ZADACI 03   (4.5. 2010.)

① $\quad P = 4.5 \qquad K = 36$

$\quad r = 12\% \qquad T = 3 \text{ mj}$

$S(T) = ?$

isplata $= (K - S(T))^+$

$\Rightarrow \quad 4.5 \cdot e^{0.12 \cdot \frac{3}{12}} + 3 = 36 - S(T)$

$\qquad\qquad S(T) = 33 - 4.5 \cdot e^{0.03}$

$\qquad\qquad \boxed{S(T) = 28.3629546}$

② $\quad C = e^{-rT} \cdot E^*[(S(T) - K)^+]$

$\qquad P = e^{-rT} \cdot E^*[(K - S(T))^+]$

$\Rightarrow C - P = e^{-rT} \cdot (E^*[(S(T) - K)^+] - E^*[(K - S(T))^+])$

$\qquad = e^{-rT} \cdot E^*[(S(T) - K)^+ - (K - S(T))^+]$

$\qquad = e^{-rT} \cdot E^*[S(T) - K]$

$\qquad = e^{-rT} \cdot E^*[S(T)] - e^{-rT} \cdot K$

$\qquad = E^*\left[\frac{S(T)}{e^{-rT}}\right] - e^{-rT} \cdot K$

$\qquad = S(0) - e^{-rT} \cdot K$

$\qquad\qquad\qquad\qquad Q.E.D.$

$$C > P + S(0) - K \cdot e^{-rT}$$

a) Da, postoji mogućnost arbitraže
jer iz call-put pariteta proizlazi
da je nearbitražna cijena call opcije
$= P + S(0) - K \cdot e^{-rT}$.

b) $t=0$ Kako je $C >$ od nearbitražne vrijednosti
napravimo short sell 1 call opcije
$\Rightarrow$ dobijemo $C > P + S(0) - K \cdot e^{-rT}$
$\Rightarrow$ kupimo 1 put opciju za iznos P,
1 dionicu za iznos $S(0)$,
posudimo iz banke $K \cdot e^{-rT}$ novca
$\Rightarrow$ time je zapravo kupljen
replicirajući portfelj za call
opciju!!
$\Rightarrow$ ostatak novca $(C - P - S(0) + K \cdot e^{-rT} > 0)$
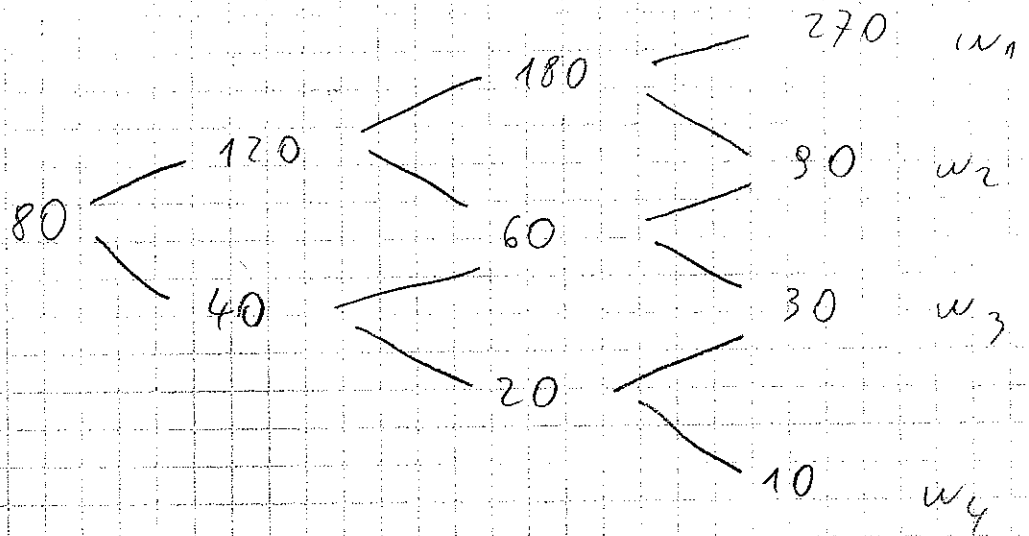oročimo po kam. stopi r

$t=T$ Replicirajući portfelj vrijedi
točno koliko je obveza povratka
short - sellingom call opcije
(dok za:

$$vrij. \ repl. portfelja =$$
$$= (K - S(T))^+ + S(T) - K$$
$$= (S(T) - K)^+$$
$$\qquad \qquad (S(T))$$

$\Rightarrow$ $\boxed{naš \ profit \ (C - P - S(0) + K \cdot e^{-rT}) \cdot e^{rT} > 0}$

(4.)

a)

