ZI 2011/2012 — fale ptalice — 1

2

$$X \sim \begin{pmatrix} -50 & x-50 & 2x-50 \\ \frac{30}{50} & \frac{15}{50} & \frac{5}{50} \end{pmatrix}$$

$$E[S_{n+1} \mid F_n] = S_n$$

$$E[S_n + x_{n+1} \mid F_n] = S_n$$

$$S_n + E[x_{n+1} \mid F_n] = S_n$$

(x_{n+1} neg.)

$$-50 \cdot \frac{30}{50} + (x-50) \cdot \frac{15}{50} + (2x-50) \cdot \frac{5}{50} = 0$$

$x = 100$ dobitak za ter igru

b) kuća dobiva ako $x < 100$

3 — PUT $K = 50$

$$50 \begin{cases} 55 & g = 10\% \\ 40 & r = -20\% \end{cases}$$

a) $$p^* = \frac{r-d}{q-d} = 0.833$$

b) $$X = (K - S(1))^+ = \begin{cases} K - S(0)(1+g) \\ K - S(0)(1+d) \end{cases} = \begin{cases} 0 \\ 10 \end{cases}$$

$$P(X) = \frac{1}{1+r} \cdot (p^* \cdot 0 + (1-p^*) \cdot 10)$$

$$= \frac{1}{1.05} \cdot 1.666 = 1.5873 \text{ ter cijena.}$$

c) stvarna cijena < fer cijena = repliciroJući portfelj

pa prodati portfelj za 1.5873
kupiti opciju za 1.12
ostatak od 0.4073 oročiti

d) da li je opcija <u>dostižna</u>?

DEF: zahtjev X je dostižan ako postoji:
repliciroJući portfelj h tako da $V^h(1) = X$
tj. treba naći x, y

$$x \cdot S(0)(1+g) + y(1+r) = f(g)$$
$$x \cdot S(0)(1+d) + y(1+r) = f(d)$$

$$55x + 1.05y = 0$$
$$40x + 1.05y = 10$$

Dostižan je.

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4 $A(0) = 100$, $A(1) = 110$, $r = 10\%$.

$S(0) = 100$ → 120, 115

$g = 20\%$, $d = 15\%$ $\Big\}$ $d < r < g$

Postoji strategija arbitraže:

$r < d$ kupiti dionicu

$r > g$ kratko prodati

$t = 0$ posudim 100 kn, kupim dionicu

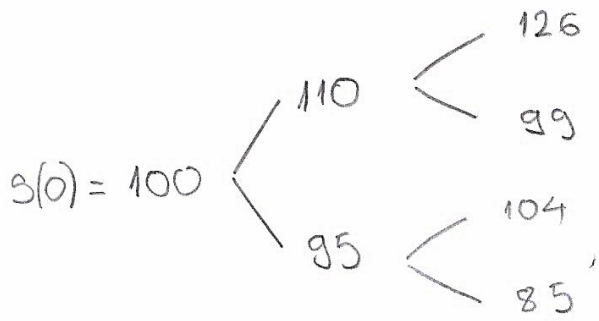
$V(0) = 0$ $(x(1), y(1)) = (1, -100)$

$t = 1$ prodam dionicu za 120 ili 115 kn

vratim banki 110 kn

$$V(1) = \begin{cases} 10, & S(1) = 120 \\ 5, & S(2) = 115 \end{cases}$$

5



$r = 5\ \%$

a) $g = 10\ \%$, $gg = 14.54\ \%$, $dg = 9.47\ \%$
$d = -5\ \%$, $gd = -10\ \%$, $dd = -10.52\ \%$

$d < r < g$ je zadovoljeno u svakom čvoru, ne postoji mogućnost arbitraže.

b) CALL OPCIJA , $K = 100$, $T = 2$

$$p^* = \frac{r-d}{g-d} = 77{,}63\ \%$$

$$X = (S(2) - K)^+ = \begin{cases} 104 - 100 \\ 85 - 100 \end{cases} = \begin{cases} 4 \\ 0 \end{cases}$$

$$P(X) = \frac{1}{1+r}\left(p^* \cdot 4 + (1-p^*)\cdot 0\right)$$

$$= 2.9573 //$$

c) $$p^* = \frac{r-d}{g-d}$$

$$x = (S(1) - K)^+ = \begin{cases} 110 - 100 \\ 95 - 100 \end{cases} = \begin{cases} 10 \\ 0 \end{cases}$$

$$P(x) = \frac{1}{1+r}\left(p^* \cdot 10\right) //$$

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6 $S(0) = 50$

$r = 6\%$

$D_1 = 5$ za $t = 0.5$

$D_2 = 1.5 D_1$ za $t = 1$

$F(0,T)$ u slučaju neisplate $= ?$

$$F(0,T) = E^*[S(T)]$$

$$F(0,T) = [S(0) - e^{-rT} \cdot D] \cdot e^{rT}$$

$$= \left[50 - 5e^{-0.06 \cdot \frac{1}{2}} - 7.5e^{-0.06}\right] \cdot e^{0.06}$$

$$= 40.439 //$$

7

$S(0) = 115$

$r = 6\ \%$

$r_D = 2\ \%$ (neprekidna dividenda)

$F(0, \frac{3}{12}) = 119$

$S(0) \cdot e^{(r - r_D) \cdot \frac{3}{12}} = 115\, e^{0.04 \cdot \frac{3}{12}} = 116.1557$

$119 > 116.1557 \Rightarrow$ Postoji mogućnost arbitraže.

