



# Feature detection and description

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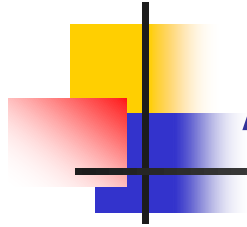


# A feature of an image ?

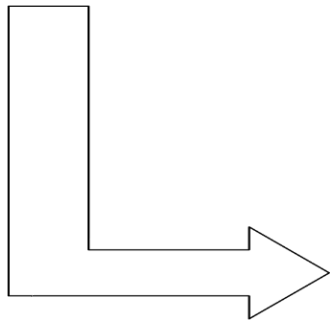
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- Characterizes its visual content.
  - A part of an organization on describing a region
  - A higher level organization than a pixel
- A point in a multidimensional space
  - feature vector: n-Dimensional vector:  $x \in \mathbb{R}^n$
- Represents
  - a point within a neighborhood
  - a pattern
  - a patch
  - an object
  - the whole image

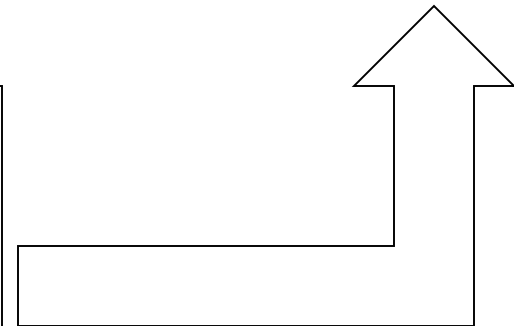
# Role of features in Image Analysis



Pre-processing

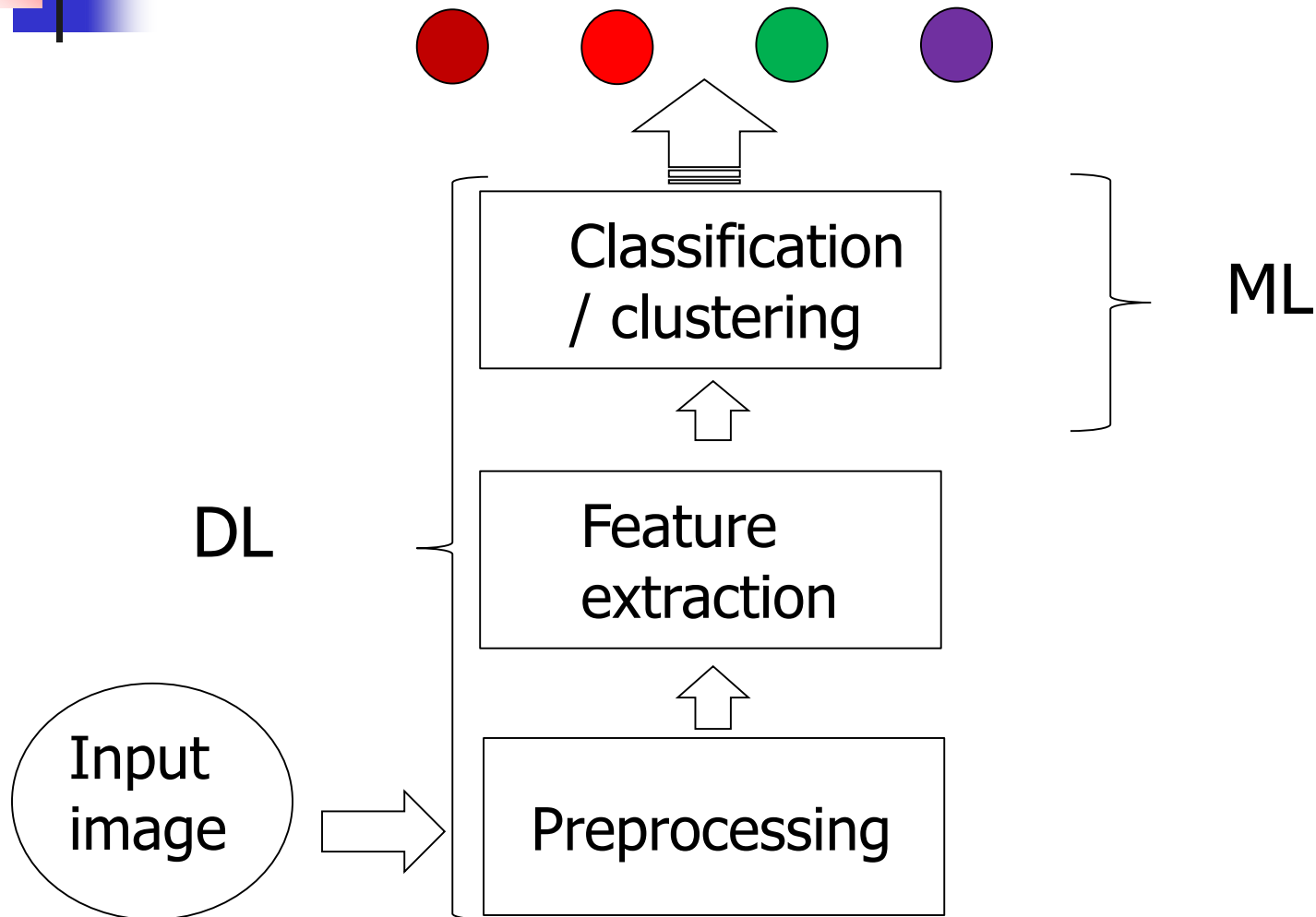


Feature Extraction



End Analysis

# Role of features in learning visual content



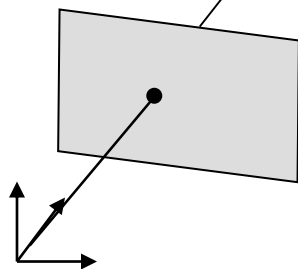
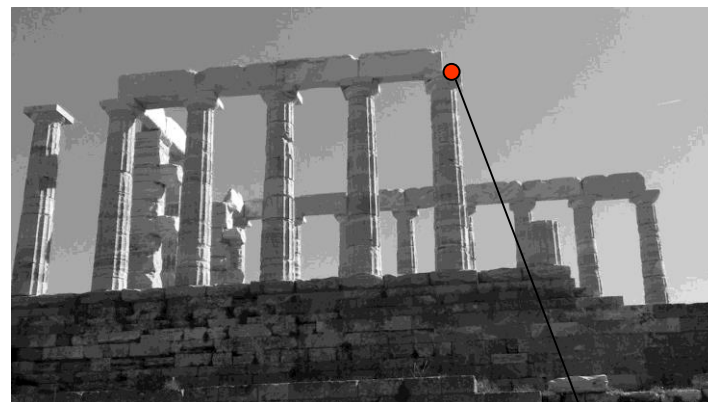


# A few typical problems

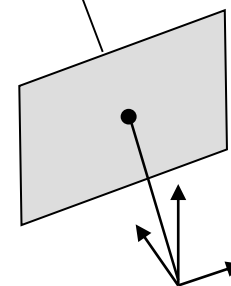
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- To determine structure / geometry of objects.
- To determine material properties of objects.
- To determine an object's position, category, and its role / interaction with other objects.
- To find similar images from a library given a query image

# Example: Structure Recovery



Given two views of the scene  
recover the unknown camera  
displacement and 3D scene  
structure



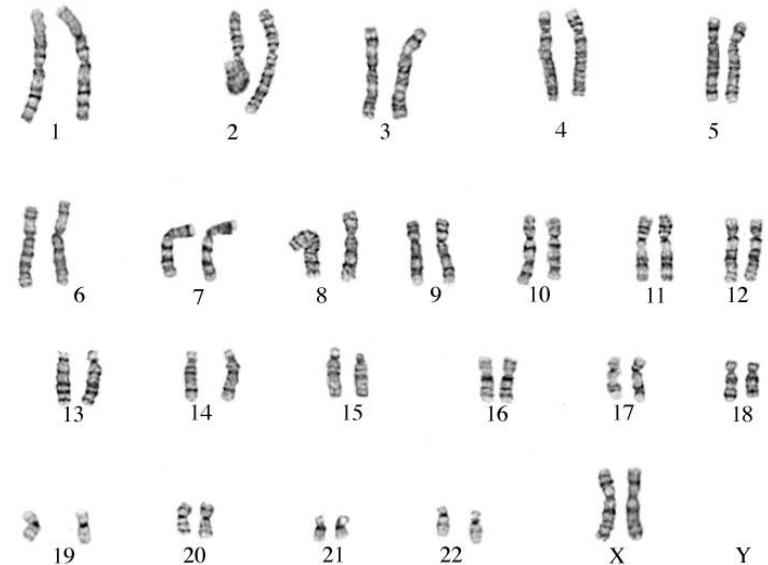
*Adapted from [www.cs.gmu.edu/~kosecka/lect4.ppt](http://www.cs.gmu.edu/~kosecka/lect4.ppt)*

# Example: Karyotyping

Pairing homologous chromosomes.



ZWK9904 KEY



# Example: Material Property from Remote Sensing Images



Courtesy: [www.sal.ocean.washington.edu/teaching/lectures/classification.ppt](http://www.sal.ocean.washington.edu/teaching/lectures/classification.ppt)



# Example: Document Analysis



# Example: Scene matching



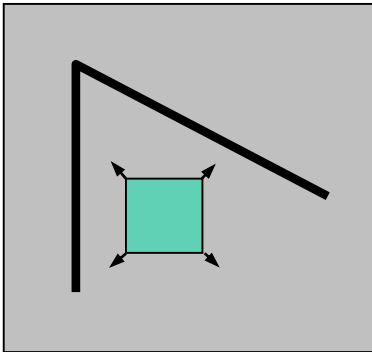
- Detection
- Description
- Matching

Translation, rotation, scale ..

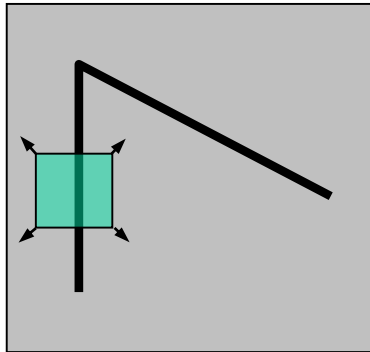
# Feature detection

A local measure for uniquely identifying the feature.

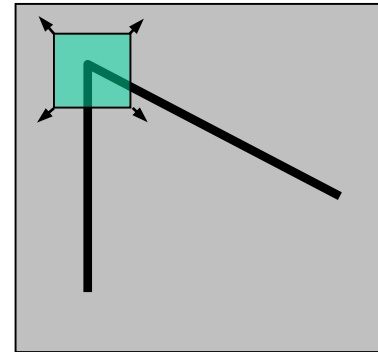
- How does the measure change on shifting windows at different points?



