

Note  
If an artificial variable appears in the basis at non-zero level.  
There exists a pseudo optimal soln. to the problem!

## Two phase Method

Another method to deal with the artificial variables whereas L.P.P. is solved with two phases.

Phase I  
Step 1 Express in standard form by introducing slack, surplus & artificial variables

Step 2 Formulate an artificial objective fn

$$Z^* = -A_1 - A_2 \dots - A_m$$

by assigning (-1) cost to each of the artificial variables.

$A_i$  and zero cost to all other variables.

Step 3 Max.  $Z^*$  subject to constraints of original problem using simplex method.

3 cases arise

-9a

a)  $\text{Max } Z^* < 0$  and at least one artificial variable appears in the optimal basis at a +ve level.

→ Original problem doesn't possess any feasible soln & procedure comes to an end

b)  $\text{Max } Z^* = 0$  & no artificial variable appears in optimal basis

→ Basic feasible soln. O.K.

Proceed to phase II.

c)  $\text{Max } Z^* = 0$  and at least one artificial variable appears in optimal basis at zero level

Here feasible soln. to auxiliary L.P.P. is also a feasible soln. to the original problem with all artificial variables set = 0.

To obtain basic feasible soln., we prolong phase I for pushing all artificial variables out of basis (without proceeding to phase II)

## Phase II

↳ Basic feasible soln. found at the end of phase I is used as starting soln. for original problem in this phase.  
and artificial objective function is replaced by the original objective function.

Ex Use two-phase method to

Minimize  $Z = \frac{15}{2}x_1 - 3x_2$

s.t.  $3x_1 - x_2 - x_3 \geq 3 \rightarrow -s_1, A_1$   
 $x_1 - x_2 + x_3 \geq 2 \rightarrow -s_2, A_2$

## Phase I

Step 1 Express in standard form

Introduce surplus variable  $s_1, s_2$  +  
artificial variable  $A_1$  +  $A_2$

Max.  $Z^* = 0x_1 + 0x_2 + 0x_3 + 0s_1 + 0s_2 - A_1 - A_2$

s.t.  $3x_1 - x_2 - x_3 - s_1 + 0s_2 + A_1 + 0A_2 = 3$

$x_1 - x_2 + x_3 + 0s_1 - s_2 + 0A_1 + A_2 = 2$

$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$

Step 2 Find an initial basic feasible soln.

$$\text{set } x_1 = x_2 = x_3 = s_1 = s_2 = 0$$

$$\text{where } A_1 = 3, A_2 = 2, Z^* = -5$$

Simplex Table

$C_j$		0	0	0	0	0	-1	-1		
	Basic	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$A_1$	$A_2$	$b$	$\theta$
-1	$A_1$	<span style="border: 1px solid black; padding: 2px;">3</span>	-1	-1	-1	0	1	0	3	$3/3 \leftarrow \textcircled{2}$
-1	$A_2$	1	-1	1	0	-1	0	1	2	$2/1$
$Z_j = \sum C_B a_{ij}$		-4	2	0	1	1	-1	-1		
$C_j - Z_j$		4	-2	0	-1	-1	0	0		

↑  
 $\textcircled{1}$

as  $C_j$  is +ve under  $x_1$ , this soln. is not optimal

~~Rep~~ Incoming  $x_1$  & out going  $A_1$

$$R_1' = R_1 / 3$$

$$R_2' = R_2 - R_1'$$

	$C_j$	0	0	0	0	0	-1	-1		
	Basis	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$A_1$	$A_2$	$b$	$\theta$
0	$x_1$	1	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{1}{3}$	0	1	-3
-1	$A_2$	0	$-\frac{2}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$	-1	$-\frac{1}{3}$	1	1	$\frac{3}{4} \leftarrow$

$$Z_j^* \quad 0 \quad \frac{2}{3} \quad -\frac{4}{3} \quad -\frac{1}{3} \quad 1 \quad \frac{1}{3} \quad -1 \quad -1$$

$$G = -\frac{4}{3}$$

$$0 \quad -\frac{2}{3} \quad \frac{4}{3} \quad \frac{1}{3} \quad -1 \quad -\frac{1}{3} \quad 0$$

