## Service Level model

This model is preferred where the different cost factor involved with inventory are not known exactly, it is based upon probability theory and the amount of safety stock is kept according to the level of service management want to achieve.

$$\text{Service level, } S = \left(\frac{\text{No. of units supplied without delay}}{\text{Total no. of units demanded}}\right)_{LT}$$

$S \to 0$ to $1$

$S\% \to 0$ to $100\%$.

95% service level is the ~~ekto~~ standard value and it means that 95% of the customers order on an average are fulfilled during lead time and only 5% of the customers order on an avg are rejected during lead time.

When the demand during lead time may be approximated by a normal distribution with certain avg ( $\bar{X}$ or $\mu$) and std deviation ($\sigma$) then the reorder level is given by

$$\boxed{ROL = \bar{X} + Z\cdot\sigma}$$

$$\boxed{SS = Z\cdot\sigma}$$



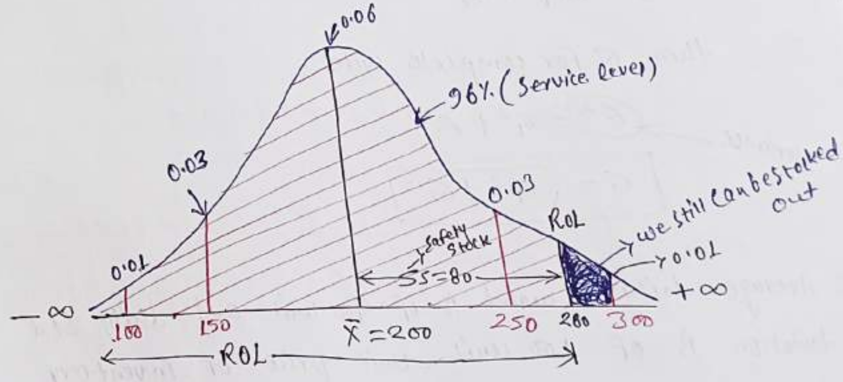
where $\bar{X}$ = avg demand during lead time.

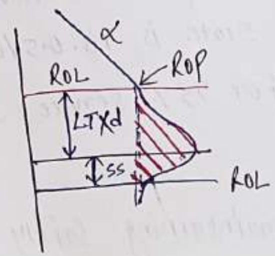
$$\boxed{\bar{X} = ADDLT = LT \times d}$$

$\sigma$ = std deviation for the demand variation during lead time

$Z$ = std normal variate whose value depend upon the service level required.

| Z | Service level(%) |
|---|---|
| 0 | 50% |
| 0.84 | 80% |
| 1.28 | 90% |
| 1.645 | 95% (very imp) |
| 2.33 | 99% |



$$\boxed{SS = Z \cdot \sigma}$$

$x_1 \quad x_2 \quad x_3$

$$\bar{x} = \frac{x_1 + x_2 + x_3}{3}$$

$$\sigma = \sqrt{\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2}{3}}$$

NOTE :- σ represent variation or fluctuation in the demand during lead time. Lead time is one complete cycle and σ should be always corresponding to lead time while computing safety stock → see next page highly highlited

160, 20 → $\mu = 60$

80, 40 → $\mu = 60$

70, 50 → $\mu = 60$

60, 60 → $\mu = 60$

One cycle consist of 2 points.

1st half − $\sigma_1$

2nd half − $\sigma_2$

then σ for complete cycle

Variance → $\sigma^2 = \sigma_1^2 + \sigma_2^2$

$$\boxed{\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}}$$

Q. Average daily demand is of 400 unit and daily std deviation is of 100 unit, unit price of inventory is Rs.40 and the holding rate is Rs. 0.5/unit/month. Lead time is of 4 days for 95% service level. Determine (i) Safety stock
(2) ROL
(3) Annual cost of maintaining safety stock

$d = 400$ units/day $\qquad C_o = Rs\ 40$

$\sigma = 100$ units/day $\qquad C_h = Rs\ 0.5$

Daily Std deviation

Lead time $LT = 4$ day.

As LT is of 4 days and σ is given daily, so converting σ corresponding to LT

$\sigma'^2 = \sigma^2 + \sigma^2 + \sigma^2 + \sigma^2$ — coz lead time is 4 day

$\sigma'^2 = 4\sigma^2$

$\sigma' = 2\sigma = 200$ unit

Now

(i) Safety stock

$SS = Z \cdot \sigma'$ → from service level data for 95%

$= 1.645 \times 200$

$= 329$ units

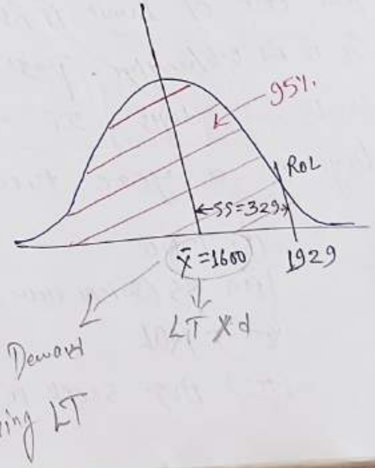
(2) ROL $= LT \times d + SS$

$= 4 \times 400 + 329$

$= 1929$ units

(3) $\frac{SS \times C_h}{329 \times 0.5 \times 12}$

$=$ Rs 1974



[illegible]

50 units /

Q) For a production system annual demand is 20,000 unit and cost of 1 unit is Rs. 20, ordering cost is Rs 60/order, Ch is Rs [illegible]/unit/yr. Past lead time are 20, 15, 18, 22, 19, 27 days. If there are 250 working days in a year then calculate.

(i) EOQ
(ii) SS (safety stock)
(III) ROL
(IV) Avg stock in inventory

(3) Annual co[illegible]

---



(1) $Q^* = 2000$ → EOQ

(2) SS = Max. DDLT − Avg DDLT
= max. LT×d − Avg LT×d
= (max LT − Avg LT) d

$d = \frac{20,000}{250} = 80$ units/day

SS = (27 − 19) × 80
= 640 units

(3) ROL = Avg LT × d + ss
= 19 × 80 + 640 ⇒ 2160 units

(4) $Q_{avg} = \frac{Q^*}{2} + SS$
= 1640 units

# ORDERing Policy for probabilistic

(i) Lot For Lot production

In this method the ordering qty is exactly equal to the demand for a particular period and this policy is preffered where ordering cost is less or holding cost is more.

(1) Lot For Lot production Example

LT = 1 week, $C_h$ = Rs 1/unit/week, $C_o$ = Rs 500/order

TIC → O.C 6×500 = Rs 3000
+ H.C 0

| Week | Demand |
|---|---|
| 1 | 600 |
| 2 | 200 |
| 3 | 100 |
| 4 | 700 |
| 5 | 200 |

600 - 600 = 0

200 - 200 = 0

↓ 0

0 - 600
1 - 200
2 - 100
3 - 700
4 - 200
5 - 800

(2) Least cost technique :- In this method the best ordered quantity is where the total inventory cost is minimum. It is a hit and trial method in which we need to check all diff. combinations to get the best ordered quantity.

Lot For Lot production

LT = 1 week, $C_h$ = Rs 1/unit/week

$C_o$ = Rs 500/order

TIC → O.C + H.C

1500 + 600

= Rs 2100

| Week | Demand |
|---|---|
| 1 | 600 |
| 2 | 200 |
| 3 | 100 |
| 4 | 700 |
| 5 | 200 |
| 6 | 800 |

0 - 900
3 - 900
5 - 800

900 - 600 = 300

300×1

300 - 200 = 100

100×1

100 - 100 = 0

900 - 700 = 200

200×1

200 - 200 = 0

800 - 800 = 0

# Part-Period Total cost Balancing

In this method the best ordering qty is where ordering and holding cost are as close as possible. The best thing they should be equal. This method follows EOQ to get the best ordered qty.

1st Q = 600
O.C = H.C
500 0

2nd Q = 800
O.C H.C
500 200

3rd Q = 900
O.C H.C
500 400

4th Q = 1600
O.C H.C
500 2500

| Week | Demand |
|---|---|
| 1 | 600 |
| 2 | 200 |
| 3 | 100 |
| 4 | 700 |
| 5 | 200 |
| 6 | 800 |

1600 - 600 = 1000

1000×1

1000 - 200 = 800

800×1

800 - 100 = 700

700×1

700 - 700 = 0

# Inventory classification and control.

(i) ABC control

(Pareto Law or 80-20 Law)

| | Usage % | Items % |
|---|---|---|
| A | 50-60% | 10-20% |
| B | 30-40% | 30-40% |
| C | 10-20% | 50-60% |

| Items | Item% | Demand (D) | unit price (C) | usage value | usage% |
|---|---|---|---|---|---|
| 1 | 10% | 200 | 100 | 20,000 | $\frac{20,000}{\Sigma x}\times 100$ |
| 2 | 10% | 70 | 60 | 4200 | $\frac{4200}{\Sigma x}\times 100$ |
| 3 | 10% | 400 | 300 | 1.2 lakh | $\frac{1.2\ lakh}{\Sigma x}\times 100$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 10 | 10% | | | | |
| | 100% | | | $\Sigma x$ | 100% |

usage%:

34%, 22% → 56% A

18%, 9%, 6% → 33% B

5.2%, 4.1%, ⋮ → 11% C

In ABC control inventry items are classified into A, B and C category, Depending upon their usage value For A category items inventory is kept almost nill but frequent review is done. For C category item large amount of inventory is kept and it is reviewed after a long period.

~~ABC contr~~

(I) Safety stock is kept ×

(II) NO SS is kept ② ✓

(IV) Low SS is kept ① ✓

② Vital Essential and Desirable (VED)

Inventory items are classified on the basis of importance of inventory for the producⁿ system.

③ High medium Low (HML)

Inventory are classified on the basis of unit price of inventory ↳ (Rs/unit)

④ ~~Scare~~ Score difficult and Easy

Inventory are classified on the basis of availability of inventory for the producⁿ system

Inventory Turn over ratio (ITR)

It is a term used to indicate effectiveness or efficiency of inventory system, It Indicates how many times avg inventory is sold out or rotated in a year. A higher value is preferred as it represents more profit and less holding

$$ITR = \frac{\text{cost of Goods sold or total sales}}{\text{Average Inventory value}}$$

e.g. $\frac{Rs\ 72\ lakh}{Rs\ 8\ lakh} = 9$

Avg. time to sell inventory → day or month $= \frac{365}{ITR}$

Date 19-03-21

# 3 SEQUENCING

In Sequencing our aim is to find the order in which different Jobs are to be processed on the different machines so that the Ideal Time is minimized and utilization is optimized. It is essential for smooth flow of material and effective utilization of man power and machine.

• Rule or assumption in Sequencing :-

(1) If nothing is mentioned, processing order for m/c remain fixed

(2) one Job on one m/c at a time.

(3) once a Job is started it must be fully completed.

(4) Time taken by the Job from one machine to another is negligible.

(5) IRRespective of order, Processing time for the Job remain constant



# N Jobs on one machine :-

(1) Job Flow time :- It is the time from some starting point untill that particular Job is completed.

(2) Make Span Time (MST) :- It is a time from when processing begin on the first Job in the set untill the last Job is completed.

(3) Tardiness or Lateness :- It is the amount of time by which a Job is delayed beyond its due date.

(4) Avg. no. of Jobs in System :- It is a term used to represent avg no. of Jobs present all the time within the system untill one set of Job is completed.

It is a ratio of total Job Flow time, over make span time (MST).

# Sequencing rules for N Jobs for one m/c :-

(i) Shortest Processing time (SPT)

In this rule Jobs are sequenced in increasing order of their processing time, i.e minimum time (Ist) and maximum time in the last.

| ST | Jobs | Processing Time | Due Date | EDD |
|---|---|---|---|---|
| 3 | 1 | 8 | 28 | 2 |
| 1 | 2 | 6 | 39 | 3 |
| 4 | 3 | 9 | 58 | 5 |
| 5 | 4 | 11 | 40 | 4 |
| 2 | 5 | 7 | 21 | 1 |

(2) Earliest Due Date (EDD) :- Jobs are sequent in increasing order of their due date.

(3) Critical Ratio Rule (CR) :-

$$CR = \frac{\text{Due Date}}{\text{processing time}}$$

Jobs are sequenced in increasing order of their critical ratio.

(4) Slack time remaining (STR) :-

$$\boxed{STR = \text{Due Date} - \text{Processing time}}$$

Jobs are sequenced in increasing order of slack time remaining.

Q. A set of jobs are to be processed on a single m/c. Obtain a sequence using SPT and EDT rule also Determine make span Time, Job flow time for each Job, avg Job flow time per Job, avg tardiness per Job, avg no. of Jobs in system and no. of Tardy Job.



| Jobs | Processing Time | Due Date | DD |
|---|---|---|---|
| A | 13 | 51 | |
| B | 8 | 72 | |
| C | 19 | 79 | |
| D | 10 | 21 | |
| E | 6 | 43 | |
| F | 17 | 84 | |
| G | 11 | 34 | |
| H | 15 | 61 | |

By SPT method/Rule

| Jobs | P.T | D.D | Job Flow time | Tardiness |
|---|---|---|---|---|
| E | 6 | 43 | 0+6 = 6 | 0 |
| B | 8 | 72 | 6+8 = 14 | 0 |
| D | 10 | 21 | 14+10 = 24 | 3 |
| G | 11 | 34 | 24+11 = 35 | 1 |
| A | 13 | 51 | 35+13 = 48 | 0 |
| H | 15 | 61 | 48+15 = 63 | 2 |
| F | 17 | 84 | 63+17 = 80 | 0 |
| C | 19 | 79 | 80+19 = 99 | 20 |
| | | | 369 | |

→ 43 तक रहा था D.D But 6 दिन में दे दिया इसलिए 0.

→ D.D 21 दिन का था But 24 दिन में दिया 3 दिन Late or Tardiness

1) MST = 99 min

2) E-6, B-14, D-24, ......

3) Avg Job flow time/Job = $\boxed{\frac{\text{Total Job Flow time}}{\text{No. of Jobs}}} = \frac{369}{8} = 46.125$ Min (SPT)

4) Avg Tardiness/Job = $\frac{\text{Total tardiness}}{\text{No. of Jobs}} = \frac{\overset{2+1+2+13}{26}}{8} = 3.25$ Min

5) Avg no. of Jobs in System = $\boxed{\frac{\text{Total Job Flow time}}{MST}} = \frac{369}{99} = 3.72$ Jobs (SPT)

6) No. of tardy Jobs = 4 Jobs

By EDD method/Rule

| Job | P.T | D.D | Job Flow time | Tardiness |
|---|---|---|---|---|
| D | 10 | 21 | 0+10=10 | 0 |
| G | 11 | 34 | 10+11=21 | 0 |
| E | 6 | 43 | 21+6=27 | 0 |
| A | 13 | 51 | 27+13=40 | 0 |
| H | 15 | 61 | 40+15=55 | 0 |
| B | 8 | 72 | 55+8=63 | 0 |
| C | 19 | 79 | 63+19=82 | 3 |
| F | 17 | 84 | 82+17=99 | 15 |
| | | | 397 | 18 |

1) MST = 99 min.

2) D–10, G–21, E–27, ---

3) Avg Job Flow time/Job = $\frac{\text{Total Job Flow time}}{\text{No. of Jobs}} = \frac{397}{8} = 49.625$ min

4) Avg. Tardiness/Job = $\frac{\text{Total tardiness}}{\text{No. of Jobs}} = \frac{18}{8} = 2.25$ min

5) Avg. no. of Jobs in system = $\frac{\text{Total Job Flow time}}{\text{MST}} = \frac{397}{99} = 4.01$ Jobs

6) No. of tardy Jobs = 2 Jobs.



Q. 4 Jobs are to be processed on a single m/c as per data given below
1) using EDD rule find the no. of Jobs delay
2) using SPT rule find the total Tardiness.

| Jobs | P.T (days) | D.D |
|---|---|---|
| 1 | 4 | 6 |
| 2 | 7 | 9 |
| 3 | 2 | 19 |
| 4 | 8 | 17 |

By EDD

| Jobs | P.T(day) | D.D | Job Flow time | |
|---|---|---|---|---|
| 1 | 4 | 6 | 0+4=4 | 0 |
| 2 | 7 | 9 | 4+7=11 | 2 |
| 3 | 2 | 19 | 11+2=13 | 0 |
| 4 | 8 | 17 | 13+8=21 | 4 |

2 Jobs delayed Ans(1)
8 days Ans(2)

(1) No. of tardy Job = 2.

(2)

| Jobs | P.T(day) | D.D | Job flow time | · |
|---|---|---|---|---|
| 3 | 2 | 19 | 0+2=2 | 0 |
| 1 | 4 | 6 | 2+4=6 | 0 |
| 2 | 7 | 9 | 6+7=13 | 4 |
| 4 | 8 | 17 | 13+8=21 | 4 |

→ 8 days

Ans

# N Jobs on 2 m/c

| m/c → | A | B |
|---|---|---|
| Jobs | Ai | Bi |
| 1 | | |
| 2 | | |
| 3 | | |
| ⋮ | | |
| N | | |

**e.g. 1**

A ↓ इसको minimum सबसे पहले रखोगे

B ↳ जिस में minimum जो रहेगा उसे सबसे लास्ट में रखोगे

| Jobs | A | B |
|---|---|---|
| 1 | 6 | 5 |
| 2 | 4 | 7 |
| 3 | 6 | 5 |

2  1  3

**e.g. 2**

| Jobs | A | B |
|---|---|---|
| 1 | 4 | 3 |
| 2 | 6 | 5 |
| 3 | 3 | 8 |

3  2  1

**e.g. 3**

| | A | B |
|---|---|---|
| 1 | 5 | 4 |
| 2 | 3 | 6 |
| 3 | 3 | 4 |

3  2  1

A के लिए पहले देख रहे।

**e.g. 4**

| | A | B |
|---|---|---|
| 1 | 7 | 4 |
| 2 | 5 | 8 |
| 3 | 5 | 4 |

2  1  3

B के लिए पहले देख रहे

**e.g. 5** Case of alternate option

| Jobs | A | B |
|---|---|---|
| 1 | 4 | 7 |
| 2 | 8 | 6 |
| 3 | 4 | 7 |

1  3  2

OR

3  1  2

**e.g. 6**

| Jobs | A | B |
|---|---|---|
| 1 | 4 | 3 |
| 2 | 2 | 2 |
| 3 | 5 | 4 |

2  3  1

AND

3  1  2

# N Jobs on 2 m/c

These problems are sold by Johnson's rule or Johnson's algorithm and the steps involved are

(1) find out the minimum of Ai and Bi

(2) If the minimum is for a particular Job on machine A then perform that Job at the start

(3) If the minimum is for a particular Job on m/c B then perform that Job in the last

(74)

4) Strikeoff the Job which is aligned so that it can't be considerd again.

5) continue in the similar manner untill all the Jobs are asigned.

