Image Transforms

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$$f(x,y) = \sum_j \sum_i \lambda_{ij} b_{ij}(x,y)$$

Image Transform

- Image in continuous form: $f(x,y)$: A 2-D function, where (x,y) in R^2.
- Let B be a set of basis functions:

Properties of basis functions can be extended in the analysis.

$$B=\{b_i(x,y) \mid i=..,-1,0,1,2,3,\ldots.\},\ b_i(x,y) \text{ in } R \text{ or } C.$$

- Let $f(x,y)$ be expanded using B as follows:

$$f(x,y) = \sum_i \lambda_i b_i(x,y)$$

Coefficients of transform

The **transform** of f w.r.t. B is given by $\{\lambda_i | i = \cdots.-1,0,1,2,3,\ldots\}$.

Indexing may be multidimensional say, λ_{ij}.

Orthogonal Expansion and 1-D Transforms

$$f(x) = \sum_i \lambda_i b_i(x)$$

- Inner product: $< f, g >= \int f(x) g^*(x) dx$
- Orthogonal expansion: If B satisfies :

$$\langle b_i, b_j \rangle = 0,\ for\ i \neq j$$
$$= c_i\ Otherwise\ (for\ i = j), where\ c_i > 0$$

- Transform coefficients in O.E.: $\lambda_i = \frac{1}{c_i} \langle f, b_i \rangle$
- If c_i=1, it becomes orthonormal expansion.

Forward transform $\lambda_i = \langle f, b_i \rangle$

- Inverse transform: $f(x) = \int_{i=-\propto}^{\propto} \lambda_i b_i(x) di$

Fourier transform

Complete base

$$B = \{e^{-j\omega x} | -\infty < \omega < \infty\}$$

Unit impulse function

Orthogonality: $\int_{-\infty}^{\infty} e^{j\omega x} dx = \begin{cases} 2\pi\delta(x), & for\ \omega = 0 \\ 0, & otherwise. \end{cases}$

Fourier Transform: $\mathcal{F}(f(x)) = \hat{f}(j\omega) = \int_{-\infty}^{\infty} f(x) e^{-j\omega x} dx$

Inverse Transform: $f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(j\omega) e^{j\omega x} dx$

Full reconstruction

$$e^{-j\omega x} = \cos(\omega x) - j\sin(\omega x)$$

$$\hat{f}(j\omega) = \int_{-\infty}^{\infty} f(x)(\cos(\omega x) - j\sin(\omega x))\, dx$$

$C = \{\cos(\omega x) | -\infty < \omega < \infty\}$ $\quad$ $S = \{\sin(\omega x) | -\infty < \omega < \infty\}$

Orthogonal $\quad$ But not complete!

Even and odd functions

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

- Even: $f(-x)=f(x)$ for all x.
- Odd: $f(-x)=-f(x)$ for all x. $\rightarrow f(0)=0$.
- For even $f(x)$: $\int_{-\infty}^{\infty} f(x)(\sin(\omega x))\,dx = 0$
- For odd $f(x)$: $\int_{-\infty}^{\infty} f(x)(\cos(\omega x))\,dx = 0$
- Full reconstruction possible with cosines (sines) only if it is even (odd).

Discrete representation

- Discrete representation of a function:

$$f(n) = \{f\ (nX_0)|n\epsilon\mathbb{Z}\}$$

Set of integers

Sampling interval

- Can be considered as a vector in an infinite dimensional vector space.
- In our context, it is of a finite dimensional space, e.g. $\{f(n),\ n=0,1,..N\text{-}1\}$, or
- $\boldsymbol{f}=[f(0)\ f(1)\ \dots\ f(N\text{-}1)]^T$.

Discrete Linear Transform: A general form

- For n-dimensional vector X any linear transform,
  - e.g. $Y_{mx1}=B_{mxn}X_{nx1}$
  - X_{nx1}: *A column vector of dimension n.*
  - Y_{mx1}: *A column vector of dimension m.*
  - B_{mxn}: *A matrix of dimension mxn.*
- Has inverse transform if B is a square matrix and invertible.

Basis vectors

- B is the transformation matrix.
- Rows of B are called basis vectors.

$$B = \begin{bmatrix} \boldsymbol{b}_0^{*T} \\ \boldsymbol{b}_1^{*T} \\ \vdots \\ \boldsymbol{b}_n^{*T} \end{bmatrix}$$

- $Y(i) = <\boldsymbol{b}_i^{*T}.\, X>$

dot product or inner product.

- Orthogonality condition:

$$< \boldsymbol{b}_i^{*T} . \boldsymbol{b}_j > = 0 \; if \; i \neq j$$
$$= \; c_i, \qquad otherwise$$

Discrete Fourier Transform (DFT)

$$b_k(n) = \frac{1}{\sqrt{N}} e^{j2\pi\frac{K}{N}n}, \ \text{ for } 0 \le n \le N-1, and\ 0 \le k \le N-1$$

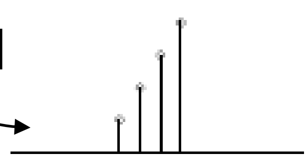
$$\hat{f}(k) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi\frac{k}{N}n}, for\ 0 \le k \le N-1$$

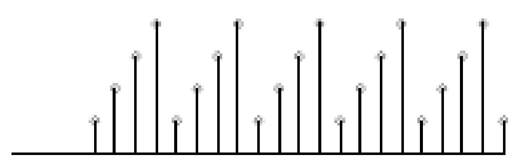
$$\hat{f}(N+k) = \hat{f}(k)$$

k/N: Normalized frequency

$$f(n) = \frac{1}{N}\sum_{k=0}^{N-1} \hat{f}(k) e^{j2\pi\frac{K}{N}n}, for\ 0 \le n \le N-1$$

A single period





Fundamental frequency: $1/(NX_0)$

$f(n+N)=f(n)$

DFT: Fourier series of a periodic function

$F(k) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi\frac{k}{N}n}, for\ 0 \le k \le N-1$

DFT: A linear transform

$$\begin{bmatrix} F(0) \\ F(1) \\ \vdots \\ F(N-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi\frac{1}{N}} & \cdots & e^{-j2\pi\frac{N-1}{N}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi\frac{N-1}{N}} & \cdots & e^{-j2\pi\frac{(N-1)^2}{N}} \end{bmatrix} \begin{bmatrix} f(0) \\ f(1) \\ \vdots \\ f(N-1) \end{bmatrix}$$

$$\mathcal{F}_N = \left[e^{-j2\pi\frac{k}{N}n}\right]_{0 \le (k,n) \le N-1}$$

$$\boldsymbol{F} = \mathcal{F}_N \boldsymbol{f}$$

$$\boldsymbol{f} = \mathcal{F}_N^{-1} \boldsymbol{F}$$

Hermitian transpose

$$\mathcal{F}_N^{-1} = \mathcal{F}_N^{H}$$

Generalized Discrete Fourier Transform (GDFT)

$$b_k^{(\alpha,\beta)}(n) = \frac{1}{\sqrt{N}} e^{j2\pi\frac{k+\alpha}{N}(n+\beta)}, for\ 0 \le n \le N-1, and\ 0 \le k \le N-1$$

$$\hat{f}_{\alpha,\beta}(k) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi\frac{k+\alpha}{N}(n+\beta)}, for\ 0 \le k \le N-1$$

$$f(n) = \frac{1}{N} \sum_{k=0}^{N-1} \hat{f}_{(\alpha,\beta)}(k) e^{j2\pi\frac{k+\alpha}{N}(n+\beta)}, for\ 0 \le n \le N-1$$

