INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

THEORY ASSIGNMENT 1
(SWITCHING CIRCUITS AND LOGIC DESIGN)
GROUP 17

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Chapter 1

Switching Circuits and Logic Design Theory Assignment 1

1.1 Q1

Answer with justification:

1. Is the 2-out-of-5 code a cyclic code?

   Answer:

   2 out of 5 code is a Numeric Code that stores one decimal digit in five binary digits in which two of the bits are always 0 or 1 and the other three are always in the opposite state.

   A binary code C is cyclic if it is a linear [n, k] code and if for every code word:

   $$<c_{n1}, ..., c_1, c_0> \in C$$

   $$<c_{n2}, ..., c_1, c_0, c_{n-1}> \in C$$

   is also a code word in C. For a Code to be linear:-

   (a) The zero vector is always a code word.
   (b) The sum or difference of two code words is another code word (the modulo 2 sum is the bit-wise $\oplus$ operation).

   Now, coming down to 2-out-of-5 code. Below are the Valid Code-Words according to the 2-out-of-5 Scheme:-

2-out-of-5 code					
N	0	1	2	4	7
0	0	0	0	1	1
1	1	1	0	0	0
2	1	0	1	0	0
3	0	1	1	0	0
4	1	0	0	1	0
5	0	1	0	1	0
6	0	0	1	1	0
7	1	0	0	0	1
8	0	1	0	0	1
9	0	0	1	0	1

From the Table we can conclude the Code Word:- Zero Vector for the given Code System is $(00000)_2$. The Zero Vector is **NOT** a part of the given System. Hence, the Code-2-out-of-5 is **NOT linear**. Hence, we can conclude that 2-out-of-5 Code **is NOT CYCLIC**.

2. Is the even parity BCD code a cyclic code?

Answer:

Following the definition of the **Cyclic Codes** above we see:-

Even Parity BCD					
N	8	4	2	1	p
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0

The Code is **NOT Linear**:-

(a) As $(00000)_2$ is a valid code word according the Even-Parity BCD Scheme.

(b) For checking if the sum and difference of two code words is another code, we can consider 6^{th} and the 9^{th}.
$(01100)_2 \oplus (10010)_2 \rightarrow (11110)_2$
By observation, $(11110)_2$ is **NOT** a valid code-word. Hence, we know that Even Parity BCD is **NOT Linear**, hence **NOT CYCLIC**.

3. Is the Hamming Code a cyclic code?

Answer:

No, the Hamming Code is **NOT** a Cyclic Code. It is Linear but it does not Satisfy the Cyclic Property. A Simple Example to Prove this Statement right is:-
Let us take the Hamming Code for the message 1100 which is 0111100. After one left shift we get 1111000 which is **NOT** a valid code-word in the Hamming Code since, the third parity bit value i.e P4 is wrong. Hence, Hamming Code is **NOT** Cyclic.

1.2 Q2

1. Determine the generator polynomial for the Gray code of n-bits.

Answer:

We know that the n-bit Gray Code is simply the set of all n-bit binary numbers with a special ordering such that any 2 consecutive binary numbers have a hamming distance of 1.
Also, we know that a minimum-degree polynomial which divides each and every code polynomial present in the code is known as the generating polynomial.

Knowing these 2 facts, it is easy to conclude that the generating polynomial of n-bit gray code is simply '**g(x) = 1**'.
Reason→
Multiplying n-bit messages with 1 will output all n-bit binary numbers, which we know are the code-words present in Gray Code! Example : 0011 $\Rightarrow (x^1 + 1) * (1) \rightarrow (x^1 + 1) \rightarrow 0011$

2. The Gray code remains cyclic after applying a fixed number (say m) of left rotations – let it be called the H-code. Determine the generator polynomial for the n-bit H-code.

Answer:

Now, since gray code is a cyclic code, the new code 'H' generated after m left-shift rotations will also be cyclic. Now let's consider this new generated code → 'H' code.

- Since 'H' code is cyclic, right shifting a code-word m times will lead to a new code-word also present in the 'H' code. (Property of cyclicity)
- Since 'H' code was generated by m left-shifts on Gray Code, performing m right-shifts will take us back to Gray Code.
- The above 2 statements signify all code-words present in 'Gray' code are present in 'H' code, and similarly we can prove the converse, i.e all code-words present in 'H' code are also present in Gray Code. This proves that the Gray Code and 'H' code are identical codes, since a code-word present in 'H' code will also be present in Gray code and vice-versa.
- Thus, the generating polynomial of Gray Code is the same as that of the 'H' code.

So, the generating polynomial of 'H' code is also 'g(x) = 1'.

1.3 Q3

Consider length 7 binary cyclic codes.

1. How many cyclic codes are possible?

Answer:

The potential generator polynomials that are the monic divisors of binary cyclic codes of length 7 are in one to one relationship with the possible generator polynomials.