“flat” region:  
no change in  
all directions



“edge”: no  
change along  
the edge  
direction

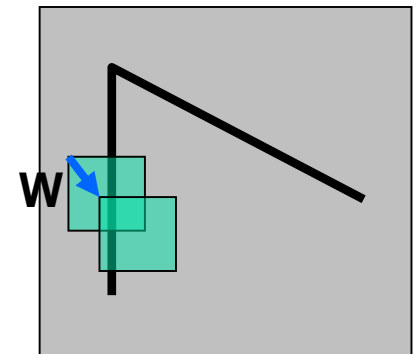


“corner”:  
significant  
change in all  
directions

# Feature detection

Consider shifting the window **W** by  $(u, v)$ .

- how do the pixels in **W** change?
- compare each pixel before and after by summing up the squared differences (SSD).
- this defines an SSD “error” of  $E(u, v)$ .



$$E(u, v) = \sum_{(x, y) \in W} [I(x + u, y + v) - I(x, y)]^2$$



# Small motion assumption

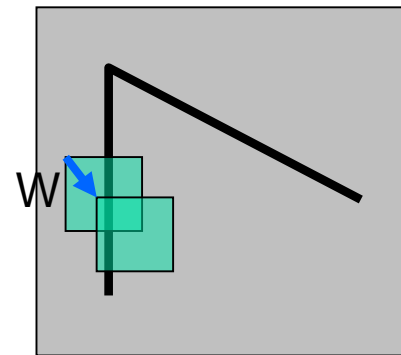
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$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

For small  $u$  and  $v$

$$\begin{aligned} I(x+u, y+v) &\approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \\ &\approx I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \\ \text{shorthand: } I_x &= \frac{\partial I}{\partial x} \end{aligned}$$

# Feature detection



$$I(x + u, y + v) \approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v \approx I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(u, v) = \sum_{(x, y) \in W} [I(x + u, y + v) - I(x, y)]^2$$

$$\approx \sum_{(x, y) \in W} [I(x, y) + [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} - I(x, y)]^2$$

$$\approx \sum_{(x, y) \in W} \left[ [I_x \ I_y] \begin{bmatrix} u \\ v \end{bmatrix} \right]^2$$

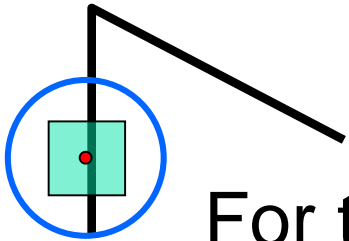
$$E(u, v) = \sum_{(x, y) \in W} [u \ v] \begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\begin{aligned} & \left( \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \right)^T \left( \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \right) \\ &= [u \ v] \begin{bmatrix} I_x \\ I_y \end{bmatrix} \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \\ &= [u \ v] \begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

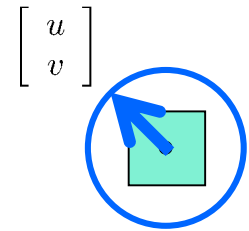
$$E(u, v) = \sum_{(x,y) \in W} [u \ v] \begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

# Feature detection

$$E(u, v) = \sum_{(x,y) \in W} [u \ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$



For the example above



- Suppose the center of the green window moved to anywhere on the blue unit circle.
- Directions for the largest and the smallest  $E$  values?
  - Eigenvectors of  $H$ .

# Quick eigenvalue/eigenvector review

$$Ax = \lambda x$$

The **eigenvectors** of a matrix  $A$  are the vectors  $x$  that satisfy:

$$\det(A - \lambda I) = 0 \quad \det \begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} = 0$$

The scalar  $\lambda$  is the **eigenvalue** corresponding to  $x$

- The eigenvalues are found by solving:

$$\lambda_{\pm} = \frac{1}{2} \left[ (h_{11} + h_{22}) \pm \sqrt{4h_{12}h_{21} + (h_{11} - h_{22})^2} \right]$$

- For eigen vector solve the following:

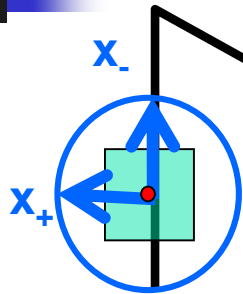
$$\begin{bmatrix} h_{11} - \lambda & h_{12} \\ h_{21} & h_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Once you know  $\lambda$ , you find eigen vector  $[x \ y]^T$  by solving the above.



$$E(u, v) = \sum_{(x, y) \in W} [u \ v] \underbrace{\begin{bmatrix} I_x^2 & I_x I_y \\ I_y I_x & I_y^2 \end{bmatrix}}_H \begin{bmatrix} u \\ v \end{bmatrix}$$

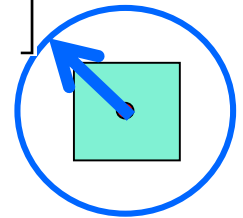
# Feature detection



$$H x_+ = \lambda_+ x_+$$

$$H x_- = \lambda_- x_-$$

$$H \begin{bmatrix} u \\ v \end{bmatrix}$$



## Eigenvalues and eigenvectors of $H$

- Define shifts with the smallest and largest change ( $E$  value)
- $x_+$  = direction of **largest** increase in  $E$ .
- $\lambda_+$  = amount of increase in direction  $x_+$
- $x_-$  = direction of **smallest** increase in  $E$ .
- $\lambda_-$  = amount of increase in direction  $x_-$



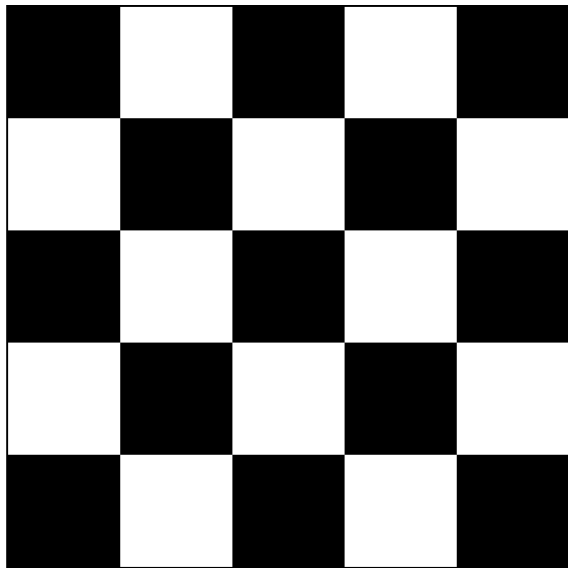
# Feature scoring function

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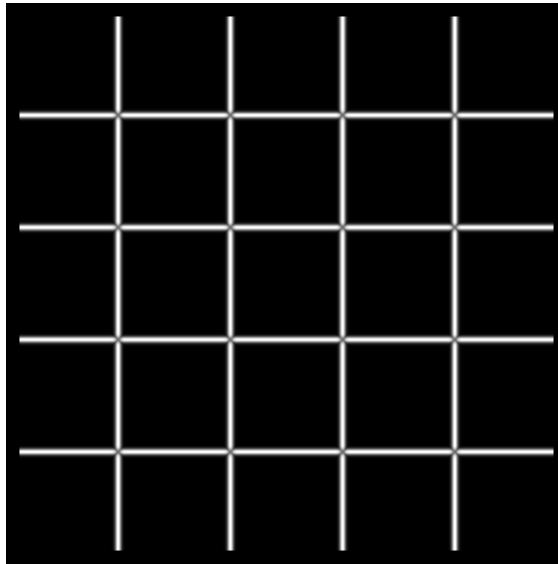
$E(u, v)$  to be **large** for small shifts in **all** directions

- the *minimum* of  $E(u, v)$  should be large, over all unit vectors  $[u \ v]$ .
- this minimum is given by the smaller eigenvalue ( $\lambda_-$ ) of  $H$

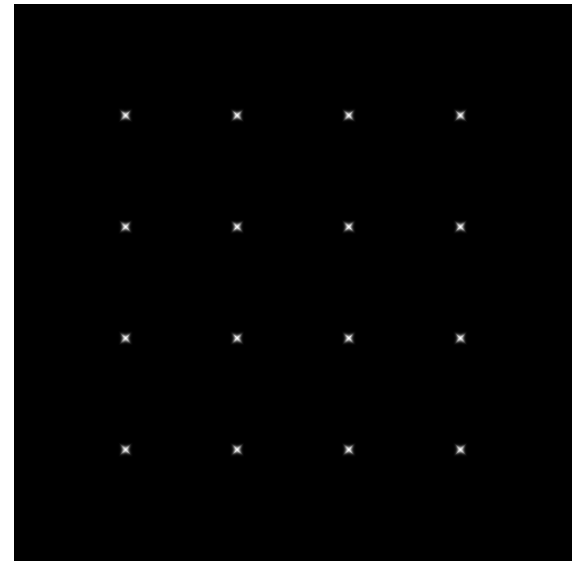
# Feature detection



$I$



$\lambda_+$



$\lambda_-$



# Feature detection: Algorithm

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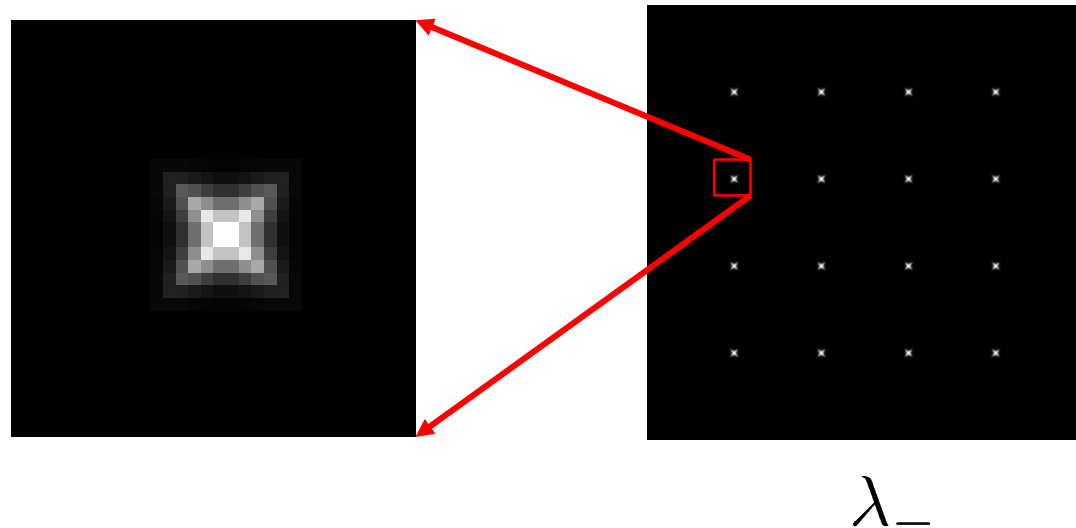
- Compute the gradient at each point in the image.
- Obtain the  $H$  matrix from the entries in the gradient.

$$H = \begin{bmatrix} \overline{I_x^2} & \overline{I_x I_y} \\ \overline{I_y I_x} & \overline{I_y^2} \end{bmatrix}$$

- Compute eigenvalues of  $H$ .
- Locate points with large response ( $\lambda_- > \text{threshold}$ )
- Select points where  $\lambda_-$  is a local maximum as features.

