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20182002 3

Formal language and Automata Theory:

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1. $\sum = \{a, b\}$

$L = \{ u v w u^r \mid u, v, w \in \Sigma^+, |v| = |w| + 1 \}$

where u^r stands for reverse of string u $|x| \rightarrow$ length of string.

(a) Regular Expression for L: (i a)

~~$a(@ - \varepsilon)^2 \# a + b(@ - \varepsilon)^2 \# b$~~

where :→

ε is L^0.

Evaluation :→

$$\boxed{a @^2 \#^3 a + b @^2 \#^3 b}$$

or $(a \# (\#\#)^+ a) + (b \# (\#\#)^+ b)$

Proof: → for Regular expressions all string that satisfy the above r. exp can be $u v w u^*$

(take $u = u^* = a$ or $u = u^* = b$)

$|v| + |w| \in (2k+1)$

splitting

$\#(\#\#)^+$ into

$(l+1)$ (l)

lengths for:

u v

all strings of the form: $u v w u^*$ since u is either a or b other symbols of the form come under $\#(\#\#)^*$

$$\boxed{\#(\text{symbols}) = |v| + |w| + 2(|u| - 1)}$$

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$\boxed{b}$ In the above given ε-NFA
initial symbol:
$\text{symbol}_{init} = a, b$

For states 0 – 9
namely: $\{0,1,2,3,4,5,6,7,8,9\}$
we need minim three
symbols for:
$\{0,1,2,3,4,5,6,7\} 8 \Rightarrow \# \# \#$
however, there is ε- loop b/w
1 and 5:
pair b/w 1 and 7:
$= (\#\#)^+ \#$
Path to Follow $(1 \to 3 \to 5 \to 7)$

Similarly when we traverse
b/w paths $(2 \to 8)$:
$(2 \to 4 \to 6 \to 8)$

path $= (\#\#)^+ \#$

$\therefore$ path from $0 \to 1 \to 3 \to 5 \to 7 \to 9$:
$\Rightarrow a (\#\#)^+ \# a$

$\therefore$ for $0 \to 2 \to 4 \to 6 \to 8 \to 9$
$= b (\#\#)^+ \# b$

$0 \to q$

$$a(\#\#)^+ \#a + b(\#\#)^+ \#b$$

$= L$

0 is final state:

→ ε is accepted

Here also is a transition from $q \to 0$

∴ $\bigcup_{n \geq 0} L$ is also accepted

∴ L^* is a accepted language of the ε-NFA

(to reach final state
→ ε input
→ go to q and make ε-transition)

$\boxed{c}$ for $\Delta \rightarrow$ transition func.

ε-NFA :→

$\hat{\Delta}(0, aabbaba)$

we need to find all the reachable states :→

* $\hat{\Delta}(\{0\}, aabbaba)$

$\hat{\Delta}(\hat{\Delta}(\{0\}, a), abbaba)$

from ε-NFA : $\hat{\Delta}(\{0\}, a) = 1$

$\hat{\Delta}(\{1\}, abbaba)$

$\hat{\Delta}(\hat{\Delta}(\{1\}, a), bbaba)$

from ε-NFA: $\hat{\Delta}(\hat{\Delta}\{1\}, a) = \cancel{\{1,5\}}$ $\{3\}$

$\hat{\Delta}(\{3\}, bbaba)$

from ε-NFA: $\hat{\Delta}(\{3\}, b) = \{1,5\}$

$\cancel{\hat{\Delta}(\{3\},}$

$\hat{\Delta}(\{1,5\}, baba)$

from ε-NFA : $\hat{\Delta}(\{1,5\}, b\} = \{3,7\}$

$\hat{\Delta}(\{3,7\}, aba)$

from ε-NFA, $\hat{\Delta}(\{3,7\}, a) = \{1,5,0,9\}$

$\hat{\Delta}(\{0,1,5,9\}, ba)$

from ε-NFA, $\hat{\Delta}(\{0,1,5,9\}, b) = \{2,3,7\}$

$\hat{\Delta}(\{2,3,7\}, a)$

from ε-NFA: $\hat{\Delta}(\{2,3,7\}, a)$

$= \{0,1,4,5,9\}$

$\Rightarrow \hat{\delta}(0, aabbaba)$

$= \hat{\delta}(\{0, 1, 4, 5, 9\}, \varepsilon)$

where $\varepsilon \to$ 'L' (empty string)
Thus, we have the final states for string "aababa" are

