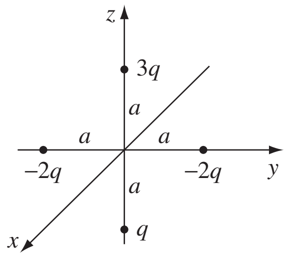
Assignment 4

1. (Griffiths 3.27) A sphere of radius R, centered at the origin, carries charge density

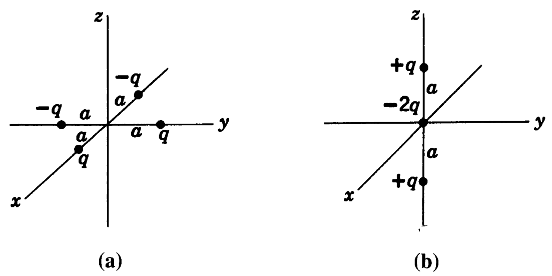
$$\rho(r,\theta) = k\frac{R}{r^2}(R-2r)\sin\theta$$

where k is a constant, and r, θ are the usual spherical coordinates. Find the approximate potential for points on the $z-$ axis, far from the sphere.

2. (Griffiths 3.29) Four particles (one of charge q, one of charge $3q$, and two of charge $-2q$) are placed as shown in Fig., each a distance a from the origin. Find a simple approximate formula for the potential, valid at points far from the origin. (Express your answer in spherical coordinates.)



3. (Jackson 4.1) Calculate the multipole moments q_{lm} of the charge distributions shown as parts a) and b). Try to obtain results for the nonvanishing moments valid for all l, but in each case find the first two sets of nonvanishing moments at the very least.



   (a) For the charge distribution of the second set b)write down the multipole expansion for the potential. Keeping only the lowest-order term in the expansion, plot the potential in the $x-y$ plane as a function of distance from the origin for distances greater than a.

   (b) Calculate directly from Coulomb's law the exact potential for b) in the x-y plane. Plot it as a function of distance and compare with the result found in part c).

4. (Jackson 4.7) A localized distribution of charge has a charge density

$$\rho(\vec{r}) = \frac{1}{64\pi} r^2 e^{-r} \sin^2\theta$$

   (a) Make a multipole expansion of the potential due to this charge density and determine all the nonvanishing multipole moments. Write down the potential at large distances as a finite expansion in Legendre polynomials.

(b) Determine the potential explicitly at any point in space, and show that near the origin, correct to r^2 inclusive

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{1}{4} - \frac{r^2}{120}P_2(\cos\theta)\right].$$

(c) If there exists at the origin a nucleus with a quadrupole moment $Q = 10^{-28}$ m^2, determine the magnitude of the interaction energy, assuming that the unit of charge in $\rho(\vec{r})$ above is the electronic charge and the unit of length is the hydrogen Bohr radius $a_0 = 4\pi\epsilon_0\hbar^2/me^2 = 0.529 \times 10^{-10}$ m. Express your answer as a frequency by dividing by Planck's constant h.

5. Show that the multiple expansion of the energy of a charge distribution in an external field (with $\Phi(\mathbf{r})$ being the external potential corresponding to this field) is given by

$$W = q\Phi(0) - \mathbf{p}\cdot\mathbf{E}(0) - \frac{1}{6}\sum_i\sum_j Q_{ij}\frac{\partial E_j}{\partial x_i}(0) + \ldots\ldots$$

Hence write down the interaction energy between two dipoles $\mathbf{p}_1$ and $\mathbf{p}_2$.