# Local maximum: illustration

- Points with  $\lambda_-$  as a local maximum.





# The Harris operator

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$\lambda_-$  is a variant of the “Harris operator” for feature detection.

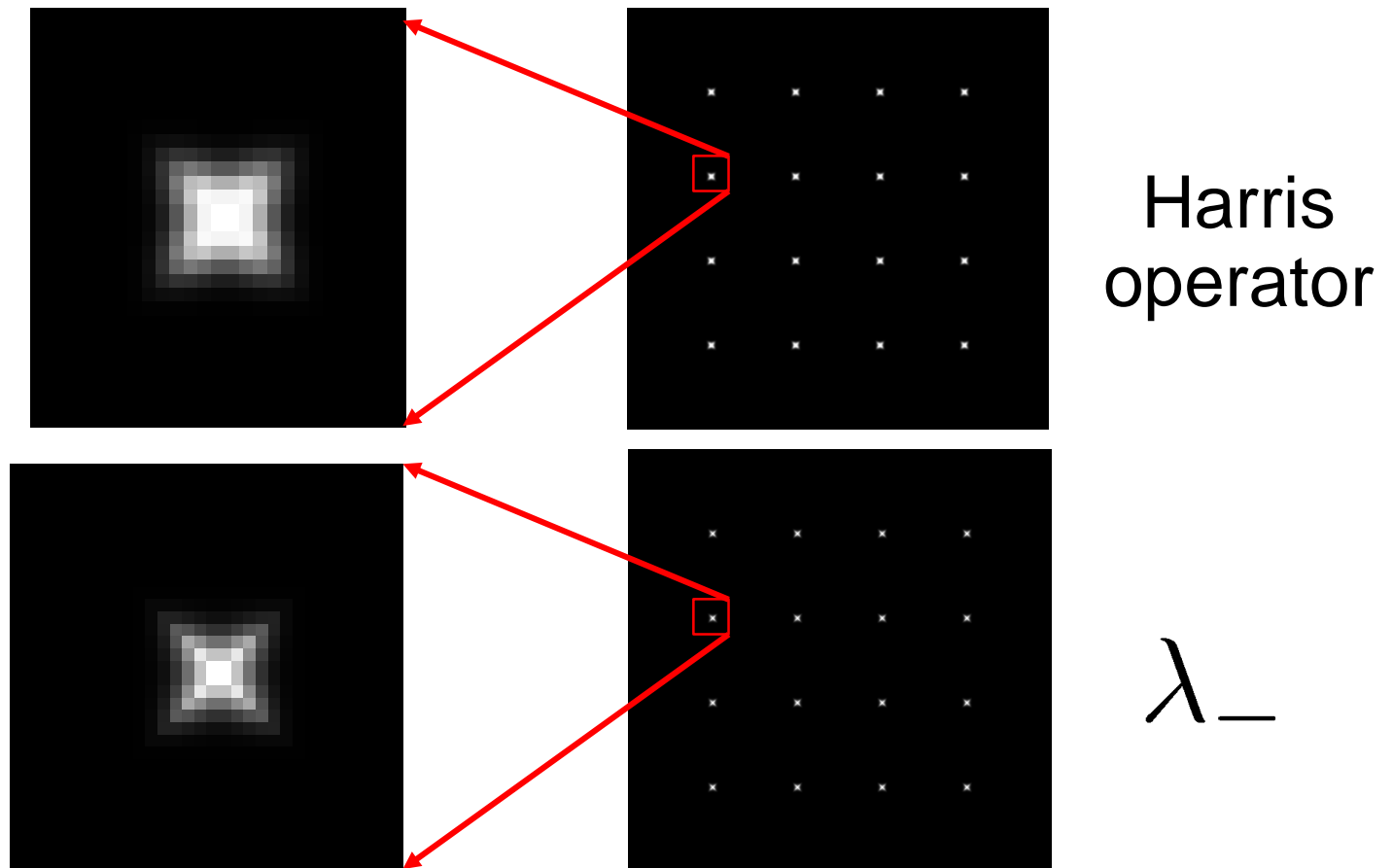
$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$
$$= \frac{\text{determinant}(H)}{\text{trace}(H)}$$

- The *trace* is the sum of the diagonals, i.e.,

$$\text{trace}(H) = h_{11} + h_{22}$$

- Very similar to  $\lambda_-$  but less expensive (no square root)
- Called the “Harris Corner Detector” or “Harris Operator”
- Lots of other detectors, this is one of the most popular.

# The Harris operator



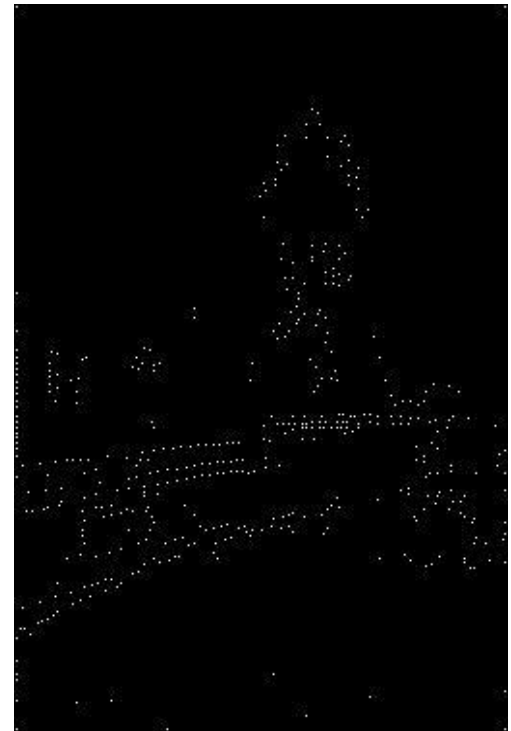
# Harris detector example



Input image



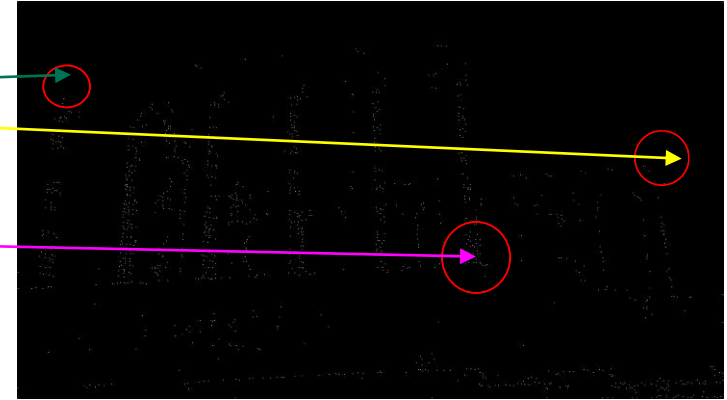
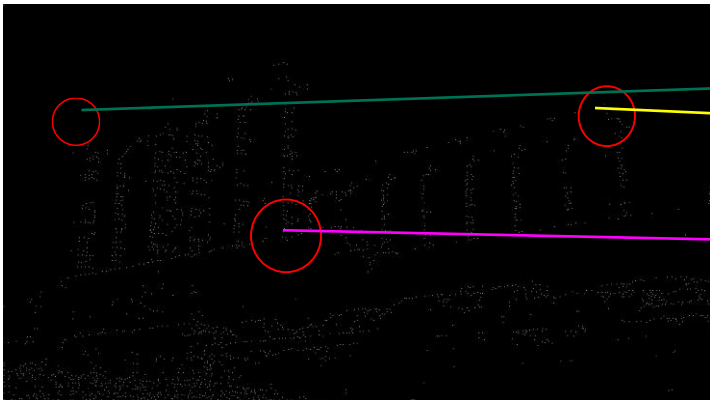
$f$ -Value



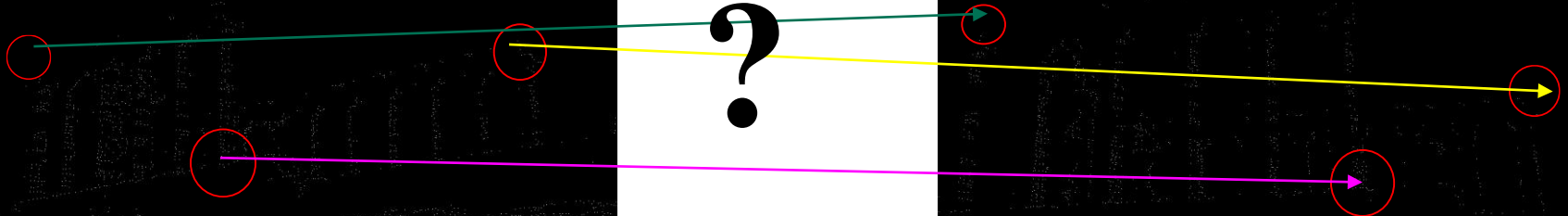
Local maxima



# Feature matching



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# Matching with Features

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- Detect feature points in both images.
- Describe them by local statistics.
- Find corresponding pairs (Matching).



# Invariance

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- Is it possible to select same features under various transformations?
  - Rotation
  - Change of illumination
  - Scale
  - ..



# Scale invariant detection

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Key idea for detecting corners

Find scale that gives local maximum of  $f$ .

- $f$  is a local maximum in both position and scale
- Common definition of  $f$ : Laplacian (or difference between two Gaussian filtered images with different s.d.'s.).



# Invariance

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Consider two images  $I_1$  and  $I_2$  so that  $I_2$  is a transformed version of  $I_1$

- For example,
  - Translation
  - Rotation
  - Scale
  - Reflection
  - Non-uniform scaling
  - Illumination
  - View ...

Transformational Invariance:  
Detection of the same  
features regardless of the  
transformation.

# Detection and description: to be invariant



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Both should be ensured.

## 1. Detector to be transformational invariant.

- Harris measure is invariant to translation and rotation.
- Scale requires handling of multi-resolution representation
  - Usual approach: Computing features at multiple scales using a Gaussian pyramid.
  - More precise computation locates features at “the best scale” (e.g., SIFT)

## 2. Feature descriptor to be transformational invariant.

- captures the information in a region around the detected feature point.
  - e.g. histogram of gradient directions in a square window centering a feature point.

# Finding Keypoints – Scale, Location

- Information content on varying scale?





# Scale Invariant Detection

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- Functions for determining scale

Convolve with image and observe extrema in 3-D.

