Assignment 6

1. (Griffiths 5.35) A circular loop of wire, with radius R, lies in the xy plane (centered at the origin) and carries a current I running counterclockwise as viewed from the positive z axis.
   (a) What is its magnetic dipole moment?
   (b) What is the (approximate) magnetic field at points far from the origin?
   (c) Show that,for points on the z axis, your answer is consistent with the exact field (Ex. 5.6), when $z \gg R$.

2. (Griffiths 5.60) A uniformly charged solid sphere of radius R carries a total charge Q, and is set spinning with angular velocity ω about the z axis.
   (a) What is the magnetic dipole moment of the sphere?
   (b) Find the average magnetic field within the sphere (see Prob. 5.59).
   (c) Find the approximate vector potential at a point (r, θ) where $r \gg R$.
   (d) Find the exact potential at a point (r, θ) outside the sphere, and check that it is consistent with (c). [Hint: refer to Ex. 5.11.]
   (e) Find the magnetic field at a point (r, θ) inside the sphere, and check that it is consistent with (b).

3. (Jackson 5.8) A localized cylindrically symmetric current distribution is such that the current flows only in the azimuthal direction; the current density is a function only of r and θ (or ρ and z): $\vec{J} = J(r, \theta)\hat{\phi}$. The distribution is "hollow" in the sense that there is a current-free region near the origin, as well as outside.
   (a) Show that the magnetic field can be derived from the azimuthal component of the vector potential, with a multipole expansion

   $$A_\phi(r, \theta) = -\frac{\mu_0}{4\pi} \sum_L m_L r^L P_L^1(\cos\theta)$$

   in the interior and

   $$A_\phi(r, \theta) = -\frac{\mu_0}{4\pi} \sum_L \mu_L r^{-L-1} P_L^1(\cos\theta)$$

   outside the current distribution.
   (b) Show that interior and exterior multipole moments are

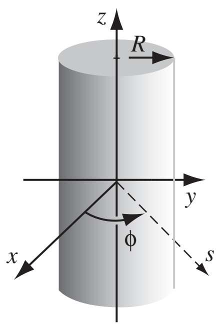
   $$m_L = -\frac{1}{L(L+1)} \int d^3x r^{-L-1} P_L^1(\cos\theta) J(r, \theta)$$

   and

   $$\mu_L = -\frac{1}{L(L+1)} \int d^3x r^L P_L^1(\cos\theta) J(r, \theta).$$

4. (Jackson 5.19) A magnetically "hard" material is in the shape of a right circular cylinder of length L and radius a. The cylinder has a permanent magnetization M_0, uniform throughout its volume and parallel to its axis.
   (a) Determine the magnetic field $\mathbf{H}$ and magnetic induction $\mathbf{B}$ at all points on the axis of the cylinder, both inside and outside.
   (b) Plot the ratios $\mathbf{B}/\mu_0 M_0$ and $\mathbf{H}/M_0$ on the axis as functions of z for $L/a = 5$.

5. (Griffiths 6.8) A long circular cylinder of radius R carries a magnetization $M = ks^2\hat{\phi}$, where k is a constant, s is the distance from the axis, and $\hat{\phi}$ is the usual azimuthal unit vector (Fig. 6.13). Find the magnetic field due to M, for points inside and outside the cylinder.



6. (Taken from Jackson text) Consider a spherical shell of inner (outer) radius $a(b)$, made of material of permeability μ, and placed in a formerly uniform constant magnetic induction $\mathbf{B}_0$, as shown in the figure. Find the fields $\mathbf{B}$ and $\mathbf{H}$ everywhere in space. What can you say about the inner field for a high permeability material ?