台灣人工智慧學校

機器學習基礎與演算法

Chapter 2　迴歸 (Regression)

講師投影片
課程投影片
資料與程式碼
今日 Playlist

「版權聲明頁」

課程內容

2. 迴歸 (Regression)

2-1 [實作課程] Linear Regression

2-2 [實作課程] Weight Regularization

Code 放在Hub中的course內

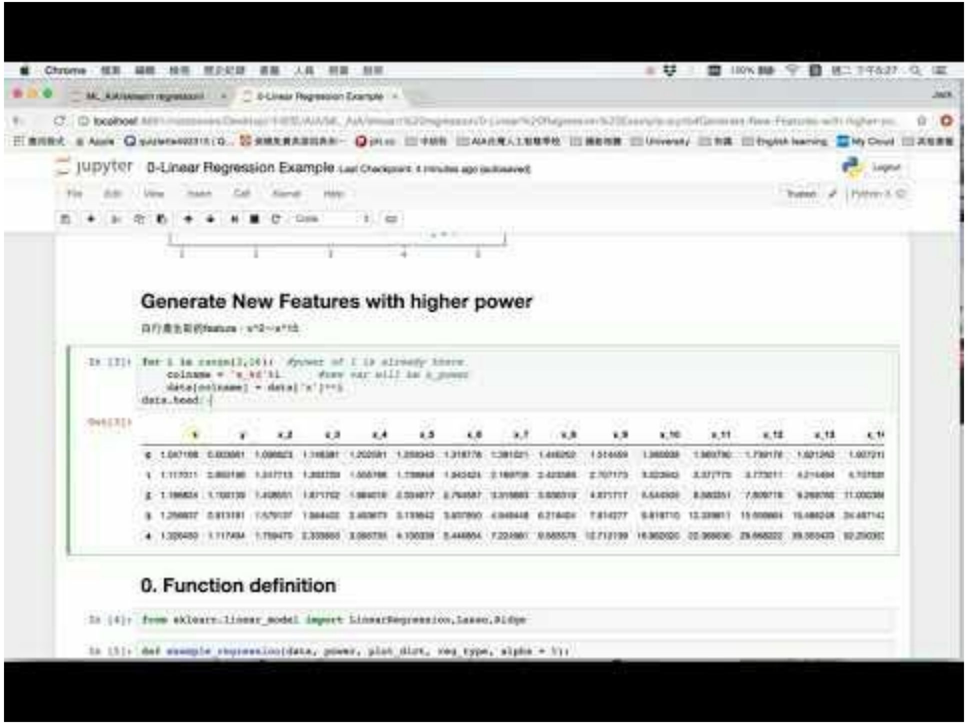
- 為維護課程資料，courses中的檔案皆為read-only，如需修改請cp至自身環境中
- 打開terminal，輸入

```
cp -r courses-tpe/ML/Chapter2 <存放至本機的名稱>
```

Chapter 2　迴歸(Regression)

- 範例程式(example)的檔名會以**藍色字體**顯示且旁邊附上
- 練習(exercise)的檔案以**紅色字體**顯示且旁邊附上

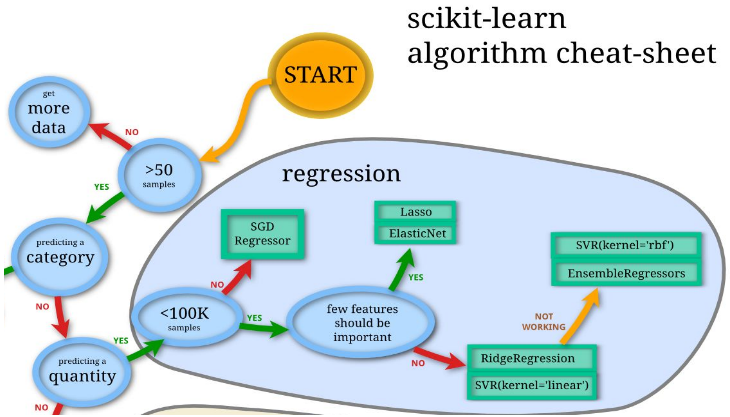
Section 2-1 [實作課程] Linear Regression - part 1



