Simplex method

1) While solving L.P.P. graphically, region of feasible solution was found to be convex
→ the optimal soln. occured at some vertex.

If the optimal soln was not unique, optimal points were on an edge.

→ Most commonly used method for locating the optimal vertex is the simplex method.

This method consists of moving step by step from one vertex to the adjacent one.

The one giving better value of $f(x)$ (objective fn) is chosen.

Since the no. of vertices is finite. Simplex method leads to an optimal vertex in a finite no. of steps

2) infinite no. of soln. is reduced to finite no. of promising soln. by following :-

1) m constraints
   m+n variables (m being $\leq n$)
   decision ↗ ↖ slack

Starting soln. is found by setting n variables equal to zero. & then solving m equation, provided soln. exists & is unique.

The n zero variables are known as non-basic variables.

while remaining m variables are called basic variables.

This reduces the no. of basic solns for obtaining the optimal soln to $^{m+n}C_m$ only

→ In L.P.P., variables must be non-negative.
Some of the basic solns may contain negative variables. Such soln. are called basic infeasible soln. and should not be considered.

We start with basic soln. which is non-negative.
The next basic soln. must be non-negative.
This is ensured by the feasibility condition.
Such a soln. is known as basic feasible soln.

If all the variables in the basic feasible soln. are non-zero, then it is called non-degenrate soln. and if some of the variables are zero, it is called degenrate soln.

③ A new basic feasible soln. may be obtaind from previous one by equating one of the basic variables to zero and replacing it by one new non-basic variable.

Eliminated variable is called → leavey or outgoing variable

New variable is known → entering or incoming variable

Incoming variable should improve thef (x) which is ensured by optimality condition.

The process is repeated till no further improvement is possible.

Resulting soln is called optimal basic feasible soln. or optimal soln.

Simplex Method
based on 2 condition

1) Feasibility condition
→ Ensure that if starting soln. is basic feasible, the subsequent solns will also be basic feasible

2) Optimality condition
→ Ensure only improved soln will be obtained

→

$$\text{Max. } Z = C_1 x_1 + C_2 x_2 + \ldots + C_n x_n \qquad -①$$

$$\text{S.f. } \sum_{j=1}^{n} a_{ij} x_j + S_i = b_i \; ; \quad i = 1, 2, \ldots m \qquad -②$$

and $x_j \geq 0 \; S_i \geq 0 \; ; \quad \begin{array}{l} j = 1, 2, \ldots n \\ i = 1, 2, \ldots, m \end{array} \qquad -③$

→ Soln $x_1, x_2, \ldots, x_n$ is soln. of general L.P.P. if it satisfies constraints ②.

→ $x_1, x_2, \ldots, x_n$ is feasible soln. if it satisfies both ② constraints & ③ negativity restriction

set S of all feasible soln is called feasible region.

A linear programme is said to be infeasible when the set S is empty.

→ Basic soln is the soln. of the m basic variable when each of the n non basic variable are equated to zero.

→ Basic feasible soln. is that basic soln. which also satisfies ③.

→ Optimal soln. → that basic feasible soln—which also optimize the objective fn ①. while satisfying ② & ③.

→ Non-degenerate basic feasible soln. is that basic feasible soln. which contains exactly m non-zero basic variable.

If any of the basic variable becom zero, it is called degenerate basic feasible soln.

Ex * Find an optimal soln to follows L.P.

$$\text{Max } Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$$

s.t

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_1, x_2, x_3, x_4 \geqslant 0$$

4 variables

2 constraints

Basic soln. obtained by setting any 2 variables

equal to zero & solving resulting $= n$.

   Total no. of basic soln $= {}^4C_2 = 6$

① Basic variable: x_1, x_2

Non basic v amch $x_3, x_4 = 0$

then

$$2x_1 + 3x_2 = 8$$
$$x_1 - 2x_2 = -3$$

$$\Rightarrow x_1 = 1, x_2 = 2$$

feasible soln. as $x_1, x_2 \geqslant 0$

Value of $Z = 8$.