Q. Find the optimum sequence for the following set of Jobs to be processed on two m/c also Determine make span time, Ideal time for each machine and their percentage utilization.

| Jobs | MC-I | MC-II |
|---|---|---|
| A | 9 | 5 |
| B | 4 | 8 |
| C | 6 | 10 |
| D | 7 | 8 |
| E | 10 | 3 |
| F | 11 | 6 |
| | Total 47 | Total 40 |

Sequence

B C D F A E

| Jobs | M/C-I In | M/C-I Out | M/C-II In | M/C-II Out |
|---|---|---|---|---|
| B | 0 | 4 | 4 | 12 |
| C | 4 | 10 | 12 | 22 |
| D | 10 | 17 | 22 | 30 |
| F | 17 | 28 | 30 | 36 |
| A | 28 | 37 | 37 | 42 |
| E | 37 | 47 | 47 | 50 |
| | 3 | | 10 | MST |

1) MST = 50 min

Idle time = MST − working Time

m/c I = 50 − 47 = 3 min

m/c II = 50 − 40 = 10 min

% utilizatn = $\frac{\text{working time} \times 100}{MST}$

m/c I = $\frac{47}{50} \times 100 = 94\%$

m/c II = $\frac{40}{50} \times 100 = 80\%$

(75)

Q. Find the optimum sequence for the following set of Jobs also find make span time and Ideal time for each machine.

| Jobs | MC-I | MC-II |
|---|---|---|
| A | 2 | 6 |
| B | 5 | 8 |
| C | 9 | 4 |
| D | 4 | 7 |
| E | 8 | 3 |
| F | 8 | 9 |
| G | 7 | 3 |
| H | 5 | 8 |
| I | 4 | 11 |
| | 50 | 55 |

Sequence A D I B H F C G E H B → Don't take twice

| Jobs | MC-I In | MC-I Out | MC-II In | MC-II Out |
|---|---|---|---|---|
| A | 0 | 2 | 2 | 8 |
| D | 2 | 6 | 8 | 15 |
| I | 6 | 10 | 15 | 26 |
| B | 10 | 15 | 26 | 34 |
| H | 15 | 20 | 34 | 42 |
| F | 20 | 28 | 42 | 51 |
| C | 28 | 37 | 51 | 55 |
| G | 37 | 44 | 55 | 58 |
| E | 44 | 50 | 58 | 61 |
| ~~H~~ | ~~50~~ | ~~55~~ | ~~61~~ | ~~69~~ |
| ~~B~~ | ~~55~~ | ~~80~~ | ~~80~~ | ~~88~~ |

1) MST = 61

1) Ideal time = 61 − 50 = 11

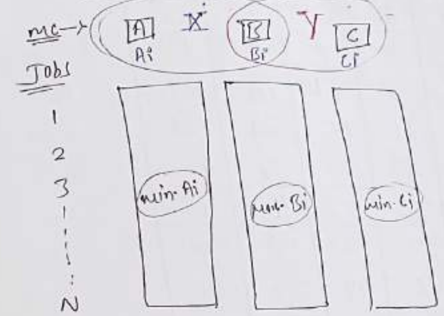
M/C II = 61 − 55 = 6

% Utilization = $\frac{\text{working time}}{MST} \times 100$

(M/C) I = $\frac{50}{61} \times 100 = 81.97$

(M/C II) = $\frac{55}{61} \times 100 = 90\%$

## N Jobs on 3 M/C




1) min. $A_i \geq$ max. $B_i$

OR

2) min. $C_i \geq$ max. $B_i$

$X_i = A_i + B_i$

$Y_i = B_i + C_i$

Q. Find the optimum sequence system to be process on 3 M/C, also find MST and Ideal time for each machine

$4+3=7$ $4+6=10$

| Jobs | M/C A | M/C B | M/C C |
|---|---|---|---|
| 1 | 3 | 4 | 6 |
| 2 | 8 | 3 | 7 |
| 3 | 7 | 2 | 5 |
| 4 | 4 | 5 | 11 |
| 5 | 9 | 1 | 5 |
| 6 | 8 | 4 | 6 |
| 7 | 7 | 3 | 12 |
| | 46 | 22 | 52 |
| Ideal time ⇒ | 59−46 = 13 | 59−22 = 37 | 59−52 = 7 |

Converting into two M/C ⇒

| Jobs | X | Y |
|---|---|---|
| 1 | 7 | 10 |
| 2 | 11 | 10 |
| 3 | 9 | 07 |
| 4 | 09 | 16 |
| 5 | 10 | 6 |
| 6 | 12 | 10 |
| 7 | 10 | 15 |

1 4 7 6 2 3 5

| Jobs | M/C A | | M/C B | | M/C C | |
|---|---|---|---|---|---|---|
| | In | out | In | out | In | out |
| 1 | 0 | 3 | 3 | 7 | 7 | 13 |
| 4 | 3 | 7 | 7 | ~~8~~ 12 | 13 | 24 |
| 7 | 7 | 14 | 14 | 17 | 24 | 36 |
| 6 | 14 | 22 | 22 | 26 | 36 | 42 |
| 2 | 22 | 30 | 30 | 33 | 42 | 49 |
| 3 | 30 | 37 | 37 | 39 | 49 | 54 |
| 5 | 37 | 46 | 46 | 47 | 54 | 59 |

# (4) PERT, CPM

It is required for the execution ~~for~~ of the project, It help us to know how much the resources and man power are required to complete the project within fix period of time.

## Project

It is a ~~prove~~ group or combination of interrelated activities that must be executed in a certain fixed order before the entire task is completed. Activities are inter-related in a logical sequence in the sense that some activity can only be started when all the activities





earlier to wait are completed.

## Event

It denotes the point of time or accomplishment occuring at a moment and is used to denote the starting and the end point of ~~end~~ an activity. event neither consume any time nor resources for its complition.

## Activity

It is a reconisable part of a project which consume time and resources for its completion and it may involve physical or mental work. when all the activities are executed then only a project get completed.

20-03-21

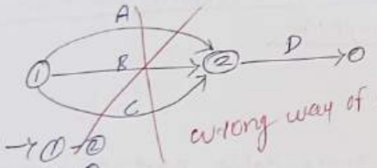
## # Network Diagram

It is a graphical representation of the logical sequence in which different activities are interrelated to each other while completing the project.

## Rules for network construction

① An activity can only be started when ~~ate~~ all the activities earlier to wait are completed.

② No two or more activities may have the same head and tail event.



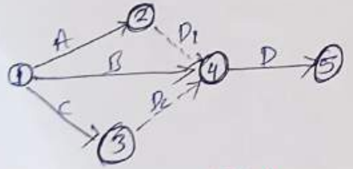

A → ①-②
B → ①-②
C → ①-②

wrong way of Network Diagram

To represent the same logic we need to use dummy activity.

### Dummy

An activity which is used to show the logic, Dependency or relationship of one activity over the other but Does not consume any time or resources for its completion is termed as dummy activity. It is represented by dotted line arrow (- - - ->)



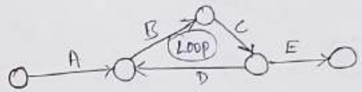

A → ①-②
B → ①-④
C → ①-③

### Rule

① Dummy activity should only be used when there is necessity but there is no restriction on the no. of dummy activity used.

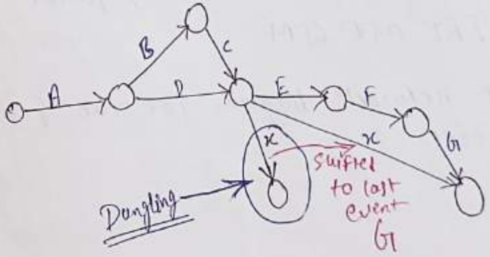
② Length and Direction of the arrow is indicative only and time flow from left to right on the network diagram.

③ There should be no looping and dangling on the network diagram



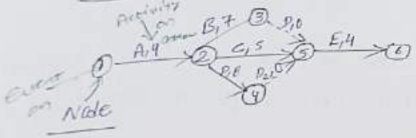

### Dangling

When activity other than the final activity then the Does not have any further activity then the situation is called dangling. Such activity should be connected to the last event of the network diagram

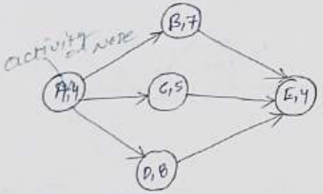


(6) Types of Network Diagram

① Event on Node (EON)
or
Activity on Arrow (AOA)




② Activity on Node (AON)

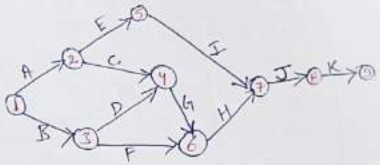



Activity on Node Diagram does not require dummy activity and it is considered to be simple and easy irrespective of these advantages event on Node Diagram is more popular and preffered in PERT and CPM

Q. Draw the network diagram for the following set of activities



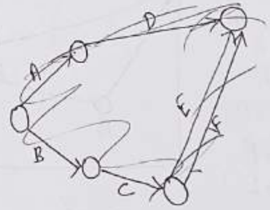
| Activity | Precedence |
|---|---|
| A | — |
| B | — |
| C | A |
| D | B |
| E | A |
| F | B |
| G | C,D |
| H | G,F |
| I | E |
| J | H, I |
| K | J |

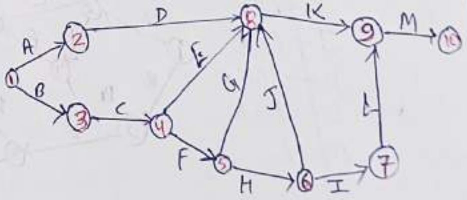



Fulkerson's Rule :- Numbering of events

Q. Draw the Network Diagram for the following set of activities

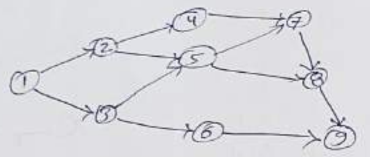
| Activity | Precedence |
|---|---|
| A | |
| B | |
| C | B |
| D | A |
| E | C |
| F | C |
| G | F |
| H | F |
| I | H |
| J | H |
| K | D, E, G, J |
| L | I |
| M | K, L |



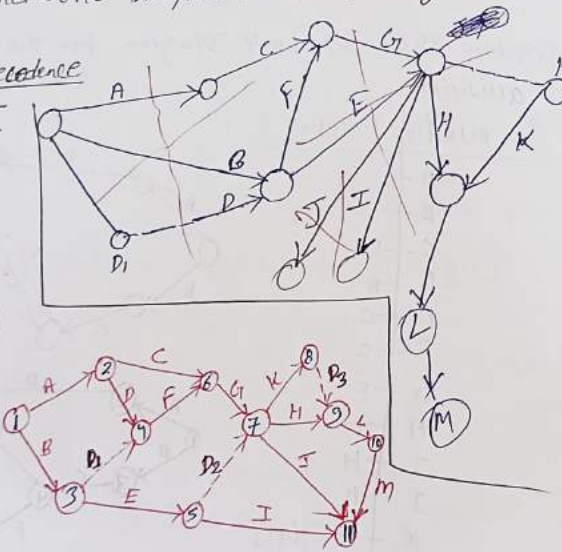
Q. Draw the network Diagram for the following set of activities.

| Activity |
|---|
| 1-2 |
| 1-3 |
| 2-4 |
| 2-5 |
| 3-5 |
| 3-6 |
| 4-7 |
| 5-7 |
| 5-8 |
| 6-9 |
| 7-8 |
| 8-9 |



Q. Draw the network Diagram for the following set of activities

| Activity | Precedence |
|---|---|
| A | — |
| B | — |
| C | A |
| D | A |
| E | B |
| F | B, D |
| G | C, F |
| H | G, E |
| I | E |
| J | G, E |
| K | G, E |
| L | H, K |
| M | L |



Difference b/w PERT and CPM

| PERT | CPM |
|---|---|
| ① Programme (project) Evaluation and Review Technique. | ① Critical Path Method |
| ② It is event oriented | ② It is activity oriented. |
| ③ It is associated with probabilistic activities | ③ associated with deterministic activities. |
| ④ It is based upon 3 time estimate to complete an activity. | ④ It is based upon single time to complete an activity. |
| ⑤ It usually does not consider cost analysis. | ⑤ It gives importance to cost analysis. and crashing is done to minimize the cost of CPM project. |
| ⑥ It is used mainly for Research and development project. | ⑥ used mainly for construction project. |

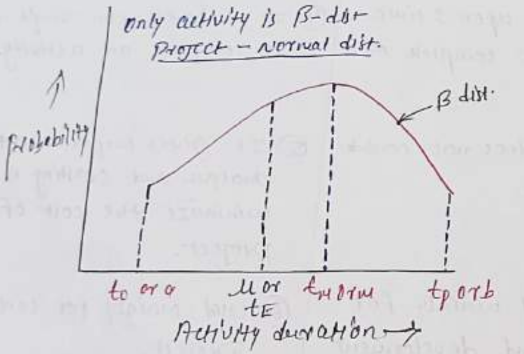
PERT

It is used for uncertain project and is based upon 3 time estimate.

(i) optimistic time (to or a) :- It is a minimum time required to complete an activity when everything goes according to the plan.

2) <u>Pessimistic time</u> ($t_p$ or $b$) :- It is a maximum time required to complete an activity when everything goes against the plan.

3) <u>Most likely time</u> ($t_m$ or $m$) :- It is a time required to complete an activity when executed under normal working conditions.




The fundamental <u>assumption</u> in Pert is that the 3 times estimate from the end point of the distribution curve and activities assumed to follow β-distribution. It is also assumed that the probability of completing activity in time $a$, $b$ is equal and the probability of completing activity in time $m$ is 4 times of either $a$ or $b$.

avg or expected time to complete an activity is given by



$$M \text{ or } t_E = \left(\frac{a+4m+b}{6}\right) = \left(\frac{t_o+4t_m+t_p}{6}\right)$$

$$\sigma = \left(\frac{b-a}{6}\right) = \left(\frac{t_p-t_o}{6}\right)$$

$$\text{Variance} = \sigma^2 = \left(\frac{b-a}{6}\right)^2 = \left(\frac{t_p-t_o}{6}\right)^2$$

<u>NOTE</u>:- variance give the measure of uncertainity of activity completion higher the value of variance larger the uncertainity will be.

<u>Critical Path</u>

It is a <u>maximum</u> time consumed path from the 1st event to the last event in a network diagram and it is a <u>minimum</u> time required to complete the project. The time taken along the critical path is termed as expected project completion time ($T_E$). The activities along the critical path are termed as critical activities and are represented by Double line arrow (⟹)

## Probability of completing project within sceduled time

If ($T_E$) is expected project completion time, $\sigma$ is std deviation along critical path then the probability of completing project within sceduled time ($T_S$) is given by

$$\boxed{Z = \frac{T_S - T_E}{\sigma}}$$

(58) where Z is standard normal variate.

$\boxed{P(T_S) = \frac{\text{Area ABS}}{\text{Area ABC}}}$

$\boxed{ROL = \bar{X} + Z \cdot \sigma}$

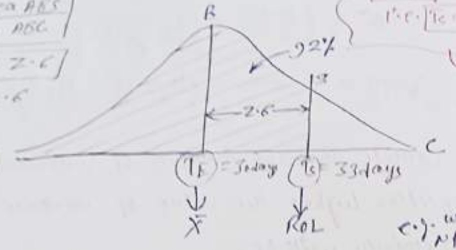
$T_S = T_E + Z \cdot \sigma$

$Z = \frac{T_S - T_E}{\sigma}$

↓

1.645

⇒ 95%




For 50% probability $\boxed{Z = 0}$ i.e. $\boxed{T_S = T_E}$

→ e.g. ws 29(P-70)

e.g. ws 15(P-18) NP 104

NOTE:- Now critical path is one complete cycle and Sigma ($\sigma$) should be always corresponding to critical path while computing probability.

$\sigma_1 \quad \sigma_2 \quad \sigma_3 \quad \sigma_4 \quad \sigma_5 \quad \sigma_6 \quad \sigma_7 \quad \sigma_8$

$\sigma = \sqrt{\text{Sum of variance along critical path}}$

$\sigma = \sqrt{\sigma_1^2 + \sigma_3^2 + \sigma_6^2 + \sigma_8^2}$

Q. For the following set of activities Draw the Network Diagram and determine.

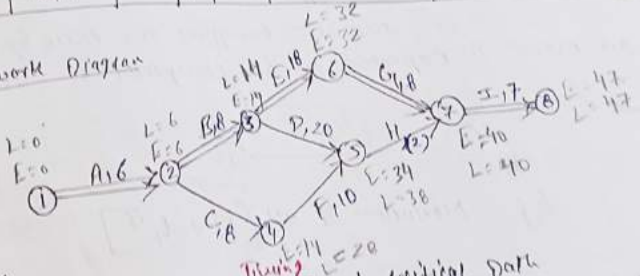
(1) Critical path and expected project completion time

(2) Find the probability of completing project in 50 days

(3) If a company makes an agreement to complete the project 50 days failing which they would pay Rs 10000/day as fine. Find the Probability that a fine must be payed paid but not exceeding Rs 50 thousand

| Activity | precedence | Time (days) a | m | b | $t_E = \left(\frac{a+4m+b}{6}\right)$ | $\sigma = \left(\frac{b-a}{6}\right)$ |
|---|---|---|---|---|---|---|
| A | — | 4 | 6 | 8 | 6 | 4/6 — Critical path |
| B | A | 5 | 7 | 15 | 8 | 10/6 — C.P |
| C | A | 4 | 8 | 12 | 8 | 6/6 |
| D | B | 15 | 20 | 25 | 20 | 10/6 |
| E | B | 10 | 18 | 26 | 18 | 16/6 → C.P |
| F | C | 8 | 9 | 16 | 10 | 8/6 |
| G | E | 4 | 8 | 12 | 8 | 8/6 → C.P |
| H | D, F | 1 | 2 | 3 | 2 | 2/6 |
| I | G, H | 6 | 7 | 8 | 7 | 2/6 → C.P |

Network Diagram




Path

Timing

1) 1-2-3-6-7-8 — 47

2) 1-2-3-5-7-8 — 42

3) 1-2-4-5-7-8 — 33

Critical path

A → B → E → G → I

$\boxed{T_E = 47}$ → expected project completion Time

(2) $\sigma = \sqrt{\left(\frac{4}{6}\right)^2 + \left(\frac{10}{6}\right)^2 + \left(\frac{16}{6}\right)^2 + \left(\frac{8}{6}\right)^2 + \left(\frac{2}{6}\right)^2}$

$\sigma = 3.496$ day

Probability $Z = \frac{T_S - T_E}{\sigma}$

$Z = \frac{50 - 47}{3.496} = 0.8581$

given in ques

⇓

80.45%

(3)


> Read the que

$$z_1 = \frac{T_S - T_E}{\sigma} \Rightarrow \frac{55-47}{3.496} = 2.238 \xrightarrow{Prob} 98.89\%$$

$$z_2 = \frac{50-47}{3.496} = 0.858 \xrightarrow{Prob} 80.45\%$$

$$- \quad 18.44\%$$

# CRITICAL PATH

The procedure for finding critical path is similar both in PERT and CPM and it consists of two phases.