↑  
①

Soln. not optimal

Incomy  $x_3$  & out goes  $A_2$

$$R_2' \rightarrow R_2 \times \frac{3}{4}$$

$$R_1' \rightarrow R_1 + R_2 \times \frac{1}{3}$$

	$C_j$	0	0	0	0	0	-1	-1		
	Basis	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$A_1$	$A_2$	$b$	$\theta$
0	$x_1$	1	$-\frac{1}{2}$	0	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{5}{4}$	
0	$x_3$	0	$-\frac{1}{2}$	1	$\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	
	$Z_j^*$	0	0	0	0	0	0	0		
	$G_j = C_j - Z_j$	0	0	0	0	0	-1	-1		

as all  $G \leq 0$ , this table give optimal soln

As  $Z_{\max} = 0$  & no artificial variable appears in the basis

Phase II

$$\text{Max. } Z' = -15/2 x_1 + 3x_2 + 0x_3 + 0s_1 + 0s_2 - 0A_1 - 0A_2$$

$$\text{s.t. } 3x_1 - x_2 - x_3 - s_1 + 0s_2 + A_1 + 0A_2 = 3$$

$$x_1 - x_2 + x_3 + 0s_1 - s_2 + 0A_1 + A_2 = 2$$

$$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$$

Using final table of phase I.

	$C_j$	-15/2	3	0	0	0	
	Basis	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	b
$C_B$							
-15/2	$x_1$	1	-1/2	0	-1/4	-1/4	5/4
0	$x_3$	0	-1/2	1	+1/4	-3/4	3/4
$Z_j$		-15/2	15/4	0	15/8	15/8	-75/8
$C_j - Z_j$		0	-3/4	0	-15/8	-15/8	

As all  $C_j \leq 0$  this soln is optimal.

$$x_1 = 5/4, x_2 = 0; x_3 = 3/4 +$$

$$\min Z = 75/8$$

$$\max Z = -75/8$$

An LPP has unbounded soln. even if  $c_j - z_j$  is +ve for a non-basic variable because all ratios are -ve or  $\infty$

An LPP yields alternative soln. if  $c_j - z_j = 0$  for basic variable &  $c_j - z_j = 0$  for at least one of the non-basic variables

An LPP will have unique soln. if  $c_j - z_j = 0$  for basic variables &  $c_j - z_j \neq 0$  for non-basic variables.

## Exceptional case

### ① Tie for incoming variable.

When more than one variable has the same largest positive value in  $C_j$  row (in max. problem) a tie for choice of incoming variable occurs.

No method to break tie.

Choose any one of the prospective incoming variables arbitrarily.

↓  
No effect on optimum

### ② Tie for outgoing variable.

When more than one variable has the same least +ve ratio under  $\theta$ -column, tie for outgoing variable occurs.

→ If equal values of said ratio are  $> 1$

Choose any one of the prospective leaving variables arbitrarily → Doesn't affect optimum

→ If equal values of ratios are zero, the simplex method fails; and one makes use of degeneracy technique.

## Degeneracy

A Basic feasible soln. is said to be degenerate if any of the basic variables vanishes.

This phenomenon of getting a degenerate basic feasible soln. is called degeneracy which may arise

- ① at initial stage  $\rightarrow$  when at least one basic variable is zero in the initial basic feasible soln.
- ② at any subsequent stage,  $\rightarrow$  least two ratios under  $\theta$ -column are equal for 2 or more rows

In this case, an arbitrary choice of one of these basic variables may result in one or more basic variables becoming zero in the next iteration.

Sometimes, same sequence of simplex iteration is repeated endlessly without improving the soln.

These are termed as cycling type of problem. Cycling occurs very rarely.

To avoid cycling, one applies perturbation procedure.

