MTH201-PRACTICE PROBLEMS 5

Q 1. Let $f(x) = x^2$. For any $n \in \mathbb{N}$, let $P_n = \{x_0, x_1, x_2, \cdots, x_n\}$ be a partition of $[0, 2]$ into equal widths.

(i) What is the width of the subinterval $[x_{k-1}, x_k]$?

(ii) What is the value of x_k for each $k = 0, 1, 2, \cdots, n$ in terms of k and n?

(iii) Find the upper sum, and the lower sum.

(iv) Calculate the limit as $n \to \infty$.

(v) Calculate the integral using the Fundamental Theorem of Calculus and see for yourself if that matches with the limit that you have calculated above.

Q 2. Show that the following functions are not integrable.

(i) $f(x) = \int_0^1 \frac{1}{x} dx$.

(ii) $f(x) = \int_0^1 \frac{1}{x^n} dx$, for any $n \in \mathbb{N}$.

Q 3. (i) Calculate $\int_{-1}^1 |t| dt$.

(ii) Calculate $F(x) = \int_{-x}^x |t| dt$ for any $x \geq 0$.

(iii) Calculate $F'(x)$.

Q 4. Find the area between the curves

(i) $f(x) = x^2 - 2x, g(x) = x - 2$.

(ii) $f(x) = x^{3/2}, g(x) = x^{2/3}$.

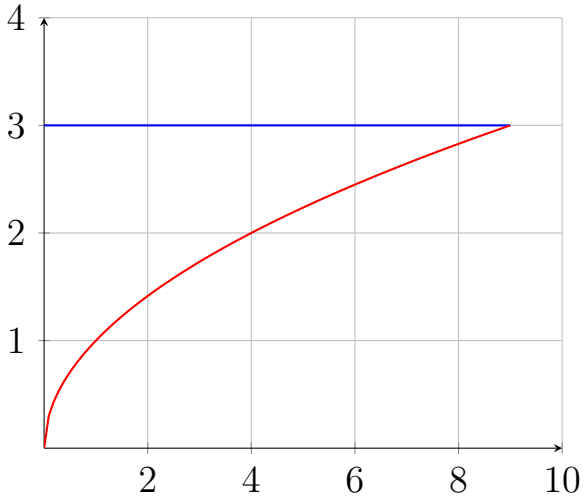
(iii) $f(x) = 3 + 2x - x^2$ and x-axis.

(iv) $f(x) = x^2 + 2, g(x) = \sin x, x = -1, x = 2.$

(v) $f(x) = \dfrac{1}{x+2}, g(x) = (x+2)^2, x = \dfrac{-3}{2}, x = 1.$

Q 5. Calculate the volume of the solid obtained by rotating the region bounded by $f(x) = \sqrt{x}, g(x) = 3$ and the y-axis about the y-axis. First, find out the cross sectional area

Plot of $f(x)$ in red and $g(x)$ in blue



The area between the blue and the red curves is then rotated about the Y-axis to get the following region.

Plot of the region