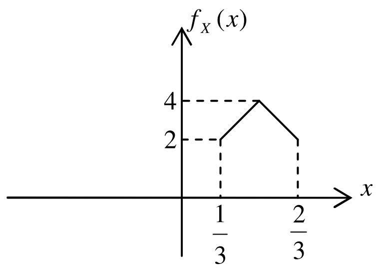
MAT271E Probability and Statistics HW #3

Instructions: Please hand in your answers toTuğba Pamay by April 18, 2016 16:00. (Use the mailbox reserved for the course in the administrative office of the Computer and Informatics faculty). Late homeworks will not be accepted. 4-5 problems will be checked in detail which will contribute 80% to the final mark. The rest will be checked for completeness which will contribute 20% to the final mark.

1. The probability density function of random variable X is given below



i) Find $E[X]$'i bulun. (5 puan)

ii) Determine and sketch $f_X(x \mid X > 0.5)$.

iii) Determine $E[X \mid X > 0.5]$

iv) Determine $\sigma_X^2 = E[(X - \overline{X})^2]$

2. Find the characteristic function for random variable X with density function

$f_X(x) = \sum_{n=0}^{4} \binom{4}{n} 0.5^4 \delta(x-n)$ and calculate first two moments by using it. (Hint: Is this a discrete r.v.?)

3. Determine $E[X \mid X > 1]$ if X is standard Normal distributed ($X \sim N(0,1)$)

4. What is $E[e^{X^2} \mid X = 1]$?

5. The probability of realization of discrete random variable X is $\Pr\{X = i\} = 2(3^{-i})$ $\mathrm{i} = 1,2,...$

a) Find $E[X]$

b) If $Y = X^2$,

i) Find $E[Y]$

ii)Find σ_Y^2

6. Density function of random variable X is given by

$$f_X(x) = \begin{cases} \frac{2(x+2)}{5}, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find variance of $g(X) = 3X$

7. If the joint probability mass function of X and Y is given by

$$f_{X,Y}(x,y)=\sum_{i=1}^{2}\sum_{j=1}^{3}\frac{i^2 j}{30}\delta(x-i)\delta(y-j)$$

Find:

a) the marginal distribution of X and Y random variables. Are X and Y statistically independent?
b) $f_X(x \mid y=1)$
c) $\Pr\{X \le 4, Y = 1\}$
d) $\Pr\{X \le 1, Y > 2\}$
e) $\Pr\{X < Y + 1\}$
f) $\Pr\{|X - Y| = 1\}$

8. Let sample space is $S=\{\varsigma_1,\ldots,\varsigma_4\}=\{-2,-1.5,-0.5,0\}$ and define two random variables as $X(\varsigma)=1/(\varsigma+1)^2$ and $Y(\varsigma)=2^{-(\varsigma+1)}$.

g) a) Are X and Y statistically dependent? Prove.
h) b) Are X and Y linearly dependent? Prove.

9. Let the joint density function of random variables X and Y be given by

$$f_{X,Y}(x,y)=\begin{cases}A(x-y) & 0<x<1\,,\ -1<y<0\\ 0 & \text{elsewhere}\end{cases}$$

a) Find A.

b) Find conditional density function of Y i.e. $f_Y(y \mid X \le 0)$.

c) Are X and Y statistically dependent? Show.

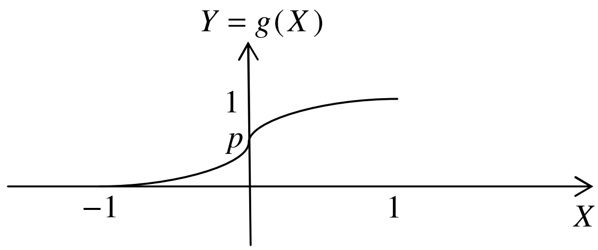
d) Find $\Pr\{0 \le X \le 0.5, 0.25 \le Y \le 0.5\}$

e) Find $\Pr\{X - Y < 1\}$

10. The joint probability function of two discrete random variables is given by
$f_{X,Y}(x,y)=0.4\delta(x)\delta(y)+0.2\delta(x)\delta(y-1)+0.4\delta(x-1)\delta(y-1)$

a) Find the two marginal distributions and draw them.
b) Find point conditional density function $f_X(x \mid Y=y)$ for all *y*.
c) Find $\Pr\{X - Y < 1\}$
d) Are X and Y statistically independent?
e) Determine the correlation coefficient between X and Y. Are X and Y linearly independent?

11. X is a uniform random variable with realizations uniformly distributed between -1 and 1 and $Y = g(X)$. See the graph below. Sketch the density and distribution function of Y as best as you can



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12. Let the joint density of X and Y random variables be given by

$$f(x, y) = \begin{cases} 24xy, & 0 \le x, 0 \le y, x + y \le 1 \\ 0, & elsewhere \end{cases}$$

i) Show that $f(x, y)$ is a valid joint density (check the properties)

ii) Find $f_X(y \mid X = 0.5)$ and evaluate $P(\frac{1}{4} < Y < \frac{1}{2} \mid X = 1)$.

iii)Find $E[Y \mid X = 1] = \int_{-\infty}^{\infty} y f_X(y \mid X = 1) dy$