EndSem for PHY402 (17/11/2025)

Maximum Marks 40

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Problem 1 Density of States for Free Electrons in d Dimensions

Derive a general expression for the density of states $g(E)$ for a free electron gas in d spatial dimensions. Express the result in terms of energy E, electron effective mass m^*, and Planck's constant $\hbar$. Check that you recover the results for $d = 1, 2, 3$. [3 + 1]

Problem 2 Temperature Dependence of Chemical Potential in a 2D Free Electron Gas

Derive the temperature dependence of the chemical potential $\mu(T)$ for a two-dimensional free electron gas. Assume the electron density remains constant with temperature. Analyze the behavior of $\mu(T)$ in the low-temperature and high-temperature limits. [3 + 1]

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Problem 3 Chemical Potential in an Intrinsic Direct Bandgap Semiconductor

Determine the position of the chemical potential μ in an intrinsic direct bandgap semiconductor at finite temperature. Express μ in terms of the conduction band minimum E_c, valence band maximum E_v, and the effective masses of electrons and holes. Discuss the limiting behavior at low temperature. [3 + 1]

Problem 4 Doping and Fermi Level Shift

Explain how doping shifts the Fermi level in a semiconductor. Quantify the shift in terms of carrier concentration. [1 + 3]

Problem 5 Hall effect in multiband metals

Derive an expression for the Hall coefficient R_H for a material in which both electrons and holes contribute to electrical conduction. Assume electron concentration n, hole concentration p, electron mobility $\mu_e = e\tau_e/m_e$, and hole mobility $\mu_h = e\tau_h/m_h$. [4 + 1] — 5

Problem 6 Bragg's Law

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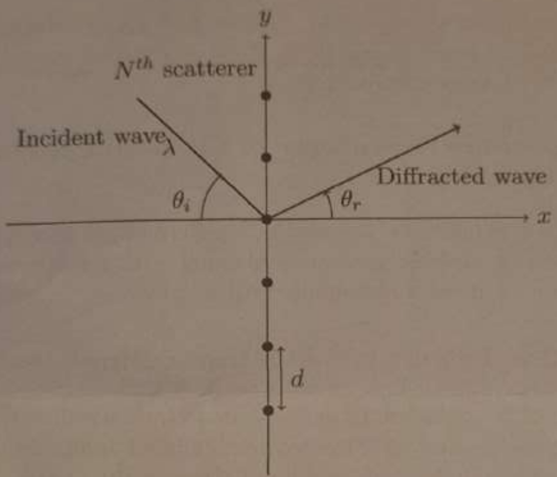
An electron is accelerated through a 25 kV potential to hit a metal target and loses all its energy to produce monochromatic x-rays. These x-rays fall on a solid with a simple cubic structure and a molar mass $M = 58.5$ g/mol and mass density $\rho = 2.165$ g/cm^3. Find the angle at which a first-order Bragg reflection will occur from the (100) planes. [3]

Problem 7 Structure factor

Find the structure factor for the (200) reflection in NaCl. [3] —3

Problem 8 Scattering from 1D crystal

Derive the expression for the intensity of a wave diffracted from a line of N equally spaced scattering centers. Assume the incident wave is monochromatic with wavelength λ, and the spacing between the centers is d. Find the condition for maximum intensity and that Bragg's law is recovered for specular reflection. [3 + 1] —4



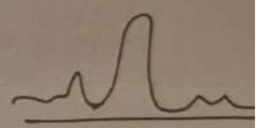


Figure 1: Wave diffraction from a line of N equally spaced scatterers. The incident wave enters from above at angle θ_i and is diffracted at angle θ_r.

Problem 9 Optical Transition and Momentum Change in Semiconductors

For a 1D semiconductor (bandgap ~ 1 eV, lattice $= 5$ Å) with the valence band maximum at the BZ center and the conduction band minima at the BZ boundary, can an optical transition be made with radiation of energy 1 eV? Explain quantitatively the conditions under which such a transition can happen. [2 + 2] —4

Problem 10 Molecular Field theory for an Antiferromagnet

Consider a two sub-lattice magnetic system with equal number of spins in both sublattices, each with Curie constant $C/2$. Suppose that a spin in each sublattice interacts with the magnetization of its own sublattice and of the other sub-lattice with mean-field exchange constants α (ferromagnetic) and β (antiferromagnetic), respectively. Find expressions for the magnetization in each sublattice in the paramagnetic state. Find the antiferromagnetic ordering temperature T_N below which there will be spontaneous magnetization in the sublattices and the Weiss temperature θ given by the Curie-Weiss law for the total susceptibility in the paramagnetic state $\chi = C/(T - \theta)$. (Use $\mu_0 = 1$