Linear Regression

- sklearn.LinearRegression使用時機：
  - Label為連續值
  - 資料量較少(<100K)
  - 假設資料的Features和Label之間有線性關係

$$y = a_1x_1 + a_2x_2 + a_3x_3 + \cdots$$



- 可以利用Features Transformation提升模型的複雜度。

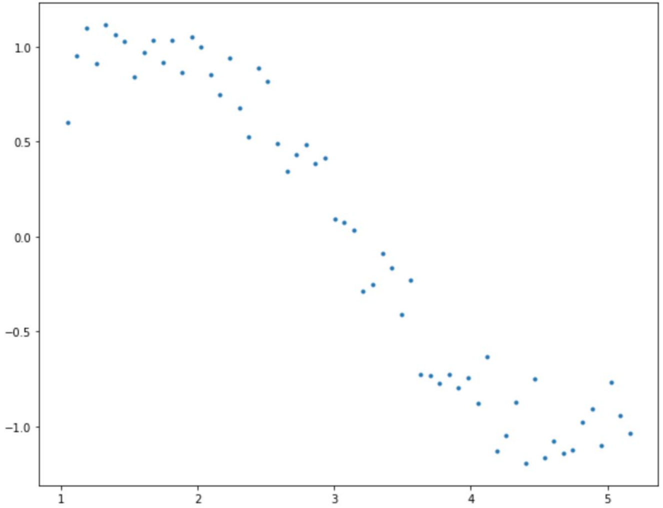
$$y = a_1x_1 + a_2x_2 + a_3x_3 + a_{11}{x_1}^2 + a_{12}x_1x_2 + \cdots$$

0-Linear Regression Example

Generate Sin(x) Dataset

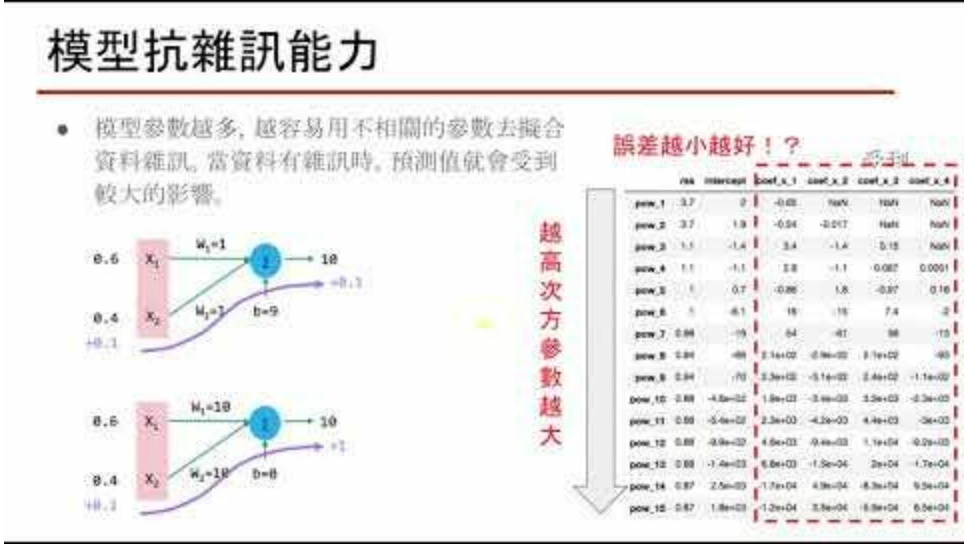
In [2]:
```
# define input array with angles from 60deg to 300deg converted to radians
x = np.array([i*np.pi/180 for i in range(60,300,4)])
np.random.seed(100)  #Setting seed for reproducibility
y = np.sin(x) + np.random.normal(0,0.15,len(x))

data = pd.DataFrame(np.column_stack([x,y]),columns=['x','y'])
plt.plot(data['x'],data['y'],'.')
```



