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BLG202E CRN:21843 Homework 2

Q1)a)

Matlab Code:

```
>> syms x;
>> y(x)=x^4-x-10;
>> g(x)=10/(x^3-1);
>> error=diff(g,x);
>> i=0;
>> x0=2;
>> while (abs(y(x0))>(10^-4)) && (abs(error(x0))<1)
i=i+1;
x0=g(x0);
vpa(x0)
end
>> i
i =
     0
>> x0
x0 =
     2
>> vpa(abs(error(x0)))
ans =
2.4489795918367346938775510204082
```

It does not converge, therefore we cannot find a root via this function.

b)

Matlab Code:

```
>> syms x;
>> y(x)=x^4-x-10;
>> g(x)=(x+10)^(1/4);
>> error=diff(g,x);
>> i=0;
>> x0=2;
>> while (abs(y(x0))>(10^-4)) && (abs(error(x0))<1)
i=i+1;
```

```
x0=g(x0);
vpa(x0)
end

ans =

1.8612097182041991978824374939665



ans =

1.8558045970397769563351377355042



ans =

1.8555931396181665093258308444551



ans =

1.8555848655790224285625663784682

>> i

i =

     4

>> vpa(y(x0))

ans =

0.0000082740391440807632644659869008921
```

We find the root as 1.8555848655790224285625663784682 with 0.000008274039144080763264465986900892 1 error

C)

Matlab Code:

```
>> syms x;

>> y(x)=x^4-x-10;

>> g(x)=((x+10)^(1/2))/x;

>> error=diff(g,x);

>> i=0;

>> x0=1.8;

>> while (abs(y(x0))>(10^-3)) && (abs(error(x0))<1)

x0=vpa(g(x0));

i=i+1;

x0

end


x0 =


1.9083960041464075829778723767147



x0 =


1.8082485920442550027104075764334



x0 =


1.9003544323640786564065045579104



x0 =


1.8152871776991452120891954106111
```

x0 =

1.893550102827184477178912268804

x0 =

1.821289367723323050635330437299

x0 =

1.887789090100565387729242577206

x0 =

1.826404942680599839549370174487

x0 =

1.8829088595893103177274313098583

x0 =

1.8307628207399336907957044371097

x0 =

1.8787729112735388411964448419786

x0 =

1.8344737436710255851241601666824

x0 =

1.8752664120437583071606940435175

x0 =

1.8376326802399104459475967063766

x0 =

1.8722926145425613797284562132397

x0 =

1.840320957363649579495492122601

x0 =

1.8697699066579860416634800584327

x0 =

1.8426081361555956111799814928141

```
x0 =

1.8676293702986127481257936278787

x0 =

1.8445536536588776668108324488113

x0 =

1.8658127540709327795929447388865

x0 =

1.8462082525879283560339059741272

x0 =

1.8642707842671977520251584999462

x0 =

1.8476152214875090451432997866964

x0 =

1.8629617537840428857804448570299
```

x0 =

1.8488114668921501076428489784575

x0 =

1.8618503403955735100310029015985

x0 =

1.8498284376939050951274821731556

x0 =

1.8609066150441301828471940257381

x0 =

1.8506929202592449264999618528583

x0 =

1.8601052081049038702054477978147

x0 =

1.8514277210512606955635894766462

x0 =

1.8594246073821116062421218114989

x0 =

1.8520522517261359912014045929836

x0 =

1.8588465662431579896041441595826

x0 =

1.852583029955686127531622348807

x0 =

1.8583556040472012875583873913716

x0 =

1.8530341076234371873264443316518

x0 =

1.8579385840681698749711091596521

```
x0 =

1.85341743657250341813166776805l8

x0 =

1.8575843565963977464799860218786

x0 =

1.8537431807578854484477680272004

x0 =

1.857283456940742340133467353068

x0 =

1.8540199824732263880665366815702

x0 =

1.8570278497320937339086413652712

x0 =

1.8542551892763964212024989028187
```