Kernels:

Laplacian

$$L = \sigma^2 (G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma))$$

Difference of Gaussians

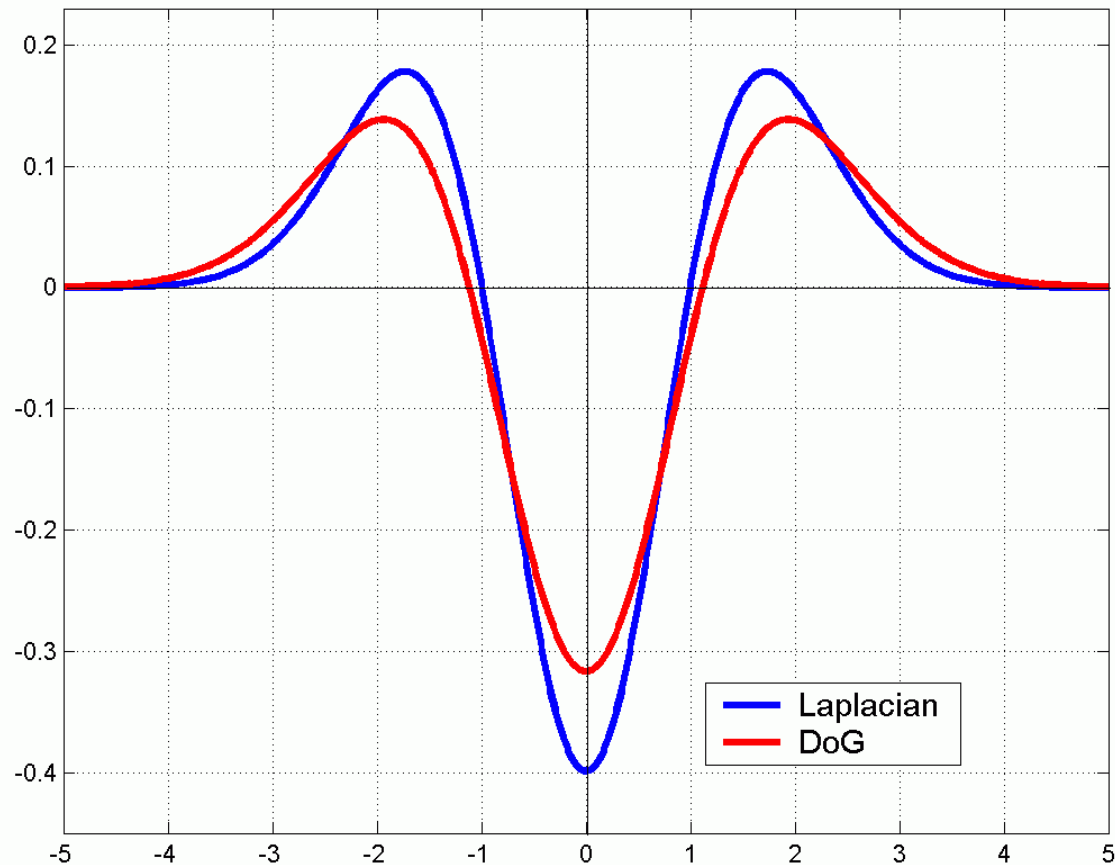
$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

where Gaussian

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



# Kernel plots in 1-D



Both kernels are invariant to *scale* and *rotation*.



# Relationship between LoG and DoG operator

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$$\frac{\partial G}{\partial \sigma} = \sigma \nabla^2 G$$

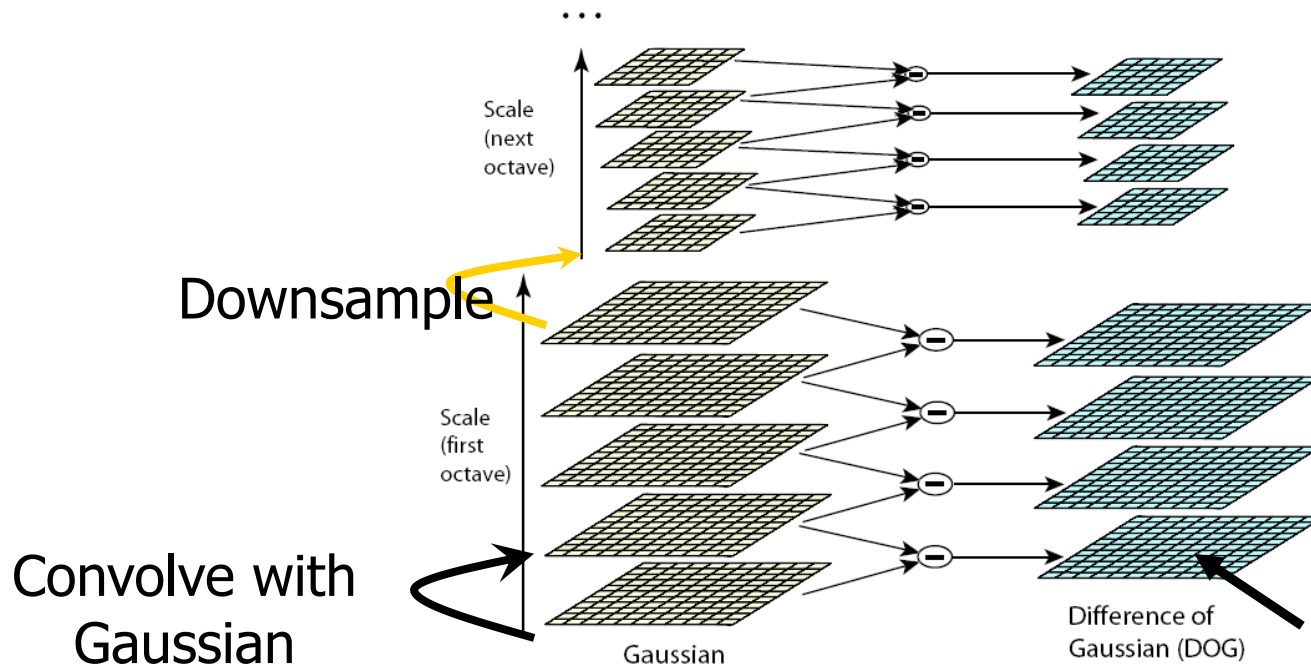
$$\sigma \nabla^2 G = \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k\sigma - \sigma}$$

$$G(x, y, k\sigma) - G(x, y, \sigma) = (k - 1)\sigma^2 \nabla^2 G$$

The factor  $(k-1)$  is kept constant across scales.  
➔ does not influence extrema locations.

# Finding Keypoints

## – Scale, Location



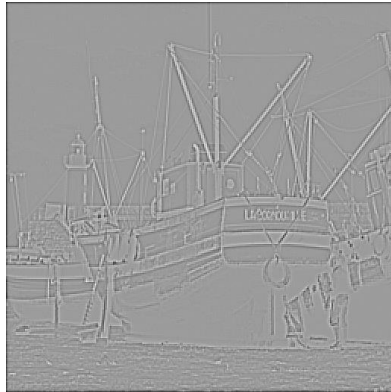
Find extrema  
in 3D DoG space

$$D(x, y, \sigma)$$

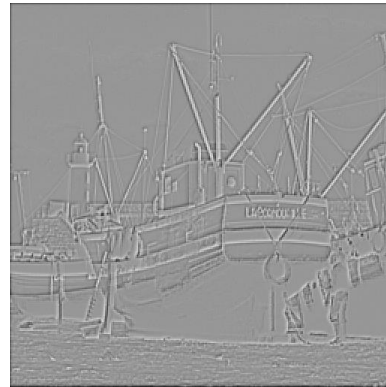
# Typical example (DOGs at different scales)



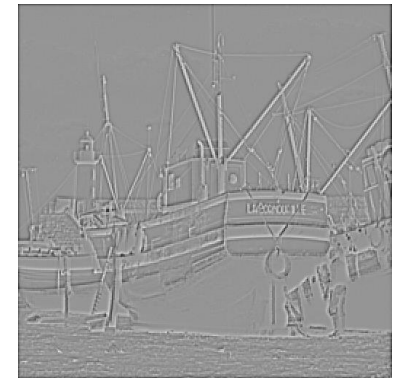
Octave:1



$$\sigma=2^{2/3}$$



$$\sigma=2^{5/6}$$



$$\sigma=2$$

Octave:2



$$\sigma=2^{7/6}$$



$$\sigma=2^{4/3}$$



$$\sigma=2^{3/2}$$

Octave:3



$$\sigma=2^{5/3}$$



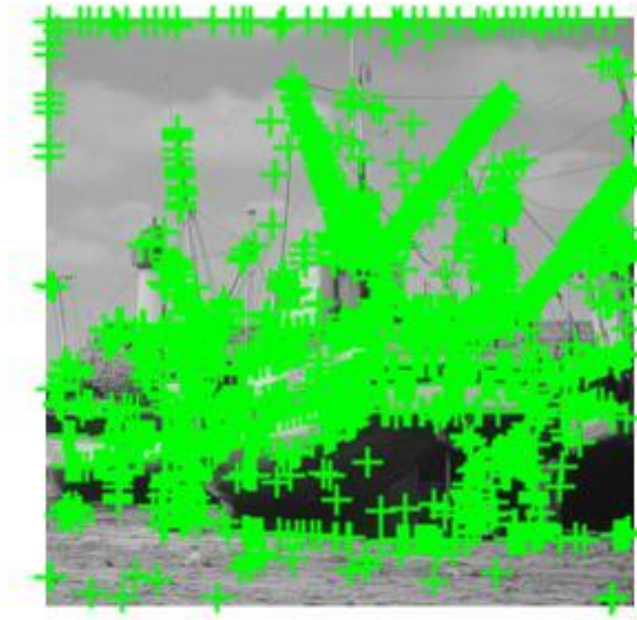
$$\sigma=2^{11/6}$$



$$\sigma=4$$

# Extrema of DOG stack: Example)

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# Keypoint localization

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- A lot of points, some of them are not good enough.
  - Edge points
- Locations may be not accurate.

# Eliminating edge points

$$H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix}$$

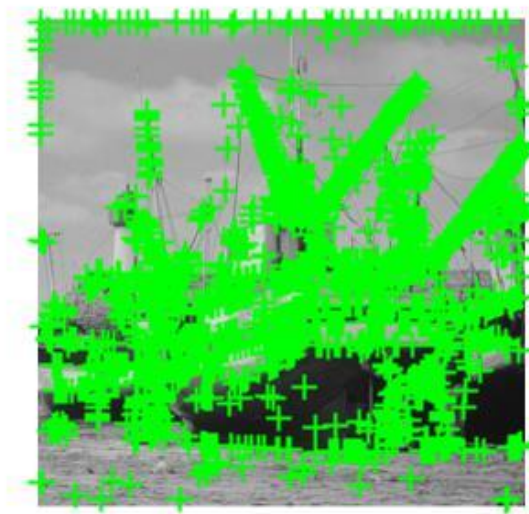
$D(.) = \text{DoG}$  of the function.

