Hertzsprung-Russell Diagrams

A winter project by-
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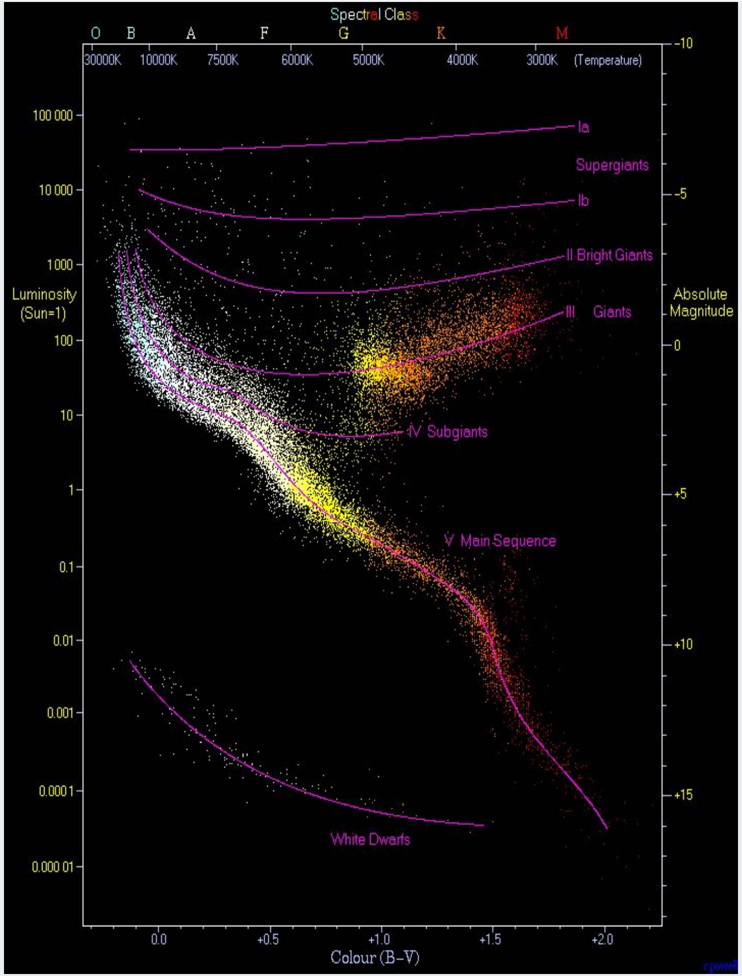


Introduction to Magnitudes and Luminosities

- **Luminosity** is the total amount of electromagnetic energy emitted per unit of time by a star, measured in watts or for ease of writing, in terms of luminosity of the sun $L_\odot$ (3.75 x 10^28 lm)
- In contrast, the term **Magnitude** in astronomy is a unitless measure of the brightness of an object
- 2 different definitions of magnitude are used-
  - **Absolute Magnitude (*M*)** describes the intrinsic luminosity emitted by an object and is defined to be equal to the apparent magnitude that the object would have if it were placed at a distance of 10 parsecs from Earth. Absolute magnitude when integrated over all wavelengths is known as bolometric magnitude.

    $$M_{\mathrm{bol1}} - M_{\mathrm{bol2}} = -2.5 \log_{10} \frac{L_1}{L_2}$$

    This is a relation between the bolometric magnitudes and luminosities of 2 objects

  - **Apparent Magnitude (m)** is the brightness of an object as it appears in the night sky from Earth. Apparent magnitude depends on an object's intrinsic luminosity, its distance, and other factors reducing its brightness.

    $$m_1 - m_{\mathrm{ref}} = -2.5 \log_{10}\left(\frac{I_1}{I_{\mathrm{ref}}}\right).$$

    The above is a relation between the Intensities and magnitudes of 2 objects, one of which is used as reference.

Continuation...

- The difference between these 2 systems can be easily understood by the example of Betelgeuse and Alpha Centauri, Betelgeuse (apparent magnitude 0.4, absolute magnitude −5.8) appears slightly dimmer in the sky than Alpha Centauri(apparent magnitude 0.0, absolute magnitude 4.4) even though it emits thousands of times more light, because Betelgeuse is much farther away.
- Absolute Magnitude and Apparent Magnitude are correlated by-

$$m - M = 5\left(\log_{10} d - 1\right).$$

where *d* is the distance to the star measured in parsecs, *m* is the apparent magnitude, and *M* is the absolute magnitude (ignoring corrections due to extinction due to absorption by interstellar dust particles).

References- https://en.wikipedia.org/wiki/Luminosity

https://en.wikipedia.org/wiki/Hertzsprung%E2%80%93Russell_diagram

https://en.wikipedia.org/wiki/Absolute_magnitude#Bolometric_magnitude

Colours for Astronomers

- Because we need to make life complicated, lets try defining a mathematical expression for colors.
- A “color” is simply a difference between magnitudes for a given source in two different bands:

$$C_{ij} = m_i - m_j = -2.5\, log_{10}(\frac{< F_\lambda >_i}{< F_\lambda >_j})$$

- Colors measure the slope of the spectral energy distribution between bands i and j (usually the B-V system) and hence the definition is such that a more positive color implies a larger flux in the second (j) band.
- From this, the effective temperatures can be calculated by assuming stars as blackbodies by the Ballestros’ formula- where x is the colour index
- $$T_{eff}(x) = 4600\,\mathrm{Kelvin} * \left(\frac{1}{0.92\,x + 1.7} + \frac{1}{0.92x + 0.62}\right).$$