## Perturbation procedure

- ① Divide each element in the tied rows by the coefficient of the key column in that row
- ② Compare resulting ratios (from left to right) first of unit  $\bar{r}_i$  + terms of the body  $\bar{r}_i$ , column by column.
- ③ Outgoing variable lies in that row which first contains the smallest algebraic ratio.

Ex

$$\text{Max. } Z = 5x_1 + 3x_2$$

$$\text{s.t. } x_1 + x_2 \leq 2$$

$$5x_1 + 2x_2 \leq 10$$

$$3x_1 + 8x_2 \leq 12$$

$$x_1, x_2 \geq 0$$

Standard form

$$\text{Max } Z = 5x_1 + 3x_2 + 0s_1 + 0s_2 + 0s_3$$

$$\text{s.t. } x_1 + x_2 + s_1 + 0s_2 + 0s_3 = 2$$

$$5x_1 + 2x_2 + 0s_1 + s_2 + 0s_3 = 10$$

$$3x_1 + 8x_2 + 0s_1 + 0s_2 + s_3 = 12$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

Initial basic feasible solution

$$x_1 = x_2 = 0 \text{ (non-basic)}$$

$$S_1 = 2, S_2 = 10, S_3 = 12 \text{ (basic)} \text{ and } Z = 0$$

Simplex table

	$C_j$	5	3	0	0	0	b	$\theta$
$C_B$	Basis	$x_1$	$x_2$	$S_1$	$S_2$	$S_3$		
0	$S_1$	1	1	1	0	0	2	2/1
0	$S_2$	<u>5</u>	2	0	1	0	10	10/5 ←
0	$S_3$	3	8	0	0	1	12	12/3

$$Z_j = \sum C_j a_{ij} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

$$G_j = C_j - Z_j \quad 5 \quad 3 \quad 0 \quad 0 \quad 0$$

↑  
①

① key column is  $x_1$   
↓  
5  
1  
3

$x_1$  is in comp.

But first 2 row has same ratio under ①.

∴, we apply perturbation method.

First column of under ① has 1 & 0 in the row.  
Divide this by corresponding elements of the key column.

we get  $1/1$  &  $0/5$ ,  $S_2$  row gives smaller ratio ∴  $S_2$  is the outgoing variable.

② Divide ~~the key column~~  $S_1$  column by key column

$$S_1 \text{ row} \rightarrow 1/1 \text{ and } 0/5$$

∴ As  $S_2$  row is less  
we use  $S_2$  as outgoing

	$C_j$	5	3	0	0	0		
$C_B$	Basis	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$b$	$\theta$
0	$s_1$	0	<span style="border: 1px solid black;">3/5</span>	1	-1/5	0	0	0 $\leftarrow$ ②
5	$x_1$	1	2/5	0	1/5	0	2	5
0	$s_3$	0	34/5	0	-3/5	1	6	15/18
$Z_j$		5	2	0	1	0		
$G = Z_j - C_j$		0	1	0	-1	0		

$G > 0$ , not optimal  $\rightarrow$  3/5 is key

incoming  $x_2$  & outgoing  $s_1$

	$C_j$	5	3	0	0	0	
$C_B$	Basis	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$b$
3	$x_2$	0	<del>3/5</del> 1	5/3	-1/3	0	0
5	$x_1$	1	<del>2/5</del> 0	-2/3	1/3	0	2
0	$s_3$	0	0	-34/3	5/3	1	6
$Z_j$		5	3	5/3	2/3	0	

$$G = 0 \quad 0 \quad -5/3 \quad -2/3$$

As  $G \leq 0$ , optimal soln

feasible soln  $\hookrightarrow x_1 = 2, x_2 = 0 \rightarrow Z_{\max} = 10$

## Duality concept

One of the interesting concept in linear programming is duality theory.

Every linear programming problem has associated with it; another linear programming problem involving same data & closely related optimal soln. Such 2 problems are said to be dual of each other.

while one of these is called primal, other the dual.

Importance is due to fact

- If primal contains a large no. of constraints & a smaller variable, labour of computation can be considerably reduced by converting it into the dual problem.
- Interpretation of dual problem from cost or economic point of view proves extremely useful in making future decisions.

