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2. $1/a_1 + 1/a_2 + 1/a_3 + \dots 1/a_n = 1$

Multiplying and dividing by K we get:
[this helps us visualize it as Harmonic Mean's inverse]

$$\frac{K}{a_1} + \frac{K}{a_2} + \frac{K}{a_3} + \dots \frac{K}{a_n} = K$$

Now we know [$a_i \in \mathbb{N}$] and

This would then be visualized as sum of divisors (n in number) :-

This are termed as perfect No's :-
Eg :- $1 + 2 + 3 = 6$
Another example is 28.

Acc^n to Euclid — Euler Theorem :-
if $2^p - 1$ is prime
then
$2^{p-1}(2^p - 1)$ is perfect.

Which satisfy the above Eq^n :-

3. $f: \mathbb{N} \to \mathbb{N}$ is:

$f(1) + f(2) + \ldots f(n) = n^2 f(n)$

$\forall n \geq 2$

$f(1) = 2022$

for $n=2$: $f(1) + f(2) = 2^2 f(2)$

$n = 3$: $\underbrace{f(1) + f(2)} + f(3) = 3^2 f(3)$

$\therefore 2^2 f(2) = (3^2-1) f(3)$

$\therefore f(3) = \left(\frac{2^2}{3^2-1}\right) f(2)$

Now $f(2)$ can be solved

$f(1) = (2^2-1) f(2)$

$\therefore f(2) = \left(\frac{1}{2^2-1}\right) f(1)$

$\therefore f(3) = \frac{2^2}{(3^2-1)} \cdot \frac{1}{2^2-1} f(1)$

Similarly for $f(4)$ we get:

$f(4) + 3^2 f(3) = 4^2 f(4)$

$\therefore f(4) = f(1)\left(\frac{3^2}{4^2-1}\right) \frac{2^2 \cdot 1^2}{(3^2-1)(2^2-1)}$

Hence generalising we get:→

~~$f(4) = 3^2$~~

$f(n) = \frac{(n-1)^2 \ldots 1^2}{(n^2-1)((n-1)^2-1)\ldots(2^2-1)} f(1)$

Solving we get:→

$f(n) = \frac{[(n-1)!]^2}{((n+1)((n-1)+1)\ldots(2+1)) \cdot [(n-1)((n-1)-1)\ldots(2-1)]}$

classmate

Date
Page

$$f(n) = \frac{[(n-1)!]^2}{\left(\frac{(n+1)!}{2\cdot 1}\right)\left((n-1)!\right)} f(1)$$

$$= \frac{2\cdot(n-1)!\, f(1)}{(n+1)!} = \frac{2}{n(n+1)} f(1)$$

$$= \frac{2(\cancel{2022})}{\cancel{2022}\cdot 2023} = \frac{2}{2023}$$

M2: from eq^n (1), (2), (3):→

$$f_n = n^2 f_n - (n-1)^2 f(n-1)$$

$$(n-1)(n+1) f_n = (n-1)^2 f(n-1)$$

$$f_n = \frac{(n-1)}{(n+1)} f(n-1)$$

Similarly for $f(n-1) = \frac{(n-2)}{(n+0)} f(n-2)$

$$f_n = \frac{(n-1)(n-2)\ldots\ldots 1}{(n+1)(n)\ldots\ldots(3)} f(1)$$

$$= \frac{(n-1)!}{\left((n+1)!/2\right)} f(1) = \frac{2 f(1)}{n(n+1)}$$

1. (a) We (2n)! players we have to divide them into n groups of 2 players each:→ [Using formula]

~~(1) Total permutation / permutation $\frac{(2n)!}{(2!)^n} \times \underbrace{\frac{1}{n!}}_{}$ AS ORDER IS NOT IMPORTANT~~

(1) $\underbrace{\frac{(2n)!}{(2!)^n}}_{\text{Total arrangement of teams.}} \times \underbrace{\frac{1}{n!}}_{\text{ORDER NOT IMPORTANT}}$