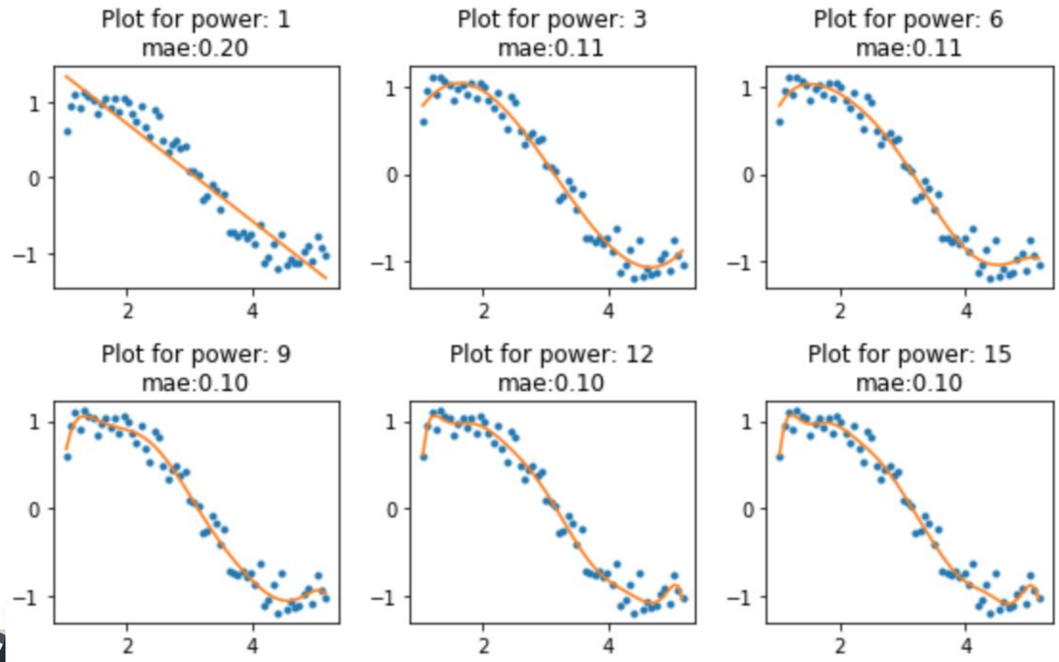
- 學習點:
    - Feature Transformation
    - Linear Regression

Section 2-1 [實作課程] Linear Regression - part 2



	rss	intercept	coef_x_1	coef_x_2	coef_x_3	coef_x_4
pow_1	3.7	0	-0.03	NaN	NaN	NaN
pow_2	3.7	1.9	-0.04	-0.017	NaN	NaN
pow_3	1.1	-1.4	3.4	-1.4	0.15	NaN
pow_4	1.1	-1.1	2.8	-1.1	0.087	0.0001
pow_5	1	0.7	-0.86	1.8	-0.37	0.16
pow_6	1	-6.1	16	-15	7.4	-2
pow_7	0.98	-19	64	-61	38	-13
pow_8	0.94	-69	2.1e+02	-2.9e+02	2.1e+02	-93
pow_9	0.94	-70	3.3e+02	-5.1e+02	2.4e+02	-1.1e+02
pow_10	0.88	-4.8e+02	1.8e+03	-3.4e+03	3.5e+03	-2.3e+03
pow_11	0.88	-5.4e+02	2.3e+03	-4.2e+03	4.4e+03	-3e+03
pow_12	0.88	-8.9e+02	4.6e+03	-9.4e+03	1.1e+04	-8.2e+03
pow_13	0.88	-1.4e+03	6.8e+03	-1.5e+04	2e+04	-1.7e+04
pow_14	0.87	2.5e+03	-1.7e+04	4.9e+04	-8.3e+04	9.5e+04
pow_15	0.87	1.8e+03	-1.2e+04	3.5e+04	-5.8e+04	6.5e+04

