

Note
If an artificial variable appears
in the basis at non-zero level.
There exists a pseudo optimal soln to the
problem.

Two phase Method

Another method to deal with the artificial
variables wherein L.P.P. is solved with two
phases.

Phase I

Step 1 Express in standard form by introducing
slack, surplus & artificial variables

Step 2 Formulate an artificial objective fn

$$Z^* = -A_1 - A_2 - \dots - A_m$$

by assigning (-1) cost to each of the artificial variable
and zero cost to all other variables.

Step 3 Max. Z^* subject to constraints of original
problem using simplex method.

3 cases arises

a) $\text{Max } Z^* < 0$ and at least one artificial variable appears in the optimal basis at a non-zero level.

→ Original problem doesn't possess any feasible soln & procedure comes to an end

b) $\text{Max } Z^* = 0$ & no artificial variable appears in optimal basis

→ Basic feasible soln. O.K.

Proceed to phase II.

c) $\text{Max } Z^* = 0$ and at least one artificial variable appears in optimal basis at zero level.
Here feasible soln. to auxiliary L.P.P. is also a feasible soln. to the original problem with all artificial variables set = 0.

To obtain basic feasible soln., we proceed phase I (for pushing all artificial variables out of basis, without proceeding to phase II)

Phase II

↳ Basic feasible soln. found at the end of phase I is used as starting soln. for original problem in this phase.

original problem in this phase.
and artificial objective function is replaced by the original objective function.

Ex Use two-phase method to

$$\text{Minimize } Z = \frac{15}{2}x_1 - 3x_2$$

$$\begin{aligned} \text{s.t. } 3x_1 - x_2 - x_3 &\geq 3 \rightarrow -s_1, A_1 \\ x_1 - x_2 + x_3 &\geq 2 \rightarrow -s_2, A_2 \end{aligned}$$

Phase I

Step 1 Express in standard form

introduce surplus variable s_1, s_2 +
artificial variable A_1, A_2

$$\text{Max. } Z^* = 0x_1 + 0x_2 + 0x_3 + 0s_1 + 0s_2 - A_1 - A_2$$

$$\text{s.t. } 3x_1 - x_2 - x_3 - s_1 + 0s_2 + A_1 + 0A_2 = 3$$

$$x_1 - x_2 + x_3 + 0s_1 - s_2 + 0A_1 + A_2 = 2$$

$$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$$

Step 2 Find an initial basic feasible soln.

set $x_1 = x_2 = x_3 = s_1 = s_2 = 0$

where $A_1 = 3, A_2 = 2, Z^* = -5$

Simplex Table

	C_j	0	0	0	0	0	-1	-1			
C_B	Basis	x_1	x_2	x_3	s_1	s_2	A_1	A_2	b	θ	
-1	A_1	3	-1	-1	-1	0	1	0	3	$3/3 \leftarrow ②$	
-1	A_2	1	-1	1	0	-1	0	1	2	$2/1$	
	$Z_j = \sum C_B a_{ij}$	-4	2	0	1	1	-1	-1			
	$C_j = c_j - Z_j$	4	-2	0	-1	-1	0	0			
	↑										
	①										

as C_j is +ve under x_1 , this soln. is not optimal

Replace incoming x_1 & outgoing A_1 .

$$R'_1 = R_1 / 3$$

$$R'_2 = R_2 - R_1'$$

	x_1	0	0	0	0	0	-1	-1		b	0
Basis	x_1	x_2	x_3	s_1	s_2	A_1	A_2				
0	x_1	1	$-1/3$	$-1/3$	0 $-1/3$	0 $1/3$	0 $1/3$	0	1	-3	
-1	A_2	0	$-2/3$	$4/3$	$-1/3$	-1	$-1/3$	1	1	$3/4$	\leftarrow

$$Z_j^* \quad 0 \quad 2/3 \quad -4/3 \quad -1/3 \quad 1 \quad 1/3 \quad -1 \quad -1$$

$$G_j = z_j - z_{j^*} \quad 0 \quad -2/3 \quad 4/3 \quad 1/3 \quad -1 \quad -1/3 \quad 0$$

