P-value Approach

Among patients with lung cancer, usually 80% or more die within three years. As a result of new forms of treatment, it is felt that this rate has been reduced. In a recent study of $n=150$ lung cancer patients, $y=128$ died within 3 years.

Is there sufficient evidence at the $\alpha = 0,05$ level, say to conclude that the death rate due to lung cancer has been reduced?

a)

$\hat{p}$: sample proportion

$$\hat{p} = \frac{128}{150} = 0,853$$

Hypotheses $\Rightarrow$ $H_0 \Rightarrow p = 0,90$ / $p \geq 0,90$

$H_1 \Rightarrow p < 0,90$

Test statistic $\Rightarrow$

$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0,853 - 0,90}{\sqrt{\frac{0,90\,(1-0,90)}{150}}} = \boxed{-1,92}$$

$\alpha = 0,05 \Rightarrow$ %95 C.I.

$\cong 0,95 - 0,5 = 0,45$

$z_{\alpha=0,05} = -1,645$

α $z_0 < z_{\alpha=0,05} \Rightarrow$ Reject H_0. So, there is sufficient evidence at the $\alpha = 0,05$ level to conclude that the rate has been reduced

* one-tailed hypothesis test in which the alternative hypothesis involved either a less than (<) or a greater than (>) sign.

b) What if we set the significance level $\alpha = 0,01$? Is there still sufficient evidence to conclude that the death rate due to lung cancer has been reduced?

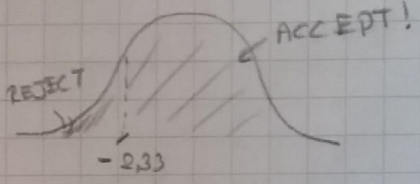
$$z_0 = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0,853 - 0,90}{\sqrt{\frac{0,90(1-0,90)}{150}}} = -1,82$$

$\alpha = 0,01 \Rightarrow$ %99 C.I. Find this value in the Z-table

$0,99 - 0,5 = 0,49$

$z_{\alpha=0,01} = -2,33$

$z_{\alpha=0,01} < z_0 \Rightarrow$ Accept the hypothesis!

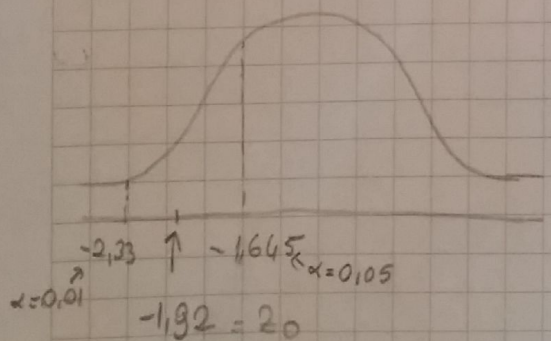


So, There is no any sufficient evidence at the $\alpha = 0,01$ level to conclude that the rate has been reduced.

c) In the 1. part of the example (a), we rejected null hypothesis when $\alpha = 0,05$. And, in the second part, we accepted the null hypothesis when $\alpha = 0,01$.

So, there must be some level of α, in which we cross threshold from rejecting to not rejecting the null hypothesis.

What is the smallest α-level that would still cause us to reject the null hypothesis?



That is, we would reject if the critical value were -1,645, -1,83, -1,82

but, we would not reject if the C.V. were -1,83.

The α-level associated with the test statistic -1,82 is called the p-value.
It is the smallest α-level that would lead to rejection.
In this case, P-value is;

$$P(z < -1,82) = 0,0274$$

$$1 - p(z) -1,82) = 1 - 0,9726 = 0,0274$$