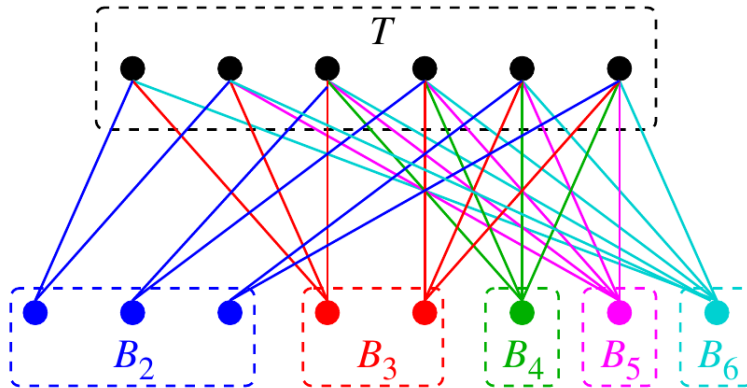


## A logarithmic approximation algorithm for MIN-VERTEX-COVER

```
Initialize  $U = \emptyset$ .
while ( $E$  is not empty) {
    Find a vertex  $u \in V$  of largest (remaining) degree.
    Add  $u$  to  $U$ .
    Delete from  $E$  all the (remaining) edges with  $u$  as one endpoint.
}
Return  $U$ .
```

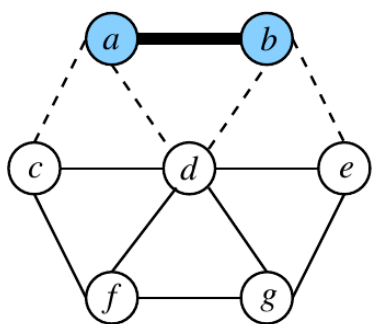
## Tightness of the approximation ratio

The logarithmic approximation factor for the greedy vertex cover algorithm is optimal

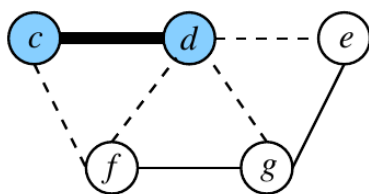


## A 2-approximation algorithm for MIN-VERTEX-COVER

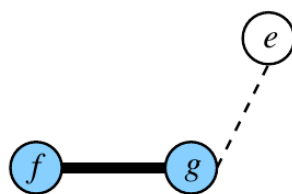
```
Initialize  $U = \emptyset$ .  
while ( $E$  is not empty) {  
    Pick any edge  $e = (u, v)$  from  $E$ .  
    Add  $u$  and  $v$  to  $U$ .  
    Remove  $u$  and  $v$  from  $V$ .  
    Remove from  $E$  all edges incident on  $u$  or  $v$ .  
}  
Return  $U$ .
```



$$U = \{ a, b \}$$



$$U = \{ a, b, c, d \}$$



$$U = \{ a, b, c, d, f, g \}$$

## A 2-approximation algorithm for ETSP

Compute a minimum spanning tree  $T$  of  $G$  under the given cost function.

Choose an arbitrary vertex  $u_0$  of  $T$ .

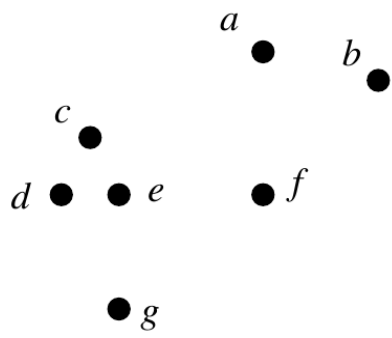
Treat  $T$  as a tree rooted at  $u_0$ .

Impose an arbitrary ordering on the children of each node.

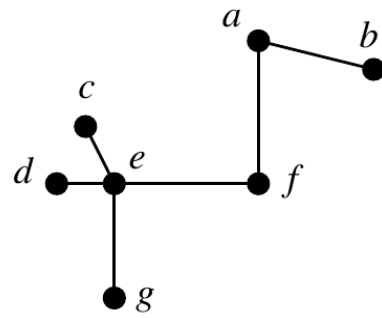
Make a pre-order traversal of  $T$  (starting at the root  $u_0$ ).

Suppose that the traversal returns the list  $u_0, u_1, u_2, \dots, u_{n-1}$  of visited nodes.

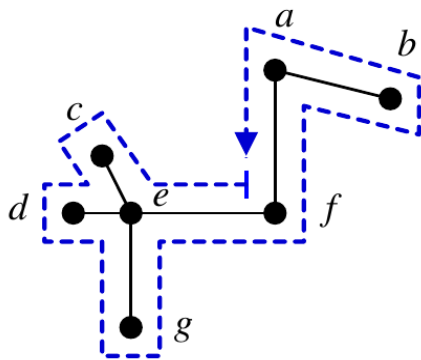
Return the Hamiltonian cycle  $Z = (u_0, u_1, u_2, \dots, u_{n-1}, u_0)$ .



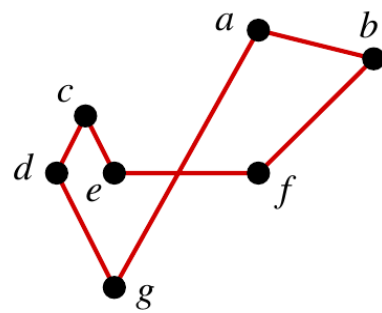
(a) Location of the cities



(b) Computation of an MST



(c) Preorder traversal of MST



(d) The TSP cycle