atlasqtl: an R package for variable selection in sparse regression with hierarchically-related responses

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Statistical studies aimed at identifying associations between a high-dimensional vector of candidate predictors and a series of correlated outcomes multiply, prompted by the proliferation of technologies capable of measuring large volumes of information, whether on health and lifestyle parameters, molecular entities or even galaxies. As well as growing in dimension, the collected datasets are also growing in complexity, which calls for elaborate and flexible modelling approaches. When coupled with robust and efficient inference, Bayesian hierarchical modelling is a powerful framework for describing intricate dependencies at the scale required by current applications, while conveying uncertainty in a coherent fashion. In this note, we present the R package `atlasqtl`, which implements a scalable hierarchical framework for variable selection in regression problems with high-dimensional predictor and response spaces.

Model and inference

The model consists of a series of hierarchically-related spike-and-slab regressions that permit borrowing information across a large number of responses. Specifically, to each pair of predictor X_s and response y_t corresponds a regression coefficient β_{st} and a spike-and-slab binary latent variable γ_{st}; posterior probabilities of association, $\text{pr}(\gamma_{st} = 1 \mid y)$, can thus be obtained and employed to implement a Bayesian false discovery rate control. The model is also tailored to the detection of *hotspots*, namely, predictors associated with multiple responses: the top-level hierarchy entails a probit submodel which involves a response-specific contribution to the spike-and-slab probability of association, via ζ_t, and a predictor-specific modulation of this contribution, via θ_s. The latter parameter also acts as the propensity of predictor X_s to be a hotspot and is assigned a horseshoe prior — the corresponding global-local specification adapts to the overall problem sparsity (thanks to the global scale σ_0), while flexibly capturing the individual hotspot effects (thanks to the Cauchy tail of the local scale λ_s). A graphical representation of the model is provided in Figure 1 and its full specification is given in Ruffieux et al. (2020).

Joint inference requires special attention as the binary latent matrix $\Gamma = \{\gamma_{st}\}$ creates a discrete search space of dimension $2^{p \times q}$, where p and q are the number of predictors and responses, respectively. To overcome this complication, `atlasqtl` relies on variational inference, with a structured mean-field factorisation and efficient batch updates. The algorithm is augmented with a simulated annealing scheme that improves the exploration of highly multimodal spaces by introducing so-called "temperature" parameters controlling the degree of separation of the modes for a series of "heated" distributions (Ruffieux et al. (2020)).

`atlasqtl` can be employed in any sparse multiple-response regression setting with Gaussian errors. Hereafter we discuss a use case in the context of expression quantitative trait locus (eQTL) analysis, in which *hotspot genetic variants*, controlling many gene expression traits at once, may be responsible for important functional mechanisms underlying specific disease endpoints — such studies aim to clarify the genetic architecture of diseases by estimating associations between several thousand responses (molecular traits, such as genomic, proteomic, metabolomic levels) and up to a few million candidate predictors (genetic variants).

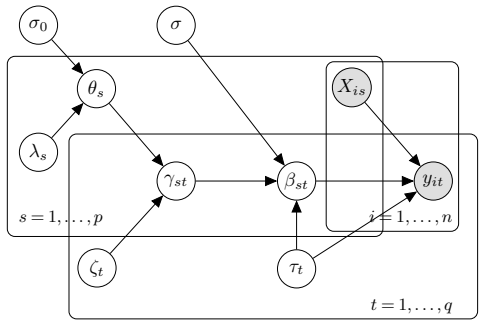

Figure 1: *Graphical representation of the model with n samples, p candidate predictors and q responses. The shaded nodes are observed, the others are inferred.*

Installation

The `atlasqtl` package is written in R with C++ subroutines. It can be installed following the instructions provided on GitHub[1].

Data and reproducibility

The material (incl. Rmarkdown document) used in the example below is publicly available[2] for replication purposes. It involves synthetic genotyping data generated using our in-house R package `echoseq`; these data emulate real genotypes (single nucleotide polymorphisms, "SNPs") located in the *LYZ* gene region, on chromosome 12. The gene expression traits consist of B-cell levels.

[1] https://github.com/hruffieux/atlasqtl

[2] https://github.com/hruffieux/software_corner_ibs_bulletin

Case study: an eQTL analysis to understand gene regulation in B cells

`atlasqtl` requires the specification of just two hyperparameters via the elicitation of a prior mean `mu_t` and variance `v_t` for the number of predictors associated with each response. The sensitivity of inference to these choices should be evaluated on a case-by-case basis; it is however low when a permutation-based Bayesian FDR threshold is employed, where the "null-case" permutation runs use the same hyperparameter specifications. The remaining model parameters are inferred by variational inference. In particular, the horseshoe prior on the hotspot propensity circumvents *ad-hoc* choices of top-level variance parameters (for which inference is prone to spurious effects). This specification also has desirable multiplicity adjustment properties in large-response settings, see Ruffieux et al. (2020).

```
require(atlasqtl)
```

```
mu_t <- 1; v_t <- 4 # hyperparameter specification

obj_atlasqtl <- atlasqtl(Y = Y, X = X, p0 = c(mu_t, v_t),
                         add_collinear_back = TRUE,
                         user_seed = 1)
```

The above code runs the analysis using a default geometric schedule on the inverse temperature, with initial temperature of 2 and grid size of 10 schedule — these choices can be altered via the argument `anneal`. The argument `add_collinear_back` is a boolean indicating whether or not to include the final posterior summaries for all possible collinear variables in X (which are removed during the run to ensure the stability of inference). A number of additional settings (e.g., seed for random number generator, tolerance for the variational algorithm, maximum number of iterations, checkpointing, etc.) can be chosen as detailed on the help page of the function, which can be accessed by running `?atlasqtl` in the console.