~~$= 2/3$~~
 ~~z_j~~

↑
①

Soln. not optimal

Incoming x_3 & outgoing A_2

$$R_2' \rightarrow R_2 + 3/4$$

$$R_1' \rightarrow R_1 + R_2 \times \frac{1}{3}$$

	x_1	0	0	0	0	0	-1	-1		b	0
Basis	x_1	x_2	x_3	s_1	s_2	A_1	A_2				
0	x_1	1	$-1/2$	0	$-1/4$	$-1/4$	y_1	y_1	y_1	$5/4$	
0	x_3	0	$-1/2$	1	$1/4$	$-3/4$	$-1/4$	$3/4$	$3/4$		
	Z_j^*	0	0	0	0	0	0	0	0		
	$G_j = z_j - z_{j^*}$	0	0	0	0	0	-1	-1			

as all $G \leq 0$, this table give optimal soln

As $Z_{\max} = 0$ & no artificial variable appears in the basis

Phase II

$$\text{Max. } Z' = -15/2x_1 + 3x_2 + 0x_3 + 0s_1 + 0s_2 - 0A_1 - 0A_2$$

$$\text{s.t. } 3x_1 - x_2 - x_3 - s_1 + 0s_2 + A_1 + 0A_2 = 3$$

$$x_1 - x_2 + x_3 + 0s_1 - s_2 + 0A_1 + A_2 = 2$$

$$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$$

Using final table of phase I.

	C_j	-15/2	3	0	0	0	
C_B	Basis	x_1	x_2	x_3	s_1	s_2	b
-15/2	x_1	1	-1/2	0	-1/4	-1/4	5/4
0	x_3	0	-1/2	1	+1/4	-3/4	3/4
Z_j		-15/2	15/4	0	15/8	15/8	-75/8
$C_j - Z_j$	0	-3/4	0	-15/8	-15/8		

As all $C_j \leq 0$ this soln is optimal.

$$x_1 = 5/4, x_2 = 0; x_3 = 3/4 +$$

$$\min Z = 75/8$$

$$\max t = -75/8$$

An LPP has unbounded soln. even if $x_j - z_j \neq 0$ for a non-basic variable because all ratios are -ve or ∞

An LPP yields alternative soln. if $x_j - z_j = 0$ for basic variables & $x_j - z_j \neq 0$ for at least one of the non-basic variables.

An LPP will have unique soln. if $x_j - z_j = 0$ for basic variables & $x_j - z_j \neq 0$ for non-basic variables.

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 1 & 2 & 3 & 4 & 0 \\ 2 & 4 & 6 & 8 & 0 \\ 3 & 6 & 9 & 12 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 1 & 2 & 3 & 4 & 0 \\ 2 & 4 & 6 & 8 & 0 \\ 3 & 6 & 9 & 12 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 1 & 2 & 3 & 4 & 0 \\ 2 & 4 & 6 & 8 & 0 \\ 3 & 6 & 9 & 12 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 1 & 2 & 3 & 4 & 0 \\ 2 & 4 & 6 & 8 & 0 \\ 3 & 6 & 9 & 12 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 1 & 2 & 3 & 4 & 0 \\ 2 & 4 & 6 & 8 & 0 \\ 3 & 6 & 9 & 12 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \hline 1 & 2 & 3 & 4 & 0 \\ 2 & 4 & 6 & 8 & 0 \\ 3 & 6 & 9 & 12 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}$$

Exceptional cases

① Tie for incoming variable.

When more than one variable has the same largest positive value in C_j row (in max. problem) a tie for choice of incoming variables occurs.

No method to break tie.

choose any one of the prospective incoming variables arbitrarily.

↓
No effect on optimality

② Tie for outgoing variable.

When more than one variable has the same least +ve ratio under θ-column, tie for outgoing variable occurs.

→ If equal values of said ratios are > 1

Choose any one of the prospective leaving variables arbitrarily → Doesn't affect optimal

→ If equal values of ratios are zero, the simplex method fails; and one makes use of degeneracy technique.

Degeneracy

A basic feasible soln. is said to be degenerate if any of the basic variables vanishes.

