The office is about fifteen minuets from my house

The office is about fifteen minuets from my house min-u-et an noun \min-ya-\weth

- : a slow, graceful dance that was popular in the 17th and 18th centuries
- : the music for a minuet

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```
min·u·et | noun \min-ya-\wet\
: a slow, graceful dance that was popular in the 17th and 18th centuries
```

: the music for a minuet

Use a Language Model

P(about fifteen **minutes** from) > P(about fifteen **minuets** from)

Probablilistic Language Models: Applications

Speech Recognition

• P(I saw a van) >> P(eyes awe of an)

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Which sentence is more plausible in the target language?

• P(high winds) > P(large winds)

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Other Applications

- Context Sensitive Spelling Correction
- Natural Language Generation
- o ...

Completion Prediction

- Language model also supports predicting the completion of a sentence.
 - Please turn off your cell ...
 - Your program does not ...

Completion Prediction

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 - Please turn off your cell ...
 - Your program does not ...
- Predictive text input systems can guess what you are typing and give choices on how to complete it.

Probabilistic Language Modeling

• Goal: Compute the probability of a sentence or sequence of words:

$$P(W) = P(w_1, w_2, w_3, \dots, w_n)$$

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$$P(w_4|w_1,w_2,w_3)$$

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• A model that computes either of these is called a language model

Computing $P(\overline{W})$

How to compute the joint probability

P(about, fifteen, minutes, from)

Computing P(W)

How to compute the joint probability

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Basic Idea

Rely on the Chain Rule of Probability

Conditional Probabilities

$$P(B|A) = \frac{P(A,B)}{P(A)}$$

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More Variables

$$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$$

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More Variables

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The Chain Rule in General

$$P(x_1,x_2,...,x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1,x_2)...P(x_n|x_1,...,x_{n-1})$$

Probability of words in sentences

$$P(w_1w_2...w_n) = \prod_i P(w_i|w_1w_2...w_{i-1})$$

P("about fifteen minutes from") =

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P("about fifteen minutes from") =

P(about) x P(fifteen | about) x P(minutes | about fifteen) x P(from | about fifteen minutes)

Estimating These Probability Values

Count and divide

P(office | about fifteen minutes from) = $\frac{Count \text{ (about fifteen minutes from office)}}{Count \text{ (about fifteen minutes from)}}$

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Count and divide

P(office | about fifteen minutes from) = $\frac{Count \text{ (about fifteen minutes from office)}}{Count \text{ (about fifteen minutes from)}}$

What is the problem

We may never see enough data for estimating these

Simplifying Assumption: Use only the previous word

P(office | about fifteen minutes from) ≈ P(office | from)

Simplifying Assumption: Use only the previous word

P(office | about fifteen minutes from) \approx P(office | from)

Or the couple previous words

P(office | about fifteen minutes from) \approx P(office | minutes from)

More Formally: kth order Markov Model

Chain Rule:

$$P(w_1w_2...w_n) = \prod_i P(w_i|w_1w_2...w_{i-1})$$

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$$P(w_1w_2...w_n) = \prod_i P(w_i|w_1w_2...w_{i-1})$$

Using Markov Assumption: only *k* previous words

$$P(w_1w_2...w_n) \approx \prod_i P(w_i|w_{i-k}...w_{i-1})$$

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Chain Rule:

$$P(w_1w_2...w_n) = \prod_i P(w_i|w_1w_2...w_{i-1})$$

Using Markov Assumption: only *k* previous words

$$P(w_1w_2...w_n) \approx \prod_i P(w_i|w_{i-k}...w_{i-1})$$

We approximate each component in the product

$$P(w_i|w_1w_2...w_{i-1})\approx P(w_i|w_{i-k}...w_{i-1})$$

P(office | about fifteen minutes from)

An N-gram model uses only N-1 words of prior context.

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Unigram: P(office)

• Bigram: P(office | from)

Trigram: P(office | minutes from)

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Markov model and Language Model

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An N-gram model uses only N-1 words of prior context.

- Unigram: P(office)
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Markov model and Language Model

An N-gram model is an N-1-order Markov Model

- We can extend to trigrams, 4-grams, 5-grams
- In general, an insufficient model of language:

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 language has long-distance dependencies:
 "The computer which I had just put into the machine room on the fifth floor crashed."

- We can extend to trigrams, 4-grams, 5-grams
- In general, an insufficient model of language:
 language has long-distance dependencies:
 "The computer which I had just put into the machine room on the fifth floor crashed."
- In most of the applications, we can get away with N-gram models

Estimating N-grams probabilities

Estimating N-grams probabilities

Maximum Likelihood Estimate

Value that makes the observed data the "most probable"

$$P(w_i|w_{i-1}) = \frac{count(w_{i-1},w_i)}{count(w_{i-1})}$$

Estimating N-grams probabilities

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$$P(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An Example

$$P(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s>I am here </s>
<s>who am I </s>
<s>I would like to know </s>

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Estimating bigrams

$$P(I|~~) =~~$$

$$P(|here) =$$

$$P(would | I) =$$

An Example

$$P(w_i|w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

<s>I am here </s>

<s>who am I </s>

<s>I would like to know </s>

Estimating bigrams

$$P(I|~~) = 2/3~~$$

$$P(|here) = 1$$

$$P(would \mid I) = 1/3$$

$$P(here \mid am) = 1/2$$

$$P(know | like) = 0$$

Bigram counts from 9222 Restaurant Sentences

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Computing bigram probabilities

Normlize by unigrams

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

Computing bigram probabilities

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Bigram Probabilities

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Computing Sentence Probabilities

$$P(\langle s \rangle | I \text{ want english food } \langle s \rangle)$$

= $P(I | \langle s \rangle) \times P(\text{want } | I) \times P(\text{english } | \text{want}) \times P(\text{food } | \text{english }) \times P(\langle s \rangle | \text{food})$

Computing Sentence Probabilities

```
P(\langle s \rangle | I \text{ want english food } \langle s \rangle)
```

- = $P(I \mid <s>) x P(want \mid I) x P(english \mid want) x P(food \mid english) x P(</s> \mid food)$
- = 0.000031

What knowledge does n-gram represent?

- P(english|want) = .0011
- P(chinese|want) = .0065
- P(to|want) = .66
- P(eat | to) = .28
- P(food | to) = 0
- P(want | spend) = 0
- P (i | <s>) = .25

Practical Issues

Everything in log space

Avoids underflow

Practical Issues

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- Avoids underflow
- Adding is faster than multiplying

Practical Issues

Everything in log space

- Avoids underflow
- Adding is faster than multiplying

$$log(p_1 \times p_2 \times p_3 \times p_4) = logp_1 + logp_2 + logp_3 + logp_4$$

Handling zeros

Use smoothing

Language Modeling Toolkit

SRILM

http://www.speech.sri.com/projects/srilm/

Google N-grams

Number of tokens: 1,024,908,267,229 Number of sentences: 95,119,665,584

Number of unigrams: 13,588,391 Number of bigrams: 314,843,401 Number of trigrams: 977,069,902

Number of fourgrams: 1,313,818,354 Number of fivegrams: 1,176,470,663

http://googleresearch.blogspot.in/2006/08/

all-our-n-gram-are-belong-to-you.html

Example from the 4-gram data

serve as the inspector 66 serve as the inspiration 1390 serve as the installation 136 serve as the institute 187 serve as the institution 279 serve as the institutional 461

Google books Ngram Data

Google books Ngram Viewer