- Edge point: large principal curvature across the edge but a small one in the perpendicular direction.
- The eigenvalues of  $H$  are proportional to the principal curvatures, so two eigenvalues shouldn't differ too much.

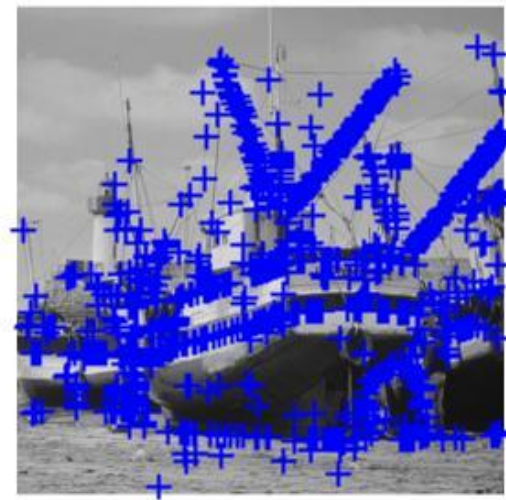
Eliminate keypoint if the ratio greater than the threshold.

$$\frac{\text{Tr}(H)^2}{\text{Det}(H)} < \frac{(r+1)^2}{r} \text{ for high value of } r \text{ (say 10)}$$

# Key-points filtering: An example



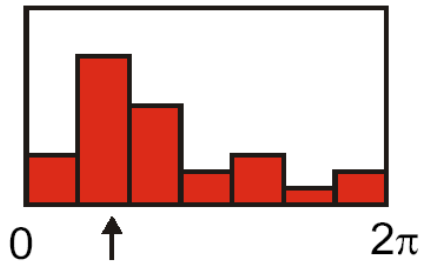
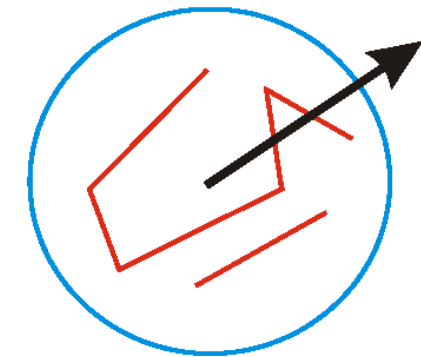
Initial set



After ratio threshold



# Orientation assignment



- Create histogram of local gradient directions at selected scale.
- Assign canonical orientation at peak of smoothed histogram.
- Each key specifies stable 2D coordinates ( $x$ ,  $y$ , scale, orientation).

If there are two major orientations, use both.



# Keypoint Descriptors

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- At this point, each keypoint has
  - location
  - scale
  - orientation
- Next is to compute a descriptor for the local image region about each keypoint that is
  - highly distinctive
  - invariant as possible to variations such as changes in viewpoint and illumination

# Scale Invariant Feature Transform (Lowe'99, ICCV)

- Take 16x16 square window (orientation corrected) around detected feature
- Compute edge orientation (angle of the gradient -  $90^\circ$ ) for each pixel
- Throw out weak edges (threshold gradient magnitude)
- Create histogram of surviving edge orientations

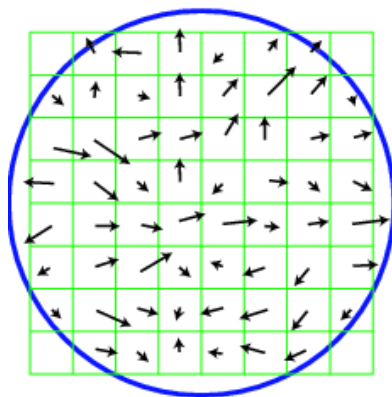
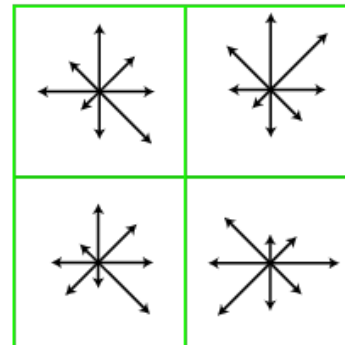
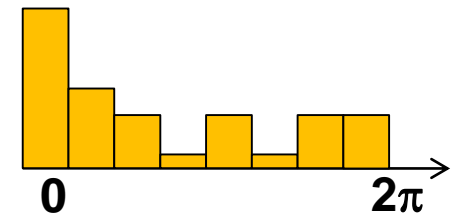


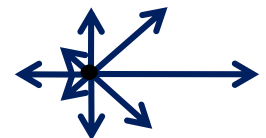
Image gradients



Keypoint descriptor

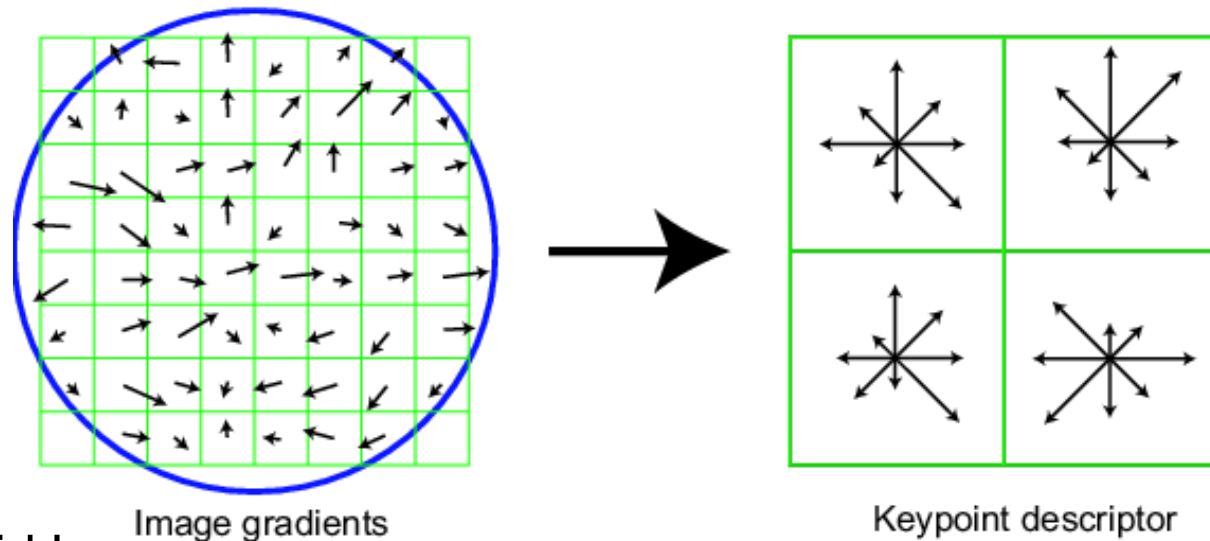


angle histogram



# SIFT descriptor

- Divide the 16x16 window into a 4x4 grid of cells (2x2 case shown below).
- Compute an orientation histogram for each cell
- 16 cells \* 8 orientations = 128 dimensional descriptor.





# Robustness

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Capable of handling

- changes in viewpoint
  - Up to about 60 degree out of plane rotation
- significant changes in illumination
  - Sometimes even day vs. night



# Speeded-Up Robust Features (SURF): Another descriptor

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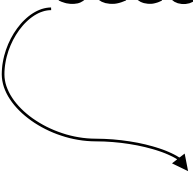
- Speeded-Up Robust Features (SURF)
  - (Bay et al. ECCV, 2006)
  - Box-type convolution filters and use of integral images to speed up the computation.
  - Use of Hessian operator for key point detection → Local maxima of  $\det(H)$ .
  - Accumulate orientation corrected Haar wavelet responses.



# Hessian Operator

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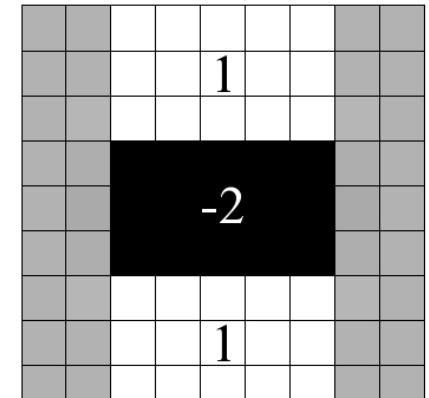
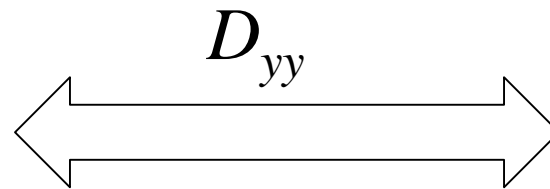
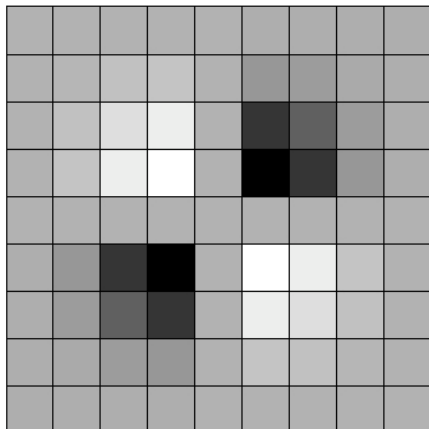
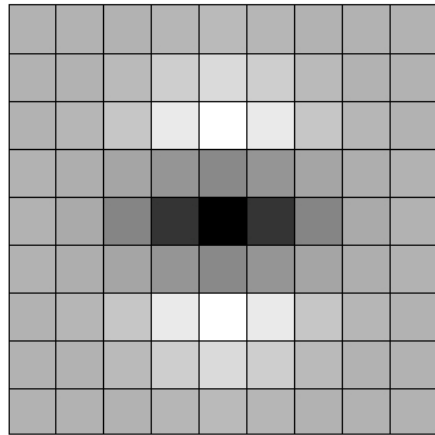
Convolution with the Gaussian second order derivative with image.


$$\mathcal{H}(\mathbf{x}, \sigma) = \begin{bmatrix} D_{xx}(\mathbf{x}, \sigma) & D_{xy}(\mathbf{x}, \sigma) \\ D_{yx}(\mathbf{x}, \sigma) & D_{yy}(\mathbf{x}, \sigma) \end{bmatrix}$$

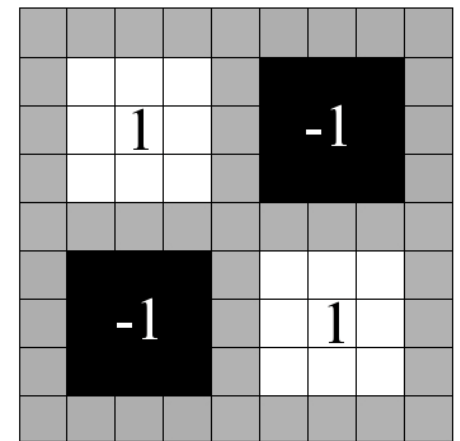
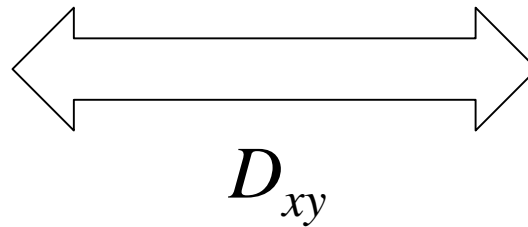
Keypoint: Maximum of  $\det(\mathcal{H}(.))$  over space and scale.