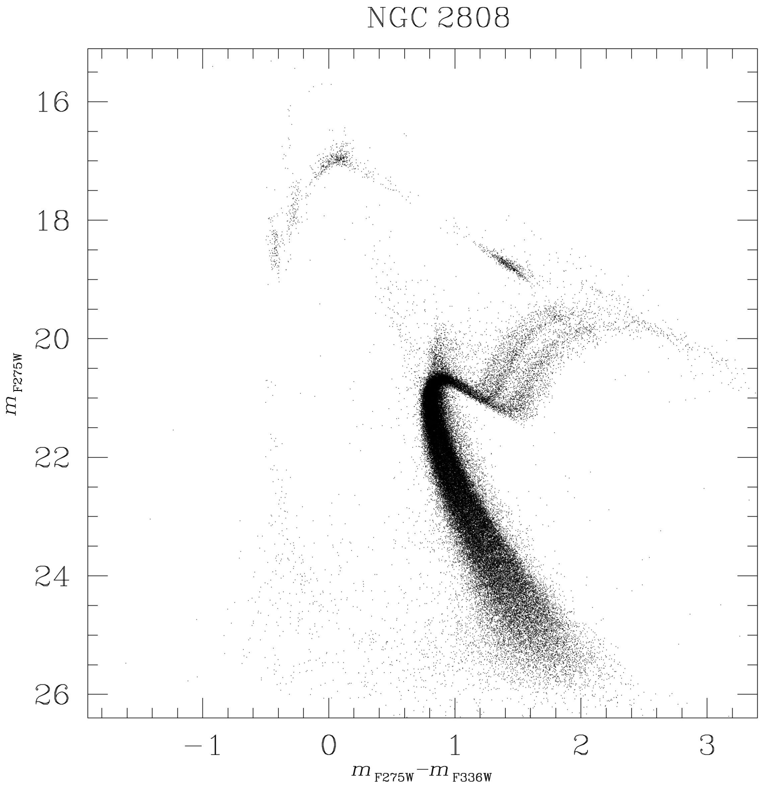
http://web.ipac.caltech.edu/staff/fmasci/home/astro_refs/ma

B−V	U−B	V−R	R−I	T_{eff} (K)
−0.33	−1.19	−0.15	−0.32	42,000
−0.30	−1.08	−0.13	−0.29	30,000
−0.02	−0.02	0.02	−0.02	9,790
0.30	0.03	0.30	0.17	7,300
0.58	0.06	0.50	0.31	5,940
0.81	0.45	0.64	0.42	5,150
1.40	1.22	1.28	0.91	3,840

Zombeck, Martin V. (1990). "Calibration of MK spectral types". *Handbook of Space Astronomy and Astrophysics* (2nd ed.). Cambridge University Press. p. 105. ISBN 0-521-34787-4.

So finally to the topic at hand...

- ❑ Now that we know what are colors and magnitudes, an HR diagram, or more simply put a CMD- Color Magnitude diagram plots the correlation between the luminosities (originally, the bolometric absolute magnitudes) and the effective temperatures (or more roughly, their spectral types) and basically plots a star's brightness against its color.
- ❑ CMD’s are pretty useful to determine the type of stars in a cluster, their ages and distances and so on.
- ❑ In the project we mostly used data from HUGS(https://archive.stsci.edu/prepds/hugs/)