## Formulation of dual Problem

$$\begin{aligned}
 \text{Max. } Z &= C_1 x_1 + C_2 x_2 + \dots + C_n x_n \\
 \text{s.t. } a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n &\leq b_1 \\
 a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n &\leq b_2 \\
 &\vdots \\
 a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n &\leq b_m \\
 x_1, x_2, \dots, x_n &\geq 0
 \end{aligned}$$

To construct dual problem: one adopts following guide-lines

- ① Max. problem in primal becomes minimization problem in the dual + vice versa.
- ② ( $\leq$ ) type of constraints in primal becomes ( $\geq$ ) type in dual + vice versa.
- ③ Coeff  $C_1, C_2, \dots, C_n$  in objective fn of primal becomes  $b_1, b_2, \dots, b_n$  in objective fn of dual.
- ④ Constraints  $b_1, b_2, \dots, b_n$  in constraint of primal become  $C_1, C_2, \dots, C_n$  constraint of the dual.
- ⑤ Primal has  $n$  variables +  $m$  constraints, dual will have  $m$  variables +  $n$  constraints. i.e. Transpose of the body of primal problem gives body of dual.
- ⑥ Variables in both primal + dual are non-negative.

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$$\text{Min. } W = b_1 y_1 + b_2 y_2 + \dots + b_m y_m$$

$$\text{s.t. } \begin{aligned} a_{11} y_1 + a_{21} y_2 + \dots + a_{m1} y_m &\geq c_1 \\ a_{12} y_1 + a_{22} y_2 + \dots + a_{m2} y_m &\geq c_2 \\ &\vdots \\ a_{1n} y_1 + a_{2n} y_2 + \dots + a_{mn} y_m &\geq c_n \\ y_1, y_2, \dots, y_m &\geq 0 \end{aligned}$$

Ex

$$\text{Min. } Z = 3x_1 - 2x_2 + 4x_3$$

$$\text{s.t. } 3x_1 + 5x_2 + 4x_3 \geq 7 \quad (1)$$

$$6x_1 + x_2 + 3x_3 \geq 4 \rightarrow (2)$$

$$7x_1 - 2x_2 - x_3 \leq 10 \rightarrow (3)$$

$$x_1 - 2x_2 + 5x_3 \geq 3 \rightarrow (4)$$

$$4x_1 + 7x_2 - 2x_3 \geq 2 \quad (5)$$

$$x_1, x_2, x_3 \geq 0$$

$$(3) \rightarrow \text{converted to } -7x_1 + 2x_2 + x_3 \geq -10$$

Let  $y_1, y_2, y_3, y_4, y_5$  be the dual variables associated with the above 5 constraints

Dual problem is given by

$$\begin{aligned} \text{Max. } W &= 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5 \\ \text{s.t. } &3y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 \leq 3 \\ &5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2 \\ &4y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \leq 4 \\ &y_1, y_2, y_3, y_4, y_5 \geq 0 \end{aligned}$$

Formulation of dual problem when primal has equality constraint.

$$\begin{aligned} \text{Max. } Z &= C_1 x_1 + C_2 x_2 \\ \text{s.t. } &a_{11} x_1 + a_{12} x_2 = b_1 \\ &a_{21} x_1 + a_{22} x_2 \leq b_2 \\ &x_1, x_2 \geq 0 \end{aligned}$$

Equality constraints written as

$$\begin{aligned} &a_{11} x_1 + a_{12} x_2 \leq b_1 \quad \text{and} \quad a_{11} x_1 + a_{12} x_2 \geq b_1 \\ \text{or } &a_{11} x_1 + a_{12} x_2 \leq b_1 \quad + \quad -a_{11} x_1 - a_{12} x_2 \leq -b_1 \end{aligned}$$

Then above problem is written as

$$\begin{aligned} \text{Max. } Z &= C_1 x_1 + C_2 x_2 \\ \text{s.t. } &a_{11} x_1 + a_{12} x_2 \leq b_1 \\ &-a_{11} x_1 - a_{12} x_2 \leq -b_1 \\ &a_{21} x_1 + a_{22} x_2 \leq b_2 \\ &x_1, x_2 \geq 0 \end{aligned}$$

