

ZAPOSIM ISPIIT 17

FIN. MATEMATIKE (24.6.2019.)
 RJEŠENJA

1) a) $100 + 200 \cdot 0.76 + 300 \cdot 0.76^2 = \frac{600}{(1+r_c)^{10}}$

$(1+r_c)^{10} = 1.490835294 / (1.1)^2 = 1$

$r_c = 0.035092348 = 3.5\% > 3\%$

(N)

b) $D < T = 3 \rightarrow$ (N)

c) $x < \begin{matrix} 992 \\ 96 \end{matrix} \quad x \cdot 0.35 = 96$

$x = 80$

$\rightarrow 80 \cdot (1+g) = 992$

$g = 14\%$

(T)

d) $K=70, T=1, S(1)=67.3123, r=0.5$

$\rightarrow (S(1) - K)^+ = \begin{cases} 3 \\ 0 \end{cases}$

$\rightarrow E(K(1)) = 0.5 \cdot 3 - 0.5 \cdot e^r = 0$

$e^r = \frac{1.5}{0.5} = 3 \Rightarrow r = 0.5878 = 58.78\%$

e) $E(S_{n+1} | F_n) = E(S_n + X_{n+1} | F_n) = S_n + 0$

(T)

f) $F(0,1) = 580.917, S(0) = 500, r = 15\%$

$500 \cdot e^{0.15} = 580.917 = F(0,1)$

(T)

g) $D=5, t=\frac{3}{12}, z=0.1, F(0,1)=34.076$

$(S(0) - 5 \cdot e^{-0.1 \cdot \frac{3}{12}}) \cdot e^{0.1 \cdot \frac{3}{12}} = 34.076 \Rightarrow S(0) = 30.00035$

(T)

$$\Rightarrow P^E \geq X e^{-rT} - S(0) \quad \checkmark$$

i) Nikoli da ni je potrošen
bonitet!

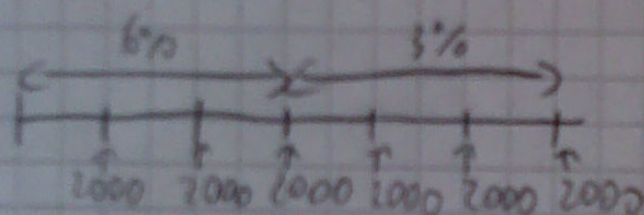
(T)

j) $\text{vega}_{CE} = \frac{\partial C^E}{\partial \sigma} > 0$

(T)

li opije na volatilitetu
dionici je kupijo

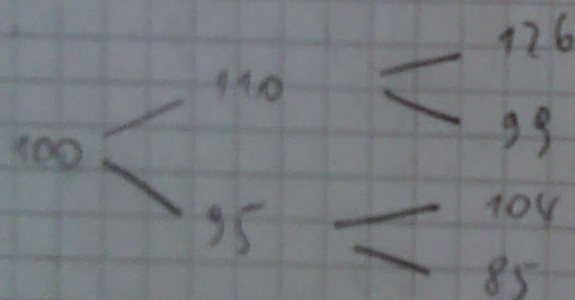
(2)



$$V_3 = 2000 \cdot 1.06^2 + 2000 \cdot 1.06 + 2000 + \frac{2000}{1.03} + \frac{2000}{1.03^2}$$

$$V_3 = 12024.42271$$

(3)



$$r = 5\%$$

a) $\left. \begin{array}{l} g = 10\% \\ d = -5\% \end{array} \right\}$

$$\left. \begin{array}{l} gg = 14.54\% \\ gd = -10\% \\ dg = 3.47\% \\ dd = -10.53\% \end{array} \right\}$$

$d < r < g$ vrsto

\Rightarrow NE POSTOJI
ARBITRAŽE

b) $K = 100, T = 2$

$$p^* \cdot 3.47\% + (1 - p^*) \cdot (-10.53\%) = 5\% \Rightarrow p^* = \dots$$

$$(K - S(2))^+ = 0 \Rightarrow P^E = \frac{1}{e^{rT}} \cdot E^*[K - S(2)]^+$$

$$b) \quad C^E - P^E = S(0) - X e^{-rT}$$

$$-P^E \leq S(0) - X e^{-rT}$$

$$\Rightarrow P^E \geq X e^{-rT} - S(0)$$

(+)

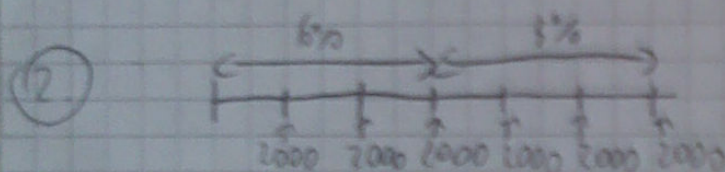
i) Risiko der kurz gestrichelten
Konten!

