Linear Algebra in Artificial Intelligence and Machine Learning

Assignment 1

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Abstract

Insert your abstract text here. Approximately 250-300 words should do the trick!

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1 Introduction

1.1 Initial engagement

1.1.1 Introduction and background

Numerical methods are extensively used in numerical weather prediction (NWP) for finding approximate solutions to partial differential equations (PDEs), particularly since many of the equations being used do not have analytic solutions. These numerical methods use approximations of the function and its derivatives at discrete points in space, on what is known as a mesh. Since many features of the atmosphere are smaller than the resolution of the meshes commonly used in NWP, the weather models solved on these meshes are not usually sensitive to these features, as they will not be well resolved by the model.

Small scale weather features could be resolved better by increasing the resolution of the mesh. However, for models such as that currently in use by the Met Office, the resolution would have to be increased for the entire mesh (Met Office, 2015). This would be very computationally expensive, and the availability of computer resources therefore limits how well small scale weather features can be resolved. Further, much of this extra computational expense may be wasted in areas of space in which the increased resolution is not needed. Mesh adaptation can be used to either add additional mesh points at or move existing mesh points to areas in which the error of the numerical solution is (or is expected to be) largest (Huang, Ren, and Russell, 1994), meaning that for a given number of mesh points, more small scale phenomena can be resolved, and the error of the approximate solution is much reduced. This makes the numerical method much more efficient, and potentially much more accurate. Mesh adaptivity can be applied to both finite difference and finite element methods (Huang, Ren, and Russell, 1994).

Whilst mesh adaptation has been researched for many years, and has been applied to problems in other areas, it has seen little application to geophysical models (models of the atmosphere, ocean and interior of the earth). One of many reasons for this is that mesh adaptation is difficult to implement on a parallel computer, and that it would be necessary to do this because the method is computationally intensive to implement (Weller et al., 2010).

1.2   Motivation for project

Numerical weather prediction is very important to many industries and also to the public. Applications of NWP include the aviation industry, where air traffic controllers use short term weather forecasts to schedule landings of arriving aircraft to avoid adverse weather conditions (National Aviation Weather Services Committee, Physical Sciences, and Technical Systems, 1995), and businesses such as energy providers, which buy gas cheaply when they receive forecasts of cold weather, when they expect demands to be higher (Inness and Dorling, 2012). Weather forecasts can also be used to help people prepare for extreme weather, and the Met Office provide severe weather warnings to give the public and businesses an indication of both the likelihood of a given weather event occurring, and the impact it could have if it were to occur (Met Office, 2014b).

In August 2004, several hours of heavy rain caused flooding at Boscastle. Two million tonnes of water flowed through Boscastle, Cornwall, a large amount of water having come from local hills. Many cars were swept away by the water, and properties were damaged. However, the Met Office had only predicted light rain. This is because the storms were positioned in a thin line, which was much too narrow to be effectively represented by the model in use at the time, which used a grid with 12km resolution. After the event, the weather data from the day was used to run a forecast of the weather around the time of the storms, using a grid with 1km resolution. This model was able to much more accurately predict the narrow row of storms that occurred (Met Office, 2014a). Since a well designed moving mesh method for solving the model equations would have relocated mesh nodes to the location in which the storms were taking place, its use may well have produced a more accurate forecast, which could accurately model the narrow line of storms which occurred.

Clearly the accuracy of weather forecasts is very important, and inaccurate forecasts could cause large costs for many businesses, or leave people unprepared for extreme weather. Concentrating mesh nodes in areas of high error could not only reduce error in the solution, but also better model phenomena that would ordinarily be smaller than the resolution of a mesh with the same number of points.

References

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