Now we form the dual using  $y_1', y_1'', y_2$

Then dual problem is

$$\text{Min } W = b_1(y_1' - y_1'') + b_2 y_2,$$

$$\text{s.t. } a_{11}(y_1' - y_1'') + a_{21} y_2 \geq c_1,$$

$$a_{12}(y_1' - y_1'') + a_{22} y_2 \geq c_2$$

$$y_1', y_1'', y_2 \geq 0$$

If  $y_1$  unrestricted in sign. then

$$\text{Max } W = b_1 y_1 + b_2 y_2$$

$$\text{s.t. } a_{11} y_1 + a_{21} y_2 \geq c_1,$$

$$a_{12} y_1 + a_{22} y_2 \geq c_2$$

$$y_2 \geq 0$$

In general of primal problem

$$\text{Max. } Z = C_1 x_1 + C_2 x_2 + \dots + C_n x_n$$

$$\text{s.t. } a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1,$$

$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n = b_2$$

$$\vdots$$

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n = b_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

Let say  $y_1, y_2, \dots, y_m$  all unrestricted sign

then one can

write dual problem as

if we use  $(y_1' - y_1'') = 0$ .  
 $(y_1' - y_1'')$  appears in both objective function + all constraints of dual. happens when equality constraint in primal. Then new variable  $y_1' - y_1'' (= y_1)$  become unrestricted sign.

$$\text{Min. } W = b_1 y_1 + b_2 y_2 + \dots + b_m y_m$$

$$\text{s.t. } a_{11} y_1 + a_{21} y_2 + \dots + a_{m1} y_m \geq C_1$$

$$a_{12} y_1 + a_{22} y_2 + \dots + a_{m2} y_m \geq C_2$$

$$\vdots$$

$$a_{1n} y_1 + a_{2n} y_2 + \dots + a_{mn} y_m \geq C_n$$

$y_1, y_2, \dots, y_m \rightarrow$  any sign unrestricted in sign

## Duality principle

If the primal & dual problems have feasible solns then both have optimal solns & the optimal value of the primal objective function is equal to the optimal value of the dual objective function i.e.

$$\text{Max. } Z = \text{Min. } W$$

This is fundamental theorem of duality.

2) Working rules for obtaining an optimal soln. to the primal (dual) problem from that of dual (primal)

Suppose one has already found an optimal soln to the dual (primal) problem by simplex method

Rule I  
→ If the primal variable corresponds to a slack starting variable in the dual problem, then its optimal value is directly given by the coefficient of the slack variable with changed sign; in the  $C_j$  row of the optimal dual simplex table. & vice-versa.

Rule II  
→ If primal variable corresponds to an artificial starting variable in the dual problem, then its optimal value is directly given by the coefficient of the artificial variable, with changed sign, in the  $C_j$  row of the optimal dual simplex table, after deleting the constant  $M$  & vice-versa.

If the primal has an unbounded soln, then dual problem will not have a feasible soln. & vice-versa.

