1. First HIT

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Quick start

For quick start of drawing something, we need to have a physical representation of surface. To not making overcomplex system from the very beginning, I would like to refer to mathematical models instead of physical ones. The most popular mathematical models are:

- Sphere
- Box
- Plane

In your training, you will deal with spheres only, adding other types of surfaces isn't hard but required addition math.

To add spheres in our world, we need to figure our what is a sphere.

Sphere

Sphere - it's a perfect model with equal radius across a whole model, and can be represented with math formula

$$x^2 + y^2 + z^2 = R^2$$

Read is as - if sum of position squares equal to square of radius. this point IS in the surface of the sphere.

We want to place our object across our world, so we need to alter formula. Let point C be a center of sphere in world, the final form will be

$$(x - Cx)^2 + (y - Cy)^2 + (z - Cz)^2 = R^2$$

Let point P be a point on sphere, with coordinates (x,y,z), our formula will be in form of

$$dot(P-C, P-C) = R^2$$

in vector form it's a

$$(P - C)* (P - C) = R^2$$

expanding it with a algebraic rules $(a-b)^2 = a^2 -2ab + b^2$,we get

$$P^2 – 2PC + C^2 = R^2$$

P - it's a point of sphere, so, this our goal to find it. Instead of P we can place point from Ray

$P = R_{pos} + R_{dir} * t$, for simplicity I will use A + Bt

$(A + Bt)^2 - 2*(A + Bt)*C + C^2 = R^2$

We know everything except 't', so let's find it

$$(A + Bt)^2 - 2*(A + Bt)*C + C^2 = R^2$$

$$A^2 + 2ABt + (Bt)^2 - (2A + 2Bt)*C + C^2 = R^2 \Rightarrow$$

$$A^2 + 2ABt + (Bt)^2 - 2AC - 2BtC + C^2 = R^2 \Rightarrow$$

Grouping some vars,

$$A^2 + 2ABt + (Bt)^2 - 2AC - 2BtC + C^2 = R^2 \Rightarrow$$

$$(Bt)^2 + 2ABt - 2BtC + (A - C)^2 = R^2 \Rightarrow$$

getting common values from '+ 2ABt - 2BtC'

$$(Bt)^2 + 2Bt(A - C) + (A - C)^2 = R^2 \Rightarrow$$

$$B^2 * t^2 + 2Bt(A - C) + (A - C)^2 = R^2$$

Placing everything into one side

$$B^2 * t^2 + 2Bt(A - C) + (A - C)^2 - R^2 = 0$$

I see a quadratic equation here! $ax^2 + bx + c = 0$, where x = t in our sample. So

$a = B^2$
$b = 2 * B * (A - C)$
$c = (A - C) * (A - C) - R^2$

Replacing back original values for A, B, C we get

$B = R_{dir}$ where $B^2 = R_{dir}^2$ what can be changed to vector forms like dot(R_{dir}, R_{dir})

$A = R_{pos}$

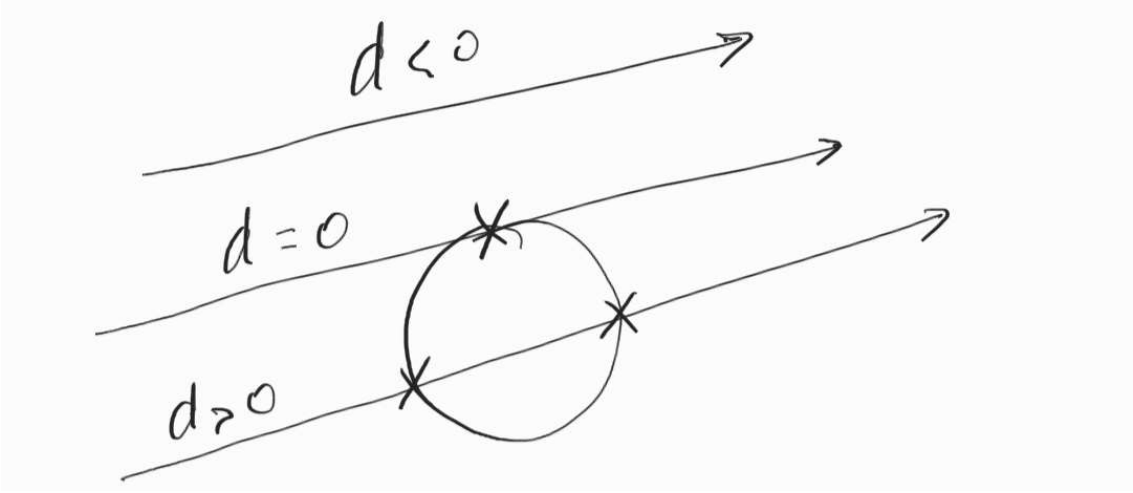
C = position of our sphere

That is, we only need to solve quadratic equation to find point in our ray and particular sphere. Sounds easy.

Discriminant can be found by formula

$$Discr = b^2 - 4ac$$

and this value gives as a geometrical representation



Solution 2 = -b + sqrt(Discr) / 2a

Code

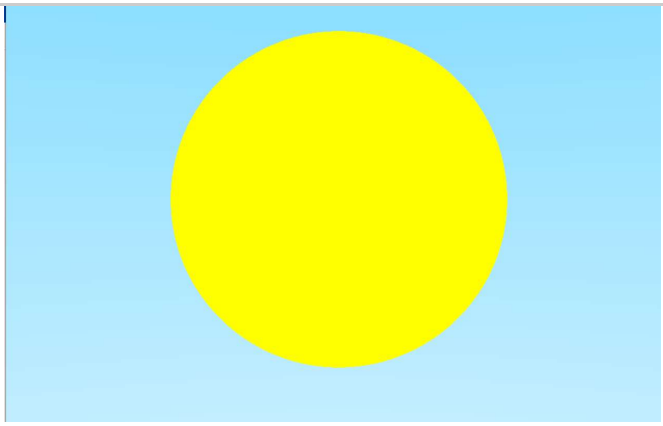
For your primal hit things, you need to implement algorithm before. Sample code below

MathBase.h | Application.cpp

Sample → Application

```
float3 Application::RayTracing( const Ray& ray ) const
{
    // hardcoded place of sphere( for now )
    float3 fSheperePos = { 0.0f, 0.0f, 1.0f };

    auto hit_sphere = []( const float3& center, float radius, const Ray& ray ) -> bool
    {
        float3 AC = ray.getPosition() - center;
        float a = float3::Dot( ray.getDirection(), ray.getDirection() );
        float b = 2.0f * float3::Dot( AC, ray.getDirection() );
        float c = float3::Dot( AC, AC ) - radius * radius;
        float discriminant = b * b - 4.0f * a * c;

        if( discriminant < 0.0f )
        {
            return false;
        }

        return true;

    };

    // if we have any distance for sphere( intersection occiured! )
    if( hit_sphere( fSheperePos, 0.5f, ray ) )
    {
        // draw yellow
        return float3( 1.0f, 1.0f, 0.0f );
    }

    // nothin' hit, horizon
    //
    return this->getHorizonColor( ray );
}
```

and result will be



My congratulations for you! you have your own ray tracer!

Tasks

Draw only silhouette of sphere

To achieve it, you can to compare discriminant value in some ranges.(After you have done it, describe by words why this approach is the worst of even possible of one)

Draw inner and outer silhouette

Let inner be in yellow color and outer in kind of green. Also, describe why outer part is mathematically wrong.

No labels