CMSC 442/653 Hrudaya TA40935

① Given $G = \begin{bmatrix} 0 & 2 & 1 & 2 & 0 \\ 2 & 1 & 1 & 0 & 2 \\ 2 & 2 & 0 & 1 & 1 \end{bmatrix}$

generator matrix for binary linear code V
span(G) = V

ⓐ Length of V is n = 5
which has 5 columns 5 bit vector

ⓑ Echelon canonical form

$G = \begin{pmatrix} 0 & 2 & 1 & 2 & 0 \\ 2 & 1 & 1 & 0 & 2 \\ 2 & 2 & 0 & 1 & 1 \end{pmatrix}$

Swaping the rows $R_1 \leftrightarrow R_3$

$\begin{pmatrix} 2 & 2 & 0 & 1 & 1 \\ 2 & 1 & 1 & 0 & 2 \\ 0 & 2 & 1 & 2 & 0 \end{pmatrix}$

$R_1 \leftrightarrow R_1 + R_2$

$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 1 & 0 & 2 \\ 0 & 2 & 1 & 2 & 0 \end{pmatrix}$

$R_2 \leftrightarrow R_2 + R_1$

$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 2 & 1 & 2 & 0 \end{pmatrix}$

$R_3 \leftrightarrow R_2 + R_3$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$$

$R_3 \leftrightarrow R_3 \div 2$

$$\begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$R_2 \leftrightarrow R_2 + R_3$

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

here no of rows in echlon form are 3
where distance $k = 3$

© Parity check matrix

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

we know that if G is of form $G = (I \mid P)$
then $H = (-P^T \mid I)$
so in order to make above $G = (I \mid P)$ we have to swap C_3 and C_5

$$G = \left(\begin{array}{ccc|cc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right)$$

$G = [I \mid P]$
then $H = (-P^T \mid I)$

$0\ 1\ 2$

$\boxed{2=-1}$

$$H=\left(\begin{array}{ccc|cc} -1 & -1 & 0 & 1 & 0 \\ -1 & -2 & 0 & 0 & 1 \end{array}\right)$$

$$H=\left[\begin{array}{ccc|cc} 2 & 2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

and at last we have to swap $C_3 \leftrightarrow C_5$

$$H=\left(\begin{array}{ccc|cc} 2 & 2 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 \end{array}\right)$$

This is the parity check matrix

we can verify $syn(c) = G \cdot H^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Hence H is correct

2) Given $G=\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$

ⓐ Given binary linear code

$R_2 \leftrightarrow R_2 + R_3$

$$G=\begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

swap $C_3 \leftrightarrow C_4$

$$G=\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right)$$

$G = (I \mid P)$

Here $P=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

length $n=6$
distance $k=3$

$k=3$, then $2^k = 2^3 = 8$

Hence no. of codework will be 8.

In order to find code words we know that

$$c = m \cdot p$$

$$[c_1 \; c_2 \; c_3] = [m_1 \; m_2 \; m_3] \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$c_1 = m_1 \oplus m_2$

$c_2 = m_1 \oplus m_3$

$c_3 = m_1 \oplus m_2 \oplus m_3$

m_1	m_2	m_3	c_1	c_2	c_3	code words	
0	0	0	0	0	0	000 000	
0	0	1	0	1	1	001 011	3
0	1	0	1	0	1	010 101	4
0	1	1	1	1	0	011 110	3
1	0	0	1	1	1	100 111	4
1	0	1	1	0	0	101 100	3
1	1	0	0	1	0	110 010	4
1	1	1	0	0	1	111 001	3

(b) The minimum distance d of v is the minimum weight like we can consider the number of 1's in a row because the less number of 1's can be compared with zero

so minimum 1's are 3

and also distance b/w the rows are 3 and 4 and minimum is 3

$d = 3$

(d) Find parity check matrix H of binary linear code v.

$$G = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right)$$

$G = (I \mid P)$ $\quad H = (-P^T \mid I)$

$$H = \left(\begin{array}{ccc|ccc} -1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 1 \end{array}\right)$$

$-1 = 1$

$$H = \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

so as we did in (a) $c_3 \leftrightarrow c_4$ swap

$$H = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$H = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

we can verify

$syn(G) = GH^T$

$$= \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

swap

$$= \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \end{bmatrix}$$

H is correct

2(c) It is easier to find minimum distance of linear code because it can be calculated by minium weight ie number of 1's in an row

And whereas in non-linear codes it is hard to find minimum distance because we have to use specialized techniques such as decoding algorithms or enumerative methods that involves checking all possible codewords

③ Given $H = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$

In order to find G:

we have to make in the form $H = (-P^T | I)$

so that we can find $G = (I | P)$

Hence C_4, C_5, C_6 should be an Identity matrix

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$R_2 \leftrightarrow R_1 + R_2$

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$R_3 \leftrightarrow R_1 + R_3$

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$R_2 \leftrightarrow R_2 + R_3$

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$R_1 \leftrightarrow R_1 + R_3$

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$C_2 \leftrightarrow C_5 \quad C_3 \leftrightarrow C_6$

$$H = \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$H = -P^T | I$

$G = (I | P)$ $\quad G = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right)$

4 (a) standard parity check matrix H for binary Hamming (15, 11) code

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$(2^r - 1,\ 2^r - r - 1) = (15, 11)$

$r = 4$

(b) syn(r)

$r = (0111\ 0011\ 0010\ 110)$

$syn(r) = r \cdot H^T$

$$= (0111\ 0011\ 0010\ 110) \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$= [0\ 0\ 1\ 0]$

(c) Based on the syndrome the code we got is 0010 which matches with 2nd column of H. So we need to change the 2nd bit as it is error.

$$= [0\ 011 \quad 0011 \quad 0010\ 110]$$