OMIS 2010 – Assignment B2 – Fall 2017

Assignment Instructions (please read carefully)

1. Submit your assignment to the link posted on Moodle.
2. Make sure to upload it to the correct link by section.
3. Work can be done by group of 2-3 students. Groups can **not** consist of students from other sections of the course.
4. Only one person uploads the assignment, adding all names on the file.
   The files names should be of the form: "A2_LastName1_LastName2_LastName3.xlsx"
   LastName1 is the last name of student 1, LastName2 is the last name of student 2, etc.
5. You are free to discuss your approach with other individuals or pairs, but your write-up should be prepared independently by the person(s) submitting the assignment. It is a violation of the rules of academic honesty to copy someone else's assignment and present it as your own.
6. The assignment cover page is posted (see next page).
7. You should upload the following:
   a. A completed cover page.
   b. A Word or PDF file with the assignment answers. Each answer file should include screenshots of supporting work (Excel model & output) to accompany the relevant question.
   c. Excel file with your work [always submit the Excel work to support your answer]. Use different tabs to identify the answers to each individual question when needed.
8. Generally, answers should be typed on the computer (graphs can be generated manually and incorporated into the assignment document).
9. The assignments should be organized by question. That means every piece of information for that question is found in the same section of the report. Use labeling and titles to make sure all is clear. If I have to look for it, it is not organized. Don't forget to add the Excel outputs if you used them to analyze the case. You may use an appendix to give further details of your model or calculations if required.
10. Late work will **not** be accepted. (You will earn a grade of 0.)

OMIS 2010 – Assignment Cover Page
Fall 2017

Schulich School of Business

Assignment #:	B2
Section:	

	Group Member 1	Group Member 2	Group Member 3
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Group can be minimum of 2 members and maximum of 3 members

Question 1

At the start of football season, the York student ticket office gets busy the day before the first game. Customers arrive at the rate of four every ten minutes. A ticket seller can service a customer in four minutes. Traditionally, there are two ticket sellers working. The university is considering an automated ticket machine similar to the airlines' e-ticket system. The automated ticket machine can service a customer in 2 minutes.

(a) What is the average length of the queue for the in-person model?

M/M/S → M/M/2

λ = 24 customers per hour, μ = 15 customers per hour, S = 2 servers working simultaneously

$$L_q = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^M}{(M-1)!(M\mu-\lambda)^2}P_o$$

we must find Po before we can solve for Lq

$$P_0 = \frac{1}{\left[\sum_{n=0}^{S-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^s \frac{S\mu}{S\mu - \lambda}} \quad \text{for } S\mu > \lambda$$

*2 * 15 > 24*

$$Po = \frac{1}{(1 + \left(\frac{1}{1!}\right)\left(\frac{24}{15}\right)^1 + (\left(\frac{1}{2!}\right)\left(\frac{24}{15}\right)^2 * \frac{2*15}{(2*15)-24}}$$

$$Po = \frac{1}{2.6 + (1.28 * 5)}$$

$$Po = \frac{1}{9}$$

$$Lq = \frac{(24)(15)\left(\frac{24}{15}\right)^2}{(2-1)!\left((2*15)-24\right)^2} * \frac{1}{9}$$

$$Lq = \frac{921.6}{36} * \frac{1}{9}$$

$$Lq = 2.84$$

(b) What is the average length of the queue for the automated system model?

M/D/1

λ=24 customers per hour, μ = 30 customers per hour

$$L_q = \frac{\lambda^2\sigma^2 + (\lambda/\mu)^2}{2(1-\lambda/\mu)}$$

; where σ = 0

$$Lq = \frac{24^2 0^2 + \left(\frac{24}{30}\right)^2}{2 * \left(1 - \left(\frac{24}{30}\right)\right)}$$

$$Lq = \frac{0.64}{0.4}$$
$$Lq = 1.6\ Customers$$

(c) What is the average time in the system for the in-person model?