② $x_1, x_3 \rightarrow x_2 = x_4 = 0$

$$2x_1 - x_3 = 8$$
$$x_1 + 6x_3 = -3$$
$$x_1 = -14/13, \quad x_3 = -67/13 \qquad \text{feasible soln. } \underline{No}$$

(2) x_1, x_4

$x_2 = x_3 = 0$

$2x_1 + 4x_4 = 8$

$x_1 - 7x_4 = -3$

$\therefore x_1 = 22/9$

$x_2 = 7/9$

all $x \geq 0$ $\qquad Z = 10.3$

(4) x_2, x_3

$x_1, x_4 = 0$

$3x_2 - x_3 = 8$

$-2x_2 + 6x_3 = -3 \qquad x_2 = 45/16$

$\qquad \qquad \qquad \qquad \qquad x_3 = 7/16 \qquad Z = 10.2$

all $x \geq 0$

(5) x_2, x_4

$x_1 = x_3 = 0$

$3x_2 + 4x_4 = 8$

$-2x_2 - 7x_4 = -3$

$x_2 = 132/39$

$x_4 = -7/13$

all $x \geq 0$ Not true

$\qquad \qquad$ as $x_4 < 0$

(6) x_3, x_4

$x_1 = x_2 = 0$

$-x_3 + 4x_4 = 8$

$6x_3 - 7x_4 = -3$

$x_3 = 44/17$

$x_4 = 45/17$

all $x \geq 0$ $\qquad$ optimal

$Z = 28.9$

Working procedure of Simplex Method

① check whether objective fn to be maximized or minimized

if $Z = C_1 x_1 + C_2 x_2 + C_3 x_3 + \ldots + C_n x_n$

is to be minimized.

then convert it to problem of maximization by

$\text{Min } Z = \text{Max}(-Z)$

$$\sum_{j=1}^{n} a_{ij} x_j + S_i = b_i \quad i=1,2,\ldots,m$$

② check all b are +ve

if any of b_i's is -ve, multiply both sides by
of the constraints by -1

② Express problem in standard form

Convert all inequalities of constraints into $=n$ by introducing slack/surplus variables in the constraints

~~steady cloths bar~~

$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \ldots + S_1 + 0 S_2 + 0 S_3 + \ldots = b_1$

③ Find an initial basic feasible soln.

If there are $m = n$ involving n unknowns, then assign O values to any $(n-m)$ of the variable for finding a soln.

Starting with a basic soln for which

$x_j \div j = 1, 2, \ldots, (n-m)$ are each zero,

find all S_i.

If all S_i are ≥ 0, the basic soln is feasible & non-degenerate.

The above information in written in simplex table

C_B	C_j Basis	C_1 x_1	C_2 x_2	C_3 x_3 $\ldots$	O S_1	O S_2	O $\ldots$ S_3 $\ldots$	b
O	S_1	a_{11}	a_{12}	$a_{13} \ldots$	1	O	O $\ldots$	b_1
O	S_2	a_{21}	a_{22}	$a_{23} \ldots$	O	1	O $\ldots$	b_2
O	S_3	a_{31}	a_{32}	$a_{33} \ldots$	O	O	1 $\ldots$	b_3
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$			

Body mᵗ   Unit mᵗ

$S_1, S_2, S_3, \ldots \rightarrow$ basic variables
$x_1, x_2, x_3, \ldots \rightarrow$ non-basic variables

Basis refers to basic variable $S_1, S_2, ..S_3 ...$

C_j row denots cofficients of variable u objectivs

C_B- colum denots cofficien of basi basic variable.
only in objective fn.

b- colum denote values of basic variables
while remaining variable will always be zero.

Cofficients of x's (decision variables) is
constraint =n constitute body $\overline{nf}$
while coeffical of slack variables constitute unit $\overline{n}$.

④ Apply optimality Test
Compute $C_j = c_j - Z_j$ where $Z_j = \sum C_B a_{ij}$

$C_j \rightarrow$ row is called net evaluation row and
indicates the per unit ↑ se in objective fn.
of variable heady column is brought into soln

If all C_j are -ve, intial basic feasible soln is
optiml.
If even one C_j is +ve, then current feasible soln is
not optimal & proceed to next

① Identify the incoming & outgoing variable

② iterate toward optimal sol.