$$\boxed{\{0, 1, 4, 5, 9\}}$$

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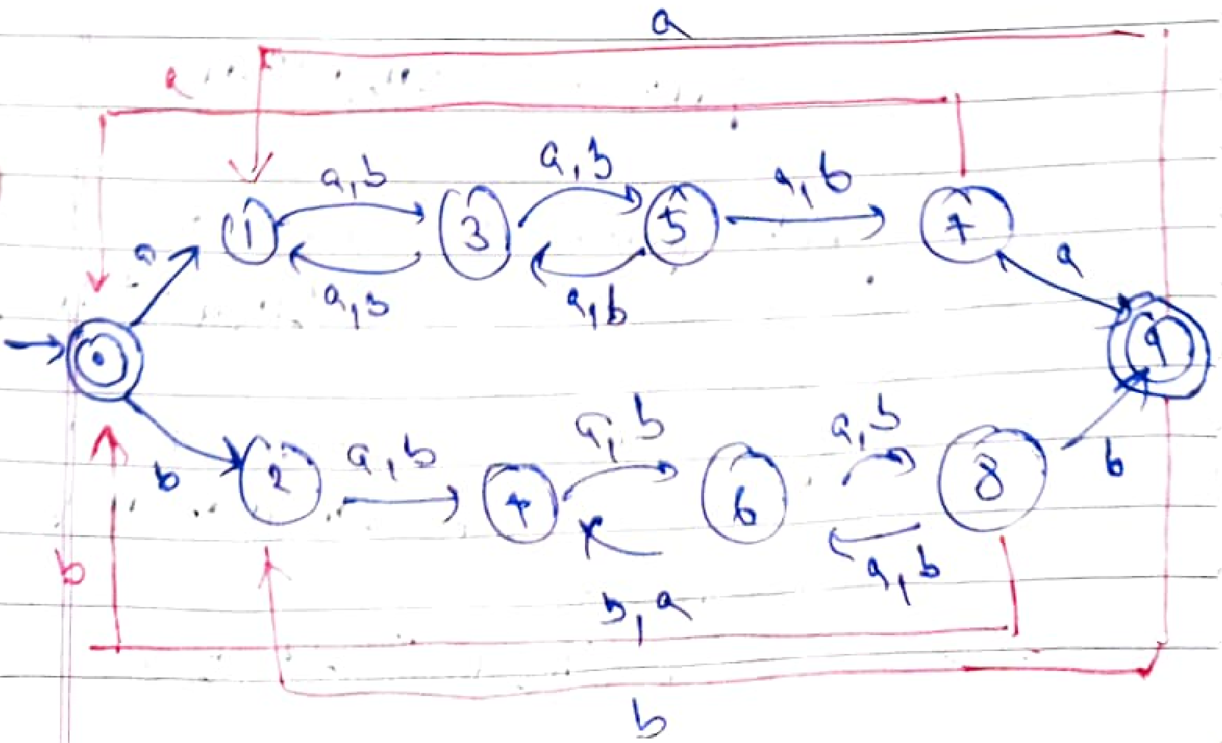
d We have here a question to convert ε-NFA to equivalent NFA:
For letter/alphabet 'a':

	ε*	a	ε*
0	0	1	1
1	1	3	3
2	2	4	4
3	3	5	1,5
4	4	6	6
5	1,5	3,7	3,7
6	6	8	4,8
7	7	9	9
8	4,8	6,ϕ	6,ϕ
9	0,9	1,ϕ	1,ϕ

Here we have implemented the ε-closure algorithm:

We started from every possible state for every letter as input and then found ε-closure for every of them and then again took ε-closure then it is again implemented for b.

	ε*	b	ε*
0	0	2	2
1	1	3	3
2	3	5	1,5
3	4	6	6
4	1,5	3,7	3,7
6	6	8	4,8
7	7	φ	φ
8	4,8	6,9	6,9
5	1,5	3,7	3,7
9	0,9	2,φ	2,φ



2. (a) L be a language over Σ :

from L we generate

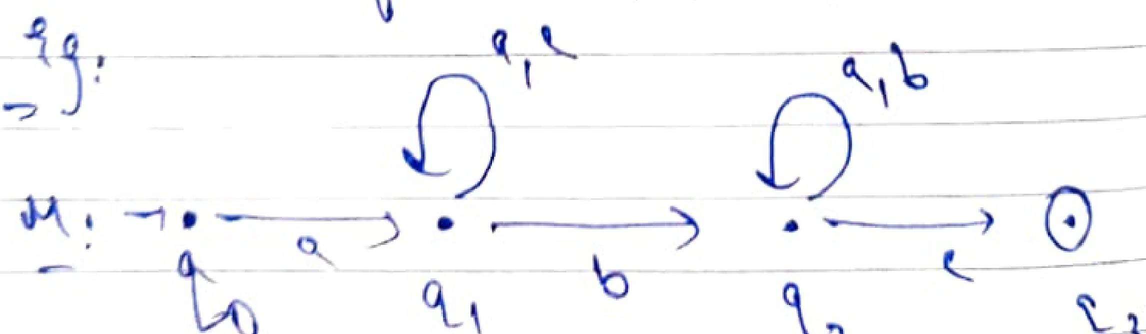
dupPrefix(L) = $\{ xy \mid y \in L$, and x is a prefix of $y \}$

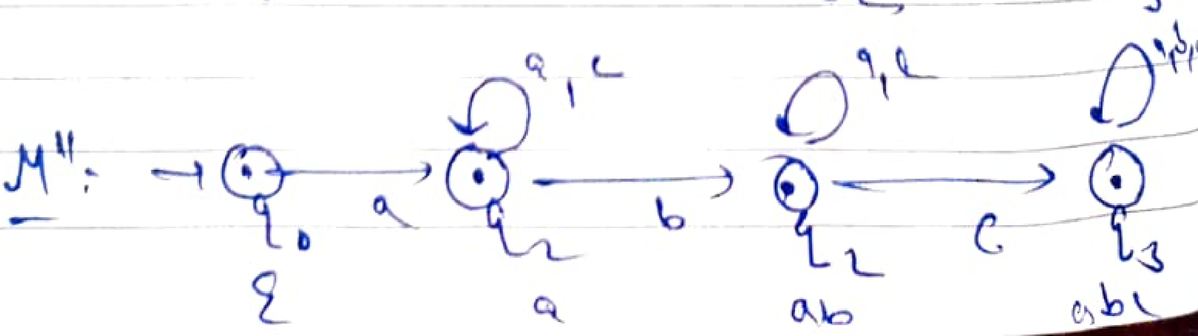
(a) To prove, if L is regular, then dupPrefix(L) must also be regular:

If $y \in L$ and L given a regular language, then $\exists$ a DFA (M) that accepts L.

We can extract M'' from M that accepts all leading prefixes of L. This can be done by making all states of M as final (F) for M''.

Eg:





Now, we can further modify M'' to M''' that accepts dupPrefix(L).

To do this we will create a copy of M'' and add to final states of M''. A M'' (DFA) accepts prefix of L and then M'' would accept dupPrefix(L).