1(c). Given objective function: $\sum_{i=1}^{N} w_i (y_i - x_i \cdot \beta_w)^2 = \| \text{diag}(w_i)(y - X\beta_w) \|_2^2$

Express in matrix form: $\sum_{i=1}^{N} w_i (y_i - x_i \cdot \beta_w)^2 = (y - X\beta_w)^T W (y - X\beta_w)$

Gradient: $f(\beta_w) = y^T W y - y^T W X \beta_w - X^T \beta_w^T W y + X^T \beta_w^T W X \beta_w$

$= y^T W y - 2\beta_w^T X^T W y + X^T \beta_w^T W X \beta_w$

F.O.C: $\frac{\partial f(\beta_w)}{\partial \beta_w} = 0 - 2X^T W y + 2X^T W X \beta_w = 0$

$X^T W y = X^T W X \beta_w$

Solution: $\hat{\beta_w} = (X^T W X)^{-1} X^T W y$