⑥ go to step ④

Using Simplex Method

Max. $Z = 5x_1 + 3x_2$

s.t. $x_1 + x_2 \leq 2$

$5x_1 + 2x_2 \leq 10$

$3x_1 + 8x_2 \leq 12$

$x_1, x_2 \geq 0$

① Maximized, and all b are +ve

② Introducing slack variable S_1, S_2, S_3

Max. $Z = 5x_1 + 3x_2 + 0S_1 + 0S_2 + 0S_3$

s.t. $x_1 + x_2 + S_1 + 0S_2 + 0S_3 = 2$

$5x_1 + 2x_2 + 0S_1 + S_2 + 0S_3 = 10$

$3x_1 + 8x_2 + 0S_1 + 0S_2 + S_3 = 12$

$x_1, x_2, S_1, S_2, S_3 \geq 0$

③ Find initial basic feasible sol

(5) There are $3 = n0$
involving 5 unknown.

assign zero to first 2 of var.

$$x_1 = 0 + x_2 = 0$$

$$S_1 = 2, S_2 = 10, S_3 = 12$$

$\therefore$ basic feasible sol^n is

$$x_1 = x_2 = 0 \qquad S_1 = 2, S_2 = 10, S_3 = 12 \ (\text{basic})$$

$\therefore$ Initial basic feasible sol^n.

C_j		5	3	0	0	0		
C_B	Basis	x_1	x_2	S_1	S_2	S_3	b	θ
0	S_1	[1]	1	1	0	0	2	2/1 ← ②
0	S_2	5	2	0	1	0	10	10/5
0	S_3	3	8	0	0	1	12	12/3
	$Z_j = \Sigma(C_B a_{ij})$	0	0	0	0	0	0	
	$C_j = c_j - Z_j$	5	3				0	

↑① key column

Divide b/ key column to get 0

x_1 coln (j=1) $Z_j = \Sigma C_B a_{i1} = 0(1) + 0(5) + 0(3) = 0$

x_2 j=2 $Z_j = \Sigma C_B a_{i2} = 0(1) + 0(2) + 0(8) = 0$

Sim $Z_j(b) = 0(2) + 0(10) + 0(12) = 0$

As for the C_j is +ve ∴ initial basic feasible soln is not optimal.

⑤ Identify is coming & outgoing variable.

Above table show that x_1 is the incoming variable as its incremental contribution $C_j (=5)$ is max. & column in which it appears is the key column.

Dividing the elements under b-column by corresponding elements of key-column, we find min. the ratio 0 & 2 in two rows.

∴ we arbitrarily choose row containing S_1 as the key row.

The element at the intersection of key row & key colum ie. (1) is the key element

S_1 is therefore, out going basic variable which will now become non-basic

Removing S_1, new basis will contain x_1, S_2 & S_3 as basic variables.

c_j		5	3	0	0	0		
C_B	Basis	x_1	x_2	s_1	s_2	s_3	b	θ
5	x_1	1	1	1	0	0	②2	
0	s_2	0	-3	-5	1	0	0	
0	s_3	0	5	-3	0	1	6	〔10〕
	$Z_j = \sum c_B a_{ij}$	5	5	5	0	0		
	$C_j = c_j - Z_j$	0	-2	-5	0	0		

As C_j is either zero or negative under all colum, above table give optimal basic feasible soln.

This optimal soln. is

$$x_1 = 2 \ , \ x_2 = 0 \ \& \ max. \ Z = 10$$

Make all other elements of key column zero

	x_1	x_2	s_1	s_2	s_3	b
Ⓐ	$5 - 5 \times 1$	$2 - 5 \times 1$	$0 - 5 \times 1$	$1 - 0 \times 5$	$0 - 0 \times 5$	$10 - 2 \times 5$
	$3 - 3 \times 1$	$8 - 3 \times 1$	$0 - 3 \times 1$	$0 - 0 \times 3$	$1 - 0 \times 3$	$12 - 2 \times 3$