This phenomenon of getting a degenerate basic feasible

sln. is called degeneracy which may arise

① at critical stage \rightarrow when at least one basic variable is zero in the initial basic feasible soln.

② at any subsequent stage, \rightarrow least three ratios under

0-column are equal for 2 or more rows

In this case, an arbitrary choice of one of these basic variable may result in one or more basic variables becoming zero in the next iteration

A sometimes, same sequence of simplex iterations is

repeated endlessly without improving the soln.

These are termed as cycling type of problem

Cycling occurs very rarely.

To avoid cycling, one apply perturbation procedure

Perturbation procedure

- ① Divide each element in the tied rows by the coefficients of the key column in that row
- ② Compare resulting ratios (from left to right) first of unit in + terms of the body in columns
- ③ Outgoing variables lie in that row which first contains the smallest algebraic ratio.

Ex Max. $Z = 5x_1 + 3x_2$

$$\text{s.t. } x_1 + x_2 \leq 2$$

$$5x_1 + 2x_2 \leq 10$$

$$3x_1 + 8x_2 \leq 12$$

$$x_1, x_2 \geq 0$$

Standard form

$$\text{Max. } Z = 5x_1 + 3x_2 + 0s_1 + 0s_2 + 0s_3$$

$$\text{s.t. } x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 2$$

$$5x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 = 10$$

$$3x_1 + 8x_2 + 0s_1 + 0s_2 + s_3 = 12$$

$$x_1, x_2, s_1, s_2, s_3 \geq 0$$

Initial basic feasible sol.

$$x_1 = x_2 = 0 \text{ (non-basic)}$$

$$S_1 = 2, S_2 = 10, S_3 = 12 \text{ (basic)} \Rightarrow Z = 0$$

Simpler form

	C_j	5	3	0	0	d	b	0
C_B	Basis	x_1	x_2	S_1	S_2	S_3		
0	S_1	1	1	1	0	0	2	$2/1$
0	S_2	<u>1/5</u>	2	0	1	0	10	$10/5$ ←
0	S_3	3	8	0	0	1	12	$12/3$

$$Z_j = \sum c_j a_{ij} \quad 0 \quad 0 \quad 0 \quad 0$$

$$G_j = g_j - Z_j \quad 5 \quad 3 \quad 0 \quad 0 \quad 0$$

↑
①

0 key column is
 x_1
 $\frac{1}{5}$
5
5

x_1 is in coming.

But first 2 rows has same ratio under 0.

∴ we apply perturbation method

First column of and if has 1 + 0 in the row.

Divide this by corresponding elements of the key column.

on get $1/1 + 0/5$, S_2 row gives smaller

ratio ∴ S_2 is the outgoing variable

② Divide ~~the whole~~ S_1 column by key column

$S_1 \rightarrow 1/1 + 0/5$ ∵ as S_2 row is less
we use S_2 as outgoing

C_j	5	3	0	0	0			
C_B	Basic	x_1	x_2	s_1	s_2	s_3	b	0
0	s_1	0	$\boxed{3/5}$	1	- $1/5$	0	0	$0 \leftarrow ②$
5	x_1	1	$2/5$	0	$1/5$	0	2	5
0	s_3	0	$34/5$	0	$-3/5$	1	6	$15/10$

Z_j	5	2	0	1	6
	0	1	0	-1	0

$$G_j = q - Z_j$$

$$0 \quad 1 \quad 0 \quad -1 \quad 0$$

$\uparrow ①$

$G_j + w$, not optimal $\rightarrow 3/5$ is key

incoming x_2 & outgoing x_1

C_j	5	3	0	0	0		
C_B	Basic	x_1	x_2	s_1	s_2	s_3	b
0	x_2	0	3/5 1	5/3	$-1/3$	0	0
3		1	2/5 0	$-2/3$	$1/3$	0	2
5	x_1	1	2/5 0	$-2/3$	$1/3$	0	2
0	s_3	0	0	$-34/5$	$5/3$	1	6

Z_j	5	3	$5/3$	$2/3$	0
	0	0	$-5/3$	$-2/3$	0

As $G_j \leq 0$, optimal soln

feasible soln $\rightarrow x_1 = 2, x_2 = 0 \leftarrow \underline{\underline{Z_{\max} = 10}}$

Duality concept

One of the interesting concept in linear programming is duality theory.

