MAT281E Linear Algebra and Applications HW 2

Instructions: Turn in your solutions (hardcopy) no later than October 26th, 2015, Monday 16:00.

(Use the mailbox reserved for the course in the administrative office of the Computer and Informatics faculty). Late homeworks will not be accepted. 4-5 problems will be checked in detail which will contribute 80% to the final mark. The rest will be checked for completeness which will contribute 20% to the final mark.

1. For each of the following i) Express the linear system in matrix form, i.e. as $\underline{\underline{A}}\,\underline{x} = \underline{b}$. Indicate dimensions of $\underline{\underline{A}}, \underline{x}, \underline{b}$.Determine the solution for $\underline{x}$ (if any) using Gauss-Jordan Elimination (i.e. row reduction).

a.

$$\begin{aligned} &x + y - 3z = 4 \\ &2x + 2z = 3 \\ &-x + y + z = 2 \end{aligned}$$

b.

$$\begin{aligned} &x + 2y - 3z = 1 \\ &x + z = 1 \\ &-x + 2y - 4z = 5 \end{aligned}$$

c.

$$\begin{aligned} &x + y - z + w = 1 \\ &2x + z + 2w = 5 \end{aligned}$$

2. Bring the following matrix to row echelon form and then to reduced row echelon form by applying elementary row operations

$$\underline{\underline{A}} = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 2 & 1 \\ -2 & -1 & 4 \end{bmatrix}$$

3. Consider $\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Is this matrix in row echelon form, reduced row echelon form or neither? Explain.

4. Show that the products $\underline{\underline{A}}\,\underline{\underline{A}}^T, \underline{\underline{A}}^T\underline{\underline{A}}$ always exist.

5. If $\underline{\underline{A}}$ is 2x4, $\underline{\underline{B}}$ is 3x4 and $\underline{\underline{C}}$ is 3x2 what are the dimensions (size) of the matrix resulting from

$$(\underline{\underline{C}}^T - \underline{\underline{A}}\,\underline{\underline{B}}^T)\underline{\underline{B}} \quad ?$$

6. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a 3x3 matrix. If $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ is a solution of the matrix-vector equation:

$$A\vec{x} = \vec{0}$$

Is A invertible? Why or why not?

7. Perform the matrix products $\underline{\underline{A}}\,\underline{\underline{B}}$ and $\underline{\underline{B}}\,\underline{\underline{A}}$ by first expressing the result in terms of the submatrices.

$$\underline{\underline{A}} = \left[\underline{\underline{A}}_1 \vdots \underline{\underline{A}}_2\right] = \left[\begin{array}{ccc:cc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \underline{\underline{B}} = \begin{bmatrix} \underline{\underline{B}}_1 \\ \cdots \\ \underline{\underline{B}}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ \cdots & \cdots \\ 1 & 2 \\ -1 & 1 \end{bmatrix}.$$

8. Write down a 4x4 matrix $\underline{\underline{A}}$ for which the elements satisfy $a_{ij} = 0$ if $|i - j| \geq 1$. Determine the inverse of this matrix. What should you care about in specifying $\underline{\underline{A}}$?

9. Prove that the inverse of matrix $\underline{\underline{A}} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\underline{\underline{A}}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ by solving a linear system of the form $\underline{\underline{A}}\,\underline{\underline{X}} = \underline{\underline{I}}$ where $X = \begin{bmatrix} x & w \\ y & z \end{bmatrix}$.

10. Let $\underline{\underline{A}}^3 - 2\underline{\underline{A}}^2 + 3\underline{\underline{A}} = 0$, and suppose that the inverse of the square matrix A exists. Write down a formula for $\underline{\underline{A}}^{-1}$ in terms of $\underline{\underline{A}}$.

11. Let a matrix have a row of zeroes. Does its inverse exist? Explain by using $\underline{A}\,\underline{A}^{-1} = \underline{I}$.

12. Under what condition can you write the following?

$$(\underline{\underline{A}}+\underline{\underline{B}})^2 = \underline{\underline{A}}^2 + 2\underline{\underline{A}}\underline{\underline{B}} + \underline{\underline{B}}^2$$

13. Find shortest sequence of row operations that will turn $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 5 & 1 \end{bmatrix}$ into $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ .Write down the elementary matrix for each operation. Is the sequence unique? Explain.

14. Is it possible to apply elementary row operations to turn an invertible matrix into a matrix with two proportional rows? Explain.

15. Try to find the inverse of the following matrices by applying Gauss Jordan elimination on augmented matrices of the form $\lfloor \underline{\underline{A}} \mid \underline{\underline{I}} \rfloor$

a) $A = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 2 \\ 1 & 1 & 0 \end{bmatrix}$ c) $A = \begin{bmatrix} 0 & 0 & a \\ 0 & b & 0 \\ c & 0 & 0 \end{bmatrix}$

16. State the elementary row operations that are needed to turn $\underline{\underline{A}} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ into $\underline{\underline{B}} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$. (Hint: For both A and B, first determine the sequence of elementary row operations that will yield the identity matrix.)

17. Is the given triangular matrix invertible? Explain by saying what happens when the matrix is reduced to a row echelon or reduced row echelon form. Do you get a row of zeroes or not? (Do not try to compute the inverse.)

$$\underline{\underline{A}} = \begin{bmatrix} 2 & 0 & 4 & -4 & -1 \\ 0 & 3 & 0 & -2 & 2 \\ 0 & 0 & 5 & 0 & 1 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

18. Determine if there are value(s) of x for which the inverse of

$\underline{\underline{A}} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & x & 3 & -1 \\ 0 & 0 & 1 & 0 \\ x+1 & 2 & 4 & 1 \end{bmatrix}$ does not exist.

19. Describe all possible r.r.e.f.s (reduced row echelon forms) of a 3 × 6 matrix.

Note: Label those entries which are not required to be specified by * in the matrix.

20. Compute the determinant of the following matrix using any method:

$$A = \begin{bmatrix} 4 & 4 & 4 & 1 \\ 2 & 3 & 8 & 2 \\ 0 & 0 & -5 & 7 \\ 0 & 0 & -7 & -2 \end{bmatrix}$$

21. Find A^{120} if

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$