PHY303 : Assignment 1

1. (Jackson 1.3) Using Dirac delta functions in the appropriate coordinates, express the following charge distributions as three dimensional charge densities $\rho(\mathbf{r})$.

   (a) In spherical coordinates, a charge Q uniformly distributed over a spherical shell of radius R.

   (b) In cylindrical coordinates, a charge λ per unit length uniformly distributed over a cylindrical surface of radius b.

   (c) In cylindrical coordinates, a charge Q spread uniformly over a flat circular disc of negligible thickness and radius R.

   (d) The same as part (c), but using spherical coordinates.

2. (Jackson 1.5) The time-averaged potential of a neutral hydrogen atom is given by

$$V = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}\left(1 = \frac{\alpha r}{2}\right)$$

   where q is the magnitude of the electronic charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

3. (Jackson 1.12) Prove Green's reciprocation theorem: If Φ is the potential due to a volume-charge density ρ within a volume V and a surface-charge density σ on the conducting surface S bounding the volume V , while Φ' is the potential due to another charge distribution ρ' and σ', then

$$\int_V \rho\Phi' d^3r + \int_S \sigma\Phi' da = \int_V \rho'\Phi d^3r + \int_S \sigma'\Phi da$$

4. (Jackson 1.14) Consider the electrostatic Green functions for Dirichlet and Neumann boundary conditions on the surface S bounding the volume V . Apply Green's theorem with integration variable $\mathbf{y}$ and $\phi = G(\mathbf{x}, \mathbf{y}), \psi = G(\mathbf{x}', \mathbf{y})$ with $\nabla_y^2 G(\mathbf{z}, \mathbf{y}) = -4\pi\delta^3(\mathbf{y} - \mathbf{z})$. Find an expression for the difference $[G(\mathbf{x}, \mathbf{x}') - G(\mathbf{x}', \mathbf{x})]$ in terms of an integral over the boundary surface S.

   (a) For Dirichlet boundary conditions on the potential and the associated boundary condition on the Green function, show that $G(\mathbf{x}, \mathbf{x}')$ must be symmetric in $\mathbf{x}$ and $\mathbf{x}'$.

   (b) For Neumann boundary conditions, use the boundary condition for $G_N(\mathbf{x}, \mathbf{x}')$ to show that $G_N(\mathbf{x}, \mathbf{x}')$ is not symmetric in general, but that $G_N(\mathbf{x}, \mathbf{x}') - F(\mathbf{x})$ is symmetric in $\mathbf{x}$ and $\mathbf{x}'$ where

$$F(\mathbf{x}) = \frac{1}{S}\int_S G_N(\mathbf{x}, \mathbf{y}) da_y.$$

(c) Show that the addition of $F(\mathbf{x}$ to the Green function does not affect the potential $\Phi(\mathbf{x})$.

5. Consider a system of two charges $+Q$ and $-Q$ placed on the $z-$axis at $z=-a$ and $z=a$ respectively. If the charges are separated far apart, the electric field is uniform and given by $E_0 = 2Q/4\pi\epsilon_0 a^2$ near $z=0$. Now bring a grounded conducting sphere of radius R at the origin.

   (a) Using the method of images, find the potential at a point P outside the sphere. Recall that for a single charge q placed at a distance a from the center of a conducting sphere of radius R, the potential at a point outside the sphere is obtained by putting an image charge $q' = -qR/a$ at a distance $b = R^2/a$ from the center of the sphere.

   (b) FInd the potential in the limit $a \gg r$, r being the distance of point P from the center. Express your result in terms of E_0.

   (c) Hence show that the surface charge density is given by $\sigma = 3\epsilon_0 E_0 \cos\theta$.