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Formal Languages and Automata Theory
End - Semester Test
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(1.)

Alphabets $\in \{a, b, \#\}$
The language is defined as:-

$$L_1 = \{ x\#y \mid x, y \in \{a, b, \#\}^*, x \neq y, |x| = |y| \}$$

To prove/disprove the above language L_1 is context-free we can apply pumping

~~Let L_1 be a con~~
Now, take pumping length = k.

Now, take u, v, w, x, y such
$z = uvwxy$, $|vx| \neq \varepsilon$, $|vwx| \leq k$

Case-1:- $|x| = |y| = k$ [assumption]

Take z at the left side
then, uv^iwx^iy take $i > 1$
then; $|x'| \neq |y'|$

hence a contradiction has:-
Hence, disproved.

Case 2:- Take z at right side then

$u v^i w x^i y$ By same procedure take $i > 1$

then $|x| \neq |y|$ [$\because |x| < |y|$]

So, contradiction raised

Take

$z = a^{n-1} a \# b b b^{n-1}$

such that

$u = a^{n-1}$ $v = a$ $w = \#$, $x = b$

$y = b^{n-1}$

for $i = 0$

$z_1 = uwy \Rightarrow a^{n-1} \# b^{n-2}$

Hence $x = a^{n-1}$

$y = b^{n-2}$

$|x| \neq |y|$

~~So~~

$z \in$

Hence, contradiction raised

$\therefore$ So L_1 is not context-free.

(2.) We need to design a DPDA (determ-
-stic pushdown automata) to accep
the language :-

$$L_2 = \{ a^m b^n \mid m,n \geq 0 \text{ and } 3m-2n$$

The DPDA should loop in only
two distinguished states t and r :-

We will construct a CFG for L_2 :-
if we fix :- $m = \left(\frac{2n+5}{3} \right)$

$\Rightarrow$ (m,n) : $(\cancel{\{1,3\}}$;
$(\{3,2\}; \{5,5\} \ldots)$

We can assume :-
$m = 2k+1$
$n = 3k-1$

$\therefore$ CFG as intended can be designe
as :-

$$S = \{ aaabb \mid aaSbbb \}$$

(3.) For a language L over the alphabet :→ $\{a, b\}$

half (L) = $\{x \mid x \in \Sigma^*$ and there exists $y \in \Sigma^*$ such that $|x| = |y|$ and $yx \in L\}$

We need to prove | disprove :→

(a) If L is content-free, then half(L) is content-free.

$S \to 0S333 \mid 0T44333$

$T \to 1T2 \mid 12$

L is $0^i 1^j 2^j 4 4 3^{3i}$

Consider the language M on $0^* 1^* 2^* 4^*$

Now, the intersection of L and M is:

$0^n 1^n 2^n 4 1^n \geq 0$

and this is half (L)

and half (L) is not content-free.

(5) (c) If L is recursive it never halts for given input unless the recursion stops. thus if $yx \in L$ with $|x| = |y|$ and $half(L) = \{x\}$ the half (L) would never halt, as well as it same as providing later part of input of L i.e. 'x' is given as input to PDA that accepts L.

(7.) $L_4 = \{ a^n w^i \mid w \in \Sigma^* ; n \geq 0$ an
$\#c(w) = n \}$

We intend to define a CFG as follows :-

$$S \to XY \qquad X \to aXBC \mid \varepsilon$$

$$CY \to Y \qquad CB \to BC$$

$$Y \to \varepsilon \qquad B \to bV \mid Vb \mid$$

$$V \to aV \mid cV \mid \varepsilon$$

Now, we are here trying to make a possible transition to show the role of non-terminal symbols

$$S \to XY \xrightarrow{*} a^n x (BC)^n Y$$

This is explain the roles played by the non-terminal symbols of your grammer.

$$\to a^n BC\ BC \ldots BC\ Y$$

$$\xrightarrow{*} a^n B^n C^n Y$$

$$\xrightarrow{*} a^n B^n Y C^n \to a^n B^n C^n$$

Now, the non-terminal symbols. As plays the role of generating $w \in \Sigma^*$ with $\#c(w) = n$

classmate

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(5) 5(a) Consider the language

$L_5 = \{ M \mid M$ (encoding of) a deterministic Turing machine that ~~halts on at least~~ loops on at most 2022 input strings $\}$

We aim to design a TM which can simulate M on all possible inputs. If any 2022 simulation loops, the Turing Machine accepts and loops. If we say 2022 strings are never checked then we parallely simulate TM., then TM never stops. Hence, we deduce that TM can be a non-deterministic TM which is simulating 2022 distinct strings.

Hence, we conclude L_5 is not recursively enumerable.