Every linear programming problem has associated with it another linear programming problem involving same data & closely related optimal soln. Such 2 problems are said to be dual of each other.

One of these is called primal,
other the dual.

Importance is due to fact

- If primal contains a large no. of constraints & a smaller variable, labour of computation can be considerably reduced by converting it into the dual problem.
- Interpretation of dual problem from cost or economic point of view proves extremely useful in making future decisions.

4a Formulation of dual Problem

$$\begin{aligned} \text{Max. } Z &= c_1 x_1 + c_2 x_2 + \dots + c_n x_n \\ \text{s.t. } a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n &\leq b_1 \\ a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n &\leq b_2 \\ \vdots \\ a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n &\leq b_m \\ x_1, x_2, \dots, x_n &\geq 0 \end{aligned}$$

To construct dual problem: one adopt following
guidelines

- ① Max. problem in primal becomes minimization problem in the dual + nice needs.
- ② (\leq) type of constraints in primal becomes (\geq) type in dual. + nice needs
- ③ Coeff c_1, c_2, \dots, c_n in objective fn of primal becomes b_1, b_2, \dots, b_n in objective fn of dual.
- ④ Constraints b_1, b_2, \dots, b_n in constraints of primal become c_1, c_2, \dots, c_n constraint of the dual.
- ⑤ Primal has n variables + m constraints, dual will have m variables + n constraints. i.e. transpose the body mth of primal problem give body mth of dual.
- ⑥ Variables in both primal + dual are non-negative.

4b

$$\text{Min. } W = b_1 y_1 + b_2 y_2 + \dots + b_m y_m$$

$$\begin{aligned} \text{s.t. } & a_{11} y_1 + a_{21} y_2 + \dots + a_{m1} y_m \geq c_1, \\ & a_{12} y_1 + a_{22} y_2 + \dots + a_{m2} y_m \geq c_2 \\ & \vdots \\ & a_{1n} y_1 + a_{2n} y_2 + \dots + a_{mn} y_m \geq c_n \\ & y_1, y_2, \dots, y_m \geq 0 \end{aligned}$$

Ex

$$\text{Min. } Z = 3x_1 - 2x_2 + 4x_3$$

$$\text{s.t. } 3x_1 + 5x_2 + 4x_3 \geq 7 \quad ①$$

$$6x_1 + x_2 + 3x_3 \geq 4 \rightarrow ②$$

$$7x_1 - 2x_2 - x_3 \leq 10 \rightarrow ③$$

$$x_1 - 2x_2 + 5x_3 \geq 3 \rightarrow ④$$

$$4x_1 + 7x_2 - 2x_3 \geq 2 \quad -⑤$$

$$x_1, x_2, x_3 \geq 0$$

$$③ \rightarrow \text{converted to } -7x_1 + 2x_2 + x_3 \geq 10$$

Let y_1, y_2, y_3, y_4, y_5 be the dual variables associated with the above 5 constraint

Dual problem is given by

$$\begin{aligned}
 \text{Max. } W &= 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5 \\
 \text{s.t.} \quad &3y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 \leq 3 \\
 &5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2 \\
 &4y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \leq 4 \\
 &y_1, y_2, y_3, y_4, y_5 \geq 0
 \end{aligned}$$

formulation of dual problem when primal has equality constraint.

$$\begin{aligned}
 \text{Max. } Z &= C_1 x_1 + C_2 x_2 \\
 \text{s.t.} \quad &a_{11} x_1 + a_{12} x_2 = b_1, \\
 &a_{21} x_1 + a_{22} x_2 \leq b_2 \\
 &x_1, x_2 \geq 0
 \end{aligned}$$

Equality constraints written as

$$\begin{aligned}
 a_{11} x_1 + a_{12} x_2 \leq b_1 \quad \text{and} \quad a_{11} x_1 + a_{12} x_2 \geq b_1, \\
 \text{or} \quad a_{11} x_1 + a_{12} x_2 \leq b_1 \quad + \quad -a_{11} x_1 - a_{12} x_2 \leq -b_1,
 \end{aligned}$$