Ex. Solve the following L.P.P. by simplex method

$$\text{Min. } Z = x_1 - 3x_2 + 3x_3$$

s.t.
$$3x_1 - x_2 + 2x_3 \le 7 \quad - \text{①}$$
$$2x_1 + 4x_2 \ge -12 \quad - \text{②}$$
$$-4x_1 + 3x_2 + 8x_3 \le 10 \quad - \text{③}$$
$$x_1, x_2, x_3 \ge 0 \quad - \text{④}$$

Soln

Convert objective fn to Max. by -ve sign

$$\text{Max } Z' = -x_1 + 3x_2 - 3x_3$$

As the ② constraint is -ve we we write as the

$$-2x_1 - 4x_2 \le 12$$

Express in standard form.
Introduce slack variable

$$\text{Max } Z' = -x_1 + 3x_2 - 3x_3 + 0s_1 + 0s_2 + 0s_3$$

s.t.
$$3x_1 - x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 7$$
$$-2x_1 - 4x_2 + 0x_3 + 0s_1 + s_2 + 0s_3 = 12$$
$$-4x_1 + 3x_2 + 8x_3 + 0s_1 + 0s_2 + s_3 = 10$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$$

Find initial basic feasible sol^n

basic feasible sol^n :

$$x_1 = x_2 = x_3 = 0 \text{ (non-basic)}$$

$$S_1 = 7; \ S_2 = 12, \ S_3 = 10 \text{ (basic)}$$

Intial basic feasible soln is given by table

c_j		-1	3	-3	0	0	0		θ
c_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	
0	S_1	3	-1	2	1	0	0	7	$7/-1$
0	S_2	-2	-4	6	0	1	0	12	$12/-4$
0	S_3	-4	$\boxed{3}_{③}$	8	0	0	1	10	$10/3 \leftarrow$
$Z_j = \Sigma c_B a_{ij}$		0	0	0	0	0	0	0	
$C_j = c_j - z_j$		-1	3 $\uparrow_{①}$	-3	0	0	0		

$$Z_1 = 0(3) + 0(-2) + 0(-4) = 0$$

$$Z_2 = 0(-1) + 0(-4) + 0(3) = 0$$

.
.

$$Z_b = 0(7) + 0(12) + 0(10) = 0$$

C_j is $+ve$ under second column.

initial basic feasible soln is not optimal

→ Identify in coming & out going variable

x_2 is in coming variable &

S_3 is out going variable &

3 ③ is the key element

Iterate again

Drop S_3 & introduce x_2 with its associated value

② 3 under column C_B column.

Convert key element to unity & make all other elements of key column zero.

Second basic feasible soln is given by

table

ʰ
R molecular p
unexplo
to extract basic inst

x_j	-1	3	-3	0	0	0		
C_B Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
0 S_1	$3-\frac{4}{3}$	$-1+\frac{3}{3}$	$2+\frac{8}{3}$	$1+0$	0	$0+\frac{1}{3}$	$7+\frac{10}{3}$	
0 S_2	$-2+\frac{16}{3}$	$-4+4\frac{3}{3}$	$8+\frac{8\cdot4}{3}$	$0+0$	0	$D+\frac{4}{3}$	$12+\frac{4\cdot10}{3}$	
3 x_2	$-4/3$	$3/3$	$8/3$	0	0	$1/3$	$10/3$	

C_j	-1	3	-3	0	0	0		θ ②
C_B Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	
0 S_1	$\boxed{5/3}$	0	$14/3$	1	0	$1/3$	$31/3$	$\frac{31}{3}\times\frac{3}{5}$ $31/5$
0 S_2	$-\frac{22}{3}$	0	$\frac{32}{3}$	0	1	$4/3$	$76/3$	$\frac{76}{3}\times\frac{3}{-22}$ $-38/11$
3 x_2	$-4/3$	1	$8/3$	0	0	$1/3$	$10/3$	$\frac{10}{3}\times\frac{3}{-4}$ $-5/2$
$Z_j=\sum C_B a_{ij}$	-4	3	$0\times\frac{14}{3}+0\times\frac{32}{3}+3\times\frac{1}{1}=8$	0	0	1		
$C_j=C_j-Z_j$	3	0	-11	0	0	-1		

① ↑

C_j is the under first column.

solⁿ is not optimal we proceed further

x_1 is incoming & S_1 is outgoing variable

key elm is $5/3$

- 3a

Drops x_1 & introduce x_2 with its associated value

−1 under C_B column.