α=0, β=0: Discrete Fourier Transform (DFT)

α=0, β=1/2: Odd Time Discrete Fourier Transform (OTDFT)

α=1/2, β=0: Odd Frequency Discrete Fourier Transform (OFDFT)

α=1/2, β=1/2: Odd Frequency Odd Time Discrete Fourier Transform (O^2DFT)

GDFT: Inverse Transforms

Transformation matrix $\boldsymbol{F}_{\alpha,\beta} = \left[e^{-j2\pi\frac{k+\alpha}{N}(n+\beta)}\right]_{0\le(k,n)\le N-1}$

Relationships of inverse transforms

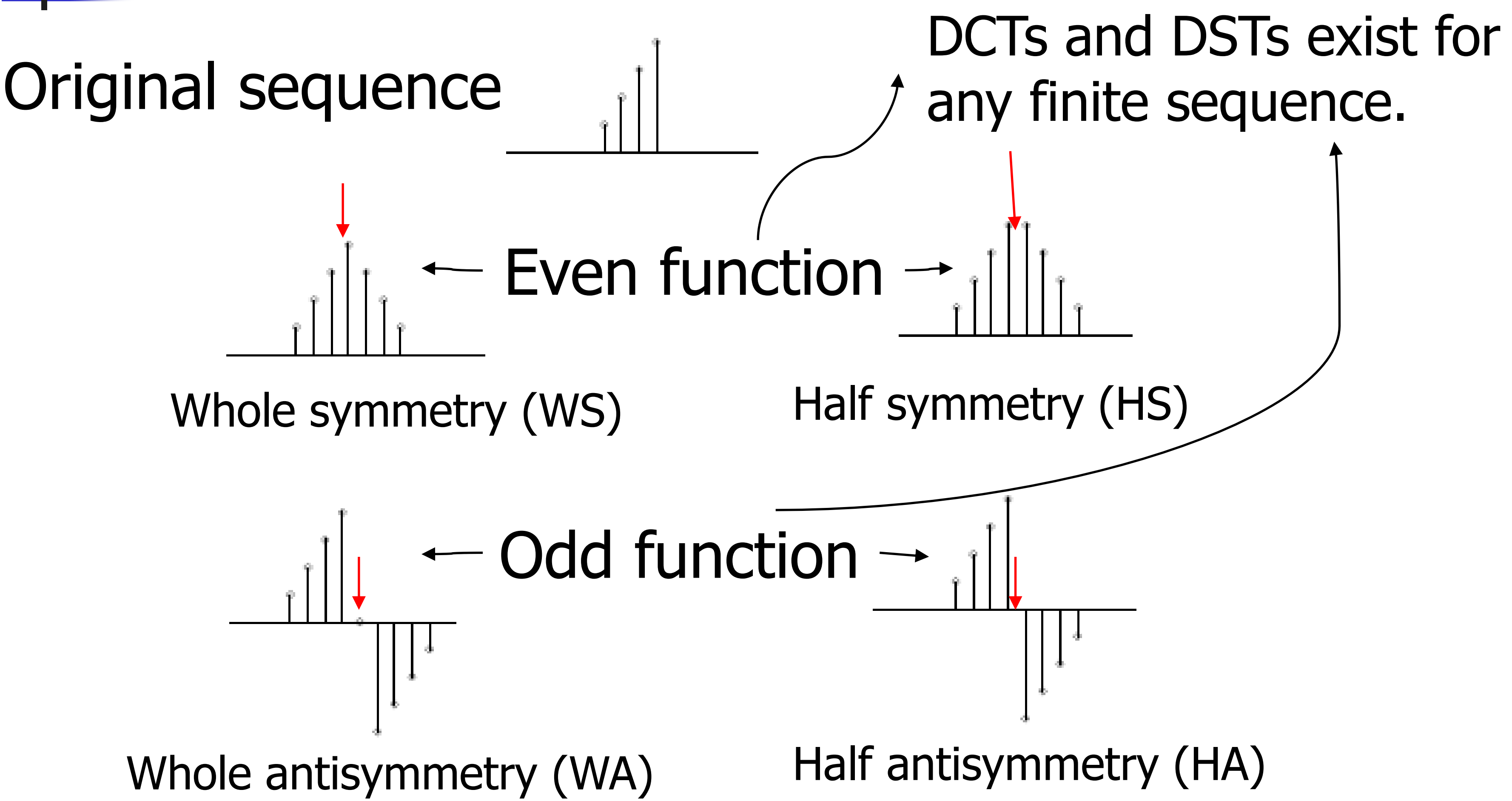
$$\boldsymbol{F}_{0,0}^{-1} = \frac{1}{N}\boldsymbol{F}_{0,0}^{H} = \frac{1}{N}\boldsymbol{F}_{0,0}^{*}$$

$$\boldsymbol{F}_{\frac{1}{2},0}^{-1} = \frac{1}{N}\boldsymbol{F}_{\frac{1}{2},0}^{H} = \frac{1}{N}\boldsymbol{F}^{*}_{0,\frac{1}{2}}$$

$$\boldsymbol{F}_{0,\frac{1}{2}}^{-1} = \frac{1}{N}\boldsymbol{F}^{H}_{0,\frac{1}{2}} = \frac{1}{N}\boldsymbol{F}_{\frac{1}{2},0}^{*}$$

$$\boldsymbol{F}_{\frac{1}{2},\frac{1}{2}}^{-1} = \frac{1}{N}\boldsymbol{F}_{\frac{1}{2},\frac{1}{2}}^{H} = \frac{1}{N}\boldsymbol{F}_{\frac{1}{2},\frac{1}{2}}^{*}$$

Symmetric / Antisymmetric extension of a finite sequence

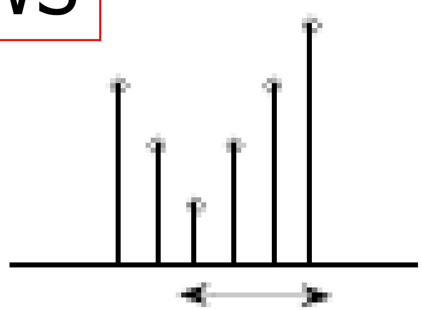


Discrete Cosine / Sine Transforms

$$\alpha(p) = \begin{cases} \sqrt{\frac{1}{2}} & p = 0 \text{ or } N \\ 1 & \text{Otherwise} \end{cases}$$

- Types of symmetric / antisymmetric extensions at the two ends of a sequence and a type of GDFT→ DCTs / DSTs

