BLG 354E Homework - 3

Yunus Güngör

May 2018

1 Answers

This homework only includes answers to given questions

1)

a)This system is not casual due to output depending on a value at $t+2$ This system is not stable since $d(x)/dt$ is not bounded if $|x(t)| < B_x$

b)This system is casual since output does not depend any future value at $t + n$ This system is stable because it can be bounded, if x is also bounded.

c)This system is casual since output does not depend any future value at $t + n$

$$\int_{-\infty}^{\inf} e^{-(t-5)}u(t-5) = \int_{t=5}^{\infty} e^{-(t-5)} = -e^{5-t} = -e^5/e^t$$

$$lim_{t\to\infty} - e^5/e^t = 0$$

$$-e^5/e^t < 0$$

This system can be bounded therefore it is stable

d)This system is casual since output does not depend any future value

at $t+n$

$$\int_{t=-\infty}^{\infty} u(t) - e^{-3t}u(t) = \int_{t=0}^{\infty} 1 - e^{-3t} = 1 + e^{\infty} - 2 = \infty$$

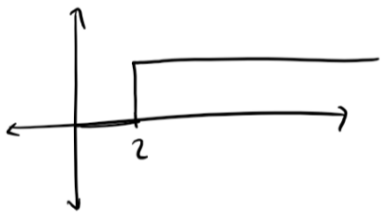
This system can not be bounded therefore it is not stable

2)

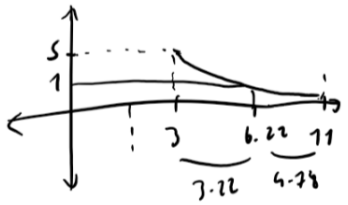
$$\int_{-\infty}^{\infty} 5e^{-0.5(t-\mathrm{T}-3)}[u(t-\mathrm{T}-3) - u(t-\mathrm{T}-11)]u(\mathrm{T}-2)d\mathrm{T} =$$

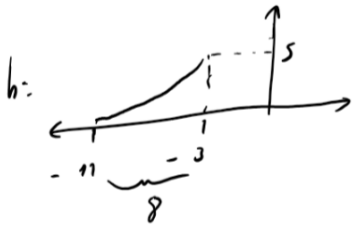
Convolution:

X:



h:





Regions:

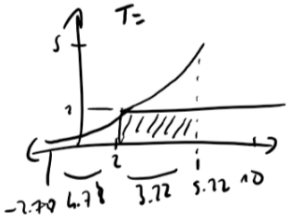
$T<5 \Rightarrow$ no overlap

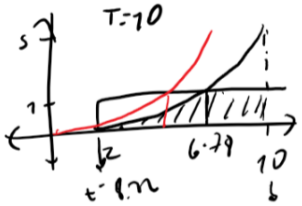
$5<T<8.22 \Rightarrow$ overlap

product $= (t-2)$

$8.22<T<13 \Rightarrow$ partial overlap

$T>13 \Rightarrow$ no overlap





$$y(t) = \begin{cases} 0 & t<5 \\ t-2 & 5<t<8.22 \\ f(t) & 8.22<t<13 \\ 0 & t>13 \end{cases}$$