x0 =

1.856810712318270525005076930693 9

x0 =

1.854455047320149374043096166308 4

x0 =

1.856626251193589673481624359001 3

x0 =

1.854624865992486136496897219815 8

x0 =

1.856469546366922658456118425741 4

x0 =

1.854769158074677753176588871325 6

x0 =

1.856336419374226630112671231616 7

x0 =

1.854891759021672270048096538700l

x0 =

1.856223321313078439346941982043

x0 =

1.854995928448903882102125494719

x0 =

1.8561272378400888102330495298778

x0 =

1.8550844364611947063755505499763

x0 =

1.8560456085455181847337438865998

x0 =

1.8551596370742444606262280459881

```
x0 =

1.8559762585179248410769616322715

x0 =

1.8552235306488347687349459870576

x0 =

1.8559173402475624040018246292797

x0 =

1.8552778169749470670318164681379

x0 =

1.8558672843006768455239516996316

x0 =

1.8553239404009934306560742714906

x0 =

1.8558247574362621841961386426692
```

x0 =

1.8553631281965826403751902023266

x0 =

1.8557886270392300645872568705202

x0 =

1.8553964231607174776402325708223

x0 =

1.8557579309151787478237318968349

x0 =

1.8554247113367291814544701170692

x0 =

1.8557318516369014353140353357689

x0 =

1.8554487455668704945900626304591

x0 =

1.8557096947555500680336603794181

x0 =

1.8554691655100929773292473316082

x0 =

1.855690870293409571771610186919

x0 =

1.8554865146533601938182079608304

x0 =

1.8556748770234329012738641710924

x0 =

1.8555012547675204252390580159667

x0 =

1.8556612891154767183308687519528

x0 =

1.8555137781912446561825374115481

x0 =

1.8556497447926168071493165252247

x0 =

1.8555244182690849948786712504103

x0 =

1.8556399366947468978729943964602

x0 =

1.8555334582208355110586271489501

x0 =

1.8556316036923409961038226768955

x0 =

1.8555411386778091595998626227849

```
x0 =

1.8556245239320278719331167447431

>> vpa(y(x0))

ans =

0.00098217939831089986542227545215 2

>> i

i =

    89

>>
```

D)

$g_1(x)$ does not converge but, $g_2(x)$ converges at step 4 with a appoximite error of 0.000008 and $g_3(x)$ converges at step 89 with a approximate error of 0.00098.

Q2)

$A = LU,\ \ I = A^{-1}LU$

$$A = \begin{bmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$U_{11} = 10$, $U_{12} = 2$, $U_{13} = -1$

$L_{21}U_{11} = -3$, $L_{21} = -0.3$, $L_{21}U_{12} + U_{22} = -6$, $-0.6 + U_{22} = -6$, $U_{22} = -5.4$, $L_{21}U_{13} + U_{23} = 2$, $U_{23} = 1.7$

$L_{31}U_{11} = 1$, $L_{31} = 0.1$, $L_{31}U_{12} + L_{32}U_{22} = 1$, $0.2 - 6L_{32} = 1$, $L_{32} = -0.1481$

$L_{31}U_{13} + L_{32}U_{23} + U_{33} = 5$, $-0.35228 + U_{33} = 5$, $U_{33} = 5.35228$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -0.3 & 1 & 0 \\ 0.1 & -0.1481 & 1 \end{bmatrix} \begin{bmatrix} 10 & 2 & -1 \\ 0 & -5.4 & 1.7 \\ 0 & 0 & 5.35228 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} b_{11} & b_{12} & b_{14} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$LU\begin{bmatrix}b_{11}\\b_{21}\\b_{31}\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix},\ U\begin{bmatrix}b_{11}\\b_{21}\\b_{31}\end{bmatrix}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix},\ L\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix},\ \begin{bmatrix}1&0&0\\-0.3&1&0\\0.1&-0.1481&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}1\\0.3\\-0.05557\end{bmatrix},\ \begin{bmatrix}10&2&-1\\0&-5.4&1.7\\0&0&5.35228\end{bmatrix}\begin{bmatrix}b_{11}\\b_{21}\\b_{31}\end{bmatrix}=\begin{bmatrix}1\\0.3\\-0.05557\end{bmatrix},\ \begin{bmatrix}b_{11}\\b_{21}\\b_{31}\end{bmatrix}=\begin{bmatrix}0.1107\\-0.0523\\-0.01038\end{bmatrix}$$