The custom `print.atlasqtl`, `summary.atlasqtl` and `plot.atlasqtl` S3 functions provide summary information and visualisation functionalities:

```
obj_atlasqtl
```

```
## ******************************************************
## Successful convergence after 117 iterations, using a
## tolerance of 0.1 on the absolute changes in the ELBO.
## ******************************************************
##
## Geometric annealing on the inverse temperature was
## applied for the first 10 iterations, with initial
## temperature of 2 (default).
##
## Number of samples: 100;
## Number of (non-redundant) candidate predictors: 200;
## Number of responses: 875;
## Prior expectation for the number of predictors
## associated with each response: 1 (sd: 2).
##
## The posterior quantities inferred by ATLASQTL can
## be accessed as list elements from the `atlasqtl` S3
## object, and a summary can obtained using the
## `summary` function.
```

A serial execution of the above analysis on a standard laptop[3] takes less than 10 seconds.

[3] 2.8GHz Quad-Core Intel Core i7 machine

The object returned by `atlasqtl` contains posterior quantities that can be employed to assess:

- pairwise associations between each pair of predictor and response: using the variational posterior probabilities of association (PPAs) stored in `gam_vb` ($p \times q$ matrix) and the variational posterior means of the regression estimates stored in `beta_vb` ($p \times q$ matrix);
- hotspot propensities: using the variational posterior mean of θ_s stored in `theta_vb` (vector of length p).

A summary for these quantities can optionally be based on a supplied Bayesian FDR threshold (here 5%):

```
thres_fdr <- 0.05
summary(obj_atlasqtl, thres = thres_fdr,
        fdr_adjust = TRUE, full_summary = FALSE)
```

```
## ******************************************************
## * ATLASQTL: posterior summary for variable selection *
## ******************************************************
##
## Using a 5% FDR control:
## -----------------------
##
## Nb of pairwise (predictor-response) associations: 499
##
## Nb of predictors associated with at least one response
## (active predictors): 14
##
## Hotspot sizes (nb of responses associated with each
## active predictor):
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    1.00    1.00    1.50   35.64   38.00  209.00
##
## Top hotspots:
## snp_103 (size 209), snp_115 (size 132), snp_171 (size 81),
## snp_54 (size 44), snp_15 (size 20), snp_75 (size 4)
```

Alternatively a $p \times q$ matrix of FDR estimates can be directly obtained using the `assign_bFDR` function.

The Manhattan-type plot indicates the position of the hotspots after Bayesian FDR control:

```
plot(obj_atlasqtl, thres = thres_fdr, fdr_adjust = TRUE)
```

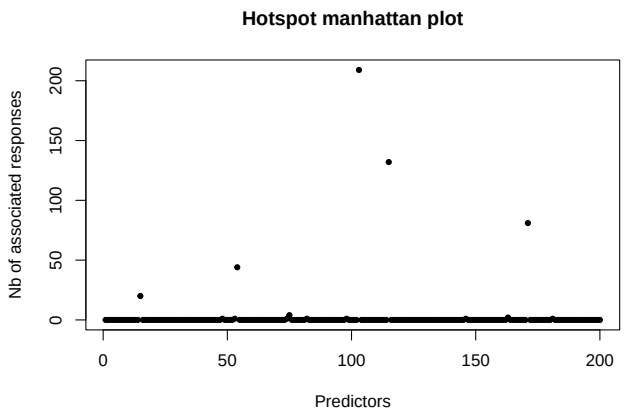

Figure 2: *Hotspot sizes, as obtained using the S3 function plot.atlasqtl.*

Thanks to the scalability of the variational implementation, a permutation-based Bayesian FDR control can also be used (Ruffieux et al. (2017) and Ruffieux et al. (2020)).

Encoding predictor-level information

We recently proposed a new approach, called `epispot`, which extends the `atlasqtl` hierarchical framework to encode and leverage information on the probability of candidate predictors to be involved in associations (Ruffieux et al. (2021)). Specifically, the effect of a (possibly high-dimensional) predictor-level vector of covariates is estimated using a secondary spike-and-slab regression at the top of the model hierarchy. The R package is available on GitHub[4].

References

Ruffieux, H., A. C. Davison, J. Hager, J. Inshaw, B. Fairfax, S. Richardson, and L. Bottolo. 2020. "A Global-Local Approach for Detecting Hotspots in Multiple Response Regression." *The Annals of Applied Statistics* 14: 905–28.

Ruffieux, H., A. C. Davison, J. Hager, and I. Irincheeva. 2017. "Efficient Inference for Genetic Association Studies with Multiple Outcomes." *Biostatistics* 18: 618–36.

Ruffieux, H., B. Fairfax, I. Nassiri, E. Vigorito, C. Wallace, S. Richardson, and L. Bottolo. 2021. "EPISPOT: an epigenome-driven approach for detecting and interpreting hotspots in molecular QTL studies." *The American Journal of Human Genetics* 108: 983–1000.

[4] https://github.com/hruffieux/epispot