WSWS



$F_{0,0}$

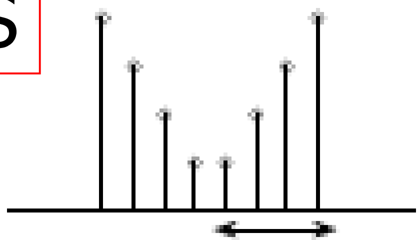
Type-I Even DCT

$$C_{1e}\big(x(n)\big) = X_{Ie}(k) = \sqrt{\frac{2}{N}}\,\alpha^2(k)\sum_{n=0}^{N} x(n)\cos\left(\frac{2\pi kn}{2N}\right), 0 \le k \le N$$

Discrete Cosine / Sine Transforms

$$\alpha(p) = \begin{cases} \sqrt{\frac{1}{2}} & p = 0 \text{ or } N \\ 1 & Otherwise \end{cases}$$

HSHS

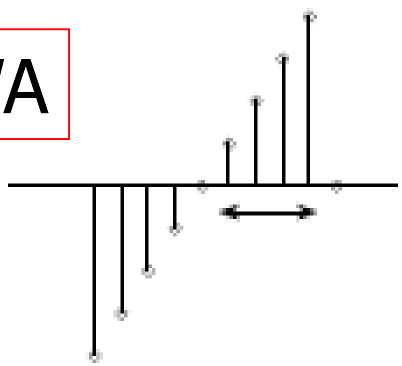


$F_{0,1/2}$

Type-2 Even DCT

$$C_{2e}(x(n)) = X_{IIe}(k) = \sqrt{\frac{2}{N}}\alpha(k)\sum_{n=0}^{N-1} x(n)\cos\left(\frac{2\pi k(n+\frac{1}{2})}{2N}\right), 0 \le k \le N\text{-}1$$

WAWA



$jF_{0,0}$

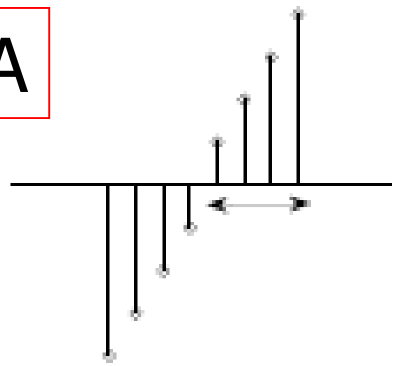
Type-1 Even DST

$$S_{1e}(x(n)) = X_{sIe}(k) = \sqrt{\frac{2}{N}}\sum_{n=0}^{N} x(n)\sin\left(\frac{2\pi kn}{2N}\right), 1 \le k \le N-1$$

Discrete Cosine / Sine Transforms

$$\alpha(p) = \begin{cases} \sqrt{\frac{1}{2}} & p = 0 \ or \ N \\ 1 & Otherwise \end{cases}$$

HAHA



$jF_{0,1/2}$

Type-2 Even DST

$$S_{2e}\big(x(n)\big) = X_{sIIe}(k) = \sqrt{\frac{2}{N}}\,\alpha(k)\sum_{n=0}^{N-1} x(n)\sin\left(\frac{2\pi k(n+\frac{1}{2})}{2N}\right), 1 \le k \le N\text{-}1$$

There exist 16 different types of DCTs and DSTs. Type-II Even DCT is used in signal, image, and video compression.

Matrix form of Type-II DCT

$$\alpha(p) = \begin{cases} \sqrt{\frac{1}{2}} & p = 0 \text{ or } N \\ 1 & Otherwise \end{cases}$$

- Matrix form:

N-point DCT

$$C_N = \left[\sqrt{\frac{2}{N}} \alpha(k) \cos(\frac{\pi k(2n+1)}{2N}) \right]_{0 \le (k,n) \le N-1}$$

$$\mathrm{X} = C_N \mathrm{x}$$

- Each row is either symmetric (even row) or antisymmetric (odd row).

$$C_N(k, N-1-n) = \begin{cases} C_N(k,n) & for\ k\ even \\ -C_N(k,n) & for\ k\ odd \end{cases}$$

$$C_N^{-1} = C_N^T$$

Convolution Multiplication Property (CMP)

$$f(t) \longrightarrow \boxed{\;h(t)\;} \longrightarrow \int_{-\infty}^{\infty} f(\tau)h(t-\tau)\,d\tau$$

$$F(j\omega) \longrightarrow \boxed{\;H(j\omega)\;} \longrightarrow H(j\omega)F(j\omega)$$

CMP for Fourier Transform

$$\widehat{f \circledast h}(k) = \hat{f}(k)\hat{h}(k)$$

CMP for DFT

CMP for DFT holds for circular convolution.

$\boxed{\text{Linear convolution}}$

$$f(n) \longrightarrow \boxed{h(n)} \longrightarrow \sum_{m=-\infty}^{\infty} f(m)\, h(n-m)$$

- Periodic convolution: Convolution between two finite sequences with periodic extension.
- It is defined if both have the same period, providing a periodic sequence with the same period.

$\boxed{\text{Circular Convolution}}$

$$f \circledast h(n) = \sum_{m=0}^{N-1} f(m)\, h(n-m)$$

$$\Rightarrow \sum_{m=0}^{n} f(m)\, h(n-m) + \sum_{m=n+1}^{N-1} f(m)\, h(n-m+N)$$

Antiperiodic extension and skew-circular convolution

- Antiperiodic function with an antiperiod N,
  - if $f(x+N)=-f(x)$.
- An antiperiodic function of antiperiod N
  - a periodic function of period $2N$.
- Skew-circular convolution: convolution between two antiperiodic extended sequences of the same antiperiod.

$$f \circledcirc h(n) = \sum_{m=0}^{N-1} f(m)\, h(n-m)$$

$$\Downarrow$$

$$\sum_{m=0}^{n} f(m)\, h(n-m) - \sum_{m=n+1}^{N-1} f(m)\, h(n-m+N)$$

$$u(n) = x(n) \circledast h(n)$$

$$w(n) = x(n) \circledcirc h(n)$$

CMPs for DCTs

$$C_{1e}\big(u(n)\big) = \sqrt{2N}C_{1e}(x(n))C_{1e}(h(n))$$

$$C_{2e}\big(u(n)\big) = \sqrt{2N}C_{2e}(x(n))C_{1e}(h(n))$$

$$C_{3e}\big(w(n)\big) = \sqrt{2N}C_{3e}(x(n))C_{3e}(h(n))$$

$$f(x,y) = \sum_j \sum_i \lambda_{ij} b_{ij}(x,y)$$

2-D Transforms

- Easily extendable if basis functions are separable, i.e. $B=\{ b_{ij}(x,y)=g_i(x).g_j(y)\}$.

They could be from two different sets, say $b(x,y)=g(x).h(y)$.

