MAT281E Linear Algebra and Applications Homework 5 – Due December 30, 2014

Turn in your solutions (hardcopy) no later than December 30th, 2014 16:00. (Use the mailbox reserved for the course in the administrative office of the Computer and Informatics faculty). Late homeworks will not be accepted.

Late submissions will not be graded.

1. Is there linear dependence among a set of 5 vectors in $\Re^6$ that form a pentagon when end-to-end connected. Explain.

2. For each of the following sets explain whether or not the set is/could be a basis for the space mentioned.

a) Five 2x3 matrices for the space of 2x3 matrices.

b) $(1, y)$ and $(1, -y)$ for any $y \neq 0$ in $\Re^2$.

c) $(1\ 2\ 1)$, $(1\ 1\ 0)$, $(-1\ 1\ 2)$ in $\Re^3$.

3. Find the coordinates of $\underline{w}$ relative to the basis $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$.

$\underline{w} = (1\ \ 1\ \ \text{-}1)$, $\underline{v}_1 = (2\ \ 0\ \ 1)$, $\underline{v}_2 = (0\ \ 1\ \ \text{-}1)$, $\underline{v}_3 = (1\ \ \text{-}3\ \ 4)$

4. Let $\underline{w} = (\text{-}1\ 1\ \ 2)$, $\underline{v}_1 = (2\ \ \text{-}1\ \ \text{-}1)$, $\underline{v}_2 = (0\ \ 1\ \ 3)$, $\underline{v}_3 = (1\ \ \text{-}1\ \ \text{-}1)$, $\underline{v}_4 = (1\ \ 0\ \ 2)$, $\underline{v}_5 = (1\ \ 2\ \ 4)$

a) Do the vectors $\{\underline{v}_1, \underline{v}_2, \underline{v}_3, \underline{v}_4, \underline{v}_5\}$ form a linearly independent set?
b) What is the dimension of the space spanned by $\{\underline{v}_1, \underline{v}_2, \underline{v}_3, \underline{v}_4, \underline{v}_5\}$?
c) Determine a suitable subset of $\{\underline{v}_1, \underline{v}_2, \underline{v}_3, \underline{v}_4, \underline{v}_5\}$ as a basis for the space spanned by $\{\underline{v}_1, \underline{v}_2, \underline{v}_3, \underline{v}_4, \underline{v}_5\}$.
d) Is vector $\underline{w}$ in the span of $\{\underline{v}_1, \underline{v}_2, \underline{v}_3, \underline{v}_4, \underline{v}_5\}$?

5. Can you determine a basis for the plane $x - 2y + z = 2$ that spans only the plane?

6. What is the dimension of the subspace of $\Re^4$ described with all vectors of the form (w, x, y, z) that satisfy

$$-x + z = 0,\ x + y - 2z - w = 0, -x + y - w = 0$$

7. Determine a basis from the following set of second degree polynomials. Does this basis span the space of the second degree polynomials? What is the dimension of the (sub)space that it spans?

$$p_1(x) = 4x + 1,\ p_2(x) = x^2 - 2x + 1, p_3(x) = x + 1, p_4(x) = x^2 - x + 1$$

(Hint: Use the standard basis for the space of second degree polynomials. Work with coordinate vectors written relative to this basis)

8.What is the dimension of the columnspace of $\underline{\underline{A}} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$? Is there a solution to the general equation $\underline{\underline{A}}x = \underline{b}$ for any given $\underline{b}$?

9. Let $\underline{\underline{A}} = \begin{bmatrix} 1 & -6 & 1 & -2 \\ -1 & -2 & 6 & -6 \\ 2 & -2 & -6 & 6 \end{bmatrix}$.

a) Is $\underline{b} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$ in the columnspace of the matrix $\underline{\underline{A}}$. If so write it as a linear combination of the column vectors of this matrix.

b) What is the rank of $\underline{\underline{A}}$?

c) What is the dimension of the nullspace (nullity) of the matrix?

d) Write down the homogeneous solution and the general solution for $\underline{b} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$?

10. Reduce $\underline{\underline{A}}$ to a row-echelon form. Appropriately select the rows of the row-reduced matrix and the columns of $\underline{\underline{A}}$ to determine bases for the rowspace and columnspace of $\underline{\underline{A}}$, respectively.

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 2 & 2 & 2 & 2 \\ 2 & 1 & 3 & 0 \end{bmatrix}$$

11. By making use of row-reduction, appropriately select the rows and columns of $\underline{\underline{A}}$ to determine bases for the rowspace and columnspace of $\underline{\underline{A}}$, respectively.