Then above problem is written as

$$\begin{aligned}
 \text{Max. } Z &= C_1 x_1 + C_2 x_2 \\
 \text{s.t.} \quad &a_{11} x_1 + a_{12} x_2 \leq b_1, \\
 &-a_{11} x_1 - a_{12} x_2 \leq -b_1, \\
 &a_{21} x_1 + a_{22} x_2 \leq b_2 \\
 &x_1, x_2 \geq 0
 \end{aligned}$$

Now we form the dual using y_1' , y_1'' , y_2

Then dual problem is

$$\text{Min } W = b_1(y_1' - y_1'') + b_2 y_2,$$

$$\text{s.t. } a_{11}(y_1' - y_1'') + a_{21} y_2 \geq c,$$

$$a_{12}(y_1' - y_1'') + a_{22} y_2 \geq c_2$$

$$y_1', y_1'', y_2 \geq 0$$

If y_1 , unrestricted in sign. Then

$$\text{Min } W = b_1 y_1 + b_2 y_2$$

$$\text{s.t. } a_{11} y_1 + a_{21} y_2 \geq c,$$

$$a_{12} y_1 + a_{22} y_2 \geq c_2$$

$$y_2 \geq 0$$

In general
of primal problem

$$\text{Max. } Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

$$\text{s.t. } a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1 \\ a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n = b_2$$

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n = b_m \\ x_1, x_2, \dots, x_n \geq 0$$

Let say y_1, y_2, \dots, y_m all unrestricted & s.t.

then one can

write dual problem as

$\begin{cases} \text{Max. } W = b_1(y_1' - y_1'') + b_2 y_2 \\ \text{s.t. } a_{11}(y_1' - y_1'') + a_{21} y_2 \geq c, \\ a_{12}(y_1' - y_1'') + a_{22} y_2 \geq c_2 \\ y_1', y_1'', y_2 \geq 0 \end{cases}$

if we see $(y_1' - y_1'')$ appears in both objectives + all constraints of dual. happen when equality constraint in primal

Then new variable $y_1' - y_1'' (= y_1)$ become unrestricted in sign

$$\begin{aligned}
 \text{Min. } W &= b_1 y_1 + b_2 y_2 + \dots + b_m y_m \\
 \text{s.t.} \quad a_{11} y_1 + a_{21} y_2 + \dots + a_{m1} y_m &\geq c_1 \\
 a_{12} y_1 + a_{22} y_2 + \dots + a_{m2} y_m &\geq c_2 \\
 &\vdots \\
 a_{1n} y_1 + a_{2n} y_2 + \dots + a_{mn} y_m &\geq c_n
 \end{aligned}$$

$y_1, y_2, \dots, y_m \rightarrow \text{any sign unrestricted in sign}$

Duality principle

If the primal & dual problems have feasible solns then both have optimal solns & the optimal value of the primal objective function is equal to the optimal value of the dual objective function i.e.

$$\text{Max. } Z = \text{Min. } W$$

This is fundamental theorem of duality.

2) Working rules for obtaining an optimal soln to the primal (dual) problem from that of dual (primal)

-6a
Suppose one has already found an optimal soln to the dual (primal) problem by simplex method

Rule I → If the primal variable corresponds to a slack starting variable in the dual problem, then its optimal value is directly given by the coefficient of the slack variable with changed sign; in the C_j row of the optimal dual simplex table. & vice-versa.

Rule II → If primal variable corresponds to an artificial starting variable in the dual problem, then its optimal value is directly given by the coefficient of the artificial variable, with changed sign, in the C_j row of the optimal dual simplex table, after deleting the constant M & vice-versa.