# Approximation of Gaussian by Box filters

Bay et al, Speeded up robust features (SURF), CVIU, 2008



9x9 Box-filters are approximation of Gaussian width 1.2.

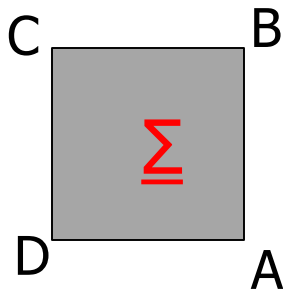


$$\det(H_{approx}) = D_{xx}D_{yy} - (wD_{xy})^2 \quad w \sim 0.9$$



# Fast computation using the integral image

$$I_{\Sigma}(\mathbf{x}) = \sum_{i=0}^{i \leq x} \sum_{j=0}^{j \leq y} I(i, j)$$

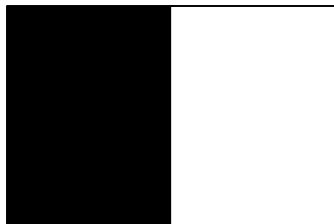


Only 3 additions and four memory access.

$$\underline{\Sigma} = A - B - C + D$$

# Haar Filter Responses

- Dominant orientation by accumulating Haar horizontal and vertical responses in a rotated sliding window (of width  $60^\circ$ ) at the scale of key point.
- The longest vector provides the dominant direction.

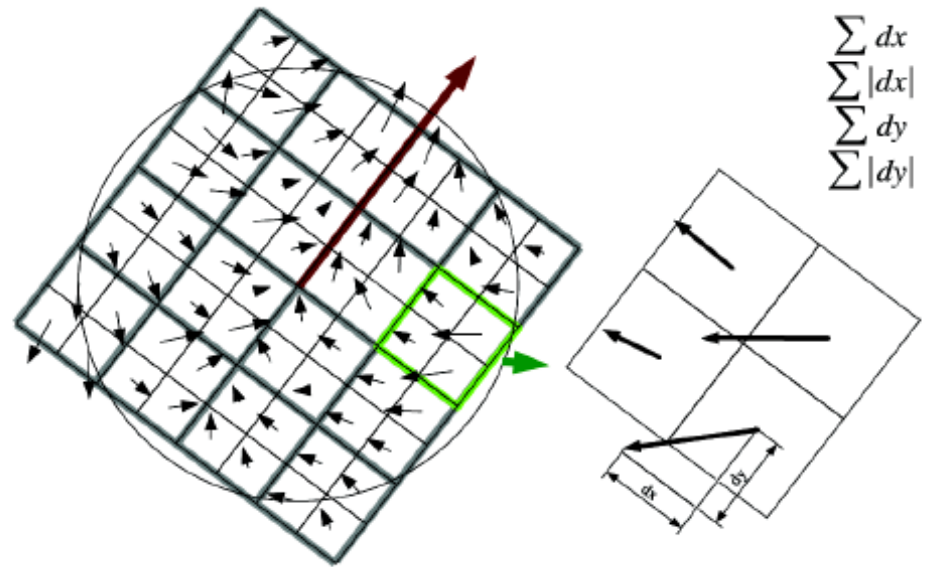


6 operations needed for  
computing each filter response  
using integral image.

Haar Filters  
Box filter implementation.

# SURF: Sum of Haar Wavelet responses

- Partitioned into 4x4 square sub-regions.
- Haar wavelet responses at regularly spaced 5x5 sample patches in each sub-region.
- Each sub-region has 4D vector.
- Concatenate them to 64 D vector.



Size of the window=20 x scale

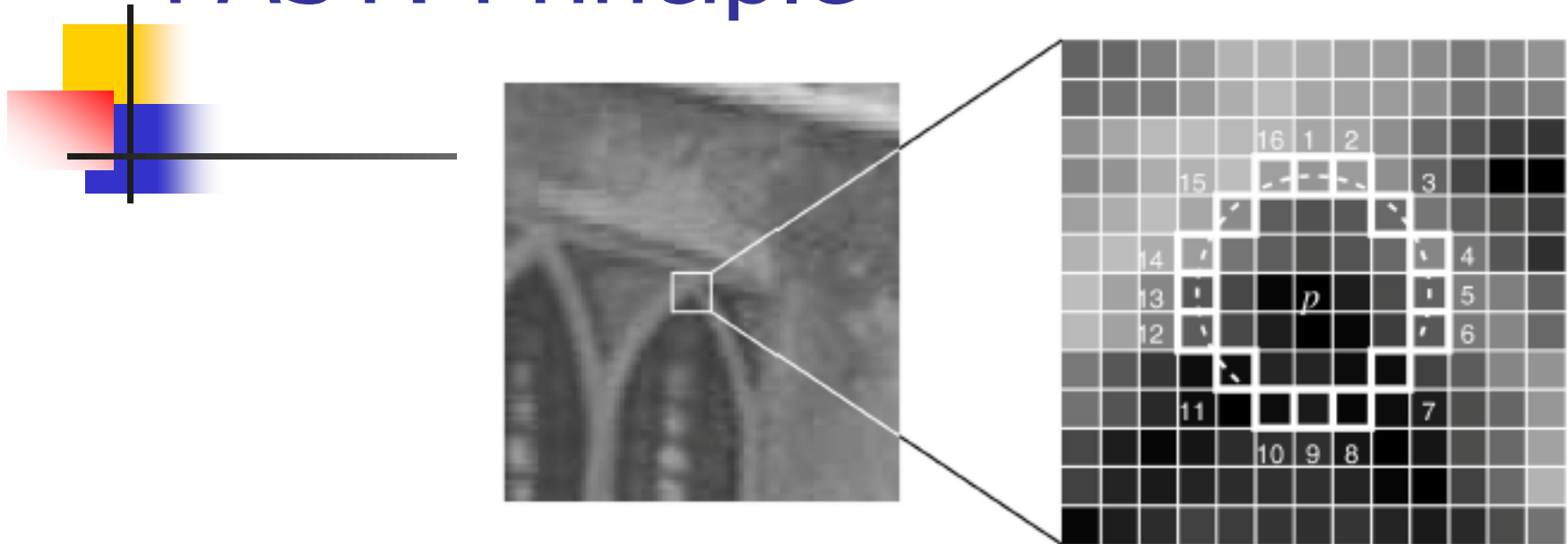


# Other types of detectors and descriptors

---

- FAST: Features from Accelerated Segment Test (Rosten et al, PAMI, 2010).
- BRIEF: Binary Robust Independent Elementary Features (Colonder et al, ECCV, 2010).
- ORB: Oriented FAST and Rotated BRIEF (Rublee et al, ICCV, 2011)

# FAST: Principle



- 12 point test: If there exists 12 consecutive points in the set of 16 points brighter than the central pixel (Rosten et al ICCV'05).
- Modified strategy: Train decision tree on the boolean conditions given labelled data.



# FAST: Partitioning of points on the circle and Training a DT

---

$$S_{p \rightarrow x} = \begin{cases} d, & I_{p \rightarrow x} \leq I_p - t & (\text{darker}) \\ s, & I_p - t < I_{p \rightarrow x} < I_p + t & (\text{similar}) \\ b, & I_p + t \leq I_{p \rightarrow x} & (\text{brighter}) \end{cases}$$

- Classification of points in three classes and create three partitions for a point.
- Train a decision tree.



# BRIEF: Principle

---

- Generate randomly a set of  $n_d$  pairs of locations  $(\mathbf{x}_i, \mathbf{y}_i)$  in a patch centered around a point.
- Perform the following boolean test.

$$\tau(\mathbf{p}; \mathbf{x}, \mathbf{y}) = \begin{cases} 1 & \text{if } \mathbf{p}(\mathbf{x}) < \mathbf{p}(\mathbf{y}) \\ 0 & \text{otherwise} \end{cases}$$

BRIEF descriptor:  $n_d$  *Dimensional binary string*

$$f_{n_d}(\mathbf{p}) = \sum_{1 \leq i \leq n_d} 2^{i-1} \tau(\mathbf{p}; \mathbf{x}_i, \mathbf{y}_i)$$

128, 256, 512



# ORB: Principle

---

- FAST does not operate across scale.
- Apply FAST detector on pyramid of smoothened images.
- Orientation by intensity centroid: The vector from the center of the patch and to a centroid considering the intensity distribution (intensity weighted center of patch).
- Rotate the patch by the angle and compute BRIEF called steered BRIEF.





# Matching

---

- Representation of a key-point by a feature vector.
  - e.g.  $[f_0 f_1 \dots f_n]^T$
- Use distance functions / similarity measures.

- $L_1$  norm

$$L_1(\vec{f}, \vec{g}) = \sum_{i=0}^n |f_i - g_i|$$

- $L_2$  norm

$$L_2(\vec{f}, \vec{g}) = \left( \sum_{i=0}^n |f_i - g_i|^2 \right)^{\frac{1}{2}}$$

- $L_p$  norm

$$L_p(\vec{f}, \vec{g}) = \left( \sum_{i=0}^n |f_i - g_i|^p \right)^{\frac{1}{p}}$$



# Region descriptors

---

- Patch descriptors
- Texture descriptors
- Image / Sub-Image global descriptors



# Patch Descriptor: Histogram of Gradients (HoG)

---

- Compute centered horizontal and vertical gradients with no smoothing.
- Compute gradient orientation and magnitudes,
- For color image, pick the color channel with the highest gradient magnitude for each pixel.
- For a 64x128 image, divide the image into 16x16 blocks of 50% overlap.  $\rightarrow 7 \times 15 = 105$  blocks in total.



# Histogram of Gradients (HoG)

---

- Each block: 2x2 cells with size 8x8.
- Quantize the gradient orientation into 9 bins.
- The vote is the gradient magnitude.
- Interpolate votes between neighboring bin center.
- The vote can also be weighted with Gaussian to down-weight the pixels near the edges of the block.
- Concatenate histograms.
  - Feature dimension:  $105 \times 4 \times 9 = 3,780$



# Object detection with patch descriptors.

---

- Typical examples:
  - Pedestrian detection
  - Character recognition
- Detection as a classification task.
  - Generate labeled sample feature descriptors.
  - Train a classifier.
    - NN, SVM, Decision Tree, Random Forest .....
  - Label an unknown patch using its descriptor.