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 4 \\ 1 & 0 & -2 \\ 0 & 2 & 5 \end{bmatrix}$$

12. If $\underline{\underline{E}}$ is an elementary matrix, do $\underline{\underline{A}}$ and $\underline{\underline{EA}}$ have the same columnspace? Explain. (Think of $\underline{\underline{A}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \end{bmatrix}$)

13. Find all 3x3 matrices for which the homogeneous system has a solution space as the line $x = 2t$, $y = -\frac{t}{2}$, $z = 0$. (Hint: Write the row reduced augmented matrix from given information.)What is the rank in this case?

14. What conditions must be satisfied for the overdetermined system below to be consistent? Do not try to solve for the solution naively. Apply row reduction. If the system is consistent does a unique solution exist?

$$\begin{aligned} &x_1 + x_2 - x_3 = b_1 \\ &2x_1 - x_2 + x_3 = b_2 \\ &-x_1 + 2x_2 + x_3 = b_3 \\ &x_1 - 2x_4 = b_4 \end{aligned}$$

15. We have a 4x6 matrix $\underline{\underline{A}}$ with maximum rank. What is the number of free variables in the solution to the system $\underline{\underline{A}}\underline{x} = \underline{0}$? For a given $\underline{b}$, are we guaranteed to have a solution to $\underline{\underline{A}}\underline{x} = \underline{b}$? If we have a solution, what is the dimension of the solution space? Explain without using an example.

16. For a 4x3 matrix, if the nullspace is a line through the origin what can you say about the row space?

17. Describe the columnspace and rowspace of matrix $\underline{\underline{A}} = \begin{bmatrix} 4 & 2 \\ 2 & 1 \\ 3 & t \end{bmatrix}$ for all possible t. Is the solution space of $\underline{\underline{A}}\underline{x} = \underline{b}$ for a given $\underline{b}$ in the span of the column vectors the same for all t? (Hint: Apply row reduction)

18. Find the characteristic equation, eigenvalues, eigenvectors and the bases for the eigenspaces of the following matrices. For each eigenvalue also find the rank of $\lambda\underline{\underline{I}} - \underline{\underline{A}}$:

a) $\underline{\underline{A}} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

b) $\underline{\underline{A}} = \begin{bmatrix} 3 & -2 & 2 \\ -2 & 3 & 0 \\ 2 & 1 & 1 \end{bmatrix}$

c) $\underline{\underline{A}} = \begin{bmatrix} 1 & -1 & -2 \\ -1 & 2 & 5 \\ 0 & 1 & 3 \end{bmatrix}$

19. Find the eigenvalues and eigenvectors of $\underline{\underline{A}}^{11}$ for matrix in part b) of problem 18. How many eigenspaces does it have? What is the dimension of each eigenspace and what are the bases vectors for it?

20. For a square matrix, is there always as many eigenvectors as the matrix dimension. Explain.

21. Let the characteristic polynomial of a matrix be $p(\lambda) = \lambda^2 - 9\lambda$.

a) What are the dimensions of the matrix?

b) What is the determinant of the matrix?

c) Suppose the matrix is invertible. What is the characteristic polynomial for the inverse matrix? Refer to exercise 20 of 7.1 in your textbook.

d) What is the characteristic polynomial of the transpose of the matrix? Explain your reasoning

22. Let $\underline{\underline{A}} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix}$

Is the matrix $\underline{\underline{A}}$ diagonalizable? If so find a matrix $\underline{\underline{P}}$ that diagonalizes $\underline{\underline{A}}$ and apply the diagonalization procedure to get a diagonal matrix.

23. Let $\underline{\underline{A}} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

Is the matrix $\underline{\underline{A}}$ diagonalizable? If so find a matrix $\underline{\underline{P}}$ that diagonalizes $\underline{\underline{A}}$ and apply the diagonalization procedure to get a diagonal matrix.

24. Does $\underline{\underline{A}} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ have a zero eigenvalue? Use the determinant to get/justify your answer.

25. Are the matrices $\underline{\underline{A}} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ and $\underline{\underline{B}} = \begin{bmatrix} -4 & 2 \\ 2 & 4 \end{bmatrix}$ orthogonally similar? If yes, determine the orthogonal matrix.

MATLAB PROBLEM on matrix similarity: Generate a random 4x4 matrix A and an invertible 4x4 matrix P and then confirm that $P^{-1}AP$ and A have the same: (i) determinant; (ii) rank; (iii) nullity; (iv) trace; (v) characteristic polynomial; (vi) eigenvalues.