(+)

$$j) \quad \text{vege } C^E = \frac{\partial C^E}{\partial \sigma} > 0$$

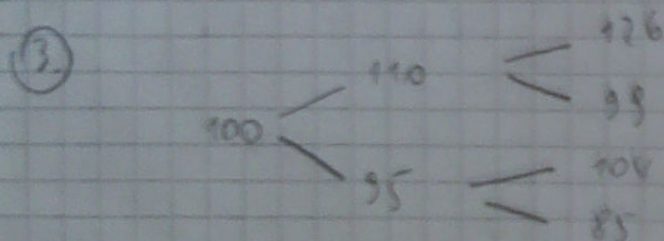
(+)

li opzij na volatilituim
dieren je laagje



$$V_3 = 2000 \cdot 1.06^2 + 2000 \cdot 1.06 + 2000 + \frac{2000}{1.06} + \frac{2000}{1.06^2} + \frac{2000}{1.06^3}$$

$$V_3 = 72024.42271$$



$$a) \quad \left. \begin{array}{l} g = 10\% \\ d = -5\% \end{array} \right\} \begin{array}{l} gg = 14.5\% \\ gd = -10\% \\ dg = 5.47\% \\ dd = -10.53\% \end{array}$$

de afsp. 11.7.1988
→ 10. 11.7.1988
11.7.1988

$$b) \quad K = 100, T = 2$$

$$r^* = 5.47\% + (1 - r^*) \cdot (1.10135) = 5.47\% + 0.051 \cdot 1.10135 = 5.53\%$$

$$(K - S(2)) = \begin{cases} 0 \\ 15 \end{cases} \Rightarrow P^E = \frac{1}{1.055} + \frac{1}{1.055^2} = 1.81$$

$$P^E = 1.81$$

$$④ \quad 100 \begin{cases} 130 \\ +0 \end{cases} \quad r = 10\%$$

a) $g = 30\%$, $d = -30\%$ $\Rightarrow d \neq -g \Rightarrow$ NO PORTFOLIO
NOB. ARB.

b) $r^* = 30\% + (1 - r^*) \cdot (-30\%) = 10\%$
 $\Rightarrow r^* = \frac{2}{3} = 0.6667$
 $1 - r^* = \frac{1}{3} = 0.3333$

c) $(S(t) - K)^+ = \begin{cases} 40 \\ 0 \end{cases}$

d) $C^E = \left(\frac{1}{1+r} \right) \cdot E^*[(S(t) - K)^+] = 24.2924$
 $\rightarrow \text{present: } e^{-rt}$

e) $\begin{cases} 130x + 1.1y = 40 \\ 70x + 1.1y = 0 \end{cases} \Rightarrow \begin{cases} 60x = 40 \Rightarrow x = \frac{2}{3} \\ y = -42.4242 \end{cases}$

$\Rightarrow (x, y) = \left(\frac{2}{3}, -42.4242 \right) \rightarrow$ greife $\frac{2}{3}$
 shorte 1

⑤ $S(0) = 50$, $r = 6\%$, $D_1 = 5$, $t_1 = 0.5$
 $D_2 = 2.5$, $t_2 = 1$, $T = 1$

$F(0, T) = (S(0) - D_1 \cdot e^{-rt_1} - D_2 \cdot e^{-rt_2}) \cdot e^{-rT}$
 $\rightarrow F(0, 1) = 40.43955466$

⑥ $S(0) = 500$, $C^E = 86.79$, $P^E = 28.9$, $r = 6\%$, $X = ?$

$C^E - P^E = S(0) - X \cdot e^{-rt}$

$86.79 - 28.9 = 500 - X \cdot e^{-0.06}$

$\Rightarrow X = 469.4485556$

$S(0) = 112$, $r = 5\%$, $T = 0.5$

$F(0, 0.5) = 112$

$F^S(0, T) = S(0) \cdot e^{(r-r_0)T}$

$\Rightarrow F^S(0, 0.5) = 111.6624371 < 112$

STRATEGIJA : u $t=0$ uđemo " kratku poziciju"
 \Leftarrow > usprizidnim ugovorom i dugu poziciju za sintetičnim ugovorom

