MAT281E Linear Algebra and Applications HW 3

Instructions: Turn in your solutions (hardcopy) to first 12 problems no later than November 11th, 2015 16:00. You may turn in the solutions to the remaining problems until November 17th, 2015 16:00. (Use the mailbox reserved for the course in the administrative office of the Computer and Informatics faculty). Late homeworks will not be accepted. 4-5 problems will be checked in detail which will contribute 80% to the final mark. The rest will be checked for completeness which will contribute 20% to the final mark.

1.Evaluate the determinant of the following matrix by cofactor expansion rows or columns of your choice. Explain your reasoning for choosing the rows or columns. $\underline{\underline{A}}_{4x4} = \begin{bmatrix} 2 & -2 & 0 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 0 & 2 & -1 \end{bmatrix}$

2. Use row reduction along with cofactor expansion to compute the determinant of

$$\underline{\underline{A}}_{4x4} = \begin{bmatrix} 2 & 1 & 4 & 1 \\ 2 & 2 & 12 & 5 \\ 1 & 1 & 6 & 1 \\ 3 & 3 & 18 & -1 \end{bmatrix}$$

3. Evaluate $\underline{\underline{A}}^{-1}$ by the method of adjoints where $\underline{\underline{A}}_{3x3} = \begin{bmatrix} 1 & 4 & 1 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

4.Let

$$\begin{aligned} &2x_1 + 4x_3 + 6x_4 = 2 \\ &x_1 + 2x_3 + 6x_5 = 1 \\ &3x_1 + -2x_2 + 4x_3 - x_4 - 5x_5 = 2 \\ &x_1 + 4x_2 + 4x_3 + x_4 - x_5 = 2 \\ &2x_1 - x_2 + 4x_3 + 4x_4 = 0 \end{aligned}$$

Determine x_1, x_3 by Cramer's method.

5. Reduce following the matrix to upper triangular form. $\underline{\underline{A}}_{3x3} = \begin{bmatrix} 2 & -2 & 1 \\ 1 & x & 5 \\ 0 & 1 & -2 \end{bmatrix}$. For what value of x is the following matrix noninvertible?

6. Let $\underline{\underline{A}}_{4x4} = \begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 2 & 2 & -3 \\ 0 & 0 & 6 & 11 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ Determine $\underline{\underline{A}}^{-1}$. Which elements of $\underline{\underline{A}}^{-1}$ require almost no computation? (Hint: Do you expect $\underline{\underline{A}}^{-1}$ to be triangular?)

7. Evaluate determinant of $\underline{\underline{A}}_{4x4} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 8 & 1 \\ 1 & 0 & -1 & 1 \\ 3 & 3 & 12 & -1 \end{bmatrix}$ by row reduction only. Reduce the matrix to row echelon form.

8. Let $\underline{\underline{A}}_{3x3} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & -8 \\ 0 & 2 & 2 \end{bmatrix}$. Determine $\det((\underline{\underline{A}}^T)^5)$.

9. By inspection explain why the following matrix is not invertible. Do not try to compute the inverse!

$$\underline{\underline{A}}_{4x4} = \begin{bmatrix} -1 & 1 & 2 & 2 \\ 1 & 1 & 2 & 0 \\ 1 & -1 & -2 & -2 \\ 0 & 2 & 4 & 2 \end{bmatrix}$$

10. Use properties of determinant to determine $\det(\underline{\underline{A}}\underline{\underline{B}}^{-1}\underline{\underline{A}}^{-1})$ if $\det(\underline{\underline{A}}) = 1$, $\det(\underline{\underline{B}}) = -2$?

11. Prove the following by applying elementary row or column operations

$$\begin{vmatrix} a & b & c & d \\ 2a & b & 2c & 2d \\ 0 & b & 2c & 2d \\ -2a & 0 & 0 & 0 \end{vmatrix} = 0$$

12. Determine $\begin{vmatrix} a+b & c \\ d+e & f \end{vmatrix}$ in terms of the determinants of $\begin{bmatrix} a & c \\ d & f \end{bmatrix}, \begin{bmatrix} b & c \\ e & f \end{bmatrix}$.

13. Let $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$ be two vectors from the origin to points P_1 and P_2. Let point Q be the midpoint of the line between P_1 and P_2. Show that $\overrightarrow{QO}$, $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$ are coplanar. (You can try to show that the projection of $\overrightarrow{QO}$ onto the plane defined by $\overrightarrow{OP_1}$ and $\overrightarrow{OP_2}$ is the same as $\overrightarrow{QO}$. Alternatively , you can try to show that $\overrightarrow{OP_1} \text{ x } \overrightarrow{OP_2}$ is orthogonal to $\overrightarrow{QO}$(evaluate a 3x3 determinant).)

14. 16. Setup a linear system and solve to get the equation of a line that contains points $\underline{x} = (2,2,0)$, $\underline{y} = (2,1,-1)$ and $\underline{z} = (-1,0,1)$.

15. In $\Re^n$ use triangular inequality for two vectors to prove or disprove $\|\underline{u} + \underline{v} + \underline{w}\| \le \|\underline{u}\| + \|\underline{v}\| + \|\underline{w}\|$? Demonstrate your answer in $\Re^2$.

16. Construct two examples in $\Re^2$ to demonstrate $|\,\|\underline{u}\| - \|\underline{v}\|\,| \le \|\underline{u} - \underline{v}\|$ with equality and inequality?

17. Find the orthogonal projection of line with direction vector $\underline{u} = (1,1,1)$ onto the plane described by equation $x - 2y + z = -2$.

18. Find a unit norm vector that is orthogonal to both $(1,0,-1)$ and $(1,2,1)$.

19. Find the equation describing the set of all points $P \in \Re^3$ such that the vector $\overrightarrow{P_0P}$ is orthogonal to the vector $\underline{v} = (1,-1,2)$ where P_0 is the origin. What is this set?

20. Find equation of all points $P \in \Re^3$ such that the vector $\overrightarrow{P_0P}$ is parallel to the vector $\underline{v} = (1,-1,2)$ where $P_0 = (0,1,2)$. What is this the equation of?

21. Given the coordinates of its three corners, describe a method to determine the largest interior angle of a triangle.

22. Two triangles (T1 corners: (1,1,-1),(-1,1,2),(0,0,0)) and (T2 corners: (1,1,-1),(-1,1,2),(2,2,2)) share an edge in $\Re^3$. Describe and apply a method to determine the dihedral angle between the two triangles. Dihedral angle is the angle between the planes of the triangles.

23. Determine the distance between the point (0,1) and the line 2x+y=1 in $\Re^2$ using the method of orthogonal projections.

24. Determine the distance between the point (0,1,-1) and the line x-y+z=-1 in $\Re^3$ using the method of orthogonal projections.

25. If $(\underline{u}+\underline{v}) \perp (\underline{u}-\underline{v})$ what can you say about the norms of the two vectors $\underline{u}$ and $\underline{v}$?. Assume that the vector dimension is arbitrary.

26. Let $\underline{u}=(u_1,u_2)$ and $\underline{v}=(v_1,v_2)$ be two vectors that form two sides of a triangle. Use the determinant to write down the area of the parallelogram in terms of u_i and v_j.

27. Let vectors (1,1,1), (1,1,-1) and (1,-1,1) form the three edges of a parallelpiped. Determine the volume of the parallelepiped by using the determinant.