0-Linear Regression Example

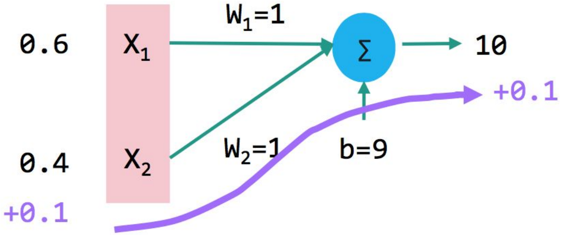


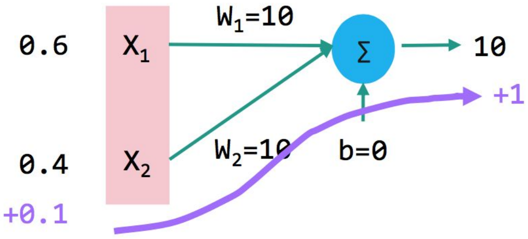
誤差越小越好！？

	rss	intercept	coef_x_1	coef_x_2	coef_x_3	coef_x_4
pow_1	3.7	2	-0.65	NaN	NaN	NaN
pow_2	3.7	1.9	-0.54	-0.017	NaN	NaN
pow_3	1.1	-1.4	3.4	-1.4	0.15	NaN
pow_4	1.1	-1.1	2.9	-1.1	0.087	0.0051
pow_5	1	0.7	-0.86	1.8	-0.97	0.18
pow_6	1	-6.1	16	-15	7.4	-2
pow_7	0.98	-19	54	-61	36	-13
pow_8	0.94	-66	2.1e+02	-2.9e+02	2.1e+02	-93
pow_9	0.94	-70	2.3e+02	-3.1e+02	2.4e+02	-1.1e+02
pow_10	0.88	-4.6e+02	1.9e+03	-3.4e+03	3.5e+03	-2.3e+03
pow_11	0.88	-5.4e+02	2.3e+03	-4.2e+03	4.4e+03	-3e+03
pow_12	0.88	-9.9e+02	4.6e+03	-9.4e+03	1.1e+04	-9.2e+03
pow_13	0.88	-1.4e+03	6.8e+03	-1.5e+04	2e+04	-1.7e+04
pow_14	0.87	2.5e+03	-1.7e+04	4.9e+04	-8.3e+04	9.5e+04
pow_15	0.87	1.8e+03	-1.2e+04	3.5e+04	-5.9e+04	6.5e+04