(i) Forward pass computation :-

In this we compute the time by which an event is expected to be completed at the earliest.

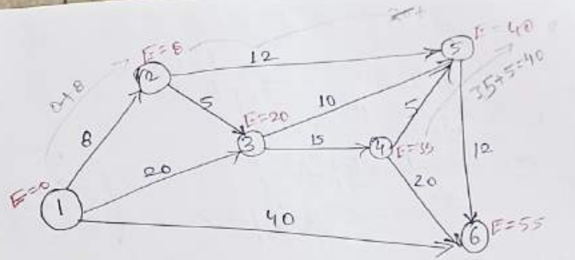
$$\overset{E_i}{(i)} \xrightarrow{t_E^{ij}} \overset{E_j}{(j)}$$

$$\boxed{E_j = \text{Maximum of all } [E_i + t_E^{ij}]}$$

where $E_i$ = earliest expected time for event $i$ ~~Ef earliest expected time~~

$E_j$ = earliest expected time for event $(j)$

$T_E^{ij}$ = expected time for equity $(ij)$




| Event | | Earliest time |
|---|---|---|
| 1 | — | 0 |
| 2 | — | 8 |
| 3 | — | Max[13, 20] 20 ← maximum |
| 4 | — | 35 |
| 5 | — | Max[20, 30, 40] 40 |
| 6 | — | max[52, 55, 40] 55 |

(ii) Backward pass computation :- In this we compute the time by which an event must be completed at the latest.

$$\overset{L_i}{(i)} \xrightarrow{t_E^{ij}} \overset{L_j}{(j)}$$

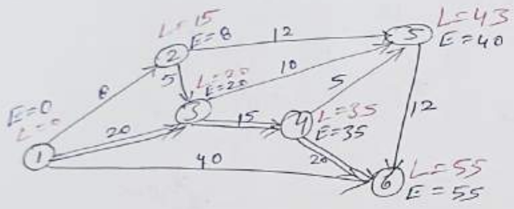
$$L_i = \text{minimum of all } [L_j - t_E^{ij}]$$

where $L_i \Rightarrow$ Latest allowable time for event $(i)$

$L_j \Rightarrow$ Latest —— " —— $(j)$

$T_E^{ij} \Rightarrow$ expected time for equity $(ij)$

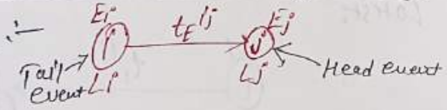
POCO M4 PRO 5G | SHIVANSHU




⇒ Critical ~~activity~~ path

| Event | Latest time |
|---|---|
| 6 | 55 |
| 5 | 43 |
| 4 | min [38, 35] 35 → selecting minimum |
| 3 | min [33, 20] 20 |
| 2 | min [31, 15] 15 |
| 1 | min [7, 0, 15] 0 |

For any activity to be critical the following 3 condition must be satisfied :-




(1) Head event slack = 0

$\boxed{L_j - E_j = 0}$

(2) Tail event slack = 0, $\boxed{L_i - E_i = 0}$

(3) $L_j - L_i = E_j - E_i = t_E^{ij}$

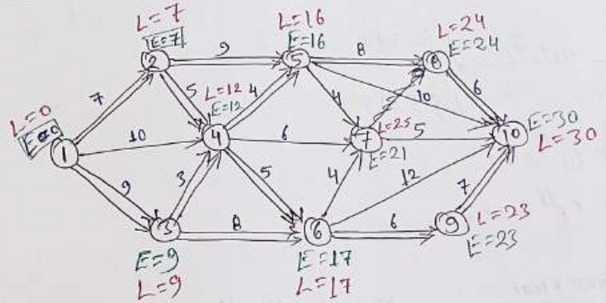


Critical path = 1-3-4-6

Project duration = 55

Critical path is termed as critical because if any activity along this path is delayed by certain amount of time the whole project is delayed by the same amount of time.

Q For the network diagram shown below find the critical path and expected project completion time




Critical path

1) 1-2-5-8-10
2) 1-2-4-5-8-10
3) 1-2-4-6-9-10
4) 1-3-6-9-10
5) 1-3-4-6-9-10
6) 1-3-4-5-8-10

NOTE :- In case of PERT if there are more than 1 critical path then in order to determine Probability we select the path having max^m Std. deviation

Date 22-03-21

## Slack and Float

Slack and Float → PERT → Event; Slack and Float → CPM → Activity

$Ei \approx EST$, $Ej \approx EFT$

$i \xrightarrow{A, t_E^{ij}} j$

$Li \approx LST$, $Lj \approx LFT$

$Ei + t_E^{ij} = Ej$

$EST + t_E^{ij} = EFT$

EST = Earliest Starting time
EFT = earliest finising time
LST = Least starting time
LFT = Least finishing time

## Slack or Event Float

$i \longrightarrow j$

$E_i$, $Ej = 18$, $Li$, $Lj = 22$

$\boxed{Si = Li - Ei}$ $\boxed{Sj = Lj - Ej}$

It denotes the amount of time by which a particular event can be delayed without delaying the project scedule

## (2) Float

### (a) Total Float (TF)

$i \xrightarrow{A, t_E^{ij}} j$ (EST, LST at i; EFT, LFT at j)

$\boxed{TF = LFT - EFT = LST - EST}$

⇒ It denotes the amount of time by which an activity can be delayed without delaying the project completion time. It is the extra time available for an activity without delaying the project scedule

If total float value is

(1) +ve → resources are surplus and can be allocated for other activities

(2) (If) zero 0 → resources are just sufficient to complete activity on time

(3) If -ve → Resources are not sufficient and activity may not complete on time.

(i) Super critical path → Highest -ve total float value. Path

(ii) Critical path → path having total float value zero.

(III) Sub-critical path → After critical path the next higher positive total float value path

### (b) Free Float (FF)

It is that part of total float which can be used without affecting the float of succeeding activity. It is the extra time by which an activity can be delayed so that the succeeding activity can be started at their earliest start time.

$\boxed{FF = TF - \text{Head event slack}}$

(96)

(c) Indipendent Float (IF)

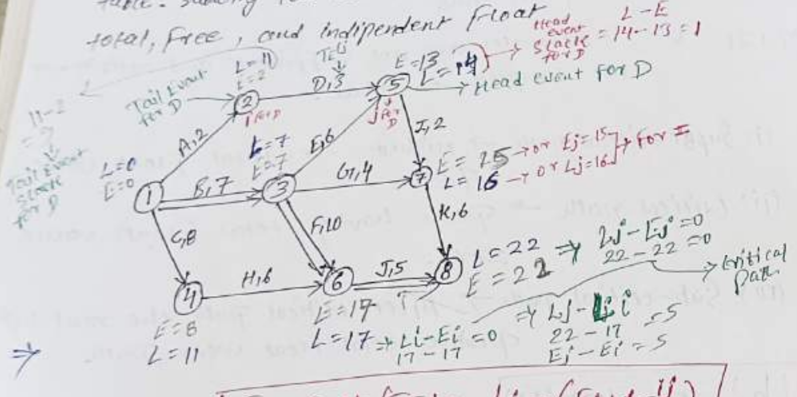
It is the amount of time which can be used without affecting either the head or tail event

$\boxed{IF = FF - \text{Tail event slack}}$

$\boxed{IF = E_j - (L_i + t_E^{ij})}$

$\boxed{TF \geq FF \geq IF}$

Q. For the Network diagram shown below find the critical path and expected project completion time draw a table. Showing the details for each activity along with total, free, and indipendent float.




$\boxed{\text{Total Float (TF)} = L_j - (E_i + t_E^{ij})}$

For exam — For objective directly from Diagram

for D: $i = 2$, $j = 5$

$= 14 - (2+3) = 9 \rightarrow$ for D

$\boxed{FF = TF - \text{Head event slack}}$

$D \rightarrow 9 - 1 = 8$

$\boxed{FF = E_j - (E_i + t_E^{ij})}$ $D \rightarrow 13 - (2+3) = 8$

| Activity (i-j) | $t_E^{ij}$ | Earliest EST | Earliest EFT | Latest LST | Latest LFT | Float Total | Float Free | Float Independent |
|---|---|---|---|---|---|---|---|---|
| A, 1-2 | 2 | 0 | 2 | 9 | 11 | 9 | 0 | 0 |
| B, 1-3 | 7 | 0 | 7 | 0 | 7 | 0 | 0 | 0 |
| C, 1-4 | 8 | 0 | 8 | 03 | 11 | 3 | 0 | 0 |
| D, 2-5 | 3 | 2 | 5 | 11 | 14 | 9 | 8 | $-1^*$ (0) |
| E, 3-5 | 6 | 7 | 13 | 08 | 14 | 1 | 0 | 0 |
| F, 3-6 | 10 | 7 | 17 | 7 | 17 | 0 | 0 | 0 |
| G, 3-7 | 4 | 7 | 11 | 12 | 16 | 5 | 4 | 4 |
| H, 4-6 | 6 | 8 | 14 | 11 | 17 | 3 | 3 | 0 |
| I, 5-7 | 2 | 13 | 15 | 14 | 16 | 1 | 0 | $-1^*$ (0) |
| J, 6-8 | 5 | 17 | 22 | 17 | 22 | 0 | 0 | 0 |
| K, 7-8 | 6 | 15 | 21 | 16 | 22 | 1 | 1 | 0 |

(→ Forward, ← backward)

$\boxed{IF = FF - \text{Tail event slack}}$

$D \rightarrow 8 - 9 = -1^* (0)$

$IF = E_j - (L_i + t_E^{ij})$

$D \rightarrow = 13 - (11 + 3)$

$= -1^* (0)$

98

## Interferring Float (InF)

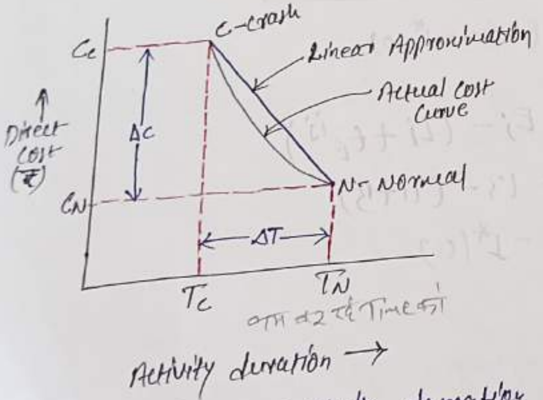
$$\boxed{InF = TF - FF}$$

# Crashing or Time cost

It is an extension of critical path method that consider a compromise b/w the time and cost required complete the project, the total cost of any project consist of direct and indirect cost involve in its execution

(i) Direct cost:-

It is the cost directly involve in the execution of an activity it include direct labour, Direct material, m/c equipment etc.




Crash time is the minimum activity duration to which an activity can be compressed by increasing the resources and hence by increasing direct cost. The slope of the line give the amount of increase in the direct cost per unit time for crashing an activity.

$$\boxed{\text{Cost time Slope} = \frac{\Delta C}{\Delta T} = \frac{C_c - C_N}{T_N - T_c}}$$

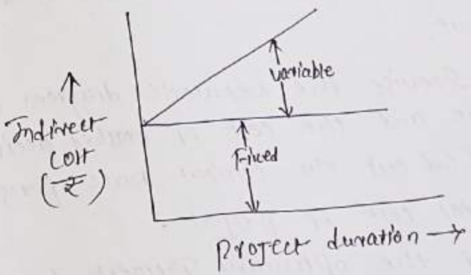
e.g. Normal — 10day Rs 8000

Crash — 6day Rs 18000

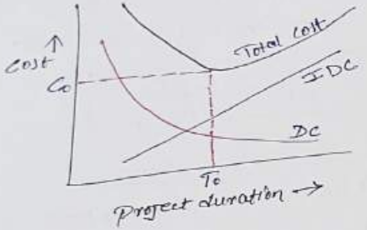
$$\frac{\Delta C}{\Delta T} = \frac{\text{Rs } 10000}{4 \text{ day}} = \boxed{\text{Rs } 2500/\text{day}}$$

# Indirect cost (Overhead cost)

It is a cost not directly involve in the execution of an activity but it is compulsory required for the safe and timely completion of project.




# Total cost




→ The objective of crashing a network is to determine optimum project duration corresponding to minimum cost of project, the steps involved are in the critical path. ① Select the critical activity having minimum cost slope.

② Reduce the duration of this activity by one time unit.

③ Revise the network diagram by adjusting the time and the cost of crashed activity.

④ Find out the critical path project duration and total cost of project.

⑤ If the optimum project duration is obtained then stop otherwise repeat from step ①.

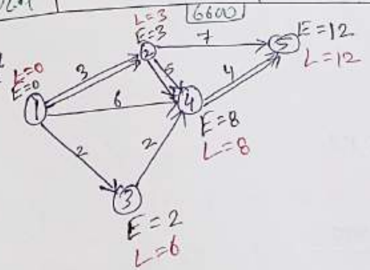
Ek gate academy lecture 10 50 min

Q. Draw the network diagram and crash the network to optimum project duration corresponding to minimum cost of project. It is given that indirect cost is Rs. 900/day.



| Activity (i-j) | Normal Time (days) | Normal Cost (₹) | Crash Time (days) | Crash Cost (₹) | $\Delta C / \Delta T = \frac{C_c - C_N}{T_N - T_c}$ |
|---|---|---|---|---|---|
| | $T_N$ | $C_N$ | $T_c$ | $C_c$ | |
| (1-2) C.P. | 3 | 500 | 2 | 1000 | 500 → $\frac{1000-500}{3-2}$ |
| 1-3 | 2 | 750 | 1 | 1500 | 750 → $\frac{1500-750}{2-1}$ |
| 1-4 | 6 | 1400 | 4 | 2600 | 600 |
| (2-4) C.P. | 5 | 1000 | 3 | 1800 | 400 |
| 2-5 | 7 | 1150 | 6 | 1450 | 300 |
| 3-4 | 2 | 800 | 2 | 800 | – ($\because \frac{0}{0}$ = indeterminate) |
| (4-5) C.P. | 4 | 1000 | 2 | 2400 | 700 |
| | | 6600 | | | |

Network Diagram




(2-4) can only be crashed max by only two days

Now $T_E = 12$ days

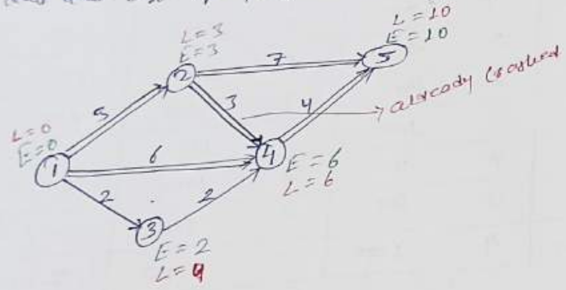
$T_C = DC + IDC$

$DC = ₹\,6600$

$IDC = 12 \times 900 = ₹\,10{,}800$ → Given in que

$T_C = ₹\,17400$

→ Now crashing the minimum cost slope activity along the critical path, It is activity (2)-(4) so crashing it by 2 Days, The revise Network Diagram and the cost of Project is as given below.




$T_E = 10$ days

$TC = DC + IDC$

$DC = ₹\,6600 + 400 \times 2$

$DC = ₹\,7400$

$IDC = 10 \times 900 = ₹\,9000$

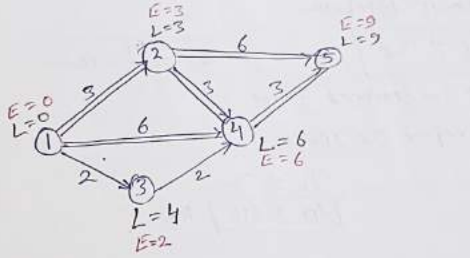
$TC = ₹\,16400$

→ Now the Network has three critical path and by crashing any one activity project duration does not change so we need to crash atleast two activity simultaneously which gives 3 options out of these we select option for which



the summation of cost slope is minimum.

1) 1-2 and 1-4 → 500 + 600 = 1100
2) 2-5 and 4-5 → 300 + 700 = 1000 minimum
3) 1-2 and 4-5 → 500 + 700 = 1200




$T_E = 9$ days

$TC = DC + IDC$ (1st crash, 2nd crash)

$DC = ₹\,6600 + 400 \times 2 + 1000$

$DC = ₹\,8400$

$IDC = 9 \times 900 = ₹\,8100$

$TC = Rs\ 16500$

Analysis Now

$TC = DC + IDC$

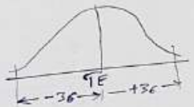
| 1st crash | 2nd crash |
|---|---|
| DC ↑ Rs 800 | DC ↑ Rs 1000 |
| IDC ↓ Rs 1800 | IDC ↓ Rs 900 |
| TC ↓ Rs 1000 | TC ↑ Rs 100 |

CW 8.1.5 (P-66)

$$\circ\overset{T}{\underset{K}{\text{———}}}\circ\overset{T}{\underset{K}{\text{———}}}\circ\overset{T}{\underset{K}{\text{———}}}\circ\overset{T}{\underset{K}{\text{———}}}\circ$$

$T_E = 4T$ and $\sigma = \sqrt{K^2+K^2+K^2+K^2} = 2K$

In Pert Project completion is assumed Normal distribution




$$\boxed{T_E \pm 3\sigma}$$

If nothing mentioned, we should prefer 3σ limits

$$T_E \pm 3\sigma$$

$$\boxed{4T \pm 6K}$$ Ans





# FORCASTING

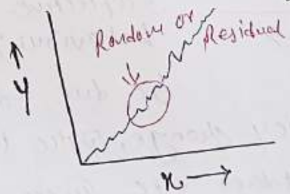
Forcasting can be termed as pridiction of Future sales or demand of a particular product. It is a projection, based upon past data and the art of human Judgement.

## # Need or benifits of forecasting :-

1. It helps in determining the vol. of production and production rate.
2. It form the basis for production budget, Labour budget material budget etc.
3. It is essential for product design and development;
4. It suggest the need for plant expansion,
5. It helps in determining the extent of marketing, advertising and destribution required.

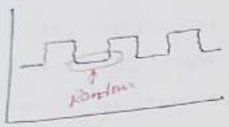
## Types of demand variatn :-

(1) Trend (T)




It shows a long term upward or downward movement in the demand pattern of a particular product

(2) Seasonal (S)

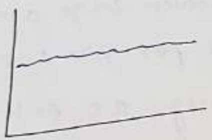



It shows a short term regular variation, and repeated after very short duration may be weekly or daily

(3) Cyclic (C)

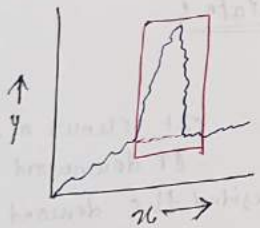

It shows long term wave like demand variation normally for a year or more

(4) Level (L) or constant



The demand almost remain constant w.r.t. some variable Normally time

(5) Irregular variation (I)




These variations are caused due to unusual circumstances which are not ~~reflected~~ reflective of normal behaviour, these may be due to Govt. Policy change, price hike, strike shutdown etc. These variations are always neglected while forecasting.