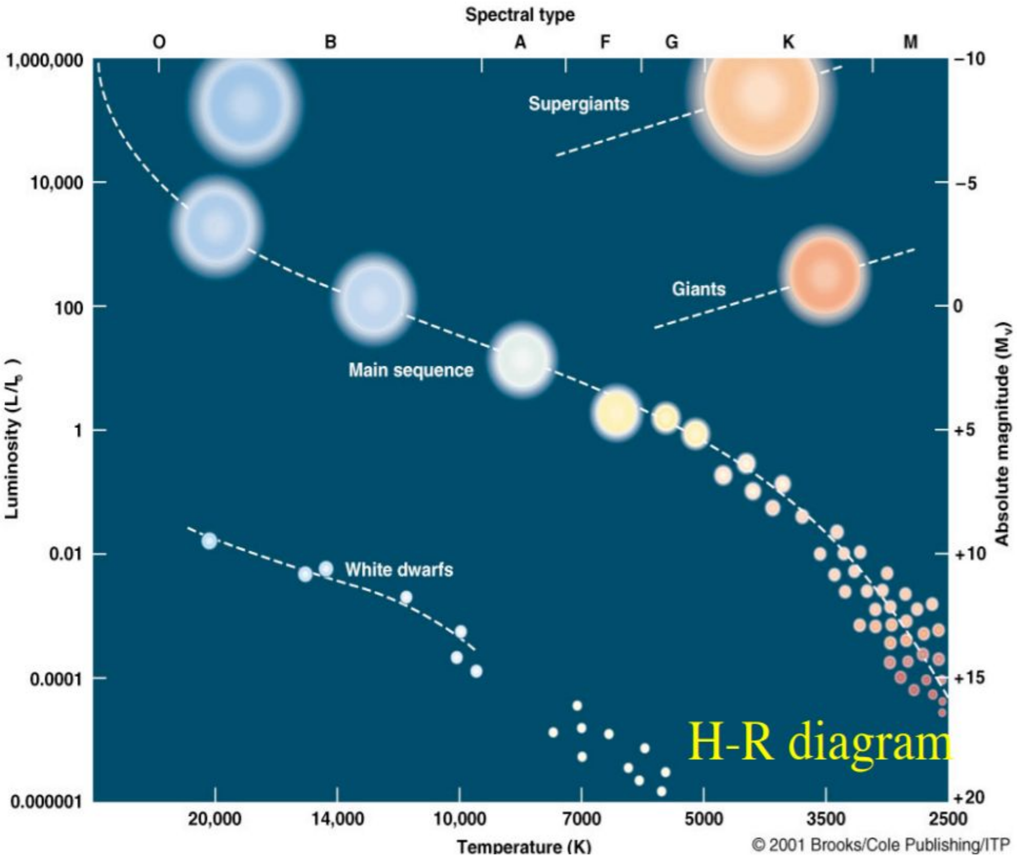


Colour Magnitude Diagrams

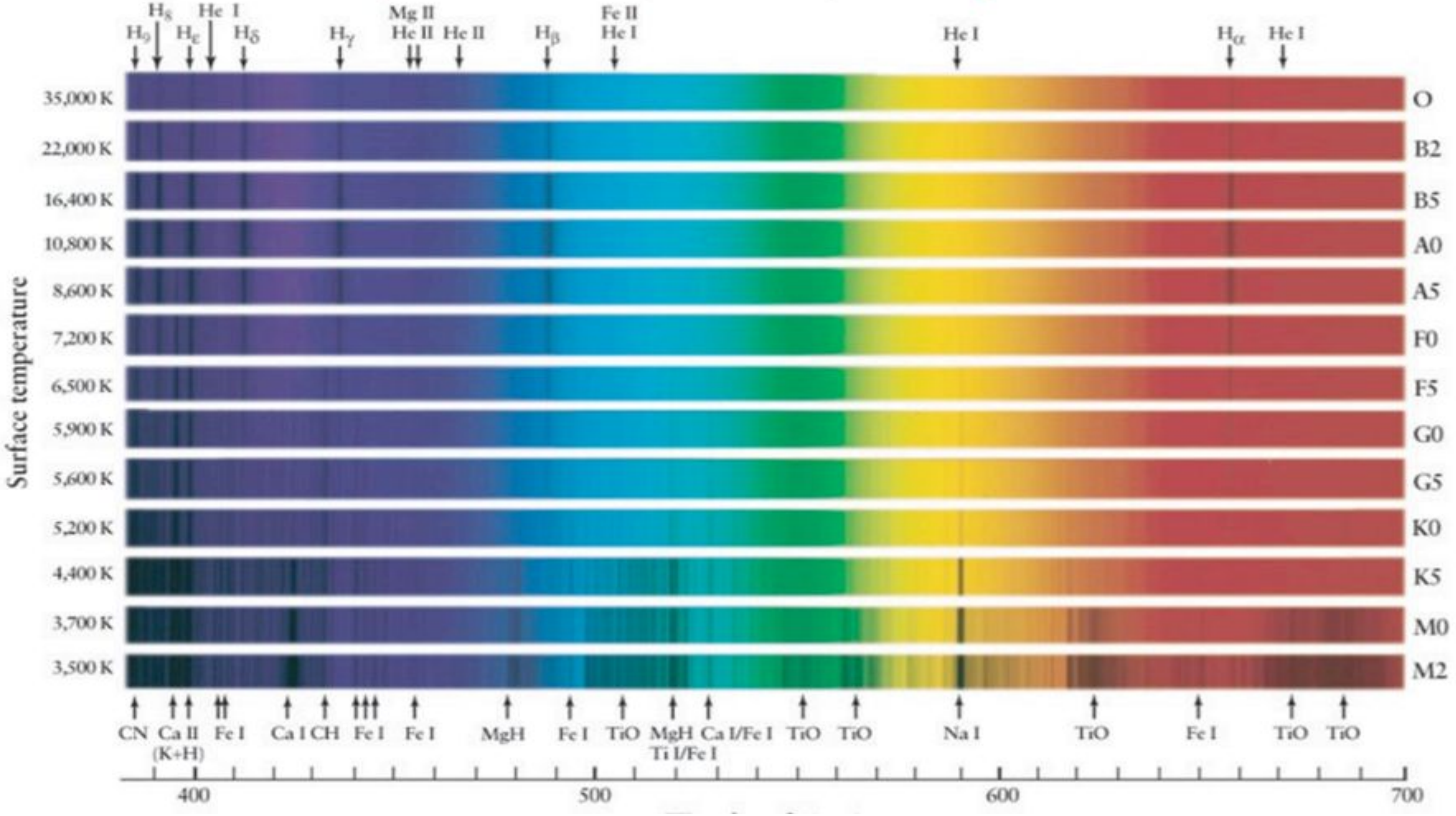
- Rather than actually measure & plot the temperature of every star, it is MUCH easier and quicker to simply measure & plot the ratio of the intensity of the star in two spectral bands, which gives us a colour magnitude diagram. This ratio is then directly related to the black body function and hence temperature, giving a HR diagram.
- For primarily historical reasons, the ratio is usually expressed as the difference (in magnitudes) between two standard (optical/IR) spectral bands and is known as the color. Traditionally, the most commonly used color is the difference between the B and V bands ,centered at 440 & 550 nm, respectively, and usually written as simply B - V.

Stellar Classification

- ❑ Astronomers classify stars based on the relative strengths of their absorption lines as O B A F G K M L T
- ❑ Stellar spectra consists of Blackbody continuum spectrum by the star's interior (hot dense gas) + a set of absorption lines given by the stellar atmosphere (cooler, low density gas).
- ❑ These lines give information about the composition of stars, their temperature etc.
- ❑ Eg-
  - ❑ He I lines: strongest in B2 stars
  - ❑ H I Balmer lines: strongest in A0 stars
  - ❑ Ca II H and K lines: strongest in K0 stars
  - ❑ Molecular absorption bands (TiO, VO): strongest in M stars



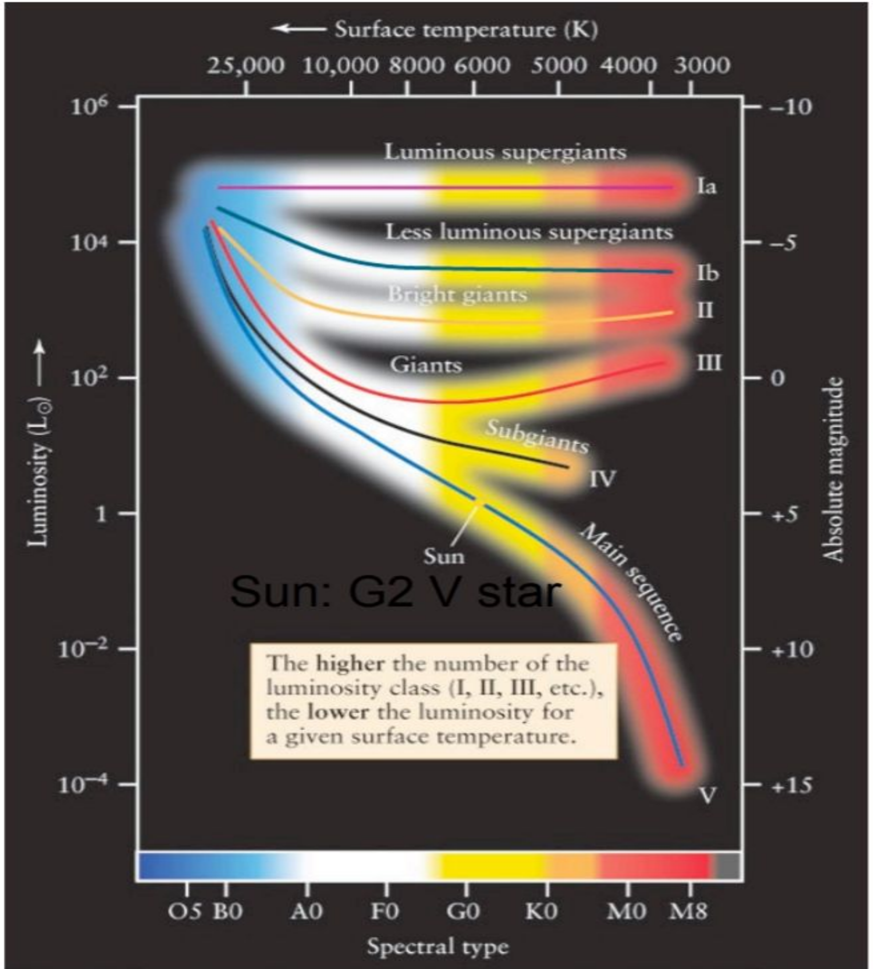
https://www.astro.princeton.edu/~burrows/classes/204/stellar.atmospheres.HR.pdf