模型抗雜訊能力

- 模型參數越多，越容易用不相關的參數去擬合資料雜訊。當資料有雜訊時，預測值就會受到較大的影響。





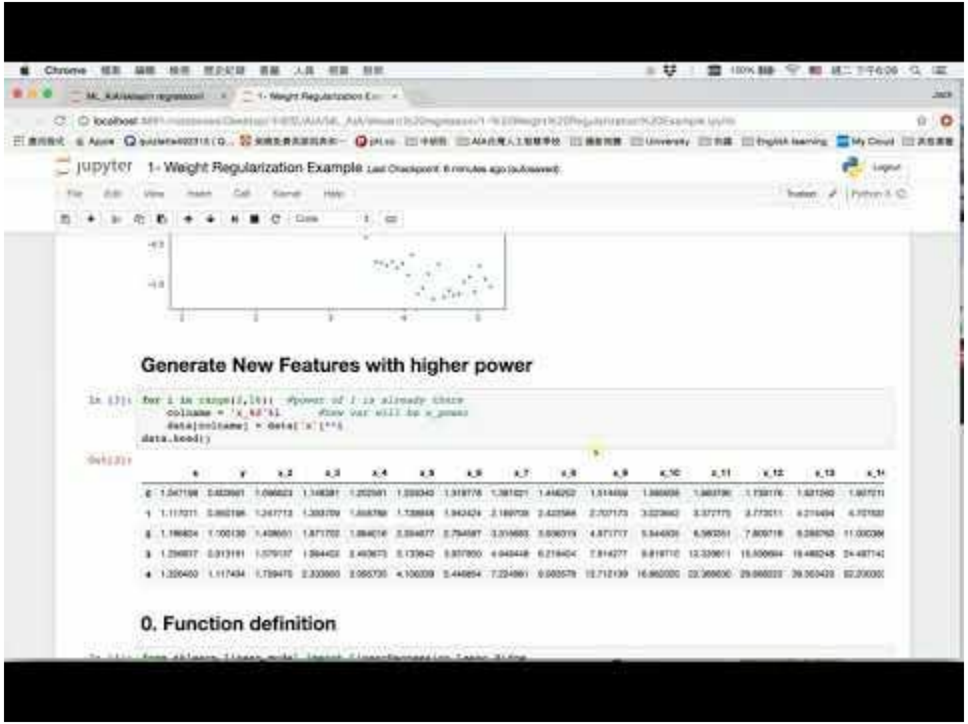
誤差越小越好！？

受到

越高次方參數越大

	rss	intercept	coef_x_1	coef_x_2	coef_x_3	coef_x_4
pow_1	3.7	2	-0.65	NaN	NaN	NaN
pow_2	3.7	1.9	-0.54	-0.017	NaN	NaN
pow_3	1.1	-1.4	3.4	-1.4	0.15	NaN
pow_4	1.1	-1.1	2.9	-1.1	0.087	0.0051
pow_5	1	0.7	-0.86	1.8	-0.97	0.18
pow_6	1	-6.1	16	-15	7.4	-2
pow_7	0.98	-19	54	-61	36	-13
pow_8	0.94	-66	2.1e+02	-2.9e+02	2.1e+02	-93
pow_9	0.94	-70	2.3e+02	-3.1e+02	2.4e+02	-1.1e+02
pow_10	0.88	-4.6e+02	1.9e+03	-3.4e+03	3.5e+03	-2.3e+03
pow_11	0.88	-5.4e+02	2.3e+03	-4.2e+03	4.4e+03	-3e+03
pow_12	0.88	-9.9e+02	4.6e+03	-9.4e+03	1.1e+04	-9.2e+03
pow_13	0.88	-1.4e+03	6.8e+03	-1.5e+04	2e+04	-1.7e+04
pow_14	0.87	2.5e+03	-1.7e+04	4.9e+04	-8.3e+04	9.5e+04
pow_15	0.87	1.8e+03	-1.2e+04	3.5e+04	-5.9e+04	6.5e+04

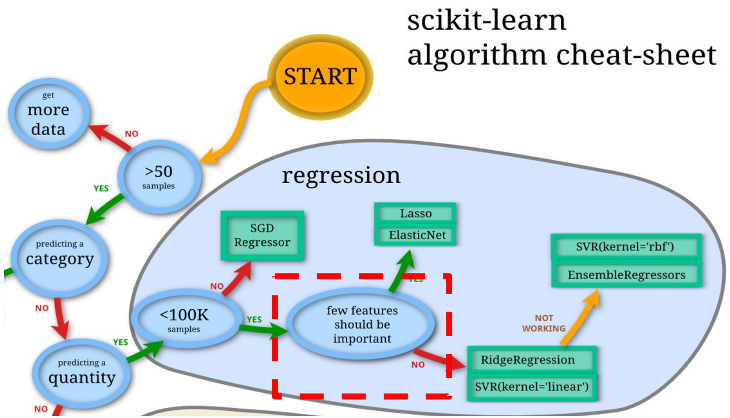
Section 2-2 [實作課程] Weight Regularization - part 1



Weight Regularization

- Ridge Regression (L2)