Ex

Construct dual of

$$\text{Max. } Z = 2x_1 + x_2$$

$$\text{s.t. } -x_1 + 2x_2 \leq 2$$

$$x_1 + x_2 \leq 4$$

$$x_1 \leq 3;$$

$$x_1, x_2 \geq 0$$

dual problem

$$\text{Min } W = 2y_1 + 4y_2 + 3y_3$$

$$\text{s.t. } -y_1 + y_2 + y_3 \geq 2 \quad -s_1, A_1$$

$$2y_1 + y_2 \geq 4 \quad \rightarrow -s_2, A_2$$

$$y_1, y_2 \geq 0$$

Express problem in standard form

$$\text{Max } W' = -2y_1 - 4y_2 - 3y_3 + 0s_1 + 0s_2 - MA_1 - MA_2$$

$$\text{s.t. } -y_1 + y_2 + y_3 - s_1 + 0s_2 + A_1 + 0A_2 = 2$$

$$2y_1 + y_2 + 0y_3 + 0s_1 - s_2 + 0A_1 + A_2 = 4$$

$$y_1 = y_2 = y_3 = s_1 = s_2 = 0 \text{ (non basic)}$$

$$A_1 = 2, A_2 = 4 \text{ (basic)}$$

## Initial Simplex table

$C_j$		-2	-4	-3	0	0	-M	-M		
$C_B$	Basis	$y_1$	$y_2$	$y_3$	$s_1$	$s_2$	$A_1$	$A_2$	b	$\theta$
-M	$A_1$	-1	1	1	-1	0	1	0	2	2/1
-M	$A_2$	2	<span style="border: 1px solid black;">1</span>	0	0	-1	0	1	1	1/1 <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">2</span>
$Z_j$		-M	-2M	-M	M	M	-M	-M		
$G_j = C_j - Z_j$		M-2	2M-4	M-3	-M	-M	0	0		

Incoming  $y_2$  & outgoing  $A_2$   $R_1' \rightarrow R_1 - R_2$

$C_j$		-2	-4	-3	0	0	-M	-M		
$C_B$	Basis	$y_1$	$y_2$	$y_3$	$s_1$	$s_2$	$A_1$	$A_2$	b	$\theta$
-M <span style="border: 1px solid black; border-radius: 50%; padding: 2px;">2</span>	$A_1$	-3	0	<span style="border: 1px solid black;">1</span>	-1	1	1	-1	1	1/1 $\leftarrow$
<del>-M</del>	$y_2$	2	1	0	0	-1	0	1	1	1/0
$Z_j$		3M-8	-4	-M	M	4-M	-M	M-4		
$G_j$		6-3M	0	M-3	-M	M-4	0	4-2M		

Incoming  $y_3$  & outgoing  $A_1$

$$R_2' \rightarrow R_2 - 0 \cdot R_1$$

	$y_j$	-2	-4	-3	0	0	$-M$	$-M$	
$C_B$	Base	$y_1$	$y_2$	$y_3$	$s_1$	$s_2$	$A_1$	$A_2$	$b$
-3	$y_3$	-3	0	1	-1	1	1	-1	1
-4	$y_2$	2	1	0	10	-1	0	1	1
$Z_j$		1	-4	-3	3	1	-3	-1	
$G_j$		-3	0	0	-3	-1	$3-M$	$1-M$	

As all  $G_j \leq 0$ , optimal solution attained

Thus the optimal solution to dual problem is

$$y_1 = 0, y_2 = 1, y_3 = 1$$

$$\text{Max. } W = -\text{Max. } (W') = 7$$

To derive optimal basic feasible solution to the primal problem,

we note that primal variables  $x_1, x_2$  correspond to artificial starting dual variables  $A_1, A_2$  respectively.

In the final simplex table of the dual problem  $G$  corresponding to  $A_1$  &  $A_2$  are 3 & 1 respectively. referring M...

Thus by rule 2, we get

$$x_1 = 3 + x_2 = 1$$

Hence optimal basic feasible soln. to the given primal is

$$x_1 = 3 + x_2 = 1, \max Z = 7$$

$$\rightarrow \max Z = \min \underline{W} = 7 \rightarrow \text{Duality theorem}$$