https://www.astro.princeton.edu/~burrows/classes/204/stellar.atmospheres.HR.pdf

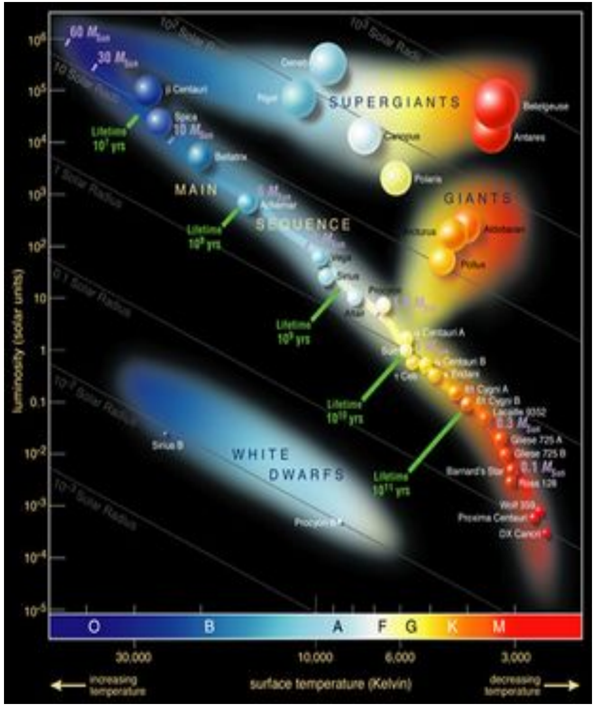
The Morgan-Keenan Luminosity Class

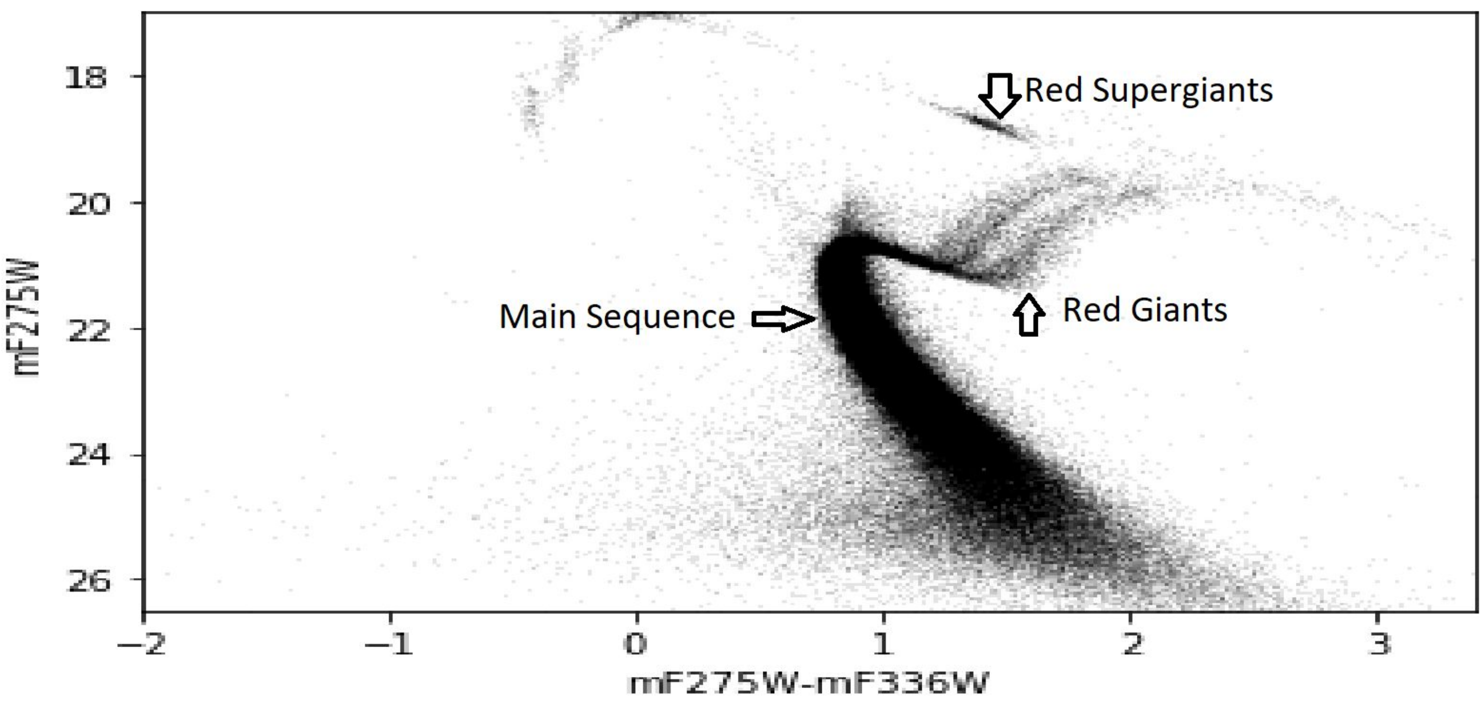
L. Class	Star
Ia	Luminous supergiant
Ib	supergiant
II	bright giant
III	giant
IV	subgiant
V	main sequence