$V(0) = 0 + 0 = 0$

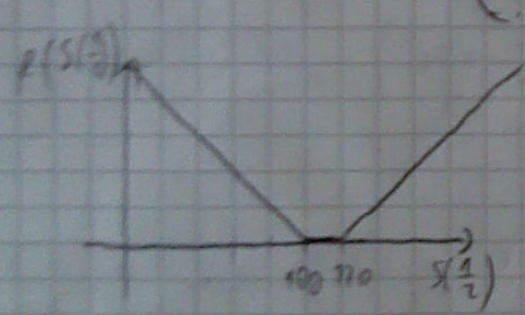
u $t=0.5$ izborimo da ugovor
 tj. kupimo (po sintetičnom ugovoru)
 dionicu za 111.6624371 ; prodamo
 ju (po direktnom usprizidnom ug.)
 za 112.

\Rightarrow direktni profit u $t=0.5$ iznosi

$V(1) = 112 - 111.6624371 = 0.3375629 > 0$

(5) $r = 5\%$, $\sigma = 0.3$, $S(0) = 100$

$f(S(\frac{T}{2})) = \begin{cases} 100 - S(\frac{T}{2}), & S(\frac{T}{2}) < 100 \\ 0, & 100 \leq S(\frac{T}{2}) \leq 120 \\ S(\frac{T}{2}) - 120, & S(\frac{T}{2}) > 120 \end{cases}$



$\Rightarrow f(S(\frac{T}{2})) = \underbrace{(100 - S(\frac{T}{2}))^+}_{f_{PE}, K=100} + \underbrace{(S(\frac{T}{2}) - 120)^+}_{f_{CE}, K=120}$

fer cijena = $e^{-r \cdot 0.5} \cdot E^*[f(S(\frac{T}{2}))] = e^{-r \cdot 0.5} \cdot E^*[f_{PE} + f_{CE}] =$
 $= P^E + C^E = (*)$

$$C^E = \underbrace{S(0)}_{100} \cdot N(d_1^{CE}) - \underbrace{K^{CE}}_{120} \cdot e^{-rT} \cdot N(d_2^{CE})$$

$$\Rightarrow d_1^{CE} = -0.6356, \quad d_2^{CE} = -0.8477$$

$$\Rightarrow C^E = 100 \cdot 0.2675 - 120 \cdot e^{-0.05 \cdot 0.5} \cdot 0.1983$$

$$= \underline{3.0415}$$

$$P^E = \underbrace{K}_{100} \cdot e^{-rT} \cdot N(-d_2^{PE}) - \underbrace{S(0)}_{100} \cdot N(-d_1^{PE})$$

$$\Rightarrow d_1^{PE} = 0.2233, \quad d_2^{PE} = 0.0198$$

$$\Rightarrow P^E = 100 \cdot e^{-0.05 \cdot 0.5} \cdot (1 - 0.5047) - 100 \cdot (1 - 0.5886)$$

$$= \underline{41.14} \quad 7.1671$$

$$\Rightarrow (*) = \underline{41.14} + 3.0415 = \underline{44.1815} \leftarrow \text{per eigene Investition}$$

⑤ $S(0) = 85, \quad r = 0.05, \quad K = 75, \quad C^E = 10, \quad T = 1$
 $S(\frac{8}{12}) = 85, \quad \sigma = 0.26$

$$V(\frac{8}{12}) = C_{t=\frac{8}{12}}^E - 10 \cdot e^{0.05 \cdot \frac{8}{12}}$$

$$C_{t=\frac{8}{12}}^E = S(\frac{8}{12}) \cdot N(d_1) - K \cdot e^{-0.05 \cdot \frac{8}{12}} \cdot N(d_2)$$

$$d_1 = 1.02, \quad d_2 = 0.87$$

$$\Rightarrow C_{t=\frac{8}{12}}^E = 85 \cdot 0.8461 - 75 \cdot e^{-0.05 \cdot \frac{8}{12}} \cdot 0.8078$$

$$= 17.33488187$$

$$\Rightarrow V(\frac{8}{12}) = \underline{1.995930835}$$

⑧ nexteinde:

$$C^E = \underbrace{S(0)}_{100} \cdot N(d_1^E) - \underbrace{K}_{120} \cdot e^{-rT} \cdot N(d_2^E)$$

$$\Rightarrow d_1^E = -0.6356, \quad d_2^E = -0.8477$$

$$\Rightarrow C^E = 100 \cdot 0.2625 - 120 \cdot e^{-0.05 \cdot 0.5} \cdot 0.1983$$

$$= \underline{3.0415}$$

$$P^E = \underbrace{K}_{100} \cdot e^{-rT} \cdot N(-d_2^E) - \underbrace{S(0)}_{100} \cdot N(-d_1^E)$$

$$\Rightarrow d_1^{P^E} = 0.2233, \quad d_2^{P^E} = 0.0118$$

$$\Rightarrow P^E = 100 \cdot e^{-0.05 \cdot 0.5} \cdot (1 - 0.5847) - 100 \cdot (1 - 0.5886)$$

$$= \underline{41.14} \quad 7.1671$$

$$\rightarrow (*) = \underline{41.14 + 3.0415 = 44.1815} \leftarrow \text{für eigene Investition}$$

