5. Prove the following identities.

(a) $\gamma^\alpha(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\gamma_\alpha = 0.$

(b) $\gamma^0 S^\dagger \gamma^0 = S^{-1}$, where $S$ is the Lorentz transformation on the Dirac field.

[5]

6. Calculate the unpolarized cross section for the process $\psi(p_1) + \bar\psi(p_2) \to \phi(P)$ at the leading order in the rest frame of $\phi$ in Yukawa theory,

$$\mathcal{L}_{\text{Yukawa}} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m_\phi^2\phi^2) + \bar\psi(i\slashed{\partial} - m)\psi - g\bar\psi\psi\phi.$$

[5]

7. Consider the leading order scattering $e^-(p_1) + \mu^-(p_2) \to e^-(p_3) + \mu^-(p_4)$ in QED,

$$\mathcal{L}_{\text{QED}} = \sum_i \bar\psi_i(i\slashed{\partial} - m_i)\psi_i - \frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} - e\bar\psi_i\gamma^\alpha\psi_i A_\alpha,$$

where $i = 1, 2$ refer to electron and muon respectively.

(a) Calculate spin-summed and averaged squared amplitude in terms of Mandelstam variables $s, t$ and $u$.

(b) Calculate differential cross section $\frac{d\sigma}{d\Omega}$ in the center-of-mass frame and express your result in terms of $\alpha = \frac{e^2}{4\pi}$, $s$ and $\cos\theta$ where $\theta$ is the angle between scattered electron and the incident electron which can be taken along $z$-axis.

Take $m_e = m_\mu = 0$. Use $u_e(p_1)$ and $u_\mu(p_2)$ for *incoming* electron and muon. Use $\bar u_e(p_3)$ and $\bar u_\mu(p_4)$ for *outgoing* electron and muon. Photon propagator can be taken as:

$$D_A^{\alpha\beta}(p) = \frac{i(-g^{\alpha\beta})}{p^2 + i\epsilon}$$

[10]

$$i\mathcal{M} = (-ig)\,\bar v(p_2)\,u(p_1)$$

$$\boxed{|\mathcal{M}|^2 = 4g^2\,(p_2\cdot p_1 - m^2)}$$

$$\int \frac{d^3x}{(2\pi)^3}\, e^{-i(p_i - p_2)x} = \delta^{(3)}(p_1 - p_2)?$$

2

$$\gamma^i\gamma^i := \begin{pmatrix} 0 & \sigma^i \\ \end{pmatrix}\begin{pmatrix} 0 & \sigma^i \\ \end{pmatrix}$$