CS 21201 Discrete Structures

20CS30023

Hardik Soni

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1. We have a C code below:

```
void now(int a, int b, int c)
{
    int r, s, t;
    while (1) {
    if ((a==b) || (b==c) || (c==a))
    break;
    r = (a+b)/2 ; s = (b+c)/2 ;
    t = (c+a)/2;
    a = r; b = s; c = t;
    }
}
```

To prove that the above C function/method terminates for all non-negative integer inputs a, b, c:

Note → The division by 2 are to considered for int variables.

(2) 65 integers are chosen from
$1, 2, 3, \ldots\ldots 2021$

To ~~P~~ Prove: there must exist four chosen integers such that:

$$a - b + c - d = \underbrace{2021k}_{\text{multiple of } 2021}$$

We know that factors of 2021 are:
$1, 43, 47$

To prove: ① By Example:

$a = 2021$
$b = 1$ $c = 2$
$d = 1$

$2021 - 1 + 2 - 1 = \underbrace{2021}_{\text{multiple}}$

② If we are choosing 65 integers from $1, 2, 3, \ldots\ldots 2021$

$(a + c) - (b + d) = 2021k$

Only possible multiple is 2021.

$(a + c) - (b + d) = 2021$

$= 45^2 - 2^2$

Can be represented as Product of Odd x Odd - Even x Even

(3) ρ and σ are two binary relations

$\rho \circ \sigma$ } a composite relation over A:

$$\rho \circ \sigma = \{(p,r) \mid \text{there exists some } q \in A \text{ such that } (p,q) \in \rho \text{ and } (q,r) \in \sigma\}$$

(a) If σ and ρ are equivalence relⁿ, then $\rho \circ \sigma$ is equivalence if $\boxed{\sigma \circ \rho = \rho \circ \sigma}$

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The condition implies:

$\sigma \circ \rho = \rho \circ \sigma$

$\rho \circ \sigma = \{(p, r) \mid$ there exists some $q \in A$ such that $(p, q) \in \rho$ and $(q, r) \in \sigma\}$

$\sigma \circ \rho = \{\cancel{q}(p, r) \mid$ ~~there exists some~~ $\cancel{q} \in A$ such that $(p, q)\ \cancel{(q, r)} \in \sigma$ and $\cancel{(r, p)}\ (q, r) \in \rho\}$

If σ is equivalence and ρ is equivalence:

$(p, p) \in \rho$

$(p, q) \in \rho \Rightarrow (q, p) \in \rho$

$(p, q), (q, r) \in \rho \Rightarrow (p, r) \in \rho$

If $\boxed{\rho \circ \sigma = \sigma \circ \rho}$ is true

then

$(p, r) \in$ range

and other properties are satisfied

Hence equivalence

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(4.) $\mathcal{P}(S)$ denotes the Power set
For a function: $f: X \to Y$

$$g: \mathcal{P}(A) \to \mathcal{P}(B)$$

$$h: \mathcal{P}(B) \to \mathcal{P}(A)$$

$g(A) = \{b \mid \exists a \in A, f(a) = b\}$ and
$h(B) = \{a \mid f(a) \in B\}$

for all $A \subseteq X$ and $B \subseteq Y$

To prove:

(a) f is injective and only if $h(g(A)) = A$ for all $A \subseteq X$

Note that,

$$g(A) = \{b \mid \exists a \in A, f(a) = b\}$$

$$g(A) = \bigcup_{a \in A} f(a)$$

This in layman terms would be understood as all those points to which f is getting mapped, i.e., range.

Also

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$h(B) = \{ a \mid f(a) \in B \}$

$$h(B) = \bigcup_{\substack{a \in A \\ B}} (f(a) \in B)$$

This in layman terms means Domain:

So for f to be injective:

for any two points in $f(S)$ like S_1 and S_2:

$$f(S_1) = f(S_2) \Leftrightarrow S_1 = S_2$$

It is said if:

$$h(g(A)) = A \text{ for all } A \subseteq X$$

~~then~~ f is one-one / injective:

Now if $g(A)$ represents range for function $f: X \to Y$ and $h(B)$ denotes all possible point in domain mapping ~~to~~ ~~set~~ B:

for condition:→

$$h(g(A)) = A \text{ for all } A \subseteq X$$

Means for every point in range we have a unique mapping from set A to it which says:

$\underbrace{g(A)}_{\text{range}} = \underbrace{h^{-1}(A)}_{\text{points in codomain with unique}}$

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Hence you can conclude
For all $A \subseteq X$

f is injective for all
$h(g(A)) = A$.

(b) f is surjective if and only if $g(h(B)) = B$ for all $B \subseteq Y$

Speaking furthermore we know:

If f is surjective, then we know that for all $b \in B$, there exists $a \in A$ such that $f(a) = b$. Therefore for all $S_2 \subseteq B$ there exists $S_1 \subseteq A$ such that $f(S_1) = S_2$.

It follows that if
$$g(h(B)) = B \quad \forall B \subseteq Y$$

$h(B)$: denotes all possible inputs of f when $f(a) \in B$

$g(A)$ :: denotes all possible outputs of f when $A \in \mathcal{P}(A)$.

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Now for every input $h(B)$ of f if we have a output in B we conclude:

for all

$s \in P(B)$ there exists $a : s \in P(A)$ for function f.

$\Rightarrow$ Hence you can conclude

For all $B \subseteq Y$ for all f is surjective

$$\boxed{g(h(B)) = B}$$