1-D basis function

- B: Orthogonal if $G=\{g_i(x), i=1,2,..\}$ is orthogonal.
- B: Orthogonal and complete if G is so.
- Reuse of 1-D transform computation.

$$\lambda_{ij} = \sum_j g_j^*(y) \left(\sum_i f(x,y) g_i^*(x) \right)$$

2D Discrete Transform

$$Y_{mxn} = B_{mxm} X_{mxn} B_{nxn}^{\mathrm{T}}$$

- Use of separability:
  - Transform columns.
  - Transform rows.
- Input: X_{mxn} 1-D Transform Matrix: *B*
- Transform columns: $[Y_1]_{mxn} = B_{mxm} X_{mxn}$
- Transform rows: $Y_{mxn} = [B_{nxn} Y_1^{\mathrm{T}}]^{\mathrm{T}}$

$$= Y_1 B_{nxn}^{\mathrm{T}}$$

$$= B_{mxm} X_{mxn} B_{nxn}^{\mathrm{T}}$$

Image Transform: DFT

Image: $f(m,n)$, of size M x N

$$F(k,l)=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f(m,n)e^{-j2\pi\frac{km}{M}}e^{-j2\pi\frac{ln}{N}}$$

$$f(m,n)=\sum_{k=0}^{M-1}\sum_{l=0}^{N-1}F(k,l)e^{j2\pi\frac{km}{M}}e^{j2\pi\frac{ln}{N}}$$

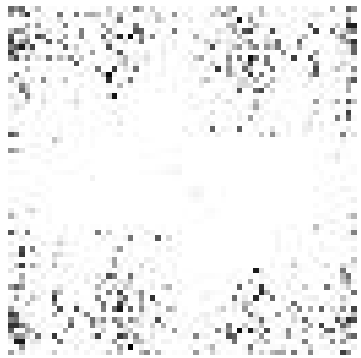
Property of separability

$$\boldsymbol{F}=\mathcal{F}_m\boldsymbol{f}\mathcal{F}_N^T$$

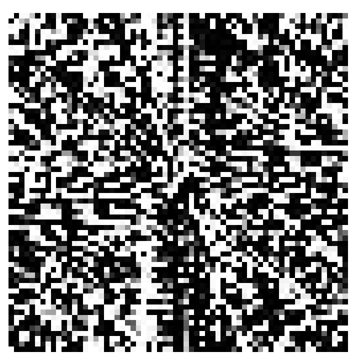
$$F(k,l)=\sum_{m=0}^{M-1}e^{-j2\pi\frac{km}{M}}\sum_{n=0}^{N-1}f(m,n)e^{-j2\pi\frac{ln}{N}}$$

DFT Examples:

Magnitude



Phase



Magnitudes and phases are shown by bringing them into displayable range, and shifting the origin at the center of image.

2D DCT

$$\alpha(p) = \begin{cases} \sqrt{\dfrac{1}{2}} & p = 0 \ or \ N \\ 1 & Otherwise \end{cases}$$

- Type-I:

$$X_{Ie}(k,l) = \frac{2}{N}\alpha^2(k)\alpha^2(l)\sum_{m=0}^{M}\sum_{n=0}^{N}x(m,n)\cos\left(\frac{\pi km}{M}\right)\cos\left(\frac{\pi kn}{N}\right),$$

$$0 \le k \le M,\ 0 \le l \le N$$

- Type-II

$$X_{IIe}(k,l) = \frac{2}{N}\alpha(k)\alpha(l)\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}x(m,n)\cos\left(\frac{\pi k(2m+1)}{2M}\right)\cos\left(\frac{\pi k(2n+1)}{2N}\right),$$

$$0 \le k \le M\text{-}1,\ 0 \le l \le N\text{-}1$$

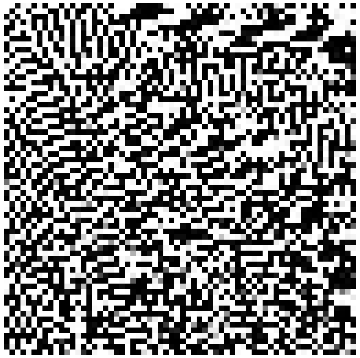
- Matrix Representation:

$$\boldsymbol{X} = C_M \mathbf{x} C_N^T$$

An example:

Input image

Discrete Cosine Transform

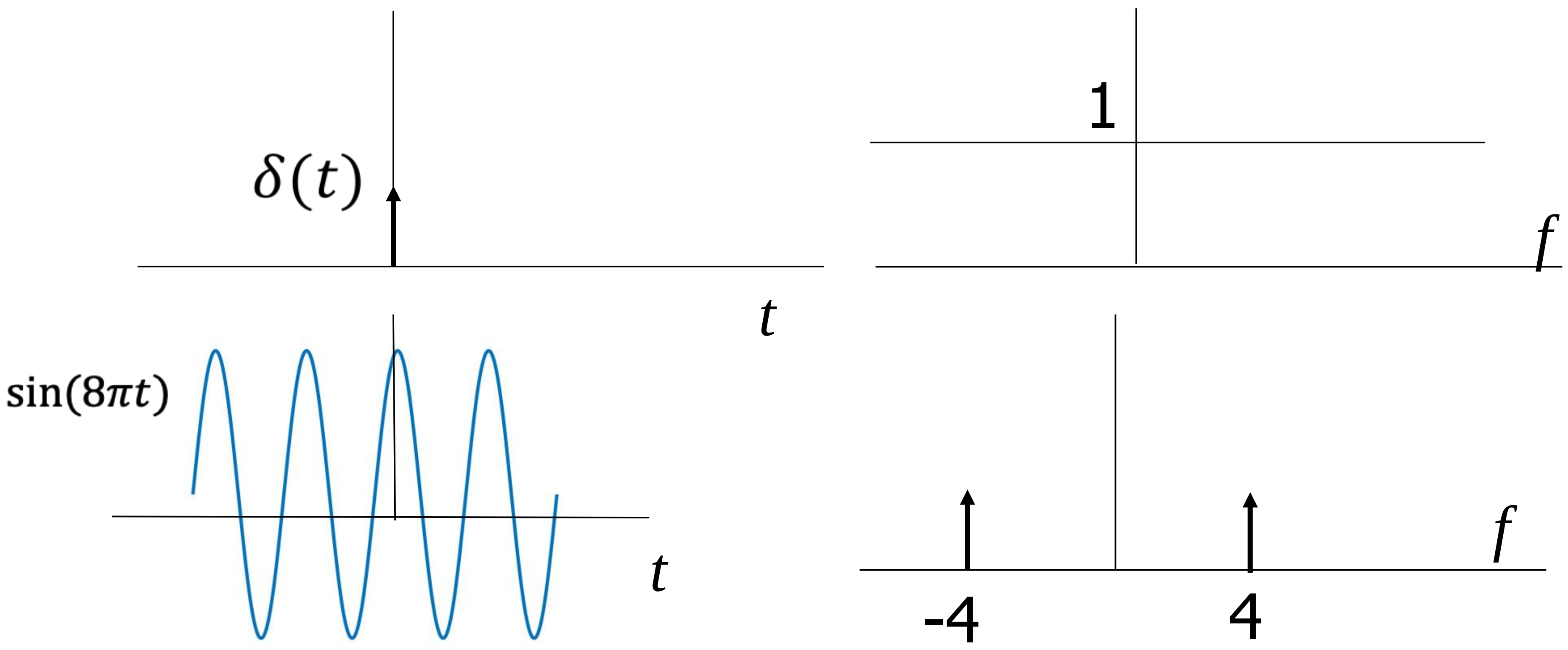


There are 16 different types of DCTs and DSTs.

Wavelets

- Functions to have *ideally* finite support in both its original domain (say, time or space) and also in the transform domain (i.e., the frequency domain).
  - No such function exists truly satisfying it.
  - Attempts to match these properties as far as possible.
- Acts as basis functions.
- Good localization property in both domain.

