarrow Cba \mid cba \\ abC \rightarrow Cba \mid cba \end{array}}$$

$$a(ba)^+C(ab)^+a \rightarrow \underbrace{\underbrace{\underbrace{(ab)^+Cba(ba)^+}_{(ab)^*Cbaba(ba)^+}}_{(ab)^*c(ba)^+ba(ba)^+} \vee (ab)^*cbaba(ba)^+}_{(ab)^*c(ba)^+ba(ba)^+} \vee (ab)^+cba(ba)^+$$

$$a(ba)^+Ca \rightarrow (ab)^+Cba \vee (ab)^+cba \rightarrow \underbrace{\underbrace{(ab)^*Cbaba}_{(ab)^*c(ba)^+ba} \vee (ab)^*cbaba}_{(ab)^*c(ba)^+ba} \vee (ab)^+cba$$

$aC(ab)^+a \rightarrow cba(ba)^+$

$aCa \rightarrow cba$

$$L(G) = \underbrace{\underbrace{\underbrace{(ab)^*c(ba)^+ba(ba)^+ \vee (ab)^*c(ba)^+ba}_{(ab)^*c(ba)^+ba(ba)^*}}_{(ab)^*c(ba)^+ba} \vee \underbrace{\underbrace{\underbrace{(ab)^+cba(ba)^+ \vee (ab)^+cba}_{(ab)^+cba(ba)^*} \vee \underbrace{cba(ba)^+ \vee cba}_{cba(ba)^*}}_{(ab)^*cba(ba)^*}}_{(ab)^*c(ba)^+}}_{(ab)^*c(ba)^+(ba \vee \Lambda)}$$

$L(G) = (ab)^*c(ba)^+$

d) $A \rightarrow abA \mid cB$

$B \rightarrow baB \mid ba$