If the primal has an unbounded soln, then dual problem will not have a feasible soln. & vice-versa

L1

Construct dual of

$$\text{Max. } Z = 2x_1 + x_2$$

$$\text{s.t. } -x_1 + 2x_2 \leq 2$$

$$x_1 + x_2 \leq 4$$

$$x_1 \leq 3;$$

$$x_1, x_2 \geq 0$$

dual problem

$$\text{Min. } W = 2y_1 + 4y_2 + 3y_3$$

$$\text{s.t. } -y_1 + y_2 + y_3 \geq 2 \quad -s_1, A_1$$

$$2y_1 + y_2 \geq 4 \quad \rightarrow -s_2, A_2$$

$$y_1, y_2 \geq 0$$

Express problem in Standard form

$$\text{Max. } W' = -2y_1 - 4y_2 - 3y_3 + 0s_1 + 0s_2 - MA_1 - MA_2$$

$$\text{s.t. } -y_1 + y_2 + y_3 - s_1 + 0s_2 + A_1 + 0A_2 = 2$$

$$2y_1 + y_2 + 0y_3 + 0s_1 - s_2 + 0A_1 + A_2 = 1$$

$$y_1 = y_2 = y_3 = s_1 = s_2 = 0 \quad (\text{non basic})$$

$$A_1 = 2, A_2 = 1 \quad (\text{basic})$$

7a

Initial Simplex Table

C_j	-2	-4	-3	0	0	-M	-M			
C_B	Bas's	y_1	y_2	y_3	s_1	s_2	A_1	A_2	b	θ
-M	A_1	-1	1	0	-1	0	1	0	2	2/1
-M	A_2	2	1	0	0	-1	0	1	1	1/1 ②
Z_j		$-M$	$-2M$	M	M	$-M$	$-M$			
$G_j = C_j - Z_j$		$M-2$	$2M-4$	$M-3$	$-M$	0	0			

↑
①

Incoming y_2 & outgoing $A_2 \rightarrow R_1' \rightarrow R_1 - R_2$

C_j	-2	-4	-3	0	0	-M	-M			
C_B	Bas's	y_1	y_2	y_3	s_1	s_2	A_1	A_2	b	θ
-M ③	A_1	-3	0	1	-1	1	1	-1	1	1/1 ←
-M	y_2	2	1	0	0	-1	0	1	1	1/0
Z_j		$3M-8$	$-a$	$-M$	M	$4-M$	$-M$	$M-4$		
G_j		$6-3M$	0	$M-3$	$-M$	$M-4$	0	$4-2M$		

↑
①

Incoming y_3 & outgoing A_1

$$1st \rightarrow R_2' \rightarrow R_2 - 0 \cdot R_1$$

y_1	-2	-4	-3	0	0	-M	-M
Base	y_1	y_2	y_3	s.	s_2	A_1	A_2
-3	y_3	-3	0	1	-1	1	b
-4	y_2	2	1	0	10	-1	0
						1	1
Z_j	1	-4	-3	3	1	-3	-1
G	-3	0	0	-3	-1	$3-M$	$1-M$

As all $G \leq 0$, optimal soln. attained

The the Optimal soln to dual problem is

$$y_1=0, y_2=1, y_3=1$$

$$\text{Max } W = -\text{Max. } (W') = 7$$

To derive optimal basic feasible soln. to the primal problem, we note that primal variables x_1, x_2 correspond to artificial starting dual variables A_1, A_2 .

we note that primal variables x_1, x_2 correspond to artificial starting dual variables A_1, A_2 respectively.

In the final simplex table of the dual problem g corresponding to A_1 & A_2 are 3 & 1 respectively. after ignoring M..

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Thus by rule 2, we get

$$x_1 = 3 + x_2 = 1$$

Hence optimal basic feasible soln. to the given
problem is

$$x_1 = 3 + x_2 = 1, \max Z = 7$$

$$\rightarrow \max Z = \min \underline{W} = 7 \rightarrow \text{Duality theorem}$$