$$\text{Cost} = \text{Prediction error} + \alpha \sum (\text{weights})^2$$



- Lasso Regression (L1)
  --- Least Absolute Shrinkage and Selection Operator

$$\text{Cost} = \text{Prediction error} + \boxed{\alpha \sum |\text{weights}|}$$

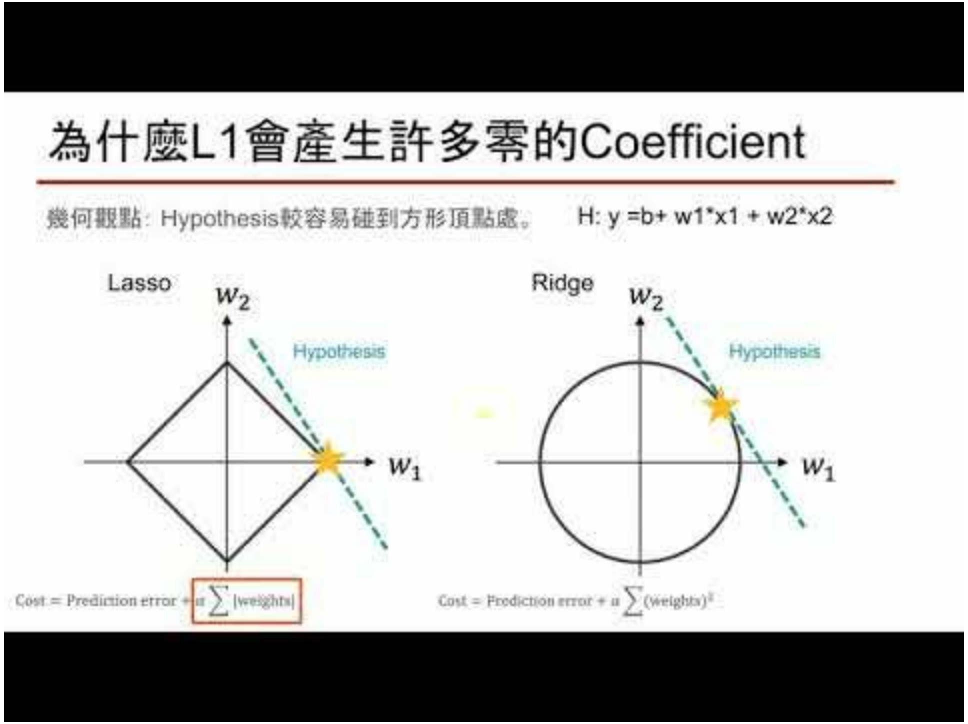
1-Weight Regularization Example

- Lasso Regularization --- L1

- Ridge Regularization --- L2

- 學習點:
  - 調整alpha並觀察結果
  - 比較L1&L2 Regularization的差異

Section 2-2 [實作課程] Weight Regularization - part 2



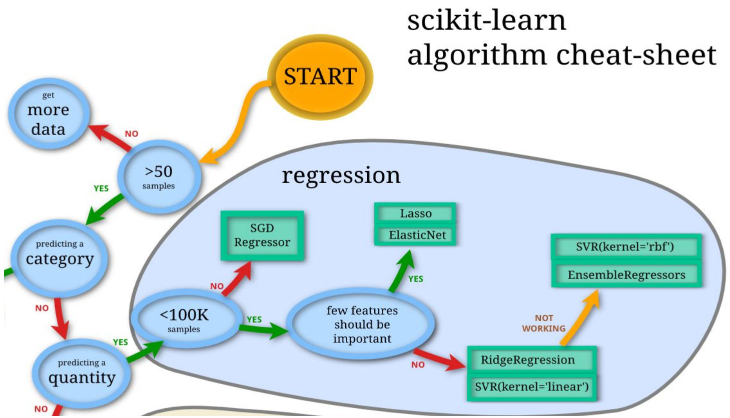
L1, L2 Regularization Summary

- 將alpha值調大對高次項的(較不相關的features)限縮越嚴謹，抵抗資料雜訊的能力越強。

參數越小

擬合程度變小

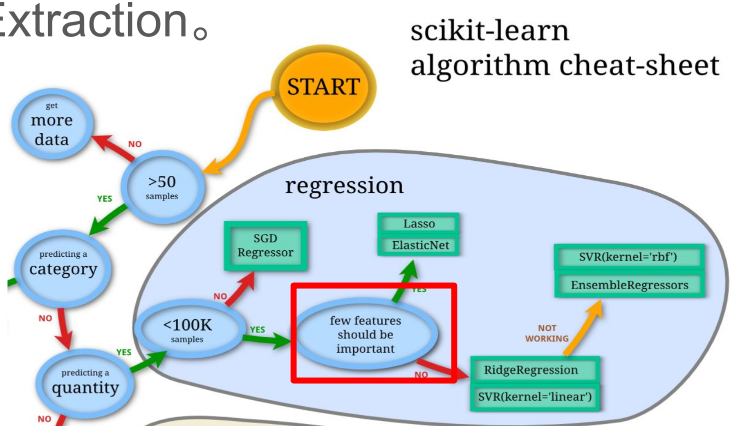
	rss	intercept	coef_x_1	coef_x_2	coef_x_3	coef_x_4	coef_x_5	coef_x_6	coef_x_7	coef_x_8	coef_x_9
alpha_1e-3	1.1	0.69	0.43	-0.11	-0.024	-0.0029	-0.00015	3.6e-05	1.5e-05	3.6e-06	6.9e-07
alpha_2e-3	1.2	0.86	0.27	-0.088	-0.02	-0.0026	-0.00018	2.1e-05	1.2e-05	3e-06	6.1e-07
alpha_3e-3	1.3	0.97	0.18	-0.078	-0.017	-0.0023	-0.00018	1.4e-05	9.6e-06	2.6e-06	5.4e-07
alpha_4e-3	1.4	1	0.12	-0.072	-0.016	-0.0021	-0.00017	1e-05	8.2e-06	2.3e-06	4.8e-07
alpha_5e-3	1.4	1.1	0.07	-0.067	-0.014	-0.002	-0.00016	7.5e-06	7.1e-06	2e-06	4.3e-07
alpha_6e-3	1.5	1.2	0.032	-0.064	-0.013	-0.0018	-0.00016	5.5e-06	6.3e-06	1.8e-06	3.9e-07
alpha_7e-3	1.5	1.2	0.0019	-0.061	-0.013	-0.0017	-0.00015	3.9e-06	5.6e-06	1.7e-06	3.6e-07
alpha_8e-3	1.6	1.2	-0.023	-0.058	-0.012	-0.0017	-0.00015	2.6e-06	5.1e-06	1.5e-06	3.4e-07
alpha_9e-3	1.6	1.2	-0.044	-0.057	-0.011	-0.0016	-0.00014	1.5e-06	4.6e-06	1.4e-06	3.1e-07
alpha_10e-3	1.6	1.3	-0.061	-0.055	-0.011	-0.0015	-0.00014	6.4e-07	4.2e-06	1.3e-06	2.9e-07
