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Q1 let $(x_1, x_2 \ldots)$ be a random sample of size n taken from a normal population with parameters : mean θ_1 & variance θ_2. Find the max parameter likelihood estimates of these 2 parameters.

Sol $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-u)^2}{2\sigma^2}}$ [normal distribution]

$\mu = \theta_1$

$\sigma^2 = \theta_2$

$f(x_i) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$.

Now, in likelihood function

$L(\theta_1, \theta_2) = \prod_{n=1}^{\theta} f(x_i)$

$L(\theta_1, \theta_2) = \prod_{n=1}^{\theta} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$

$L(\theta_1, \theta_2) = (2\pi\theta_2)^{-n/2} e^{-\frac{2}{\theta_2}\prod_{n=1}^{n}(x_i-\theta_1)^2}$ — (1)

Taking log on both sides

$\ln(L(\theta_1,\theta_2)) = \ln\left[(2\pi\theta_2)^{-n/2} e^{-\frac{2}{\theta_2}\prod_{n=1}^{n}(x_i-\theta_1)^2}\right]$

$\ln(L(\theta_1,\theta_2)) = -\frac{n}{2}\ln(2\pi\theta_2) - \frac{2}{\theta_2}\sum_{i=1}^{N}(x_i-\theta_1)^2$ — (11)

Differenciate wrt θ_1 (eq 1)

$\frac{\partial z}{\partial \theta_1} = \frac{\partial \ln(L(\theta_1,\theta_2))}{\partial\theta_1} = \frac{-2}{\theta_2} 2\sum(x_i-\theta_1)$

$\frac{\partial z}{\partial \theta_1} = 0 \Rightarrow \frac{-4}{\theta_2}\sum(x_i-\theta_1) = 0$

$\sum x_i - n\theta_1 = 0$

$\theta_1 = \frac{\sum x_i}{n} = \bar{x}_n$ —— Result 1

Differenciate wrt θ_2 (eq 2)

$$\frac{\partial(\ln(L(\theta_1,\theta_2)))}{\partial\theta_2} = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2}\sum(x_i-\theta_1)^2$$

Similarly $\frac{\partial z}{\partial\theta_2} = 0$

$$\theta_2 = \frac{1}{n}\sum(x_i-\theta_1)^2$$

equating from result 1

$$\boxed{\theta_2 = \frac{1}{n}\sum(x_i-\bar{x}_n)^2}$$

Q2 let $x_1, x_2 -- x_n$ be a random sample from $B(m,\theta)$ distribution, where $\theta \in \Phi = (0,1)$ is unknown & m is a known +ve int. Complete value of θ using MLE (max likelihood estimator)

Sol let the distribution is binomial?

$f(x) = {}^nC_x\, p^x (1-p)^{n-x}$

given $n = m$, $p = \theta$

$f(x_i) = {}^mC_{x_i}\,\theta^{x_i}(1-\theta)^{m-x_i}$

Now, ln likelihood function

$L(m,\theta) = \prod_{i=1}^{n} f(x_i)$

$L(m,\theta) = \prod_{i=1}^{n} {}^mC_{x_i}\,\theta^{x_i}(1-\theta)^{m-x_i}$

Take log on both sides.

$\ln(L(m,\theta)) = \ln\left(\prod {}^mC_{x_i}\right) + \ln\left(\theta^{\sum x_i}\right) + \ln\left[(1-\theta)^{mn-\sum x_i}\right]$

$$\frac{dl}{d\theta} = \frac{1}{\theta} \sum x_i + \left(\frac{\sum x_i - mn}{1-\theta} \right)$$

$$\frac{dl}{d\theta} = 0 \Rightarrow \frac{\sum x_i}{\theta} = \frac{\sum x_i - mn}{\theta - 1}$$

$$\frac{\theta - 1}{\theta} = 1 - \frac{mn}{\sum x_i}$$

$$\boxed{\theta = \frac{\bar{x}_n}{m}}$$ result.