Parts of an HR Diagram

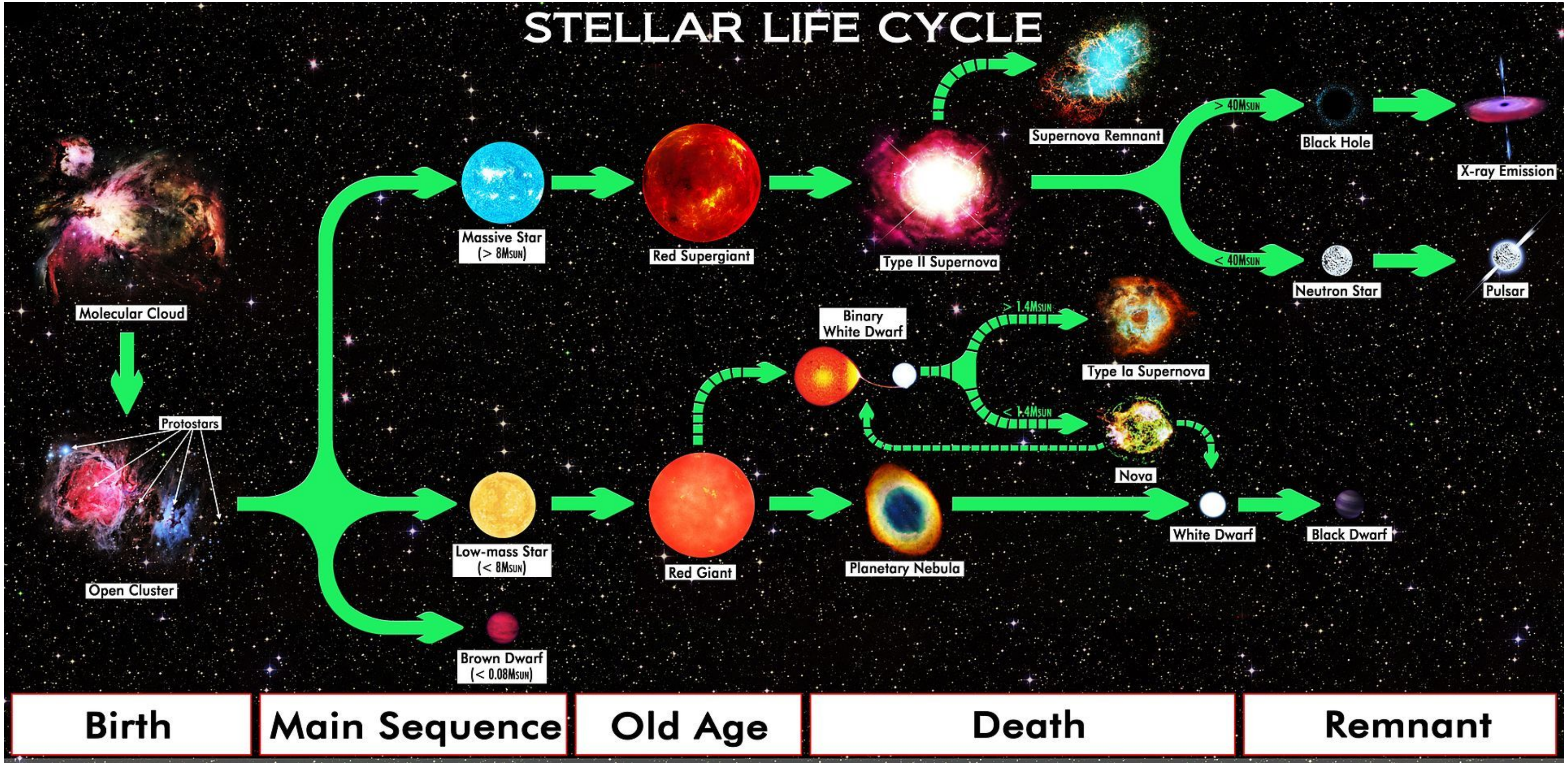
- Depending on its initial mass, every star goes through specific evolutionary stages dictated by its internal structure and how it produces energy. Each of these stages corresponds to a change in the temperature and luminosity of the star, which can be seen to move to different regions on the HR diagram as it evolves.
- There are 3 main regions (or evolutionary stages) of the HR diagram:
    - The **main sequence** stretching from the upper left (hot, luminous stars) to the bottom right (cool, faint stars) dominates the HR diagram. It is here that stars spend about 90% of their lives burning hydrogen into helium in their cores.
    - **Red giant and supergiant stars** have low surface temperatures and high luminosities which means they also have large radii. Stars enter this evolutionary stage once they have exhausted the hydrogen fuel in their cores and have started to burn helium and other heavier elements.
    - **white dwarf** stars are the final evolutionary stage of low to intermediate mass stars, and are found in the bottom left of the HR diagram. These stars are very hot but have low luminosities due to their small size





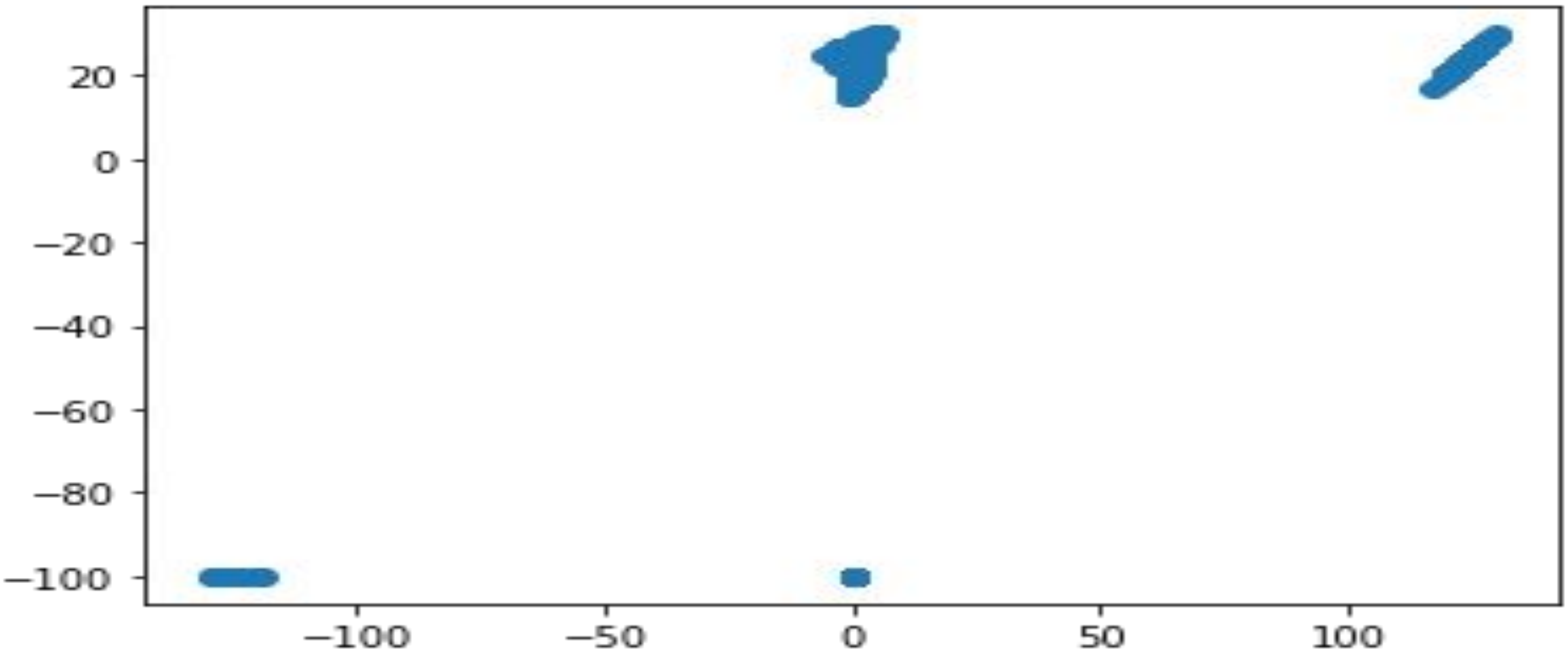
Classification of stars in our CMD of NGC-2808