Types of Forecasting

- Forecast
  - Qualitative or Subjective ← long term (2-5 yrs) New product.
    - Judgemental
      - Opinion Survey
      - Market trial
      - Market research
      - Delphi Technique
  - Quantitative or objective
    - Time Series (Passive or No change in future)
      - Past Average
      - Simple moving Avg (SMA)
      - weighted moving Avg. (WMA)
      - exponential Smoothing
    - Causal or econometric (Active change in future)
      - Corelation
      - Regression.

Judgemental

This method is based upon the art of human Judgement i.e. how well a human being can predict the demand in future. This method does not require past data or the sales figure.

(i) Opinion Survey :- In this method openions are collected from the customer, retailer and distributer regarding the demand of a product. These informations are used while forecasting

(II) Market trial :- This method is applicable for new product and in that case product is introduced b/w the limited population. In the form of free sample the response on the limited population is used to project the demand for bigger population. This method is applied for low cost consumables like toothpaste, colddrink, chocolate, cosmetic item etc.

(3) market research :- In this method the work of survey is assigned to external marketing agencies and the purpose of research is to collect information regarding the demand of a product. The details about various factor which influence the demand like customer income, location, occupation, quantity, quality etc are related to get the forecast.

Both used for long range forecasting.

(4) Delphi Technique (expert openion) :- In this method OBJ

A pannel of expert are asked a sequencial question in which the response to one question is used to produce next question. It is a step-by-step procedure in which information available to some expert, is made available to other and the final forecast is obtained by the common opinion of all the expert.

Time series OBJ

In this method past data are arranged in some time model as dependent variable and time as independent variable. Based upon these past data

It is not used for very long range forecasting



we need to project the demand in future.

(i) Past average :-

In this method forecast is given by mean or avg. of the actual demand data for the previous period

(2) simple moving avg or rolling avg.

| year | Demand | Forecast |
|---|---|---|
| 2014 | 320 ·1 — WMA | |
| 2015 | 420 ·2 | |
| 16 | 390 0.3 | |
| 17 | 470 × ·4 | |
| 18 | 540 | |
| 19 | 560 | |
| 20 | 310 | |

n = no. of period for SMA

1st forecast = $(n+1)^{th}$

e.g. $n=3$, $1^{st} = 4^{th}$

This method uses past data and calculate the ~~moving~~ rolling average for a constant period. Fresh avg is computed at the end of each period by adding the actual demand data for the most recent period and deleting the data for older period. In this method as data changes from period to period It is formed as moving avg method. [illegible]

$SMA\ (n=4),\ F_{2018} = \frac{470+390+430+320}{4}$ [illegible]

$SMA(n=4),\ F_{2018} = 0.25 \times 470 + 0.25 \times 390 + 0.25 \times 420 + 0.25 \times 320$

$WMA\ (n=4),\ F_{2019} = 0.4 \times 470 + 0.3 \times 390 + 0.2 \times 420 + 0.1 \times 320$ ← $D_{2014}$, $D_{2017}$

(5) weighted moving avg:- This method is similar to simple moving avg. The only diff this method gives unequal weight to each demand data in such a manner that summation of all weight always equal to 1. The most present date is given the highest state and the weight assigned to the oldest data will be the least.

100

Sum of digit method

n = no. of period for WMA

(1) Find the sum of n natural no.,

$$\boxed{\Sigma n = \frac{n(n+1)}{2}}$$

(2) Arrange them in decreasing order of weight as

$\frac{n}{\Sigma n}, \frac{n-1}{\Sigma n}, \frac{n-2}{\Sigma n}$ ------- $\frac{1}{\Sigma n}$

a) $n = 4$

$\Sigma n = 10$ $\left\{\frac{n(n+1)}{2}\right\}$

$\frac{4}{10}, \frac{3}{10}, \frac{2}{10}, \frac{1}{10}$

(b) $n = 5$

$\Sigma n = 15$

$\frac{5}{15}, \frac{4}{15}, \frac{3}{15}, \frac{2}{15}, \frac{1}{15}$

Q. For the given data generate a forecast for each of the three period using simple moving average for n=3 period, and weighted moving average n=4 period. also find the forecast for the 9th 10th and 11th period

| Period | Demand | SMA, n=3 | WMA, n=4 |
|---|---|---|---|
| 1 | 340 ·1 | | |
| 2 | 520 ·2 | | |
| 3 | 460 ·3 | | |
| 4 | 550 ·4 | 440 | |
| 5 | 670 | 510 | 496 |
| 6 | 790 | 560 | 577 |
| 7 | 880 | 670 | 673 |
| 8 | 1030 | 780 | 778 |
| 9 | | 900 | 901 |
| 10 | | 900 | 901 |
| 11 | | 900 | 901 |

for WMA

$\frac{340+520+460}{3}$

$\frac{520+460+550}{3}$

$550\times0.4+(460\times0.3)+(520\times0.2)+(340\times0.1) = 496$

$670\times0.4+(550\times0.3)+(460\times0.2)+(520\times0.1) = 577$

# exponential Soothing method

This method requires only the current demand and the forecasted value for the current period to give the next forecast. It is a modified form of weighted moving average which gives weight to all the previous data but the weight assigned are in exponential decreasing order

P.T.O.

General form $\rightarrow F_t = \alpha \cdot D_{t-1} + \alpha(1-\alpha) \cdot D_{t-2} + \alpha \cdot (1-\alpha)^2 \cdot D_{t-3} + \alpha (1-\alpha)^3 \cdot D_{t-4} + \cdots \infty$

$$F_t = \alpha \cdot D_{t-1} + (1-\alpha) \cdot [\alpha \cdot D_{t-2} + \alpha(1-\alpha) \cdot D_{t-3} + \alpha(1-\alpha)^2 D_{t-4} + \cdots \infty]$$

$$\boxed{F_t = \alpha \cdot D_{t-1} + (1-\alpha) \cdot F_{t-1}}$$

or

S.N $\boxed{F_t \text{ or } F_i = F_{t-1} + \alpha(D_{t-1} - F_{t-1})}$

error $= e_i = \Delta i = D_i - F_i$

$$\boxed{F_t = F_{t-1} + \alpha \cdot e_{t-1}}$$

$\boxed{\alpha = \frac{2}{n+1}}$

S.N

$\alpha \rightarrow$ Smoothing constant and is equivalent to n period of simple moving avg. and is given by $\boxed{\alpha = \frac{2}{n+1}}$

S.N

NOTE $\rightarrow$ If for the initial period forecasted value is not given

① Take the actual demand for the 1st period equal to forecast $\boxed{D_1 = F_1}$ and proceed (e.g. 110) $\rightarrow$ next page

② Take the avg of the actual demand data as the forecast for the period and proceed.

Q. The sale of a product in a showroom for 5 month is as given below. With a smoothing constant of 0.4 find the forecast for the next month.



| Month | Sales $D_i$ | $F_i$ | error $e_i = D_i - F_i$ |
|---|---|---|---|
| | | 110 | 0 |
| 1 | 110 | 110 | 30 |
| 2 | 140 | 122 | −32 |
| 3 | 90 | 109.2 | −9.2 |
| 4 | 100 | 105.52 | 14.48 |
| 5 | 120 | 111.312 | |
| 6 | | | |

See 1st NOTE

$\boxed{F_t = F_{t-1} + \alpha \cdot e_{t-1}}$

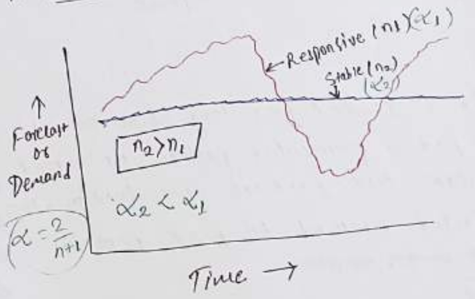
$F_i + \alpha(D_i - F_{i-1})$

$\boxed{110 + 0.4(140 - 110)}$

$122 + 0.4(90 - 122) = 109.2$

$= F_i + \alpha(D_i - F_i)$
$= 110 + \cancel{0.4(110 - 110)} = 0$
$= 110$

# Responsiveness and Stability




Responsive

Responsiveness indicate that the forecast pattern is fluctuating or swinging. It is preferred for new product and for that the no. of period is kept small.

Stability: It means that the forecast pattern is flat, smooth or has less fluctuation, it is preferred for old existing product and for that the no. of period is kept large.

S.N $\boxed{\alpha = \frac{2}{n+1}}$ $\boxed{F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1})}$

(1) If $\alpha = 0$ ($n \to \infty$) {limit of stability}

$\boxed{F_t = F_{t-1}}$

(2) If $\alpha = 1$ ($n = 1$) {limit of Responsiveness}

$\boxed{F_t = D_{t-1}}$

$0 \xleftarrow{\text{Stable}} \alpha \xrightarrow{\text{Responsive}} 1$

# Forecast error

when error is studied for long duration it becomes helpfull to find a particular pattern or trend which may regulate our future production. The most commonly used method to find forecast error, ~~mean absolute variati. Deviati~~

S.N $\boxed{e_i = D_i - F_i}$

## (1) Mean Absolute Deviation (MAD)

S.N

$$MAD = \frac{\sum_{i=1}^{n} |D_i - F_i|}{n}$$

It idicates the avg. magnitude of error made in every period without considering sign i.e. in absolute term

| S.No | $D_i$ | $F_i$ | $e_i$ $(D_i - F_i)$ |
|---|---|---|---|
| 1 | 120 | 140 | -20 |
| 2 | 180 | 150 | +30 |
| 3 | 160 | 170 | -10 |
| | | | 0 |

$MAD = \frac{60}{3} = 20$



## (2) Mean Forecast error (MFE) or Bias

$$\boxed{\text{MFE or Bias} = \frac{\sum_{i=1}^{n} (D_i - F_i)}{n}}$$

It measures the forecast error with regard to Direction and shown any tendency of over or under forecast, +ve Bias indicate understimated forecasting and −ve Bias indicate overstimated forecasting.

Running sum forecast Error (RSFE) $= \sum_{i=1}^{n} (D_i - F_i)$

S.N $\boxed{Bias = \frac{RSFE}{n}}$

## (3) Mean square error (MSE)

S.N

$$\boxed{MSE = \frac{\sum_{i=1}^{n} (D_i - F_i)^2}{n}}$$

MSE is used to compute std deviation for forecast error which is utilize to plot control chart for forecast error.

$\pm 3\sigma$ → 99.74%

$\sigma = \sqrt{MSE}$

$\sigma = 10$

(4) Mean Absolute % Error (MAPE)

$$\boxed{MAPE = \frac{\sum_{i=1}^{n} \left| \frac{D_i - F_i}{D_i} \times 100 \right|}{n}}$$ S.N

30% to 40%
↓ ↓
±15% ±20%

It is the avg of % error compare to actual demand and it is used to put error in prospective because there is diff. b/w 40 out of 100 and 40 out of 1000.

(5) Tracking Signal (TS)

$$\boxed{TS = \frac{RSFE}{MAD}}$$

It tells how well the forecaster is predicting the actual demand. A value of zero would be ideal but ±5 or ±4 is the acceptable range.

Q. The demand of a product has been shown below the expert forecasted sale of $\boxed{100 \text{ unit}}$ for the month of April with smoothing constant of 0.2. Find the forecast for the month of September also determine MAD, MSE, MAPE and Bias.



| Month | Demand $D_i$ | Forecast | $e_i$ ($D_i - F_i$) | $e_i^2$ | $\left\lvert \frac{e_i}{D_i} \times 100 \right\rvert$ |
|---|---|---|---|---|---|
| April | 120 | 100 (Given in que) | 20 | 400 | $\left\lvert \frac{20}{120} \times 100 \right\rvert = 16.6$ |
| May | 160 | 104 | 56 | 3136 | 35 |
| June | 200 | 115.2 | -15.2 | 231.04 | 15.2 |
| July | 80 | 112.16 | -32.16 | 1034.2656 | 40.2 |
| Aug | 130 | 105.728 | 24.27 | 589.03 | 18.66 |
| Sep. | n=5 | 110.58 | $\Sigma\lvert e_i\rvert = 147.63$; $\Sigma e_i = 52.91$ | $\Sigma e_i^2 = 5390.43$ | $\Sigma \left\lvert \frac{e_i}{D_i} \times 100 \right\rvert = 125.74$ |

Note

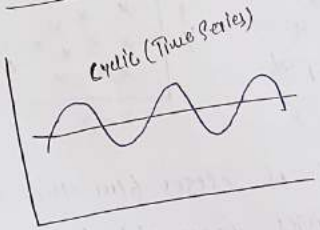
(1) $MAD = \frac{147.63}{5} = 29.526 \rightarrow \frac{\sum_{i=1}^{n} \lvert D_i - F_i \rvert}{n}$

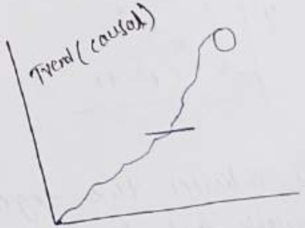
(2) $MSE = \frac{5390.43}{5} = 1078.086 \rightarrow \frac{\sum_{i=1}^{n} (D_i - F_i)^2}{n}$

(3) $MAPE = \frac{125.74}{5} = 25.148$

(4) $Bias = \frac{52.91}{5} = 10.582 \rightarrow \frac{\Sigma e_i}{n}$ $\rightarrow (D_i - F_i)\Sigma$

## # Causal or Econometric



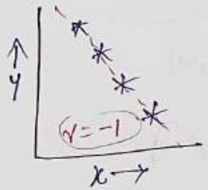
118 In this method forecaster try to establish the cause and effect relation b/w the demand of a product and any other variable on which demand is dependent. The objective is to establish a relation such that changes in one variable becomes usefull for the pridiction of other.

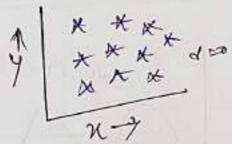
There are two method :-

① Correlation analysis :-

$r \rightarrow \boxed{+1 \; +0 \; -1}$

| x and y | |
|---|---|
| r = +0.47 | r = -0.62 |
| x ↑ 100 | x ↑ 100 |
| y ↑ 47 | y ↓ 62 |


It indicates the degree of of closes b/w the two variable and its value ranges from +1 to -ve. It is an indicator of extend to which knowledge of one variable becomes usefull for the pridiction of other. The correlation coefficient of the two variable x and y is given by

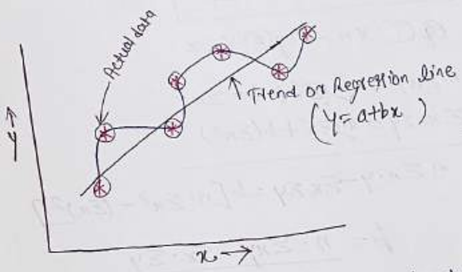
$$\boxed{r = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2 \cdot \Sigma(y-\bar{y})^2}}}$$

where

$\bar{x}$ and $\bar{y}$ are the avg value of individual x and y value

24-03-21

Linear regression




It is a mathematical technique of obtaining the line of best fit b/w the dependent variable which is usually the demand of a product and any other variable on which demand is dependent.

In regression analysis the relationship b/w some independent variable x and dependent variable y can be represented by straight line $\boxed{y = a + bx}$

where $a$ is intersept on y-axis and $b$ is the slope of line.

Dependent → $\boxed{y = a + bx}$ ← Independent — Trend line ——— ①

$n$ = no. of period of data

Taking $\Sigma$ both side of eq ①

$$\boxed{\Sigma y = an + b\Sigma x} \quad \text{——— ②}$$

eq ① × x

$$xy = a \cdot x + bx^2$$

Taking $\Sigma$ both side

$$\boxed{\Sigma x \cdot y = a \cdot \Sigma x + b \cdot \Sigma x^2} \quad \text{——— ③}$$

eq ③ × n − eq ② × $\Sigma x$

$$n \cdot \Sigma xy = an\cancel{\Sigma x} + bn\,\Sigma x^2$$

$$\Sigma x \cdot \Sigma y = an\cancel{\Sigma x} + b(\Sigma x^2)$$

$$n\,\Sigma x \cdot y - \Sigma x \cdot \Sigma y = b\left[n \cdot \Sigma x^2 - (\Sigma x)^2\right]$$

$$b = \frac{n \cdot \Sigma xy - \Sigma x \cdot \Sigma y}{n\,\Sigma x^2 - (\Sigma x)^2}$$

from eq ② $\boxed{a = \dfrac{\Sigma y - b \cdot \Sigma x}{n}}$



# Least square method

when the indipendent variable is linear and uniform and is in such a form that it can be modified to make $\Sigma x = 0$ then the calculation become very simple and the method is termed as least square method.

