IISER Mohali
[**August 2025**]
PHY424 (QFT-I)

End-Sem

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Max. Marks : 40

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1. Using the Dirac equation for a massive fermion, check the conservation of the following current,
$$j^\mu(x) = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x).$$
[2]

2. Obtain the equations of motion for,
$$\mathcal{L} = -\frac{1}{2}\partial^\mu A^\nu \partial_\mu A_\nu + \frac{1}{2}\partial^\mu A^\nu \partial_\nu A_\mu + \frac{1}{2}m^2 A^\mu A_\mu.$$
[3]

3. Apply Wick's theorem to the following time ordered products.
   (a) $T\Big(\phi(x_1)\phi(x_2)\phi(x_3)\phi^\dagger(x_4)\Big)$
   (b) $T\Big(\psi(x_1)\bar{\psi}(x_2)\psi(x_3)\bar{\psi}(x_4)\Big)$

[5]

4. Consider the usual mode expansion of a real scalar field $\phi(x)$ with the following *anti-commutation* relations.
$$\{a_{\mathbf{p}}, a^\dagger_{\mathbf{k}}\} = (2\pi)^3\delta^3(\mathbf{p}-\mathbf{k}), \{a_{\mathbf{p}}, a_{\mathbf{k}}\} = \{a^\dagger_{\mathbf{p}}, a^\dagger_{\mathbf{k}}\} = 0$$
   (a) Calculate the normal-ordered Hamiltonian,
$$H = \int d^3x\, \frac{1}{2}\Big(\pi^2 + (\nabla\phi)^2 + m^2\phi^2\Big)$$
   in terms of $a_{\mathbf{p}}$ and $a^\dagger_{\mathbf{p}}$.
   (b) Calculate the energy of zero, single and two particle states, and comment on the particle spectrum of the theory.
   (c) Test the causality for $\phi^2(x)$ operator.

[10]

5. Prove the following identities.

(a) $\gamma^\alpha(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\gamma_\alpha = 0.$

(b) $\gamma^0 S^\dagger \gamma^0 = S^{-1}$, where S is the Lorentz transformation on the Dirac field.

[5]

6. Calculate the unpolarized cross section for the process $\psi(p_1) + \bar{\psi}(p_2) \to \phi(P)$ at the leading order in the rest frame of ϕ in Yukawa theory,

$$\mathcal{L}_{\text{Yukawa}} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_\phi^2\phi^2) + \bar{\psi}(i\partial\!\!\!/ - m)\psi - g\bar{\psi}\psi\phi.$$

[5]

7. Consider the leading order scattering $e^-(p_1) + \mu^-(p_2) \to e^-(p_3) + \mu^-(p_4)$ in QED,

$$\mathcal{L}_{\text{QED}} = \sum_i \bar{\psi}_i(i\partial\!\!\!/ - m_i)\psi_i - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} - e\bar{\psi}_i\gamma^\alpha\psi_i A_\alpha,$$

where $i = 1, 2$ refer to electron and muon respectively.

(a) Calculate spin-summed and averaged squared amplitude in terms of Mandelstam variables s, t and u.

(b) Calculate differential cross section $\dfrac{d\sigma}{d\Omega}$ in the center-of-mass frame and express your result in terms of $\alpha = \dfrac{e^2}{4\pi}$, s and $\cos\theta$ where θ is the angle between scattered electron and the incident electron which can be taken along z-axis.

Take $m_e = m_\mu = 0$. Use $u_e(p_1)$ and $u_\mu(p_2)$ for *incoming* electron and muon. Use $\bar{u}_e(p_3)$ and $\bar{u}_\mu(p_4)$ for *outgoing* electron and muon. Photon propagator can be taken as:

$$D_A^{\alpha\beta}(p) = \frac{i(-g^{\alpha\beta})}{p^2 + i\epsilon}$$

[10]

$i\mathcal{M} = (-ig)\,\bar{v}(p_2)\,u(p_1)$

$$\boxed{|\mathcal{M}|^2 = 4g^2\,(p_2\cdot p_1 - m^2)}$$

$\int \frac{d^3x}{(2\pi)^3}\, e^{-i(p_1-p_2)x} = \delta^{(3)}(p_1-p_2)\,?$

$\gamma^i\gamma_i = -(0\ \sigma^i)(0\ \sigma_i)$