Any generator polynomial **g** over F_2. Any generator polynomial **g** is then of the form

$$g = (x+1)^{a1}(x^3+x+1)^{a2}(x^3+x^2+1)^{a3} \tag{1.1}$$

for some $a_i \in \{0, 1\}$ .To find how many 7-bit binary cyclic codes exist, we can consider the maximum number of binary cyclic codes possible:-

For each term in the generating polynomial we have two choices:-

$$a_i = \{0, 1\} \tag{1.2}$$

Hence, the total possible codes are:-

$$\begin{aligned} &= 2 * 2 * 2 \\ &= 8 \end{aligned} \tag{1.3}$$

2. Determine the generator polynomials for those codes.

Answer:

The List of all generating polynomial for 7-bit binary cyclic codes (by altering the values of a1,a2 and a3):-

- 1
- $(x+1)$
- (x^3+x+1)
- (x^3+x^2+1)
- $(x+1)(x^3+x+1) = (x^4+x^3+x^2+1)$
- $(x+1)(x^3+x^2+1) = (x^4+x^2+x+1)$
- $(x^2+x+1)(x^3+x^2+1) = (x^6+x^5+x^4+x^3+x^2+x+1)$
- $(x+1)(x^3+x+1)(x^3+x^2+x+1) = x^7+1$

(Note:- We get the above generating polynomials by taking modulo 2 of all coefficients)

1.4 Q4

Take the last two digits of the roll numbers of the members of your group and convert the corresponding BCD to binary using the reverse double-dabble method.

Answer:

My Teammates for the Switching Circuits and Logic Design Laboratory Group are:-

1. 20CS30023
2. 20CS30030
3. 20CS30024
4. 20CS10056

The numbers to be converted from Hence, the Code to be converted from BCD to Binary using Double-Dabble method:- '23' concatenated to '30' concatenated to '24' concatenated to '56' The Number will be:-

$$23302456$$

The Binary for 0,2,3,4,5,6 are respectively:-

$$0 : 000$$

$$2 : 010$$

$$3 : 011$$

$$4 : 100$$

$$5 : 101$$

$$6 : 110$$

Hence, the BCD to be Converted is:-

$$00100011000000100100010101110 \tag{1.4}$$

Operations	B_7	B_6	B_5	B_4	B_3	B_2	B_1	B_0
original number	0010	0011	0011	0000	0101	0110	0010	0100
right shift	0001	0001	1001	1000	0010	1011	0001	0010
sub 3 in D[5] D[4] D[2]	0001	0001	0110	0101	0010	1000	0001	0010
right shift	0000	1000	1011	0010	1001	0100	0000	1001
sub 3 in D[6] D[5] D[4] D[3] D[2] D[0]	0000	0101	1000	0010	0110	0100	0000	0110
right shift	0000	0010	1100	0001	0011	0010	0000	0011
sub 3 in D[5] D[4] D[3] D[2] D[0]	0000	0010	1001	0001	0011	0010	0000	0011
right shift	0000	0001	0100	1000	1001	1001	0000	0001
sub 3 in D[5] D[4] D[3] D[2] D[0]	0000	0001	0100	0101	0110	0110	0000	0001
right shift	0000	0000	1010	0010	1011	0011	0000	0000
sub 3 in D[5] D[4] D[3] D[2] D[0]	0000	0000	0111	0010	1000	0011	0000	0000
right shift	0000	0000	0011	1001	0100	0001	1000	0000
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0011	0110	0100	0001	0101	0000
right shift	0000	0000	0001	1011	0010	0000	1010	1000
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0001	1000	0010	0000	0111	0101
right shift	0000	0000	0000	1100	0001	0000	0011	1010
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	1001	0001	0000	0011	0111
right shift	0000	0000	0000	0100	1000	1000	0001	1011
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	0100	0101	0101	0001	1000
right shift	0000	0000	0000	0010	0010	1010	1000	1100
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	0010	0010	0111	0101	1001
right shift	0000	0000	0000	0001	0001	0011	1010	1100
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	0001	0001	0011	0111	1001
right shift	0000	0000	0000	0000	1000	1001	1011	1100
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	0000	0101	0110	1000	1001
right shift	0000	0000	0000	0000	0010	1011	0100	0100
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	0000	0010	1000	0100	0100
right shift	0000	0000	0000	0000	0001	0100	0010	0010
right shift	0000	0000	0000	0000	0000	1010	0001	0001
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	0000	0000	0111	0001	0001
right shift	0000	0000	0000	0000	0000	0011	1000	1000
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	0000	0000	0011	0101	0101
right shift	0000	0000	0000	0000	0000	0001	1010	1010
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	0000	0000	0001	0111	0111
right shift	0000	0000	0000	0000	0000	0000	1011	1011
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	0000	0000	0000	1000	1000
right shift	0000	0000	0000	0000	0000	0000	0100	0100
right shift	0000	0000	0000	0000	0000	0000	0010	0010
right shift	0000	0000	0000	0000	0000	0000	0001	0001
right shift	0000	0000	0000	0000	0000	0000	0000	1000
sub 3 in D[5] D[4] D[3] D[2] D[1] D[0]	0000	0000	0000	0000	0000	0000	0000	0101
right shift	0000	0000	0000	0000	0000	0000	0000	0010
right shift	0000	0000	0000	0000	0000	0000	0000	0001
right shift	0000	0000	0000	0000	0000	0000	0000	0000