A few examples

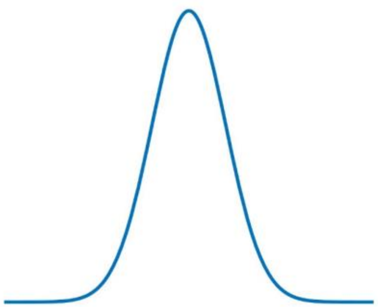


An interesting function

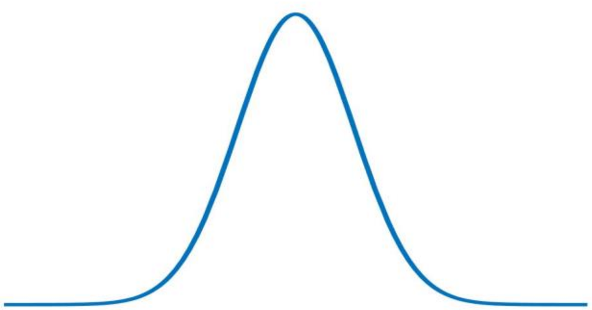
- Same form in time and frequency domain
  - Gaussian
- Analogy from Heisenberg's uncertainty principle

$$\sigma_t^2 \sigma_f^2 \geq \frac{1}{4}$$

Variance of t weighted by $g^2(t)$. Similarly for f. For any function it holds !!



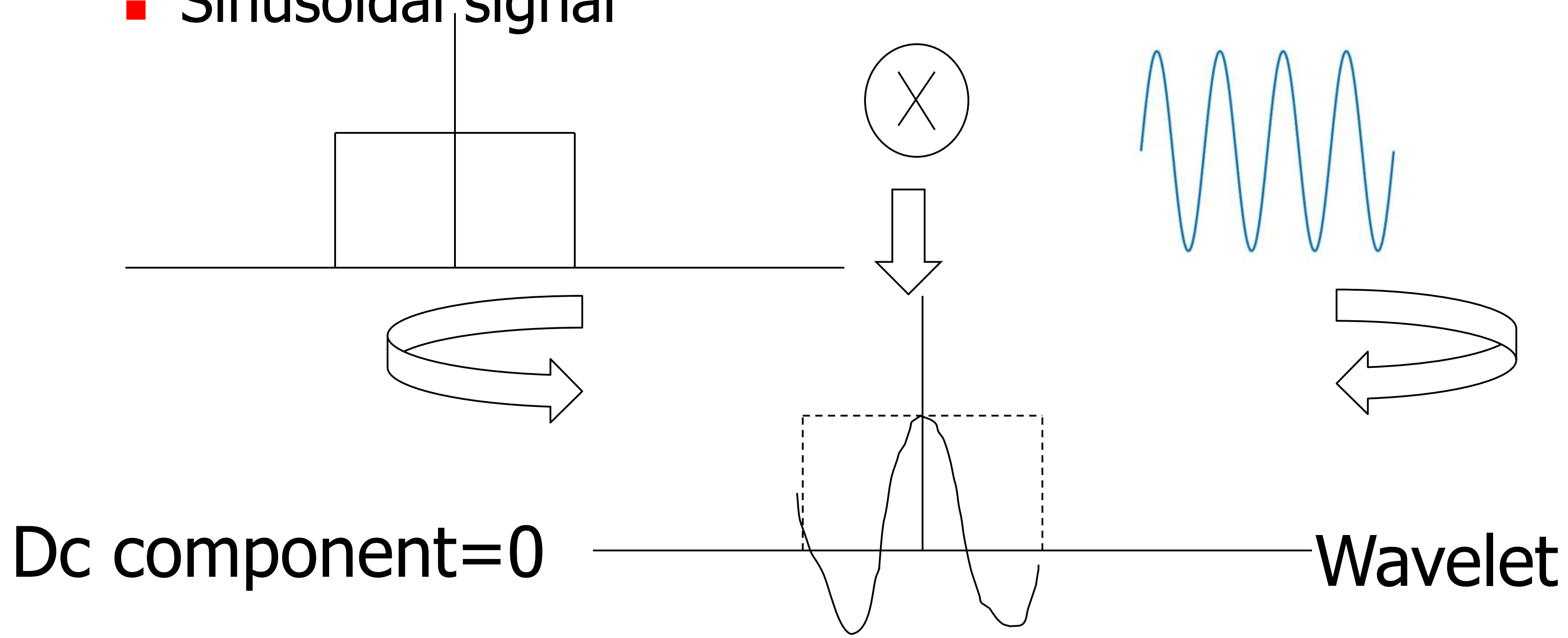
$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}}$$



$$G(\omega) = e^{-\frac{\omega^2\sigma^2}{2}}$$

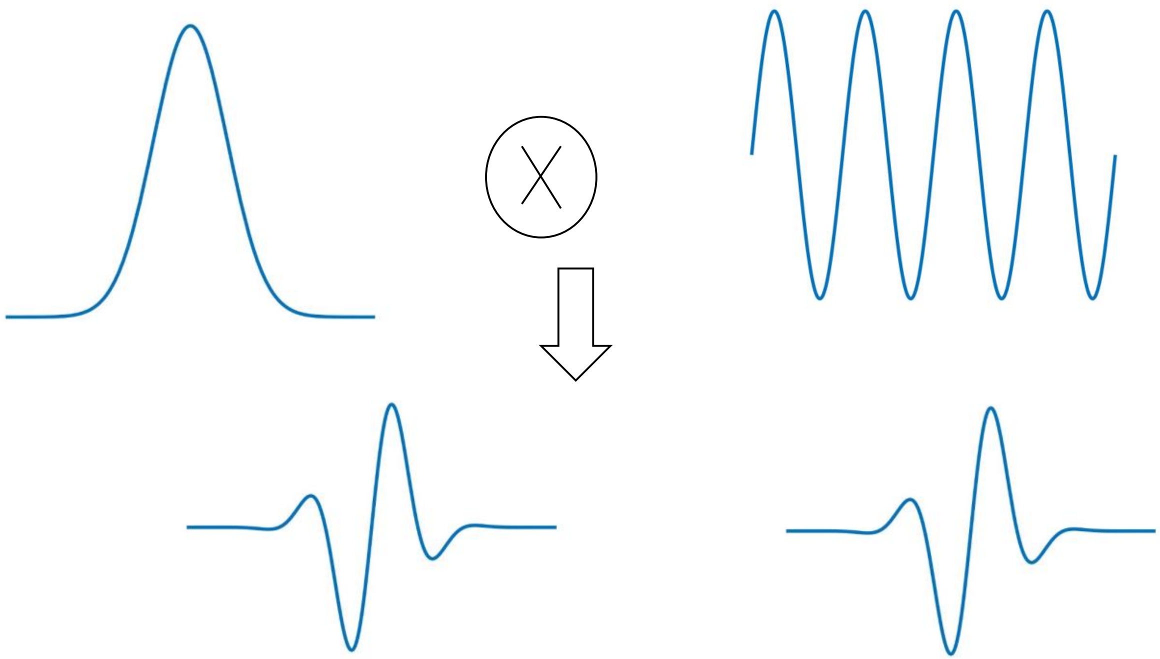
Designing wavelet: An intuitive approach

- Time limited signal:
  - Square pulse
- Band limited signal:
  - Sinusoidal signal
- Wavelet to satisfy both?
  - Multiply them!!