$$W_s = \frac{\mu(\lambda/\mu)^S}{(S-1)!(S\mu-\lambda)^2} P_0 + \frac{1}{\mu} = \frac{L_s}{\lambda}$$

$$Ws = \frac{(15)\left(\frac{24}{15}\right)^2}{(2-1)!\left((2*15)-24\right)^2} * \frac{1}{9} + \frac{1}{15}$$
$$Ws = 0.1185 + \frac{1}{15}$$
$$Ws = 0.1852\ customers$$

(d) What is the average time in the system for the automated system model?

$$W_s = W_q + \frac{1}{\mu}$$

$$W_q = \frac{L_q}{\lambda}$$

$$Wq = \frac{1.6}{24} + \frac{1}{30}$$
$$Wq = 0.1\ customers$$

(e) Assume the ticket sellers earn $8 per hour and the machine costs $20 per hour (amortized over 5 years). The wait time is only $4 per hour because students are patient. What is the total cost of each model?

Ticket Sellers

Total Cost/hour = Worker's Pay/hour + Cost of Wait Time/hour (Lq)
*Total Cost/hour = $8*2 + $4*2.84*
Total Cost/hour = $27.36/hour

Machine

Total Cost/hour = Cost of machine/hour + Cost of Wait Time/hour (Lq)
*Total Cost/hour = $20 + $4*1.6*
Total Cost/hour = $26.40/hour

(f) A ticket agency offers to take over the ticket distribution at a cost of $15/hour. The university will only accept an outside agency if (i) the cost is lower than providing the services internally (either in-person or using an automated ticket machine) and (ii) if the average queue length is lower than providing the services internally. The ticket agency promises an average service rate of 36 per hour. What standard deviation on SERVICE TIME does the agency have to achieve in order for the university to be willing to hire them?

M/G/1

λ=24 customers per hour, μ = 36 customers per hour

We must find value for std when the cost to hire or use our own machines is greater than the cost to hire the ticketing agency

i.e. we must find σ

Hire ticketers vs. hire the ticketing agency

Cost to hire ticketers/hour + cost of wait time/hour = Cost to hire ticket agency + cost of wait time/hour

*27.36 = $15/hour + $4 * Lq*

$$Lq = \frac{27.36 * 15}{4}$$

Lq = 3.09

$$\frac{\left(24^2\sigma^2 + \left(\frac{24}{36}\right)^2\right)}{2\left(1 - \left(\frac{24}{36}\right)\right.} \leq 3.09; \text{ we must find std}$$

$$\sigma \leq \sqrt{\frac{\left(3.09 * \left(2\left(\frac{24}{36}\right)\right) - \left(\frac{24}{36}\right)^2\right)}{24^2}}$$

$$\sigma \leq 0.0530$$

Therefore, when in comparison to the hired ticket sellers, the ticket agency must have a standard deviation which is less than or equal to 0.0530 if they want to be considered for hire.

Install machine vs. Hire the ticket agency

Cost to run machine/hour + cost of wait time/hour Cost to hire ticket agency + cost of wait time/hour

*26.40 = $15/hour + $4 * Lq*

$$Lq = \frac{26.40 - 14}{4}$$

Lq = 2.85

$$\frac{\left(24^2\sigma^2 + \left(\frac{24}{36}\right)^2\right)}{2\left(1 - \left(\frac{24}{36}\right)\right.} \leq 2.85; \text{ we must find std}$$

$$\sigma \leq \sqrt{\frac{\left(2.85 * \left(2\left(\frac{24}{36}\right)\right) - \left(\frac{24}{36}\right)^2\right)}{24^2}}$$

$$\sigma \leq 0.0503$$

Therefore, when in comparison to installing a machine, the ticket agency must have a standard deviation which is less than or equal to 0.0503 if they want to be considered for hire.

Question 2

The Campus Coffee Shop has very uneven demand for its service, with peak customer arrivals in the 15 minute intervals just prior to the start of classes. A team of Operations majors has studied the arrival and service times and found that the arrival rate is 8 customers per 5-minute interval while the service rate is 6 customers per 5-minute interval. Baristas are paid $21/hour and the loss of goodwill was estimated at 25 cents per minute of wait time.

