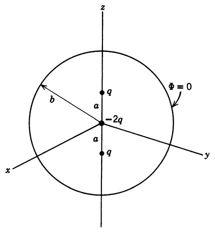
Assignment 3

1. (Jackson 3.1) Two concentric spheres have radii $a, b (b > a)$ and each is divided into two hemispheres by the same horizontal plane. The upper hemisphere of the inner sphere and the lower hemisphere of the outer sphere are maintained at potential V_0. The other hemispheres are at zero potential. Determine the potential in the region $a \le r \le b$ as a series in Legendre polynomials. Include terms at least up to $l = 4$. Check your solution against known results in the limiting cases $b \to \infty$, and $a \to 0$.

2. (Jackson 3.7) Three point charges $(q, -2q, q)$ are located in a straight line with separation a and with the middle charge $(-2q)$ at the origin of a grounded conducting spherical shell of radius b, as indicated in the sketch.



   (a) Write down the potential of the three charges in the absence of the grounded sphere. Find the limiting form of the potential as $a \to 0$, but the product $qa^2 = Q$ remains finite. Write this latter answer in spherical coordinates.

   (b) The presence of the grounded sphere of radius b alters the potential for $r < b$. The added potential can be viewed as caused by the surface-charge density induced on the inner surface at $r = b$ or by image charges located at $r > b$. Use linear superposition to satisfy the boundary conditions and find the potential everywhere inside the sphere for $r < a$ and $r > a$. Show that in the limit $a \to 0$,

$$V(r, \theta, \phi) \to \frac{Q}{2\pi\epsilon_0 r^3}\left(1 - \frac{r^5}{b^5}\right) P_2(\cos\theta).$$

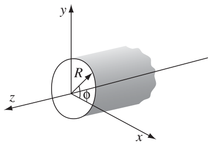
3. (Griffiths 3.19) The potential at the surface of a sphere (radius R) is given by $\Phi_0 = k\cos 3\theta$, where k is a constant. Find the potential inside and outside the sphere, as well as the surface charge density $\sigma(\theta)$ on the sphere. (Assume there's no charge inside or outside the sphere.).

4. (Griffiths 3.24) Solve Laplace's equation by separation of variables in cylindrical coordinates, assuming there is no dependence on z (cylindrical symmetry). [Make sure you find all solutions to the radial equation; in particular, your result must accommodate the case of an infinite line charge, for which (of course) we already know the answer.]

5. (Griffiths 3.26) Charge density

$$\sigma(\phi) = a \sin 5\phi$$

(where a is a constant) is glued over the surface of an infinite cylinder of radius R. Find the



potential inside and outside the cylinder. [Use your result from Prob. 3.24.]