Stellar Evolution



Project

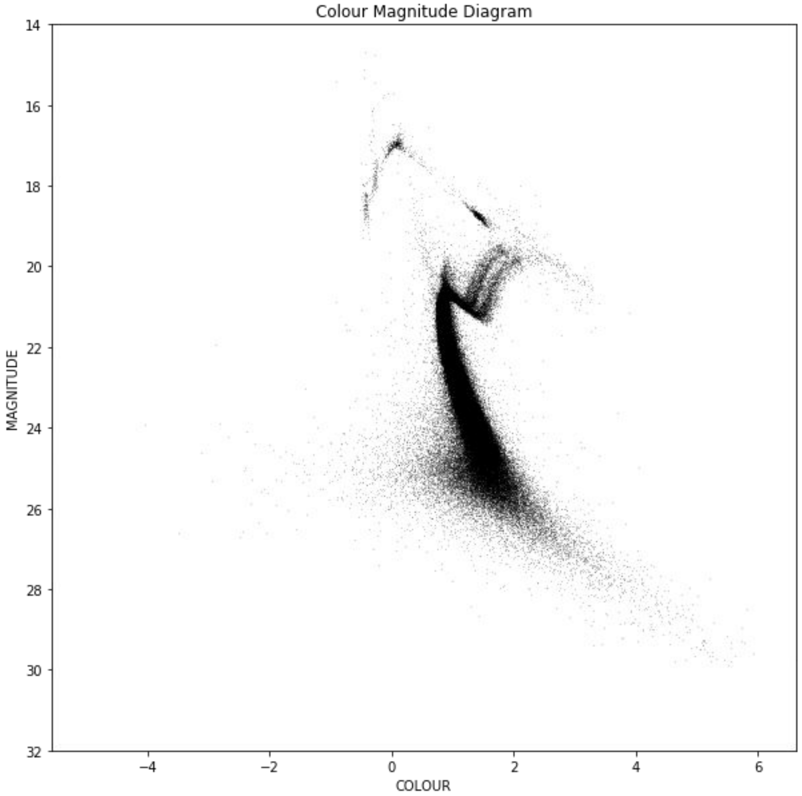
Making our own HR Diagram



After probability cuts

```
Cols. 1,2: (X,Y) stellar position in a reference system where the cluster center is in (5000,5000)
Col.  3: F275W calibrated magnitude
Col.  4: F275W photometric RMS
Col.  5: F275W quality-fit parameter
Col.  6: F275W sharp parameter
Col.  7: Number of F275W exposures the source is found in [99: saturated star]
Col.  8: Number of F275W exposures the source is well measured [99: saturated star]
Col.  9: F336W calibrated magnitude
Col. 10: F336W photometric RMS
Col. 11: F336W quality-fit parameter
Col. 12: F336W sharp parameter
Col. 13: Number of F336W exposures the source is found in [99: saturated star]
Col. 14: Number of F336W exposures the source is well measured [99: saturated star]
Col. 15: F438W calibrated magnitude
Col. 16: F438W photometric RMS
Col. 17: F438W quality-fit parameter
Col. 18: F438W sharp parameter
Col. 19: Number of F438W exposures the source is found in [99: saturated star]
Col. 20: Number of F438W exposures the source is well measured [99: saturated star]
Col. 21: F606W calibrated magnitude
Col. 22: F606W photometric RMS
Col. 23: F606W quality-fit parameter
Col. 24: F606W sharp parameter
Col. 25: Number of F606W exposures the source is found in [99: saturated star]
Col. 26: Number of F606W exposures the source is well measured [99: saturated star]
Col. 27: F814W calibrated magnitude
Col. 28: F814W photometric RMS
Col. 29: F814W quality-fit parameter
Col. 30: F814W sharp parameter
Col. 31: Number of F814W exposures the source is found in [99: saturated star]
Col. 32: Number of F814W exposures the source is well measured [99: saturated star]
Col. 33: Membership Probability [-1.0: not available]# Cols. 34,35: Right ascension (J2000, epoch 2015.0) and
```

After more refinement:



Distance of a cluster from a CMD?

- In an exception to the general rule, the universe turns out to be nice to astronomers (for once): there is a fundamental relationship between the temperature of a star and its size, which is true for most of the stars lying on the main sequence.
- This is due to the fact that they all have the property that they burn hydrogen to helium in their cores, and thus gives a relation between stars in different clusters.
- Using this property, we can find the ratio of distances to clusters by finding the difference in magnitudes of stars of the same colour in the main sequence.

