BLG 354E Homework - 1

Due 09.03.2018 23:59

Policy:

- Cheating is highly discouraged. It could mean a zero or negative grade. Please do your homework on your own. Team work is not allowed. Pattern of your solutions must be belong to only you.
- Prepare reports using LaTeX. Otherwise, you will get 0 point.
- After the deadline, your point will decrease with slope -30 according to the number of days past.
- You will get 50 points from completeness of your report and 50 points from selected 3 questions.
- In Problems 7, and 8, you will write code in Python 3.5+.
- Upload your solutions through Ninova.

For your questions: albay@itu.edu.tr

1. From signals and systems perspective, draw a simple block diagram for a system composed of a digital photograph camera and a projector. The system is capable of taking picture and projecting the image on a curtain. (Note: Read ch. 1 from your text book. You do not have to draw all the details, just give a rough diagram.)

2. Which of the followings represent a signal? Give reasoning for your answer.

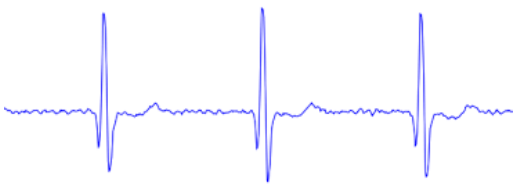

Figure 1: Heartbeat record

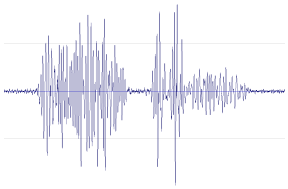

Figure 2: Voice record

Figure 3: An image

3. $z \in \mathbb{C}$, find all roots of $z^4 = j$.

4. Prove that $e^{j\theta} = \cos\theta + j\sin\theta$ using Taylor Expansion.

5. Give answer the followings

   (a) What is an odd function? Define an odd function and give an example.

   (b) What is an even function? Define an even function and give an example.

   (c) Match equalities.

a.	$\sin\theta$	_____	$\cos\theta$
b.	$\cos(\theta + 2\pi k)$	_____	$-\sin\theta$
c.	$\cos(-\theta)$	_____	1, when k is integer
d.	$\sin(-\theta)$	_____	0, when k is integer
e.	$\sin(\pi k)$	_____	$\cos\theta$, when k is integer
f.	$\cos(2\pi k)$	_____	-1, when k is integer
g.	$\cos[2\pi(k + 1/2)]$	_____	$\cos(\theta - \pi/2)$

(d) Give derivation of the following identities.

i. $\sin^2\theta + \cos^2\theta = 1$

ii. $cos(2\theta) = \cos^2\theta - \sin^2\theta$

iii. $sin(2\theta) = 2\sin\theta\cos\theta$

iv. $sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

v. $cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

6. Show that

$$\sum_{k=1}^{N} A_k \cos(\omega_0 t + \phi_k) = A\cos(\omega_0 t + \phi)$$

7. Let

$$z_1(t) = \frac{e^{j(\omega t - \frac{1}{3}\pi)} + e^{-j(\omega t - \frac{1}{3}\pi)}}{2}$$
$$z_2(t) = 3\sin(\omega t - \frac{5}{4}\pi)$$
$$z_3(t) = \mathrm{Re}\left\{2e^{j(\omega(t - \frac{4.7124}{\omega}))}\right\}$$

$x(t)$ is defined as follows:

$$x(t) = z_1(t) + z_2(t) + z_3(t)$$

(a) Express $x(t)$ in the form $x(t) = A\cos(\omega t + \phi)$ by finding the numerical values of A and ϕ. Use complex phasor manipulations to obtain the answer.

(b) Plot all the phasors used to solve the problem in (a) in the complex plane.

(c) Write a script that will plot the signal $x(t)$ using Python 3.5+. Please select suitable sampling space that makes the curve a faithful representation of the cosine function (Select suitable ω).

8. A signal is given by the equation

$$x(t) = 2 + 4\cos(500\pi t + \frac{5}{4}\pi) - 3\sin(60\pi t) + 3\cos(250\pi(t - 10^{-3}))$$

(a) Sketch the spectrum of this signal, indicating the complex size of each frequency component. Make separate plots for real/imaginary or magnitude/phase of the complex amplitudes at each frequency (Make plots using Python 3.5+).

(b) Is $x(t)$ periodic? If so, what is the period?

(c) What is the fundamental frequency of this signal? Which harmonics does $x(t)$ contain?

9. Let

$$\begin{aligned} y_1(t) &= 2\cos(10\pi t) \\ y_2(t) &= 7\sin(10000\pi t - \frac{1}{3}\pi) \\ x(t) &= y_1 y_2 \end{aligned}$$

(a) Find spectrum of $x(t)$ and express $x(t)$ as a sum of complex exponential signals.

(b) Plot the spectrum of this signal.

(c) Express $x(t)$ as a sum of two sinusoids, i.e., find the numbers A_1, A_2, ω_1, ω_2, ϕ_1 and ϕ_2 such that:

$$x(t) = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2)$$

10. What is the orthogonality of functions? Show that the following functions are orthogonal.

(a) $\sin(2\pi nft)$ and $\sin(2\pi mft)$ on $-L \leq t \leq L, n \neq m$ and m, n integer

(b) $\cos(2\pi nft)$ and $\cos(2\pi mft)$ on $-L \leq t \leq L, n \neq m$ and m, n integer

(c) $\sin(2\pi nft)$ and $\cos(2\pi mft)$ on $-L \leq t \leq L, m$ and n integer

11. What is the Gibbs phenomenon? Why does it occurs? Explain in detail.

12. (Will not be graded.) Please solve questions that is shown below from your textbook for your own good.

(a) From ch. 2: P-2.1, P-2.4, P-2.5, P-2.7, P-2.12, P-2.15, P-2.18

(b) From ch. 3: P-3.1, P-3.4, P-3.11, P-3.14, P-3.15, P-3.17