alpha_11e-3	1.7	1.3	-0.076	-0.053	-0.01	-0.0015	-0.00014	-1.3e-07	3.9e-06	1.2e-06	2.8e-07
alpha_12e-3	1.7	1.3	-0.089	-0.052	-0.01	-0.0014	-0.00013	-7.9e-07	3.6e-06	1.2e-06	2.6e-07
alpha_13e-3	1.7	1.3	-0.1	-0.051	-0.0098	-0.0014	-0.00013	-1.4e-06	3.3e-06	1.1e-06	2.5e-07
alpha_14e-3	1.7	1.3	-0.11	-0.05	-0.0095	-0.0013	-0.00013	-1.9e-06	3.1e-06	1e-06	2.4e-07
alpha_15e-3	1.7	1.3	-0.12	-0.049	-0.0093	-0.0013	-0.00013	-2.4e-06	2.9e-06	9.7e-07	2.3e-07



L1, L2 Regularization Summary

- Lasso(L1) Regularization相較於Ridge(L2) Regularization會產生較多零的coefficient，這個特性可以用來做重要Feature Extraction。

	rss	intercept	coef_x_1	coef_x_2	coef_x_3	coef_x_4	coef_x_5	coef_x_6	coef_x_7	coef_x_8	coef_x_9	coef_x_10	coef_x_11	coef_x_12	coef_x_13
pow_1	3.7	2	-0.64	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
pow_2	3.7	1.9	-0.54	-0.016	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
pow_3	3.7	1.9	-0.54	-0.016	-0	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
pow_4	3.1	1.5	-0.2	-0.11	-0	0.0015	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
pow_5	2.4	1.4	-0	-0.15	-0	0	0.00042	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
pow_6	2.2	1.4	-0	-0.15	-0	-0	0	7.8e-05	NaN	NaN	NaN	NaN	NaN	NaN	NaN
pow_7	2	1.3	-0	-0.13	-0.0025	-0	-0	0	1.6e-05	NaN	NaN	NaN	NaN	NaN	NaN
pow_8	1.9	1.3	-0	-0.12	-0.0043	-0	-0	0	0	3.1e-06	NaN	NaN	NaN	NaN	NaN
pow_9	1.8	1.3	-0	-0.12	-0.0044	-0	-0	0	0	0	5.9e-07	NaN	NaN	NaN	NaN
pow_10	1.9	1.3	-0	-0.12	-0.0025	-0	-0	0	0	0	0	1.1e-07	NaN	NaN	NaN
pow_11	1.9	1.3	-0	-0.13	-0.00044	-0	-0	0	0	0	0	0	1.9e-08	NaN	NaN