Main Sequence Matching

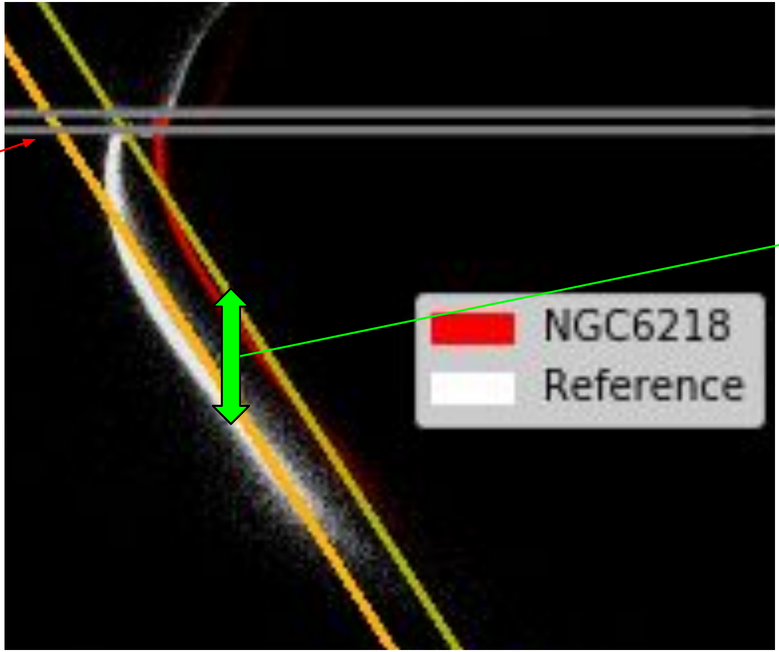
Objective: Finding the ratio of distances to 2 clusters by matching the Main Sequence. Keeping any one cluster as a reference, find the distances to all the other clusters.

So, basically we take a cluster as a reference, we have taken NGC 5272(Messier 3), and find the ratio of distances of a cluster to the reference cluster. Thus, knowing the distance of the reference cluster, multiplying it by the ratio yields the distance of the other cluster.

$$\frac{r}{r_{ref}} = ratio = 10^{distance\ modulus/5}$$

So, the question essentially boils down to finding the distance modulus. This is equal to the amount by which the cluster's MS needs to be shifted vertically to match with the reference.

We need horizontal subgiant branch so that we can select the main sequence by limiting the magnitudes only till this limit by a horizontal line.



Distance Modulus

(Color index as F336W - F606W and the Y-axis as F336W).

Method:

1.) Write a function that finds the horizontal subgiant branch, and limit the dataset till this point. Thus, we have selected the main sequence.

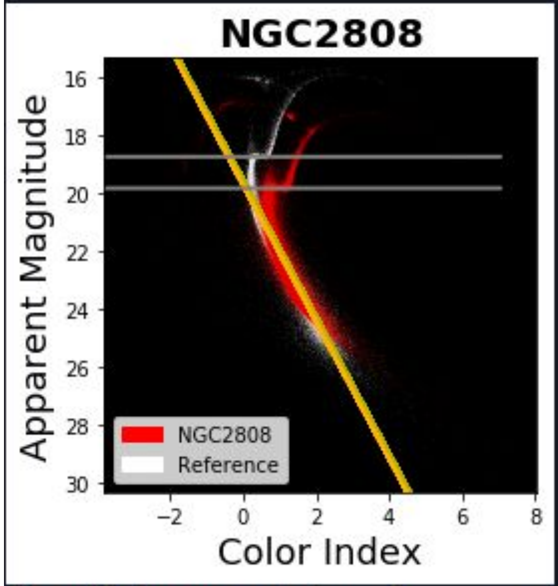
2.) Use `scipy.optimize.curve_fit` to fit a straight line to the selected points.

3.) The slopes of the two lines obtained are similar and differ by a very small amount. Find their average.

4.) Now, rotate the two lines about their geometric centres such that after rotation, their new slopes are equal to the average slope.

5.) Now, the vertical distance can be calculated easily as the new lines are parallel.

```
Enter the numeric part of the cluster's name: 2808
```




```
The distance modulus is     :  0.2366296959438614
The ratio of distances is   :  0.8967555244290127
The distance of the cluster:  9.326257454061732  kpc
Magnitude limit is          :  19.8278
```

Isochrone Fitting

An **isochrone** is a curve on the HR diagram, representing a population of stars of the same age.

Since isochrones mimic **simple stellar populations(SSPs)**, they are commonly used to derive the basic parameters (age, metallicity, distance, etc.) for star clusters.

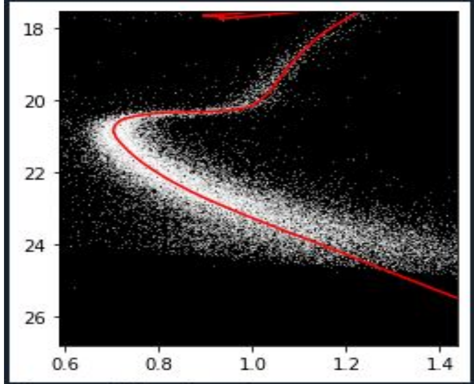
An isochrone is plotted on a CMD as a trendline where the actual cluster data should lie.

Isochrones are generated using PARSEC: PAdova and TRieste Stellar Evolution Code.

Basically, it's a web interface dealing with stellar isochrones. You need to input the metallicity of the cluster, and the range of the ages for which you want the isochrones for.

The magnitudes given by this are absolute in nature and must be converted to apparent for fitting by adding the distance modulus and extinction and reddening to the x-values.

```
Numeric part of the cluster's name           : 6934
The distance modulus is                      : 15.97

Enter the expected age of the cluster (Gyr) : 10

Enter the extra correction factor            : 0.7

Enter the reddening value                    : 0.2
```



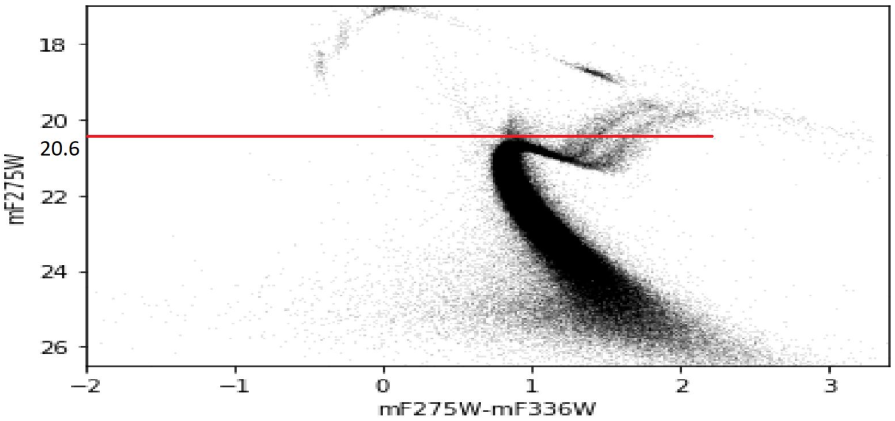
```
The reddening is : 0.2
The y - shift is : 16.67
The age is       : 10.0 Gyr

Is this fit good enough? Yes: 1, No: 0

1
```

INTER-IIT 2018

1. Calculate the half-light radius of the cluster. Consider only the stars with probability of membership greater than 90%. Report your answer in light years.