① $n$ = odd

| year | Demand ($y$) | ($x$) |
|---|---|---|
| 2014 | | −2 |
| 2015 | | −1 |
| 2016 | | 0 |
| 2017 | | +1 |
| 2018 | | +2 |
| | | $\Sigma x = 0$ |

uniform gap of 1

Similarly uniform

② $n$ = even

| year | Demand | $x$ | $x$ |
|---|---|---|---|
| 2015 | | −2.5 | −5 |
| 2016 | | −1.5 | −3 |
| 2017 | | −0.5 | −1 |
| 2018 | | +0.5 | +1 |
| 2019 | | +1.5 | +3 |
| 2020 | | +2.5 | +5 |
| | | $\Sigma x = 0$ | $\Sigma x = 0$ |

Q. A car manufacturer recently held road side car exhibition for the new model of car, the no. of salesman requested at each exhibition and the no. of car booked is as given below fit a linear regression equation and estimate the no. of car booked if 10 salesmen are employed in a exhibition.

| No. of Salesman (x) | No. of cars booked (y) | $x^2$ | $x \cdot y$ |
|---|---|---|---|
| 5 | 144 | 25 | |
| 8 | 176 | 64 | |
| 6 | 152 | 36 | |
| 8 | 182 | 64 | |
| 5 | 138 | 25 | |
| 9 | 196 | 81 | |
| 3 | 128 | 09 | |
| 6 | 158 | 36 | |
| 4 | 132 | 16 | |
| 6 | 146 | 36 | |
| $\Sigma x = 60$, $n = 10$ | $\Sigma y = 1554$ | $\Sigma x^2 = 392$ | $\Sigma x \cdot y = 9698$ |

$\boxed{\Sigma y = a \cdot n + b \cdot \Sigma x}$ — ②

$\boxed{\Sigma x \cdot y = a \cdot \Sigma x + b \Sigma x^2}$ — ③

$\boxed{1554 = 10 \cdot a + 60 \cdot b}$ — from ①

$\boxed{9698 = 60 \cdot a + 392 \cdot b}$ — from ②

$a = 85.275$

$b = 11.6875$

$y = 85.275 + 11.6875x$

$x = 10 \rightarrow$ Given 10 Salesmen

$y = \underline{202.56}$

WBIS (P-73)

Q. The sales of an automobile company is as given below. Forecast the demand for the next two year using least square method.

| Year | Sales (Cr) (y) | x | $x^*$ | $x^2$ | $x^* \cdot y$ |
|---|---|---|---|---|---|
| 2011 | 30 | -4.5 | -9 | | -270 |
| 2012 | 33 | -3.5 | -7 | | -231 |
| 2013 | 37 | -2.5 | -5 | | -185 |
| 2014 | 39 | -1.5 | -3 | | -117 |
| 2015 | 42 | -0.5 | -1 | | -42 |
| 2016 | 46 | +0.5 | +1 | | 46 |
| 2017 | 48 | +1.5 | +3 | | 144 |
| 2018 | 50 | +2.5 | +5 | | 250 |
| 2019 | 55 | +3.5 | +7 | | 385 |
| 2020 | 58 | +4.5 | +9 | | 522 |
| $n = 10$ | $\Sigma y = 438$ | $\Sigma x = 0$ | $\Sigma x = 0$ | $\Sigma x^2 = 330$ | $\Sigma x \cdot y = 502$ |

$\Rightarrow -844$

$\Rightarrow 1347$

$b = \frac{\Sigma x \cdot y}{\Sigma x^2} = \frac{502}{330} = 1.52$

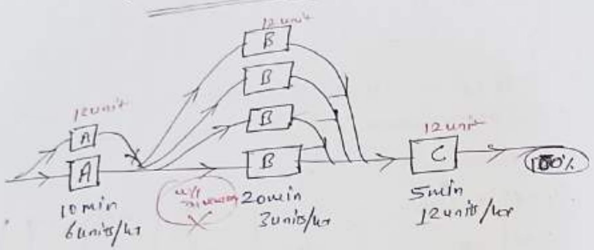
$a = \frac{\Sigma y}{n} = 43.8$

$\boxed{y = 43.8 + 1.52x}$

2021, $x = +11$, $y = 60.52$ Cr

2022, $x = +13$, $y = 63.56$ Cr

# (6) LINE BALANCING




$$\eta_{system} = \frac{\text{Actual output}}{\text{Installed output}}$$

The aim of assembly line is to devide total work content into different work station in such a manner that ideal time is minimized and utilization is optimized.

## Advantages

1. Decrease in work in process inventory.
2. Reduction in material handling cost.
3. effective utilization of man power and M/C.
4. uniform rate of production.
5. easy production control.



(1) Work element :-

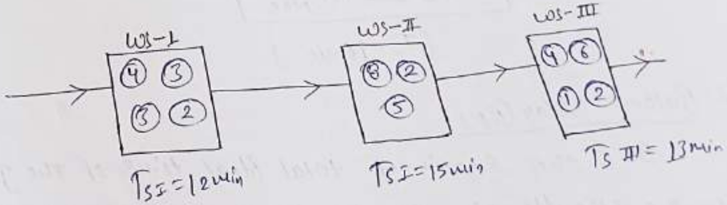
Every job is completed by a set of operation and each operation which is performed on the job is called work element.

(2) Task Time :- ($T_c$)

It is the std time required to complete work element.

(3) Work Station :-

It is the specific location on the assembly line where the given amount of work elements are completed within a fix period of time.




(4) Station Time:-

It is a time required to complete work element assigned in a work station.

126

## (5) Total work content (TWC)

It is the time required to complete one set of Job It is given by either the summation of all the elemental task time or the summation of all the station time.

$$\boxed{TWC = \Sigma T_{si} = \Sigma T_i = 40min}$$

## (6) Cycle time ($T_c$)

It is the ammount of time for which a Job that is to be assembeled remains in a work station. It is the timegap b/w two successive product coming out from the assembly line.

$$\boxed{T_c \geq (T_{si})_{max}}$$

$(T_c = 16min)$

## (7) Balance Delay (BD)

It is the ratio of total ideal time of the Job on the assembly line to the total time spend by the Job on the assembly line.

$$\boxed{BD\% = \left(\frac{n \cdot T_c - TWC}{n \cdot T_c}\right) \times 100}$$

where
$n$ = no. of work station
$T_c$ = cycle time
$T_{wc}$ = total work content

## (8) Line efficiency $\eta(\eta_L)$

S.N

$$\boxed{\eta_L = \frac{TWC}{n \cdot T_c} \times 100}$$

$$\boxed{\eta_L = 100 - BD\%}$$

## (9) Smoothness Index (SI)

It is a term used to represet the distribution of load b/w the different workstation compared to station consuming max^m time

$$SI = \sqrt{\sum_{i=1}^{n} (\text{Maximum Station time} - \text{Station time})^2}$$

S.N $$SI = \sqrt{\sum_{i=1}^{n} \left[(T_{si})_{max} - T_{si}\right]^2}$$

- Minimum no. of workstation required

$$\boxed{n_{min} = \frac{TWC}{T_c}}$$

e.g.
$n_{min} = \frac{40}{16} = 2.5 \approx 3$

## Method of line balancing

### (1) Longest candidate rule :-

**Step ①**
List all the element in the decreasing order of their task time

**Step ②**
To assign an element in a work station start from the begining of the list moving downward searching first feasible element which can be placed in a work station.

A feasible element is one that satisfy presidence requirement and when that element is placed in a workstation, the total time of workstation should not exceed the cycle time.

**Step ③**
Strike of the element which is assigned so that it cannot be considered again.

**Step ④**
Continue in the similar manner untill all the elements are assigned.

Q. For the following set of elements. Draw the precedence diagram, Balance the line and determine Balance delay, line efficiency, and smoothness index. Take the cycle time as 1min

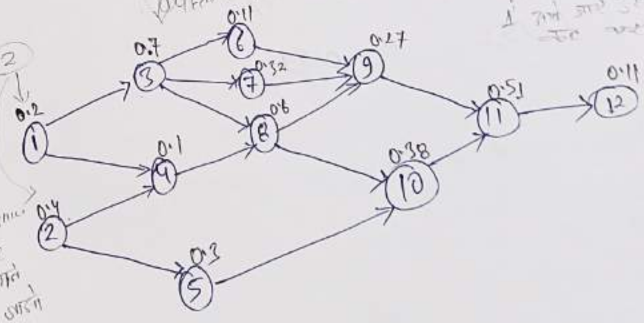
| Element | Ti(min) | Precedence |
|---|---|---|
| 1 | 0.2 | — |
| 2 | 0.4 | — |
| 3 | 0.7 | 1 |
| 4 | 0.1 | 1,2 |
| 5 | 0.3 | 2 |
| 6 | 0.11 | 3 |
| 7 | 0.32 | 3 |
| 8 | 0.6 | 3,4 |
| 9 | 0.27 | 6,7,8 |
| 10 | 0.38 | 5,8 |
| 11 | 0.51 | 9,10 |
| 12 | 0.11 | 11 |

③ Decreasing order

| Element | Ti(min) | Precedence |
|---|---|---|
| 3 | 0.7 | 1 |
| 8 | 0.6 | 3,4 |
| 11 | 0.51 | 9,10 |
| 02 | 0.4 | — |
| 10 | 0.38 | 5,8 |
| 7 | 0.32 | 3 |
| 5 | 0.3 | 2 |
| 9 | 0.27 | 6,7,8 |
| 1 | 0.2 | — |
| 6 | 0.11 | 3 |
| 12 | 0.11 | 11 |
| 4 | 0.1 | 1,2 |

④ work station without exceeding

⇒ ②



130

| WS | Element | $T_{ei}$ | $T_{si}$ | Idle time |
|---|---|---|---|---|
| I | 2<br>5<br>1<br>4 | 0.4<br>0.3<br>0.2<br>0.1 | 1.0 | 0 |
| II | 3<br>6 | 0.7<br>0.11 | 0.81 | 0.19 |
| III | 8<br>10 | 0.6<br>0.38 | 0.98 | 0.02 |
| IV | 7<br>9 | 0.32<br>0.27 | 0.59 | 0.41 |
| V | 11<br>12 | 0.51<br>0.11 | 0.62 | 0.38 |

maximum cycle time can be taken if not mentioned in objective

Here:, $n = 5$, $T_c = 1\text{ min}$, $T_{wc} = 4\text{ min}$

(1) Balance Delay $BD = 20\%$

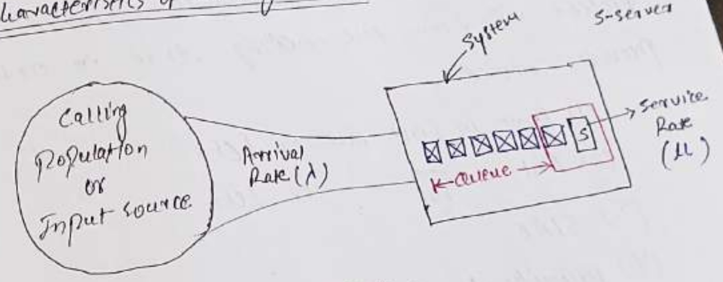
(2) $\eta_L$ (Line efficiency) $= 80\%$

(3) $SI = 0.59$

# 7 Queuing or Waiting line

The aim of queuing theory is achievement of an economic balance b/w the cost of providing service and the cost associated with the weight required for that service. It is applied to service oriented organization like production shop, work shop, repair shop, restaurant, Bank, ATM etc.

## Characteristics of queing model




## Arrival rate or Arrival pattern :-

The no of customer arriving per unit time is termed as arrival rate, customer arrival is random and therefore it is assumed to follow poison's distribution

Service rate or service pattern :- No. of customer serviced per unit time is known as service rate and it is assumed to follow exponential distribution (−ve exponential),


This graph is –ve exponential

✓ +ve
∵ expo [illegible]

## Service Rule or Service order

It gives information about the queue discipline which means the order by which customers are picked up from the waiting line in order to provide service.

(1) First in First out or FCFS → First come First serve
(2) Last ~~Last~~ First ——— or LCFS.
(3) SIRO
(4) priority treatment

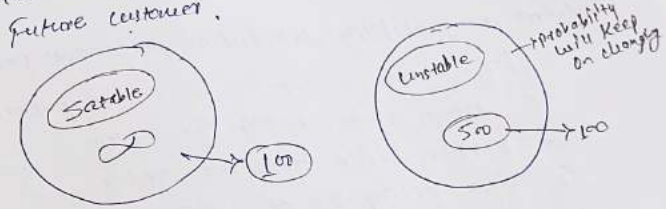
## System and calling population

System is the place or facility where customer arrive in order to get service and it's capacity may be finite or infinite.



The entire sample of customer from which few visit the system is termed as calling population or input source. It's capacity may be finite or infinite.

It is infinite when the arrival of few customer does not have any effect on the arrival of future customer.




Customer's Attitude
- Patient
- Impatient
  - Jockey
  - Balking
  - Reneging
  - Cheater

**Jockey**
customer keep on changing queque in hope to get service faster.

**Balking**
customer does not Join the queque and leave the system as queque is very long.

**Reneging** → customer Join the queque for short duration and when queque is moving very slow, then leave the system.

**Cheater** → customer take illegal means like fighting, bribing etc in hope to get service faster.

134

## Representation of queuing model

~~The~~ queuing model are represented by Kendall and Lee notations whose general form is Kendal and Lee Notations

$(a/b/c) : (d/e/f)$

where a = probability distribution for arrival pattern

b = ———— " ———— service pattern

c = No. of server within the system

d = service rule or service order

e = size or capacity of system

f = size or capacity of incoming population.

### Symbols

**a & b**

M — markovian (poisson) ⇒ for arrival pattern or exponential service pattern.

E — Erlangian (Gamma) ⇒ for arrival or service pattern

D — Deterministic ⇒ Arrival or service pattern.

Symbols

c

1, 2, 3, 4, 5 ...

135

**d**

(1) FIFO or FCFS

(2) LIFO or LCFS

(3) SIRO

(4) GD

**e and f**

N — Finite

$\infty$ — Infinite

Arrival Rate = $\lambda$ (poisson)

↳ $\lambda = 15$ customer/hr

Inter-Arrival Rate (exponential) $= \frac{1}{\lambda} = \frac{1}{15}$ hr/cust $=$ 4 min/cust.

Service Rate $= \mu$ (exponential)

↳ $\mu = 20$ cust/hr

Inter-service Rate $= \frac{1}{\mu} = \frac{1}{20}$ hr/cust $=$ 3 min/cust

Poison

(1) If $\lambda > \mu$ → Queue length will keep on increasing and after certain period of time incoming population will not get service. In these condition we can not find sol^n as ultimately system failed.

($\lambda$ → Arrival rate, 50 cust/hr; $\mu$ → service rate, 20 cust/hr)

(2) $\boxed{\lambda \le \mu}$ → System works

$\boxed{\lambda < \mu}$ → prefer

$$\boxed{\rho = \frac{\text{Arrival Rate}}{\text{Service Rate}} = \frac{\lambda}{\mu}}$$

rho

$$\boxed{\mu = 20 \text{ cust/hr}}$$

(1) $\lambda = 5$, $\rho = \frac{5}{20} = 0.25$

(2) $\lambda = 10$, $\rho = \frac{10}{20} = 0.50$

(3) $\lambda = 15$, $\rho = \frac{15}{20} = 0.75$

(4) $\lambda = 20$, $\rho = \frac{20}{20} = 1.00$

The ratio of arrival to service rate indicate the % time server is busy and is known as utilization factor, avg utilization, system utilization, channel efficiency and clearing ratio.

It also indicates the probability that a new customer need to wait.

formulae

① probability that the system is Ideal or probability of zero customer in the system.

$$\boxed{P_0 = 1 - \rho}$$

② Probability of having exactly ~~in~~ n customer in the system

$$\boxed{P_n = \rho^n \cdot P_0}$$

$$P_0 + P_1 + P_2 + P_3 + \dots = 1$$

$$P_3 + P_4 + \dots = 1 - (P_0 + P_1 + P_2)$$

$$= 1 - (P_0 + \rho^1 P_0 + \rho^2 \cdot P_0$$

③ Avg. No. of customer in the system:- In this we include both the customers waiting in the queue along with those getting service

$$\boxed{L_s = \sum_{n=0}^{\infty} n \cdot P_n}$$

or

$$\boxed{L_s = \frac{\rho}{1-\rho}}$$

or

④ Avg. No. of customer in the queue:- In this we don't include the customer getting serviced.

$$\boxed{L_q = \sum_{n=2}^{\infty} (n-1) \cdot P_n}$$

$$\boxed{L_q = \frac{\rho^2}{1-\rho} = L_s * \rho = L_s \cdot \rho}$$

158

**Little's Law** :- For a stable system avg. no. of customer in the system or queque is given by avg. customer arrival rate multiplied by avg waiting time of the customer in the system or queque

$$\boxed{L_s = \lambda \cdot W_s} \rightarrow \boxed{W_s = \frac{L_s}{\lambda}}$$

$$\boxed{L_q = \lambda \cdot W_q} \qquad \boxed{W_q = \frac{L_q}{\lambda} = W_s - \frac{1}{\mu}}$$

where $W_s$ = avg waiting time of the customer in system.

$W_q$ = avg waiting time of the customer in the queque.

If not mentioned in que.

Average → system

mean → queue

WB-13

Q. The no. of persons arriving in the service center is 8 cust/hr and the service provider takes 5 min/cust on an avg then determine.

(i) $L_s$ and $L_q$

(II) $W_s$ and $W_q$

⇒ ~~Lets~~ $\lambda = 8$ cust/hr

$\frac{1}{\mu} = 5$ min/cust

$\mu = 12$ cust/hr {60 min × }

$\rho = \frac{8}{12} = \frac{2}{3}$ → arrival rate / → service rate

(1) $L_s = 2$ → $L = \frac{\rho}{1-\rho}$  (2) $W_s = 15$ min

$L_q = \frac{4}{3}$  $W_q = 10$ min

25-03-21

Q. A Shopekeeper service 10 customer/hour and the customer arrival is 6customer/hr find the probability that atleast two customer waiting in the que.

⇒ Atleast 2 in the queue means atleast 3 in the system.

$P_3 + P_4 + \text{------} = 1 - (P_0 + P_1 + P_2)$

$= 1 - (\underset{0.4}{P_0} + \underset{0.4\times0.6}{\rho^1 \cdot P_0} + \underset{0.6^2\times0.4}{\rho^2 \cdot P_0}) = \rho^n$

Probability of atleast n the customer in the system

$= 0.216$

$\rho = 0.6$

$P_0 = 1-\rho = 1-0.6 = 0.4$

Q.14 page 70 W/B

$\lambda = 6$ mc/hr, Idle mc. Rate = $\frac{Rs15}{hr \cdot mc}$

~~All~~ $\frac{1}{\mu_A} = 6$ min/mc, $\mu_A = 10$ mc/hr

$\rho_A = \frac{\lambda}{\mu_A} = \frac{6}{10} = 0.6$, $(\text{Salary})_A = Rs\ 8/hr$

**B** $\frac{1}{\mu_B} = 5\,min/mc$, $\mu_B = 12\,mc/hr$

$\rho_B = \frac{\lambda}{\mu_B} = \frac{6}{12} = 0.5$, $(salary)_B = Rs\,10/hr$

8hr-shift

Total cost = cost of Idle mc + salary

**A** $(Ls)_A = \frac{\rho_A}{1-\rho_A} = \frac{0.6}{0.4} = 1.5\,mc$

$\left(\begin{matrix}\text{Idle cost}\\ \text{of m/c}\end{matrix}\right)_A = (Ls)_A \times \underset{\text{Rate}}{\text{Idle m/c}} \times \text{shift duration}$

$= 1.5\,mc \times \frac{Rs\,15}{hr \cdot mc} \times 8\,hr = Rs\,180$

$(Salary)_A = Rs\,8/hr \times 8\,hr = Rs\,64$

$(T.C)_A = Rs\,244$

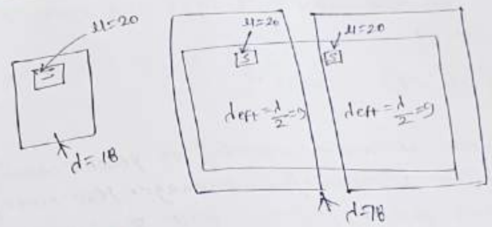
**B** $(Ls)_B = \frac{\rho_B}{1-\rho_B} = 1\,mc$

$\left(\begin{matrix}\text{Idle cost}\\ \text{of mc}\end{matrix}\right)_B = 1\,mc \times \frac{Rs\,15}{hr \cdot mc} \times 8\,hr = Rs\,120$

$(Salary)_B = Rs\,10/hr \times 8\,hr = Rs\,80$

$\boxed{(TC)_B = Rs\,200}$






**Imp Formulae**

① Avg length of non ~~ATQ~~ - empty Queue or avg length of queue containing atleast one customer

$$\boxed{Lq' = \frac{1}{1-\rho}}$$

② Probability of n arrival in the system during period T

within

$$\boxed{P(n,T) = \frac{(exp)^{-\lambda T} \cdot (\lambda T)^n}{n!}}$$

③ Probability that more than capability time period is required to service a customer.

$$\boxed{P = (exp)^{-\mu \cdot T}}$$ → for more than

④ Probability that the waiting time in the queue is greater than T

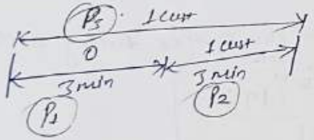
$$\boxed{P(W_q > T) = \rho \cdot (exp)^{-\frac{T}{W_s}}}$$

(5) probability that the waiting time in the system is greater than T

$$P(W_s > T) = (\exp)^{-\frac{T}{W_s}}$$

(6) Customer arrive at a shop according to poison's distribution with a mean of 10 customer/hr. Manager notes that no customer arrived for the first 3 minute when the shop opens. The probability that a customer arrives within the next three minute

⇒ $$\boxed{P(n,T) = \frac{(\exp)^{-\lambda T}\cdot(\lambda\cdot T)^n}{n!}}$$




$P_1 \times P_2 = P_3$ —— (1)

Now

$\boxed{n=0}$

P(0, 3min)

$\lambda\cdot T = 10 \times \frac{3}{60} = 0.5$

$P_1 = \frac{(\exp)^{-0.5}\cdot(0.5)^0}{0!} = 0.606$

P(1, 6min)

$\lambda T = \frac{10\times 6}{60} = 1$

$P_3 = \frac{(\exp)^{-1}\cdot(1)^1}{1!} = 0.367$

From eq (1)

$P_1 \times P_2 = P_3$

$P_2 = \frac{0.367}{0.606} = 0.6056$

Ans



# 8 LINEAR PROGRAMMING

LP is used for optimization of limited resources when there are no. of alternate soln possible for the problem. It is a mathematical technique and the term linear is used for the variable and it simply means that the relationship b/w different variables are such that they can be represented in the form of st. line.

