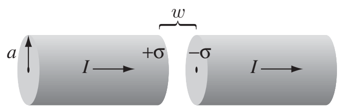
Assignment 6

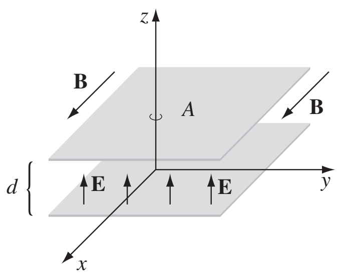
1. *Maxwell's equations in matter.* In order to write the Maxwell's equation in matter, you need to note that electric polarization $\boldsymbol{P}$ produces a bound charge density $\rho_b = -\boldsymbol{\nabla} \cdot \boldsymbol{P}$ and magnetization $\boldsymbol{M}$ leads to a bound current density $\boldsymbol{J}_b = -\boldsymbol{\nabla} \times \boldsymbol{M}$. There is also a polarization current due to the linear motion of charge when the electric polarization changes: $\boldsymbol{J}_p = \partial \boldsymbol{P}/\partial t$. Using this information, write down the Maxwell's equation in matter.

2. (Griffiths 7.34)A fat wire, radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in Fig.. Find the magnetic field in the gap, at a distance $s < a$ from the axis.



3. (Griffiths 8.2) Consider the charging capacitor in Prob. 7.34.
   (a) Find the electric and magnetic fields in the gap, as functions of the distance s from the axis and the time t. (Assume the charge is zero at $t = 0$.)
   (b) Find the energy density u_{em} and the Poynting vector $\mathbf{S}$ in the gap. Note especially the direction of $\mathbf{S}$. Check that Eq. 8.12 is satisfied.
   (c) Determine the total energy in the gap, as a function of time. Calculate the total power flowing into the gap, by integrating the Poynting vector over the appro- priate surface. Check that the power input is equal to the rate of increase of energy in the gap (Eq. 8.9—in this case W = 0, because there is no charge in the gap). [If you're worried about the fringing fields, do it for a volume of radius b ¡ a well inside the gap.]

4. (Griffiths 8.6) A charged parallel-plate capacitor (with uniform electric field $\mathbf{E} = E\hat{\mathbf{z}}$) is placed in a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{x}}$, as shown in Fig.



   (a) Find the electromagnetic momentum in the space between the plates.
   (b) Now a resistive wire is connected between the plates, along the z axis, so that the capacitor slowly discharges. The current through the wire will experience a magnetic force; what is the total impulse delivered to the system, during the discharge?

5. (Griffiths 8.14) An infinitely long cylindrical tube, of radius a, moves at constant speed v along its axis. It carries a net charge per unit length λ, uniformly distributed over its surface. Surrounding it, at radius b, is another cylinder, moving with the same velocity but carrying the opposite charge $(-\lambda)$. Find:
   (a) The energy per unit length stored in the fields.
   (b) The momentum per unit length in the fields.
   (c) The energy per unit time transported by the fields across a plane perpendicular to the cylinders.