Required:

For k = 1, 2, and 3 servers, determine the following values, filling in the table with your answers. Except for the Utilization Factor, do not report non-meaningful values, but use "nm" instead. Calculate all values manually. Show ALL your work.

λ = arrival rate = 96 customers per hour
σ = service rate = 72 customers per hour

	k = 1	k = 2	k = 3
a) Utilization Factor	133.33%	66.67%	44.44%
b) Probability of no units in system	Nm	20%	25.42%
c) Average Queue Length	Nm	1.066 customers	0.1446 customers
d) Average Number in the System	Nm	2.4 customers	1.4779 customers
e) Average Wait Time	Nm	0.0111 hours	0.0015 hours
f) Average Time in the System	Nm	0.025 hours	0.0154 hours
g) Total Cost (Wait + Server)	nm	$58/hour	$65.17/hour

K=1

Part A

$$M/M/1 \frac{\lambda}{S\mu} = \frac{96}{72 \times 1} = 1.\overline{33}$$

Service Rate < Arrival Rate; $S\mu < \lambda$, therefore, does not follow steady rate of assumption
Non-meaningful values for rest

K=2

Part A

$$M/M/S; \frac{\lambda}{S\mu} = \frac{96}{72 \times 2} = 0.\overline{66}$$

Part B

$$P_0 = \frac{1}{\left(\sum_{n=0}^{S-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right)+\frac{1}{S!}(\frac{\lambda}{\mu})^s \times \frac{S\mu}{S\mu-\lambda}}\ for\ S\mu > \lambda$$

$$Check\ for\ S\mu > \lambda, 72 \times 2 > 96, 144 > 96$$

$$P_0 = \frac{1}{\left(\frac{1}{0!}\left(\frac{96}{72}\right)^0+\frac{1}{1!}\left(\frac{96}{72}\right)^1\right)+\frac{1}{2!}\left(\frac{96}{72}\right)^2 \times \frac{2\times 72}{2*72-96}}$$

$$P_0 = 0.2 = 20.00\%$$

Therefore, 20% of the time, the server isn't that busy

Part C

$$L_Q = L_S - \frac{\lambda}{\mu}$$

$$L_Q = 2.4 - \frac{96}{72}$$

$$L_Q = 1.0\overline{66}$$

Part D

$$L_S = \frac{\lambda\mu(\frac{\lambda}{\mu})^S}{(S-1)!\,(S\mu-\lambda)^2}P_0 + \frac{\lambda}{\mu}$$

$$L_S = \frac{96\times 72(\frac{96}{72})^2}{(2-1)!\,(2\times 72-96)^2}0.2 + \frac{96}{72}$$

$$L_S = 2.4\ average\ \#\ of\ customers\ in\ system$$

Part E

$$W_Q = \frac{L_Q}{\lambda}$$

$$W_Q = \frac{1.066}{96}$$

$$W_Q = 0.0111\ hours$$

Part F

$$W_S = \frac{L_S}{\lambda}$$

$$W_s = \frac{2.4}{96}$$

$$W_s = 0.025\ hours$$

Part G

$$Total\ cost\ per\ hour =\ L_Q\ cost\ per\ hour + x\ worker\ cost\ per\ hour$$

$$= 1.0667 \times \$15 + 2 \times \$21$$

Total cost per hour = $58.00 per hour

K=3

Part A

$$M/M/1\frac{\lambda}{S\mu}=\frac{96}{72\times 3}=0.\overline{44}$$

Part B

$$P_0=\frac{1}{\left(\sum_{n=0}^{S-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right)+\frac{1}{S!}\left(\frac{\lambda}{\mu}\right)^s\times\frac{S\mu}{S\mu-\lambda}}\ for\ S\mu>\lambda$$

$$Check\ for\ S\mu>\lambda, 72\times 3>96, 216>96$$

$$P_0=\frac{1}{\left(\frac{1}{0!}\left(\frac{96}{72}\right)^0+\frac{1}{1!}\left(\frac{96}{72}\right)^1+\frac{1}{2!}\left(\frac{96}{72}\right)^2\right)+\frac{1}{3!}\left(\frac{96}{72}\right)^3\times\frac{3\times 72}{3*72-96}}$$