## Requirement of LP

(1) objective function

It is the main funcn which we need to optimize and it should be clearly identifiable and measurable in quantitative term like maximization of profit, sales or minimization of cost.

(2) Constraint or condition or restriction

These are the resistriction or limited resources within which we need to optimize our objective function.

(3) All the variable should be linear and not -ve

Law or rule in LP

P+0

① Law of certainty :- 144

In LP model the various parameter like objective function coefficient, constraints and resources are known exactly and their value does not change with time.

② Law of proportionality

③ Addition

④ Law of continuity of divisibility

In LP model descision variables are continuous that they are permited to take any non -ve value that satisfy all the constaints.

## General statement of Linear Programming :-

objective funct^n → $\max Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$ ← or profit coefficient

Resource values

Contraint or Condition
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le b_m \end{cases}$$

non -ve condition → $x_1, x_2 \dots x_n \ge 0$

क्योंकि -ve में produce नहीं कर सकते

where $a_{ij}, b_i, c_j$ are constant and $x_j$ is variable

$i = 1, 2, \dots m$

$j = 1, 2 \dots n$

where $a_{ij}$ are technological coefficient or substitution

$b_i$ → resource value (optimise की मात्रा)

$c_j$ → profit coefficient

① अगर sell कर रहे हैं और profit कमा रहे हैं तो Maximize करेंगे Profit को।

$x_j$ → descision or choice variable

② अगर कुछ चीज purchase कर रहे हैं तो minimise करेंगे।

Graphical Steps → used only for linear variable (only used for variables)

① Identify the problem and define objective function and constraint.

② Draw a graph that include all the constraint and Identify the common feasible reason

③ Find out the point within the feasible region that optimises the objective fn. This point gives the final sol^n.

Q. 36 page 84 w/B.

| M/C → / product ↓ | P | Q | R | Profit/unit |
|---|---|---|---|---|
| $x_1$ – A | 10 | 06 | 5 | 60 |
| $x_2$ – B | 7.5 | 9 | 13 | 70 |
| max hrs / week | 75 | 54 | 65 | |

→ $Z = 60x_1 + 70x_2$

Step ① Key descision is to determine no. of units produced of product A and B in a week let these are $x_1$ and $x_2$ respectively.

PTO

(3) feasible alternative are all the value of $x_1, x_2$ greater than equals to zero.

(5) objective is to maximize weekly profit when the profit per unit is given so the objective $f^n$,

$x_1, x_2, x_3, x_4 \to$ Decision variable

$\to$ profit coefficient

Max. $\boxed{Z = 60x_1 + 70x_2}$

objective $f^n$

(4) restriction is on the maximum machining time available for the three machines in a week so the contraints are

constraints

$\to$ for $x_2$, Let $x_1 = 0$, $x_2 \to (0,10)$ for $x_1$, Let $x_2 = 0$, $x_1 \to (7.5, 0)$

Towards origin

$P \to 10x_1 + 7.5x_2 \le 75$

$Q \to 6x_1 + 9x_2 \le 54$

$R \to 5x_1 + 13x_2 \le 65$

Resource value

$10x_1 + 7.5x_2 = 75$

e.g. for (B) point in graph the line that cut each other are (I), III so, By eqn a.R

$30x_1 + 45x_2 = 270$

$30x_1 + 78x_2 = 390$

$-33x_2 = -120$

$\boxed{x_2 = 3.63}$

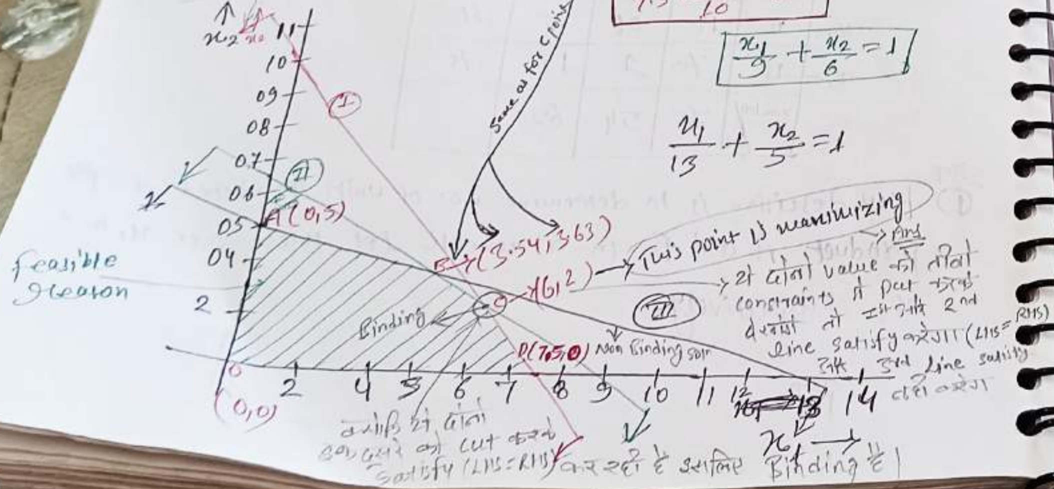
$\boxed{x_1 = 3.54}$

(6) all the contraints are ploted on a graph to get the feasible reagion

$$\boxed{\frac{x_1}{7.5} + \frac{x_2}{10} = 1}$$

$$\boxed{\frac{x_1}{9} + \frac{x_2}{6} = 1}$$

$$\frac{x_1}{13} + \frac{x_2}{5} = 1$$




Optibility

Now input the values of the corner point of the feasible reason in the objective fn the point which optimise the objective function give the final soln

$z(A) = 60 \times 0 + 70 \times 5 = 350$

$z(B) = 60 \times 3.54 + 70 \times 3.63 = 466$

$z(C) = 60 \times 6 + 70 \times 2 = \underline{500}$ — we have to maximize so 500 is maximum profit here

$z(D) = 60 \times 7.5 + 70 \times 0 = 450$

$z(O) = 60 \times 0 + 70 \times 0 = 0.$

To find feasible region will be inside or outside put $x_1, x_2 = 0$ in every line of P, Q, R and verify that it is inside equal or greater Resource value

$\boxed{x_1 = 6}$, $\boxed{x_2 = 2}$

one of the vertex of the feasible region give the final soln because objective function is straight line with a constent slope. and as it moves away from the ~~region~~ origin its value increases and the optimum value will be at one of the extream point. objective funcn will be tangent to that point and give the optimum soln

$x_1 = 6$, $x_2 = 2$ जो value (1st और 2nd line) को satisfy कर रहे हैं so these are Binding line

## Binding and Non-Binding constraints

when we put the values of optimum soln in the constraint and LHS = RHS the contraints is termed as Binding otherwise non Binding.

146

Final soln is always obtained from the binding constraint.

Q. Solve the following (L.P.) fn

$$Min\ Z = 6x_1 + 4x_2$$

away from origin

P → $3x_1 + 3x_2 \geq 40$

Q → $3x_1 + x_2 \geq 40$

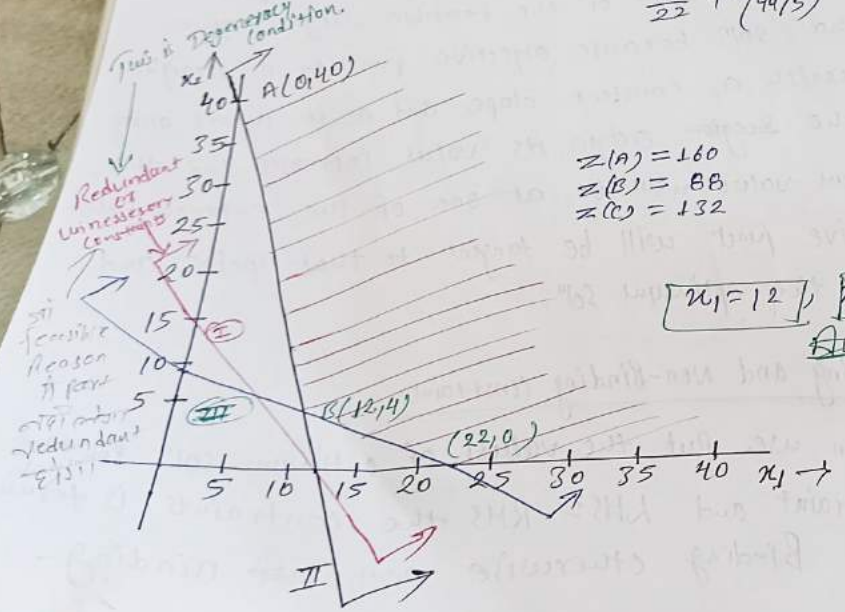
R → $2x_1 + 5x_2 \geq 44$

⇒ (P) $3x_1 + 3x_2 \geq 40$

P: $\boxed{\frac{x_1}{40/3} + \frac{x_2}{40/3} = 1}$

Q: $\boxed{\frac{x_1}{40/3} + \frac{x_2}{40} = 1}$

$$\frac{x_1}{22} + \frac{x_2}{(44/5)} = 1$$




$Z(A) = 160$

$Z(B) = 88$

$Z(C) = 132$

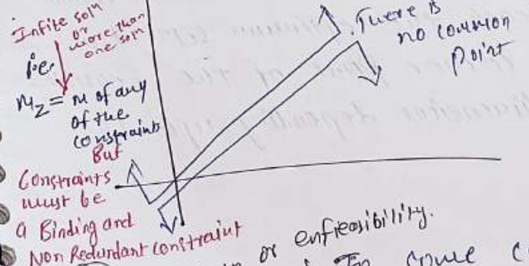
$\boxed{x_1 = 12}$, $\boxed{x_2 = 4}$ Ans

## Redundant

Constraint which does not become part of the boundary making a feasible region is termed redundant constraint. The inclusion or exclusion of such constraint does not have any effect on the final optimum soln of the problem.

## Special cases

① Infinite or multioptimum soln :- It means we get the same optimum value of objective function for different varying variable. We always get a unique soln when the slope of objective fn is different from constraint. Infinite no. of soln is obtained when the slope of objective fn becomes equal to one of the binding constraint.

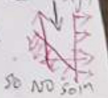



Unique soln →

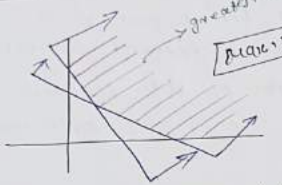
Slope of Z: $m_Z \neq$ slope of any of the constraints.

See in Nb 130

② No soln or infeasibility → In some condition constraint may be inconsistant in such a manner it is not possible to find a feasible soln with satisfy all the constraint. There is no soln to such problem.



(5) unbounded solⁿ :-




In some condition the highest value ~~for~~ of objective fⁿ goes upto infinite and it simply means that the common feasible ~~reation~~ region is not bounded by the limit on the constraints. It is termed as unbounded solⁿ

# Simplex

It is a step by step procedure in which we proceed in a systematic manner from an initial feasible solⁿ with an improve upon the initial solⁿ until in certain no. of steps we reach the optimum solⁿ this method also check the corner point of the feasible region but in multidimensions depending upon the no. of variables.

Std Form for simplex

(1) All the resource value for the given constraint should be non -ve

(2) All the inequalities of the given constraints should be converted into equalities.

$3x_1 + 2x_2 \le 70$

$3x_1 + 2x_2 + s_1 = 70$

$s_1$ ↑ slack variable

$4x_1 + 5x_2 \ge 85$

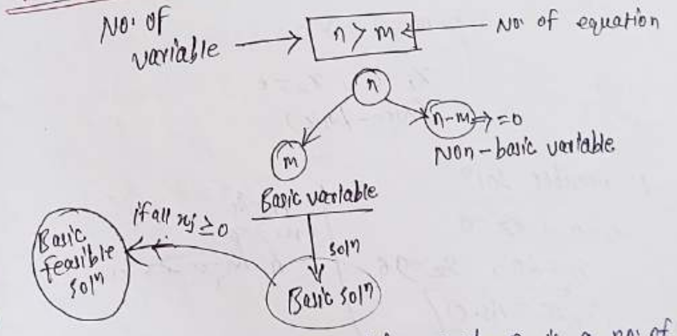
$4x_1 + 5x_2 - s_2 = 85$

$s_2$ ↑ surplus variable

(3) each of the decision variable for objective fⁿ and constraint should be linear and non -ve

all $\boxed{x_j \ge 0}$

No. of solutions




If there are m equation and n is a no. of variable and n is greater than m then we need to put n-m variable equal to zero (n-m = 0) known as non basic variable and solve the remaining m basic variable

152

variable to get basic solⁿ this step reduces the no. of alternate solⁿ whose maxᵐ limit is given by

Variable → $\boxed{{}^nC_m = \dfrac{n!}{m!(n-m)!}}$ ← Equation

Q. 38 P-55

⇒ $\max. z = 40x_1 + 35x_2$

$\max. z = 40x_1 + 35x_2 + 0S_1 + 0S_2$ — ① → adding slack and surplus variable to convert in equality.

zero = coefficient of $S_1$ and $S_2$

R.M (kg) → $2x_1 + 3x_2 \leq 60$

Lab (hrs) → $4x_1 + 3x_2 \leq 96$

$2x_1 + 3x_2 + S_1 = 60$ — ①

$4x_1 + 3x_2 + S_2 = 96$ — ②

No. of variables $x_1, x_2, S_1, S_2$

$n = 4$

$m = 2$ → No. of constraint (eqⁿ)

$n - m = 2 \Rightarrow 0$

$x_1 = 0$, $x_2 = 0$

(Non-basic)

4 variables है 2 ही eqn तो 2 eqn से दो ही variable Solve किये जा सकते है इसलिए बाकी दो variable को हमको 0 (zero) रखना पड़ता है। जिसकी value zero रखते हैं या जिनकी value zero होती है उन्हें Non-Basic कहते हैं और जिनकी value zero नहीं होती उन्हें Basic कहते हैं।

1st feasible solⁿ

$x_1 = 0$, $x_2 = 0$

$S_1 = 60$, $S_2 = 96$

$\therefore \boxed{Z = Rs. 0}$

$n = 4$

$m = 2$

$n - m = 2 \Rightarrow 0$

153

| $C_i$ | Basis | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $b_i$ | $\theta_i = \frac{b_i}{a_{ij}}$ |
|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 2 | 3 | 1 | 0 | 60 | 30 |
| 0 | $S_2$ | (4)* | 3 | 0 | 1 | 96 | 24 ← Key Row |
| | $C_j$ | 40 | 35 | 0 | 0 | | |
| | $Z_j = \Sigma C_i \cdot a_{ij}$ | 0 | 0 | 0 | 0 | | |
| | $\Delta_j = C_j - Z_j$ | 40 | 35 | 0 | 0 | | |

coeff. of $S_1$, coeff. of $S_2$

$\theta_i$ ← Replacement Ratio

24 → minimum (outgoing variable)

$C_j$ → max. of z from eq (0)

$Z_j = 2\times0 + 4\times0 = 0$ → for $x_1$, similarly for $x_2$

NER — Net evaluation Row or net opportunity cost Row

↑ Key column — maximize

⇒ Calculate $\Delta_j$ value as the diff of $C_j$ and $Z_j$ rows and it is termed as Net evaluation row or Net opportunity cost row. The value of $\Delta_j$ row give the amount of increase or decrease in the objective funcⁿ that would occur if 1 unit represented by the column head is brought into the current solⁿ. A simplex table indicate the current solⁿ to be optimum when all the values in the $\Delta_j$ row are

① −ve or zero when LP is for maximizatⁿ.

② +ve or zero when LP is for minimization.

The current problem is for maximizatⁿ so we select highest +ve value in a $\Delta_j$ row. The selected column is called Key column and the variable in the column head as incoming variable. Now Divide $b_i$ value by corresponding element of Key column to get replacement ratio column. In this column we select the minimum +ve value, the selected row is called key row, and the variable in the row as outgoing variable. The element at the intersection of key column and key row is termed as key element.