Ex Using duality solve the ~~problem~~

$$\text{Min. } Z = 0.7x_1 + 0.5x_2$$

$$\text{s.t. } x_1 \geq 4$$

$$x_2 \geq 6$$

$$x_1 + 2x_2 \geq 20$$

$$2x_1 + x_2 \geq 18$$

$$x_1, x_2 \geq 0$$

Dual of the given problem is

$$\text{Max. } W = 4y_1 + 6y_2 + 20y_3 + 18y_4$$

$$\text{s.t. } y_1 + y_3 + 2y_4 \leq 0.7$$

$$y_2 + 2y_3 + y_4 \leq 0.5$$

$$y_1, y_2, y_3, y_4 \geq 0$$

Express problem in

Introduce slack Variable

dual problem in standard form

$$\text{Max. } W = 4y_1 + 6y_2 + 20y_3 + 18y_4 + 0s_1 + 0s_2$$

$$\text{s.t. } y_1 + 0y_2 + y_3 + 2y_4 + s_1 + 0s_2 = 0.7$$

$$0y_1 + y_2 + 2y_3 + y_4 + 0s_1 + s_2 = 0.5$$

$$y_1, y_2, y_3, y_4 \geq 0$$

Find initial basic feasible soln

$$y_1 = y_2 = y_3 = y_4 = 0 \text{ (non-basic)}$$

$$s_1 = 0.7 \text{ [basic]}$$

$$s_2 = 0.5$$

Since basic variable  $s_1, s_2 > 0$ , initial basic soln is feasible & non-degenerate

	$C_j$	4	6	20	18	0	0		
$C_B$	Base	$y_1$	$y_2$	$y_3$	$y_4$	$s_1$	$s_2$	$b$	$\theta$
0	$s_1$	1	0	1	2	1	0	0.7	0.7/1
0	$s_2$	0	1	<span style="border: 1px solid black;">2</span>	1	0	1	0.5	0.5/2 $\leftarrow$ ②
	$Z_j$	0	0	0	0	0	0		
	$G_j = 4-4$	+4	+6	20	18	0	0		
				$\uparrow$					
				①					

4a

$$R_2' \rightarrow R_2 / 2$$

$$R_1' \rightarrow R_1 - R_2'$$

outgoing  $s_2$  , incoming  $y_3$

	$y_1$	$y_2$	$y_3$	$y_4$	$s_1$	$s_2$	b	$\theta$
0	$s_1$	1	$-\frac{1}{2}$	0	$\frac{3}{2}$	$1 - \frac{1}{2}$	$\frac{9}{20}$	$\frac{3}{10} \leftarrow$
20	$y_3$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{2}$

$$Z_j \quad 0 \quad 10 \quad 20 \quad 10 \quad 0 \quad 10$$

$$C_j - C_B \quad 4 \quad -4 \quad 0 \quad 8 \quad 0 \quad -10$$

$\uparrow$   
0

$$R_1' \rightarrow R_1 / (3/2)$$

$$R_2' \rightarrow R_2 - R_1' \cdot \frac{1}{2}$$

outgoing  $s_1$   
incoming  $y_4$

	$y_1$	$y_2$	$y_3$	$y_4$	$s_1$	$s_2$	b	$\theta$
18	$y_4$	$\frac{2}{3}$	$-\frac{1}{3}$	0	1	$\frac{2}{3} - \frac{1}{3}$	$\frac{3}{10}$	
20	$y_3$	$-\frac{1}{3}$	$\frac{2}{3}$	1	0	$-\frac{1}{3} + \frac{2}{3}$	$\frac{1}{10}$	

$$Z_j \quad 16\frac{2}{3} \quad 22\frac{2}{3} \quad 20 \quad 18 \quad 16\frac{2}{3} \quad 22\frac{2}{3}$$

$$C_j - C_B \quad -4\frac{1}{3} \quad -4\frac{1}{3} \quad 0 \quad 0 \quad -16\frac{2}{3} \quad -22\frac{2}{3}$$

As all  $G \leq 0$ ,  
take gives optim sol.

$$y_1 = 0, y_2 = 0, y_3 = 1/10, y_4 = 3/10$$

$$\max W = 7.4$$

Optim sol. to prim.

primel ~~rate~~ variables  $x_1, x_2$  correspond to  
slack status ~~and~~ dual variables  $s_1, s_2$   
respectively.

$G$  value corresponding to  $s_1$  &  $s_2$  are  ~~$-16/3$  &  $-22/3$~~

Thus by rule I

$$\text{Opt } x_1 = 16/3 \text{ \& } \text{Opt } x_2 = 22/3$$

$$x_1 = 16/3 \text{ \& } x_2 = 22/3$$

$$\underline{\underline{\min. Z = 7.4}}$$