$80 \to 120 \to 180 \to 270 \quad w_1$

$80 \to 40 \to 60 \to 90 \quad w_2$

$\to 30 \quad w_3$

$\to 20 \to 10 \quad w_4$

b) NE, jer je $d < r < g$

c) $120 \cdot p^* + 40 \cdot (1 - p^*) = 80 \cdot 1.25$

$$80 p^* + 40 = 100$$
$$80 p^* = 60$$
$$p^* = \frac{3}{4}, \quad 1 - p^* = \frac{1}{4}$$

d) Isplate put opcije u $t = T = 3, (K = 90)$:

$w_1: \quad (90 - 270)^+ = 0$

$w_2: \quad (90 - 90)^+ = 0$

$w_3: \quad (90 - 30)^+ = 60$

$w_4: \quad (90 - 10)^+ = 80$

u $t = 2$

$w_1, w_2: \quad C_2 = E^* \left[\frac{C_3}{1 + r} \right] =$

$$= \frac{1}{1.25} \left(\frac{3}{4} \cdot 0 + \frac{1}{4} \cdot 0 \right) = 0$$

$w_2, w_3: \quad C_2 = \frac{1}{1.25} \left(\frac{3}{4} \cdot 0 + \frac{1}{4} \cdot 60 \right) = 12$

$w_3, w_4: \quad C_2 = \frac{1}{1.25} \left(\frac{3}{4} \cdot 60 + \frac{1}{4} \cdot 80 \right) = 52$

u $t = 1$ $\quad w_1, w_2, w_3: \quad C_1 = E^* \left[\frac{C_2}{1+r} \right] = \frac{1}{1.25} \left(\frac{3}{4} \cdot 0 + \frac{1}{4} \cdot 12 \right) = 3.2$

$w_2, w_3, w_4: \quad C_1 = \frac{1}{1.25} \left(\frac{3}{4} \cdot 12 + \frac{1}{4} \cdot 52 \right) = 17.6$
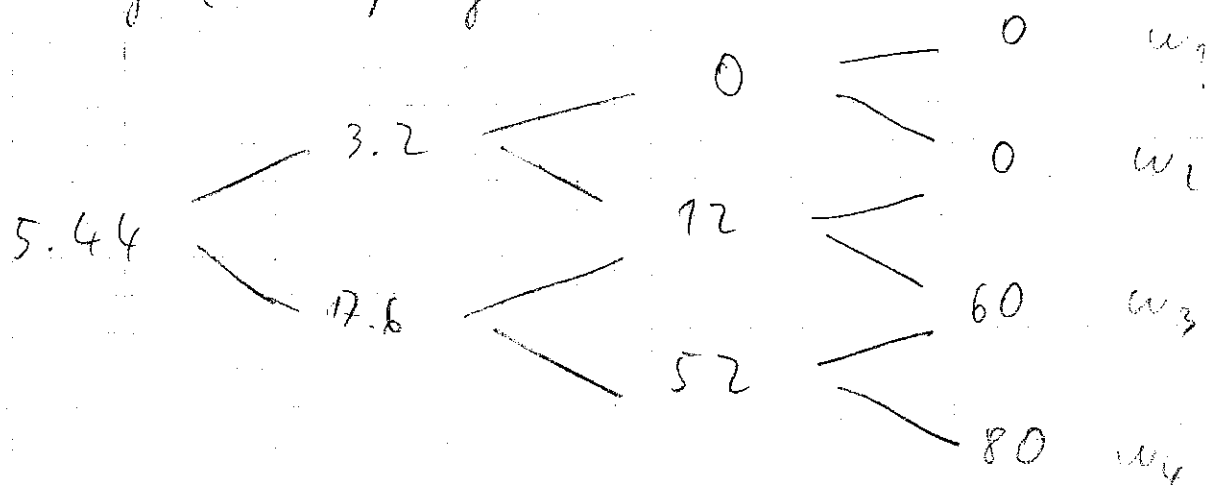
u $t=0$:

$$C_0 = E^x\left(\frac{C_1}{1+r}\right)$$

$$= \frac{1}{1.25}\left(\frac{3}{4}\cdot 3.2 + \frac{1}{4}\cdot 17.6\right) =$$

$$\boxed{C_0 = 5.44}$$

$\Rightarrow$ Stablo cijene opcije:

e) replicirajući portfelj može se računati
bilo kojim redoslijedom (od poč. prema kraju ili obratno)