# Applications

---

## ■ Pedestrian Detection



N.Dalal and B. Triggs, Histograms of oriented gradients for human detection, CVPR-2005



# Non-maximal suppression

---

- Expected to get a high detection score with neighboring overlapping patches.
  - Select the patch with locally maximal score.
- A greedy approach:
  - Select the best scoring window
    - It is expected to cover the target object.
  - Suppress the windows that are too close to the selected window.
  - Search next top-scoring windows out of the rest.

# Character Spotting (E-PURALEKHAK)



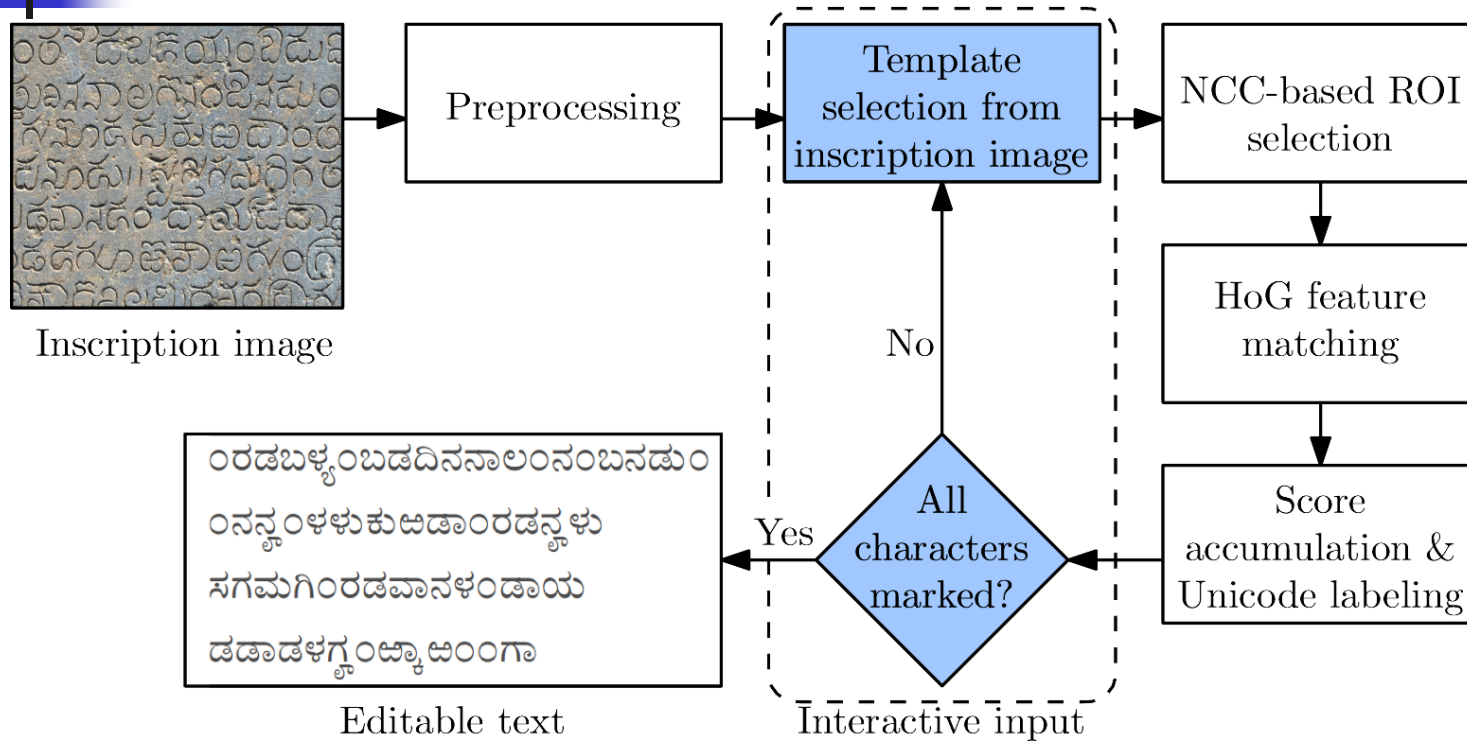
---

- A tool for digital paleography
  - Aids analysis of inscription
  - Converts inscribed substrate to editable text
- Processing pipeline
  - Preprocessing – denoising and normalization ([0 255])
  - Search possible locations of user indicated character by cross correlation
  - Prune the search results by HoG feature matching
  - Parse the Unicode file – editable text

Shashaank. M. Aswatha et al., A Method for Extracting Text from Stone Inscriptions using Character spotting, ACCV, 2014.



# Character Spotting (E-PURALEKHAK)



## Character spotting – process flow

Shashaank. M. Aswatha et al., A Method for Extracting Text from Stone Inscriptions using Character spotting, ACCV, 2014.



# Texture descriptor



- Texture: spatial arrangement of the colors or intensities in an image
  - A quantitative measure of the arrangement of intensities in the region.



# Texture descriptors

---

- Edge density and direction
- Local Binary Pattern (LBP).
- Co-occurrence Matrix.
- Laws' texture energy features.



# Edge density and direction

---

- Compute gradient at each pixel.
  - The descriptor: normalized histograms of magnitudes and directions of gradients over a region.
    - $(H_R(\text{mag}), H_R(\text{dir}))$ 
      - Normalized histogram of magnitudes.
      - Normalized histogram of directions.
  - Numbers of bins in histograms kept small (e.g. 10).
    - Use L1 norm between the feature vectors as a distance measure.
- Normalized histogram  $\rightarrow$  Area = 1;



# Local Binary Pattern (LBP).

3	2	1
4	<i>c</i>	0
5	6	7

$$b(i) = \begin{cases} 1 & \text{if } (I(i) > I(c)) \\ 0 & \text{Otherwise} \end{cases}$$

$$LBP(c) = \sum_{i=0}^7 b(i)2^i$$

You may have  
different ordering of  
neighbors.

- Values range from 0 to 255.
- Obtain normalized histogram over a region.
- Not rotational invariant.
- Invariant to illumination and contrast.



# Variations of LBP

---

- Making it rotational invariant.
  - A circular neighborhood of radius  $R$ , with  $P$  pixels at equal intervals of angles.
  - Use interpolation if does not belong to the discrete grid.

$$LBP_{P,R}(c) = \sum_{i=0}^{P-1} b(i)2^i \qquad LBP_{8,1} \leftarrow \rightarrow LBP$$

$$LBP_{P,R}^{ri}(c) = \min\{ROR(LBP_{P,R}(c), i) \mid i = 0, 1, 2, \dots, P-1\}$$

where  $ROR(x, i)$  performs a circular bit-wise right shift on the  $P$ -bit number  $x$ ,  $i$  times.

36 distinct values for  $LBP_{8,1}^{ri}$ .



# Variations of LBP

---

## ■ Uniform pattern

- Not more than 2 spatial transitions in the bit sequence.

- $U(11111111)=0$

- $U(11101111)=2$

- $U(10001001)=4$

Use rotation invariant value  
by computing minimum  
applying ROR operator.

$$LBP_{P,R}^{riu2}(c) = \begin{cases} \sum_{i=0}^{P-1} b(i)2^i & \text{if } U(LBP_{p.R}(c)) \leq 2 \\ P + 1 & \text{Otherwise} \end{cases}$$

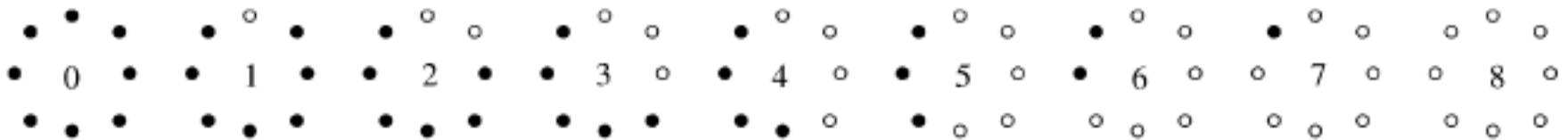
Exactly  $P+1$   
uniform  
patterns, hence  
 $P+2$  distinct  
values.



# Nine uniform patterns of

$LBP_{8,R}^{riu2}$

- The nine uniform patterns and the numbers inside them correspond to their unique codes.





# Variance Measures of the Contrast

---

- Local variance of intensities of uniform pattern.
- Normalized histogram of local variances.

$$var_{P,R}(c) = \frac{1}{P} \sum_{p=0}^{P-1} (g_p - \mu)^2 \quad \mu = \frac{1}{P} \sum_{p=0}^{P-1} g_p$$

Another robust representation:  $\frac{LBP_{P,R}^{riu2}}{var_{P,R}}$



# Co-occurrence Matrix ( $C_r$ )

---

- $C_r(x, y)$ : How many times elements  $x$  and  $y$  occur at a pair of pixels related spatially (designated by  $r$  in the notation).
- e.g.  $\mathbf{p} \ r \ \mathbf{q}$  denotes  $\mathbf{q}$  is shifted from  $\mathbf{p}$  by a translation of  $\mathbf{t}=(a,b)$ , i.e.  $\mathbf{q}=\mathbf{p}+\mathbf{t}$ .
  - $C_{(a,b)}(x, y)$ : Number of cases in an image where  $I(\mathbf{p})=x$  and  $I(\mathbf{p}+\mathbf{t})=y$ .



# Co-occurrence Matrix ( $C_r$ )

0	0	1	1
0	0	1	1
1	1	0	0
1	1	0	0

	0	1
0		
1		

$C_{(0,1)}$

	0	1
0		
1		

$C_{(1,0)}$

	0	1
0		
1		

$C_{(0,1)}$



# Co-occurrence Matrix ( $C_r$ )

0	0	1	1
0	0	1	1
1	1	0	0
1	1	0	0

	0	1
0	4	2
1	2	4

$C_{(0,1)}$

	0	1
0	4	2
1	2	4

$C_{(1,0)}$

	0	1
0	2	2
1	2	3

$C_{(1,1)}$



# Normalized Co-occurrence Matrix ( $N_r$ )

Divide by the sum of frequencies in a matrix.