- Key element is converted by multiplying or dividing key row by a common multiplying factor. All the element in the key column are made zero except key element which will be unity this is done by adding or subtracting by proper multiples of key row from other row. In the new table outgoing variable is replaced by incoming variable.

| $C_i$ | Basic | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $b_i$ | $\theta_i$ |
|---|---|---|---|---|---|---|---|
| 0 | $s_1$ | 0 | $3/2$* | 1 | $-1/2$ | 12 | $\frac{12}{3/2}$ = 8 ← Key Row → outgoing |
| 40 | $x_1$ | 1 | $3/4$ | 0 | $1/4$ | 24 | 32 → $\frac{24}{3/4}$ |

Key element

$R_1 - 2R_2$ $(60 - 2\times 24)$

$R_2 \rightarrow R_2 \div 4$

$R_1 \rightarrow R_1 - 2\times R_2$

2nd feasible soln maxm

$x_1 = 24,\ x_2 = 0$

$S_1 = 12,\ S_2 = 0$

$Z = Rs\ 9600$

$[0\times 0 + 1\times 40]$

| | | | | |
|---|---|---|---|---|
| $C_j$ | 40 | 35 | 0 | 0 |
| $Z_j = \Sigma c_i a_{ij}$ | 40 | 30 | 0 | 10 |
| $\Delta_j = C_j - Z_j$ | 0 | 5 | 0 | $-10$ |

Key Column



| $C_i$ | Basis | $x_1$ | $x_2$ | $s_1$ | $s_2$ | $b_i$ |
|---|---|---|---|---|---|---|
| 35 | $x_2$ | 0 | 1 | $2/3$ | $-1/3$ | 8 |
| 40 | $x_1$ | 1 | 0 | $-1/2$ | $1/2$ | 18 |

$R_1 \rightarrow R_1 \times \frac{2}{3}$

$R_2 \rightarrow R_2 - \frac{3}{4}\cdot R_1$

3rd feasible soln

$x_1 = 18,\ x_2 = 8$

$S_1 = 0,\ S_2 = 0$

$Z = Rs\ 1000$

| | | | | |
|---|---|---|---|---|
| $C_j$ | 40 | 35 | 0 | 0 |
| $Z_j$ | 40 | 35 | $10/3$ | $25/3$ |
| $\Delta_j = C_j - Z_j$ | 0 | 0 | $-10/3$ | $-25/3$ |

ये zero आना चाहिए अगर नहीं आ रहा तो कहीं तो गलती कर रहे हो।

$\because\ x_2 - s_1 \rightarrow 2/3$

$8 - \frac{2}{3} = \frac{22}{3} \times 3 = 22\ kg$

+

$x_1 - s_1 \rightarrow -\frac{1}{2}$

$18 - \left(-\frac{1}{2}\right) = \frac{37}{2} \times 2 = \frac{37\ kg}{59\ kg}$

$x_1 - s_2 \rightarrow \frac{1}{2}$

$18 - \frac{1}{2} = \frac{35}{2} \times 4 = 70\ hrs$

+

$x_2 - s_2 \rightarrow -\frac{1}{3}$

$8 - \left(-\frac{1}{3}\right) = \frac{25}{3} \times 3 = \frac{25\ hrs}{95\ hrs}$

## Big M Method

It is a modified form of simplex and is always required whenever the constraints are $\geq$ or $=$ type irrespective of wether the problem is for maximization or for minimization, In these condition we introduce an artificial variable in the current soln to get an initial working matrix. These artificial variable must not appear in the final soln and this is ensured by providing extreamly -ve value to their profit coefficient in the objective function.

for $Max = -M \cdot A_1$

for $min = +M \cdot A_1$

where M is a no. higher than any finite no..

### Special cases

(1) **Infinite or multi optimum soln** :- When a non basic variable in an optimum soln has zero value for $\Delta_j$ row then the soln is not unique and it indicates that The problem has ~~unbounded~~ Infinite no. of soln.



| Basis | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $S_3$ (Non-basic) | $b_i$ |
|---|---|---|---|---|---|---|---|
| $x_3$ | | | | | | | |
| $S_2$ | | | | | | | |
| $x_1$ | | | | | | | |
| $\Delta_j$ | 0 | | 0 | | 0 | 0 | |

$\theta_i = b_i/a_{ij}$

$\boxed{\text{-ve or } \infty}$

(2) **unbounded soln** :- If in a case all the variables in the replacement ratio column are either -ve or $\infty$ then the soln terminate and it indicates that the problem has unbounded soln.

(3) **NO soln or infeasibility** :- when in the final soln artificial variable remains in the basis then there is no feasible soln to the problem.

| Basis | $x_1$ $x_2$ $S_1$ $S_2$ | $b_1$ |
|---|---|---|
| $A_1$ | | |
| $S_2$ | $\geq$ or $=$ | |
| $\Delta_j$ | -ve or 0 | |

(4) **Degenerate soln**

when one or more of the basic variable becomes equal to zero during calculation then the soln is called degenerate. in degenerate soln the no. of basic variable become less than constraint.

## Duality

$$\boxed{\text{Primal}} \xleftrightarrow{\text{Transpose}} \boxed{\text{Dual}} \rightarrow ②$$

★ OBJ.
The initial given problem is known as primal and the problem obtained by transposing row and column the problem obtained by transposing row and column but having the same optimum value of objective fn is termed as ~~dual~~ dual.

Primal

(1) Maxini
$\leq$ type constraint

(2) min
$\geq$ type constraints

| Primal | | Dual |
|---|---|---|
| max | $\longleftrightarrow$ | min |
| $n$ | $\longleftrightarrow$ | $m$ |
| $m$ | $\longleftrightarrow$ | $n$ |
| $b_i$ | $\longleftrightarrow$ | $c_j$ |
| $c_j$ | $\longleftrightarrow$ | $b_i$ |
| $\leq$ type constraint | $\longleftrightarrow$ | Non-ve variable |
| = type constraint | $\longleftrightarrow$ | Unrestricted sign variable |

Q. Find the dual for the following LP problem.

$n=3$, $m=5$ $\quad$ $n=5$, $m=3$

$$\text{min. } Z = 8x_1 - 13x_2 + 9x_3$$

PTO



$$2x_1 - x_2 + 5x_5 \geq 7$$
$$3x_2 - 2x_3 \leq 6 \longrightarrow -3x_2 + 2x_3 \geq -6$$
$$5x_1 + 4x_3 \geq 8$$
$$3x_1 + 2x_2 - 3x_3 = 0$$
$$\rightarrow 3x_1 + 2x_2 - 3x_3 = 10$$
$$\rightarrow 3x_1 + 2x_2 - 3x_3 \leq 10 \longrightarrow -3x_1 - 2x_2 + 3x_3 \geq -10$$

$$\text{max. } W = 7y_1 - 6y_2 + 8y_3 + 10y_4 - 10y_5$$
$$2y_1 + 5y_3 + 3y_4 - 3y_5 \leq 8$$
$$-y_1 - 3y_2 + 2y_4 - 2y_5 \leq -13$$
$$5y_1 + 2y_2 + 4y_3 - 3y_4 + 3y_5 \leq 9$$
$$y_1, y_2, y_3, y_4, y_5 \geq 0$$

☹ correctn

$$\text{max. } w = 7y_1 - 6y_2 + 8y_3 + 10y_6$$
$$2y_1 + 5y_3 + 3y_6 \leq 8$$
$$-y_1 - 3y_2 + 2y_6 \leq -13$$
$$5y_1 + 2y_2 + 4y_3 - 3y_6 \leq 9$$
$$y_1, y_2, y_3 \geq 0, \; y_6 - \text{unrestricted sign variable}$$

26-03-21

# Transportation

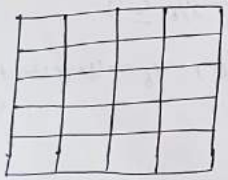
The aim of transportation problem is to meet the demand and supply requirement in a most optimum and effective manner ~~known as total~~ to minimize total transportation cost.

| | $D_1$ | $D_2$ | Destination ----- | $D_n$ | supply |
|---|---|---|---|---|---|
| $F_1$ | $c_{11}$ $x_{11}$ | $c_{12}$ $x_{12}$ | ---- | $c_{1n}$ $x_{1n}$ | $a_1$ |
| $F_2$ | $c_{21}$ $x_{21}$ | $c_{22}$ $x_{22}$ | ---- | $c_{2n}$ $x_{2n}$ | $a_2$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | $c_{m1}$ $x_{m1}$ | $c_{m2}$ $x_{m2}$ | ---- | $c_{mn}$ $x_{mn}$ | $a_m$ |
| Demand | $b_1$ | $b_2$ | ------ | $b_n$ | |

Factory or Source

No. of variable $= m \times n$

No. of equation $= m+n-1$



5x4

No. of variable → n, $C_m$ =

equation

$^{20}C_8 = 125970$

$^4C_2 = 6$

$$\boxed{Min.\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij}}$$

$$\boxed{\sum a_i = \sum b_j}$$



(1) Feasible soln :-

A set of non -ve individual allocation will satisfy all the given constant is termed as feasible soln.

(2) Basic Feasible soln :-

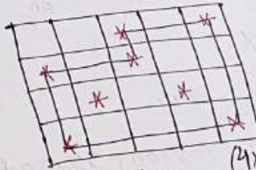
A set of non(-ve) individual allocation will in a m×n transportation problem is termed basic feasible if the total no. of allocatn is exactly equal to (m+n-1)

(3) Non-degenerate basic feasible soln

For a m×n transportatn proble soln is called non degenerate when the following two conditions are satisfied :-

(1) Total no. of allocation exactly equals to m+n-1.

(2) These m+n-1 allocations must be at indipendent positions.



(4×5)

Dependent

By indipendent position we mean that it is always impossible to form a closed loop by joining these allocation by a series of horizontal and vertical line from one allocated shell to another.

**Note:** optimality test can only be performed when the initial soln is non degenerated.

## Balanced and unbalance transportation problem:

If the total supply equals to total demand problem is called balanced otherwise unbalanced if the given problem is unbalanced. Balance it by adding dummy source or destination

Q. A complete transport produce from three factories to 4 destination as given in the cross metric below find the optimum allocation to minimize total transportation cost.

| | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| $P_1$ | $30_{50}$ | $27_{20}$ | 60 | 40 | 70/20/0 |
| $P_2$ | 80 | $70_{100}$ | 50 | 45 | 100/0 |
| $P_3$ | 50 | $35_{30}$ | $70_{70}$ | $22_{80}$ | 180/150/80/0 |
| | 50/0 | 150/130/30/0 | 70/0 | 80/0 | |

### Initial soln

(1) North west corner rule

$Z = 30\times50 + 27\times20 + 70\times100 + 35\times30 + 70\times70 + 22\times80$

$Z = Rs\ 16750$

(2) Row minima

$Z = Rs\ 15290$



| | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| $P_1$ | 30 | $27_{70}$ | 60 | 40 | 70/0 |
| $P_2$ | 80 | 70 | $50_{20}$ | $45_{80}$ | 100/20/0 |
| $P_3$ | $50_{50}$ | $35_{80}$ | $70_{50}$ | 22 | 180/130/50/0 |
| | 50/0 | 150/80/0 | 70/50/0 | 80/0 | |

(3) Column Minima

| | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| $P_1$ | $30_{50}$ | $27_{20}$ | 60 | 40 | 70/20/0 |
| $P_2$ | 80 | 70 | $50_{70}$ | $45_{30}$ | 100/30/0 |
| $P_3$ | 50 | $35_{130}$ | 70 | $22_{50}$ | 180/50/0 |
| | 50/0 | 150/130/0 | 70/0 | 80/50/0 | |

$Z = Rs\ 12540$

(4) Least cost method
or
Method of matrix minima

| | 1 | 2 | 3 | 4 | |
|---|---|---|---|---|---|
| $P_1$ | 30 | $27_{70}$ | 60 | 40 | 70/0 |
| $P_2$ | $80_{30}$ | 70 | $50_{70}$ | 45 | 100/30/0 |
| $P_3$ | $50_{20}$ | $35_{80}$ | 70 | $22_{80}$ | 180/100/20/0 |
| | 50/30/0 | 150/80/0 | 70/0 | 80/0 | |

$Z = Rs\ 13350$

164

## (5) Vogel's Approximation (VAM)
or
Unit cost Penalty

| | 1 | 2 | 3 | 4 | | Penalties Row wise |
|---|---|---|---|---|---|---|
| P1 | 30 $_{30}$ | 27 $_{20}$ | 60 | 40 | 70/20/0 | 3, 13, 13, 13 |
| P2 | 80 | 70 $_{30}$ | 50 $_{70}$ | 45 | 100/30/0 | 5, 5, 20, 20 |
| P3 | 50 | 35 $_{100}$ | 70 | 22 $_{80}$ | 180/100/0 | 13, 13, 35 ← |
| | 50/0 | 150/50/30/0 | 70/0 | 80/0 | | |
| Penalties column wise → | 20 ↑ | 8, 8, 8, 45 ↑ | 10, 10, 10, 10 | 18, 18 ↑ | | |

इस तरह से पहले Row की penalty निकालो फिर coloumn की फिर Row और coloumn की penalty में देखो कौन max. है उसे choose करो उसके corresponding row और colum को solve करो minimum वाले के corres. ... Northwest वाले ... again penalty ... zero ...

$Z = Rs\ 12900$

In this metho we take a diff b/w smallest and 2nd Smallest ~~variable~~ element in each row and column and Right them below rows and column then we select the highest individual difference and the maximum possible allocation is done in the minimum cost shell of the selected row ~~and~~ or colum. The row column whose requirement become Zero is striked off so that it cannot be considered again continuing the similar manner untill all the allocations are done.

165

### Optimility

As the no. of allocation is exactly equal to $m+n-1 = 6$ and at indipendent position so optimility test can be performed

### (1) Stepping stone method

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| P1 | 30 | 27 | 60 | 40 |
| P2 | 80 | −30 70 $_{30}$ | 50 $_{70}$ | +30 45 |
| P3 | 50 | +30 35 $_{100}$ | 70 | −30 22 $_{80}$ |

| | | | |
|---|---|---|---|
| +70 | +50 | +60 | +45 |
| −35 | −30 | −50 | −22 |
| +70 | +27 | +70 | +35 |
| −50 | −35 | −27 | −70 |
| +55 | +12 | +53 | −12 |

In this method we allocate 1 unit in an unallocated empty shell and compute the effect on the cost of matrix, it is a hit and trial method and for bigger matrix chances of making error are more.

### (2) Modified Distribution (MODI-method)
or
U-V method

166

Step 1) Develop cost matrix for allocated shells only

$V_j \rightarrow$, $u_i \downarrow$

| | $V_1$ (0) | $V_2$ (−3) | $V_3$ (−23) | $V_4$ (−16) |
|---|---|---|---|---|
| 30 $u_1$ | 30 | 27 | 7 | 14 |
| 73 $u_2$ | 73 | 70 | 56 | 57 |
| 38 $u_3$ | 38 | 35 | 15 | 22 |

$\boxed{V_1 = 0}$

$u_1 + V_1 = 30$, $u_1 + V_2 = 27$

$u_2 + V_2 = 70$, $u_2 + V_3 = 50$

$u_3 + V_2 = 35$, $u_3 + V_4 = 22$

Computing $u_i$ and $v_j$ value by taking $V_1 = 0$

Step 2) Develop $u_i + v_j$ matrix for unallocated cell by entering the summation of $u_i$ and $v_j$ value for unallocated cell.

Step 3) Subtract the cell values of $u_i + v_j$ matrix for unallocated cell from original cost matrix to get cell evaluation matrix.

| | | | |
|---|---|---|---|
| | | 53 | 26 |
| 7 | | | −12 |
| 12 | | 55 | |

Cell evaluation matrix

167

- If any of the cell value in the cell evaluation matrix is −ve then the current soln is not optimum.
- In the cell evaluation matrix identify the cell with the most −ve value, mark it and it is termed as identified cell. Trace a path in the matrix such that it start from the identified cell and all the corner of the path should already have allocation.
- Make identified cell as +ve and each other cell at the corner of the path alternatevily −ve, +ve, −ve and so on. Make a new allocation in the identified cell by entering the smallest allocation on the path that has been assigned −ve sign. The basic cell where allocation become zero leave the soln.

| | | | | |
|---|---|---|---|---|
| | | 53 | 26 | |
| 7 | −30 | | $\bar{36}$ 2 | |
| 12 | +30 | 55 | −30 | |

| | 0 | −3 | −11 | −16 |
|---|---|---|---|---|
| 30 | ~~30~~ | ~~27~~ | 19 | 14 |
| 61 | 61 | 58 | ~~50~~ | ~~45~~ |
| 38 | 38 | ~~35~~ | 27 | ~~22~~ |

$U_i + V_j$ - matrix for unallocated cells.

POCO M4 PRO 5G | SHIVANSHU

1) Degeneracy

When the no. of allocation become less than $m+n-1$ then optimility test can not be performed and such a solⁿ is called degenerate and the condition is known as degeneracy.

Maximization problem

$Z = 24\times80 + 30\times60 + 42\times90 + 10\times10 + 28\times70 + 22\times100$

maxim:

| | | | |
|---|---|---|---|
| 80 | 30 | 60 | 90 |
| 50 | 10 | 70 | 40 |
| 70 | 20 | 100 | 30 |

100-matrix

min:

| | | | |
|---|---|---|---|
| 20 (24) | 70 | 40 (30) | 10 (42) |
| 50 | 90 (10) | 30 (28) | 60 |
| 30 | 80 | 0 (22) | 70 |

Maximization problem are solved by converting it into minimization and this is done by subtracting from the highest element all the elements of matrix.

Q. Determine the initial feasible solⁿ using Vogel's approximation and find the optimum distribution possible.

WB 35

| | D | E | F | G | H | |
|---|---|---|---|---|---|---|
| A | 44 | 50 | 40 | 39 | 0 | 180 |
| B | 42 | 51 | 54 | 53 | 0 | 170 |
| C | 41 | 40 | 42 | 45 | 0 | 200 |
| | 90 | 100 | 120 | 180 | | |

| | D | E | F | G | H | | | |
|---|---|---|---|---|---|---|---|---|
| A | 44 | 50 | 40 | $39_{180}$ | 0 | 180/0 | 39 | 1, 1, 1 |
| B | $42_{90}$ | 51 | $54_{20}$ | 53 | $0_{60}$ | 170/110/20/0 | 42← | 9, 11, 1 |
| C | 41 | $40_{100}$ | $_{100}42$ | 45 | 0 | 200/100/0 | 40 | 1, 1, 3 |
| | 90/0 | 100/0 | 120/20/0 | 180/0 | 60/0 | | | |
| | 1 | 10 | 2 | 6 | 0 | | | |
| | | 10 | 2 | 6 | | | | |
| | | | 2 | 6 | | | | |
| | | | 2 | 6 | | | | |
| | | | | 6 | | | | |

$\boxed{Z = Rs\ 20080}$

As the total no. of allocation is 6 which is less than $m+n-1 = 7$ so the current solⁿ is degenerate
(source, 3 column)

Now allocating infinitely small but the value $\epsilon$ (Apsilan) (minimum cost cell) such that all allocation remain at indipendent position in the final solⁿ we put $\epsilon = 0$

| | D | E | F | G | H |
|---|---|---|---|---|---|
| A | 44 | 50 | 40 ($\epsilon$) | 39 | 0 ← ($\epsilon$) |
| B | $42_{90}$ | $51_{20}$ | 54 0 ← (20) | 53 | $0_{60}$ |
| C | 41 | 40 80 ← (100) | 42 100 ← (100) | 45 | 0 |

$Z = Rs\ 20080$

$\downarrow -20$

$Z = Rs\ 20060$

+51
−54
+42
−40
(−1)

## Assignment

(1) Square matrix
$(n=m)$

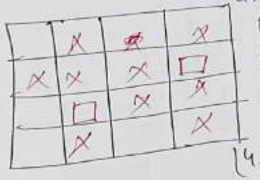
(2) All $x_{ij} = 0$ or $1$
all $a_i = 1$
and all $b_j = 1$

□ — Allocation
X — Non-allocation.