$$P_0=0.2542=25.42\%$$

Therefore, 25.42% of the time, the server isn't that busy

Part C

$$L_Q=L_S-\frac{\lambda}{\mu}$$

$$L_Q=1.4779-\frac{96}{72}$$

$$L_Q=0.1446$$

Part D

$$L_S=\frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^S}{(S-1)!\,(S\mu-\lambda)^2}P_0+\frac{\lambda}{\mu}$$

$$L_S=\frac{96\times 72\left(\frac{96}{72}\right)^3}{(3-1)!\,(3\times 72-96)^3}0.2542+\frac{96}{72}$$

$$L_S=1.4779\ average\ \#\ of\ customers\ in\ system$$

Part E

$$W_Q=\frac{L_Q}{\lambda}$$

$$W_Q=\frac{0.1446}{96}$$

$$W_Q=0.0015\ hours$$

Part F

$$W_S=\frac{L_S}{\lambda}$$

$$W_s = \frac{1.4779}{96}$$
$$W_s = 0.0154\ hours$$

Part G

$$Total\ cost\ per\ hour =\ L_Q\ cost\ per\ hour + x\ worker\ cost\ per\ hour$$
$$= 0.1446 \times \$15 + 3 \times \$21$$

$Total$ cost per hour = $65.17 per hour

Question 3

Here is a Driver's License Office process
Step 1: Check application (15 sec), step 2: Process Payment (30 sec), Step 3: Check for violations (60 sec), Step 4: Conduct eye test (40 sec), Step 5: Photograph applicant (20 sec) & Step 6: Issue new license (30 Sec)
The office has 6 clerks, one for each step.

Required:
Part I:

(a) Draw the process flow and include the flow time for each step.
(b) How long does it take to process one applicant's driver license?
(c) What is the capacity rate for each step*?
(d) What is the capacity rate of the whole process?
(e) What is the throughput rate of the whole process?
(f) Compute the utilization for each step in the process*.
(g) Which step/s are the bottleneck in the process?

*Summarize your results for (c) and (f) in a table.

Part II:
Redo Part I (a)-(g) for additional clerk in step 1.
(h)What does increasing the capacity of the bottleneck process do to the utilization of the bottleneck station?

Part III:
Redo Part I (a)-(g) for additional clerk in step 3.

Part I
a)

	Clerk 1		Clerk 2		Clerk 3		Clerk 4		Clerk 5		Clerk 6	
→	Check Application	→	Process Payment	→	Check For Violations	→	Conduct Eye Test	→	Photograph Applicant	→	Issue New License	→
	15 seconds		30 seconds		60 seconds		40 seconds		20 seconds		30 seconds	

b)

Flow Time = 15 seconds +30 seconds + 60 seconds + 40 seconds + 20 seconds + 30 seconds
Flow Time = 195 seconds = 3 minutes and 15 seconds

c) Formula used: *(60/flow time) * 60*

		Check Application	Process Payment	Check For Violations	Conduct Eye Test	Photograph Applicant	Issue New License
	Flow time (Seconds)	15	30	60	40	20	30
C	Capacity Rate (per hour)	240	120	60	90	180	120
F	Utilization	25.00%	50.00%	100.00%	66.67%	33.33%	50.00%
				BOTTLENECK			

d) The capacity rate of the process is defined by the bottleneck, which is when the clerk checks for violations; i.e. The capacity rate of the process is 60 licenses per hour.

