Assignment –1 PDE MA20203,20103

*** Assignment is one component of grading (20% weightage) F.M. = 60M**

NO CREDIT FOR NOT SHOWING DETAILED STEPS.

Symbols: $p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}, r \equiv \frac{\partial^2 z}{\partial x^2}, s \equiv \frac{\partial^2 z}{\partial x \partial y}, t \equiv \frac{\partial^2 z}{\partial y^2},$

1. **[6+3+3=12M]**

a) For each of the following PDE, write down the order and degree $[6 \times 1 = 6\text{M}]$

(i) $p + q = z + xyz,$ (ii) $\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^3 z}{\partial y^3} = 2x\frac{\partial z}{\partial x},$ (iii) $r + (1+p)^2 = 1,$

(iv) $\frac{\partial^2 z}{\partial x^2} = \left(1 + \frac{\partial z}{\partial y}\right)^{1/2},$ (v) $y(p^2 + q^2) = qz,$ (vi) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = xyz$

b) Form a PDE by eliminating arbitrary constants a and b from $z = axe^y + \frac{1}{2}a^2 e^{2y} + b$.

c) Form a PDE by eliminating arbitrary functions f and g from $z = yf(x) + xg(y)$.

2. **[12+6=18M]**

a) Solve the following PDEs by Lagrange method: $[6 \times 2 = 12\text{M}]$

i. $(x + 2z)p + (4zx - y)q = 2x^2 + y$

ii. $\frac{y^2 z}{x}p + xzq = y^2, x \neq 0$

b) Find the integral surface of $x^2p + y^2q + z^2 = 0$ which passes through the curve $xy = x + y, z = 1$.

3. **[5+10=15M]**

Consider a family of PDE $(p^2 z^a + q^2)y^\alpha = b + \beta q z^\gamma, \alpha, \beta, \gamma, a, b \in \mathbb{R}$. The complete integral of this PDE-family is denoted by $z = z(x, y; c, d)$, where c, d are arbitrary constants.

a) For $\alpha = \beta = a = 0, b = 25, \gamma \in \mathbb{R}$, given that $z(x, y; c, d) > 10, \forall x, y, c, d \in \mathbb{R}$, where c is the value of p at any point (x, y), and d is the value of the solution at $(x, y) = (0,0)$. Compute the value of $|z(2,1; 4,1)|$. [5M]

b) For $\alpha = \beta = \gamma = 1, a = b = 0$, given that $|z(x, y; c, d)| < 3, \forall x, y, c, d \in \mathbb{R}$, where c is the value of sum of squares of p and q at any point (x, y), and d^2 is the value of the square of the solution at $(x, y) = (0,0)$. Find the value of $|z(1,1; 2, -1)|^2$. [10M]

4. **[10+5=15M]**

Consider the pair of PDEs $\alpha(px - qy) - \beta z = 0$ $(\alpha\beta \neq 0)$, $x^2yp + xy^2q - xyz = 0$.

a) Show that two PDEs are compatible for all non-zero real values of α, β. [10M]

b) If $z(x, y; c)$ is the common solution where c is the value of the square of the solution at $(x, y) = (1,1)$, then for all real α, β, compute the value/s of $z(\alpha\beta, \alpha\beta; \frac{1}{\alpha^2\beta^2})$.

THE END