2022년 2학기

Computational Statistics

HW#8

222STG10

김희숙

Problem

6.3 Consider finding σ^2 = E{X^2} when X has the density that is proportional to q(x) = exp{−|x|^3/3}.

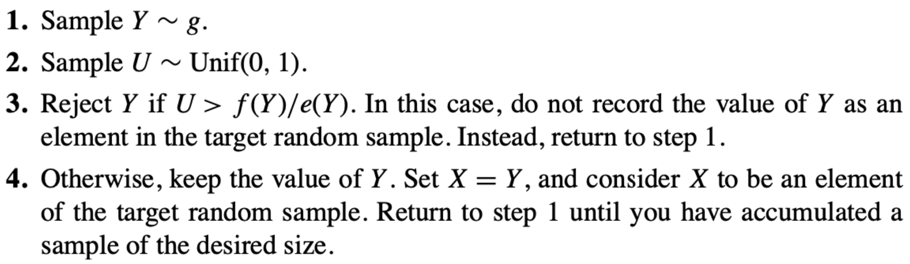
a. Estimate σ^2 using importance sampling with standardized weights.

$$\mu = \int h(\mathbf{x}) f(\mathbf{x})\, d\mathbf{x} = \int h(\mathbf{x}) \frac{f(\mathbf{x})}{g(\mathbf{x})} g(\mathbf{x})\, d\mathbf{x}$$

$$\mu = \frac{\int h(\mathbf{x}) f(\mathbf{x})\, d\mathbf{x}}{\int f(\mathbf{x})\, d\mathbf{x}} = \frac{\int h(\mathbf{x})[f(\mathbf{x})/g(\mathbf{x})] g(\mathbf{x})\, d\mathbf{x}}{\int [f(\mathbf{x})/g(\mathbf{x})] g(\mathbf{x})\, d\mathbf{x}},$$

σ^2 = E{X2}, q(x) = exp{−|x|3/3}일 때, importance sampling with standardized weights 을 이용해 σ^2 를 추정한 결과는 **0.818262** 이다.

b. Repeat the estimation using rejection sampling.



1. Sample $Y \sim g$.
2. Sample $U \sim$ Unif(0, 1).
3. Reject Y if $U > f(Y)/e(Y)$. In this case, do not record the value of Y as an element in the target random sample. Instead, return to step 1.
4. Otherwise, keep the value of Y. Set $X = Y$, and consider X to be an element of the target random sample. Return to step 1 until you have accumulated a sample of the desired size.

rejection sampling 을 이용해 σ^2 를 추정한 값은 **0.8989728** 이다.

# of total iteration	# of acceptance	# of rejection	acceptance ratio
1012	1000	12	0.9881423

c. Philippe and Robert describe an alternative to importance-weighted averaging that employs a Riemann sum strategy with random nodes [506, 507]. When draws X1,...,Xn originate from f, an estimator of E{h(X)} is

$$\sum_{i=1}^{n-1} (X_{[i+1]} - X_{[i]}) h(X_{[i]}) f(X_{[i]}), \qquad (6.86)$$

where X[1] ≤ · · · ≤ X[n] is the ordered sample associated with X1, . . . , Xn. This estimator has faster convergence than the simple Monte Carlo estimator. When *f = cq* and the normalization constant c is not known, then

$$\frac{\sum_{i=1}^{n-1}(X_{[i+1]} - X_{[i]})h(X_{[i]})q(X_{[i]})}{\sum_{i=1}^{n-1}(X_{[i+1]} - X_{[i]})q(X_{[i]})} \tag{6.87}$$

estimates E{h(X)}, noting that the denominator estimates 1/c. Use this strategy to estimate σ^2, applying it post hoc to the output obtained in part (b).

Philippe and Robert의 방법을 이용해 σ^2를 추정한 값은 **0.7808458**로 (b)보다 작은 값을 갖는다.

d. Carry out a replicated simulation experiment to compare the performance of the two estimators in parts (b) and (c). Discuss your results.

	min.	1st Qu.	median.	mean.	3rd Qu.	max.	sd..
rejection sampling	0.7691	0.8626	0.8848	0.8866	0.9093	1.0067	0.03530781
Philippe & Robert	0.7544	0.7703	0.7727	0.7727	0.7752	0.8039	0.004125182

rejection sampling 과 Philippe and Robert 을 1000 번 반복했을 때, rejection sampling 의 sd 는 **0.03530781**, Philippe and Robert 의 sd 는 **0.004125182** 로 훨씬 작은 값으로 추정된다.

Code appendix

```
# 6.3
# a
q <- function(x) exp(-abs(x)^3/3)
x <- seq(-4, 4, by=0.01)
qx <- q(x)
IS <- function(n) {
  x <- rnorm(n)
  est <- sum(x^2*q(x)/dnorm(x))/sum(q(x)/dnorm(x))
  return(est)
}
IS(1000)

# b
rs <- function(n, print=TRUE) {  # n: sample 개수
  niter <- 0
  reject_num <- 0
  accept_num <- 0
  samples <- c()

  while (accept_num < n) {
    z <- rnorm(1)   # proposal density
    u <- runif(1)   # sample for rejection
    t <- q(z)/dnorm(z)
    if (u > t) {
      reject_num <- reject_num + 1
    }
    else {
      samples <- append(samples, z)
      accept_num <- accept_num + 1
    }
    niter <- niter+1
  }
  if (print == TRUE) {
    cat(
        "# of total iteration:", niter, "\n",
        "# of acceptance:", accept_num, "\t", "# of rejection:", reject_num, "\n",
        "acceptance ratio:", accept_num/niter, "\n")
  }
  return(samples)
}
rsdata <- rs(1000)
mean(rsdata^2)

# c
prmethod <- function(x) {
  x <- sort(x)
  n <- c()
  d <- c()
  for (i in 1:length(x)-1) {
    n <- append(n, (x[i+1]-x[i])*x[i]^2*q(x[i]))
    d <- append(d, (x[i+1]-x[i])*q(x[i]))
  }
  return(sum(n)/sum(d))
}
prmethod(rsdata)

# d
result <- data.frame(n = 1:1000, rs=rep(1, 1000), pr=rep(1, 1000))
for (i in 1:nrow(result)) {
  samples <- rs(1000, print=FALSE)
  result[i, 2] <- mean(samples^2)
  result[i, 3] <- prmethod(samples)
}
sd(result$rs)
sd(result$pr)
summary(result$rs)
summary(result$pr)
```