Jiali Liang
Hannah Willy

C.

$$\sum_{i=1}^{N} \omega_i (y_i - x_i \cdot \beta_\omega)^2 = \|\operatorname{diag}(\omega_i)(y - X\beta_\omega)\|_2^2$$

$$= (y - X\beta_\omega)^T W (y - X\beta_\omega)$$

$$= y^T W y - y W X \beta_\omega - \beta_\omega^T X^T W y + \beta_\omega^T X^T W X \beta_\omega$$

$$f(\beta_\omega) = y^T W y - 2\beta_\omega^T X^T W y + \beta_\omega^T X^T W X \beta_\omega$$

$$0 = \frac{\partial f(\beta_\omega)}{\partial \beta_\omega} = 0 - 2X^T W y + 2X^T W X \beta_\omega$$

$$\therefore \quad 2X^T W X \beta_\omega = 2X^T W y$$

$$X^T W X \beta_\omega = X^T W y$$

$$(X^T W X)^{-1} X^T W X \beta_\omega = (X^T W X)^{-1} X^T W y$$

$$I \beta_\omega = (X^T W X)^{-1} X^T W y$$

$$\boxed{\beta_\omega = (X^T W X)^{-1} X^T W y}$$