Convert key element to unity & make all other elements of key column zero.

Third basic feasible soln is :−

C_j		−1	3	−3	0	0	0	
C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b
$\frac{5}{8}B-1$	x_1	$\frac{5}{3} \times \frac{3}{5}$	$0 \times \frac{3}{5}$	$\frac{14}{3} \times \frac{3}{5}$	$1 \times \frac{3}{5}$	0	$\frac{1}{3} \times \frac{2}{5}$	$\frac{31}{3} \times \frac{3}{5}$
0	S_2	$-\frac{22}{3} + \frac{22}{3} \times 1$	$0 + \frac{22}{3} \times 0$	$\frac{32}{3} + \frac{22}{3} \times \frac{14}{5}$	$0 + \frac{22}{3} \times \frac{3}{5}$	$1+0$	$\frac{4}{3} + \frac{22}{3} \times \frac{1}{5}$	$\frac{76}{3} + \frac{22}{3} \times$
3	x_2	$-\frac{4}{3} + \frac{4}{3} \times 1$	$1 + \frac{4}{3} \times 0$	$\frac{8}{3} + \frac{4}{3} \times \frac{14}{5}$	$0 + \frac{4}{3} \times \frac{3}{5}$	$0 + 0$	$\frac{1}{3} + \frac{4}{3} \times \frac{1}{5}$	$\frac{10}{3} + \frac{4}{3} \times$

C_j		−1	3	−3	0	0	0		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
−1	x_1	1	0	$\frac{14}{5}$	$3/5$	0	$1/5$	$31/5$	
0	S_2	0	0	$\frac{156}{5}$	$\frac{22}{5}$	1	$14/5$	$354/5$	
3	x_2	0	1	$\frac{32}{5}$	$4/5$	0	$3/5$	$58/5$	
	z_j	−1	3	$82/5$	$9/5$	0	$8/5$	(143/5)	
	C_j $= C_j - z_j$	0	0	$-97/5$	$-9/5$	0	$-8/5$		

Now since each $G \leq 0$.

$\therefore$ it gives optimal sol.

$x_1 = 31/5$

$x_2 = 58/5$

$x_3 = 0$ (non-basic)

$Z'_{max} = 143/5$

$\therefore Z_{min} = -143/5$

a

Max $Z = 22x_1 + 6x_2 + 2x_3$

$$Z = 22x_1 + 6x_2 + 2x_3$$

s.t.

$$10x_1 + 2x_2 + x_3 \leq 100$$
$$7x_1 + 3x_2 + 2x_3 \leq 72$$
$$2x_1 + 4x_2 + x_3 \leq 80$$

and $x_1, x_2, x_3 \geq 0$

Soln

Max $Z = 22x_1 + 6x_2 + 2x_3 + 0S_1 + 0S_2 + 0S_3$

s.t.

$$10x_1 + 2x_2 + x_3 + S_1 + 0S_2 + 0S_3 = 100$$
$$7x_1 + 3x_2 + 2x_3 + 0S_1 + S_2 + 0S_3 = 72$$
$$2x_1 + 4x_2 + x_3 + 0S_1 + 0S_2 + S_3 = 80$$

and $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Find initial basic feasible soln

$x_1 = x_2 = x_3 = 0$ (non-basic)

$S_1 = 100, S_2 = 72, S_3 = 80$ (basic)

Scanned with OKEN Scanner

Initial basic feasible soln is given by

Iteration 1

C_B	Basis	C_j x_1 22	x_2 6	x_3 2	S_1 0	S_2 0	S_3 0	b	θ
0	S_1	10 ③	2	1	1	0	0	100	100
0	S_2	7	3	2	0	1	0	72	72
0	S_3	2	4	1	0	0	1	80	80/

$Z_j = \varepsilon C_B a_{ij}$

| | | 0 | 0 | 0 | 0 | 0 | 0 | — |

$0(10)+0(7)+0(2)$ $0(2)+0(3)+0(4)$ . . . .