Gabor wavelet (1-D)

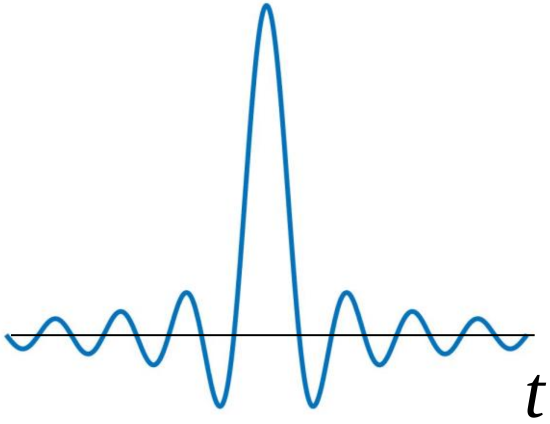
$$g(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{t^2}{2\sigma^2}} \qquad e^{j2\pi f t}$$

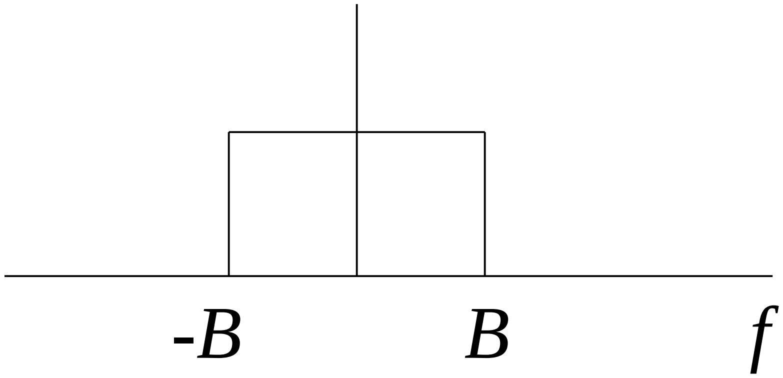


Real part Imaginary part

Shannon wavelet

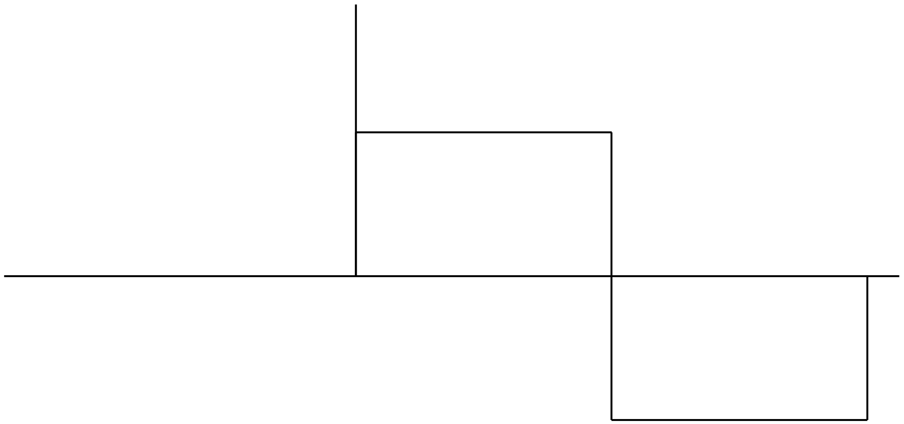
$$h(t) = 2B\frac{sin(2\pi Bt)}{2\pi Bt} = 2Bsinc(2Bt)$$





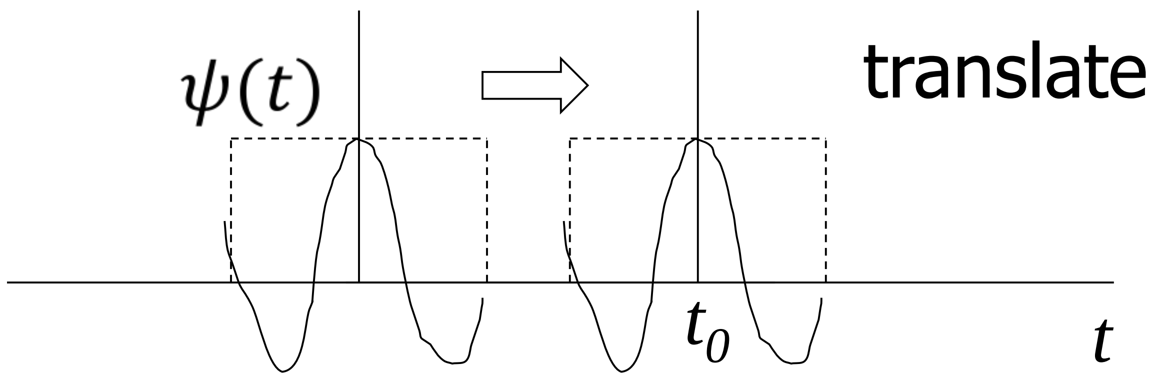
Haar Wavelet

$$\psi(t) = \begin{cases} 1, & 0 \leq t < \frac{1}{2} \\ -1 & \frac{1}{2} < t \leq 1 \\ 0 & Otherwise \end{cases}$$



Family of wavelets

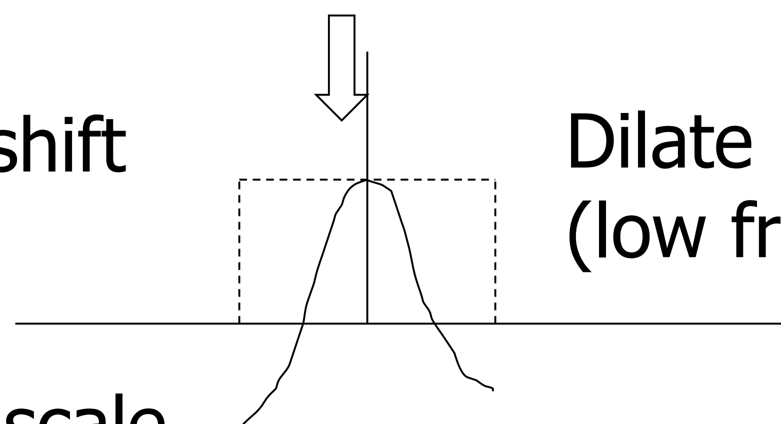
- Translate and dilate a mother wavelet



shift

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

scale



Continuous wavelet transform

- Forward transform

From 1-D representation to 2-D representation.

$$W(s,\tau) = \int f(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-\tau}{s} \right) dt$$

How correlated at that instance with the wavelet fn.

Reveals structure of function at multiple resolution.