~~Ex~~ Using duality solve the ~~given~~

$$\text{Min. } Z = 0.7x_1 + 0.5x_2$$

$$\text{s.t. } x_1 \geq 4$$

$$x_2 \geq 6$$

$$x_1 + 2x_2 \geq 20$$

$$2x_1 + x_2 \geq 18$$

$$x_1, x_2 \geq 0$$

Dual of the given problem is

$$\text{Max. } W = 4y_1 + 6y_2 + 20y_3 + 18y_4$$

$$\text{s.t. } y_1 + y_3 + 2y_4 \leq 0.7$$

$$y_2 + 2y_3 + y_4 \leq 0.5$$

$$y_1, y_2, y_3, y_4 \geq 0$$

Express problem

Introducing slack Variable

Convert problem in standard form

dual problem in standard form

$$\text{Max. } W = 4y_1 + 6y_2 + 20y_3 + 18y_4 + 0s_1 + 0s_2 = 0.7$$

$$\text{s.t.} \quad y_1 + 0y_2 + y_3 + 2y_4 + s_1 + 0s_2 = 0.5$$

$$0y_1 + y_2 + 2y_3 + 0y_4 + 0s_1 + s_2 = 0$$

$$y_1, y_2, y_3, y_4 \geq 0$$

Find initial basic feasible soln.

$$y_1 = y_2 = y_3 = y_4 = 0 \text{ (non-basic)}$$

$$s_1 = 0.7 \quad [\text{basic}]$$

$$s_2 = 0.5$$

Since basic variables $s_1, s_2 > 0$, initial basic soln.

feasible & non-degenerate

C_j	4	6	20	18	0	0	b	θ
C_B	Base	y_1	y_2	y_3	y_4	s_1	s_2	
0	s_1	1	0	1	2	1	0	0.7 0.7/1
0	s_2	0	1	2	0	1	0.5	0.5/2 \leftarrow ②

$$Z_j = 0 \cdot 4 + 0 \cdot 6 + 0 \cdot 20 + 0 \cdot 18 + 0 \cdot 0 + 0 \cdot 0$$

$$C_j = 4 - 4 + 6 + 20 + 18 + 0 + 0$$

↑
①

$$R_2' \rightarrow R_2 / 2$$

$$R_1' \rightarrow R_1 - R_2'$$

out going s_2 , incoming y_3

$$\begin{array}{ccccccccc}
 g & 4 & 6 & 20 & 18 & 0 & 0 \\
 \text{Base} & y_1 & y_2 & y_3 & y_4 & s_1 & s_2 & b & \theta \\
 0 & s_1 & 1 & -y_2 & 0 & 3/2 & 1 - y_2 & 9/20 & 3/10 \leftarrow \\
 20 & y_3 & 0 & y_2 & 1 & y_2 & 0 & y_4 & 1/2
 \end{array}$$

$$z_j \quad 0 \quad 10 \quad 20 \quad 10 \quad 0 \quad 10$$

$$G = g - z_j \quad 4 \quad -4 \quad 0 \quad 8 \quad 0 \quad -10$$

↑
①

$$R_1' \rightarrow R_1 / (3/2)$$

out going s_1

$$R_2' \rightarrow R_2 - R_1 \cdot \frac{1}{2}$$

incoming y_3

$$\begin{array}{ccccccccc}
 g & 4 & 6 & 20 & 18 & 0 & 0 \\
 \text{Base} & y_1 & y_2 & y_3 & y_4 & s_1 & s_2 & b & \theta \\
 18 & y_4 & 2/3 & -y_3 & 0 & 1 & 2/3 & -y_3 & 3/10 \\
 20 & y_3 & -y_3 & 2/3 & 1 & 0 & -y_3 & 2/3 & 1/10
 \end{array}$$

$$z_j \quad 16/3 \quad 22/3 \quad 20 \quad 18 \quad 16/3 \quad 22/3$$

$$g \quad -4/3 \quad -y_3 \quad 0 \quad 0 - 16/3 \quad -22/3$$

As all $g_i \leq 0$,

take y_1 as optimal soln.

$$y_1 = 0, y_2 = 0, y_3 = 1/10, y_4 = 3/10$$

$$\therefore W = 7.4$$

Optimal soln. to prim.

Primal variables x_1, x_2 correspond to slack starting ~~and~~ dual variables s_1, s_2 respectively.

Given corresponding to s_1, s_2 are ~~-16/3~~ and $-22/3$

Thus by rule I

$$\text{Opt } x_1 = 16/3 + 0 + x_2 = 22/3$$

$$x_1 = 16/3 + x_2 = 22/3$$

$$\text{min. } \underline{Z = 7.4}$$