$C_j = C_j - Z_j$

| | | 22 | 6 | 2 | 0 | 0 | 0 |

① ↑ mox - the

x_1 is incoming & S_1 is out going var ch

key element is ⬛ 10

$R_1' = R_1 / 10$

$R_2' = R_2 - 7 R_1'$

$R_3' = R_3 - 2 R_1'$

iter 2

C_B	C_j	(22)	6	2	0	0	0		
	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
[22]	x_1	1	$1/5$	$1/10$	$1/10$	0	0	10	$\frac{10}{1/5}=50$
0	S_2	0	$\boxed{3-7/5 = 8/5}$	$13/10$	$-7/10$	1	0	2	$\frac{2}{8/5}=\frac{5}{4}$ ←③
0	S_3	0	$18/5$	$4/5$	$-1/5$	0	1	60	$\frac{60}{18/5}=\frac{50}{3}$
	$Z_j=\sum C_B a_{ij}$	22	$22/5$	$11/5$	$11/5$	0	0		
	$C_j-c_j-Z_j$	0	$\frac{8}{5}$	$-1/5$	$\frac{-11}{5}$	0	0		

↑① (under x_2 column)

max +v

~~Incoming~~

In coming variable is x_2 & out going S_2

Pivot is $8/5$

$R_2' = R_2 \times 5/8$

$R_1 = R_1 - \dfrac{R_2'}{5}$

$R_3 = R_3 - \dfrac{18}{5} R_2'$

Sheet 3

C_B	Basis	C_j x_1	22 x_2	⑥ x_3	2 S_1	0 S_2	0 S_3	0 b	②
22	x_1	1	0	$-1/16$	$3/16$	$-1/8$	0	$39/4$	
⑥	x_2	0	1	$13/16$	$-7/16$	$5/8$	0	$5/4$	
0	S_3	0	0	$-\dfrac{17}{8}$	$1/8$	$\dfrac{-9}{4}$	1	$11\tfrac{1}{2}$	

$Z_j = \Sigma c_B a_{ij}$ 22   6 $\dfrac{7}{2}$ $3/2$ 1   0

$\overline{C_j} = I_j - 2_j$ 0   0 $-3/2$ $-3/2$ -1 0

Since all of $C_j \nleqslant 0$

Optimal soln. is arrived with↑ value of variable

$$x_1 = \dfrac{39}{4} \; ; \; x_2 = \dfrac{5}{4} \; ; \; x_3 = 0$$

→ Simplex method to solve LPP's. with availability constraints of ≤ type.

→ Add slack variables to convert them into equalities.

→ If constraint is ≥ type → convert to ≤ by × -1.

→ But this method is neither suitable nor applicable in certain situation.

     Particularly when there is more than one constraint and/or if we get negative sign in the right hand side. we need surplus variable instead of slack variable

Artificial variable technique

Introduction of slack / surplus variables provide initial basic feasible soln.

But there are some problem wherein at least one of the constraint is of (≥) or (=) type and slack variables fail to give soln.

Then One can then use:

M-method or Method of penalities or Big-M method

Given by A. Charnes & consist of steps [Charnes penalty method]

① Express problem in standard form.

② Add non-negative variables to the L.H.S. of all those constraint which you are of (≥) or (=) type.

Such new variables are called artificial variables.

→ Their addition causes violation of the corresponding constraints.

→ One should get rid of these variables and should not allow them to appear in final solution.

For this purpose,

Assign a very large penality $(-M)$ to these artificial variables in the objective function

Step 3 -

Solve modified L.P.P by simplex method

3 cases

Case i → No artificial variable in basis.

Case ii → Atleast one artificial variable in basis at zero level (with zero value in b-column)

Soln. is degenerate optimal basic feasible soln

Case iii → Atleast one artificial variable in basis at non-zero level (with +ve value in b-column).

Problem has no feasible soln.