Alternatively:→

(2) $\left[\underbrace{{}^{2n}C_2}_{\text{select 2 people}} \cdot \underbrace{{}^{2n-2}C_2}_{\text{Next 2}} \cdots \, {}^{2}C_2\right]$ ORDER IMPORTANT hence divide by $n!$

$$\frac{(2n)! \, \cancel{(2n-2)!} \cdots 2!}{(2!)\cancel{(2n-2)!}\, 2! \, \cancel{(2n-4)!} \cdots 0!}$$

$$= \boxed{\frac{(2n)!}{(2!)^n} \times \frac{1}{n!}}$$

(b) Using part (a):→

M1: Put 2 as m and ~~divide~~
divide (i) m people into n groups
(ii) n people into m groups

(i) $\underbrace{\frac{mn!}{(m!)^n} \times \frac{1}{n!}}_{\text{division into groups}}$

(ii) $\underbrace{\frac{mn!}{(n!)^m} \times \frac{1}{m!}}_{\text{division into groups}}$

(i) × (ii) = $\underbrace{\frac{(mn!)^2}{(m!)^n (m!) . (n!)^m . n!}}$ ∈ Z

Z × Z

$$\boxed{\therefore \frac{(mn!)^2}{(m!)^{n+1} (n!)^{m+1}} \in Z}$$

5. (a) $d = \gcd(61^{610}+1, 61^{671}-1)$

Let $x = 61^{610}+1$

$= (61^{61})^{10}+1$

$y = 61^{671}-1$

$= (61^{61})^{11}-1$

Let 61^{61} be a $\Rightarrow$

$x = a^{10}+1 \qquad y = a^{11}-1$

$y = (x)(a) - a - 1$

$y = a(x-1) - 1 =$

$y+1 = a(x-1)$

$d = \gcd(x, ax - a - 1)$

Using Euclidean Algorithm

$d = \gcd(x, (ax-a-1) \% x)$

$= \gcd((x-a-1), (x))$

$= \gcd(\underbrace{(a+1)}_{\text{even}}, \underbrace{x-a-1}_{\text{even}})$

$= \gcd(\underbrace{\left(\frac{a+1}{2}\right)}_{\text{odd}}, \left(\frac{x-a-1}{2}\right))$

as $(61^{61}+1) = 2(2k+1)$

$$x(-a) - a - 1 - x + x$$
$$x(a-1) + x - a - 1$$

$$y = ax - a - 1 + x - x$$
$$= x(a-1) + x - a - 1$$
$$= x(a-1) + x - (a-1) - 2$$
$$= (x-1)(a-1) + \underline{(x-2)}$$

$$y = a(x-1) + (x-1) - x$$
$$y = (a+1)(x-1) - x$$

$$(x+y) = (a+1)(x-1)$$

a is odd $\rightarrow$ $(a+1)$ is even

$$(x-1) = [(61)^{91}]^{10}$$

$$\underbrace{(ax-y)}_{\text{even}} = \underbrace{(a+1)}_{\text{even}}$$

6. To show that:

$$a^3 + b^3 + c^2 = d^4$$

has infinitely many +ve integers a, b, c, d with no common divisor > 1

Let's say we have it as p
And prove by contradiction

$a = a'p \quad b = b'p \quad c = c'p$
$d = d'p$

$$(a'p)^3 + (b'p)^3 + (c'p)^2 = (d'p)^4$$

$$a'^3 p + b'^3 p + c'^2 = (d'^4) p^2$$

$$(p)\{(a'+b')(a'^2 + b'^2 - a'b')\}$$
$$= (d'^2 p - c')(d'^2 p + c')$$

See here on L.H.S we have p; but on R.H.S we know how that ~~$d'p^2$~~ $d'^2 p$ & c are coprime
[Condition before contradiction]

Hence contradiction prevails :)

The eq^n has infinite sol^n whose a, b, c, d have no common divisor > 1.