$$\text{binary of N} = 1011000111001110110011000 \tag{1.5}$$

1.5 Q5

Find out the total number of equivalence relations on the set $A = \{1, 2, 3, 4\}$ and also write down the equivalence classes.

Answer:

An equivalence relation is Reflexive, Symmetric and Transitive. Before counting the number of possible equivalence relations on a set:

$$|A| = n$$

Let:

$$A = \{1, 2, 3, 4\}$$

be a set and

$$R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$$

be an equivalence relation on A. We see here that the total relation:

$$T = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$$

over the set:

$$C1 = \{1, 2\}$$

There are five integer partitions of 4:

- 4
- 3+1
- 2+2
- 2+1+1
- 1+1+1+1

which is the subset of

$$A \subset R$$

And also there is no such total relation

$$\bar{T} >= T$$

over set

$$\bar{C}_1 >= C_1$$

which is present in R. Hence we found an equivalence class $E_1 = \{1, 2\}$ over relation R.
The total number of partitions of the set is the total number of equivalence relations over the set. The Partitions of A along with the equivalence classes are:

$\{1\}, \{2\}, \{3\}, \{4\} \Rightarrow [1], [2], [3], [4]$
{1},{2},{3,4} $\Rightarrow$ [1], [2], [3]
{1},{3},{2,4} $\Rightarrow$ [1], [3], [2]
{1},{4},{2,3} $\Rightarrow$ [1], [4], [2]
{2},{3},{1,4} $\Rightarrow$ [2], [3], [1]
{2,4},{1,3} $\Rightarrow$ [2], [1]
{3,4},{1,2} $\Rightarrow$ [3], [1]

$$\{1\},\{2,3,4\} \Rightarrow [1],[2]$$
$$\{2\},\{1,3,4\} \Rightarrow [2],[1]$$
$$\{3\},\{1,2,4\} \Rightarrow [3],[1]$$
$$\{4\},\{1,2,3\} \Rightarrow [4],[1]$$
$$\{1,2\},\{3,4\} \Rightarrow [1],[3]$$
$$\{1,3\},\{2,4\} \Rightarrow [1],[2]$$
$$\{1,4\},\{2,3\} \Rightarrow [1],[2]$$
$$\{1,2,3,4\} \Rightarrow [1]$$

Hence, in all there are 15 equivalence classes.

1.6 Q6

Consider sets $A_1, \ldots, A_k$ and all possible intersections of those sets or their complements; then consider the resulting meet and joins of those; what kind of algebraic structure is that?

Answer:

Let $\mathcal{A} = \{A_i | 1 \leq i \leq k\} \quad \cup \quad \{A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_m} \cap \bar{A}_{j_1} \cap \bar{A}_{j_2} \cap \ldots \cap \bar{A}_{j_n} | i_1, i_2, \ldots, i_m, j_1, j_2, \ldots, j_n \in I_k \ \ \& \ \ 0 \leq m, n \leq k \& m + n \geq 2\}$

Define a relation $\rho = \ \subseteq$ over $\mathcal{A}$. It can be proved that $(\mathcal{A}, \subseteq)$ is a partially ordered set (POSET).

1. **Reflexive:** $\forall A \in \mathcal{A}, [\,(A, A) \in (\mathcal{A}, \subseteq)]$ since $A \subseteq A$
2. **Anti-symmetric:** $\forall A, B \in \mathcal{A}, [\,(A, B), (B, A) \in (\mathcal{A}, \subseteq) A = B]$ since $A \subseteq B \& B \subseteq A A = B$
3. **Transitive:** $\forall A, B, C \in \mathcal{A}, [\,(A, B), (B, C) \in (\mathcal{A}, \subseteq)(A, C) \in (\mathcal{A}, \subseteq)]$ since $A \subseteq B \& B \subseteq C A \subseteq C$

$\forall A, B \in \mathcal{A}$, meet of $\{A, B\}$ is $(A \cap B) \in \mathcal{A}$ since $\mathcal{A}$ contains all possible intersections of sets $A_1, A_2, \ldots, A_k$ or their complements.Therefore, We conclude that $\mathcal{A}$ is a **meet semi-lattice**