Assignment 8

1. On a nylon filament 0.01 cm in diameter and 4 cm long there are 5×10^8 extra electrons distributed uniformly over the surface. What is the electric field strength at the surface of the filament :

   (a) In the rest frame of the filament ?

   (b) In a frame in which the filament is moving at a speed 0.9c in a direction parallel to its length ?

2. Consider the trajectory of a charged particle which is moving with speed 0.8c in the x direction when it enters a large region in which there is a uniform electric field in the y direction. Show that the x velocity of the particle must actually decrease. What about the x component of momentum?

3. Two protons are moving parallel to one another a distance r apart, with the same velocity βc in the lab frame. At the instantaneous position of one of the protons the electric field strength caused by the other is $\gamma e/4\pi\epsilon_0 r^2$. But the force on the proton measured in the lab frame is not the same. Verify this by finding the force in the proton rest frame and transforming that force back to the lab frame. Show that the discrepancy can be accounted for if there is a magnetic field β times as strong as the electric field, accompanying this proton as it travels through the lab frame.

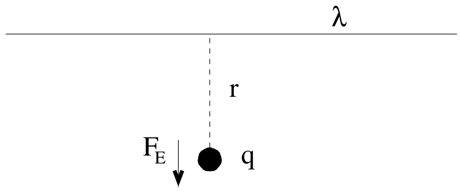
4. Consider a composite line charge consisting of several kinds of carriers, each with its own velocity. For one kind, k, the linear density of charge measured in frame F is λ_k and the velocity is $\beta_k c$ parallel to the line. The contribution of these carriers to the current in F is then $I_k = \lambda_k \beta_k c$. How much do these k-type carriers contribute to the charge and current in a frame F' which is moving parallel to the line at velocity $-\beta c$ with respect to F ? You should be able to show that

$$\lambda'_k = \gamma\left(\lambda_k + \frac{\beta I_k}{c}\right) \qquad I'_k = \gamma(I_k + \beta c \lambda_k)$$

   If each component of the linear charge density and current transforms in this way, then so must the total λ and I :

$$\lambda' = \gamma\left(\lambda + \frac{\beta I}{c}\right) \qquad I' = \gamma(I + \beta c \lambda).$$

5. A charge q is at rest a distance r from a long rod with linear charge density λ, as shown in the Fig. The charges in the rod are also at rest. The electric field due to the rod takes the standard form of $E = \lambda/2\pi\epsilon_0 r$, so the force on the charge q in the lab frame is simply $F = qE = q\lambda/2\pi\epsilon_0 r$. This force is repulsive, assuming q and λ have the same sign. Now



   consider the setup in the frame that moves to the left with speed v. In this frame both the charge q and the charges in the rod move to the right with speed v. What is the force on the charge q in this new frame ? Solve this in three different ways.

   (a) Transform the force from the lab frame to the new frame.

   (b) Directly calculate the electric and magnetic forces in the new frame.

(c) Transform the fields using the Lorentz transformations.

6. In the neighborhood of the origin in the coordinate system x, y, z, there is an electric field $\mathbf{E}$ of magnitude 100 V/m, pointing in a direction that makes angles of 30 degrees with the x axis and 60 degrees with the y axis. The frame F' has its axes parallel to those just described, but is moving, relative to the first frame, with a speed 0.6c in the positive y direction. Find the direction and magnitude of the electric field that will be reported by an observer in F'. What magnetic field does the observer report ?

7. (Griffiths 12.45)

   (a) Charge q_A is at rest at the origin in system S; charge q_B flies by at speed v on a trajectory parallel to the $x-$ axis, but at $y = d$. What is the electromagnetic force on q_B as it crosses the y axis?

   (b) Now study the same problem from system S' which moves to the right with speed v. What is the force on q_B when q_A passes the y' axis? [Do it two ways: (i) by using your answer to (a) and transforming the force; (ii) by computing the fields in S' and using the Lorentz force law.

8. (Griffiths 12.47)

   (a) Show that $(\mathbf{E} \cdot \mathbf{B})$ is relativistically invariant.

   (b) Show that $(E^2 - c^2 B^2)$ is relativistically invariant.

   (c) Suppose that in one inertial system $\mathbf{B} = 0$ but $\mathbf{E} \neq 0$ (at some point P). Is it possible to find another system in which the *electric* field is zero at P?