$-S(1)_{gornje} \quad \text{dobi. norca}$

$t=1 \qquad x(1)\cdot 120 + y(1)\cdot 1.25 = 3.2$

$\qquad x(1)\cdot 40 + y(1)\cdot 1.25 = 17.6$

$\qquad S(1)donje$

$$x(1)(120-40) = 3.2 - 17.6$$

$$\Rightarrow \boxed{x(1) = -0.18 \Rightarrow y(1) = 19.84}$$

$t=2 \quad w_1, w_2, w_3: -S(2)$ gornje u gornjem podstablu

$\qquad x(2)\cdot 180 + y(2)\cdot 1.25 = 0 \quad \boxed{x(2) = -0.1}$

$\qquad x(2)\cdot 60 + y(2)\cdot 1.25 = 12 \quad \boxed{y(2) = 14.4}$

$\qquad "S(2)donje$ u gornjem podstablu

$w_2, w_3, w_4: -S(2)$ gornje u donjem podstablu

$\qquad x(2)\cdot 60 + y(2)\cdot 1.25 = 12 \quad \boxed{x(2) = -1}$

$\qquad x(2)\cdot 20 + y(2)\cdot 1.25 = 52 \quad \boxed{y(2) = 57.6}$

$\qquad S(2)donje u donjem podstablu$

$t=3$ w_1, w_2:

$$X(3) \cdot 270 + y(3) \cdot 1.25 = 0$$
$$X(3) \cdot 90 + y(3) \cdot 1.25 = 0$$

$$\boxed{\begin{aligned} X(3) &= 0 \\ y(3) &= 0 \end{aligned}}$$

w_2, w_3:

$$X(3) \cdot 90 + y(3) \cdot 1.25 = 0$$
$$X(3) \cdot 30 + y(3) \cdot 1.25 = 60$$

$$\boxed{\begin{aligned} X(3) &= -1 \\ y(3) &= 72 \end{aligned}}$$

w_3, w_4:

$$X(3) \cdot 30 + y(3) \cdot 1.25 = 60$$
$$X(3) \cdot 10 + y(3) \cdot 1.25 = 80$$

$$\boxed{\begin{aligned} X(3) &= -1 \\ y(3) &= 72 \end{aligned}}$$

Stablo cijena call opcije i repl. portfelja:
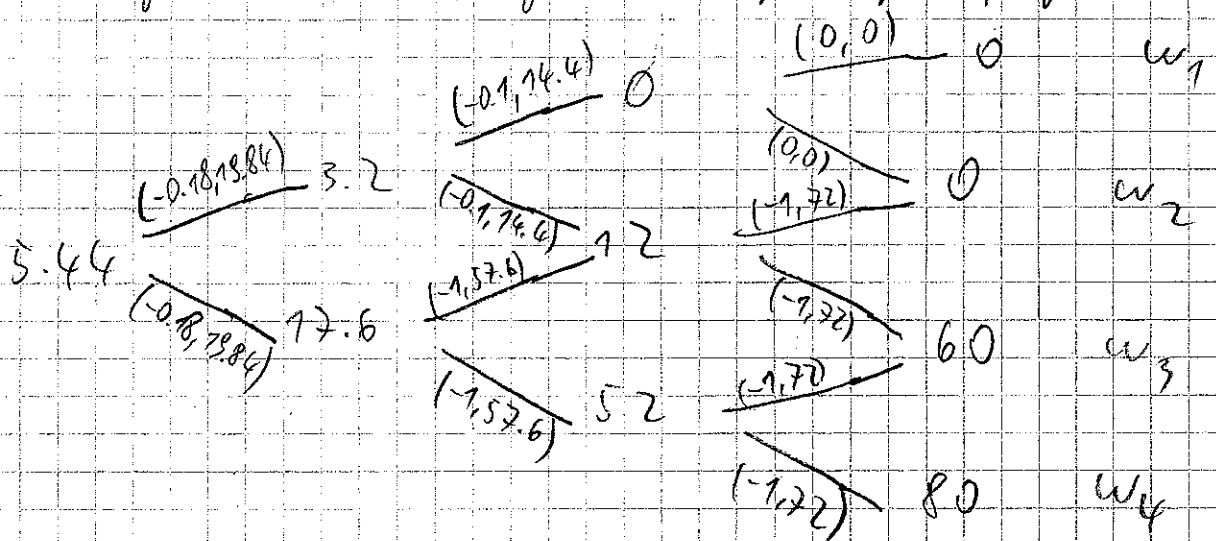
f) Kako je $1.8 < 5.44 =$ nearbitražne cijene opcije u $t=0$, onda prodamo replicirajući portfelj za 5.44 (efektivno to znači kupimo 0.18 DIONICA i posudimo 19.84 u novcu), kupimo opciju za 1.8, a razliku od $5.44 - 1.8 = 3.64$ oročimo po 25%.

U $t=1$ prodani repl. portfelj i opciju vrijede isto (iz otkrivenog prava opcije nanovno olvorim prodanog repl. portfelja)
(zoista: $-0.18 \cdot 120 + 19.84 \cdot 1.25 = 3.2$ tj.
$\qquad -0.18 \cdot 40 + 19.84 \cdot 1.25 = 17.6$) .

Stoga smo u $t=1$ otvorili rizicõn profit jer mamo sigurnih $3.64 \cdot 1.25 = \underline{\underline{4.55}}$.