MAT281E Linear Algebra and Applications HW 4

Instructions: Turn in your solutions (hardcopy) no later than December 9th, 2015 16:00. (Use the mailbox reserved for the course in the administrative office of the Computer and Informatics faculty). Late homeworks will not be accepted. 5-6 problems will be checked in detail which will contribute 80% to the final mark. The rest will be checked for completeness which will contribute 20% to the final mark.

1. Find a vector orthogonal to the plane spanned by $\underline{u} = (1\ 0\ 1)$ and $\underline{v} = (1\ 2\ 3)$.
2. Find all unit vectors in the intersection of the plane P1 determined by vectors $\underline{u} = (0\ 1\ 1)$ and $\underline{v} = (2\ \text{-}1\ 3)$ and P2 determined by vectors $\underline{w} = (5\ 7\ \text{-}4)$ and $\underline{z} = (1\ 2\ \text{-}1)$.
3. Find equation(s) of the plane that contains the point $P = (1\ \text{-}1\ 1)$ and is perpendicular to the plane $x + y + 2z = 1$
4. Determine whether the line described by $x = 1 + 2t, y = -t, z = 1 - t$ and the plane $x + 2y = 3$ are perpendicular or paralel or neither.
5. Find parametric equations for the line of intersection of the planes with normal vectors $\underline{n}_1 = (5\ 1\ \text{-}2)$ and $\underline{n}_2 = (5\ 1\ \text{-}2)$ that go through the point $(1\ 2\ 1)$.
6. Determine whether the lines $r_1 = 1 - 2t, r_2 = 1 - t, r_3 = 5 - 2t$ and $r_1 = 3s, r_2 = 1 + s, r_3 = 1 + 2s$ intersect in $\Re^3$? (Hint: can you solve for the parameters at the point of intersection? Is the linear system consistent?)
7. Verify the Cauchy Scwartz inequality for the following vectors $\underline{u} = (1\ 1\ \text{-}1\ 1)$ and $\underline{v} = (2\ 1\ 1\ 0)$
8. Is the set of all pairs of real numbers of the form (x, y) a vector space if
   a. $x = -y$
   b. $x \geq 0$
   c. $x + y = 1$
9. Is the set of all 2x2 matrices of the form $\begin{bmatrix} \mathrm{a}+1 & \mathrm{a}+2 \\ \mathrm{a}+3 & \mathrm{a}+4 \end{bmatrix}$ a vector space? (Assume that matrices are added and scalar multiplied in the usual way.)Explain.
10. Is the set of all real numbers with the following definitions of addition and scalar multiplication a vector space. Explain why or why not.
    a. $x + y \equiv +\sqrt{x^2 + y^2}$, $kx \equiv +\sqrt{k}x$
    b. (Hint: Discuss the existence of a negative.)
11. Do the set of singular matrices with dimension nxn form a subspace of the set of matrices with dimension nxn? Do the set of matrices with dimension nxn whose trace is zero form a subspace of the set of matrices with dimension nxn.
12. Do the set of nxn triangular matrices form a subspace of the space of nxn matrices? Do the set of nxn non-triangular matrices form a subspace of the space of nxn matrices?
13. For each part below describe the solution space (subspace) of the homogeneous system
    a. $\underline{\underline{A}}\underline{x} = \underline{0}$
    b. a) $\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ 1 & -1 & 3 \end{bmatrix}$ b) $\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix}$ c) $\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ 0 & 1 & 8 \end{bmatrix}$ d) $\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ -1 & 0 & 2 \end{bmatrix}$
    c. Specifically find equation(s) that describe the subspace
14. Do the vectors $\underline{v}_1 = (1\ 0\ 1), \underline{v}_2 = (2\ 1\ 1), \underline{v}_3 = (0\ \text{-}3\ 3),$

(a) span $\Re^3$. Explain.

(b) span a subspace that includes $\underline{w} = (1\ \text{-}1\ 2)$. Explain.

15. Are the vectors $\underline{v}_i$ in prob.7 linearly independent? Solve a linear system by row reduction to show your answer.

16. . Can you determine *a* such that scalars c_1, c_2, c_3 exist that satisfy

$$c_1(1\ 0\ 2\ 0) + c_1(1\ \text{-}2\ \text{-}2\ 0) + c_1(0\ 1\ 1\ \text{-}2) = (\text{a}\ \text{-}1\ 3\ 1).$$

Check condition for consistency.