$F(0,T) > S(0)e^{rT}$ — kupiti dionicu, sklopiti ugovor, KRATKA POZICIJA

$F(0,T) < S(0)e^{rT}$ — staviti $S(0)$ u banku, sklopiti ugovor DUGA POZICIJA

$t = 0$

posuditi od banke $S(0)$

kupiti dionicu

ući u kratku poziciju ugovorom

$t = \frac{3}{12}$

prodajem po 119

$V(1) = 119 - 116.1557$ //

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8 $r = 5\%$

$\sigma = 30\%$

$S(0) = 100$

$$f\left(S\left(\tfrac{1}{2}\right)\right) = \begin{cases} 100 - S\left(\tfrac{1}{2}\right), & S\left(\tfrac{1}{2}\right) < 100 \\ 0, & 100 \le S\left(\tfrac{1}{2}\right) \le 120 \\ S\left(\tfrac{1}{2}\right) - 120, & S\left(\tfrac{1}{2}\right) > 120 \end{cases}$$

$$\rightarrow f\left(S\left(\tfrac{1}{2}\right)\right) = \underbrace{\left(100 - S\left(\tfrac{1}{2}\right)\right)^{+}}_{\text{OCITO PVT}} + \underbrace{\left(S\left(\tfrac{1}{2}\right) - 120\right)^{+}}_{\text{CALL}}$$

b) $f_{PE}, X = 100$ a) $f_{CE}, X = 120$

fer cjena $= P^E + C^E$

a) $\boxed{C^E = S_0 \cdot N(d_1) - X e^{-rT} \cdot N(d_2)}$

$d_1 = -0.6355$; $N(d_1) = 0.2625$

$d_2 = -0.847$; $N(d_2) = 0.1983$

$C^E = 100 \cdot 0.2625 - 120 \cdot e^{-0.05 \cdot \frac{1}{2}} \cdot 0.1983$

$= 3.0415$ //

b) $\boxed{P^E = X e^{-rT} \cdot N(-d_2) - S_0 \cdot N(-d_1)}$

$d_1 = 0.2239$ $N(-d_1) = 1 - N(d_1) = 0.4114$

$d_2 = 0.0118$ $N(-d_2) = 1 - N(d_2) = 0.4953$

$P^E = 100 \cdot e^{-0.05 \cdot \frac{1}{2}} \cdot 0.4953 - 100 \cdot 0.4114$

$= 7.167$ //

fer cjena $= 3.0415 + 7.167$

$= 10.1815$ //

9] $T = 8/12$

$C^E = 10$, $\sigma = 26$, $T = 1$ godina, $X = 75$

danas $\left(\frac{8}{12}\right)$ $S\left(\frac{8}{12}\right) = 85$

---

da li sam danas u profitu?

$$C^E = S_0 N(d_1) - X e^{-rT} \cdot N(d_2)$$

$d_1 = 1{,}02 \Rightarrow N(d_1) = 0.8461$

$d_2 = 0.87 \Rightarrow N(d_2) = 0.8078$

$t = \frac{4}{12}$

$C^E = 85 \cdot 0.8461 - 75 \cdot e^{-0.05 \cdot \frac{4}{12}} \cdot 0.8078$

$C^E = 12.33$ ter cijena opcije danas

$C^E(0) = 10 \Rightarrow$ da sam prodala: $10 \cdot e^{0.05 \cdot \frac{8}{12}} = 10.338$ //

$C^E\left(\frac{8}{12}\right) = 12.33$ //

U profitu sam: $12.33 - 10.338 = 1.99$ //

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10 $S(0) = 2$

$\sigma = 0.15$

$r = 0.05$

1000 CALL : $z = -1000$, $X = 2.1$, $T = 0.5$

500 PUT : $d = -500$, $X = 1.9$, $T = \frac{3}{4}$

DELTA NEUTRALNI PORTFELJ $(x, y, z, d) = ?$

1 $V(S) = xS + y + zC_E + dP_E = 0$

2 $\frac{\partial V(S)}{\partial S} = x + z \underbrace{\frac{\partial C_E}{\partial S}}_{N(d_1)} + d \underbrace{\frac{\partial P_E}{\partial S}}_{N(\hat{d}_1)} = 0$

$d_1 = -0.713$, $d_2 = -0.2773$

$\hat{d}_1 = 0.7484$, $\hat{d}_2 = 0.6186$

$N(d_1) = 0.4320$, $N(d_2) = 0.3908$

$N(\hat{d}_1) = 0.7729$, $N(\hat{d}_2) = 0.7319 \Rightarrow N(-\hat{d}_1) = 0.2271$, $N(-\hat{d}_2) = 0.2681$

$C^E = 0.06358$

$P^E = 0.03644$

iz 2 $\Rightarrow$ $x = -z \cdot N(d_1) - d \cdot N(d_1)$

$= 1000 \cdot 0.4320 + 500 \cdot (-0.2271)$

$x = 318.45$

iz 1 $\Rightarrow$ $y = -xS - zC_E - dP_E$

$= -555.12$

$(x, y, z, d) = (318.45, -555.12, -1000, -500)$ //