- Find center of cluster (median of RA and Dec columns after sorting)
- Find total luminosity from the magnitudes of stars
- Use np.where() to eliminate stars outside the circle of a given radius((RA-sortRA[34174])**2+(dec-sortdec[34174])**2)>0.0001641)
- Vary the radius till half the value of total luminosity is obtained (using for loop or hit and trial)
- Convert to light years.

2. Estimate the age of the cluster from its turn off point. Assume that the time spent by a star in the main sequence phase is proportional to ratio of its mass and luminosity. The main sequence lifetime of sun is 10 billion years. Report the turn off point magnitude as well as the estimated age of the cluster.



- Find turn off point by trial and error.
- Given luminosity proportional to mass^3.8.
- Given main sequence lifetime proportional to mass/luminosity
- Main sequence lifetime and absolute bolometric magnitude of sun given.
- Using these relations and given information, we could easily find the age of the cluster.

Inter IIT 2018, Part 4

4. X-ray observations of this cluster bring to light quite a few prominent X ray sources. Find the optical counterparts corresponding to the given 2 X ray sources in the cluster. The RA Dec and Error Radius are all in degrees. [30 pts]

S. no.	RA°	Dec°	Error Radius°
1	205.5407	28.3798	1e-4
2	205.5729	28.359	4.4e-4

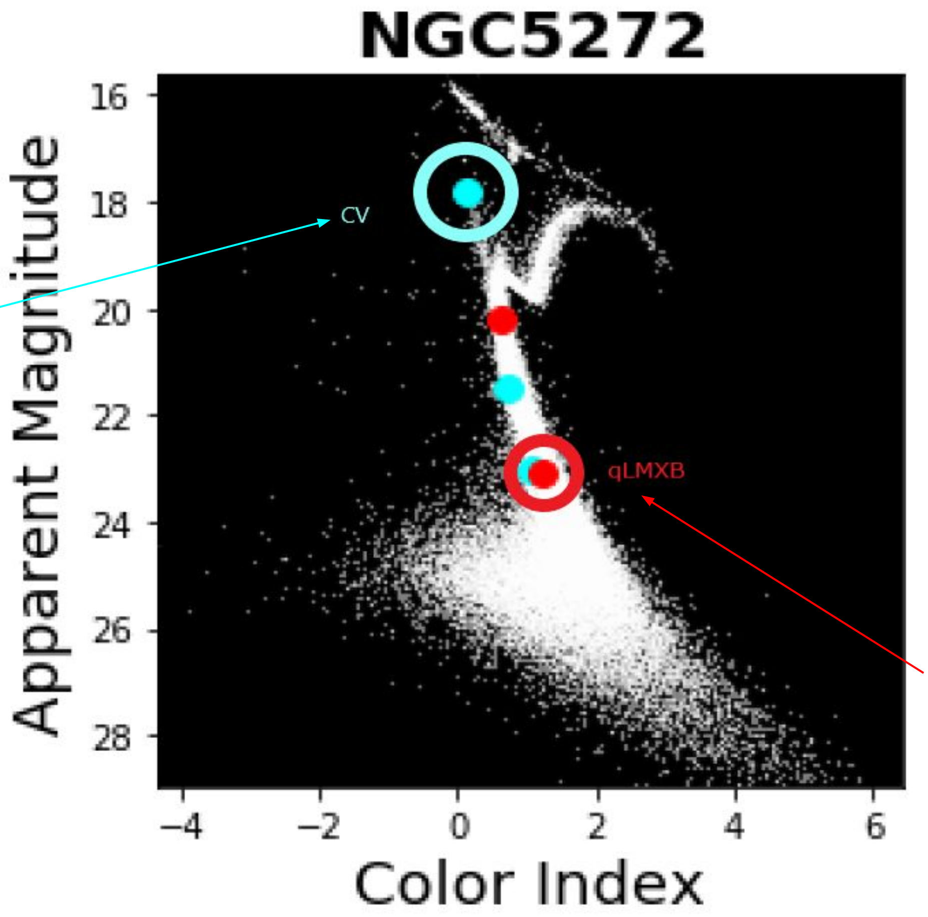
Method:

1.) Find the number of stars in the given regions. These are the potential optical counterparts. Plot them on a CMD, to further analyse their properties.
2.) Identify the region in which they're present (On the MS for example). Construct different CMDs to further analyse their variability.

Now, comparing the results obtained by us and in the paper - Zhao, Yue et al. "Identifications of faint Chandra sources in the Globular Cluster M3." Monthly Notices of the Royal Astronomical Society 483.4 (2018) we can see that the results obtained by their extensive research matches quite well with ours.
(a) From the section 4.1 of the paper we can identify the star in region 1 to be CX1, a cataclysmic variable.
(b) From the section 4.2 of the paper we can identify the star in region 2 to be CX2, a quiescent LMXB.

Thus we identify optical counterparts for the given sources.
https://arxiv.org/abs/1812.05130

Cataclysmic variables (CVs) are binary star systems that have a white dwarf and a normal star companion, in this case, it could be possibly a subgiant.



A **low-mass X-ray binary** (**LMXB**) is a binary star system where one of the components is either a black hole or neutron star
Quiescent: in a state of inactivity or dormancy.

Summary

- Basically, we first read about and got introduced to what an HR Diagram was.
- We learnt basic Python along with two of its libraries, numpy and matplotlib.
- We used Python for the data analysis of clusters from the HUGS directory, which included plotting a CMD, improving upon it, plotting the actual cluster, finding distances and age of the clusters.
- Along with these programming tasks, we learnt a lot about stars in general, such as magnitude systems, colour indices, classification, life cycle, spectrography, etc.
- At the end, we became acquainted with using Python for problem solving and became pretty familiar with many things related to stars.

Future plans

- ❏ A statistical way to fit isochrones
- ❏ Numerical derivation of Ballesteros' formula
- ❏ To further study color vs Temperature relation for real stars.
- ❏ Extracting branches of stars using colour indices (eg: Blue Horizontal Branch, Red Clump Stars etc)
- ❏ Plotting HR Diagrams for large surveys (Gaia, Pan-STARSS) to study local stellar properties (for example, arxiv:1901.02900 found two populations from Gaia DR2)

THANK YOU