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BLG311E – Formal Languages and Automata

Spring 2017

**Quiz 3**

For $\Sigma = \{0,1\}$, prove that $L = \{0^n 10^n \mid n > 0\}$ is not a regular language.

**Duration:** 20 mins.

**Solution:**

**Remember: Pumping Lemma**

If $L$ is a regular language with unrestricted word length then in this language we can build any word longer than $n$ using substrings $u, v, w$ in following form: $x = uv^i w$. The following conditions should apply:

1. $|uv| \leq n$
2. $v \neq \Lambda$
3. $\forall x \in L \;\; x = uv^i w \wedge i \geq 0$

Assumptions:

- Suppose that there exists a finite automaton $M$ having $k$ states and accepting $L$.
- Choose $x = 0^k 10^k$ so $x \in L$ and $|x| \geq k$.

By pumping lemma:

- $x = uvw$, $|v| > 0$ and $|uv| \leq k$.
- For all possible splits that satisfy these rules: $v = 0^l$ where $1 \leq l \leq k$.
- Lemma states that for a regular language all $uv^i w, i \geq 0$ must belong to the language.
- Consider any string $uv^i w$ where $i \neq 1$.
- $(i = 0)\; uw = 0^{k-l}10^k$ → The string does not belong to $L$ since $k - l \neq k$.
- $(i > 1)\; uv^i w = 0^{k+(i-1)l}10^k$→ The string does not belong to $L$ since $k + (i-1)l \neq k$.
- This is a contradiction so $L$ is not regular.