$$h(t) = 1$$

$$5e^{-0.5(t-3)} = 1$$

$$-0.5(t-3) = \ln(1/5)$$

$$-0.5(t-3) \approx -1.61$$

$$t-3 \approx 3.22$$

$$t = 6.22$$

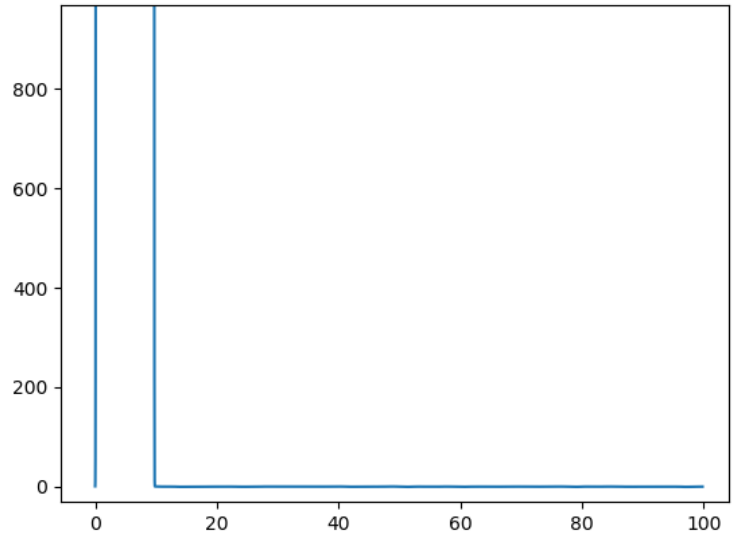
$$f(t) = \int_{t-8.22}^{t-13} 5e^{-t/2}\,dt + \int_{6.78}^{10} 1\,dt = -10e^{-t/2}\Big|_{t-8.22}^{t-13} + 3.22$$

$$f(t) = -10e^{\frac{-t+13}{2}} + 10e^{\frac{-t+8.22}{2}} + 3.22$$

3)You can find solution in attached files

4)Code for this question can be found in attachment.

Result:



Results have infite value for $0 < t < 1$

5)

a)

$$H(jw) = \int_{-\infty}^{\infty} h(t)e^{-jwt}dt$$

$$H(jw) = \int_{-\infty}^{\infty} (\delta(t-2) - 0.2e^{-0.2(t-2)}[u(t-2)])e^{-jwt}dt$$

$$\int_{-\infty}^{\infty} \delta(t-2)e^{-jwt}dt - 0.2\int_{-\infty}^{\infty} u(t-2)e^{-jwt-0.2(t-2)}dt$$

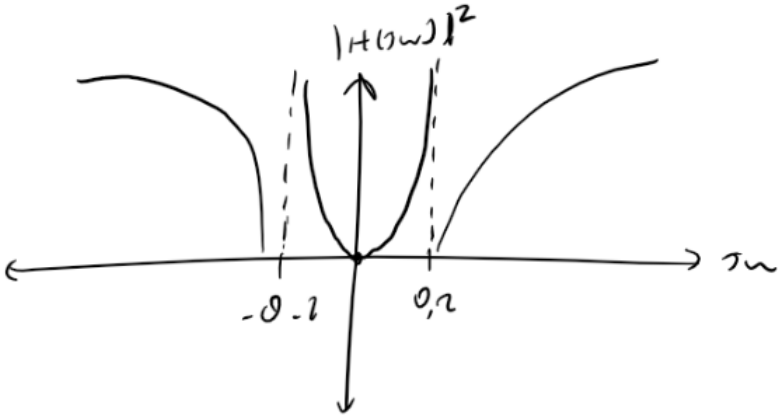
$$e^{-2jw} - 0.2\int_{2}^{\infty} e^{-jwt-0.2(t-2)}dt$$

$$e^{-2jw} - 0.2\int_{2}^{\infty} e^{(-jw-0.2)t-0.4)}dt$$

$$H(jw) = e^{-2jw} - 0.2\frac{e^{2jw}}{jw + 0.2}$$

b)

$$|H(jw)|^2 = H(jw)H^*(jw)$$
$$= (e^{-2jw} - 0.2\frac{e^{2jw}}{jw + 0.2})(e^{2jw} - 0.2\frac{e^{-2jw}}{-jw + 0.2})$$
$$= 1 - 0.2\frac{e^{4jw}}{jw + 0.2} + 0.2\frac{e^{-4jw}}{jw - 0.2} + 0.04\frac{1}{w^2 + 0.04}$$



c)

$$y(t) = h(t)x(t)$$

$$x(t) = ax_1(t) + bx_2(t)$$

$$y_1(t) = h(t)x_1(t)$$

$$y_2(t) = h(t)x_1(t)$$

$$y(t) = ay_1(t) + by_2(t)$$

6)

a)

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{sin(10w)^2}{2w^2} e^{iwt} dw$$

b)

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{25+w^2}e^{iwt}dw$$

c)

$$\int_{-\infty}^{\infty}e^{-a(t-w)}u(t-2)cos(w_0t)e^{iwt}dt$$