Assignment 7

1. (Griffiths Example 10.1) Find the charge and current distributions that would give rise to the potentials

$$V = 0, \quad \mathbf{A} = \begin{cases} \frac{\mu_0 k}{4c}(ct - |x|)^2 \hat{\mathbf{z}}, & \text{for } |x| < ct, \\ \mathbf{0}, & \text{for } |x| > ct, \end{cases}$$

where k is a constant, and $c = 1/\sqrt{\epsilon_0 \mu_0}$.

2. Show that the differential equations for V and $\mathbf{A}$ can be written in the more symmetrical form

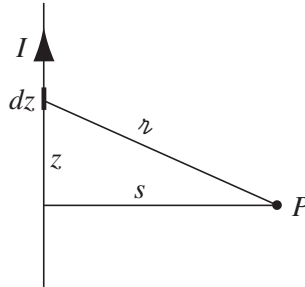$$\Box^2 V + \frac{\partial L}{\partial t} = -\frac{1}{\epsilon_0} \rho$$

$$\Box^2 \mathbf{A} - \boldsymbol{\nabla} L = -\mu_0 \mathbf{J},$$

where $\Box^2 \equiv \nabla^2 - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2}$ and $L \equiv \boldsymbol{\nabla} \cdot \mathbf{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}$.

3. (Griffiths 10.4) Suppose $V = 0$ and $A = A_0 \sin(kx - \omega t)\hat{\mathbf{y}}$, where A_0, ω, and k are constants. Find $\mathbf{E}$ and $\mathbf{B}$, and check that they satisfy Maxwell's equations in vacuum. What condition must you impose on ω and k?

4. (Griffiths 10.8) The vector potential for a uniform magnetostatic field is $\mathbf{A} = -\frac{1}{2}(\mathbf{r} \times \mathbf{B})$. Show that $d\mathbf{A}/dt = -\frac{1}{2}(\mathbf{v} \times \mathbf{B})$, in this case, and confirm that Eq. 10.20 yields the correct equation of motion.

5. (Griffiths Ex 10.2) An infinite straight wire carries the current

$$I(t) = \begin{cases} 0, \text{for } t \le 0, \\ I_0, \text{for } t > 0. \end{cases}$$

That is, a constant current I_0 is turned on abruptly at $t = 0$. Find the resulting electric and magnetic fields.

6. (Griffiths 10.24) Suppose you take a plastic ring of radius a and glue charge on it, so that the line charge density is $\lambda_0 |\sin(\theta/2)|$. Then you spin the loop about its axis at an angular velocity ω. Find the (exact) scalar and vector potentials at the center of the ring.