⑨ $S(0) = 85, \quad r = 0.05, \quad K = 75, \quad C^E = 10, \quad T = 1$
 $S\left(\frac{8}{12}\right) = 85, \quad \sigma = 0.26$

$$V\left(\frac{8}{12}\right) = C_{r=\frac{8}{12}}^E - 10 \cdot e^{0.05 \cdot \frac{8}{12}}$$

$$C_{r=\frac{8}{12}}^E = S\left(\frac{8}{12}\right) \cdot N(d_1) - K \cdot e^{-0.05 \cdot \frac{8}{12}} \cdot N(d_2)$$

$$d_1 = 1.02, \quad d_2 = 0.17$$

$$\Rightarrow C_{r=\frac{8}{12}}^E = 85 \cdot 0.8461 - 75 \cdot e^{-0.05 \cdot \frac{8}{12}} \cdot 0.8078$$

$$= 12.33488137$$

$$\Rightarrow \underline{V\left(\frac{8}{12}\right) = 1.995930135}$$

10) $S(0) = 2$, $\sigma = 0.15$, $r = 0.05$

(x, y, z, w) , $z = -1000$, $w = -500$, $K_{CE} = 2.1$, $T_{CE} = 0.5$

$K_{PE} = 1.9$, $T_{PE} = \frac{3}{12}$

$K = 1.9$, $T = \frac{3}{12}$

verifichi:

$$V(S(0)) = x \cdot S(0) + y + z \cdot C^E + d \cdot P^E = 0$$

$$\text{delta}_V = \frac{\partial V}{\partial S} = x + z \cdot \underbrace{\frac{\partial C^E}{\partial S}}_{\text{delta}_{CE}} + d \cdot \underbrace{\frac{\partial P^E}{\partial S}}_{\text{delta}_{PE}} = 0$$

$$\text{delta}_{CE} = N(d_1^{CE}), \quad d_1^{CE} = \frac{\ln \frac{2}{2.1} + (0.05 + 0.5 \cdot 0.15^2) \cdot 0.5}{0.15 \cdot \sqrt{0.5}}$$

$$d_2^{CE} = -0.2773 \quad \Leftarrow \quad d_1^{CE} = -0.1713$$

$$\text{delta}_{PE} = -N(-d_1^{PE}), \quad d_1^{PE} = \frac{\ln \frac{2}{1.9} + (0.05 + 0.5 \cdot 0.15^2) \cdot \frac{3}{12}}{0.15 \cdot \sqrt{\frac{3}{12}}}$$

$$d_2^{PE} = 0.6186 \quad \Leftarrow \quad d_1^{PE} = 0.7485$$

$$\begin{aligned} C^E &= S(0) \cdot N(d_1^{CE}) - K^{CE} \cdot e^{-rT^{CE}} \cdot N(d_2^{CE}) = \\ &= 2 \cdot 0.4320 - 2.1 \cdot e^{-0.05 \cdot 0.5} \cdot 0.3908 \\ &= \underline{0.063582661} \end{aligned}$$

$$\begin{aligned} P^E &= K^{PE} \cdot e^{-rT^{PE}} \cdot N(-d_2^{PE}) - S(0) \cdot N(-d_1^{PE}) = \\ &= 1.9 \cdot e^{-0.05 \cdot \frac{3}{12}} \cdot (1 - 0.7315) - 2 \cdot (1 - 0.7774) \\ &= 0.450641604 - 0.4542 \\ &= \underline{0.036441604} \end{aligned}$$

verifichi: $2x + y - 1000 - 0.063582661 - 500 \cdot 0.036441604 = 0$

$x - 1000 \cdot 0.4320 - 500 \cdot (-0.2771) = 0$

$$\Rightarrow x = 398.45$$

$$y = -555.098537$$

$$\Rightarrow (x, y, z, w) = (398.45, -555.098537, -1000, 1)$$