17. Is the following transformation linear? Explain.

$$\begin{aligned} w_1 &= x_1 + 2x_2 - x_3 \\ w_2 &= x_1 + x_2 - x_3 \\ w_3 &= x_1 + 1 \end{aligned}$$

18. Find the domain, codomain, range and the standard matrix for the linear transformation defined by the equations

$$\begin{aligned} w_1 &= 2x_1 + 3x_2 - x_3 \\ w_2 &= x_1 - 3x_2 - x_3 \\ w_3 &= x_2 \end{aligned}$$

(Hint: To determine the range apply row reduction on the linear system and read the constraint on w_i in the row echelon form)

19. Use matrix multiplication to find
    a. The reflection of $(1\ 1\ \text{-}3)$ about the y-axis
    b. The orthogonal projection of $(1\ 1\ 0)$ onto the xy plane
    c. The orthogonal projection of $(1\ 1\ 0)$ onto the z axis
    d. The rotation of $(1\ \text{-}2\ 3)$ around the z-axis by 30° clockwise
    e. The rotation of $(1\ \text{-}2\ 3)$ around the z-axis by 30° clockwise followed by contraction with factor 2, followed by projection onto $x = 0$ plane. What is the standard matrix for the stated composition?
    f. The orthogonal projection of $(1\ \text{-}2\ 3)$ onto the z-axis followed by dilation with factor 2, followed by rotation around x axis by 90°. What is the Standard matrix for the stated composition?

20. Let T_1 be the reflection with respect to the xz plane in $\Re^3$, T_2 be the rotation around the x-axis by θ. Is it true that $T_1 \circ T_2 = T_2 \circ T_1$? Explain.

21. Let $T(\underline{x}) = proj_{\underline{u}}\, \underline{x}$. Determine the dot product $T(\underline{x}) \cdot (\underline{x} - T(\underline{x}))$

22. For each part below:Is the linear transformation given below one-to-one? What is the inverse hhhhhhtransformation? Determine the range of the linear transformation.

a) $$\begin{aligned} w_1 &= x_1 + 3x_2 - x_3 \\ w_2 &= 2x_1 - 3x_2 - x_3 \\ w_3 &= 5x_1 + 6x_2 - 4x_3 \end{aligned}$$

b) $$\begin{aligned} w_1 &= x_1 + x_2 + x_3 \\ w_2 &= x_1 - x_2 - x_3 \\ w_3 &= 2x_1 - 3x_3 \end{aligned}$$

23. Let a linear transformation $T : \Re^3 \to \Re^3$ scale (dilate) the x and y components of a vector by a factor of 2 followed by a rotation around the x-axis of the dilated vector by $\theta = 30^\circ$ counterclockwise. Determine the standard matrix for this transformation from the images of the standard basis vectors.

24. Let T_1 and T_2 be two linear transformations $T_1 : \Re^3 \rightarrow \Re^3$, $T_2 : \Re^3 \rightarrow \Re^3$, if either T_1 or T_2 is not one-to-one, can the compositions $T_1 \circ T_2$, $T_2 \circ T_1$ be one-to-one? Prove your answer by using the determinants of the standard matrices T_1, T_2 and the determinant of the composition transformation.

Matlab Problems: solve the following problems in Matlab.

Read about dot, rank,

1. Let u be a vector in R^{100} (Euclidean space whose dimension is 100).
The ith component of u is i. Let v be the vector in R^{100} whose ith component is 1/(i+1). Find the dot product of u and v.

2. Find the angles that a diagonal of a box with dimensions 10cm x 11cm x 25cm makes with the edges of the box.

3. Find the decomposition of the vector u = (2,3,1,2) in the form: u = w1 + w2, where w1 is a scalar multiple of a = (-1,0,2,1) and w2 is orthogonal to a.

4. Devise a method to determine whether a set of vectors in R^n is linearly independent. Then use your method to determine whether following vectors are linearly independent or not.
V1 = (4,-5,2,6); V2=(2,-2,1,3); V3=(6,-3,3,9); V4=(4,-1, 5,6);

5. Devise a method to determine whether a given vector V5=(1 2 3 4) is in the span of the four vectors in problem 4. (Hint: In this problem and problem 4. You might want to consider to use the rank function).