O construct the multiplication table of the group of symmetries of the equilaleral triangle given by

the presentation

(1,6:13=1,6=1,6=00)

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Assume that the group distinct elements are

1,1,1,5,6,6,66

For an equilateral triangle (D3)
(1,6:13=1,0=1, 10=01)

Set of elements of D3 is

In order to construct multiplication table we have to perform certain calculations which is shown below

111	ران	12	6	Po 1	26	-
1, 4	1	e ²	6	16	66	-> Identity soseme
0	2	P3=1	16 2	= 120	26	7.
2	p3	19 p	126	17(86)	12(826) = 86	m + }
1	= 1	7612	1	6(92)2	P	
6	= 26	7 100	1 1	= 62	Pr	
186	6	16	P	-		
120	16	6	6	1 6	1	_
	1 1 P	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 1 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	1 1 1 2 6 1 1 1 2 6 1 1 1 2 6 1 1 1 2 6 1 1 1 2 6 1 2 2 6 1 2 2 6 1 2 2 6 1 2 2 6 1 2 2 6 1 2 2 6 1 2 2 6 1 2 2 6 1	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

Identity so same

- (1) e.T = e
- 3 o.P= Po (given from the above relations)
- 6) 0.1= 80 (given) (01) 1 = 12) 1= 12(12) = 80
- (o. c = c=1
- 0.10=0.002= 202= p2 (wxt po=00 4 0=1
- 6 6. Po=6.69= 01=(1)P (wort 0.6=6=1 from about)

5th row if we consider

- 0 60 = 602 ox 60
- @ Prop = for P(1) (WIT' ==1-)

= P.P6 = P6 = 0 (WKT 0 P= P6 9 P3=1)

- 3 po, p = op. p= op. p= op = po (wxT po= 6p2 06=62:13=1
- Po. 0 = Po2 = P (62=1)
- Po. 10 = 0 pp = 0 p6 = 0(1)0=6=1 WKT PO= 6 P2; P3=150=1
- for po = opt po = o. po = o. po = op= 2(1)=p WET 10=012, 13=1,0=1

For 6th 2000

Of (1) = pt

Of (1) = pt

Of (1) = pt (1) =

	1	1	1	6	Pr	16
10	Joseph	•	12	6	10	12
P	P	12	1	96	P2-	6
1	PZ	1	P	60	6	Po
6	6	1280	PE	1	12	1
16	16	6	12	P	eqt'	P
re	16	16	6	p2	e	1

Finally

2 construct the multiplication table of the group of Qu symmetries of the square given by the presentation (1,6:14=1,62=1,16=673)

Assume that the distinct group elements are;

$$P_{4} = (P, \sigma | P^{4} = 1, \sigma^{2} = 1, P_{0} = \sigma P^{3})$$

$$= (P, \sigma | P^{4} = 1, \sigma^{2} = 1, P_{0} = \sigma P^{3})$$

Mutiplication table

)							9
	1	1	12	p3	•	10	62	P36
1	1	P	12	13	0	16	10	P36
-	P	PZ	13	1	16	126	136	16
2	12	ρ3	1	P	18	P36	6	le
13	13	1	P ,	12	P36	6	10	12
6	6	136	Pro	Po	1	p3	12	f
16	P.6	6	36	120	r	1	ρ3	12
16	120	60	6	120	12.	P	1	P3
Po	136	P6	96	6	p3	12	P	1

2nd row (1) = 1 1(1) = 12 1(12) = 13 @ 1(p3)=p9=1 (with 14=1) 9 p(0): 10 (10) = 12 1 1(130) = Pto = 0 (UKT P= 1) (8) 1(p26) = 136 3rd row 0 P2(1) = 12 @ 12(1) = P3 @ p2(p2) = p4 = 1 (wkt p4 = 1) (12(P3) = P4.P= P (WKT 11=1) B 12(0) = 10 D 12(10) = 13 € 1 126)= 146 = 6 (WITPY=1) B 12(136) = 150 = 14 (16) = 1616) = 16 words 0 P(1) = 13 @ 13(1)=14=1 LWKT 14-71) B p3(p2) = e4.P= 1.P=e (UKT p1=1) (9 p3(p3) = 14.p2 = 1.p2 (WKT P4=1) (3 (0) = 130 @ p3(10) = p40 = 0(1) (WKT 14=1) @ 13(10)= 14(10)= 7(10) [MIT (1=1) 13(P30)= P'(10)=1(10)=10 LWKT 14=1)