- Inverse transform:

$$f(t) = \frac{1}{C_\psi} \int_0^\infty \int_{-\infty}^{\infty} W(s,\tau) \frac{1}{\sqrt{s}} \psi \left(\frac{t-\tau}{s} \right) d\tau \frac{ds}{s^2}$$

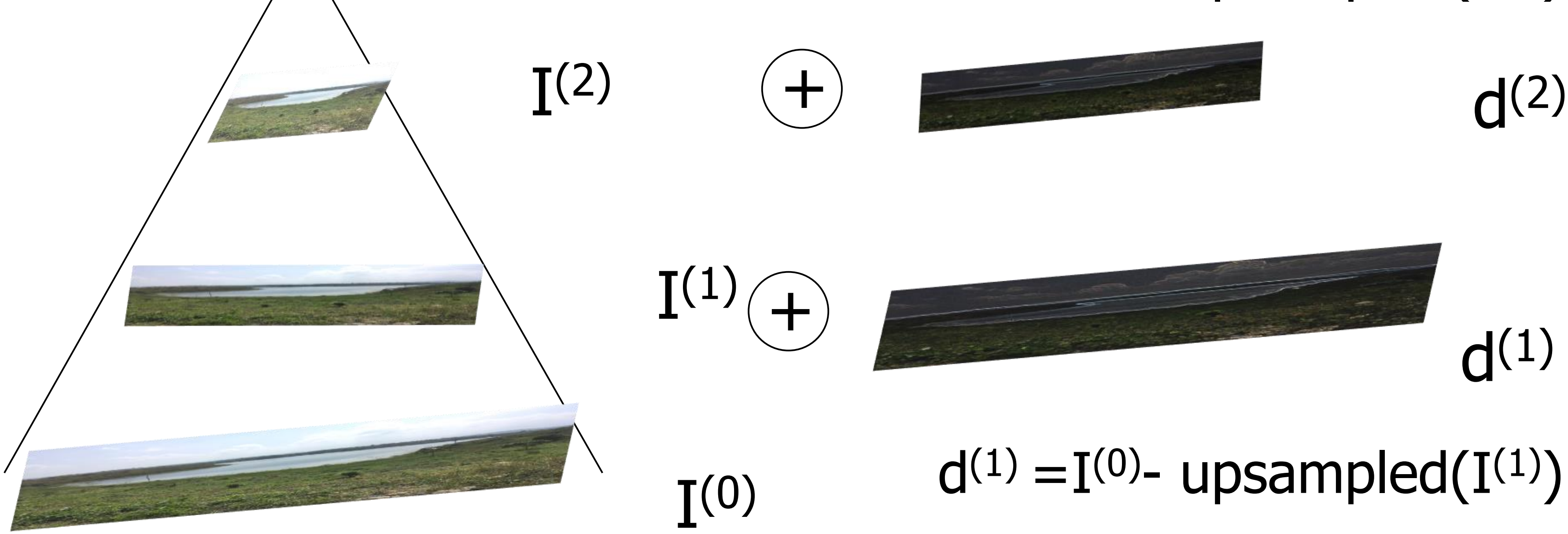
Fourier transform

where $C_\psi = \int_{-\infty}^{\infty} \frac{\left|\hat{\psi}(\omega)\right|^2}{|\omega|} d\omega$

Multiresolution representation

- Gaussian Pyramid

$d^{(2)} = I^{(1)} - \text{upsampled}(I^{(2)})$



$d^{(1)} = I^{(0)} - \text{upsampled}(I^{(1)})$

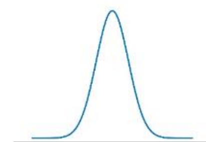
$I(0) = us(us\,(I^{(2)}) + d^{(2)}) + d^{(1)}$

Approximation   Details

$d^{(i)} = I^{(i-1)} - \text{upsampled}(I^{(i)})$

Gaussian Pyramid: Wavelet analysis

$I^{(2)}$

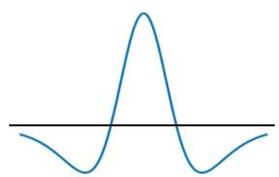


Obtained by convolution with G(x,y) and downsampling at successive stages.

Scaling function

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-((x-x_c)^2+(y-y_c)^2)}{2\sigma^2}}$$

$d^{(2)}$



Obtained by convolution with DOG(x,y) and downsampling at successive stages.

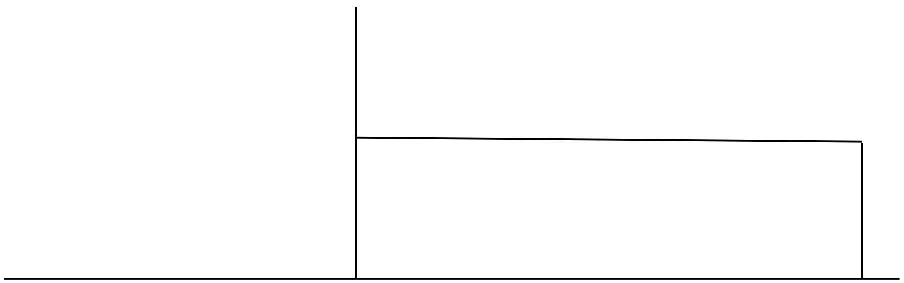
Wavelet function

$d^{(1)}$

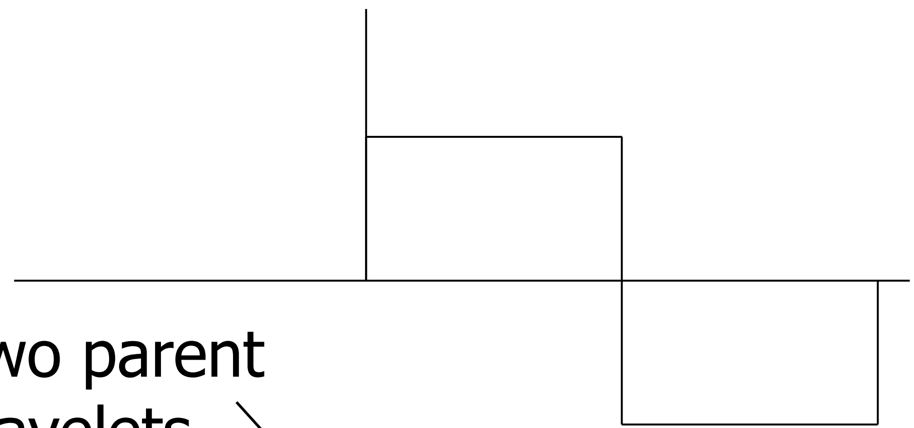
Filtering and transformation equivalent!!

Haar Wavelet transform

- Scaling function



- Wavelet function



Two parent wavelets

$$\varphi(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0 & Otherwise \end{cases}$$

$$\psi(t) = \begin{cases} 1, & 0 \leq t < \frac{1}{2} \\ -1 & \frac{1}{2} < t \leq 1 \\ 0 & Otherwise \end{cases}$$

Family of translated and dilated functions from the both forms the basis.