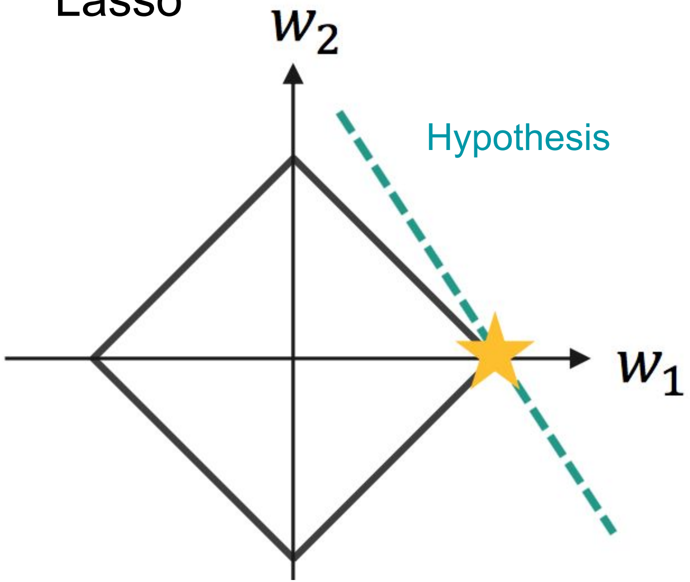


為什麼L1會產生許多零的Coefficient

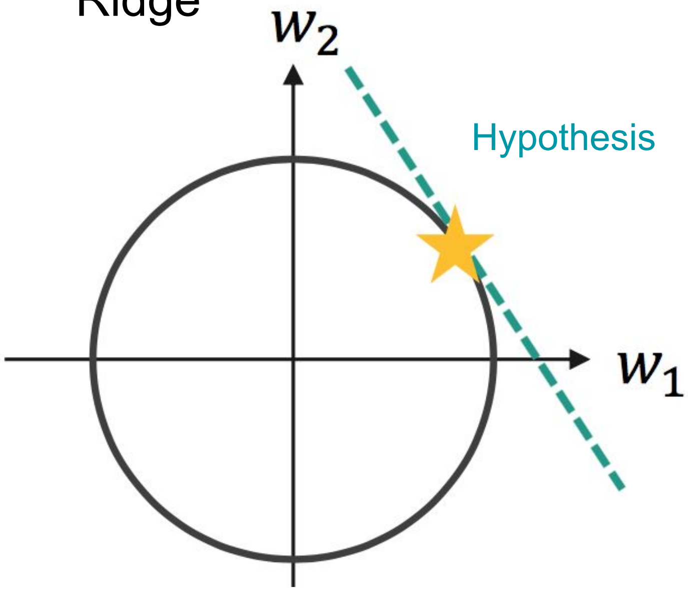
幾何觀點: Hypothesis較容易碰到方形頂點處。

H: y =b+ w1*x1 + w2*x2

Lasso



Ridge



$$\text{Cost} = \text{Prediction error} + \boxed{\alpha \sum |\text{weights}|}$$

$$\text{Cost} = \text{Prediction error} + \alpha \sum (\text{weights})^2$$

Short Summary

- 用 Linear Regression 解連續值的預測
- 利用 Feature Transform 增加模型複雜度
- 利用 L1&L2 Regularization 控制模型複雜度

Linear Regression Exercise 動手時間

- Boston house price prediction
  - 學習點: 1. Feature Transform 2. Weight Regularization
  - 其他: 1. 解讀重要Features 2. 選擇Alpha參數

```
Notes
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Data Set Characteristics:

    :Number of Instances: 506

    :Number of Attributes: 13 numeric/categorical predictive

    :Median Value (attribute 14) is usually the target

    :Attribute Information (in order):
        - CRIM     per capita crime rate by town
        - ZN       proportion of residential land zoned for lots over 25,000 sq.ft.
        - INDUS    proportion of non-retail business acres per town
        - CHAS     Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
        - NOX      nitric oxides concentration (parts per 10 million)
        - RM       average number of rooms per dwelling
        - AGE      proportion of owner-occupied units built prior to 1940
        - DIS      weighted distances to five Boston employment centres
        - RAD      index of accessibility to radial highways
        - TAX      full-value property-tax rate per $10,000
        - PTRATIO  pupil-teacher ratio by town
        - B        1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
        - LSTAT    % lower status of the population
        - MEDV     Median value of owner-occupied homes in $1000's
```