# MTH201-PRACTICE PROBLEMS 6

**Q 1.** (i) Give the projection of $P = (-9, 1, 5)$ onto the the $X, Y, Z$ coordinate planes.

(ii) What is the equation of the line passing through the points $(1, 2, 1)$ and $(2, 1, -1)$? Write this in vector form also.

(iii) Write the parametric form of the line joining $(1, 2, 1)$ and $(2, 1, -1)$. Write this in vector form also.

(iv) The line through the point $(1, -7, 14)$ and parallel to the line given by $x = 6t, y = 9, z = 8 - 16t$.

(v) Check if the line given by $r_1(t) = (4 - 7t, -10 + 5t, 21 - 4t)$ and the line given by $r_2(t) = (-2 + 3t, 7 + 5t, 5 + t)$ are perpendicular.

**Q 2.** Calculate the limit.

(i) $\lim_{t\to 0}(\cos(2t)\vec{i} - e^{4-t}\vec{j} + (t^2 + 3t - 9)\vec{k})$

(ii) $\lim_{t\to 4}\left(\frac{t-4}{t^2-3t-4}, \frac{t^2-4t}{16-t^2}\right)$

(iii) $\lim_{t\to -8}\left(\frac{e^{t^2-64}-1}{t+8}\hat{i} + \frac{\sin(t+8)}{t+8}\hat{j} - \hat{k}\right)$

(iv) $\lim_{t\to -\infty}\left(\frac{5t^2-8t+1}{2+5t^2}, \frac{2+t^3}{1+t^2+t^4}\right)$

(v) $\lim_{t\to\infty}\left(\ln(1 - 4/t), e^{1/t^2}, 2\right)$

**Q 3.** Calculate the derivative. Here the derivative is the coordinatewise derivative.

(i) $r(t) = \left(\sqrt{3}t, \frac{1}{4}t, 12t\right)$

(ii) $r(t) = \cos(2t)\vec{i} - \sin(2t)\vec{j} + \ln(2t)\vec{k}$

**Q 4.** Calculate the (indefinite) integral. This is the antiderivative, and the same constant appears for all the coordinates.

(i) $\int r(t)dt$, where $r(t) = (t^3 - t^3, 5t, \frac{1}{6}t - 8t^4)$

(ii) $\int_0^1 r(t)dt$, where $r(t) = (t\cos(\pi t), 8t - 2, 12t^3 - e^{2t})$

(iii) $\int r(t)dt$, where $r(t) = (\log(6t), e^{1-t}, 5t)$

**Q 5.** Find the unit tangent vector.

(i) $r(t) = t^2\hat{i} - \cos(8t)\hat{j} + \sin(8t)\hat{k}$

(ii) $r(t) = (8t, 2 - t^6, t^4)$

**Q 6.** Find the tangent line to the following curves.

(i) $\mathbf{r}(t) = (3 + t^2, t^4, 6)$ at $t = -1$

**Q 7.** Calculate the unit normal at a point on the following curve.

(i) $r(t) = (e^{4t}\sin(t), e^{4t}\cos(t), 2)$

(ii) $\vec{r}(t) = 2t\vec{i} + 12t^2\vec{j} + \ln(t^2)\vec{k}$

**Q 8.** Determine the arc length.

(i) $r(t) = 4\cos(2t)\vec{i} + 3t\vec{j} - 4\sin(2t)\vec{k}$ from $0 \leq t \leq 3\pi$.

(ii) $r(t) = (9 - 2t, 4 + 2t, \sqrt{2}t^2)$ from $0 \leq t \leq 1$.

**Q 9.** Calculate the curvature.

(i) $r(t) = \langle t, 3\sin t, 3\cos t\rangle$ at each point of the curve as a function of $t$.
Ans: $3/10$.

(ii) $r(t) = t^2\,\vec{i} + t\,\vec{k}$ at each point of the curve as a function of $t$.
Ans: $\kappa = 2/(4t^2 + 1)^{3/2}$

**Q 10.** Calculate the limit.

(i) $\lim_{(x,y)\to(1,1)} \frac{2x^2-xy-y^2}{x^2-y^2}$. Ans: 3/2

(ii) $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+3y^4}$. Ans: Does not exist. Calculate limit along the paths $x = 0, y = 0, y = x$. Along $y = x$, you get the limit $1/4$, while it is 0 along $x = 0$ and $y = 0$.

(iii) $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2}$. Ans: Does not exist. Calculate limit along $y = x$ and $y = x^3$ to get different answers.

(iv) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^6}{xy^3}$. Ans: Limit does not exist. Calculate taking $x = y$ and $x = y^3$.

**Q 11.** Check if $f(x, y) = \frac{x+y}{y-x}$ is increasing or decreasing in the interval $(7, -3)$:

(i) if we keep $y$ fixed and allow $x$ to vary. (Hint: Take partial derivative w.r.t $x$)

(ii) if we keep $x$ fixed and allow $y$ to vary. (Hint: Take partial derivative w.r.t $y$)

**Q 12.** By using Chain rule, calculate the derivatives w.r.t $t$.

(i) $f(x, y) = xe^{xy}, x = t^2, y = t^{-1}$.

(ii) Let $f(x, y)$ be a function where $y = g(x)$. Then show that $\dfrac{df}{dx} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dx} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dx}$.

(iii) $f(x, y) = xy$, where $y = x^4$.

**Q 13.** Find the dimensions of the box with largest volume if the total surface area is $54cm^2$. (Hint: Maximize $f(x, y, z) = xyz$, subject to $2(xy+yz+zx) = 54$, using Lagrange multiplier method.)

**Q 14.** Find the maximum and minimum values of $f(x, y) = 36x^2+y^2$ subject to $4x^2 + y^2 = 6$, by using the method of Lagrange multiplier.

**Q 15.** Calculate the following integrals using Fubini's theorem.

(i) $S = [-5, 4] \times [0, 3]$, $\iint_S (2x - 4y^3)$. Ans: -756

(ii) $S = [-2, -1] \times [0, 1]$, $\iint_S (x^2y^2 + \cos(\pi x) + \sin(\pi y))$. Ans:$7/9 + 2/\pi$

(iii) $S = [0, 1] \times [1, 2]$, $\iint_S \dfrac{1}{(2x+3y)^2} dxdy$. Ans:$\dfrac{-1}{6}(\log 8 - \log 2 - \log 5)$

(iv) Show that, for $S = [a, b] \times [c, d]$,

$$\iint_S f(x)g(y)dxdy = (\int_c^d g(y)dy)(\int_a^b f(x)dx).$$

(v) Calculate the volume of the region that lies under the surface $f(x, y) = 9x^2 + 4xy$ and above the rectangle riven by $[-1, 1] \times [0, 2]$. Hint: $\iint_R (9x^2 + 4xy)dxdy$, use Fubini's theorem.

**Q 16.** Let $f(x, y, z) = xy + yz$ and $C$ is any path starting at $(1, 1, 2)$ and terminating at $(2, 1, 1)$. Calculate $\int_C (\nabla f) \cdot dr$.

**Q 17.** Let $C : r(t) = (t, t^2), 2 \leq t \leq 3$, and $f(x, y) = x(2 - y)$. Calculate $\int_C (\nabla f) \cdot dr$.