0	0	1	1
0	0	1	1
1	1	0	0
1	1	0	0

	0	1
0	1/3	1/6
1	1/6	1/3

$C_{(0,1)}$

	0	1
0	1/3	1/6
1	1/6	1/3

$C_{(1,0)}$

	0	1
0	2/9	2/9
1	2/9	1/3

$C_{(1,1)}$

# Symmetric Co-occurrence Matrix ( $S_r$ )

$$S_r(x,y) = C_r(x,y) + C_{-r}(x,y)$$

0	0	1	1
0	0	1	1
1	1	0	0
1	1	0	0

	0	1
0	4+4	2+2
1	2+2	4+4
	$C_{(0,1)} + C_{(0,-1)}$	

	0	1
0	4+4	2+2
1	4+4	2+2
	$C_{(1,0)} + C_{(-1,0)}$	

	0	1
0	2+2	2+2
1	2+2	3+3
	$C_{(1,1)} + C_{(-1,-1)}$	



# Features from Normalized Co-occurrence Matrix

---

$$Energy = \sum_x \sum_y N_r^2(x, y)$$

$$Entropy = - \sum_x \sum_y N_r(x, y) \log_2 N_r(x, y)$$

$$Contrast = \sum_x \sum_y (x - y)^2 N_r(x, y)$$

$$Homogeneity = \sum_x \sum_y \frac{N_r(x, y)}{1 + |x - y|}$$

$$Correlation = \frac{\sum_x \sum_y N_r(x, y)xy - \mu_x \mu_y}{\sigma_x \sigma_y}$$





# Features from Normalized Co-occurrence Matrix

$$\text{Correlation} = \frac{\sum_x \sum_y (x - \mu_x) (y - \mu_y) N_r(x, y)}{\sigma_x \sigma_y}$$

Mean and s.d.  
of row sums

$$f(x) = \sum_y N_r(x, y)$$

Mean and s.d.  
of column sums

$$g(y) = \sum_x N_r(x, y)$$



# Laws' texture energy features

- A set of 9 5x5 masks used to compute texture energy.

L5 (Level):  $[1 \ 4 \ 6 \ 4 \ 1]^T$

E5 (Edge):  $[-1 \ -2 \ 0 \ 2 \ 1]^T$

S5 (Spot):  $[-1 \ 0 \ 2 \ 0 \ -1]^T$

R5 (ripple):  $[1 \ -4 \ 6 \ -4 \ 1]^T$

Computation with mask:

**Convolution**

A mask: Outer  
product of any pair.  
e.g. E5L5:  $E5.L5^T$

$$\begin{bmatrix} -1 \\ -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix}$$

K. Laws, "Rapid Texture Identification", in SPIE Vol. 238: Image Processing for Missile Guidance, 1980, pp. 376-380.



# Laws' texture energy features

- A set of 9 5x5 masks used to compute texture energy.

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S5 (Spot):  $[-1 \ 0 \ 2 \ 0 \ -1]^T$

R5 (ripple):  $[1 \ -4 \ 6 \ -4 \ 1]^T$

Take  
average of  
responses  
of two  
masks.

L5E5 and E5L5

L5R5 and R5L5

E5S5 and S5E5

L5S5 and S5L5

E5R5 and R5E5

S5R5 and R5S5

S5S5

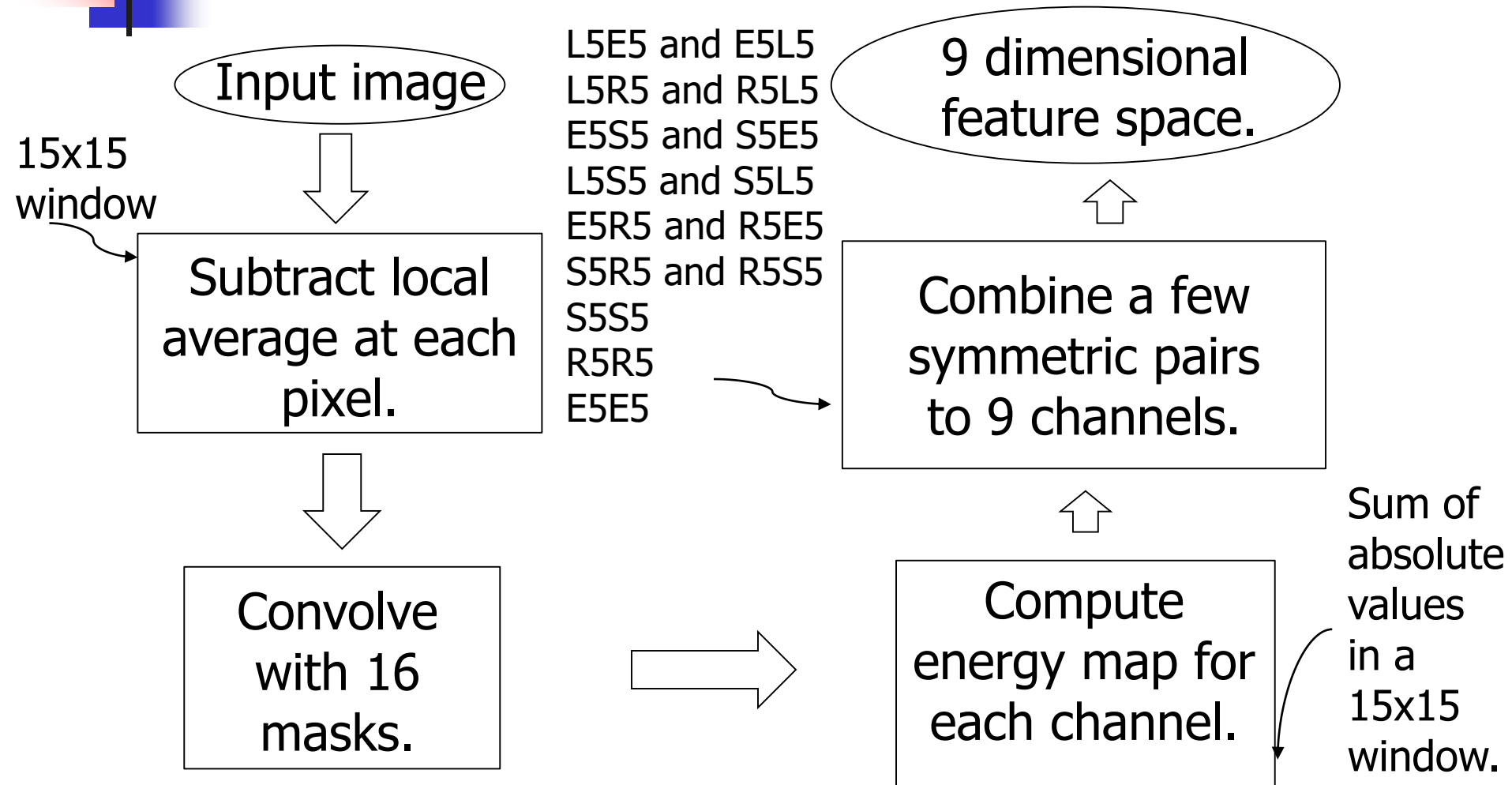
R5R5

E5E5

16 such masks possible.

Combine a few pairs to make 9 masks.

# Laws' texture energy





# Use of texture descriptors

---

- Detection of object patches represented by textured patterns.
- Segmentation of images.
- Classification / Matching
  - Generate a Library of labelled feature descriptors.
  - Detection of classes (class labels).
    - Matching to the nearest texture descriptor.



# Image / Object Descriptor

---

- Bag of visual words

- Compute key-point based feature descriptors over a library of images.
- Quantize them (clustering) to form a finite set of representative descriptors (visual words).
- For an image assign the nearest visual word corresponding to the feature descriptor of a key point.
- Represent by each image by a histogram of visual words.

K-means  
clustering



# Vector of locally aggregated descriptors (VLAD)

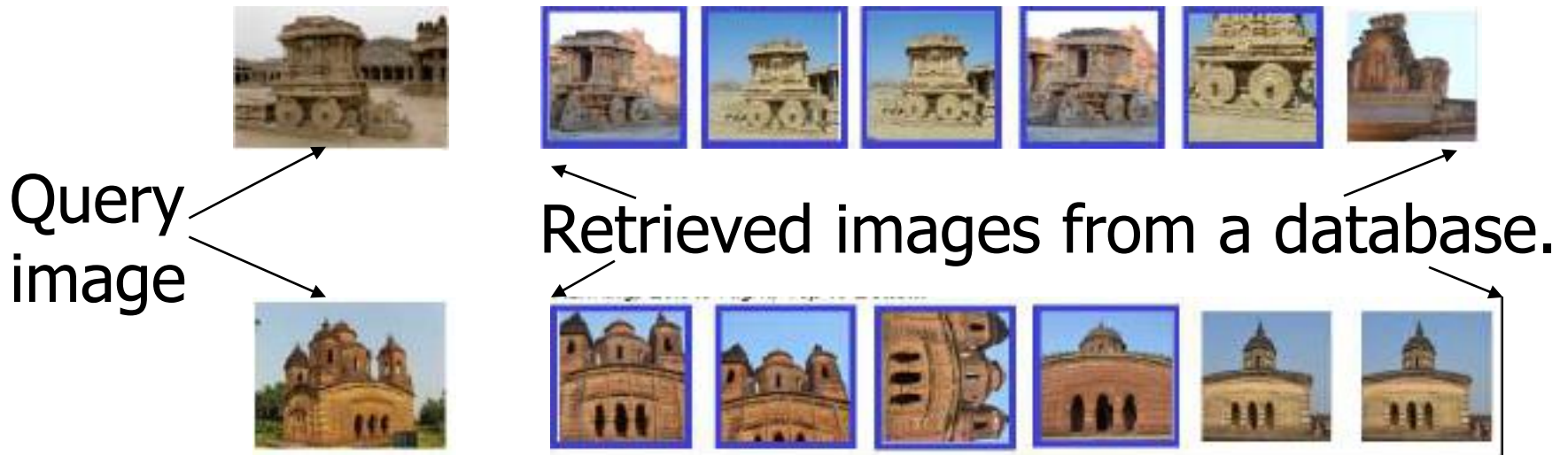
- Form the codebook of visual words as in BoVW representation.
  - $C_1, C_2, \dots, C_k$  ← Cluster centers of dimension  $D$  (say).
- Each local descriptor  $x$  in an image is associated to one of these visual words.
- Accumulate the differences w.r.t. the corresponding cluster center.

$$v_i = \sum_{x \text{ assigned to } C_i} (x - C_i)$$

- Form  $V = [v_1 \ v_2 \ \dots \ v_k]$
- VLAD descriptor  $= V / ||V||$  ← Dimension:  $k.D$

# Application of global image descriptor

- Content based image retrieval
  - Image search based on visual content







# Summary of Techniques

---

- Scale and transformation invariant feature detection:
  - Harris corner-Laplacian Maximum.
  - DOG Maximum.
  - Intensity weighted FAST.
- Scale and transformation invariant Feature descriptor:
  - SIFT, SURF, ORB



# Summary of Techniques

---

- Region and texture descriptors.
  - HoG
  - Edge density
  - LBP
  - Co-occurrence matrix
  - Laws' texture energy
- Image global descriptor
  - BoVW
  - VLAD
- A few applications
  - Key point descriptor: Matching corresponding points of a scene.
  - Region descriptor: Object detection
  - Global descriptor :Image retrieval.



Thank you!