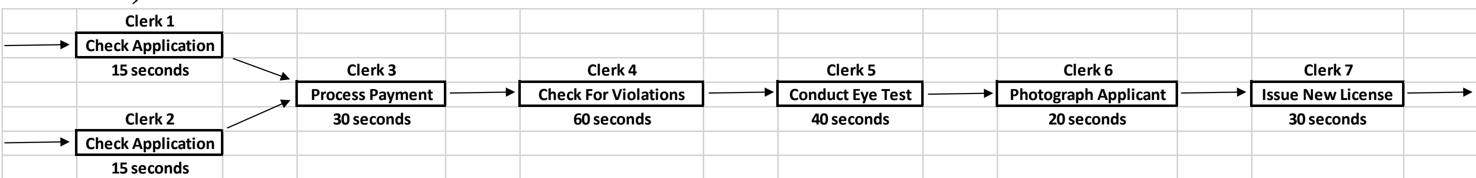
e) Following the assumption that demand is high, the throughput rate is equal to the capacity rate of the system, and therefore the throughput rate is 60 clients per hour.

f) Formula used: *(capacity rate i / capacity rate for bottleneck).*

g) The bottleneck in this case is highlighted in yellow; where the clerk checks for violations.

Part II

a)



b)

Flow Time = 15 seconds +30 seconds + 60 seconds + 40 seconds + 20 seconds + 30 seconds

Flow Time = 195 seconds = 3 minutes and 15 seconds

c) Formula used; *(60/flow time) * 60 * # of servers*

***Note - because there are now two servers checking the application, the capacity rate per hour is doubled from 240 per hour to 480 per hour.**

		Check Application	Process Payment	Check For Violations	Conduct Eye Test	Photograph Applicant	Issue New License
	Flow time (Seconds)	15	30	60	40	20	30
C	Capacity Rate (per hour)	480	120	60	90	180	120
F	Utilization	12.50%	50.00%	100.00%	66.67%	33.33%	50.00%
				BOTTLENECK			

d) The capacity rate of the process is defined by the bottleneck, which is when the clerk checks for violations; i.e. The capacity rate of the system is 60 licenses per hour.

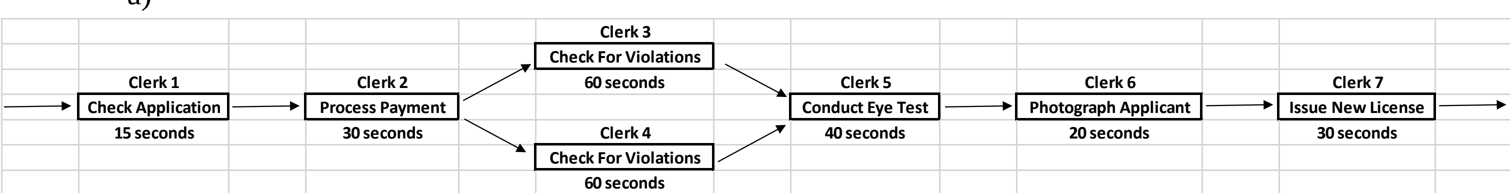
e) Following the assumption that demand is high, the throughput rate is equal to the capacity rate of the system, and therefore the throughput rate is 60 clients per hour.

f) Formula used: *(capacity rate i / capacity rate for bottleneck).*

g) The bottleneck in this case is highlighted in yellow; where the clerk checks for violations.

Part III

a)



b)

Flow Time = 15 seconds +30 seconds + 60 seconds + 40 seconds + 20 seconds + 30 seconds

Flow Time = 195 seconds = 3 minutes and 15 seconds

c) Formula used; *(60/flow time) * 60.*

***Note – because there are now two servers checking for violations, we double the capacity rate for checking for violations from 60 per hour to 120 per hour.**

		Check Application	Process Payment	Check For Violations	Conduct Eye Test	Photograph Applicant	Issue New License
	Flow time (Seconds)	15	30	60	40	20	30
C	Capacity Rate (per hour)	240	120	120	90	180	120
F	Utilization	37.50%	75.00%	75.00%	100.00%	50.00%	75.00%
					BOTTLENECK		

d) The capacity rate of the process is defined by the bottleneck, which is when the clerk conducts the eye test; i.e. the capacity rate of the system is 90 clients per hour.

e) Following the assumption that demand is high, the throughput rate is equal to the capacity rate of the system, and therefore the throughput rate is 90 clients per hour.

f) Formula used: *(capacity rate i / capacity rate for bottleneck).*

g) The bottleneck in this case is highlighted in yellow; where the clerk conducts the eye test.