No. of variable $= n^2$

No. of equation $= 2n-1$

$$\boxed{min.\ z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} \cdot x_{ij}}$$




Assignment Problems are special case of transportation where matrix must be a square matrix and in every row every column only one allocation is possible

Q. Four Jobs are to be Performed Find the optimum allocation to minimize total work time

PTO



| | | | |
|---|---|---|---|
| 20 | 36 | 31 | 27 |
| 24 | 34 | 45 | 22 |
| 22 | 45 | 38 | 18 |
| 37 | 40 | 35 | 28 |

## Hungarian method (Flood's tech)

Step 1 Develop opportunity cost matrix.

1→ Subtract the smallest element in each row from every element of corresponding row

| | | | |
|---|---|---|---|
| 0 | 16 | 11 | 7 |
| 2 | 12 | 23 | 0 |
| 4 | 27 | 20 | 0 |
| 9 | 12 | 07 | 0 |

Step 2

Subtract the smallest element in each column from every element of corresponding column

| | | | |
|---|---|---|---|
| 0 | 4 | 4 | 7 |
| 2 | 0 | 16 | 0 |
| 4 | 15 | 13 | 0 |
| 9 | 0 | 0 | 0 |

Opportunity cost matrix

Step 5 make allocation in the opportunity cost matrix.

| | | | |
|---|---|---|---|
| $\boxed{0}$ | 4 | 4 | 7 |
| 2 | $\boxed{0}$ | 16 | $\cancel{0}$ |
| 4 | 15 | 13 | $\boxed{0}$ |
| 9 | $\cancel{0}$ | $\boxed{0}$ | $\cancel{0}$ |

Opportunity cost matrix

Row → Column → Row → Column

As the no. of allocation is exactly equal to the size of matrix so the current soln is optimum

Q. Solve the following assignment problem for minimization.

| | | | |
|---|---|---|---|
| 20 | 30 | 40 | 50 |
| 40 | 50 | 60 | 70 |
| 70 | 80 | 90 | 80 |
| 30 | 50 | 80 | 40 |

→ 200 = Z

⇒

| | | | |
|---|---|---|---|
| $\cancel{0}$ | $\boxed{0}$ | $\cancel{0}$ | 20 |
| $\cancel{0}$ | $\cancel{0}$ | $\boxed{0}$ | 20 |
| $\cancel{0}$ | $\cancel{0}$ | $\cancel{0}$ | $\boxed{0}$ |
| $\boxed{0}$ | 10 | 30 | $\cancel{0}$ |

Z = 200

Q. Solve the following assignment for minimization

| | | | | | |
|---|---|---|---|---|---|
| 9 | 22 | 58 | 11 | 19 | 27 |
| 43 | 78 | 72 | 50 | 63 | 48 |
| 41 | 28 | 91 | 37 | 45 | 33 |
| 74 | 42 | 27 | 49 | 39 | 32 |
| 36 | 11 | 57 | 22 | 25 | 18 |
| 3 | 56 | 53 | 31 | 17 | 28 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $\cancel{0}$ | 13 | 49 | $\boxed{0}$ | $\cancel{0}$ | 13 | 3b ✓ |
| $\cancel{0}$ | 35 | 29 | 5 | 10 | $\boxed{0}$ | ✓ (1) |
| 13 | $\cancel{0}$ | 63 | 7 | 7 | $\cancel{0}$ | |
| 47 | 15 | $\boxed{0}$ | 20 | 2 | $\cancel{0}$ | |
| 25 | $\boxed{0}$ | 46 | 09 | 4 | 2 | — 3b |
| $\boxed{0}$ | 53 | 50 | 26 | 4 | 20 | ✓ (5) |
| (4) | ✓ 2a | | | | 2b | |

As the no. of allocation is less than the size of matrix so the current soln is not optimum.

→ No we need proceed to find the minimum no. of lines required to cover all zero atleast once the steps involved are

POCO M4 PRO 5G | SHIVANSHU 💗🔥

174

Step ① mark all row for which allocatn is not -(3rd row).

② mark all column which have unallocated zero in the marked row. (2nd and 6th column)

③ mark all row which have allocation in the marked column (2nd and 5th row)

④ continue step (2 and 3) until chain of marking is completed.

⑤ Draw the minimum no. of lines through unmarked row and through marked column to cover all zero atleast once.

⑥ Select the smallest element that do not have line through them, subtract it from all the element that do no not have line through them add to every element at the intersectn of two lines and leave the remaining elements of a matrix unchanged make allocation in the New opportunity cost matrix.

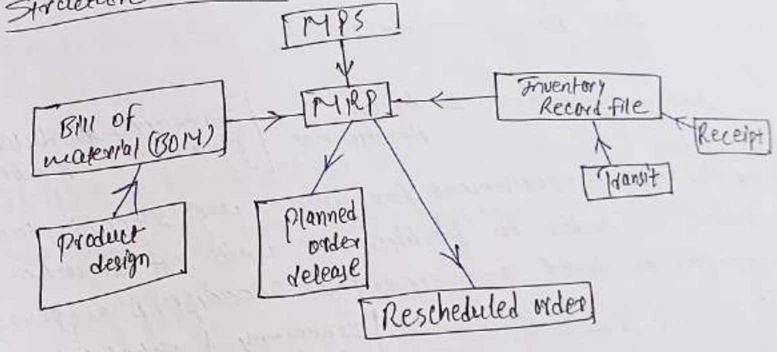
| | | | | | |
|---|---|---|---|---|---|
| 4 | 17 | 49 | $\boxed{0}$ | ~~0~~ | 17 |
| $\boxed{0}$ | 35 | 25 | 1 | 6 | ~~0~~ |
| 13 | ~~0~~ | 59 | 3 | 3 | $\boxed{0}$ |
| 51 | 19 | $\boxed{0}$ | 20 | 2 | 4 |
| 25 | $\boxed{0}$ | 42 | 5 | ~~0~~ | 2 |
| ~~0~~ | 53 | 46 | 22 | $\boxed{0}$ | 20 |

size of mat = 6

# 4 MRP (material requirement planning)

MRP is a method of working out the production plan in a multistage production system that produces many product and required their raw material and sub assembly. It is used to so that all the things needed should be available with the prodn system at an appropriate time and production can be carried out without any delay. Today MRP is a computer based information system for production, sceduling and purchasing of dependent demand item

Structure of MRP




MPS (master productn scedule) It is the complete time table of our sceduled productn in future it gives information about which product is to be produced, when it is to be produced and in what qty.

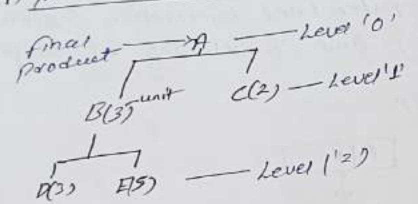
Inventory record File 176

This file gives complete and up to date information about on hand inventory, transit inventory, planned order and sceduled receipt.

Bill of material

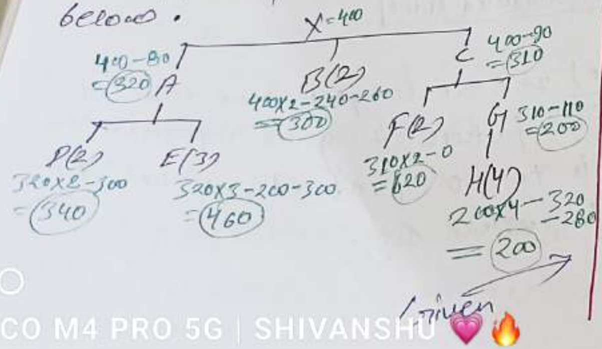
This file gives informaton about how ~~final~~ each final product is manufacture, specifying all sub component items and their sequence of buildup in the final product.

(i) product structure




$$\text{Net requirement} = \text{Gross Requirement} - \left[\text{Inventory on hand} + \text{Scheduled Receipt}\right]$$

Q. Find the net requirement for which we should place an order in order to produce 400 units of x when the inventory on hand and sceduled receipt is as given below.




| Inventory on Hand | Scheduled Receipt |
|---|---|
| A - 80 | B - 260 |
| B - 240 | D - 300 |
| C - 90 | E - 300 |
| E - 200 | H - 280 |
| G - 110 | |
| H - 320 | |



Advantages of MRP

(1) It help us to know when and how much to Order it helps in Inventory reduction

(2) It helps to avoid Delay in production

(3) It gives timely information to marketing department about expected delivery time

JIT → Just in time

It provides for a cost effective production in an organization to produce only the necessary part in Right qty at the right time with minimum of resources. This system work with very less inventory and most of the time finished inventory is also kept low. This system beleive that space is waisted for storing inventory and it also lock the capital and material.

Kan ban system (Token)

It is a Japanese word which means card or signal the purpose is to signal the need for more part to support further assembly it is a physical control system consisting of card and container there are mainly two types of Kan ban

(1) P (production) Kan ban → It signals the need to produce more inventory to support further assembly.

(2) C (conveyance) Kan ban → It signals the need to bring more inventory to support further assembly.

17. The no. of card or container is given by

$$\boxed{N = \frac{D \cdot T(1+x)}{C}}$$

N = No. of card or container
D = Demand rate
T = avg waiting time for production or conveyance
x = Safety factor
C = Container capacity

---

Q. PYB (1.4) P-732)

| S.N | Di | Fi | $D_i - F_i$ | $(D_i - F_i)^2$ |
|---|---|---|---|---|
| 14 | 100 | 75 | 25 | 625 |
| 15 | 100 | 87.5 | 12.5 | 156.25 |
| 16 | 100 | 93.75 | 6.25 | 39.06 |

$$MSE = \frac{\Sigma (D_i - F_i)^2}{n} = \frac{625 + 156.25 + 39.06}{3}$$

$$= 273.44$$

---

PYB (3.4) (P-748)

Q. A company produces two types of toys: P and Q. Production time of Q is twice that of P and the company has a maximum of 2000 time units per day. The supply of raw material is just sufficient to produce 1500 toys (of any type) per day. Toy type Q requires an electric switch which is available at 600 pieces per day only. The company makes a profit of Rs 3 and Rs 5 on type P and Q respectively. For maximization of profit the daily production quantities of P and Q toys should respectively be. (A) 1000, 500 (B) 500, 1000 (C) 800, 600 (d) 1000, 1000

| | $x_1$ P | $x_2$ Q | Resource |
|---|---|---|---|
| Time | 1 | 2 | 2000 |
| Capacity | 1 | 1 | 1500 |
| Switch | 0 | 1 | 600 |
| Profit/unit | 3 | 5 | |

Objective function $Z = 3x_1 + 5x_2$

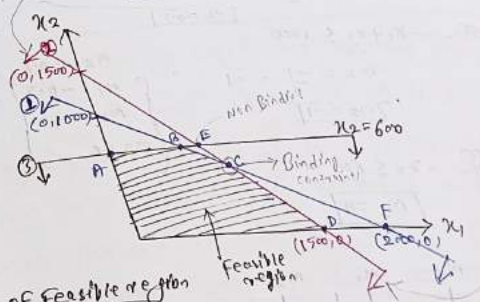
1st line $\rightarrow x_1 + 2x_2 \le 2000 \rightarrow$ at $(x_1 = 0) \Rightarrow (0, 1000)$
at $(x_2 = 0) \Rightarrow (2000, 0)$

2nd line $\rightarrow x_1 + x_2 \le 1500 \rightarrow$ at $(x_1 = 0) \Rightarrow (0, 1500)$
at $(x_2 = 0) \Rightarrow (1500, 0)$

3rd line $\rightarrow 0x_1 + 1x_2 \le 600 \rightarrow$ at $(x_2 = 600)$

$\boxed{x_1, x_2 \ge 0} \rightarrow$ Because cant make in -ve

for feasible region of 1st line put $\boxed{x_1 = x_2 = 0}$ we get $0 \le 2000$ vice versa for other lines




Points of Feasible region

A $\rightarrow$ (0, 600)

B $\rightarrow$ By the intersection of line ① and ③
line 1 $\rightarrow x_1 + 2x_2 \le 2000$
line 3 $\rightarrow \boxed{x_2 \le 600}$ put we get (800, 600)

C $\rightarrow$ By line ① and ② (1000, 500)

D $\rightarrow$ (1500, 0)

PTO

objective fn में इन value को check करके देखो कि कहाँ पे max profit मिल रहा → Ans होगा

180

$Z = 3x_1 + 5x_2$

$Z(0, 600) = 3000$ Rs

$Z(800, 600) = 5400$ Rs

$Z(1000, 500) = 5500$ Rs ← maxm profit Ans point → C(1000, 500) option A is correct Ans

$Z(1500, 0) = 4500$ Rs.

---

Shortcut method — Slope method

Limitations :- ① There will be no redundant in the problem.


we know $y = mx + c$, m → slope

$Z = 3x_1 + 5x_2$

slope of Z → $m_Z = \frac{-3}{5} = -0.6$

1st line → $x_1 + 2x_2 \le 2000$

$m_1 = \frac{-1}{2} = -0.5$ $\boxed{m_1 > 0.5}$

line ② → $x_1 + x_2 \le 1500$

$m_2 = \frac{-1}{1} = -1$

$\boxed{m_2 = -1}$

line ③ → $x_2 \le 600$

$\boxed{m_3 = 0}$

$m_Z = -0.6$

$\left\{ \begin{array}{l} m_1 = -0.5 \\ m_2 = -1 \\ m_3 = 0 \end{array} \right\}$

Objective fn का slope किसी के बराबर नहीं है।

Here $m_Z \ne$ slope of any contraints

So, it is case of unique soln.

$m_3$ — $m_1$ — $m_2$ on a number line: $0$, $-0.5$, $-0.6$, $-1$

↑ objective fn lies b/w ①st and ②nd line

So, $x_1 + 2x_2 \le 2000$
$x_1 + x_2 \le 1500$
$x_2 = 500$
$x_1 = 1000$

(1000, 500) ← Ans Direct ☺

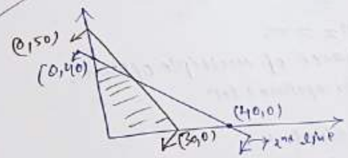
---

Q. Two models P and Q of a product earn profit of Rs 100 and Rs 80 per piece, production times for P and Q are 5 hours and 3 hours, while the total production time available is 150 hours. For a total batch size of 40, to maximize profit, the no. of units of P to be produced is —

⇒ $\boxed{Z = 100x_1 + 80x_2}$

$5x_1 + 3x_2 \le 150$ → at $(x_1 = 0) \Rightarrow (0, 50)$, $(x_2 = 0) \Rightarrow (30, 0)$

$P + Q \le 40$ → $(x_1 = 0) \Rightarrow (0, 40)$, $(x_2 = 0) \Rightarrow (40, 0)$




Slope method

$m_Z = \frac{-100}{80} = -1.25$

$m_1 = \frac{-5}{3} = -1.66$

$m_2 = -1$

$m_2$ $-1$, $m_Z$ $-1.25$, $m_1$ $-1.60$

By ① and ②nd line

$5x_1 + 3x_2 \le 150$
$5x_1 + 5x_2 \le 200$
$-2x_2 = -50$
$\boxed{x_2 = 25} = Q$
$\boxed{x_1 = P = 15}$ → Ans

182

Q. Consider the following Linear programming Problem (LPP):

Maximize $Z = 3x_1 + 2x_2$

Subject to $3x_1 + 2x_2 \le 18$ — ① $x_1 \le 4$

$\hookrightarrow$ at $(x_1 = 0) \Rightarrow (0, 9)$ $x_2 \le 6$

$(x_2 = 0) \to (6, 0)$ $x_1, x_2 \ge 0$

(A) The LPP has a unique optimal solⁿ

(B) The LPP is infeasible → No solⁿ

(C) The LPP is unbounded

(D) The LPP has multiple optimal solⁿ

slope

$m_Z = -\frac{3}{2}$

$m_1 = -\frac{3}{2}$

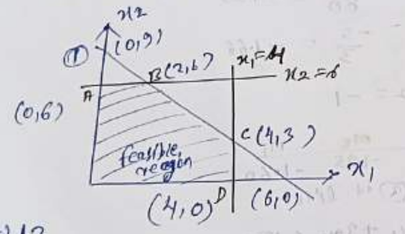
Here $m_Z = m_1$

Chance of multiple optimal.

condition ⊛ for optimal solⁿ

① Non Redundant

② Binding Contraint




$Z_A = \cancel{(0,6)}\ 12$

$Z_B = 18$ ⎤ → optimal (at $B \Rightarrow (2,6)$ ⎤ → put in eq①

$Z_C = 18$ ⎦ $C \Rightarrow (4,3)$ ⎦

$(18)\ Z_B = Z_B\ (18)$

$(18)\ Z_C = Z_C\ (18)$

$Z_D = 12$

So the contraint is Binding and Non redundant, both

so, more than one solⁿ are there

It is case of optimal solⁿ

183

Q. LPP

P4B 3.19 maximize $Z = 3x_1 + 2x_2$

P-750 Subject to $-2x_1 + 3x_2 \le 9$ — ① → $x_1 = 0 \Rightarrow (0,3)$, $x_2 = 0 \Rightarrow (-\frac{9}{2}, 0)$ $(-4.5, 0)$

$x_1 - 5x_2 \ge -20$ — → can't be −ve

$x_1, x_2 \ge 0$

The problem has

~~(a)~~ unbounded solⁿ

(b) Infeasible solⁿ

(c) Alternate optimal solⁿ

(d) Degenerate solⁿ

कोई भी चीज Time, Resource, Raw material (−ve) में use नहीं कर सकते

So, multiply both side with (−ve sign)

$\boxed{-x_1 + 5x_2 \le 20}$ — ②

$\hookrightarrow (x_1 = 0) \Rightarrow (0, 4)$

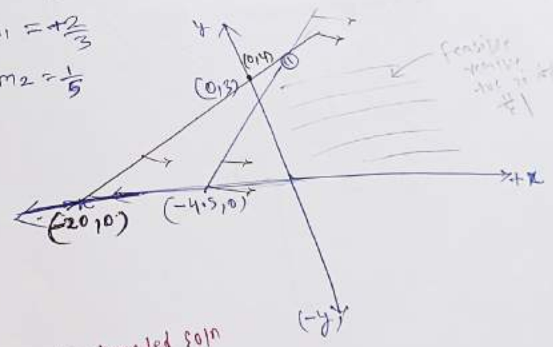
$(x_2 = 0) \Rightarrow (-20, 0)$

solⁿ = 4

$m_Z = -\frac{3}{2}$

$m_1 = +\frac{2}{3}$

$m_2 = \frac{1}{5}$




Unbounded solⁿ