O 0 (1):6 (D) e(b) : 61 = 636 3 -(p2) = -12 9 0(P3)=10 (WET P6=613) 0(0) = 0 = 1 0 o(PE) = -(EPS) = EPS = 1(PS) NKT == 1 c (10) = (cp) (10) = (30) (po) = p3 (130) c = p4 (10) c= 1(12) (B) WKL eb= 62c 64=1 6=1 = 6= 3 Q o(136): (01) 10: (p30) p20: 13(p) p20: 13(136) 120 = 14(pa) pa = 3(pe 1) pr = pe (pe) 13(136) 16 o(130) = (01) 10 = (130) 12 = p3 (01) 10= = 14 (620)(10) = 12 (01)0= 12 (130)0= 14 (10)0= 16(0)=1 8 WET (14=1, 0=1, 10=0 p3) 6th row D 60(T) = 60 (D) 1-(1)=1(-1)= 1(p30)= 140= 1(r)=6 3 10(p2) = p(01)1= 1(130)1= 14(01)= 7(13)=62 Q Po((3) = p(o1) p2 = 1(p30) 1= p401= 10p=61) 1= 200 = 13(01) = 13(130) = 14(120) = 120 6) 10(0) = po = p(1)=pwx1 (02-1) 6 Po(Po) = P(P3o) 0 = P40 = 1 (P=1, 0=1) 19 19(1,0): 6[06)10: 1(130)10: 060: (130)10: 130, = 130, () be(b3e) = b (eb)be= 1(b3e)be= 2 0 = (b3e) 1e= To From 5th now @ ans=

7th now Opr (1) = 10 (120(P) = PEI) = 17(130) = 10 (3 pro(12) = prot) = prof = prof = 6 = 6 = 6 @ 16 (13) = 12 (81) P= 12 (136) 6= (16) P= (13) 1= 61= 136 (5) Po(0)= 12(02) = P2 (12(16): 12(136)16: 15 = 1 € 10(10) = 10(13016 = 1010 = From throw 6 = 1 B 12(136)= 12(136)16 = (16)16 = 1(36)16 = 616 = 13 8th now (13-11) = p36 @ 136 (1) = 13(01) = p3(136) = 136 (3) 13-(12) = 13(01) 1= p3(136) 1= pop= 12(p36) = for (D) 130(63) = 63(06) 1= 13(620) 1= 1265-2 Esom 4 th 200 (3 = 0 (3) 130(0) = 1302 = 12(10)0-12(0p3)0= (6) (30(16) = p3(p36)6= p21)=12 (1) 136(120) = p3(p36)16= 120 pc = From 4th now@p (3) 120(p36) - 12(p36)p26 = p30p6 = From 7th 200 (3 =1

Blet S be a set with an associate binary operation sisks as let ube a left identity of sie (eis = sts is) and let ex be a right identity of sie sier = sts es)

O prove that elect

Given that

let sbe a set with binary operation

·: 5x5 -> 5

let et be a left identity and ex be a right identity

et. s=s lAs et is left identity)

ser=s (Aser is right identity)

Given S is a set with an associate broamy operation which means $a_{\times}(b_{\times}c) = (a_{\times}b)_{\times}c$

er (s.er) = (er.s).er there eris left identity er is right identity

1. (s.ep) = (e.s). 1

(S.ep) = (el.s) => s.=s right identity left identity

As above sis a set

(07) el·ep=ep becg elis left id -0 el·ep=el becg epis right identy -0 From OPDel=el Q.E.D @ Also prove that scan have atmost one 2-sided

Given that to prove

s can have atmost 2 sided identify consider let shas two sided identify say exander which means

S. Pr = Pr. S = S

so if we consider

s.PL = S then PR.PL = PR - 0

and s. p = s = ep.sthen prof = ep. p = ep. p

therefore it has only one 2-sided identity which means it can have atmost 2-sided identity.

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(B) S be a set with an associative binary operation (B)

:; SKS->S and a 2 sided identity e and let ses

let si and sp be elements of S such that

Sc. s= e = s. Sp Prove that Sc= Sp

> Given that si and si be element of s $3_L \cdot S = l = S \cdot S_R$

and given sis a set of associate binary operation

 $0:SXS \rightarrow S$ ax(bxc) = (axb)xL S(.(S.SP)) = (S(.S).SP)p given

51.8= P.SP vnow from identity
se spe vnow from identity
element

Hence we proved

Ote