Discrete wavelet transform (DWT)

- Translated only at discrete grid points.
  - k=0, ± 1, ± 2, ...
  - Finite sequence: A finite number of basis functions.
- Scaled by powers of 2: 2^j, j=0,1,...
  - Downsampling takes care of dilation of wavelets and allows to use the same function at that level.
- Family of scaling and wavelet functions:

$$\varphi_{j,k}(n) = 2^{-\frac{j}{2}}\varphi\left(2^{-j}n - k\right),\ j = 0,1,\ldots k = 0,1,\ldots,M$$

$$\psi_{j,k}(n) = 2^{\frac{-j}{2}}\psi\left(2^{-j}n - k\right),\ j = 0,1,\ldots k = 0,1,\ldots,M$$

M≤N (length of sequence)

Haar wavelets in discrete grid

- N=8

$$\varphi(n) = \frac{1}{\sqrt{2}}(1,1,0,0,0,0,0,0)$$

$$\psi(n) = \frac{1}{\sqrt{2}}(1,-1,0,0,0,0,0,0)$$

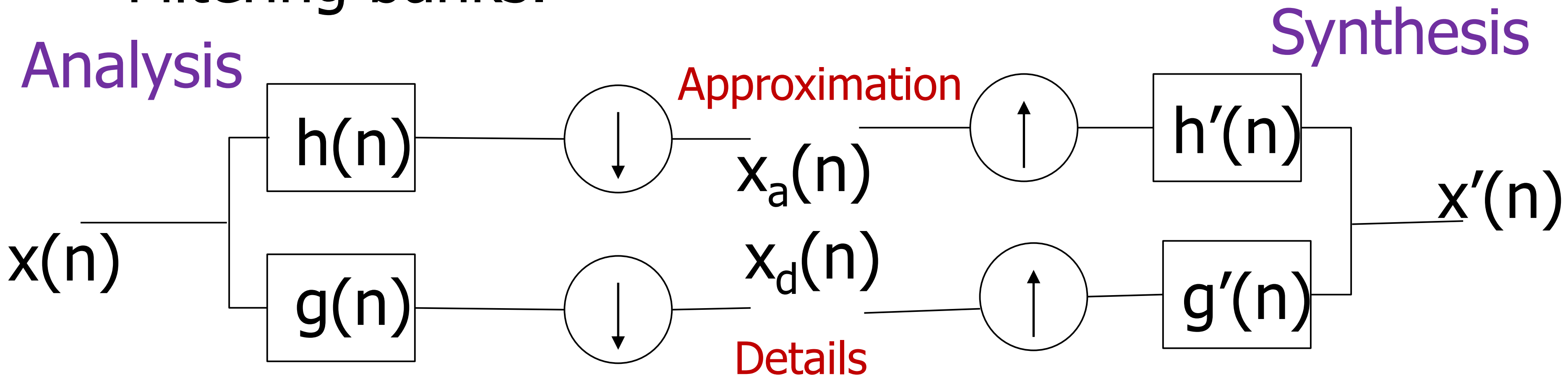
Transformation matrix:

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

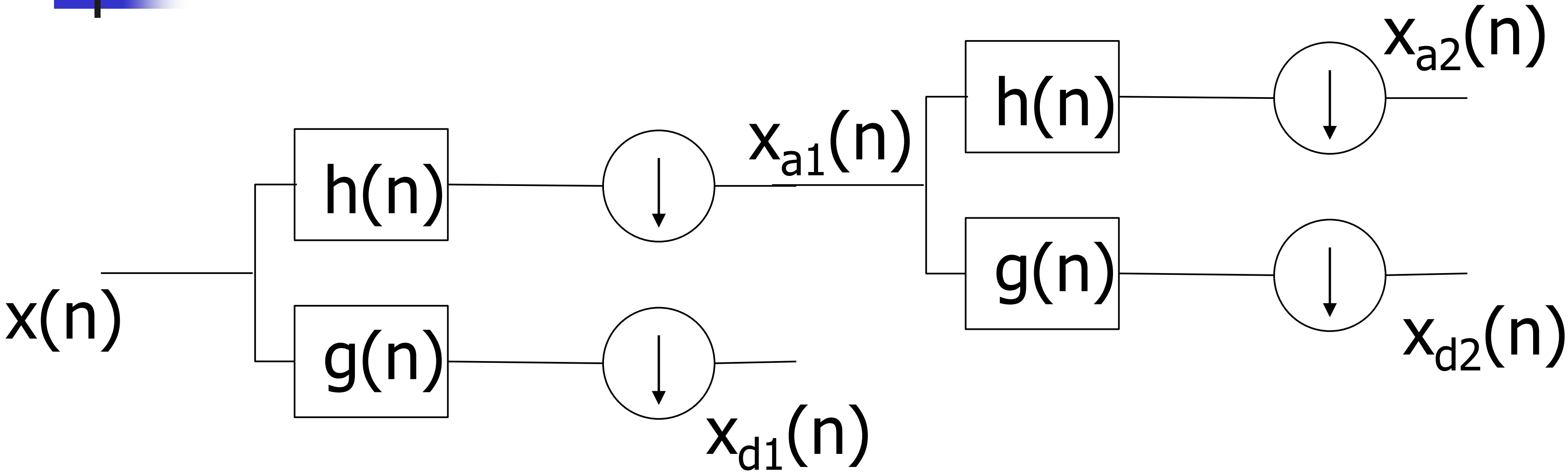
Courtesy: “Image and video processing in the compressed domain”, J. Mukhopadhyay, CRC Press, 2011.

DWT

- Translated only at discrete grid points.
- Scaled by powers of 2: 2^j, $j=0,1,\ldots$
  - Downsampling takes care of dilation of wavelets and allows to use the same function at that level.
  - Filtering by the filter of same impulse response.
- Filtering banks.



Dyadic decomposition



- At each level sample size is halved
  - Equivalent of scaling by 2.
- Total number of samples remain the same.

Typical wavelet filters

Daubechies 9/7 filters

n	Analysis Filter Bank		Synthesis Filter Bank	
	h(n)	g(n-1)	h′(n)	g′(n+1)
0	0.603	1.115	1.115	0.603
±1	0.267	-0.591	0.591	-0.267
±2	-0.078	-0.058	-0.058	-0.078
±3	-0.017	0.091	-0.091	0.017
±4	0.027			0.027

Le Gall 5/3 filters

n	Analysis Filter Bank		Synthesis Filter Bank	
	h(n)	g(n-1)	h′(n)	g′(n+1)
0	6/8	1	1	6/8
±1	2/8	-1/2	1/2	-2/8
±2	-1/8			-1/8

Courtesy: “Image and video processing in the compressed domain”, J. Mukhopadhyay, CRC Press, 2011.

2-D DWT

- Separable filters.
- Transform rows, then transform columns.

By 5/3 Analysis filters

Applications:

- Compression
- Denoising
- Feature representation
- Image fusion

…

Summary

- Alternative representation provides other insights of structure of images.
  - low frequency and high frequency components.
- May become useful for providing more compact representation.
  - A few transform coefficients.
  - Selective quantization of components, considering their effect on our perception.
    - Image compression.
- Sometimes convenient for processing.
  - Filtering, enhancement, ….

Summary

- Wavelets represent the scale of features in an image, as well as their positions.
  - Time-scale, Space-Scale representation
- Fast computation of forward and inverse transform
- Provides multiresolution representation.
  - Enables progressive and scalable processing
- Lossy and lossless reconstruction possible.
- useful for a number of applications including image compression.
  - JPEG2000

Thank you!