Such soln satisfies constraints but does n'l optimize the objective fn.

∴ called pseudo optimal soln.

Step 4   Continue simplex method.

Ex

$$\text{Min. } Z = 2x_1 + x_2$$

$$\begin{aligned}
\text{St.} \quad & 3x_1 + x_2 = 3 && \rightarrow A_1 \\
& 4x_1 + 3x_2 \geqslant 6 && \rightarrow -S_1 \; \alpha \; A_2 \\
& x_1 + 2x_2 \leqslant 3 && \rightarrow +S_2 \\
& x_1, x_2 \geqslant 0
\end{aligned}$$

Soln   Express in standard form

$$\text{Max } Z' = -2x_1 - x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$$

$$3x_1 + x_2 + 0S_1 + 0S_2 + A_1 + 0A_2 = 3$$

$$4x_1 + 3x_2 - S_1 + 0S_2 + 0A_1 + A_2 = 6$$

$$x_1 + 2x_2 + 0S_1 + S_2 + 0A_1 + 0A_2 = 3$$

$$x_1, x_2, S_1, S_2, A_1, A_2 \geqslant 0$$

Initial basic feasible soln.

$$x_1 = x_2 = 0,$$

Since surplus variable S_1 is not basic as S_1 come to be -6. To avoid it to be $-m$, make it 0.

$$S_1 = 0$$

$$A_1 = 3, \quad A_2 = 6, \quad S_2 = 3.$$

c_j		-2	-1	0	0	$-M$	$-M$		
C_B	Basis	x_1	x_2	S_1	S_2	A_1	A_2	b	θ
$-M$	A_1	$\boxed{3}$	1	0	0	1	0	3	$3/3 \leftarrow$ ②
$-M$	A_2	4	3	-1	0	0	1	6	$6/4$
0	S_2	1	2	0	1	0	0	3	$3/1$
$Z_j = \Sigma c_b a_j$		$-7M$	$-4M$	M	0	$-M$	$-M$	$-9M$	
$C_j = C_j - Z_j$		$7M-2$	$4M-1$	$-M$	0	0	0		

$\uparrow$ ①

Incoming $-x_1$
outgoing A_1

Iterate   drop $\underline{A_1\ column}$

$R_1' \rightarrow R_1/3$

$R_2'' \rightarrow R_2 - 4 \cdot R_1'$

$R_3' \rightarrow R_3 - 3 \cdot R_1'$

c_j		-2	-1	0	0	$-M$		
C_B	Basis	x_1	x_2	S_1	S_2	A_2	b	θ
-2	x_1	1	$1/3$	0	0	0	1	3
$-M$	A_2	0	$\boxed{5/3}$	-1	0	1	2	$6/5 \leftarrow$
0	S_2	0	$5/3$	0	1	0	2	$6/5$
Z_j		-2	$-\frac{2}{3} \frac{-5M}{3}$	M	0	$-M$		
C_j		0	$\frac{5M}{3} \frac{-1}{3}$	$-M$	0	0		

$\uparrow$ ①

Incoming x_2, out going A_2

Agau Iterate

$$R_2' \rightarrow R_2/(5/3) \rightarrow R_2 \times \frac{3}{5}$$

$$R_1 \rightarrow R_1 - R_2' \cdot \frac{1}{3}$$

Doop x_2 colum

$$R_3 \rightarrow R_3' - R_2' \cdot \frac{5}{3}$$

	c_j	-2	-1	0	0	b
c_B	Basis	x_1	x_2	S_1	S_2	
			0	1/5	0	3/5
-2	x_1	1	0			6/5
			1	-3/5	0	
-1	x_2	0	1		1	0
	S_2	0	0	1		-12/5
				1/5	0	
	z_j	-2	-1	1/5	0	
	c_j	0	0	-1/5	0	

Since none of c_j is +ve, this is optimal sol.

Thus, an optimal basic feasible soln is

$$x_1 = \frac{3}{5} \quad ; \quad x_2 = 6/5$$

$$\text{Max } Z' = -12/5$$

$$\text{Min. } Z = -\text{Max. } Z' = 12/5$$