# MTH201-PRACTICE PROBLEMS 4

**Q 1.** Calculate the derivative.

(i) $f(x) = x^x, \forall x \in \mathbb{R}$ (take log).

(ii) $f(x) = x^{e^x}$ (take log).

**Q 2.** Show that the following functions are not differentiable at the points indicated

(i) $f(x) = |\sin(x)|$ at $x = 0$.

(ii) $f(x) = \begin{cases} \sin(x)/x, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ at $x = 0$.

(iii) $f(x) = |x|^{1/3}$ at $x = 0$.

(iv) $f(x) = \begin{cases} \dfrac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ at $x = 0$.

**Q 3.** Use L'Hospital's rule to calculate the limits.

(i) $\lim_{x\to 2} \dfrac{x^3 - 7x^2 + 10x}{x^2 + 10x - 6}$.

(ii) $\lim_{\theta\to -4} \dfrac{\sin(\pi\theta)}{\theta^2 - 16}$.

(iii) $\lim_{t\to\infty} \dfrac{\log(3x)}{x^2}$.

(iv) $\lim_{x\to -\infty} \dfrac{x^2}{e^{1-x}}$.

(v) $\lim_{x\to\infty} (e^x + x)^{1/x}$.

(vi) $\lim_{x\to 1^+}(x-1)\tan(\pi x/2)$.

(vii) $\lim_{x\to\infty}(ax)^{\frac{b}{cx}}, \quad a \neq 0 \neq c$ .

**Q 4.** Show that L'Hospital's rule applies to the limit $\lim_{x\to\infty}\dfrac{x+\cos x}{x-\cos x}$, but it is of no use. Evaluate the limit directly.

**Q 5.** Show that the limit $\lim_{x\to 0}\dfrac{\sin x}{x}$ can be evaluated geometrically. However, it can be evaluated directly using L'Hospital's rule. Explain why doing so involves circular reasoning.

**Q 6.** The limit $\lim_{x\to a}\dfrac{\sqrt{2a^3x-x^4}-a\sqrt[3]{a^2x}}{a-\sqrt[4]{ax^3}}$, where $a>0$ is a constant, is the first example that L'Hospital used to demonstrate his theorem on finding limits. Show that the limit is 0.

**Q 7.** Using Taylor's theorem, show that $1-x^2/2 \leq \cos x, \forall\, x\in\mathbb{R}$.

**Q 8.** Consider the function $f(x) = \begin{cases} -(x\sin(1/x))^2 & \text{if } x\neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Show that the function has a local maximum at $c=0$. Show that $f$ is not increasing on the left side of $c$, nor decreasing on the right side of $c$.

**Q 9.** Show that the following functions are convex.

(i) $f(x)=x^2$ on any open interval

(ii) $f(x)=\sin x$ on $(\pi, 2\pi)$

(iii) $f(x)=\log x$ on $(0,\infty)$

**Q 10.** Construct an example of a function $f$ that has a maximum at $c$, and is twice differentiable at $c$, but $f''(c)\geq 0$.

**Q 11.** Find the extremum points of $f(x)=\dfrac{1}{x^4-2x^2+7}, \forall x\in\mathbb{R}$. Is it easy to determine that $x=0$ is a local minimum and $x=-1,1$ are local maxima for $f$, using the second derivative test. Check this by calculating the second derivative. Is it better to check the sign of $f'(x)$ in a small open interval around 0 to say about the nature of the extremum points? Check this.