$$LU\begin{bmatrix}b_{12}\\b_{22}\\b_{32}\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix},\ U\begin{bmatrix}b_{12}\\b_{22}\\b_{32}\end{bmatrix}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix},\ L\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix},\ \begin{bmatrix}1&0&0\\-0.3&1&0\\0.1&-0.1481&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\1\\0.1481\end{bmatrix},\ \begin{bmatrix}10&2&-1\\0&-5.4&1.7\\0&0&5.35228\end{bmatrix}\begin{bmatrix}b_{12}\\b_{22}\\b_{32}\end{bmatrix}=\begin{bmatrix}0\\1\\0.1481\end{bmatrix},\ \begin{bmatrix}b_{12}\\b_{22}\\b_{32}\end{bmatrix}=\begin{bmatrix}-0.0325\\-0.1765\\0.02767\end{bmatrix}$$

$$LU\begin{bmatrix}b_{13}\\b_{23}\\b_{33}\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix},\ U\begin{bmatrix}b_{13}\\b_{23}\\b_{33}\end{bmatrix}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix},\ L\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix},\ \begin{bmatrix}1&0&0\\-0.3&1&0\\0.1&-0.1481&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix},\ \begin{bmatrix}10&2&-1\\0&-5.4&1.7\\0&0&5.35228\end{bmatrix}\begin{bmatrix}b_{13}\\b_{23}\\b_{33}\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix},\ \begin{bmatrix}b_{13}\\b_{23}\\b_{33}\end{bmatrix}=\begin{bmatrix}0.0069\\0.059\\0.187\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}0.1107&-0.0325&0.0069\\-0.0523&-0.1765&0.059\\-0.01038&0.02767&0.187\end{bmatrix}$$

$$Ax=\begin{bmatrix}27\\-61.5\\-21.5\end{bmatrix},\ \ x=A^{-1}\begin{bmatrix}27\\-61.5\\-21.5\end{bmatrix},\ \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0.46886\\7.73529\\-6.8408\end{bmatrix}$$

Q3)

$$A=U\Sigma V^T$$

$UU^T=I$, $VV^T=I$ (u and v diagonal) $\Sigma^T=\Sigma$ (it only has elements on diagonal.)

$U^T=U^{-1}$, $V^T=V^{-1}$

$$AA^T=V\Sigma^T U^T U\Sigma V^T$$

$$AA^T=V\Sigma^T I\Sigma V^T$$

$$AA^T=V\Sigma^2 V^T$$

$B=X\Lambda X^{-1}$ (Λ is eigen vector), therefore;

$\sigma_i=\sqrt{\lambda_i}$ where σ_i is the singular values of A and λ_i eigenvectors of A^TA.

Q4)

To get the smallest singular value σ_{k+1} k must be as large as possible. To act reasonable, getting k as 256 σ_{k+1} gets close enough to zero.

For $k=1$ $\quad \frac{\sigma_{k+1}}{\sigma_1}=0.0970 \quad \frac{((m+n).k)}{m.n}=0.02344$

For $k=10$ $\quad \frac{\sigma_{k+1}}{\sigma_1}=0.0479 \quad \frac{((m+n).k)}{m.n}=0.07812$

For $k=20$ $\quad \frac{\sigma_{k+1}}{\sigma_1}=0.0286 \quad \frac{((m+n).k)}{m.n}=0.156$

For $k=100$ $\quad \frac{\sigma_{k+1}}{\sigma_1}=0.0051 \quad \frac{((m+n).k)}{m.n}=0.7812$

Q5)

Matlab Code:

```
>> load('C:\Users\yunus\Desktop\2015-2016 Bahar\BLG202\HW2\A.mat')

>> B=A*transpose(A);

>> [V,D] = eig(B); %Get eigenvalues by eigenvalue decomposition

>> v0 = rand(500,1); %Create random v0

>> for k=1:500 %PIM Method

w=B*v0;

v=w/norm(w);

eigValue(k)=v'*B*v;

v0=v;

end

>>max(eigValue) %find dominant eigenvalue


ans =


          4464.70498441718
```

Power iteration method only found a range of eigenvalues with highest values. But eigenvalue decomposition find all the eigenvalues.