

Assignment 2 -- Iterations

Purpose of this task is to write a C++ program which takes a positive real number (called "square" in the following) and tries to find ("guess") the square root (called "root") of this number by repeating a try-and-test algorithm.

Because the square-root function is continuous, we can use the mean value theorem: if we know a lower bound for "root" ($\text{lower} < \text{root}$) and an upper bound ($\text{upper} > \text{root}$), so the arithmetic mean of lower and upper bounds will provide a guess for the root. The error tolerance (call this variable "error") for this guess is half the difference of upper-lower.

As long as the error is not sufficiently small (start with choosing 0.001) we simply can reduce our guessing intervall by using the current root value as new lower or upper bound for the next pass, depending on whether $\text{root} \cdot \text{root}$ is greater than square or smaller ("delta" in the subsequent example is the result of $\text{root} \cdot \text{root} - \text{square}$).

CAUTION! -- the numbers in the following program output are NOT correct, please think for yourself!

What would be a good first lower bound for the number entered? What about the upper bound? Start building and testing your program with a moderate tolerance, and then, after getting trust, you can improve the tolerance value. Remember: the data type double is capable of holding 15 to 16 significant decimal digits.

square-root computation by iteration (by Hans Huber, V-1.23)

enter a positive number: 2.0

pass 1:	lower=1.0	upper=2.0	root=1.0	delta=-1.0
pass 2:	lower=1.0	upper=1.5	root=1.5	delta=0.25
pass 3:	lower=1.25	upper=1.5	root=1.25	delta=-0.4375
pass 4:	lower=1.375	upper=1.5	root=1.375	delta=-0.109375
pass 5:	lower=1.375	upper=1.4375	root=1.4375	delta=0.06640625
pass 6:	lower=1.40625	upper=1.4375	root=1.40625	delta=-0.02246094
pass 7:	lower=1.40625	upper=1.42187	root=1.421875	delta=0.0217285156
pass 8:	lower=1.41406	upper=1.42187	root=1.4140625	delta=-0.000427246

the square root of 2.0 is 1.4140625, with a margin of error 0.0035

To develop and also to present your program, an algorithmic sketch would be helpful -- use a structogram for this.